

Ripasso

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DIMOSTRAZIONE 1 ($x^n < y$).

$$x^n < y \iff \exists \epsilon \in \mathbb{R}, \epsilon > 0 : (x + \epsilon)^n < y \quad (1)$$

Sia $\epsilon \in]0, 1[$

$$(x + \epsilon)^n = ((x + \epsilon)^n - x^n) + x^n = ((x + \epsilon) - x)((x + \epsilon)^{n-1} + \dots + x^{n-1}) + x^n \quad (2)$$

$$\Downarrow \quad (3)$$

$$\epsilon((x + \epsilon)^{n-1} + \dots + x^{n-1}) + x^n \leq \epsilon \cdot n \cdot (x + 1)^{n-1} + x^n \quad (4)$$

$$\Downarrow \quad (5)$$

$$\epsilon n(x + 1)^{n-1} + x^n < y \iff \epsilon < \frac{y - x^n}{n(x + 1)^{n-1}} \stackrel{def.}{=} \epsilon > 0 \quad (6)$$

$$0 < \epsilon < \frac{y - x^n}{n(x + 1)^{n-1}} \quad (7)$$

Questo è valido per: $\epsilon \in \mathbb{R}$