

Definizione 1 (Teorema dei 2 carabinieri):

Se $\underbrace{\{a_n\}, \{b_n\}}_{\text{convergono a } l}, \{c_n\}$

è ovvio che: $a_n \leq c_n \leq b_n \implies c_n \text{ converge a } l$ (1)

DIMOSTRAZIONE 1.

$$\forall \epsilon > 0, \exists \overline{n_1}, \overline{n_2} \in \mathbb{N} : \quad (2)$$

\Downarrow

$$l - \epsilon < a_n < l + \epsilon \quad \& \quad l - \epsilon < b_n < l + \epsilon \quad (3)$$

se $n > \max\{\overline{n_1}, \overline{n_2}\}$

\Downarrow

$$l - \epsilon < a_n \leq c_n \leq b_n < l + \epsilon \quad \forall n > \overline{n} \quad (4)$$

$$\underbrace{l - \epsilon < c_n < l + \epsilon}_{|c_n - l| < \epsilon} \implies \lim_{n \rightarrow +\infty} c_n = l \quad (5)$$