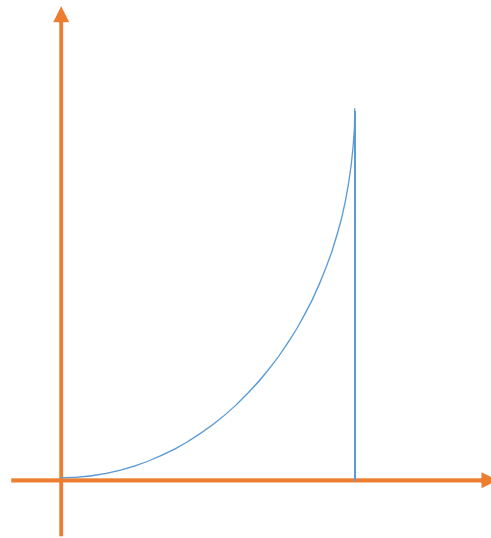


Exercício 7

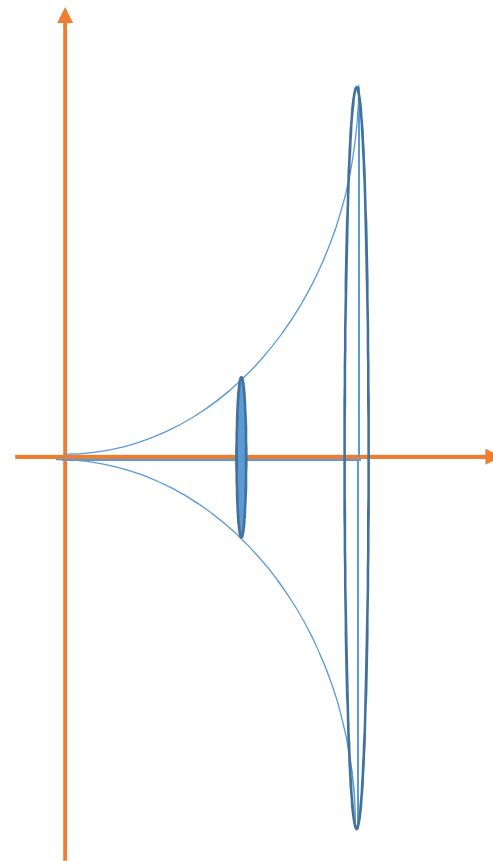
Calcule o volume do sólido gerado pela rotação da região limitada pela curva $y = x^3$ o eixo x e a reta $x = 2$ em torno do(a):

- a) Eixo x
- b) Reta $x = 4$
- c) Reta $y = 8$
- d) Eixo y

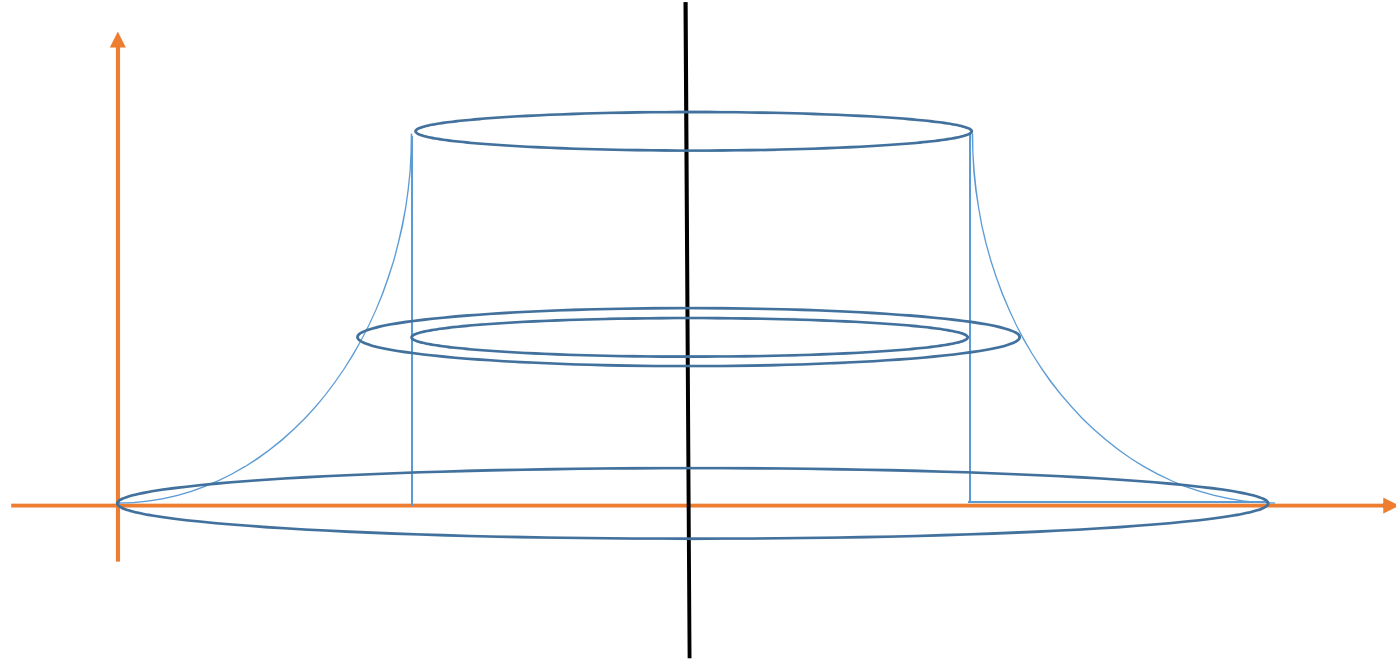


a)

$$V = \pi \int_0^2 (x^3)^2 dx = \pi \int_0^2 x^6 dx = \pi \frac{x^7}{7} \Big|_0^2 = \frac{2^7}{7} \pi = \frac{128}{7} \pi$$

