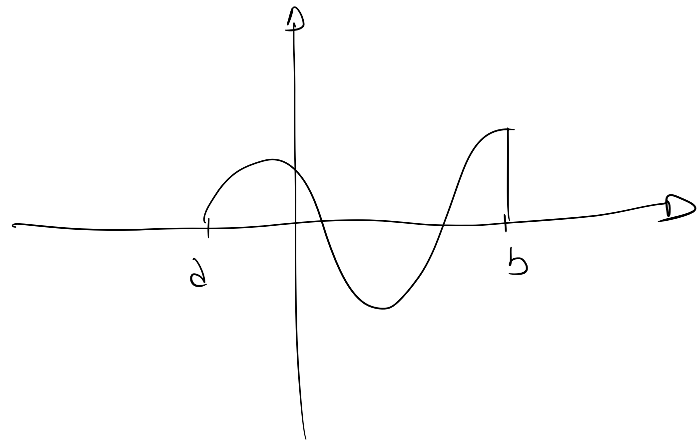
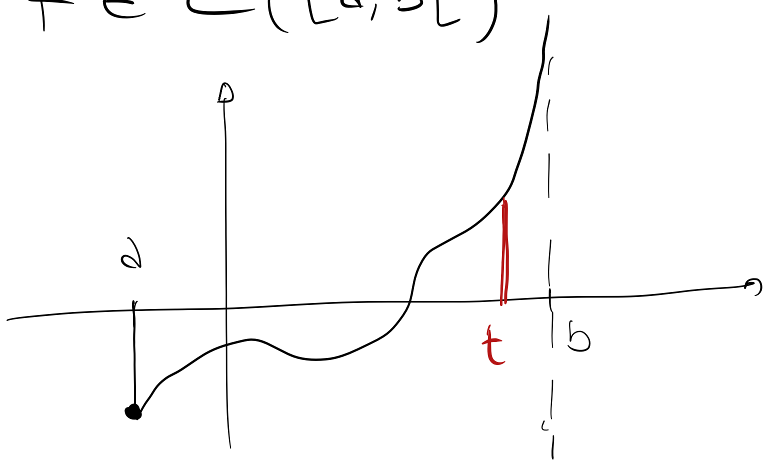
$$\int f(x)\,dx = F(x) + c$$

$$\int_a^b f(x)\,dx \in \mathbb{R}$$



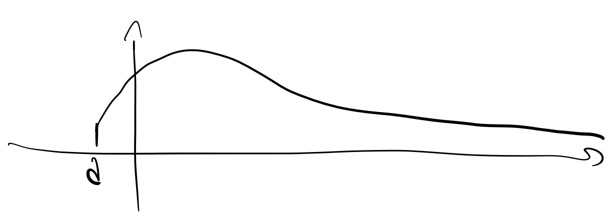
$$f \in \mathcal{C}([a,b[)$$



Possiamo calcolare $\int_a^b f(x)\,dx = ??$

$$\int_a^b f(x)\,dx \overset{def}{\equiv} \lim_{t \to b^-} \int_a^t f(x)\,dx$$

$$f \in \mathcal{C}([a,+\infty[)$$



$$\int_a^{+\infty} f(x)\,dx \equiv \lim_{t\to+\infty} \int_a^t f(x)\,dx$$

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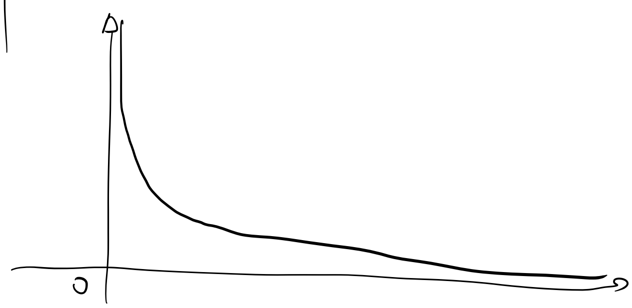
① $\displaystyle\int_0^{+\infty} \frac{dx}{\sqrt{x}+x^3}\,dx$

$$\lim_{x\to 0^+} \frac{1}{\sqrt{x}+x^3} = +\infty$$



Integrabile in senso gen: $\int_a^b f(x)\,dx$ ISG

Assolutamente int in senso gen: $\int_a^b |f(x)|\,dx$ AISG

Vale che: AISG $\Rightarrow$ ISG

$f(x) = \dfrac{1}{\sqrt{x}+x^3}$ è sempre positiva in $[0,+\infty[$

Quindi AISG $\Leftrightarrow$ ISG in quanto abbiamo che $|f(x)| = f(x)$

$$\int_0^{+\infty} \frac{1}{\sqrt{x}+x^3}\,dx$$

Studiamo cosa succede in $x=0$

$$\int_{a \to 0}^{1} \frac{1}{\sqrt{x}+x^3}\,dx \qquad f(x) \in \mathcal{C}(]0,1])$$

