

Lista 4

Aluno: Artur Arthur Silva de Lima

Matrícula: 508402

1.

a) $\tilde{z}_x - 0,6\tilde{z}_{x-1} = a_x \quad \therefore$

$$\tilde{z}_x - 0,6B\tilde{z}_x = a_x \quad \therefore$$

$$a_x = (1 - 0,6B)\tilde{z}_x \quad ; \quad AR(1)$$

b) $\tilde{z}_x = a_x + 0,8a_{x-1} \quad \therefore$

$$\tilde{z}_x = a_x + 0,8Ba_x \quad \therefore$$

$$\tilde{z}_x = (1 + 0,8B)a_x \quad ; \quad MA(1)$$

c) $\tilde{z}_x = 0,3\tilde{z}_{x-1} - 0,6\tilde{z}_{x-2} + a_x \quad \therefore$

$$\tilde{z}_x = 0,3B\tilde{z}_x - 0,6B^2\tilde{z}_x + a_x \quad \therefore$$

$$\tilde{z}_x = (0,3B - 0,6B^2)\tilde{z}_x + a_x \quad \therefore$$

$$a_x = (1 - 0,3B + 0,6B^2)\tilde{z}_x \quad ; \quad AR(2)$$

d) $\tilde{z}_x - 0,4\tilde{z}_{x-1} = a_x - 0,3a_{x-1} + 0,8a_{x-2} \quad \therefore$

$$\tilde{z}_x - 0,4B\tilde{z}_x = a_x - 0,3Ba_x + 0,8B^2a_x \quad \therefore$$

$$(1 - 0,4B)\tilde{z}_x = (1 - 0,3B + 0,8B^2)a_x \quad ; \quad ARMA(1,2)$$

e) $z_x = 1,5z_{x-1} - 0,75z_{x-2} + a_x + 4 \quad \therefore$

$$z_x - 4 = \tilde{z}_x = 1,5\tilde{z}_{x-1} - 0,75\tilde{z}_{x-2} + a_x \quad \therefore$$

$$\tilde{z}_t = 1,5B\tilde{z}_t - 0,75B^2\tilde{z}_t + a_t \quad \therefore$$

$$a_t = (1 - 1,5B + 0,75B^2)\tilde{z}_t ; \text{AR}(2)$$

$$b) \tilde{z}_t = 0,3a_{t-1} + 0,6a_{t-2} + a_t \quad \therefore$$

$$\tilde{z}_t = 0,3Ba_t + 0,6B^2a_t + a_t \quad \therefore$$

$$\tilde{z}_t = (1 + 0,3B + 0,6B^2)a_t ; \text{MA}(2)$$

2. USANDO A FUNÇÃO **POLYROOT()** DO R, PODEMOS ENCONTRAR AS RAÍZES DOS POLINÔMIOS $\pi(B)$ E $\psi(B)$ DOS MODELOS, E PARA TODOS ELES, VEMOS QUE ESTAS ESTÃO FORA DO CÍRCULO UNITÁRIO, FAZENDO COM QUE OS PROCESSOS SEJAM TODOS **INVERTÍVEIS** E **ESTACIONÁRIOS**, SABENDO QUE PARA ESTACIONARIEDADE, $\psi(B)$ DEVE CONVERGIR PARA $|B| \leq 1$, E PARA INVERTIBILIDADE, $\pi(B)$ DEVE CONVERGIR PARA $|B| \leq 1$.

3.

