·AP·I· PRIMEIRA CHAMADA · SOLUÇÕES·

01 $w(x,y,z) = x^2y - y^2x + y^2z - z^2y + z^2x - x^2z$;

$$\frac{\partial w}{\partial x} = 2xy - y^2 + z^2 - 2xz;$$
$$\frac{\partial w}{\partial y} = x^2 - 2yx + 2yz - z^2;$$
$$\frac{\partial w}{\partial z} = y^2 - 2zy + 2xz - x^2;$$

$\longrightarrow \dfrac{\partial w}{\partial x} + \dfrac{\partial w}{\partial y} + \dfrac{\partial w}{\partial z} = 0$; Sim, satisfaz!

02 $u(x,y) = (\operatorname{senh} x).(\operatorname{sen} y) \therefore u_x = (\cosh x).\operatorname{sen} y \therefore u_{xx} = \cancel{(\operatorname{senh} x)}.\operatorname{sen} y$

$u_y = (\operatorname{senh} x).(-\cos y) \therefore u_{yy} = \cancel{(\operatorname{senh} x)}.(-\operatorname{sen} y)$

$u_{xx} + u_{yy} = 0$; Sim, satisfaz!

03 $u = F(x,y)$; $x(r,s) = e^r\cos(s)$ e $y(r,s) = e^r\operatorname{sen}(s)$;

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial r} = \frac{\partial u}{\partial x}e^r\cos(s) + \frac{\partial u}{\partial y}e^r\operatorname{sen}(s) = e^r\left(\frac{\partial u}{\partial x}\cos(s) + \frac{\partial u}{\partial y}\operatorname{sen}(s)\right);$$
$$\frac{\partial u}{\partial s} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial u}{\partial y}\frac{\partial y}{\partial s} = \frac{\partial u}{\partial x}e^r(-\operatorname{sen}(s)) + \frac{\partial u}{\partial y}e^r\cos(s) = e^r\left(\frac{\partial u}{\partial x}(-\operatorname{sen}(s)) + \frac{\partial u}{\partial y}\cos(s)\right);$$
$$e^{-2r}\left[\left(\frac{\partial u}{\partial r}\right)^2 + \left(\frac{\partial u}{\partial s}\right)^2\right] = e^{-2r}\left[e^{2r}\left(\frac{\partial u}{\partial x}\cos(s) + \frac{\partial u}{\partial y}\operatorname{sen}(s)\right)^2 + e^{2r}\left(\frac{\partial u}{\partial y}\cos(s) - \frac{\partial u}{\partial x}\operatorname{sen}(s)\right)^2\right]$$
$$= \underbrace{e^{-2r}.e^{2r}}_{=1}\left[\left(\frac{\partial u}{\partial x}\cos(s) + \frac{\partial u}{\partial y}\operatorname{sen}(s)\right)^2 + \left(\frac{\partial u}{\partial y}\cos(s) - \frac{\partial u}{\partial x}\operatorname{sen}(s)\right)^2\right]$$
$$= \left(\frac{\partial u}{\partial x}\right)^2\cos^2(s) + \cancel{2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\cos(s)\operatorname{sen}(s)} + \left(\frac{\partial u}{\partial y}\right)^2\operatorname{sen}^2(s) + \left(\frac{\partial u}{\partial y}\right)^2\cos^2(s) - \cancel{2\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}\cos(s)\operatorname{sen}(s)} + \left(\frac{\partial u}{\partial x}\right)^2\operatorname{sen}^2(s)$$
$$= \left(\frac{\partial u}{\partial x}\right)^2\underbrace{(\cos^2(s) + \operatorname{sen}^2(s))}_{=1} + \left(\frac{\partial u}{\partial y}\right)^2\underbrace{(\operatorname{sen}^2(s) + \cos^2(s))}_{=1} = \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2;$$

04 $\nabla P(x,y,z) = P_x i + P_y j + P_z k = (10x - 3y + yz)i + (-3x + xz)j + (xy)k$;

