$$\lim_{x\to -1} \frac{x+1}{\underbrace{\sqrt[3]{x}+1}_{\to 0}}^{\to 0} \cdot \frac{(\sqrt[3]{x})^2-\sqrt[3]{x}+1}{\underbrace{(\sqrt[3]{x})^2-\sqrt[3]{x}+1}} = (*)$$

$\sqrt[3]{-1} = -1$

$$a^3+b^3 = \underbrace{(a+b)}\underbrace{(a^2-ab+b^2)} \quad \cdot \quad a=\sqrt[3]{x}, \; b=1$$

$$a^2-b^2=(a+b)(a-b)$$

$$(*) = \lim_{x\to -1} \frac{\cancel{(x+1)}((\sqrt[3]{x})^2-\sqrt[3]{x}+1)}{\cancel{(x+1)}} = (\sqrt[3]{-1})^2-\sqrt[3]{-1}+1 =$$

$$= (-1)^2-(-1)+1 = 3$$

$$\cdot \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} \cdot \frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}} =$$

$$= \lim_{x\to 0} \frac{1+x-(1-x)}{x\cdot(\sqrt{1+x}+\sqrt{1-x})} = \lim_{x\to 0} \frac{1+x-1+x}{x(\sqrt{1+x}+\sqrt{1-x})} =$$

$$= \lim_{x\to 0} \frac{2\cancel{x}}{\cancel{x}(\sqrt{1+\underset{\to 0}{x}}+\sqrt{1-\underset{\to 0}{x}})} = \frac{2}{1+1} = 1$$

$$(1+x)^a = 1+ax+\frac{a(a-1)}{2}x^2+o(x^2) \quad \Bigg| \quad \sqrt{1+x}=(1+x)^{\frac{1}{2}}$$

$$\cdot \lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim_{x\to 0} \frac{1+\frac{1}{2}x+o(x)-(1+\frac{1}{2}(-x)+o(x))}{x} =$$

$$= \lim_{x \to 0} \frac{\frac{1}{2}x + \frac{1}{2}x + o(x)}{x} = \lim_{x \to 0} \frac{\cancel{x}\left(1 + \frac{o(x)}{x}\right)^{\to 0}}{\cancel{x}} = 1$$

$$\lim_{x \to 0} \frac{e^{1-\cos(x)} - e^{\sin(x)}}{\log(1+3x)} = (*)$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)$$

$$\sin(x) = x - \frac{x^3}{6} + o(x^3)$$

$$\log(1+x) = x - \frac{x^2}{2} + o(x^2) \quad \Big| \log(1+3x) = 3x - \frac{(3x)^2}{2} + o(x^2)$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

$$(*) = \lim_{x \to 0} \frac{e^{1-(1-\frac{x^2}{2}+o(x^2))} - e^{x+o(x)}}{3x + o(x)} =$$

$$= \lim_{x \to 0} \frac{e^{\frac{x^2}{2}+o(x^2)} - e^{x+o(x)}}{3x + o(x)} =$$

$$= \lim_{x \to 0} \frac{1 + \frac{x^2}{2} + o(x^2) - (1 + x + o(x))}{3x + o(x)} =$$

$$= \lim_{x \to 0} \frac{\frac{x^2}{2} + o(x^2) - x + o(x)}{3x + o(x)} = \lim_{x \to 0} \frac{-x + o(x)}{3x + o(x)}$$

$$= \lim_{x\to 0} \frac{\cancel{x}\left(-1+\underbrace{\frac{o(x)}{x}}_{\to 0}\right)}{\cancel{x}\left(3+\underbrace{\frac{o(x)}{x}}_{\to 0}\right)} = -\frac{1}{3}$$

$x^n \in o(x^m)$ se $n > m$ (per $x \to 0$)

$$3x^2 + o(x) = o(x)$$

$$f(x) \in o_0(g(x)) \iff \lim_{x\to 0} \frac{f(x)}{g(x)} = 0$$

$\sin(x) = x + o(x) = x + f(x)$ con $f(x) \in o(x)$

$$\lim_{x\to 0} \frac{o(x)}{x} = 0$$

$$\lim_{x\to 0} \frac{e^{\sin(x)} - \log(1+x) - 1}{x^2} =$$

$$= \lim_{x\to 0} \frac{e^{x+o(x)} - (x+o(x)) - 1}{x^2} =$$

$$= \lim_{x\to 0} \frac{1 + x + o(x) - x + o(x) - 1}{x^2} = \lim_{x\to 0} \frac{o(x)}{x^2} \quad ?!$$

