Ripasso

Corso Informatica UNIPD A.A 2021/2022

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1 PROPRIETA' POTENZE

DIMOSTRAZIONE 1 (IPOTESI: $P(n)$ è vera).

$$P(n) \;=\; (a+b)^n \;=\; \sum_{k=0}^{n} \tbinom{k}{n}\; a^k\; b^{n-k} \tag{1}$$

Per il principio di induzione dovremmo essere capaci di dimostrare anche: $P(n+1)$ come vera.

$$P(n+1) \;=\; (a+b)^{n+1} \;=\; \sum_{k=0}^{n+1} \tbinom{k}{n+1}\; a^k\; b^{n+1-k} \tag{2}$$

$$(a+b)^{n+1} \;=\; (a+b)(a+b)^n \;\Rightarrow\; (a+b)\sum_{k=0}^{n} \tbinom{k}{n}\; a^k\; b^{n-k} \tag{3}$$

Ricordiamo la proprietà distributiva: $(a+b)c \;=\; ac+bc$

$$\sum_{k=0}^{n} \tbinom{k}{n}\; a^{k+1}\; b^{n-k} \;+\; \sum_{k=0}^{n} \tbinom{k}{n}\; a^k\; b^{n+1-k} \tag{4}$$

$$\Downarrow \tag{5}$$

$$\sum_{k=1}^{n+1} \tbinom{k-1}{n}\; a^k\; b^{n+1-k} \;+\; \sum_{k=0}^{n} \tbinom{k}{n}\; a^k\; b^{n+1-k} \;+\tbinom{0}{n}\; a^0\; b^{n+1-0} \tag{6}$$

$$\Downarrow \tag{7}$$

$$\sum_{k=0}^{n+1} \left[\tbinom{k-1}{n} + \tbinom{k}{n}\right]\; a^k\; b^{n+1-k} \tag{8}$$

$$\Downarrow \tag{9}$$

$$\frac{n!}{(k-1)!(n-k+1)!} + \frac{n!}{k!(n-k)!} = \frac{\underline{n!}}{(\underline{k-1)!}(n-k+1)(\underline{n-k)!}} + \frac{\underline{n!}}{k\underline{(k-1)!}\ \underline{(n-k)!}} \tag{10}$$

$$\Downarrow \tag{11}$$

$$\frac{n!}{(k-1)!(n-k)!} \left(\frac{1}{(n-k+1)} + \frac{1}{k} \right) \tag{12}$$

$$\Downarrow \tag{13}$$

$$\frac{n!}{(k-1)!(n-k)!} \cdot \frac{\cancel{k} + n - \cancel{k} + 1}{k(n-k+1)} \tag{14}$$

$$\Downarrow \tag{15}$$

$$\overbrace{\frac{n!}{(k-1)!(n-k)!} \cdot \frac{n+1}{k(n-k+1)}} = \frac{(n+1!)}{k!((n+1)-k)!} \tag{16}$$

L'ultima semplificazione è conseguenza di: $(n+1)! = (n+1) \cdot n!$ e di $(n-1)! \cdot n = n!$

In conclusione abbiamo che:

$$\sum_{k=0}^{n+1} \left[\tbinom{k-1}{n} + \tbinom{k}{n}\right] a^k\ b^{n+1-k} = \sum_{k=0}^{n+1} \tbinom{k}{n+1}\ a^k\ b^{n+1-k} = (a+b)^{n+1} \tag{17}$$