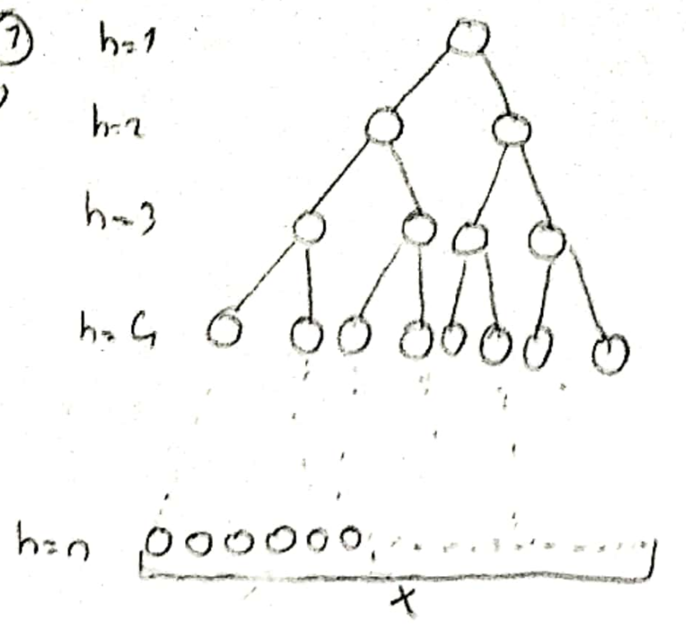
1) a)



Number of nodes	depth
1	1
2	2
4	3
8	4
2^{h-1}	h

The number of nodes at the last height could be different than 2^{h-1}

Since the complete binary tree is a perfect binary tree to the last height

General formula is $\sum_{i=1}^{h-1} 2^{i-1} \cdot i$

Last height formula is $x \cdot h$

Let's Combine

$$\sum_{i=i}^{h-1} 2^{i-1} \cdot i + x \cdot h$$

(b) There is a comprasion at evry height of complete binary search tree

so height = number of comparisons

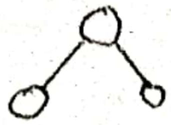
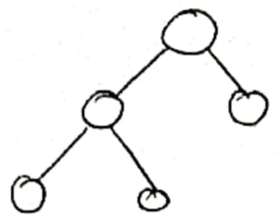
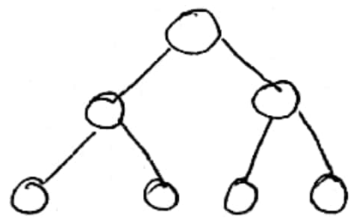
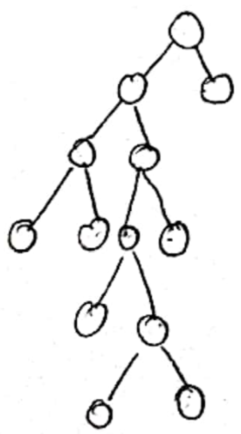
number of nodes = $\sum_{i=1}^{h} 2^{i-1}$

$n = 2^0 + 2^1 + 2^2 \cdots 2^h$

$\log_2 n = h$

$\log_2 n$ is avarge number of comparisons

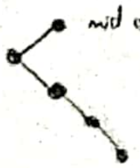
(c)

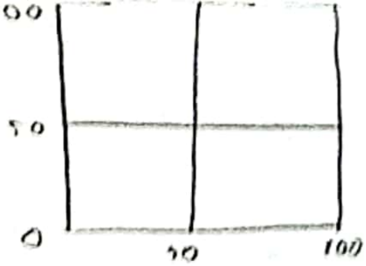
	Total Number of Nodes	internal nodes	Leaves
	1	0	1
	3	1	2
	5	2	3
	7	3	4
	13	6	7
General	n	$\frac{n-1}{2}$	$\frac{n+1}{2}$

