Radio Science Experiments with Onboard Arraying

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We propose a data processing technique applicable to interplanetary and near-Earth radio science experiments performed with one spacecraft but a network of N ground stations. Our technique relies on an onboard multichannel digital receiver, which separately distinguishes the N uplink signals from the N antennas, and further processes them on the spacecraft. The signals from each uplink element are distinguished by a small frequency offset so that their phases and amplitudes can be separately recorded onboard. By measuring, digitizing, time-tagging, and recording onboard the N uplink signals' amplitudes and phases, it is then possible to (i) combine the N onboard phase measurements with those made on the ground to minimize or entirely remove the effects of common Doppler noise sources affecting radio science experiments, and (ii) coherently combine the N uplink signals to maximize the ratio between the received microwave power over the measurement thermal noise (i.e., the SNR). Our onboard arraying technique allows perfect cancellation of the frequency fluctuations due to the onboard frequency reference for synthesized two-way Doppler, and suppression by a factor of \sqrt{N} of the atmospheric phase scintillation, the frequency fluctuations induced by mechanical vibration of the ground antennas, the ground microwave amplifiers, and the onboard and ground electronic frequency noises affecting the radio links. Depending on the number N of ground antennas used, the resulting improvement in Doppler sensitivity achievable with onboard arraying over that with a single antenna can be significant, and we derive an expression for the relative Doppler sensitivity enhancement that is independent of the particular radio science experiment considered.

I. Introduction

Measurements of the relative velocity between Earth and an interplanetary spacecraft, by means of two-way coherent microwave tracking, have allowed radio science studies of solar system bodies [1,2,3], tests of relativistic gravity [4,5], and searches for low-frequency gravitational radiation [6,7,8]. In the frequency band 10^{-6} to 10^{-2} Hz, typical deep-space tracks are limited by phase scintillation caused by random refractivity variations induced by the solar wind and the ionosphere [8,9]. The most sensitive deep-space two-way coherent Doppler observations to date, however, calibrate and largely remove these noises [5,10,11],

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and are then limited by residual postcalibration troposphere scintillation (i.e., Doppler fluctuations caused by refractive index fluctuations in Earth's atmosphere) [12] and antenna mechanical vibrations (unmodeled motion of the phase center of the ground antenna). The most sensitive observations hit the limit identified by these noise sources with an Allan standard deviation [13] of about 3×10^{-15} for integration times of a few thousand seconds. Improved sensitivity would benefit the science disciplines listed above, but antenna mechanical noise, in particular, has seemed irreducible at reasonable cost since it would require a large, moving, steel structure much more rigid than that of the current ground tracking stations.

In the case of radio occultations, which have been used extensively to probe the physical states and processes occurring within atmospheres and rings surrounding planets and natural satellites [3], an additional and even larger noise source than Earth's atmosphere and mechanical vibration of the ground antenna has been the onboard frequency reference driving these one-way Doppler measurements. In these experiments, a radio signal referenced to the onboard oscillator is transmitted from the spacecraft to Earth, with the radio beam crossing the atmosphere/ionosphere/rings surrounding a planet. These experiments, implemented with essentially all planetary probes, use information about phase and amplitude variations imposed by the near-planetary environment on the radio beam as it propagates through it. For example, ionospheric and atmospheric refractive indices bend the radio ray and induce fluctuations in the amplitude and Doppler shift observed on the ground. By inverting these data, it is possible to infer pressure/temperature profiles, electron density profiles (in the case of ionospheres), and waves and turbulence in planetary atmospheres [3,14,15,16]. Because it relies on amplitude and Doppler observables, the quality of the one-way data is limited both by link SNR (i.e., power in the carrier relative to the noise power) and the stability of the frequency and timing system (FTS) on the spacecraft (as the ground FTS is several orders of magnitude better). FTS stability is specified in terms of fractional frequency fluctuation, $y(t) \equiv \Delta v(t)/v_0$, where $\Delta v(t)$ are the random deviations of the microwave signal's frequency from its nominal center frequency, ν_0 . The Allan deviation, σ_{uv} quantifies frequency stability: on the relevant time scales for occultation experiments, typical flight-qualified frequency standards — often referred to as ultrastable oscillators (USOs) — have $\sigma_{\nu} \approx 10^{-13}$, while ground FTS systems have Allan deviations more than 100 times better. Because the frequency stability of the ground system is so much better than that of the spacecraft, the instrumental quality of these one-way Doppler experiments is currently largely determined by the stability of the spacecraft USO.

