Dimostrazione 1 (Limiti).
Se $\{a_n\}_{n\in\mathbb{N}}$ converge $l \in \mathbb{R} \quad \Longrightarrow \quad \{a_{k_n}\}_{k_{n\in\mathbb{N}}}$ converge $l \in \mathbb{R}$

$$\Downarrow$$

Si ha che:

$$\forall \epsilon > 0 \; \exists \overline{n} \in \mathbb{N} \; : \; n > \overline{n} \implies |a_n - l| < \epsilon \tag{1}$$

$$\forall \epsilon > 0 \; \exists \overline{n} \in \mathbb{N} \; : \; n > \overline{n} \implies |a_{k_n} - l| < \epsilon \tag{2}$$

$$\lim_{n\to\infty} a_{k_n} \; = \; l \tag{3}$$

Esempio 1.

$$\lim_{n\to+\infty} \frac{1}{n} = 0 \qquad \& \qquad k = 2, \lim_{k_n\to+\infty} \frac{1}{k_n} = 0 \tag{4}$$