Number of node has to be odd

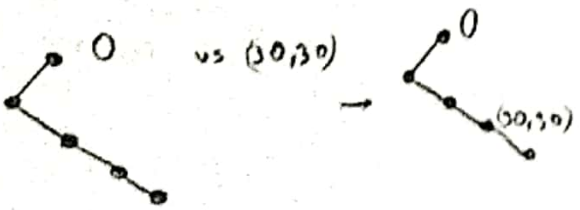
$n = 2k+1$

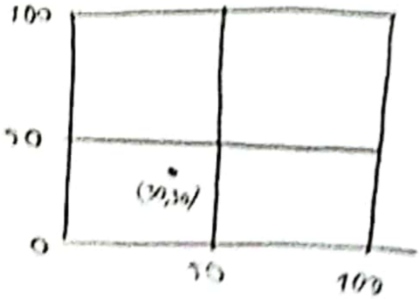
2) mid of the map O (50,50)



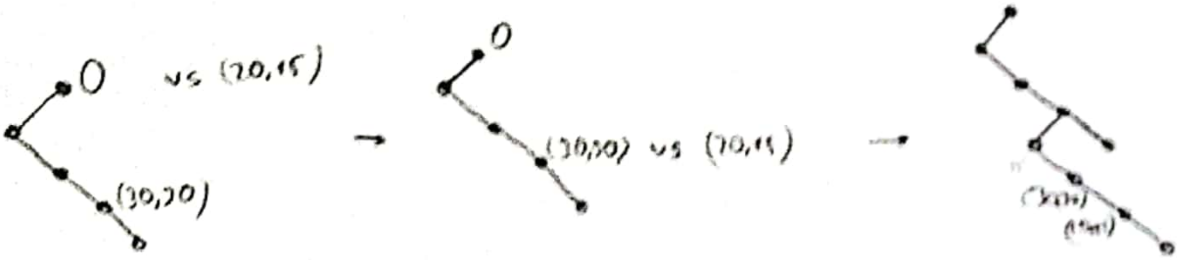


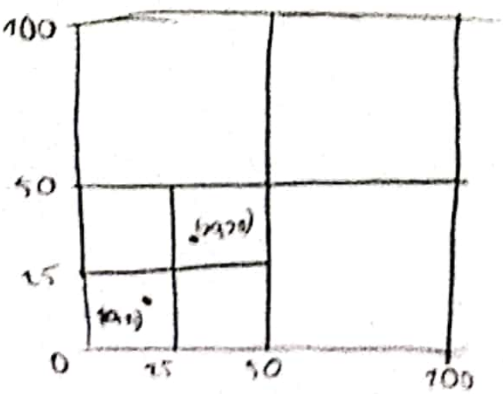
Insert (30,30)