The quality of the radiometric data measured during occultation experiments can be markedly improved, however, if the effective stability of the frequency reference and the signal-to-noise ratio (SNR) of the radio links are improved. To enhance both SNR and frequency stability, it has been noticed [7,17] that this can be done by adding an onboard digital receiver. Besides improving the SNR of the signal received onboard (as first pointed out by [18]), it is also possible to apply time-delay interferometry (TDI) [7,17,19] to the two one-way Doppler measurements (taken onboard and at the ground, respectively). This results in a two-way Doppler observable that is unaffected by the noise of the USO and retains the frequency fluctuations induced by the occulted media. The design of a single-channel digital receiver has been studied extensively at Stanford University for the uplink for NASA's

New Horizons mission [20] and at JPL for forthcoming interplanetary missions.¹ Because NASA's Deep Space Network (DSN), for reasons of cost and telecommunications capability, is moving to an architecture of arrays of ground stations as opposed to larger single radio antennas [21], the possibility of further improving the quality of radio science experiments by relying on arrays of ground antennas has been under study in recent years. Although experimental verifications have shown that it is indeed feasible to array several ground radio antennas for coherent uplink communication to an interplanetary spacecraft [22,23,24], it is less clear whether a communication-like arraying implementation could meet the more stringent radio science amplitude, phase, and frequency stability requirements [25]. We will not address these questions in this article, as we propose an alternative way of performing radio science experiments with a network of *N* DSN stations.

Our scheme requires the use of an onboard receiver for simultaneously processing multiple uplink radiometric measurements. Our data analysis technique, which we shall refer to as onboard arraying, allows us to perform radio science experiments with an arbitrary number, N, of ground antennas that simultaneously (but not coherently) transmit to/receive from the same spacecraft (whether it is interplanetary or near-Earth). By separately distinguishing the uplink signals from the N antennas via slight frequency offsets, and by processing and optimally combining the signals measured onboard by a multichannel digital receiver, we show that it is possible to improve the quality of the radiometric data over single-antenna experiments. This is possible because the effects of the physical phenomenon on the radiometric data are essentially all equal, while some of the noise sources affecting the measurements are uncorrelated. Averaging the measurements therefore results in an enhancement of the SNR and Doppler stability over those in a single-channel experiment. It should be mentioned that the simple averaging of the data presumes all the noises in each data set to be uncorrelated and comparable in magnitude. In practice, it is always possible to perform an optimal combination of the measurements that accounts for different noise levels in each data set, and possible correlations among them (see [26] as an example). We will return to this point in Section III, where our analysis is presented.

This article is organized as follows. In Section II, we present a general and unified description of all radio science experiments in terms of the one-way radiometric measurements performed at the ground and onboard. We show how the coherence of the *N* uplink radiometric measurements can be established onboard by using our proposed multichannel receiver, thereby enhancing the received SNR by a factor of *N* over the SNR of a single-channel. In this section we also show that coherent two-way Doppler data can be reconstructed by properly time-delaying and linearly combining the one-way measurements [7,17] performed onboard and at the ground (as the onboard data is digitized, recorded, and transmitted to the ground at a later time during the mission for implementing this processing). After providing the expressions of the 2*N* Doppler one-way data in terms of the relative frequency fluctuations due to the physical effect sought for and the leading noise sources, in Section III we derive the optimal combination of the 2*N* Doppler data, i.e., one that maximizes the energy (in each Fourier frequency bin of the observable band) of the Doppler fluctuations due to the "signal" sought for relative to the energy of the resulting noise. This allows us to compare the Doppler sensitivity of the onboard arraying technique

