

## PART - 2 [Regression].

• Regression models (linear & non linear) are used for predicting a real value (like salary).

- If indep. var. is time, we are forecasting future values otherwise we are predicting present but unknown values.

Regressions:

- (1) Simple Linear R.
- (2) Multiple Linear R.
- (3) Polynomial R.
- (4) Support vector R. (SVR)
- (5) Decision Tree R.
- (6) Random Forest R.

### (1) Simple Linear R.

$$\begin{array}{c} \text{Dependent variable} \rightarrow \hat{y} = \underbrace{b_0}_{\substack{\uparrow \\ \text{y-intercept (constant)}}} + \underbrace{b_1}_{\substack{\downarrow \text{ slope-coefficient.} \\ \text{indep. variable/predictor}}} X_1 \end{array}$$



- $y_i$  is the expected output
- $\hat{y}_i$  is the calculated output

There ~~are~~ is a difference all or most of time. Nothing can be perfect.

$$\text{residual} = y_i - \hat{y}_i$$

$$\hat{y} = b_0 + b_1 x_1$$

Best eq<sup>n</sup> is the one such that ( $b_1$  &  $b_0$  are such that)  $\sum (y_i - \hat{y}_i)^2$  is minimized.

## ② Multiple Linear R.

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + \dots + b_n x_n$$

- Not all features are necessary. Some have more impact than we -  
5 methods of building models? [decide by p-value of each feature]

- ① All in
- ② Backward elimination
- ③ Forward selection
- ④ Bi-Directional elimination
- ⑤ All possible Models

- We will be using backward elimination here in all models as it is the fastest.

- In multiple linear regression there is no need to apply feature scaling. Since the coefficients (weights) are multiplied by features. They get compensated.

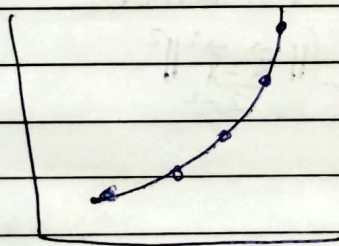
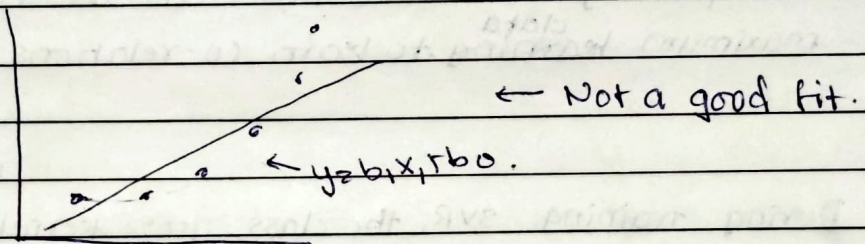
- Used with same LinearRegression library & identifies automatically as multiple Regression.



### ③ Polynomial Regression. (Linear)

$$y = b_0 + b_1x_1 + b_2x_1^2 + \dots + b_nx_1^n$$

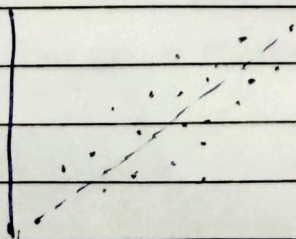
↓  
Same variable with powers.



- Since target is increasing exponentially it'll be better to use Poly. Regression

- Again used with same class.

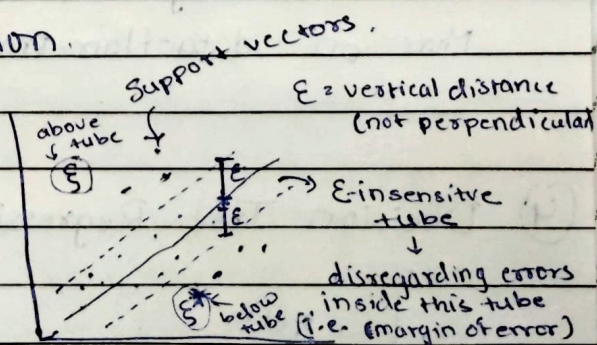
### ④ Support Vector Regression



(a) Linear

ordinary least square method.

$$\sum (y - \hat{y})^2 \rightarrow \text{minimize}$$



(b) Support Vector

$$\frac{1}{2} \|w\|^2 + c \sum_{i=1}^m (\xi_i + \xi_i^*) \rightarrow \min$$



- Slightly more advanced since we would have to play a lot with feature scaling.

we learn - (1) feature scaling transformation

(2) Inverse transformation (to go back to original scaling).

- Need to apply feature scaling since we don't have coefficients multiplying to features in SVR.

- NO splitting in train dataset as we want to leverage maximum <sup>data</sup> learning to learn co-relations in ~~pos~~ <sup>dataset</sup>.

functions

- During training SVR, the class uses kernels ~~too~~ <sup>as</sup> as args.

One of the kernel, Gaussian RBF kernel

$$K(\vec{x}, \vec{x}') = e^{-\frac{\|\vec{x} - \vec{x}'\|^2}{2\sigma^2}}$$

Some more kernel functions

(1) Polynomial Kernel

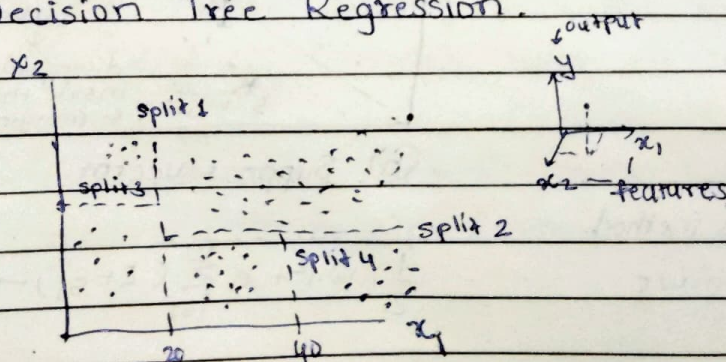
(2) Gaussian K. & Gaussian RBF K. <sup>(Recommended)</sup> for SVR

(3) Laplace RBF K.