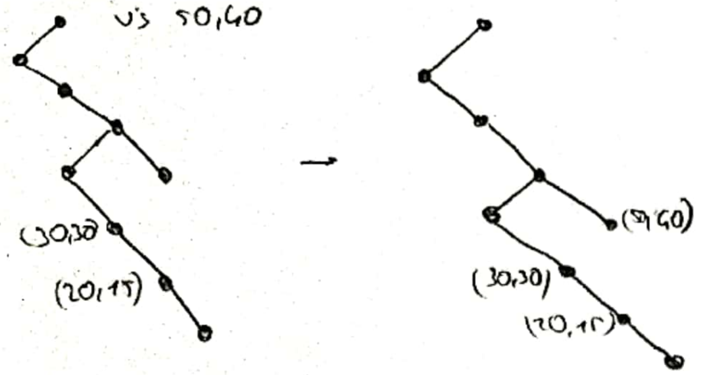


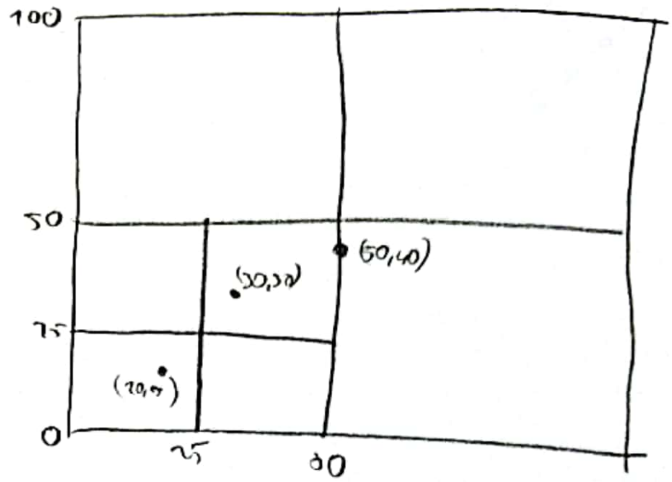
Insert (20,15)



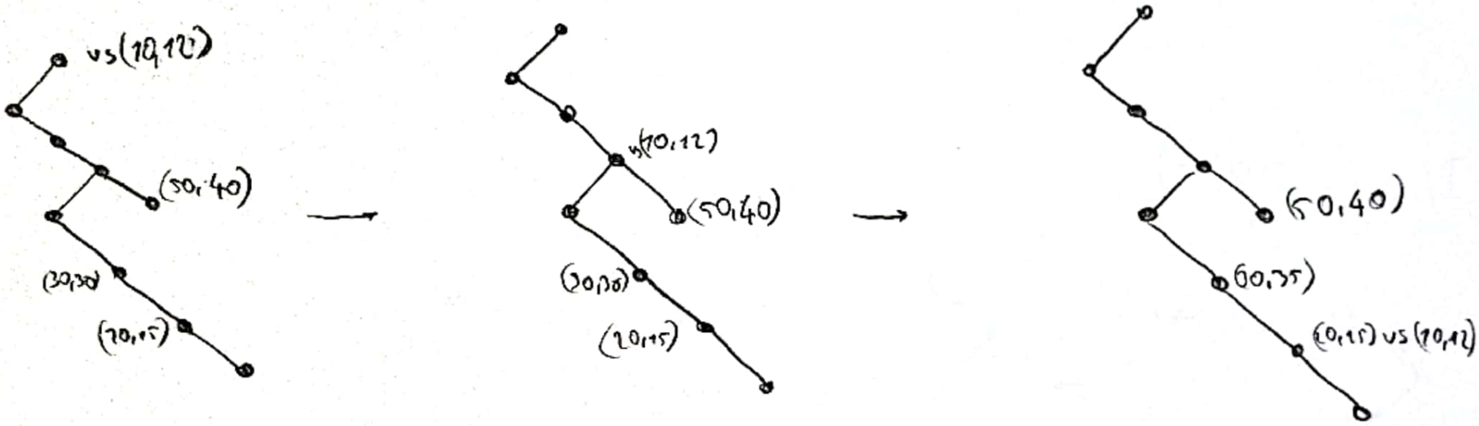


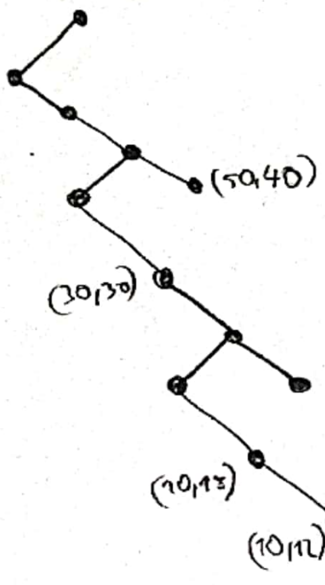
Insert (50,40)

