Lista Álgebra Matricial

Questão 1

$$A = \begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 4 & -3 \\ 1 & -2 \\ -2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} 5 \\ -4 \\ 2 \end{pmatrix}$$

```
A = matrix(c(-1,3,4,2), nrow=2, byrow=T)
B = matrix(c(4,-3,1,-2,-2,0), nrow=3, byrow=T)
C = matrix(c(5,-4,2), nrow=3, byrow=T)
```

a) BA

```
B%*%A
```

```
     [,1] [,2]
[1,]  -16    6
[2,]   -9   -1
[3,]    2   -6
```

b) $A^\top B^\top$

```
t(A)
```

```
     [,1] [,2]
[1,]   -1    4
[2,]    3    2
```

```
t(B)
```

```
     [,1] [,2] [,3]
[1,]    4    1   -2
[2,]   -3   -2    0
```

```
t(A)%*%t(B)
```

```
     [,1] [,2] [,3]
[1,]  -16   -9    2
[2,]    6   -1   -6
```

c) $C^\top B$

```
t(C)
```

```
     [,1] [,2] [,3]
[1,]    5   -4    2
```

```
t(C)%*%B
```

```
     [,1] [,2]
[1,]   12   -7
```

d) $\text{tr}(A)$ e $\text{tr}(A^\top)$

```
sum(diag(A))
```

```
[1] 1
```

```
sum(diag(t(A)))
```

```
[1] 1
```

e) $\det(A)$ e $\det(A^\top)$

```
det(A)
```

```
[1] -14
```

```
det(t(A))
```

```
[1] -14
```

f) A^{-1}

```
solve(A)
```

```
           [,1]       [,2]
[1,] -0.1428571 0.21428571
[2,]  0.2857143 0.07142857
```

g) $\det(A^{-1})$ e $\frac{1}{\det(A)}$

```
det(solve(A))
```

```
[1] -0.07142857
```

```
1/det(A)
```

```
[1] -0.07142857
```

h) $A \otimes B$

```
kronecker(A,B)
```

```
     [,1] [,2] [,3] [,4]
[1,]   -4    3   12   -9
[2,]   -1    2    3   -6
[3,]    2    0   -6    0
[4,]   16  -12    8   -6
[5,]    4   -8    2   -4
[6,]   -8    0   -4    0
```

Quetão 2

$$A = \begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix}$$

a) $(A^\top)^\top$ = A

```
t(A)
```

```
     [,1] [,2]
[1,]   -1    4
[2,]    3    2
```

```
t(t(A))
```

```
     [,1] [,2]
[1,]   -1    3
[2,]    4    2
```

b) $(A^\top)^{-1} = (A^{-1})^\top$

```
solve(t(A))
```

```
           [,1]       [,2]
[1,] -0.1428571 0.28571429
[2,]  0.2142857 0.07142857
```

```
t(solve(A))
```

```
           [,1]       [,2]
[1,] -0.1428571 0.28571429
[2,]  0.2142857 0.07142857
```

Questão 3

$$A = \begin{pmatrix} 9 & -2 \\ -2 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 4 & 8 & 8 \\ 3 & 6 & -9 \end{pmatrix}$$

```
library(Matrix); library(matrixcalc)
A = matrix(c(9,-2,-2,6), nrow=2, byrow=T)
B = matrix(c(4,8,8,3,6,-9), nrow=2, byrow=T)
```

a) A é simétrica?

```
t(A)
```

```
     [,1] [,2]
[1,]    9   -2
[2,]   -2    6
```

b) $\mathrm{rk}(A), \quad \mathrm{rk}(B), \quad \mathrm{rk}(B^\top)$

```
rankMatrix(A)[1]
```

```
[1] 2
```

```
rankMatrix(B)[1]
```

```
[1] 2
```

```
t(B)
```

```
     [,1] [,2]
[1,]    4    3
[2,]    8    6
[3,]    8   -9
```

```
rankMatrix(t(B))[1]
```

```
[1] 2
```