Corollario ① del teo del confronto

$$\frac{1}{\sqrt{x}+x^3} = O\left(\frac{1}{(x-\underbrace{a}_{0})^{\alpha}}\right), \quad \alpha \in \mathbb{R}$$

$$= O\left(\frac{1}{x^{\alpha}}\right)$$ e dobbiamo cercare se $\exists\, \alpha < 1$

allora f è AISG in $]0,1]$

$$f(x) \in O_0(g(x)) \iff \frac{|f(x)|}{|g(x)|} \leq M \quad \text{in un intorno di } 0$$

$$f(x) = \frac{1}{\sqrt{x}+\cancel{x^3}} = O\left(\frac{1}{\sqrt{x}}\right)$$

$$\frac{\frac{1}{\sqrt{x}+x^3}}{\frac{1}{\sqrt{x}}} = \frac{\sqrt{x}}{\sqrt{x}+x^3} = \frac{\sqrt{x}}{x^{\frac{1}{2}}\left(1+x^{\frac{5}{2}}\right)} = \frac{1}{1+x^{5/2}} \leq 1$$

In 0: $f(x) \in O\left(\frac{1}{\underbrace{\sqrt{x}}_{x^{\frac{1}{2}}}}\right) \Rightarrow \alpha = \frac{1}{2} \Rightarrow f$ è AISG in $]0,1]$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

A $+\infty$:

$$\int_1^{+\infty} \frac{1}{\sqrt{x}+x^3}\,dx$$

$$\frac{1}{\sqrt{x}+x^3} = O\left(\frac{1}{x^\alpha}\right)$$

$$\frac{1}{\sqrt{x}+x^3} = O\left(\frac{1}{x^3}\right)$$

- Corollario 2 del teo del confronto:

Se $f \in \mathcal{C}([a,+\infty[)$ e $\exists\ \alpha > 1$ tale che $f(x) \in O\left(\frac{1}{x^\alpha}\right)$, allora $f(x)$ è AISG in $[a,+\infty[$

Se $x \to +\infty$, allora $\sqrt{x} \in o(x^3)$

$\Rightarrow f(x)$ è ISG in $[1,+\infty[$

$f(x) \overset{x\to+\infty}{\sim} g(x) \iff \lim_{x\to+\infty} \frac{|f(x)|}{|g(x)|} = M \in \mathbb{R}$

$$\frac{|f(x)|}{|g(x)|} \leq M+\varepsilon$$

$\Rightarrow f(x) \in O(g(x))$

$$\frac{1}{\sqrt{x}+x^3} = O\left(\frac{1}{\sqrt{x}}\right) \quad \text{se } x \to 0^+$$

$$\frac{1}{1+x^{5/2}} \leq 1 \iff 1 \leq 1+x^{5/2} \iff x^{5/2} \geq 0$$

$$\frac{1}{\sqrt{x}+x^3} = \frac{1}{\sqrt{x}+o(\sqrt{x})} \sim \frac{1}{\sqrt{x}} \Rightarrow \frac{1}{\sqrt{x}+x^3} = o\left(\frac{1}{\sqrt{x}}\right)$$

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

Attenzione! Non si può fare sempre!

$$\int_a^c f(x)\,dx = +\infty \quad \text{e} \quad \int_c^b f(x)\,dx = -\infty$$

$$\Rightarrow \int_a^b f(x)\,dx = \underset{\text{F.I.}}{\boxed{+\infty - \infty}}$$

$$\int_{-\infty}^{+\infty} x\,dx = \underbrace{\int_{-\infty}^{0} x\,dx}_{\to -\infty} + \underbrace{\int_{0}^{+\infty} x\,dx}_{\to +\infty}$$

non è ISG

$$\int_0^{+\infty} \frac{(x^\alpha - 1)\arctan(x)}{(x^\beta + 1)}\,dx \qquad \alpha, \beta \in \mathbb{R}$$

