Q01. Verifique a validade da expressão **5.1.1**, isto é,

$$\sum_{i=1}^{n} \frac{(X_i-\mu)^2}{\sigma^2} = \sum_{i=1}^{n} \frac{\left(X_i-\bar{X}\right)^2}{\sigma^2} + n\,\frac{\left(\bar{X}-\mu\right)^2}{\sigma^2}$$

Solução: Basta provar que:

$$\sum_{i=1}^{n} (X_i-\mu)^2 = \sum_{i=1}^{n} \left(X_i-\bar{X}\right)^2 + n\,\left(\bar{X}-\mu\right)^2.$$

Note que:

$$X_i-\mu = X_i-\bar{X}+\bar{X}-\mu.$$

Elevando ao quadrado temos:

$$(X_i-\mu)^2 = \left(X_i-\bar{X}\right)^2 + \left(\bar{X}-\mu\right)^2 + 2\left(\bar{X}-\mu\right)\,\left(X_i-\bar{X}\right).$$

Aplicando somatório temos:

$$\sum_{i=1}^{n} (X_i-\mu)^2 = \sum_{i=1}^{n} \left(X_i-\bar{X}\right)^2 + \sum_{i=1}^{n} \left(\bar{X}-\mu\right)^2 + 2\left(\bar{X}-\mu\right)\,\sum_{i=1}^{n} \left(X_i-\bar{X}\right).$$

Mas

$$\sum_{i=1}^{n} \left(\bar{X}-\mu\right)^2 = n\,\left(\bar{X}-\mu\right)^2,$$

e

$$\sum_{i=1}^{n} \left(X_i-\bar{X}\right) = \sum_{i=1}^{n} X_i - n\bar{X} = n\bar{X}-n\bar{X} = 0.$$

Assim

$$\sum_{i=1}^{n} (X_i-\mu)^2 = \sum_{i=1}^{n} \left(X_i-\bar{X}\right)^2 + n\,\left(\bar{X}-\mu\right)^2.$$

Dividindo por σ^2 temos o resultado:

$$\sum_{i=1}^{n} \frac{(X_i-\mu)^2}{\sigma^2} = \sum_{i=1}^{n} \frac{\left(X_i-\bar{X}\right)^2}{\sigma^2} + n\,\frac{\left(\bar{X}-\mu\right)^2}{\sigma^2}.$$