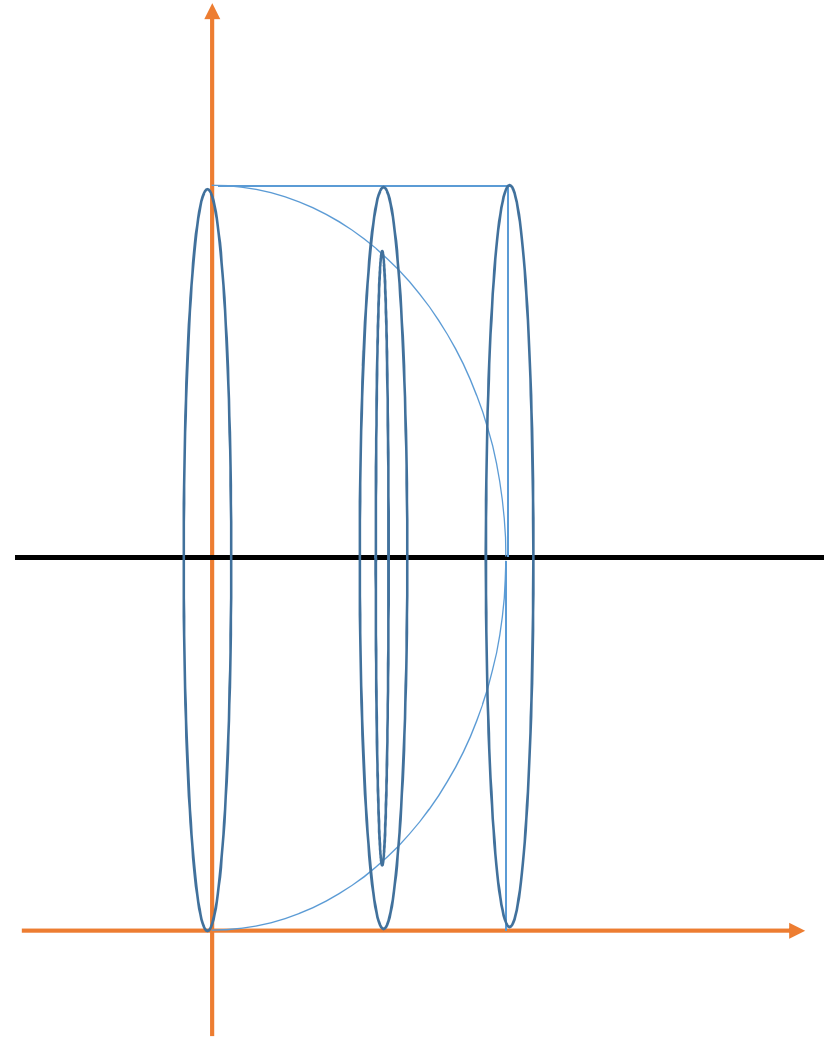


b)



$$\begin{aligned} V &= \pi \int_0^8 \left[\left(4 - y^{\frac{1}{3}} \right)^2 - 4 \right] dy = \pi \int_0^8 \left(16 - 8y^{\frac{1}{3}} + y^{\frac{2}{3}} - 4 \right) dy = \\ &= \pi \left(12y - 8 \cdot \frac{3}{4} \cdot y^{\frac{4}{3}} + \frac{3}{5} \cdot y^{\frac{5}{3}} \right) \Big|_0^8 = \pi \left(12 \cdot 8 - 6 \cdot 8^{\frac{4}{3}} + \frac{3}{5} \cdot 8^{\frac{5}{3}} \right) = \\ &= \left(96 - 96 + \frac{3}{5} \cdot 2^5 \right) \pi = \frac{96}{5} \pi \end{aligned}$$

$$\begin{aligned}
 \text{c) } V &= \pi \int_0^2 [64 - (8 - x^3)^2] dx = \\
 &\pi \int_0^2 (64 - 64 + 16x^3 - x^6) dx = \\
 &\pi \int_0^2 (16x^3 - x^6) dx = \\
 &\pi \left(16 \frac{x^4}{4} - \frac{x^7}{7} \right) \Big|_0^2 = \\
 &\pi \left(4 \cdot 2^4 - \frac{2^7}{7} \right) = \pi \left(64 - \frac{128}{7} \right) = \frac{320}{7} \pi
 \end{aligned}$$



d)

Solução 1

$$V = 2\pi \int_0^2 x \cdot x^3 dx =$$

$$2\pi \int_0^2 x^4 dx = 2\pi \frac{x^5}{5} \Big|_0^2 = \frac{64}{5} \pi$$

Solução 2

$$V = \pi \int_0^8 \left[4 - \left(y^{\frac{1}{3}} \right)^2 \right] dy = \pi \int_0^8 \left(4 - y^{\frac{2}{3}} \right) dy = \pi \left(4y - \frac{3}{5} y^{\frac{5}{3}} \right) \Big|_0^8$$

$$\pi \left(32 - \frac{96}{5} \right) = \frac{64}{5} \pi$$