¹ R. Navarro, J. W. Armstrong, S. Rogstad, S. Asmar, G. Taylor, C. Miyatake, D. Kahan, J. Klose, and M. Tinto, "The Digital Design and Testing of A Prototype Radio Atmospheric Sounding and Scattering Instrument," *The Interplanetary Network Progress Report*, submitted for publication, 2010.

against that of a single channel. We find that, over most of the accessible frequency band, onboard arraying implies a root-mean-squared (RMS) Doppler sensitivity that is \sqrt{N} times better than that with a single channel. Our comments and conclusions are then presented in Section IV, where we emphasize that our technique not only provides a simplification of ground network operations but also results in the ability of optimally combining radiometric data for sensitivity enhancement of radio science experiments.

II. Radio Science with Multiple Ground Antennas

Let us consider N ground antennas that transmit simultaneously (but not coherently) N microwave signals, slightly different in frequency, to the same spacecraft (see Figures 1 and 2); let us also assume that the N antennas use a common clock for accurately and precisely generating the frequencies of the transmitted microwave beams. If we denote with $r_i(t)$, i=1,2,...,N the distance of the i-antenna to the spacecraft at time t, the phases of the microwave signals received at the spacecraft can be written in the following form:

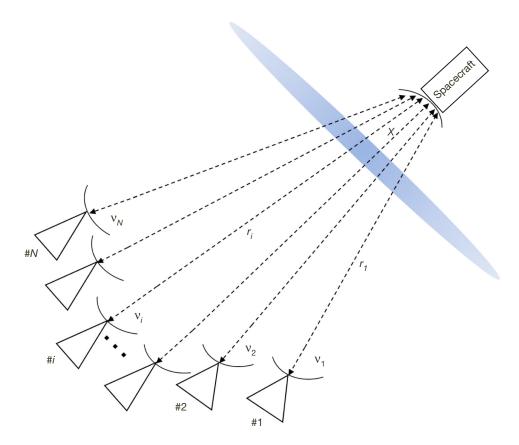


Figure 1. Schematic representation of a radio science experiment (a planetary atmosphere occultation in this case) performed with N ground antennas. The spacecraft is at distance r_i from antenna i, and its mean distance from the planet's atmosphere is denoted by x. The ground antennas are transmitting simultaneously but not coherently to the spacecraft, where N amplitudes and N Doppler measurements are performed by an onboard multichannel digital receiver. The microwave signals transmitted by the ground antennas are intentionally (slightly) different in frequencies (here denoted by $v_1, ..., v_N$ in order to distinguish them during the onboard Doppler measurements.

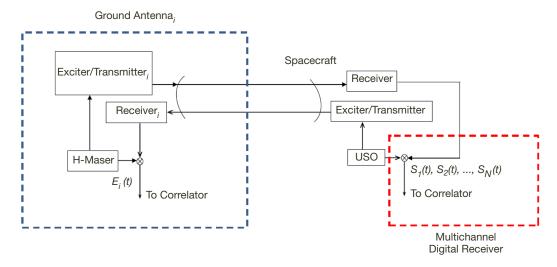


Figure 2. Diagram representing the radio arraying configuration at each of the ground antennas of the DSN and on the spacecraft. The red-dotted box represents the multichannel digital receiver instrument, which provides the onboard one-way measurements, $S_i(t)$. The blue-dotted box encloses all the hardware at the i_{th} antenna that is used for measuring the one-way Doppler data, $E_i(t)$. See Equations (3) and (4) for the expressions of the one-way Doppler measurements $E_i(t)$ and $S_i(t)$.