$$\boxed{\begin{array}{l} o(x) + o(x) \in o(x) \\ o(x) - o(x) \in o(x) \end{array}}$$

$$\lim_{x\to 0} \frac{e^{x+o(x)} - \left(x - \frac{x^2}{2} + o(x^2)\right) - 1}{x^2} = (\star)$$

$$= \lim_{x\to 0} \frac{1 + x + o(x) - x + \frac{x^2}{2} + o(x^2) - 1}{x^2} =$$

$$= \lim_{x\to 0} \frac{o(x)}{x^2} \quad ?!$$

$$(\star) = \lim_{x\to 0} \frac{1 + x + \frac{x^2}{2} + o(x^2) - \left(x - \frac{x^2}{2} + o(x^2)\right) - 1}{x^2} =$$

$$= \lim_{x\to 0} \frac{x^2 + o(x^2)}{x^2} = 1$$

$$x^2 + \overbrace{2x\,o(x)}^{\in o(x^2)} + o(x^2)$$

$$e^{x+o(x)} = 1 + (x+o(x)) + \frac{(x+o(x))^2}{2} + o\left((x+o(x))^2\right) =$$

$$e^{x} = 1 + x + \frac{x^2}{2} + o(x^2)$$

$$= 1 + x + o(x) + \frac{x^2}{2} + o(x^2) + o(x^2)$$

$$= 1 + x + o(x)$$

$$\sin(x) = x + o(x) \overset{?}{=} x + o(x^2)$$

$$\sin(x) = x - \frac{x^3}{6} + o(x^3)$$

$$\cos(x) = 1 - \frac{x^2}{2} + o(x^2) = 1 - \frac{x^2}{2} + o(x^3)$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)$$

- $x^2 \in o(x) \Rightarrow x^2 \to o(x)$

$$\lim_{x \to 0} \frac{x^2}{x} \overset{??}{=} 0$$

$$\Updownarrow$$

$$\lim_{x \to 0} \frac{x^2}{x} = \lim_{x \to 0} x = 0$$

- Sviluppare fino al terzo ordine

$$e^{x+x^2} \qquad \Big|\; e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + o(x^3)$$

$$e^{x+x^2} = 1 + x + x^2 + \frac{(x+x^2)^2}{2} + \frac{(x+x^2)^3}{6} + o((x+x^2)^3)$$

$$e^{x+x^2} = 1 + x + x^2 + o(x+x^2) = 1 + x + x^2 + o(x) = 1 + x + o(x)$$

$$e^{x+x^2} = 1 + x + x^2 + \frac{(x+x^2)^2}{2} + o((x+x^2)^2) =$$

$$= 1 + x + x^2 + \frac{x^2+2x^3+x^4}{2} + o(x^2+2x^3+x^4)$$

$$= 1 + x + x^2 + \frac{x^2}{2} + o(x^2)$$

$$\lim_{x\to-\infty} \underbrace{\sqrt{x^2+3x+2}}_{\to+\infty} \underbrace{+x}_{-\infty}$$

$$\bullet \lim_{x\to-\infty} x^2+3x+2 = \lim_{x\to-\infty} \overbrace{\boxed{x^2}}^{\to+\infty}\left(1+\underbrace{\boxed{\frac{3}{x}}}_{\to 0}+\underbrace{\boxed{\frac{2}{x^2}}}_{\to 0}\right) = +\infty$$

$$\bullet \lim_{x\to-\infty} \left(\sqrt{x^2+3x+2}+x\right)\cdot\frac{\sqrt{x^2+3x+2}-x}{\underbrace{\sqrt{x^2+3x+2}}_{\to+\infty}\underbrace{-x}_{+\infty}} =$$

$$= \lim_{x\to-\infty} \frac{x^2+3x+2-x^2}{\sqrt{x^2+3x+2}-x} = \lim_{x\to-\infty} \frac{3x+2}{\sqrt{x^2\left(1+\frac{3}{x}+\frac{2}{x^2}\right)}-x} =$$

$$= \lim_{x\to-\infty} \frac{3x+2}{|x|\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}-x} = \lim_{x\to-\infty} \frac{3x+2}{-x\sqrt{1+\frac{3}{x}+\frac{2}{x^2}}-x} =$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(3+\overbrace{\frac{2}{x}}^{\to 0}\right)}{\cancel{x}\left(-\sqrt{1+\underbrace{\frac{3}{x}}_{\to 0}+\underbrace{\frac{2}{x^2}}_{\to 0}}-1\right)} = \frac{3}{-1-1} = -\frac{3}{2}$$

$$|-3| = 3 = -(-3)$$

$|x| = -x$ se $x<0$!!

$|x| = x$ se $x>0$