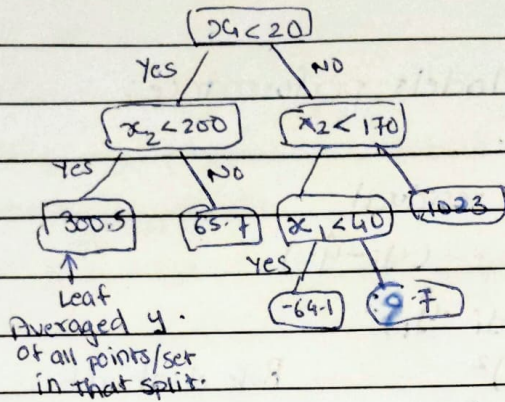
(4) Sigmoid K. e.t.c.

More on [data-flair.training/blogs/svm-kernel-functions/](#)

#### (4) Decision Tree Regression.







- The decision Tree works on information gain & Entropy (have mathematical formulas) & decide splits where information gain is maximum.
- As for the regression task, the averaged output is taken from tree at leaf. (averaged o/p of all datapoints inside split)
- We don't need to scale values here, since predictions are resulting from random splits of data & not some eq<sup>ns</sup>. So there is no problem of a feature being too overwhelming with large values.
- Recommended only for High dimension dataset (more features).

### ⑤ Random Forest Regression. (Also recommended for high dimension data i.e. mostly feature set.)

- Random forest is one of method of ensemble learning. One other method is gradient boosting.
- Ensemble learning is when we take <sup>multiple algs. or</sup> same algo. multiple times & put it together to make better model than the single original model.

- STEPS:
- ① Pick  $K$  random data points from training set.
  - ② Build Decision Tree  $k$  associated with these  $K$  data pts.
  - ③ Choose  $N$  trees to build & repeat Step 1 & 2.
  - ④ For predict, Predict  $y$  value from  $N$  models & take avg. of all  $y$ .



## # Evaluating Regression Models performance:

- (USING R SQUARED).

$SS_{res}$   $\Rightarrow$  sum of squared residual

$$\text{residual/loss} = (y_i - \hat{y}_i)$$

$$\text{sq. loss} = (y_i - \hat{y}_i)^2$$

$$SS_{res} = \sum (y_i - \hat{y}_i)^2$$

$$SS_{tot} = \sum (y_i - y_{avg})^2 \quad \text{from train set data points.}$$

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

Rule of thumb:

$$R^2 = 1.0 = \text{Perfect fit (suspicious)}$$

$$\sim 0.9 = \text{very good}$$

$$< 0.7 = \text{Not great}$$

$$< 0.4 = \text{Terrible}$$

$$< 0 = \text{Model makes no sense for given data.}$$

~~$R^2$~~   $SS_{res}$   $\Rightarrow$  lesser the  $SS_{res}$  the better the model is.

- (Adjusted R Squared).

Problem:

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 \quad \leftarrow (+ b_3 x_3)$$

we add a 3rd feature or more extra features.

- ~~But~~ The  $SS_{tot}$  doesn't change ( $SS_{tot} = \sum (y_i - y_{avg})^2$ )
- but  $SS_{res}$  changes & will decrease or stay same & never increase.

$\therefore$  [This is coz of ordinary least sq. method:  $SS_{res} \rightarrow (\min)$ ]

$$\text{Adj } R^2 = 1 - (1 - R^2) \times \frac{n-1}{n-k-1}$$

- $k$  = no. of independent variables

- $n$  = sample size.

look for  $k$ .  
if it increases, ratio increases & adj  $R$  decreases (subtraction)

- For selecting Best Model, simply train all model and compare their  $r^2$  scores.



## SUMMARY

### ① Simple Regression

- Simple regression on single feature minimizing SSres

\* IMPORT: from sklearn.linear\_model import LinearRegression

### ② Multiple Regression

- Multiple features

- May need to preprocess, for e.g. encode data

\* IMPORT: from sklearn.linear\_model import LinearRegression

- Can't plot graph since its multidimensional.

### ③ Polynomial Regression

- Output scales polynomially with input

- Need to decide degree of polynomial.

$$\hat{y} = b_0 + b_1x_1 + b_2x_2^2 + b_3x_3^3 + \dots + b_nx_n^n$$

- If  $n$  too large it'll try to overfit through all pts.

\* IMPORTS:

- for model we need to create more features of  $x$ , for each feature of  $x$

from sklearn.preprocessing import PolynomialFeatures

poly-reg = PolynomialFeatures(degree = 4)

x-poly = poly-reg.fit\_transform(x)

from sklearn.linear\_model import LinearRegression

### ④ Support Vector

- Need to rescale all features & dependent variable in SVR.

\* IMPORT: from sklearn.svm import SVR

reg = SVR(kernel = 'rbf')

- Uses Kernel functions

- Need transform scale to train & inverse transform output scale for proper output.

- Rescale with their scaler for  $x$  & diff. scaler for  $y$ .

## ⑤ Decision Tree.

- works on entropy & splits on highest.

\* IMPORTS: from sklearn.tree import DecisionTreeRegressor  
dt = DecisionTreeRegressor(random\_state=0)

## ⑥ Random Forest

- ensemble method

\* IMPORTS: from sklearn.ensemble import RandomForestRegressor  
rf = RandomForestRegressor(n\_estimators=10, random\_state=0)

## Evaluating Performance:

- from sklearn.metrics import r2\_score