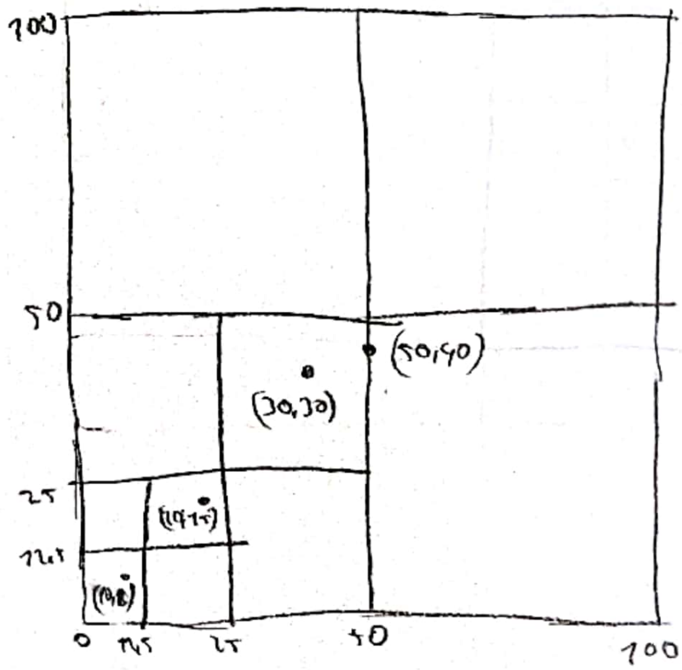




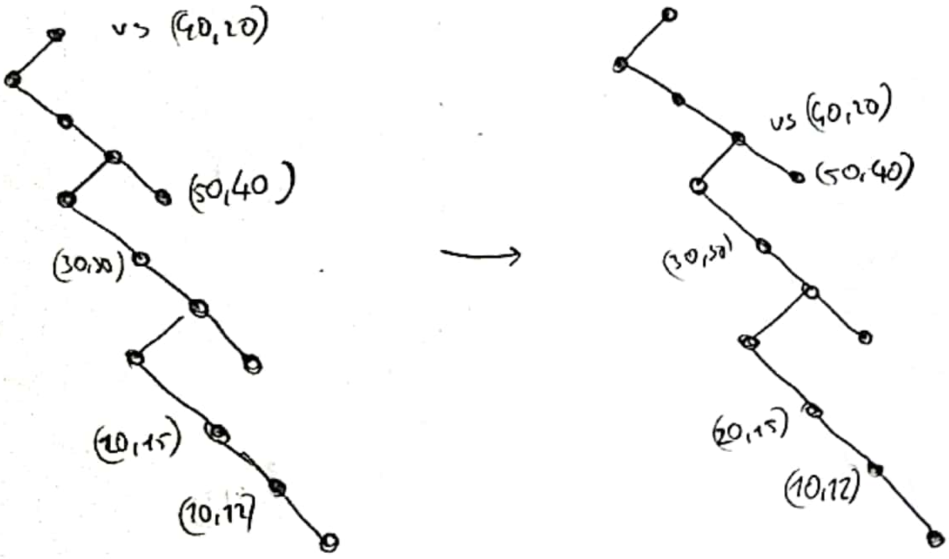
Insert (10,12)