Logo, $\nabla P(1,1,1) = 8i - 2j + k$; sabemos que a direção e o sentido da taxa de variação máxima são as do $\nabla P(1,1,1)$; ou seja $u = \dfrac{1}{\|\nabla P(1,1,1)\|}\nabla P(1,1,1)$;

Como $\|\nabla P(1,1,1)\| = \sqrt{64+4+1} = \sqrt{69}$, temos: $u = \dfrac{1}{\sqrt{69}}(8i - 2j + k)$;

05 Definamos $F(x,y,z) = x^3 + y^3 + z^3 - xyz$; observemos que $r(t_0) = (x_0, y_0, z_0)$;

E como $t_0 = 2$, $r(2) = \left(\frac{8}{4} - 2\right)i + (2-3)j + \cos 0\, k = 0i - j + k$; Logo $(x_0, y_0, z_0) = (0,-1,1)$;

$\nabla F(x,y,z) = F_x i + F_y j + F_z k = (3x^2 - yz)i + (3y^2 - xz)j + (3z^2 - xy)k \therefore$

$\nabla F(0,-1,1) = i + 3j + 3k$;

E como $r(t) = \left(\frac{t^3}{4} - 2\right)i + \left(\frac{4}{t} - 3\right)j + \cos(t-2)k \therefore r'(t) = \frac{3t^2}{4}i - \frac{4}{t^2}j - \operatorname{sen}(t-2)k$

Logo, $r'(2) = 3i - j - 0k = 3i - j$; e $r'(2).\nabla F(0,-1,1) = (3i - j).(i + 3j + 3k) = 3 - 3 = 0$;

Portanto, a curva e a superfície são tangentes quando $t_0 = 2$;

·API·SEGUNDA CHAMADA·SOLUÇÕES·

01 $\dfrac{\partial(e^{xy}\operatorname{senh}z - z^2x+1)}{\partial x} = \dfrac{\partial(0)}{\partial x} \therefore \dfrac{\partial(e^{xy}\operatorname{senh}z)}{\partial x} - \dfrac{\partial(z^2x)}{\partial x} + \underbrace{\dfrac{\partial(1)}{\partial x}}_{=0} = 0;$

$\dfrac{\partial(e^{xy})}{\partial x}.\operatorname{senh}z + \dfrac{\partial(\operatorname{senh}z)}{\partial x}.e^{xy} - \left[\dfrac{\partial(z^2)}{\partial x}.x + \dfrac{\partial(x)}{\partial x}.z^2\right] = 0;$

$ye^{xy}\operatorname{senh}z + e^{xy}\cosh z\dfrac{\partial z}{\partial x} - 2xz\dfrac{\partial z}{\partial x} - z^2 = 0 \therefore \dfrac{\partial z}{\partial x} = \dfrac{z^2 - ye^{xy}\operatorname{senh}z}{e^{xy}\cosh z - 2xz}, \forall (x,y) / e^{xy}\cosh z - 2xz \neq 0;$

$\dfrac{\partial(e^{xy}\operatorname{senh}z - z^2x+1)}{\partial y} = \dfrac{\partial(0)}{\partial y} \therefore \dfrac{\partial(e^{xy}\operatorname{senh}z)}{\partial y} - \dfrac{\partial(z^2x)}{\partial y} + \underbrace{\dfrac{\partial(1)}{\partial y}}_{=0} = 0;$

$\dfrac{\partial(e^{xy})}{\partial y}.\operatorname{senh}z + \dfrac{\partial(\operatorname{senh}z)}{\partial y}.e^{xy} - x.\dfrac{\partial(z^2)}{\partial y} = 0;$

$xe^{xy}\operatorname{senh}z + e^{xy}\cosh z\dfrac{\partial z}{\partial y} - 2xz\dfrac{\partial z}{\partial y} = 0; \dfrac{\partial z}{\partial y} = \dfrac{-xe^{xy}\operatorname{senh}z}{e^{xy}\cosh z - 2xz}, \forall (x,y) / e^{xy}\cosh z - 2xz \neq 0;$

