Ex Calcolare $\lim_{x\to 0^+} f(x)$ dove

$$\frac{\sqrt{2(1-\cos x)} - x e^{x^2}}{x - \sin x} = f(x) = \frac{N(x)}{D(x)} .$$

Sol.

- $\cos x = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4) \quad , \quad x \to 0$
- $e^{x^2} = 1 + x^2 + o(x^2) \qquad (e^y = 1 + y + o(y) \text{ per } y \to 0)$

Allora $D(x) = x - \left(x - \frac{x^3}{3!} + o(x^3) \right) = \frac{x^3}{6} + o(x^3)$

$$N(x) = \sqrt{2\left(1 - \left[1 - \frac{x^2}{2} + \frac{x^4}{4!} + o(x^4)\right]\right)} - x\left(1 + x^2 + o(x^2)\right)$$

$$= \sqrt{x^2\left(1 - \frac{x^2}{12} + o(x^2)\right)} - x\left(1 + x^2 + o(x^2)\right)$$

$$= x \Big[\underbrace{\sqrt{1 - \frac{x^2}{12} + o(x^2)} - \left(1 + x^2 + o(x^2)\right)}_{= g(x)} \Big] = x \cdot g(x)$$

$$\sqrt{1+y} = 1 + \frac{y}{2} + o(y) \implies$$

$$g(x) = \sqrt{1 \underbrace{- \frac{x^2}{2} + o(x^2)}_{y}} - \left(1 + x^2 + o(x^2)\right)$$

$$= 1 + \frac{1}{2}\left(-\frac{x^2}{2} + o(x^2)\right) - \left(1 + x^2 + o(x^2)\right)$$

$$= x^2 \underbrace{\left(-\frac{1}{4} - 1\right)}_{= -\frac{5}{4}} + o(x^2)$$

Perciò $N(x) = x\,g(x) \sim -\frac{5}{4}x^2$ e $D(x) \sim \frac{x^3}{6} \implies$

$$f(x) \sim \frac{-\frac{5}{4}x^3}{\frac{1}{6}x^3} = -\frac{5}{24}.$$