$$a) a_t = (1 - 0,6B)\tilde{z}_t , \text{AR}(1)$$

$$\gamma_3 = \phi_1 \gamma_{3-1} , 3 > 0 ;$$

$$\rho_3 = \phi_1^2 \rho_{3-1} , 3 > 0 ;$$

$$\gamma_0 = \frac{6a^2}{1 - 0,36} = \frac{6a^2}{0,64}$$

$$\gamma_1 = \phi_1 \gamma_0 = \frac{0,66a^2}{0,64} = 0,93756a^2$$

$$\gamma_2 = \phi_1 \gamma_1 = 0,6 \cdot 0,93756a^2 = 0,56256a^2$$

$$\gamma_3 = \phi_1 \gamma_2 = 0,6 \cdot 0,56256a^2 = 0,33756a^2$$

$$\phi_{11} = \rho_1 = \phi_1 \rho_0 = \phi_1 = 0,6$$

$$\phi_{22} = \phi_{33} = 0 , \text{ pois } K > P = 1$$

$$b) \tilde{z}_t = (1 + 0,8B)a_t , \text{MA}(1)$$

$$\gamma_0 = 6^2(1 + 0,8^2) = 1,646^2$$

$$\gamma_1 = \sigma^2 \cdot 0,8 = 0,8\sigma^2$$

$$\gamma_2 = \gamma_3 = 0, \text{ pois } \gamma(s) = 0 \quad \forall s > q=1$$

$$\phi_{11} = \rho_1 = \frac{0,8}{1+0,8^2} \approx 0,4878$$

$$\phi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2}; \quad \rho_2 = 0, \text{ pois } s > q = 1.$$

$$\phi_{22} = -\frac{0,4878^2}{1 - 0,4878^2} \approx -0,3122$$

$$\phi_{33} = \frac{\begin{vmatrix} 1 & 0,4878 & 0,4878 \\ 0,4878 & 1 & 0 \\ 0 & 0,4878 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0,4878 & 0 \\ 0,4878 & 1 & 0,4878 \\ 0 & 0,4878 & 1 \end{vmatrix}} \approx 0,2215$$

$$c) (1 - 0,3B + 0,6B^2)\tilde{e}_k = a_k, \text{ AR}(2)$$

$$\gamma_0 = \frac{\sigma_a^2}{1 - \phi_1\rho_1 - \phi_2\rho_2}$$

$$\rho_1 = \frac{\phi_1}{1 - \phi_2} \quad \text{e} \quad \rho_2 = \phi_1\rho_1 + \phi_2 = -0,54375$$

$$= 0,1875$$

$$\gamma_0 = \frac{\sigma_a^2}{1 - 0,3 \cdot 0,1875 - 0,6 \cdot (-0,54375)} = \frac{\sigma_a^2}{0,6175}$$

$$\gamma_1 = \phi_1\gamma_0 + \phi_2\gamma_1 \quad \therefore (1 - \phi_2)\gamma_1 = \phi_1\gamma_0 \quad \therefore \gamma_1 = \frac{\phi_1\gamma_0}{1 - \phi_2}$$

$$= \frac{0,3\gamma_0}{0,6175(1+0,6)} \approx 0,3036\sigma_a^2$$

$$\gamma_2 = \phi_1\gamma_1 + \phi_2\gamma_0 \approx 0,3 \cdot 0,3036\sigma_a^2 - \frac{0,6\sigma^2}{0,6175} \approx -0,8806\sigma_a^2$$

$$\gamma_3 = \phi_1\gamma_2 + \phi_2\gamma_1 \approx 0,3 \cdot (-0,8806\sigma_a^2) + (-0,6) \cdot 0,3036\sigma_a^2 \approx -0,4462\sigma_a^2$$

$$\phi_{11} = \rho_1 = 0,1875$$

$$\phi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} = -0,6$$

$$\phi_{33} = 0, \text{ pois } K > p = 2$$

$$2) (1 - 1,5B + 0,75B^2) \tilde{z}_t = a_t, \text{ AR}(2)$$

$$\gamma_0 = \frac{\sigma_a^2}{1 - \phi_1 \rho_1 - \phi_2 \rho_2} ;$$

$$\rho_1 = \frac{\phi_1}{1 - \phi_2} = \frac{1,5}{1 + 0,75} \approx 0,8571$$

$$\rho_2 = \phi_1 \rho_1 + \phi_2 \rho_0 = \phi_1 \rho_1 + \phi_2 \approx 1,5 \cdot 0,8571 - 0,75 \\ \approx 0,5357$$