02 $u_x = e^{-t}.\left[\cos\left(\frac{x}{a}\right).\frac{1}{a} - \operatorname{sen}\left(\frac{x}{a}\right).\frac{1}{a}\right] \therefore u_{xx} = e^{-t}.\left[-\operatorname{sen}\left(\frac{x}{a}\right).\frac{1}{a^2} - \cos\left(\frac{x}{a}\right).\frac{1}{a^2}\right];$

$u_t = -e^{-t}.\left[\operatorname{sen}\left(\frac{x}{a}\right) + \cos\left(\frac{x}{a}\right)\right] = a^2.\left[e^{-t}\left(-\operatorname{sen}\left(\frac{x}{a}\right).\frac{1}{a^2} - \cos\left(\frac{x}{a}\right).\frac{1}{a^2}\right)\right] = a^2.u_{xx};$

Sim, satisfaz à equação de Fourier;

03 $u = f(x)$ e $x(r,s,t) = a.r + b.s + c.t$; Então, pela Regra da Cadeia, temos:

$\dfrac{\partial u}{\partial r} = \dfrac{du}{dx}.\dfrac{\partial x}{\partial r} = \dfrac{du}{dx}.a;$

$\dfrac{\partial u}{\partial s} = \dfrac{du}{dx}.\dfrac{\partial x}{\partial s} = \dfrac{du}{dx}.b;$

$\dfrac{\partial u}{\partial t} = \dfrac{du}{dx}.\dfrac{\partial x}{\partial t} = \dfrac{du}{dx}.c;$

$\Rightarrow \dfrac{\partial u}{\partial r} + \dfrac{\partial u}{\partial s} + \dfrac{\partial u}{\partial t} = \dfrac{du}{dx}.a + \dfrac{du}{dx}.b + \dfrac{du}{dx}.c = \dfrac{du}{dx}.(a+b+c);$

Sim, satisfaz;

04 $F(x,y,z) = \operatorname{sen}(xy+z); \nabla F(x,y,z) = F_x i + F_y j + F_z k = (y\cos(xy+z))i + (x\cos(xy+z))j + \cos(xy+z)k;$

Logo, $\nabla F(0,-1,\pi) = (-1)\cos\pi\, i + 0.\cos\pi\, j + \cos\pi\, k = i + 0j - k = i - k;$

Sabemos que a taxa de variação mínima ocorre na mesma direção, porém no sentido contrário ao do $\nabla F(0,-1,\pi)$; ou seja: $u = \dfrac{-1}{\|\nabla F(0,-1,\pi)\|}\nabla F(0,-1,\pi);$

Portanto, para $u = \dfrac{-1}{\sqrt{2}}(i-k) = \dfrac{-1}{\sqrt{2}}i + \dfrac{1}{\sqrt{2}}k;$

05 Temos, $r(t) = \ln t\, i + (t\ln t)j + tk$; Façamos $F(x,y,z) = xz^2 - yz + \cos xy$;

Como nosso ponto é $(0,0,1)$, ele é alcançado na curva $r(t)$, quando: $(\ln t, t\ln t, t) = (0,0,1)$, ou seja quando $t=1$;

E como $r'(t) = \frac{1}{t}i + (\ln t + 1)j + k$, $r'(1) = i + j + k$;

Por outro lado, $\nabla F(x,y,z) = F_x i + F_y j + F_z k$; Ou seja:

$\nabla F(x,y,z) = (z^2 - y\operatorname{sen}xy)i + (-z - x\operatorname{sen}xy)j + (2xz - y)k;$

E, $\nabla F(0,0,1) = (1-0)i + (-1-0)j + (0-0)k = i - j;$

Por fim, $r'(1).\nabla F(0,0,1) = (i+j+k).(i-j) = 1-1 = 0;$

Logo, a curva e a superfície dadas, são tangentes em $(0,0,1)$;