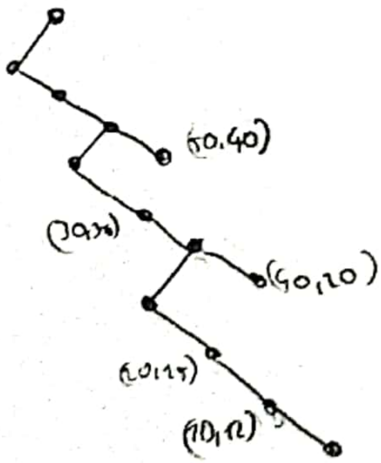




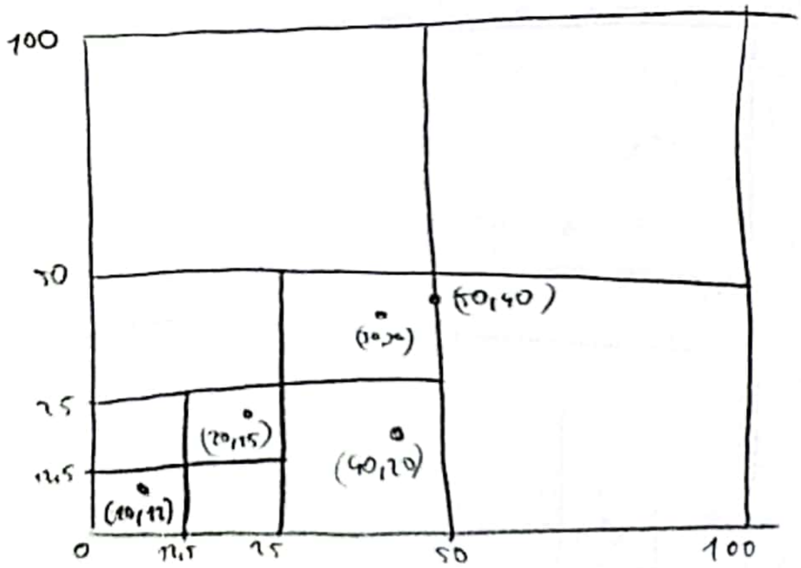
Insert (40,20)



vs (40,20)
(50,40)
(30,30)
(20,15)
(10,12)
vs (40,20)
(50,40)
(30,30)
(20,15)
(10,12)



(50,40)
(30,30)
(40,20)
(20,15)
(10,12)



100
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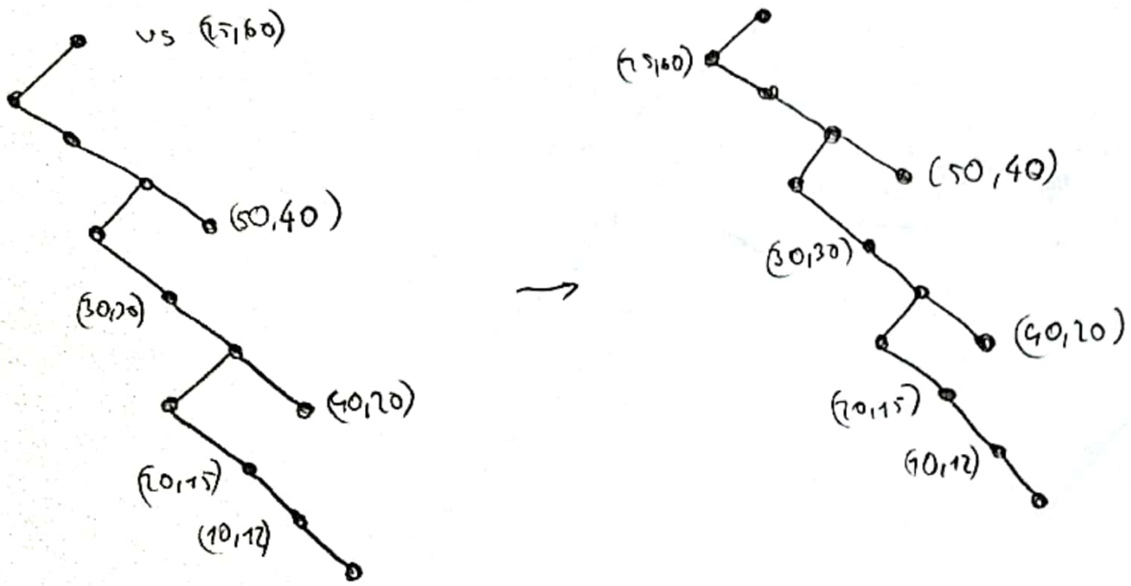
25
12,5
(50,40)
(30,30)
(20,15)
(40,20)
(10,12)
0
12,5
25
50
100