$$\phi_i(t) = 2\pi \nu_i(t - r_i(t)) + \phi_{i0} \tag{1}$$

where for the sake of simplicity we have disregarded all the phase fluctuations introduced by the various noise sources affecting the links. In Equation (1), v_i is the frequency of the microwave signal transmitted by antenna i, ϕ_{i0} is an arbitrary phase of the signal at the moment of transmission (t=0) at antenna i, and units are such that the speed of light is equal to 1. At the spacecraft, each signal is received and downconverted to the same intermediate frequency. This is possible because the values of the transmitted frequencies are known in advance onboard the spacecraft. The resulting phase differences measured onboard by each channel are then equal to

$$\delta \phi_i(t) = 2\pi (\nu_i - \bar{\nu}_i)t - 2\pi \nu_i r_i(t) + \phi_{i0} - \bar{\phi}$$
 (2)

where $\bar{\phi}$ is the phase of the signal used to downconvert the N received signals. Note that this is independent of the particular frequency \bar{V}_i as all the downconverting signals are referenced to the same frequency generator (the onboard USO). Since the frequencies \bar{V}_i are chosen in such a way that $(v_i - \bar{v}_i) \equiv \Delta v = const$, it is then possible to perform cross-correlations of the downconverted data to accurately identify the time-shifts needed to establish phase coherence among them. Once this is done, the N signals can be added coherently in order to enhance the SNR over that of a single channel. It is easy to see that the resulting SNR will be N-times larger than that in a single channel as the thermal noises in different channels are uncorrelated due to the different frequencies of the received beams.

Once the relative coherence of the uplinks and (similarly) downlinks is established, the resulting 2N one-way Doppler measurements performed onboard, $S_i(t)$, and at the ground, $E_i(t)$, i = 1, 2, ..., N, can be written in terms of the contributions from the physical effect

sought for by the experiment and the various noises [7,27] in the following way (see Figure 3):

$$E_{i}(t) = E_{h}(t) + C_{s}(t-L) - C_{E}(t) + T_{i}(t) + T_{s}(t-L) + A_{s}(t-L) + EL_{E_{i}}(t)$$
(3)

$$S_{i}(t) = S_{h}(t) + C_{E}(t-L) - C_{s}(t) + T_{i}(t-L) + T_{s}(t) + A_{E_{i}}(t-L) + EL_{s_{i}}(t)$$

$$(4)$$

In Equations (3) and (4), $E_h(t)$, $S_h(t)$ are the relative frequency shifts caused by a soughtfor signal in the one-way Doppler measurement recorded at the ground and onboard, respectively; they carry the science information and are the desired quantities to measure with high precision. Note that the signal entering in the one-way uplink data is in general different from that appearing in the downlink data. $C_E(t)$, $C_S(t)$ are the random processes associated with the relative frequency fluctuations of the ground and spacecraft frequency standards respectively; $T_s(t)$ is the frequency noise due to unmodeled stochastic motion of the spacecraft; and $T_i(t)$ is the sum of frequency noise due to unmodeled stochastic motion of the i_{th} ground antenna and frequency noise imposed on the link by scintillation in Earth's atmosphere (since they enter with the same transfer function). Both these noises are independent of the transmitted frequency; $A_{E_i}(t)$, $A_S(t)$ are the frequency noises of the ground and spacecraft transmitters; $EL_{E_i}(t)$, $EL_{s_i}(t)$ are the frequency noises of the ground and spacecraft receiver electronics. Note that, as a consequence of having assumed the frequencies of the transmitted beams to be slightly different, the thermal noises in the received channels will be in general different and uncorrelated. Finally, we have introduced a fiducial distance L between the Earth and the spacecraft, as the difference between it and the various spacecraft–antenna distances, r_i , would result in negligible corrections to our modeling of the Doppler data.

