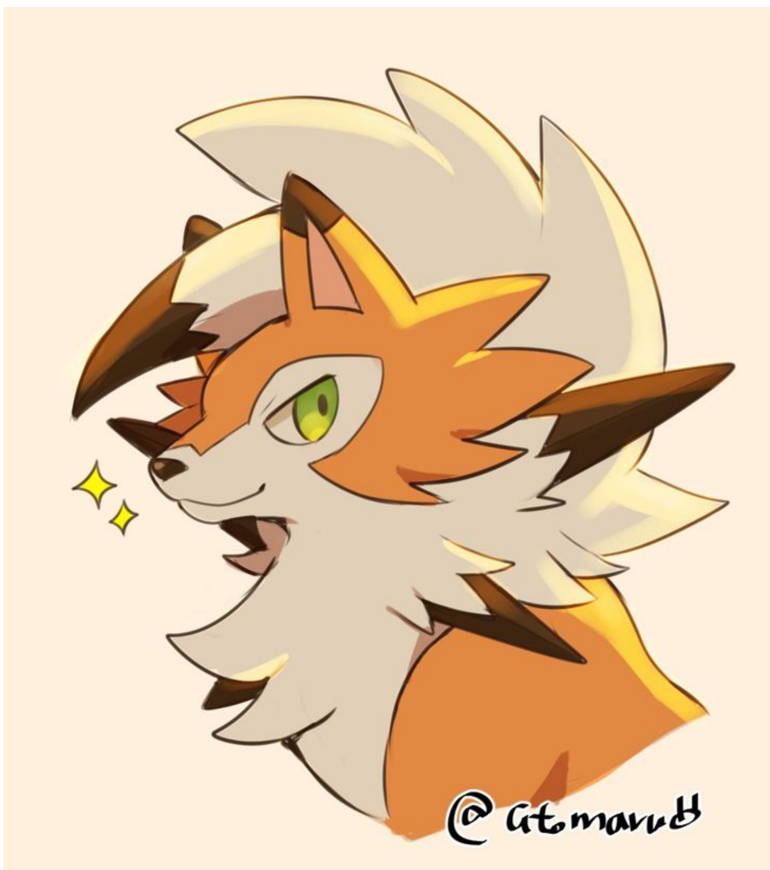
OWOMO

PROBLEMS

June 2024



1 Info

- These guys ARE integers.
- At worst the questions are like AIME question 5 difficulty probably
- They increase in difficulty?? maybe???
- Have fun :)

Problem 1

$S = \{a, b, c, d\}$ where a, b, c and d are distinct, positive integers. For all odd $s \in S$, there exists a $6s \in S$. What is the smallest value of $a + b + c + d$?

Problem 2

The average of a series is given by k:

$$k = \frac{5 + 9 + ... + 4n + 1}{n}$$

Find the largest integer n such that $k = n^2 - 41n + 225$.

Problem 3

Shapes are friends if in total, they have 33 sides. The probability that two shapes with different number of sides (less than 33) will be friends is given by $\frac{m}{n}$. Where m and n are relatively prime, what is $(m \cdot n)$?

Problem 4

Suppose a, b, and c are all unique integers and roots of $f(x)$, where $f(x) = ax^3 + bx + c$. What is $\underline{a^2}\ \underline{b^2}\ \underline{c^2}$?

Problem 5

Consider a 4×4 array of points where any 2 distinct points may be chosen at once. Call the bottom left point A and the bottom right point B, and call the two randomly selected points C and D. This means C and D may not be at points A or B or on each other. The probability that $\overline{AC}$ intersects $\overline{BD}$ is given by $\frac{m}{n}$ where m and n are relatively prime. Find $m + n$.

Problem 6

Let $y = \binom{x-1}{x-3} \cdot \binom{x-2}{x-4} \cdot \binom{x-3}{x-5} \cdot ... \cdot \binom{x-2023}{x-2025}$, it sure makes a polynomial. How many times does it hit $y = -tanh(x)$?

Problem 7

There is a equilateral triangle inscribed within a unit circle, and a circle inscribed within the triangle. If we continue inscribing triangles and circles until there are n circles, the smallest circle will have radius $f(n)$. Find the largest $f(n)$ such that $\frac{1}{f(n)} \leq 666$.

Problem 8

A *n-pinwheel* consists of n many isosceles triangles with area A where $\frac{A_{i+1}}{A_i} = \frac{1}{4}$ such that A_n is smallest. If the areas of a *3-pinwheel* and a *4-pinwheel* are equal, the ratio of their perimeters is given by $\frac{perimeter_3}{perimeter_4} = \frac{m}{n} \cdot \sqrt{\frac{170}{21\sqrt{3}}}$ where m and n are relatively prime. Find $\lceil m + n \rceil$.