

- $f(x) = |x|$

Non esiste derivata di $f(x)$ in $x_0 = 0$

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'_+(x_0) := \lim_{x \rightarrow x_0^+} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'_-(x_0) := \lim_{x \rightarrow x_0^-} \frac{f(x) - f(x_0)}{x - x_0}$$

Se $\exists f'(x_0)$ Allora $f'_+(x_0) = f'_-(x_0)$

$$\lim_{x \rightarrow 0} \frac{|x| - |0|}{x - 0} = \lim_{x \rightarrow 0} \frac{|x|}{x} \text{ non esiste}$$

$$\left. \begin{aligned} \lim_{x \rightarrow 0^+} \frac{|x|}{x} &= \lim_{x \rightarrow 0^+} \frac{x}{x} = 1 \\ \lim_{x \rightarrow 0^-} \frac{|x|}{x} &= \lim_{x \rightarrow 0^-} \frac{-x}{x} = -1 \end{aligned} \right\} f'_+(x_0) \neq f'_-(x_0)$$

$$\Rightarrow \nexists f'(x_0)$$

$$\lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh(x^2) + x^{7/3} \log(x)} =$$

$$e^{x-2x^2} = 1 + (x-2x^2) + o(x-2x^2)$$

$$o(x-2x^2) = o(x)$$

$$f(x) \in o(x-2x^2)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{x-2x^2} = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \underbrace{\frac{f(x)}{x}}_{\rightarrow 0} \cdot \underbrace{\frac{1}{(1-2x)}}_{\rightarrow 1} = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{x} = 0$$

$$\Rightarrow f(x) \in o(x)$$

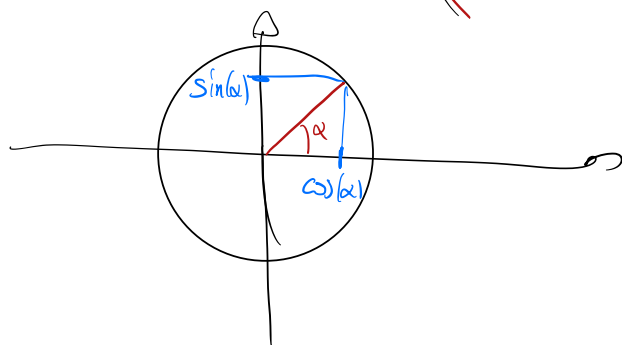
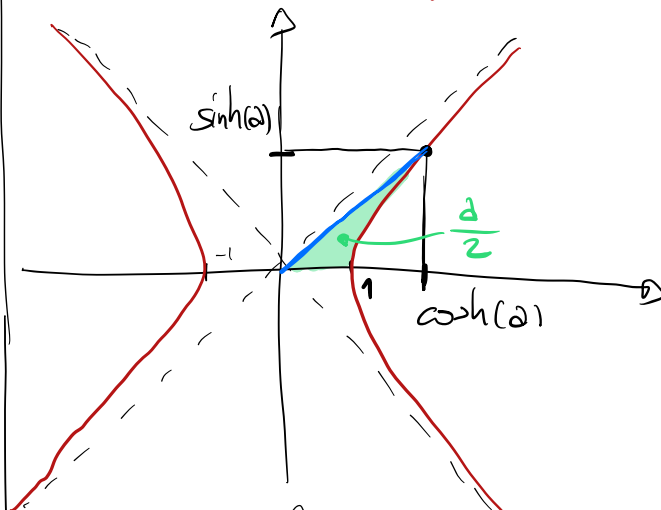
In generale,

$$o(x^m + x^{m+1} + \dots) = o(x^m)$$

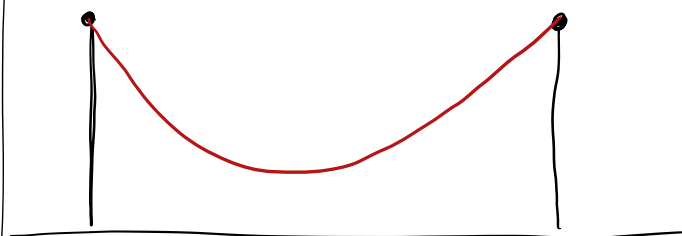
$$\begin{aligned} \Rightarrow e^{x-2x^2} &= 1 + (x-2x^2) + o(x-2x^2) = \\ &= 1 + x - 2x^2 + o(x) \\ &= 1 + x + o(x) \end{aligned}$$

$$\cosh^2(x) - \sinh^2(x) = 1$$

$$x^2 - y^2 = 1$$



$f(x) = a \cdot \cosh\left(\frac{x}{a}\right)$, $a > 0$
si chiamo estensaria



$$e^y = 1 + y + \frac{y^2}{2} + o(y^2)$$

Sviluppamento fino al secondo ordine:

$$\begin{aligned}
 e^{x-2x^2} &= 1 + (x-2x^2) + \frac{(x-2x^2)^2}{2} + o((x-2x^2)^2) = \\
 &= 1 + x - 2x^2 + \frac{x^2 - 4x^3 + 4x^4}{2} + o(x^2 - 4x^3 + 4x^4) = \\
 &= 1 + x - 2x^2 + \frac{x^2}{2} - 2x^3 + 2x^4 + o(x^2) \\
 &= 1 + x - \frac{3}{2}x^2 + o(x^2)
 \end{aligned}$$