One-way measurements, whether performed in the uplink or downlink mode, suffer, however, from the frequency fluctuations of the onboard oscillator. Space-qualified oscillators,

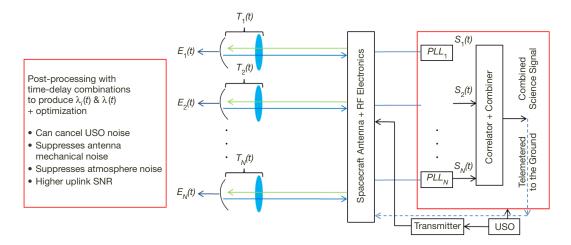


Figure 3. Schematic representation of the ground and onboard arraying radio science instrumentation. The red boxes enclose the hardware and data processing that encompass the technique proposed here. By processing simultaneously multiple uplinks and downlinks, the extra information can be used for suppressing several important noise sources, while simultaneously improving the uplink and downlink SNRs.

so-called USOs, typically have frequency stability that is more than two orders of magnitude worse than that of ground-based frequency standards for integration times of about 1000 s. It has been shown [7,17] that, for single uplinks and downlinks, it is possible to synthesize an improved Doppler data set by linearly combining the two one-way Doppler measurements, simultaneously performed onboard and at the ground, in such a way as to remove the frequency fluctuations of the USO. This procedure is called time-delay interferometry (TDI) [19]. It is easy to see that by taking (pair-wise) the following linear combination of the i_{th} one-way onboard and ground Doppler data, the USO noise exactly cancels:

$$\lambda_i(t) \equiv E_i(t) + S_i(t - L) \tag{5}$$

After substituting Equations (3) and (4) into Equation (5), we get the following expression for $\lambda_i(t)$, the synthesized two-way Doppler:

$$\lambda_{i}(t) = E_{h}(t) + S_{h}(t-L) + C_{E}(t-2L) - C_{E}(t) + T_{i}(t-2L) + T_{i}(t) + 2T_{s}(t-L) + A_{s}(t-L) + A_{E_{i}}(t-2L) + EL_{E_{i}}(t) + EL_{s_{i}}(t-L)$$
(6)

Equation (6) shows explicitly that the frequency fluctuations due to the relatively poor spacecraft USO (C_s) are removed, and that the new observable $\lambda_i(t)$, obtained by delaying and combining the two one-way measurements, has only the frequency noise of the much better ground frequency standard (C_E). The data combination $\lambda_i(t)$ shows a resulting frequency noise level that is 1 to 2 orders of magnitude smaller than that in the two one-way data, depending on integration time, while the two-way information encoded onto the radio waves by the signal is preserved [7,17].

Although the expression for $\lambda_i(t)$ derived above assumes the relative distance L between Earth and spacecraft to be constant, its ability to cancel the USO noise still holds with straightforward modification to the realistic case in which L changes with time. In principle, there are fundamental limitations to the procedure described above due to (i) the accuracy in the determination of the distance L, and (ii) the accuracy in the synchronization of the ground and onboard clocks. These will not be practical problems, however, since a very modest ranging accuracy of about 1.5×10^5 km, and a corresponding time-synchronization of about 0.5 s, is sufficient to cancel USO noise to below the remaining noises entering into $\lambda_i(t)$ [17]. Since ranging measurements to interplanetary spacecraft are routinely performed by the DSN at the few meters level — and clock synchronization can be performed at the submicrosecond level — our USO-cancellation scheme can be implemented successfully.

III. Onboard Arraying Doppler Sensitivity

The advantage of onboard arraying becomes particularly evident when considering, for instance, that the accuracy in the reconstruction of the profile of a medium through which a radio signal propagates depends both on the SNR of the microwave link and its amplitude and frequency stability. By establishing the coherence among the N received uplink signals, it is possible to enhance the SNR over that of a single channel by a factor of N. It is also possible to enhance the Doppler stability over that of a single channel by processing the data in the following way:²

² The following analysis reflects the assumptions that (1) all the noises with an index i are uncorrelated to each other, and (2) noises of the same kind but different index i are of comparable magnitude.

