

$$|\sin(x)| \leq |x|$$

$$\lim_{x \rightarrow 0} x \cdot \sin(x) =$$

$$|\sin(x)| \leq 1$$

$$|\sin(x)| \leq |x|$$

↓

$$0 \leq |x \sin(x)| \leq |x^2|$$

per il teo dei
2 card.

$$\Rightarrow \lim_{x \rightarrow 0} x \sin(x) = 0$$

$$-|x^2| \leq x \sin(x) \leq |x^2|$$

$$\bullet |f(x)| \leq g(x) \Leftrightarrow -g(x) \leq f(x) \leq g(x)$$

• Sia $f(x)$ tale che $\lim_{x \rightarrow 0} f(x) = 0$. f si dice

infinitesimo di ordine $\alpha > 0$ se

$$\lim_{x \rightarrow 0} \frac{f(x)}{x^\alpha} = l \neq 0, l \in \mathbb{R}$$

① Trovare ordine di infinitesimo di $f(x) = \sin(x)$ in 0.

$$\boxed{\alpha > 0} \quad \lim_{x \rightarrow 0} \frac{\sin(x)}{x^\alpha} = \lim_{x \rightarrow 0} \frac{x + o(x)}{x^\alpha} = \begin{cases} 0 & 0 < \alpha < 1 \\ 1 & \alpha = 1 \\ \text{non esiste} & \alpha > 1 \end{cases}$$

Se $0 < \alpha < 1$: $\frac{x}{x^\alpha} = x^{1-\alpha} \xrightarrow{x \rightarrow 0} 0$

Se $\alpha = 1$: $\frac{x}{x} = 1$

Se $\alpha > 1$: $\frac{x}{x^\alpha} = \frac{1}{x^{\alpha-1}} \xrightarrow{x \rightarrow 0} \text{non esiste}$

$\Rightarrow \sin(x)$ ha ordine di infinitesimo 1 in 0.

② $f(x) = \ln(\cos(x))$

$$\lim_{x \rightarrow 0} \frac{\ln(\cos(x))}{x^\alpha} = \lim_{x \rightarrow 0} \frac{\ln(1 - \frac{x^2}{2} + o(x^4))}{x^\alpha} = (\star)$$

$$\begin{aligned} \ln(1+x) &= x + o(x) \\ \ln(1 - \frac{x^2}{2} + o(x^4)) &= -\frac{x^2}{2} + o(x^4) + o(\frac{x^2}{2} + o(x^4)) = o(x^2) \\ &= -\frac{x^2}{2} + o(x^4) \end{aligned}$$

$$(\star) = \lim_{x \rightarrow 0} \frac{-\frac{x^2}{2} + o(x^4)}{x^\alpha} = \begin{cases} 0 & \text{se } 0 < \alpha < 2 \\ -\frac{1}{2} & \text{se } \alpha = 2 \\ \text{non esiste} & \text{se } \alpha > 2 \end{cases}$$

$\Rightarrow f(x) = \ln(\cos(x))$ ha ordine di infinitesimo 2 in $x \rightarrow 0$.

$$\lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh^2(x) + x^{3/2} \log(x)} =$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

$$e^{x-2x^2} = 1 + x - 2x^2 + o(x-2x^2) \stackrel{?!}{=} 1 + x - 2x^2 + o(x^2)$$

NO!!!

$$f(x) \in o(x-2x^2) \stackrel{!}{\Rightarrow} \lim_{x \rightarrow 0} \frac{f(x)}{x-2x^2} = 0$$

$$\Rightarrow o(x-2x^2) = o(x) \quad \text{!!!}$$

$$\star e^{x-2x^2} = 1 + x - 2x^2 + o(x) = 1 + x + o(x)$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

$$\begin{aligned} e^{x-2x^2} &= 1 + x - 2x^2 + \frac{(x-2x^2)^2}{2} + o((x-2x^2)^2) = \\ &= 1 + x - 2x^2 + \frac{x^2 - 4x^3 + 4x^4}{2} + o(x^2 - 4x^3 + 4x^4) = \\ &= 1 + x - 2x^2 + \frac{1}{2}x^2 + o(x^2) = 1 + x - \frac{3}{2}x^2 + o(x^2) \end{aligned}$$

