A MECHANISM FOR PULSAR RADIATION

By Krishna M. V. Apparao

(*Tata Institute of Fundamental Research, Bombay–5, India*)

Received July 24, 1970

(Communicated by Prof. R. R. Daniel, F.A.SC.)

Abstract

It is shown that the low frequency radiation emitted by a magnetic neutron star with the axis of its dipole field at an angle to the rotation axis, is periodic. This low frequency radiation accelerates electrons and protons to high energies. The synchrotron radiation emitted by the high energy particles in the ambient magnetic field is shown to be the pulsar radiation.

The central object of a pulsar is generally believed to be a magnetic rotating neutron star.[1] Attempts are recently being made to account for pulsar characteristics by assuming that the magnetic axis of the star makes an angle with the rotation axis (oblique rotator).[2] It is known that such a star emits low frequency electromagnetic radiation.[3] We shall show that this radiation in any given direction in space is periodic. Gunn and Ostriker[4] have shown that this low frequency radiation accelerates high energy electrons and protons along its direction of propagation. The high energy particles give out synchrotron radiation in the ambient magnetic field and we shall show that the electron synchrotron radiation is the pulsar radiation.

It has been shown by Deutsch[3] that the magnetic field of an oblique rotator rotates rigidly with the star up to a radius $r_0 = c/w$ (light circle), where w is the angular velocity of the pulsar and c is velocity of light. In the wave zone ($r > c/w$), the magnetic and electric fields vary with time. The radial component of the Poynting vector can be evaluated using expressions given by Deutsch and is

$$\mathrm{I}_r = \frac{\mathrm{K}}{r^2} \sin \chi \left[\mathrm{Sin}^2 \left(wt - \phi - \frac{wr}{c} \right) + \cos^2 \theta \cos^2 \left(wt - \phi - \frac{wr}{c} \right) \right].$$

In the above expression K is a constant involving the radius of the star, the value of the internal magnetic field of the star, and some other constants.

r is the radial distance from the star. χ is the angle which the magnetic axis makes with the rotation axis; θ is the angle measured with respect to the rotation axis, ϕ is the aximuthal angle measured with respect to a fixed direction and t is the time. The intensity of radiation per unit area as a function of time is given by

$$J(t) = \frac{1}{\Delta T} \int_{t}^{t+\Delta T} I_r \, dt$$

$$= \frac{K}{r^2} \frac{\sin \chi}{2} \left[1 - \frac{\cos 2wt}{2} + \cos \theta \left(1 + \frac{\cos 2wt}{2} \right) \right]. \tag{2}$$

For a given angle χ, the value of J_r is plotted in arbitrary units as a function of time in Fig. 1, for three different angles of directions of observation. It is seen that the radiation is periodic for $\theta > 0$.

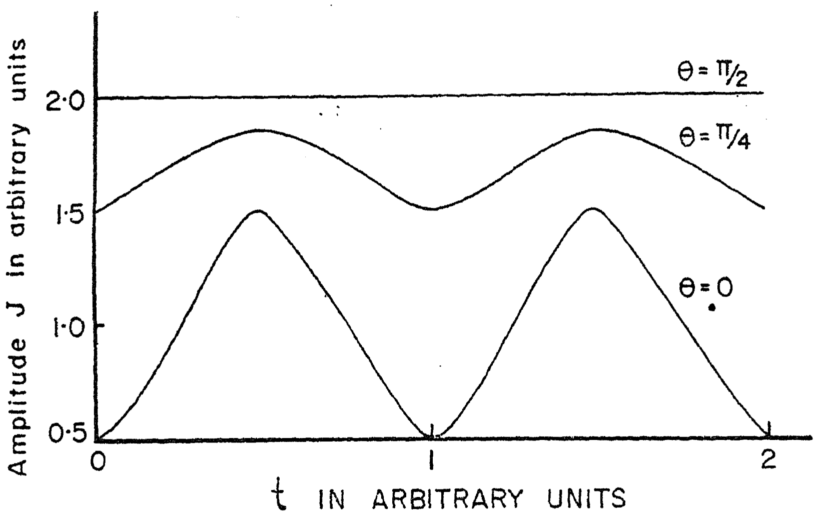

FIG. 1. The intensity J_r of the low frequency electromagnetic radiation as a function time for a given azimuth angle at three different zenith angles measured from the rotation axis.