If we add the *N* Doppler measurements, $\lambda_i(t)$, we derive the following expression for the resulting Doppler observable, $\lambda(t)$:

$$\lambda(t) = \sum_{i=1}^{N} \lambda_{i}(t) = N \left[E_{h}(t) + S_{h}(t-L) \right] + N \left[C_{E}(t-2L) - C_{E}(t) \right]$$

$$+ \sum_{i=1}^{N} \left[T_{i}(t-2L) + T_{i}(t) \right] + 2NT_{s}(t-L) + NA_{s}(t-L)$$

$$+ \sum_{i=1}^{N} \left[A_{E_{i}}(t-2L) + EL_{E_{i}}(t) + EL_{s_{i}}(t-L) \right]$$
(7)

To understand the enhancement in frequency stability offered by the onboard arrayed combination, $\lambda(t)$, over each individual combination, $\lambda_i(t)$, let us take the Fourier transform of both sides of Equations (6) and (7), modulus-square them, and take ensemble averages of both resulting expressions over many noise realizations. For this illustration, we assume the random processes associated with the noises from each antenna to be uncorrelated to each other; it is then easy to derive the following expression of the Doppler power density SNRs, $\rho_i(f), \rho(f)$ at the Fourier frequency f in the measurements λ_i and λ , respectively:

$$\rho_{i}(f) \equiv \frac{|\widetilde{E}_{h}(f)|^{2} + |\widetilde{S}_{h}(f)|^{2} + 2Re\left[\widetilde{E}_{h}(f)\widetilde{S}_{h}^{*}(f)\right]}{S_{\lambda_{i}}(f)}$$
(8)

$$\rho(f) \equiv \frac{|\widetilde{E}_h(f)|^2 + |\widetilde{S}_h(f)|^2 + 2Re\left[\widetilde{E}_h(f)\widetilde{S}_h^*(f)\right]}{S_{\lambda}(f)}$$
(9)

where we have denoted with $S_{\lambda_i}(f)$ and $S_{\lambda}(f)$ the following combinations of the noise power spectral densities:

$$S_{\lambda_i}(f) \equiv 4\sin^2(2\pi f L)S_{C_E}(f) + 4\cos^2(2\pi f L)S_T(f) + 4_{T_s}(f) + S_{A_s}(f) + S_{EL_E}(f) + S_{EL_S}(f)$$
(10)

$$S_{\lambda}(f) \equiv 4\sin^{2}(2\pi f L)S_{CE}(f) + 4S_{T_{s}}(f) + S_{A_{s}}(f) + \frac{4\cos^{2}(2\pi f L)S_{T}(f)}{N} + \frac{S_{AE}(f) + S_{ELE}(f) + S_{EL_{s}}(f)}{N}$$
(11)

The above two equations clearly summarize the main advantage brought by the onboard arraying technique. For a given radio science experiment, characterized by the signals $E_h(t)$, $S_h(t)$ in the uplink and downlink one-way measurement, respectively, the effective Doppler noise affecting the combined arrayed data is characterized by having the RMS frequency fluctuations due to Earth's atmosphere, the mechanical vibrations of the ground antennas, the ground antennas' microwave amplifiers, and the ground and onboard electronic noises reduced by a factor of \sqrt{N} . Furthermore, the noise of the USO affecting the one-way measurements has been entirely removed [see Equation (6)], while improving by a factor of N the microwave uplink SNR over that achievable with a single antenna.

In Figure 4, we plot the noise spectra $S_{\lambda_i}(f)$, $S_{\lambda}(f)$ over the band 10^{-5} to 1 Hz for the particular case of a spacecraft out to a distance L = 5.5 AU, approximately equal to the Earth–Saturn distance and representative of the Cassini mission radio science experiments.

