$$a > 1 \quad a^n > 1 \quad \forall n \in \mathbb{Q}, n > 0 \qquad p, q \in \mathbb{N} \ \wedge \ q \neq 0 \ : n = \frac{p}{q} \tag{1}$$

$$a^n = \sqrt[q]{a^p} \geq \sqrt[q]{a} > \sqrt[q]{1} = 1 \tag{2}$$

Questo perchè $a^n > 1 \wedge a > 1 \ \Rightarrow \ 1,000000000001^n > 1$
Dalle proprietà delle potenze ricordiamo che:

$$x < y \ \Rightarrow \ a^x < a^y \ \Rightarrow \ \frac{1}{a^x} > \frac{1}{x^y} \tag{3}$$

$$x - y < 0 \ \overset{\cdot(-1)}{\Longrightarrow} \ y - x > 0 \tag{4}$$

$$a^x - a^y < 0 \ \overset{\cdot(-1)}{\Longrightarrow} \ a^y - a^x > 0 \tag{5}$$

$$\Downarrow \tag{6}$$

$$\underbrace{\underbrace{a^x}_{>0}\underbrace{(a^{y-x} - 1)}_{a^n > 1}}_{>0} > 0 \tag{7}$$