$$\lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh(x^2) + x^{7/3} \log(x)} =$$

$$\begin{aligned}
 \sinh(x) &= x + o(x) \\
 &= x + \frac{x^3}{6} + o(x^3)
 \end{aligned}$$

$$= \lim_{x \rightarrow 0^+} \frac{1 + x - \frac{3}{2}x^2 + o(x^2) - 1 - x}{x^2 + o(x^2) + x^{7/3} \log(x)} = \bullet \log(1+x) = x - \frac{x^2}{2} + o(x^2)$$

$$\rightarrow \log(1 + \boxed{x-1}) \quad x \rightarrow 0 \rightarrow -1$$

$$x^{7/3} \log(x) \in o(x^2) \quad ??$$

$$\lim_{x \rightarrow 0^+} \frac{x^{7/3} \log(x)}{x^2} = \lim_{x \rightarrow 0^+} x^{7/3-2} \log(x) = \lim_{x \rightarrow 0^+} x^{1/3} \log(x) = 0$$

$$\Rightarrow x^{7/3} \log(x) \in o(x^2)$$

$$\lim_{x \rightarrow 0^+} x^\beta \log(x) = \lim_{y \rightarrow +\infty} \frac{\log(\frac{1}{y})}{y^\beta} = \lim_{y \rightarrow +\infty} \frac{-\log(y)}{y^\beta} = 0$$

$$\beta > 0$$

$$y = \frac{1}{x}$$

per la gerarchia degli infiniti

$$\boxed{\log\left(\frac{a}{b}\right) = \log(a) - \log(b) \quad | \quad \log(a \cdot b) = \log(a) + \log(b)}$$

$$\log\left(\frac{1}{b}\right) = \log(1) - \log(b) = -\log(b)$$

$$\log(a^x) = x \cdot \log(a)$$

$$\log\left(\frac{1}{b}\right) = \log(b^{-1}) = -1 \cdot \log(b)$$

$$\begin{aligned} & \bullet \lim_{x \rightarrow 0^+} \frac{1+x-\frac{3}{2}x^2+o(x^2)-1-x}{x^2+o(x^2)+x^{\frac{7}{2}}\log(x)} = \\ & \lim_{x \rightarrow 0^+} \frac{-\frac{3}{2}x^2+o(x^2)}{x^2+o(x^2)} = \lim_{x \rightarrow 0^+} \frac{\cancel{x^2}\left(-\frac{3}{2} + \frac{o(x^2)}{x^2}\right)}{\cancel{x^2}\left(1 + \frac{o(x^2)}{x^2}\right)} = \\ & = -\frac{3}{2} \end{aligned}$$

$$\begin{aligned} & \bullet \lim_{x \rightarrow 0^+} \frac{-\frac{3}{2}x^2+o(x^2)}{x^2+o(x^2)+x^2\log(x)} = \quad , \quad x^2\log(x) \not\sim o(x^2) \\ & = \lim_{x \rightarrow 0^+} \frac{\cancel{x^2}\left(-\frac{3}{2} + o(1)\right)}{\cancel{x^2}\left(1 + o(1) + \log(x)\right)} = 0 \end{aligned}$$

$$\bullet f(x) = \sin(x)^{\cos(x)} = e^{\cos(x) \cdot \log(\sin(x))} \quad \left| \begin{array}{l} x^x = e^{x \log(x)} \end{array} \right.$$

$$g(x) = e^x \quad h(x) = \cos(x) \cdot \log(\sin(x))$$

$$g(h(x)) = e^{\cos(x) \log(\sin(x))} = f(x)$$

$$D(f(x)) = D(g(h(x))) = g'(h(x)) \cdot h'(x) = \bullet$$

$$g'(x) = e^x$$

$$\begin{aligned} h'(x) &= -\sin(x) \cdot \log(\sin(x)) + \cos(x) \cdot D(\overset{K(x)}{\log}(\overset{j(x)}{\sin}(x))) = \\ &= -\sin(x) \log(\sin(x)) + \cos(x) \cdot \frac{1}{\sin(x)} \cdot \cos(x) = \\ &= -\sin(x) \cdot \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \end{aligned}$$

$$\begin{aligned} \bullet f'(x) &= g'(h(x)) \cdot h'(x) = \\ &= e^{\cos(x) \log(\sin(x))} \cdot \left(-\sin(x) \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right) = \\ &= \sin(x)^{\cos(x)} \cdot \left(-\sin(x) \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right) \end{aligned}$$

$$\bullet D(x^n) = n \cdot x^{n-1}$$

- $D([f(x)]^n) = n(f(x))^{n-1} \cdot f'(x)$

! $D((f(x))^{g(x)}) \stackrel{!!}{=} g(x)(f(x))^{g(x)-1} \cdot f'(x) \cdot g'(x)$ No!

$$D((f(x))^{g(x)}) = D(e^{g(x) \log(f(x))}) = \dots$$

$$= (f(x))^{g(x)} \left[g'(x) \log(f(x)) + \frac{g(x) \cdot f'(x)}{f(x)} \right]$$

$$D((\sin(x))^{\cos(x)}) = \sin(x)^{\cos(x)} \left[-\sin(x) \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right]$$