The radiation emitted is of very low frequency which is equal to the frequency of rotation of the star. It is shown by Gunn and Ostriker[4] that this electromagnetic wave accelerates particles to an energy

$$E = mc^2 \left[\frac{\Omega}{w} \frac{3}{2} \ln \left(\frac{r}{r_0} \right) \sin \theta_0 \right]^{\frac{2}{3}}. \tag{3}$$

In this expression m is the mass of the particle, Ω is the gyrofrequency of the particles, r is the distance from the star, and ϕ_0 is a phase angle. The energy which a particle acquires depends on the phase ϕ_0 at which the particle is injected. The maximum energy acquired at a given r is when

$\sin\phi_0 = 1$. In the case of the pulsar NP 0532 in the Crab nebula using a surface magnetic field $B_S = 10^{13}$ gauss, radius $R = 10^6$ cm. and the period $\tau = 3\cdot3 \times 10^{-2}$ seconds, the maximum energy possible for protons γ^P_{max} and for electrons γ^e_{max} both in units of rest energy at a given r becomes

$$\gamma^P_{max} = 4\cdot6 \times 10^5 \left[ln\left(\frac{r}{r_0}\right)\right]^{\frac{2}{3}} \tag{4}$$

$$\gamma^e_{max} = 7\cdot5 \times 10^7 \left[ln\left(\frac{r}{r_0}\right)\right]^{\frac{2}{3}}. \tag{5}$$

The increase in the proton energy γ_P^{max} with r is shown in Fig. 2. The electrons however do not reach γ^e_{max} as given by (5). The electrons lose energy by synchrotron radiation due to their passage through the ambient magnetic field and the the maximum energy it can acquire depends on the efficiency of acceleration and the synchrotron loss. The rate of synchrotron

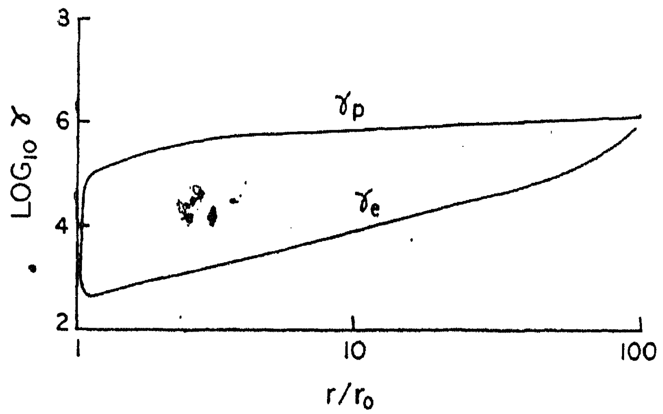

FIG. 2. The energy of protons and electrons as a function of distance from the neutron. star. The energy is expressed in units of rest mass and the distance in units of r_0 the radius of the light circle.

loss for electrons in a magnetic field H (which is a function of r and proportional to $1/r^2$) is given by

$$\left(\frac{d\gamma^e}{dt}\right)_{syn} = \frac{10^{-3}\, H^2\, \gamma_e{}^2}{m_e c^2}. \tag{6}$$

The rate of energy gain due to acceleration using expression (5) is,

$$\left(\frac{d\gamma^e}{dt}\right)_{acc} = 5 \times 10^7\, c\, \frac{1}{r\left[\left(ln\,\frac{r}{r_0}\right)\right]^{\frac{1}{3}}}. \tag{7}$$

The maximum possible energy for the electron is given by equating (6) and (7)

$$\gamma^{e}_{max} = \left[\frac{5 \times 10^{10}\, m_e c^3}{H^2 r \left\{ln \frac{r}{r_0}\right\}^{\frac{1}{3}}}\right]^{\frac{1}{4}}. \tag{8}$$

The actual γ^{e}_{max} is given by the least of γ^{e}_{max} from (5) and (8) and is plotted in Fig. 2. It seems some electrons acquire a $\gamma^{e}_{max} \simeq 10^3$ within a few meters of the light circle then decrease to about 3×10^2 and then increase slowly to about 10^8. The γ^{P}_{max} on the other hand increases continuously according to (4) and synchrotron loss on the acceleration is negligible. It should be emphasized that the γ_{max} given in Fig. 2 is the maximum energy possible at a given r, while actually there will be a distribution in the energy of the particles with a cut-off at γ_{max}. The nature of the energy spectrum depends on the injection mechanism.