Insert (25,60)


vs (25,60)
(50,40)
(30,30)
(40,20)
(20,15)
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(25,60)
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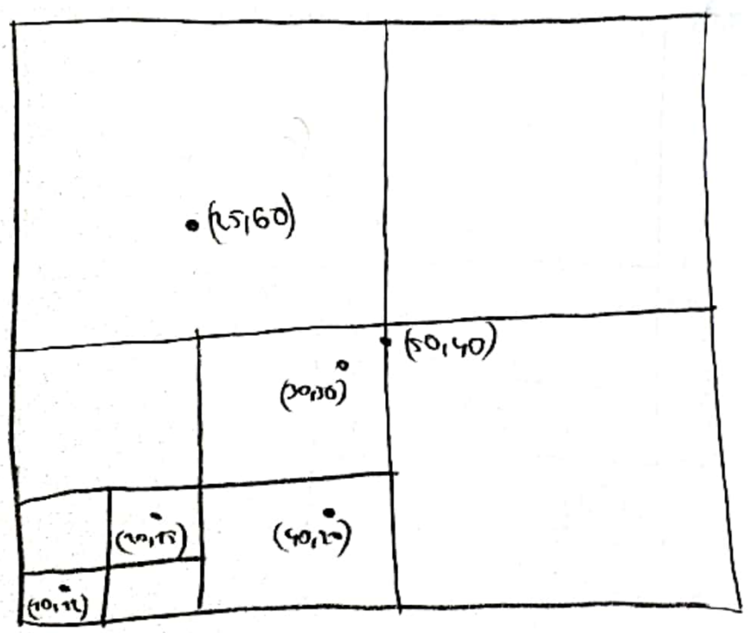
(20,15)
(10,12)


(25,60)
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(20,15)
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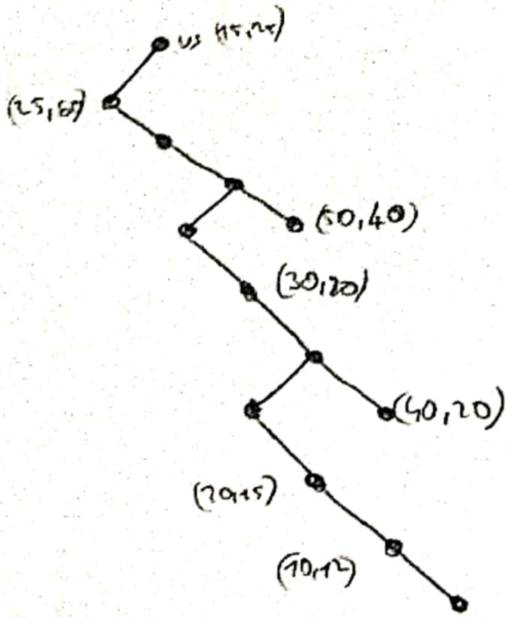
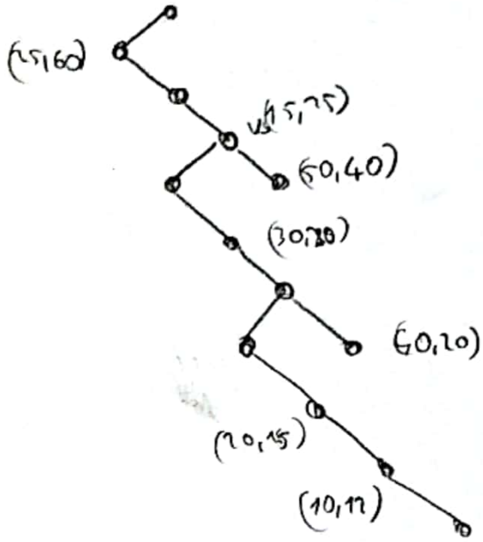
(10,12)

Insert (15,25)


vs (15,25)
(25,60)
(50,40)
(30,20)
(60,20)
(20,15)
(10,12)