1. Se $\alpha > 0$, $x^\alpha \xrightarrow{x \to 0^+} 0$ e $x^\alpha \xrightarrow{x \to +\infty} +\infty$
2. Se $\alpha = 0$, $x^\alpha = 1$
3. Se $\alpha < 0$, $x^\alpha \xrightarrow{x \to 0^+} +\infty$ e $x^\alpha \xrightarrow{x \to +\infty} 0$

1) $\alpha > 0$

- $\begin{cases} \beta > 0 \\ \beta = 0 \\ \beta < 0 \end{cases}$ $\qquad f(x) = \dfrac{(x^\alpha - 1)\arctan(x)}{(x^\beta + 1)}$

In 0^+: $\qquad \arctan(x) = x + o(x) \Rightarrow \arctan(x) \sim x$

$$f(x) \sim \begin{cases} \dfrac{-1 \cdot x}{1} & \alpha > 0,\ \beta > 0 \\[2ex] \dfrac{-1 \cdot x}{2} & \alpha > 0,\ \beta = 0 \\[2ex] \dfrac{-1 \cdot x}{x^\beta} & \alpha > 0,\ \beta < 0 \end{cases}$$

$x^\beta + 1 \overset{+o(x^\beta)}{\sim} x^\beta$ se $\beta < 0$

$\underbrace{x^\beta}_{o(1)} + 1 \sim 1$ se $\beta > 0$

In 0^+

$$f(x) \sim \begin{cases} -x & \alpha>0,\ \beta>0 \\ -\frac{x}{2} & \alpha>0,\ \beta=0 \\ -\frac{1}{x^{\beta-1}} & \alpha>0,\ \beta<0 \end{cases}$$

Per il corollario 1 del teo del confronto abbiamo che

f è AISG se $\alpha>0,\ \beta\geq 0$

Se $\alpha>0, \beta<0$, f è AISG $\Leftrightarrow \beta-1<1$
$\Leftrightarrow \beta<2$

$\Rightarrow f$ è AISG se $\alpha>0$ e $\forall \beta\in\mathbb{R}$

In 0^+:

2) $\alpha=0$ $\quad f(x) = \dfrac{(1-1)\arctan(x)}{(x^\beta+1)} = 0$

$\Rightarrow f$ è ISG $\forall\beta\in\mathbb{R}$

$f(x) = \dfrac{(x^\alpha-1)\arctan(x)}{x^\beta+1}$

Se $\alpha<0$, $x^\alpha+1\sim x^\alpha$

Se $\beta>0$, $x^\beta+1\sim 1$

Se $\beta<0$

$x^\beta+1\sim x^\beta$

3) $\alpha<0$:

$$f(x) \sim \begin{cases} \frac{x^\alpha\cdot x}{1} & \alpha<0,\ \beta>0 \\ \frac{x^\alpha\cdot x}{2} & \alpha<0,\ \beta=0 \\ \frac{x^\alpha\cdot x}{x^\beta} & \alpha<0,\ \beta<0 \end{cases}$$

$$\Rightarrow f(x) \sim \begin{cases} x^{\alpha+1} & \alpha<0,\ \beta>0 \\ \dfrac{x^{\alpha+1}}{2} & \alpha<0,\ \beta=0 \\ \dfrac{1}{x^{\beta-\alpha-1}} & \alpha<0,\ \beta<0 \end{cases}$$

$$\boxed{\begin{array}{l} \text{Se } x \to 0^+ \text{ e } \beta<0, \text{ allora} \\ x^\beta \underbrace{+1}_{o(x^\beta)} \sim x^\beta \end{array}}$$

$x^{\alpha+1} = \dfrac{1}{x^{-(\alpha+1)}}$ è AISG in 0^+

$\Leftrightarrow -(\alpha+1) < 1 \Leftrightarrow \alpha+1>0 \Leftrightarrow \alpha > -1$

Se $-1<\alpha<0$ e $\beta>0$ allora f è AISG

Se $-1<\alpha<0$ e $\beta=0$ " " "

Se $\alpha<0,\ \beta<0$:

$f(x) \sim \dfrac{1}{x^{\beta-\alpha-1}} \Rightarrow \beta-\alpha-1<1 \Leftrightarrow \beta-2<\alpha<0$

In 0^+: f è AISG $\Leftrightarrow$

- $\alpha>0,\ \beta\in\mathbb{R}$; • $\alpha=0,\ \beta\in\mathbb{R}$; • $-1<\alpha<0,\ \beta\geq 0$