The synchrotorn frequency in cycles per second is given by

$$\nu^{P}_{max} = 7 \times 10^2\, H \nu^{p^2}_{max} \tag{9}$$

$$\nu^{e}_{max} = 1 \cdot 3 \times 10^2\, H \nu^{e^2}_{max} \tag{10}$$

From the expression (9) and (10) and Fig. 1, it is seen that both electron and protons start emitting radio, optical and X-radiation within a short distance of the light circle. In the case of NP0532, much of the optical radio and X-ray emission of radiation takes place within about 10^8 cm. of the light circle, the reason being the magnetic field decreases rapidly with distance ($\propto 1/r^2$) from the light circle and the strength of the synchrotron radiation is proportional to the square of the magnetic field.

The rate of radiation of energy by protons is given by

$$P^{max}_{P} = 10^{-9}\, H^2\, \gamma^{s^2}_{max}\ \text{ev/sec.} \tag{11}$$

The highest energy electrons lose energy according to expression (6) which is the same as (7). Then

$$P^{max}_{e} = \frac{5 \times 10^7 \cdot c \cdot m_e c^2}{r\left(ln \frac{r}{r_0}\right)^{\frac{1}{3}}}\ \text{ev/sec.}$$

$$\simeq \frac{7 \times 10^{23}}{r\left(ln \frac{r}{r_0}\right)^{\frac{1}{3}}}\ \text{ev/sec.} \tag{12}$$

It can be easily seen that the electrons radiate at a much a larger rate than the protons at any given distance from the light circle.

It is possible to show that the observed X-ray emission from the pulsar can arise from the accelerated particles, if we use an estimate of the number of accelerated particles N given by Gunn and Ostriker.[4]

$$N = 2\cdot 7 \times 10^{33} \left(\frac{w}{200}\right)^2 \times \left(\frac{B_s}{10^{12}}\right) \times \left(\frac{R}{10^6}\right)^2 \text{ particles/sec.}$$

If we use the parameters used earlier the number of accelerated electrons becomes about 10^{33} per pulse. Using expression (12) and the distance given above through which the electrons radiate the X-ray energy radiated per pulse is

$$\begin{aligned} E_X &\simeq 10^{33}\cdot 7\cdot 10^{16}\cdot \frac{10^8}{3 \times 10^{10}} \text{ ev} \\ &\simeq 2 \times 10^{35} \text{ ergs.} \end{aligned}$$

The observed[5] value is 3×10^{34} ergs. It is also possible to estimate the energy emitted at optical frequencies if we make assumptions as to the energy spectrum of the accelerated electron. Thus the observed optical energy emission can be explained if we assumed an energy spectrum of the form $N(\gamma_e) \propto \gamma_e^{-1}$.

From the formula (10) and Fig. 2, it is seen that the radio radiation arises very close to the light circle. The bunching that is needed for coherent synchrotron radio emission can probably occur near the light circle.

The chief merit of the suggestion in this paper is that it provides a means of transfer of the rotational energy of the neutron star into pulsar radiation. In addition it does not demand a very special angle of observation with respect to the magnetic axis, or rotational axis, as in the case with other models using oblique rotators. However, it still remains to be explored whether, using this model, the other observed pulsar characteristics can be explained.

The idea of using 'Gunn and Ostriker' particles to produce pulsar radiation seems to have occurred to J. Trumper of Max Planck Institute and A. J. Dean and M. J. L. Turner of Milan. Our treatment differs in several aspects from the above authors and most importantly demonstrates the periodicity of this radiation.

I wish to thank Profs, R. R. Daniel and J, E, Felten for several comments,

REFERENCES

1. Gold, T. .. *Nature*, 1969, **221**, 25.
2. Komesaroff, M. M. .. *Ibid.*, 1970, **225**, 612.
3. Deutsch, A. .. *Annales d'Astrophysique*, 1955, **18**,
4. Gunn, J. E. and Ostriker, J. P. *Phys. Rev. Lett.*, 1969, **14**, 728.
5. Bradt, S., Rappaport, S., Mayer, W., Nather, R. E., Warner, B., Macfarlane, M. and Kristian, J. *Nature*, 1969, **222**, 728.