Mathematical Model
Starling Murmuration

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1 Introduction

A murmuration of starlings is an amazing sight, a swooping mass of thousands of birds whirling in the sky above. It's basically a mass aerial stunt - thousands of birds all swooping and diving in unison. It's completely breathtaking to witness. We think that starlings do it for many reasons. Grouping together offers safety in numbers – predators such as peregrine falcons find it hard to target one bird in the middle of a hypnotising flock of thousands.

Our Project would be to simulate this fascinating phenomenon. Our objective will be to measure from a realistic simulation the average energy spend by each bird, the angular momentum and the force that each bird has to withstand in a typical flight ritual.

2 Mathematical Modelling

Each bird would have some characteristics and we will computationally simulate the phenomenon by modelling each bird as an independent agent communicating and cooperating with other neighbouring agents. Briefly each bird has a position and a heading. Rules for each boid are to

$R1$ - Steer towards the average position of neighbours.

$R2$ - Steer towards average heading of the neighbours.

$R3$ - Steer to avoid crowding neighbours.

2.1 Characteristics of a bird

Each bird would have some characteristics namely,

1. Mass : Mass of a bird would remain constant.

2. Position : Position of the bird would depend on velocity and can be calculated by

$$x_{new} = x_{old} + v_x \delta t$$

Similarly for y and z directions.

3. Direction Vectors : It is the vector in the direction the bird moves towarsds.

4. Neighbours : Each bird would have a set of neighbours within a certain radius R.

5. Speed : Speed of the bird changes depending on various factors because of the three rules stated above.

2.2 How to decide changes in direction ?

Three rules mentioned above causes change in direction and we call them $F1$, $F2$, $F3$ respectively.

1. Each bird at (x_i, y_i, z_i) would want to move towards COM (x_c, y_c, z_c) of it's neighbours and therefore it would want to move towards direction $(x_c - x_i, y_c - y_i, z_c - z_i)$

2. Each bird would want to move towards direction where it's neighbours are moving and therefore it's direction would be average direction of all it's neighbours.

3. Since if two birds get very close to each other they would want to seperate out and therefore change in direction due to this would be calculated in the following fashion. Let there be m birds inside a small radius, bird i would want to go away from the rest birds. We calculate, distance vector

$$\vec{d_{ik}} = (x_i - x_k, y_i - y_k, z_i - z_k)$$
$$weight_{ik} = radius - |d_{ik}|$$
$$newdirection = \frac{\Sigma weight_{ik} * d_{ik}}{\Sigma weight_{ik}}$$

Now we need to decide weights for each rule, we use some heuristics such as if the birds are very close to each other then we need to give higher weight to seperate them out, if not, that is number of birds in some predefined sphere are zero then we need not give weight to that factor. Therefore we chose values of $w_3 = 0.8; w_1, w_2 = 0.1$ in the former case and $w_3 = 0; w_1 + w_2 = 1.0$ in the latter, where the weights are calculated in the following manner

$$w_1 = \frac{d_{com}}{R}$$

$$w_2 = 1 - \frac{d_{com}}{R}$$

2.3 How to vary speed?

We can fix some range to the possible values of speed, from $|v|_{min}$ to $|v|_{max}$. There are two possible scenarios where speed needs to be increased

1. When the birds get too close. In this case $F3$ would have large value.
2. When the birds get too far and factor $F1$ would have a large value in this case.

Therefore we can calculate speed using the following approach

$$|v|_{new} = |v|_{min} + (|v|_{min} - |v|_{max}) * [F1 + F3]$$