$$S = \left\{ 1 + \sqrt{\frac{n}{n+1}} \;\middle|\; n \in \mathbb{N} \right\}$$

sup(S)

$$1 + \sqrt{\frac{n}{n+1}} \geq 1 \quad \Leftarrow \quad \sqrt{\frac{n}{n+1}} \geq 0$$

S é inferiormente limitato da 1

$$n = 0 : \quad 1 + \sqrt{\frac{0}{0+1}} = 1 \quad \Rightarrow 1 \in S$$

$$\Rightarrow \inf(S) = \min(S) = 1$$

$$\sup(S) = 2 \; ?? \quad \begin{cases} \forall a \in S : a \leq 2 & (1) \\ \forall \varepsilon > 0 \;\; \exists a \in S \text{ t.c. } 2 - \varepsilon < a & (2) \end{cases}$$

(1) $$1 + \sqrt{\frac{n}{n+1}} \leq 2 \iff \sqrt{\frac{n}{n+1}} \leq 2 - 1$$

eleviamo alla seconda

$$\underset{\substack{n \in \mathbb{N} \\ n \geq 0}}{\iff} \sqrt{\frac{n}{n+1}} \leq 1 \iff \frac{n}{n+1} \leq 1 \iff$$

$$\iff n \leq n + 1 \iff 0 \leq 1 \quad \text{sempre vera}$$

Quindi: $1 + \sqrt{\frac{n}{n+1}} \leq 2 \iff 2$ é maggiorante di S

(2) $$\forall \varepsilon > 0 \quad \exists n \in \mathbb{N} \text{ t.c. } 2 - \varepsilon < 1 + \sqrt{\frac{n}{n+1}}$$

$$\iff \sqrt{\frac{n}{n+1}} > 1 - \varepsilon$$

- Se $\varepsilon > 1$: $1-\varepsilon < 0 \Rightarrow \sqrt{\frac{n}{n+1}} > 1-\varepsilon \quad \forall n \in \mathbb{N}$
- Se $0 < \varepsilon \leq 1$: $1-\varepsilon \geq 0$

$$\sqrt{\frac{n}{n+1}} > 1-\varepsilon \iff \frac{n}{n+1} > (1-\varepsilon)^2$$

$$\iff n > (1-\varepsilon)^2 (n+1)$$

$$\iff n > (1-\varepsilon)^2 \cdot n + (1-\varepsilon)^2$$

$$\iff n - (1-\varepsilon)^2 n > (1-\varepsilon)^2$$

$$\iff [1-(1-\varepsilon)^2]\cdot n > (1-\varepsilon)^2 \quad (\star)$$

$$\boxed{1-(1-\varepsilon)^2 \geq 0} \iff (1-\varepsilon)^2 \leq 1 \iff$$

$$\iff {}^{-1}\,-1 \leq^{-1} 1-\varepsilon \leq 1\,{}^{-1} \iff -2 \leq -\varepsilon \leq 0 \overset{\cdot(-1)}{\iff}$$

$$\iff 2 \geq \varepsilon \geq 0 \iff \boxed{0 \leq \varepsilon \leq 2}$$

$\cup$/

Per noi $\boxed{0 < \varepsilon \leq 1} \Rightarrow 1-(1-\varepsilon)^2 \geq 0$

$$(\star) \quad n > \frac{(1-\varepsilon)^2}{1-(1-\varepsilon)^2}$$

$$n_\varepsilon = \left\lfloor \frac{(1-\varepsilon)^2}{1-(1-\varepsilon)^2} \right\rfloor + 1$$

$$\lfloor \cdot \rfloor : \mathbb{R} \to \mathbb{N}$$
$$x \to \lfloor x \rfloor$$

$\lfloor 1,5 \rfloor = 1$

$\lfloor 2,5 \rfloor = 2$

$\lfloor 2,8 \rfloor = 2$

$x^2 - 2x + 1 > 0 \iff (x-1)^2 > 0 \iff$

9° pdf esercizi

$\Delta = 2^2 - 4 \cdot 1 \cdot 1 = 0$ $\quad \forall x \in \mathbb{R}$ t.c. $x \neq 1$

B.2.9 Sia $f : [0,5[\to]-3,1]$ una funzione. Quali affermazioni sono vere:

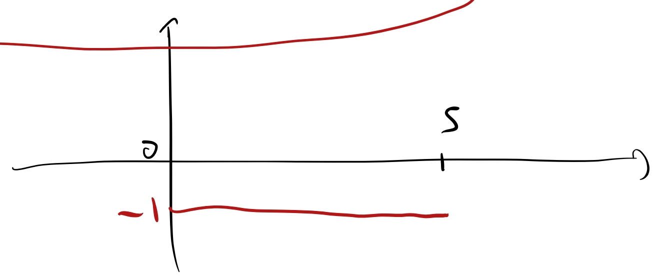
- $\inf_{x \in [0,5[} f(x) = -3$

$D = [0,5[$, $f(D) \cancel{=}]-3,1]$, $f(D) \subseteq]-3,1]$

$f(x) = -1 \cdot \mathbb{1}_{[0,5[}(x)$

$$\mathbb{1}_{[0,5[}(x) = \begin{cases} 1 & \text{se } x \in [0,5[\\ 0 & \text{altrimenti} \end{cases}$$