$$\sinh^2(x) = [\sinh(x)]^2 = [x + o(x)]^2 = x^2 + 2x o(x) + [o(x)]^2 =$$

$$1 = x^2 + o(x^2)$$

$$x^2 o(x^3) = o(x^5)$$

$$\lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh^2(x) + x^{3/2} \log(x)} = \lim_{x \rightarrow 0^+} \frac{1 + x - \frac{3}{2}x^2 - 1 - x}{x^2 + o(x^2) + \underbrace{x^{3/2}}_{\rightarrow 0} \underbrace{\log(x)}_{\rightarrow -\infty}} = \star$$

$$x^{3/2} \log(x) \in o(x^2) ?$$

$$\begin{aligned} (\Rightarrow) \lim_{x \rightarrow 0^+} \frac{x^{3/2} \log(x)}{x^2} &= \lim_{x \rightarrow 0^+} x^{1/2} \log(x) \stackrel{y = \frac{1}{x} \sim x = \frac{1}{y}}{=} \\ &= \lim_{y \rightarrow +\infty} \left(\frac{1}{y}\right)^{1/2} \log\left(\frac{1}{y}\right) = \lim_{y \rightarrow +\infty} \frac{-\log(y)}{y^{1/2}} = 0 \end{aligned}$$

$$\lim x^a \log(x) = 0 \quad \forall a > 0$$

$$\Rightarrow x^{3/2} \log(x) \in o(x^2)$$

$$\begin{aligned} \star &= \lim_{x \rightarrow 0^+} \frac{-\frac{3}{2}x^2 + o(x^2)}{x^2 + o(x^2)} = \lim_{x \rightarrow 0^+} \frac{\cancel{x^2} \left(-\frac{3}{2} + \underbrace{\left[\frac{o(x^2)}{x^2}\right]}_{\rightarrow 0}\right)}{\cancel{x^2} \left(1 + \underbrace{\left[\frac{o(x^2)}{x^2}\right]}_{\rightarrow 0}\right)} = \\ &= -\frac{3}{2} \end{aligned}$$

$$\lim_{x \rightarrow +\infty} \underbrace{3^{x+1}}_{\rightarrow +\infty} - \underbrace{3^{\sqrt{x^2+1}}}_{\rightarrow \infty} = \lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(\frac{3^{x+1}}{3^{\sqrt{x^2+1}}} - 1 \right) =$$

$$= \lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(3^{x+1-\sqrt{x^2+1}} - 1 \right)$$

$$x+1-\sqrt{x^2+1} = (x+1-\sqrt{x^2+1}) \cdot \frac{x+1+\sqrt{x^2+1}}{x+1+\sqrt{x^2+1}} =$$

$$(a+b)(a-b) = a^2 - b^2$$

$$= \frac{(x+1)^2 - (x^2+1)}{x+1+\sqrt{x^2+1}} =$$

$$= \frac{x^2+2x+1-x^2-1}{x+1+\sqrt{x^2+1}} =$$

$$= \frac{2x}{x+1+\sqrt{x^2+1}} \stackrel{x \rightarrow +\infty}{\sim} \frac{2x}{x+1+x\sqrt{1+\frac{1}{x^2}}}$$

$$= \frac{2x}{x\left(1+\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}\right)} = \frac{2}{1+\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}}$$

$$\lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(3^{1+\frac{1}{x}-\sqrt{1+\frac{1}{x^2}}} - 1 \right) = \lim_{x \rightarrow +\infty} \underbrace{3^{\sqrt{x^2+1}}}_{\rightarrow +\infty} \left(\underbrace{3^{1+\frac{1}{x}-\sqrt{1+\frac{1}{x^2}}}}_{\rightarrow 3-1=2} - 1 \right)$$

$$= +\infty$$