c) $\det(A \otimes A)$

```
kronecker(A,A)
```

```
     [,1] [,2] [,3] [,4]
[1,]   81  -18  -18    4
[2,]  -18   54    4  -12
[3,]  -18    4   54  -12
[4,]    4  -12  -12   36
```

```
det(kronecker(A,A))
```

```
[1] 6250000
```

d) $\det(A \oplus A)$

```
AA = direct.sum(A,A); AA
```

```
     [,1] [,2] [,3] [,4]
[1,]    9   -2    0    0
[2,]   -2    6    0    0
[3,]    0    0    9   -2
[4,]    0    0   -2    6
```

```
det(AA)
```

```
[1] 2500
```

e) $\det(A \oplus A \oplus A)$

```
AAA = direct.sum(AA,A);AAA
```

```
     [,1] [,2] [,3] [,4] [,5] [,6]
[1,]    9   -2    0    0    0    0
[2,]   -2    6    0    0    0    0
[3,]    0    0    9   -2    0    0
[4,]    0    0   -2    6    0    0
[5,]    0    0    0    0    9   -2
[6,]    0    0    0    0   -2    6
```

```
det(AAA)
```

```
[1] 125000
```

f) Autovalores e Autovetores de A

```
eigen(A)
```

```
eigen() decomposition
$values
[1] 10  5

$vectors
           [,1]       [,2]
[1,] -0.8944272 -0.4472136
[2,]  0.4472136 -0.8944272
```

Questão 5

$$A = \begin{pmatrix} 1 & 1 \\ 2 & -2 \\ 2 & 2 \end{pmatrix}$$

```
        A = matrix(c(1,1,2,-2,2,2), nrow=3, byrow=T)
```

a) $A^\top A$ e autovalores e autovetores

```
        t(A)
```

```
     [,1] [,2] [,3]
[1,]    1    2    2
[2,]    1   -2    2
```

```
        ata = t(A) %*% A; ata
```

```
     [,1] [,2]
[1,]    9    1
[2,]    1    9
```

```
        eigen(ata)
```

```
eigen() decomposition
$values
[1] 10  8

$vectors
          [,1]       [,2]
[1,] 0.7071068 -0.7071068
[2,] 0.7071068  0.7071068
```

b) $AA^\top$ e autovalores e autovetores

```
        aat = A%*%t(A); aat
```

```
     [,1] [,2] [,3]
[1,]    2    0    4
[2,]    0    8    0
[3,]    4    0    8
```

```
        eigen(aat)
```

```
eigen() decomposition
$values
[1] 1.000000e+01 8.000000e+00 3.552714e-15

$vectors
           [,1] [,2]       [,3]
[1,] -0.4472136    0  0.8944272
[2,]  0.0000000   -1  0.0000000
[3,] -0.8944272    0 -0.4472136
```

Questão 6

$$A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
        A = matrix(c(4,0,0,0,9,0,0,0,1), nrow=3, byrow=T)
```

a) A^{-1}

```
        a1 = solve(A); a1
```

```
     [,1]      [,2] [,3]
[1,] 0.25 0.0000000    0
[2,] 0.00 0.1111111    0
[3,] 0.00 0.0000000    1
```

b) Autovalores e autovetores de A

```
        eigen(A)
```

```
eigen() decomposition
$values
[1] 9 4 1

$vectors
     [,1] [,2] [,3]
[1,]    0    1    0
[2,]    1    0    0
[3,]    0    0    1
```

c) Autovalores e autovetores de A^{-1}

```
        eigen(a1)
```

```
eigen() decomposition
$values
[1] 1.0000000 0.2500000 0.1111111

$vectors
     [,1] [,2] [,3]
[1,]    0    1    0
[2,]    0    0    1
[3,]    1    0    0
```