$$\gamma_0 \approx \frac{\sigma_a^2}{0,1161}$$

$$\gamma_1 = \frac{\phi_1 \gamma_0}{1 - \phi_2} \approx 7,3816 \sigma_a^2$$

$$\gamma_2 = \phi_1 \gamma_1 + \phi_2 \gamma_0 \approx 4,6136 \sigma_a^2$$

$$\gamma_3 = \phi_1 \gamma_2 + \phi_2 \gamma_1 \approx 0,9966 \sigma_a^2$$

$$\phi_{11} = \rho_1 = 0,8571$$

$$\phi_{22} = \frac{\rho_2 - \rho_1^2}{1 - \rho_1^2} \approx -0,75$$

$$\phi_{33} = 0, \text{ pois } K > 3 = 2$$

$$8) \tilde{z}_t = (1 + 0,3B + 0,6B^2) a_t, \text{ MA}(2)$$

$$\gamma_0 = \sigma_a^2 (1 + 0,3^2 + 0,6^2) = 1,45 \sigma_a^2$$

$$\gamma_1 = \sigma_a^2 (1 \cdot 0,3 + 0,3 \cdot 0,6) = 0,48 \sigma_a^2$$

$$\gamma_2 = \sigma_a^2 (1 \cdot 0,6) = 0,6 \sigma_a^2$$

$$Y_3 = 0, \text{ pois } \delta(\delta) = 0 \forall \delta > 2$$

$$\phi_{11} = p_1 = \frac{0,3 + 0,3 \cdot 0,6}{1 + 0,3^2 + 0,6^2} \approx 0,331$$

$$p_2 = \frac{0,6}{1 + 0,3^2 + 0,6^2} \approx 0,4137$$

$$\phi_{22} = \frac{p_2 - p_1^2}{1 - p_1^2} \approx 0,3417$$

$$p_3 = 0, \text{ pois } \delta > q = 2$$

$$\phi_{33} = \frac{\begin{vmatrix} 1 & 0,331 & 0,331 \\ 0,331 & 1 & 0,4137 \\ 0,4137 & 0,331 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & 0,331 & 0,4137 \\ 0,331 & 1 & 0,331 \\ 0,4137 & 0,331 & 1 \end{vmatrix}} \approx 0,2526$$

4. a) $p_s = \phi_1 p_{s-1} \therefore$

$$p_1 = \phi_1 p_0 = \phi_1 = 0,6$$

$$p_2 = \phi_1 p_1 + \phi_2 p_0 = \phi_1 p_1 + \phi_2 = 0,6 \cdot 0,6 = 0,36$$

b) $p_s = \phi_1 p_{s-1} + \phi_2 p_{s-2} \therefore$

$$p_1 = \phi_1 p_0 + \phi_2 p_1 \therefore p_1 = \frac{\phi_1 p_0}{1 - \phi_2} = \frac{\phi_1}{1 - \phi_2} = \frac{1,5}{1 + 0,75} \approx 0,8571$$

$$p_2 = \phi_1 p_1 + \phi_2 p_0 = \phi_1 p_1 + \phi_2 \approx 1,5 \cdot 0,8571 - 0,75 \approx 0,5357$$

5. a) $\pi_1 = 0,6 ; \pi_2 = \pi_3 = 0$

$$\psi_s = \phi^s \therefore$$

$$\psi_1 = 0,6^1 = 0,6 ; \psi_2 = 0,6^2 = 0,36 ; \psi_3 = 0,6^3 = 0,216$$

b) $\psi_1 = 0,8 ; \psi_2 = \psi_3 = 0$

$$\pi_1 = -(-0,8) = 0,8 ; \pi_2 = -(-0,8)^2 = -0,64 ; \pi_3 = -(-0,8)^3 = 0,512$$

$$c) \quad \pi_1 = 0,3; \quad \pi_2 = -0,6; \quad \pi_3 = 0$$

$$\psi_1 = 0,3; \quad \psi_2 = 0,3^2 - 0,6 = -0,51; \quad \psi_3 = 0,3^2 - 0,6^2 = -0,333$$

$$d) \quad \pi_1 = 1,5; \quad \pi_2 = -0,75; \quad \pi_3 = 0$$

$$\psi_1 = 1,5; \quad \psi_2 = 1,5^2 - 0,75 = 1,5; \quad \psi_3 = 1,5 \cdot 0,75 = 1,125$$

$$e) \quad \psi_1 = 0,3; \quad \psi_2 = 0,6; \quad \psi_3 = 0$$

$$\pi_1 = -(-0,3) = 0,3; \quad \pi_2 = -(-0,3)^2 + 0,6 = 0,51$$

$$\pi_3 = -(-0,3)^3 - (-0,6)^2 = -0,333$$