(25,60)
vs (15,25)
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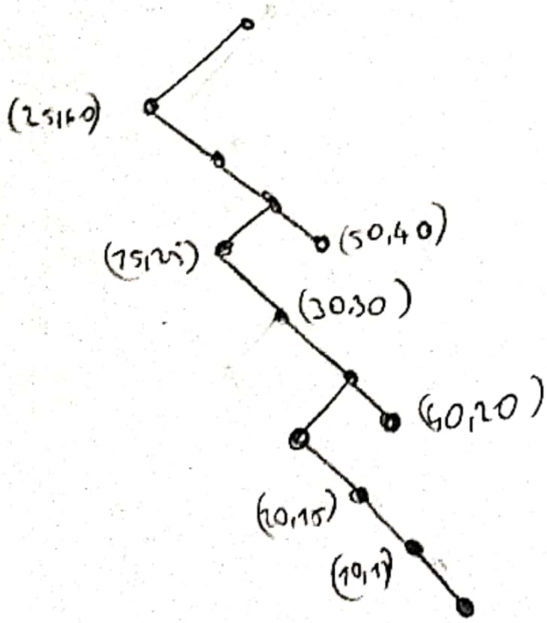
(30,20)
(60,20)
(20,15)
(10,12)


(25,60)
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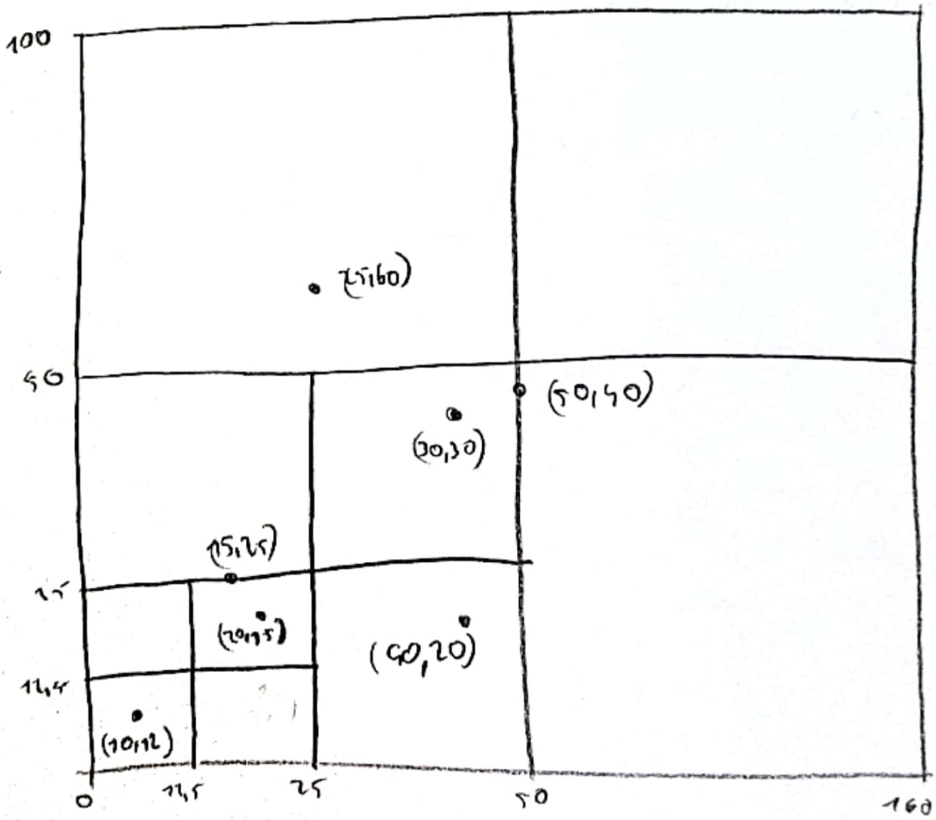
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(20,15)
(60,20)
(10,12)
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50
100