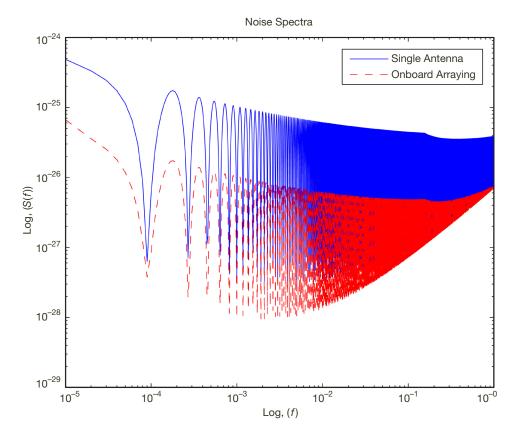


Figure 4. Comparison of the frequency stabilities achievable with onboard arraying, and conventional single-antenna radio science experiments. We have considered a spacecraft out to Saturn, the ground and onboard microwave configurations equal to those used during the Cassini radio science experiments campaign, and the frequency stability of the onboard digital receiver is comparable to that of the receiver at the ground. Furthermore, we have assumed N = 10. Onboard arraying shows a Doppler relative stability that is about a factor of N better than that with a single antenna.

We have also taken N=10, as this might be a possible number of 34-m beam-wave guide antennas available at each DSN complex in the near future. The individual noise power spectral densities used in calculating these functions also correspond to the onboard and ground configuration of the Cassini radio science experiments, while the frequency stability specifications for each channel of our onboard receiver are given in Table 1 together with a summary of its most important requirements. The Doppler sensitivity offered by onboard arraying is clearly superior to that of a single channel, with a gain of about N over most of the accessible frequency band. The degradation in sensitivity enhancement shown at higher frequencies is due to the ground frequency and timing subsystem (FTS), which is the dominant noise source in this part of the frequency band and is not suppressed by the onboard arraying technique [see Equations (10) and (11)].

IV. Conclusions

We presented a time-domain procedure for coherently combining radiometric data measured simultaneously at the ground and onboard a spacecraft that is tracked by a network

Table 1. Summary of the most important requirements for each channel of the onboard receiver.

Attribute	Specification	Driver
Maximum spurious signal	< –68 dBc/Hz at 1 Hz	Ring occultation, surface
levels	< –78 dBc/Hz at 10 Hz	scattering
	$<$ –88 dBc/Hz at 10^{-10} kHz	
Frequency stability at	$< 1.0 \times 10^{-14} \text{ at 1 s}$	All radio science
32 GHz (Ka-band)	$< 4.7 \times 10^{-15}$ at 10 s	experiments
	$< 1.5 \times 10^{-15}$ at 100 s	
	$< 5.6 \times 10^{-16}$ at 1000 s	
Maximum digital bandwidth	25 kHz	Ring occultation, surface
		scattering
One-sided power spectral	$2.0 \times 10^{-29}~{\rm Hz^{-1}}$	All radio science experiments
density of the relative		
frequency fluctuations at		
32 GHz (Ka-band)		
Onboard data storage	> 21 GB onboard (for	Long-duration experiments
	recording more than 1 day	
	at 25 kHz)	
Bit depth	12+ bits	All radio science experiments
	Enough to achieve required	
	spurious signal level, and cancel-	
	lation of USO noise while retain-	
	ing signal properties	
Maximum input frequency	Track with 15 kHz in 12 min	Assumes pass occurs when
dynamics	(UHF at Mars) and 315 kHz in	frequency is changing the
	12 min (X-band at Mars)	fastest

of N ground stations. By suitably offsetting, differencing, and combining the 2N time series measured onboard and at the ground, the common noise due to the USO is canceled exactly [Equation (6)], while most of the remaining noises are suppressed by a factor equal to the square root of the number N of tracking antennas. Onboard arraying results in an SNR enhancement over that achievable with a single antenna that is equal to the number N of transmitting stations, and provides a potentially significant simplification of ground operations for coherently tracking a spacecraft. Lastly, it provides the capability of selectively combining the 2N Doppler measurements for maximizing science data quality and return.

The future advent of large arrays, the improvement in reference phase stability and, simultaneously, in measurement SNR achievable by the onboard arraying technique, should open the door to fundamentally new radio science capabilities. A quantitative analysis of the science return improvements that each radio science experiment will be able to experience with the proposed technique will be the topic of a forthcoming article.

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