Ripasso

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DIMOSTRAZIONE 1 $(x^n < y)$.

$$x^n < y \iff \exists \epsilon \in \mathbb{R}, \epsilon > 0 \,:\, (x+\epsilon)^n < y \tag{1}$$

Sia $\epsilon \in]0,1[$

$$(x+\epsilon)^n = ((x+\epsilon)^n - x^n) + x^n = ((\not{x}+\epsilon)-\not{x})((x+\epsilon)^{n-1} + \ldots + x^{n-1}) + x^n \tag{2}$$

$$\Downarrow \tag{3}$$

$$\epsilon((x+\epsilon)^{n-1} + \cdots + x^{n-1}) + x^n \leq \epsilon \cdot n \cdot (x+1)^{n-1} + x^n \tag{4}$$

$$\Downarrow \tag{5}$$

$$\epsilon n(x+1)^{n-1} + x^n < y \iff \epsilon < \frac{y-x^n}{n(x+1)^{n-1}} \stackrel{def.}{:=} \epsilon > 0 \tag{6}$$

$$0 \,<\, \epsilon \,<\, \frac{y-x^n}{n(x+1)^{n-1}} \tag{7}$$

Questo è valido per: $\epsilon \in \mathbb{R}$