$f(x) = -1$



$\inf_{x \in [0,5[} f(x) = -1$

- $\sup_{x \in [0,5[} f(x) \geq 0$ $\quad f(x) = -1$ Controesempio!
- $\sup_{x \in [0,5[} f(x) = 1$

E se f fosse stata suriettiva?

f suriettiva $\Rightarrow$ $f(D) =]-3, 1]$

$\quad\quad\quad\quad\quad\quad\quad \| $

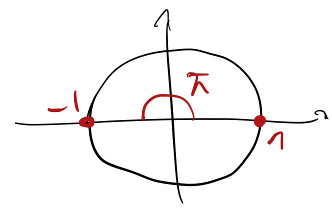
$\quad\quad\quad\quad\quad\quad \mathrm{Im}(f)$

$$E = \left\{ \frac{n}{n+1} \left(\cos(n\pi) - 1 \right) : n \in \mathbb{N} \right\}$$

Per me $0 \in \mathbb{N}$

$\inf(E)$, $\sup(E)$ e dire se sono max, min.

$\cos(n\pi) = ?$ $\quad n \in \mathbb{N}$



$n=0$: $\cos(0 \cdot \pi) = \cos(0) = 1$

$n=1$: $\cos(1 \cdot \pi) = \cos(\pi) = -1$

$n=2$: $\cos(2\pi) = \cos(2\pi) = 1$

$n=3$: $\cos(3\pi) = -1$

$$\cos(n\pi) = \begin{cases} 1 & \text{se } n \text{ è pari} \\ -1 & \text{se } n \text{ è dispari} \end{cases}$$

$$\boxed{\cos(n\pi) = (-1)^n} \quad !$$

$$E = \left\{ \frac{n}{n+1} \left((-1)^n - 1 \right) : n \in \mathbb{N} \right\}$$

- n pari: $n = 2K$, $\exists K \in \mathbb{N}$

$$\frac{2K}{2K+1} \left((-1)^{2K} - 1 \right) = \frac{2K}{2K+1} (1 - 1) = \frac{2K}{2K+1} \cdot 0 = 0$$

• n dispari : $n = 2K+1$, $\exists K \in \mathbb{N}$

$$\frac{2K+1}{2K+1+1}\left((-1)^{2K+1}-1\right) = \frac{2K+1}{2K+2}(-1-1) =$$

$$= \frac{2K+1}{2K+2}\cdot(-2) = -2\cdot\frac{2K+1}{2(K+1)} = -\frac{2K+1}{K+1}$$

$\sup(E) = 0 = \max(E)$

n pari : $\frac{n}{n+1}\left((-1)^n-1\right) = 0 \leq 0$ ✓

n dispari : $\frac{n}{n+1}\left((-1)^n-1\right) = -\frac{2K+1}{K+1} \leq 0$ ✓

0 é maggiorante e $0 \in E \Rightarrow 0 = \max(E)$

$\inf(E) = -2$??

- -2 é minorante ①
- ② $\forall \varepsilon > 0$ $\exists a \in E$ t.c. $a < -2+\varepsilon$

① Se n é pari : $-2 \leq 0$

Se n dispari : $-2 \leq -\frac{2K+1}{K+1}$

$\cdot(-1)$ $\qquad\qquad\qquad\qquad \cdot(K+1)$

$\Leftrightarrow 2 \geq \frac{2K+1}{K+1} \Leftrightarrow 2(K+1) \geq 2K+1$

$\Leftrightarrow 2K+2 \geq 2K+1 \Leftrightarrow 1 > 0$ sempre vera

$\Rightarrow -2$ é minorante

② $\forall \varepsilon > 0 \quad \exists K \in \mathbb{N}$ t.c. $-\frac{2K+1}{K+1} < -2+\varepsilon$

$\overset{\cdot(-1)}{\iff} \frac{2K+1}{K+1} > 2-\varepsilon \quad \overset{\cdot(K+1)}{\iff} 2K+1 > (2-\varepsilon)(K+1)$

$\iff 2K+1 > 2K+2-\varepsilon K-\varepsilon \iff$

$\iff 2K-2K+\varepsilon K > 2-\varepsilon-1 \iff$

$\iff \varepsilon \cdot K > 1-\varepsilon \iff K > \frac{1-\varepsilon}{\varepsilon}$

$\Rightarrow \inf(E) = -2$. È anche minimo?

$-\frac{2K+1}{K+1} = -2 \iff 2K+1 = 2(K+1)$

$\iff 2K+1 = 2K+2 \iff 0 = 1$ falso

$\Rightarrow -2$ non è minimo