- $\alpha < 0$, $\beta < 0$ e $\beta - 2 < \alpha$

$$\Rightarrow \begin{cases} \alpha \geq 0 \\ \beta \in \mathbb{R} \end{cases}, \quad \begin{cases} -1 < \alpha < 0 \\ \beta \geq 0 \end{cases}, \quad \begin{cases} \alpha < 0 \\ \beta < 0 \\ \beta - 2 < \alpha \end{cases}$$

$$\lim_{x \to +\infty} \arctan(x) = \frac{\pi}{2} \Rightarrow \arctan(x) \sim \frac{\pi}{2}$$

$$f(x) = \frac{(x^\alpha - 1)\arctan(x)}{x^\beta + 1}$$

A $+\infty$: $(x \to +\infty)$

$x^\alpha - 1 \sim x^\alpha$ se $\alpha > 0$

$x^\alpha - 1 \sim -1$ se $\alpha < 0$

$x^\beta + 1 \sim x^\beta$ se $\beta > 0$

$x^\beta + 1 \sim 1$ se $\beta < 0$

$\arctan(x) \sim \frac{\pi}{2}$

$x^\alpha - 1$ se $\alpha = 0$

$x^\beta = 1$ se $\beta = 0$

- $\alpha > 0$

$$f(x) \sim \begin{cases} \dfrac{x^\alpha \cdot \frac{\pi}{2}}{x^\beta} & \alpha > 0, \beta > 0 \\ \dfrac{x^\alpha \cdot \frac{\pi}{2}}{2} & \alpha > 0, \beta = 0 \\ \dfrac{x^\alpha \cdot \frac{\pi}{2}}{1} & \alpha > 0, \beta < 0 \end{cases}$$

$$f(x) \sim \begin{cases} \dfrac{1}{x^{\beta-\alpha}} \cdot \dfrac{\pi}{2} & \alpha>0, \beta>0 \\ \dfrac{1}{x^{-\alpha}} \cdot \dfrac{\pi}{4} & \alpha>0, \beta=0 \\ \dfrac{1}{x^{-\alpha}} \cdot \dfrac{\pi}{2} & \alpha>0, \beta<0 \end{cases}$$

- $\alpha>0, \beta>0$ f é AISG $\iff \beta-\alpha>1$
- $\alpha>0, \beta\leq 0$ // $\iff -\alpha>1 \iff \alpha<-1$ impossibile

2) $\alpha=0 \Rightarrow f(x)=0 \Rightarrow f$ é AISG $\forall \beta\in\mathbb{R}$

3) $\alpha<0$

$$f(x) \sim \begin{cases} \dfrac{-1 \cdot \frac{\pi}{2}}{x^{\beta}} & \alpha<0, \beta>0 \\ \dfrac{-1 \cdot \frac{\pi}{2}}{2} & \alpha<0, \beta=0 \\ \dfrac{-1 \cdot \frac{\pi}{2}}{1} & \alpha<0, \beta<0 \end{cases}$$

- Se $\alpha<0, \beta>0$, f é AISG $\iff \beta>1$
- Se $\alpha<0, \beta\leq 0$ f non é AISG a $+\infty$

cost. $\dfrac{1}{x^0}$ e $0<1$

$\Rightarrow$ A $+\infty$ f é AISG $\iff \begin{cases} \alpha>0 \\ \beta>0 \\ \beta-\alpha>1 \end{cases}, \begin{cases} \alpha=0 \\ \beta\in\mathbb{R} \end{cases}, \begin{cases} \alpha<0 \\ \beta>1 \end{cases}$

$\underline{\text{In } 0^+}$ $\begin{cases} \alpha \geq 0 \\ \beta \in \mathbb{R} \end{cases}$, $\begin{cases} -1 < \alpha < 0 \\ \beta \geq 0 \end{cases}$, $\begin{cases} \alpha < 0 \\ \beta < 0 \\ \beta - 2 < \alpha \end{cases}$

$\underline{A\ +\infty}$: $\begin{cases} \alpha > 0 \\ \beta > 0 \\ \beta - \alpha > 1 \end{cases}$, $\begin{cases} \alpha = 0 \\ \beta \in \mathbb{R} \end{cases}$, $\begin{cases} \alpha < 0 \\ \beta > 1 \end{cases}$

f è AISG in $]0, +\infty[$ $\iff$

$$\begin{cases} \alpha > 0 \\ \beta > 0 \\ \beta - \alpha > 1 \end{cases} \cup \begin{cases} \alpha = 0 \\ \beta \in \mathbb{R} \end{cases} \cup \begin{cases} -1 < \alpha < 0 \\ \beta > 1 \end{cases}$$