$$\lim_{x\to 0} \frac{\sin(x) - e^{\frac{x}{2}} \cdot \log(1+x)}{x^2} \qquad R = \frac{1}{2}$$

$$e^x = 1 + o(1)$$
$$= 1 + x + o(x)$$
$$= 1 + x + \frac{x^2}{2} + o(x^2)$$

$$\sin(x) - e^{\frac{x}{2}} \log(1+x) =$$

$$= x + o(x^2) - (1 + o(1))\left(x - \frac{x^2}{2} + o(x^2)\right)$$

$$= x + o(x^2) - x + o(x) + \frac{x^2}{2} + o(x^2)$$

$$= o(x)$$

$$\sin(x) - e^{\frac{x}{2}} \log(1+x) = x + o(x^2) - \left(1 + \frac{x}{2} + o(x)\right)\left(x - \frac{x^2}{2} + o(x^2)\right)$$

$$= x + o(x^2) - x + \frac{x^2}{2} + o(x^2) - \frac{x^2}{2} = o(x^2)$$

$$= x - \frac{x^3}{6} + o(x^3) + \left(1 + \frac{x}{2} + \frac{x^2}{8} + o(x^2)\right)\left(x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)\right)$$

$$\lim_{x\to 0} \frac{o(x^2)}{x^2} = 0$$

$$x^2 \cdot o(x^3) \in o(x^5)$$

$$\lim_{x\to 0} \frac{x^2 o(x^3)}{x^5} = \lim_{x\to 0} \frac{o(x^3)}{x^3} \overset{\text{per definizione}}{=} 0$$

$$\int \frac{\cos(x)}{1+\sin(x)}\,dx =$$

$f(x) = 1+\sin(x)$
$f'(x) = \cos(x)$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|+c$$

$$= \ln|1+\sin(x)| + c$$

$$\int \frac{1}{\cosh(x)}\,dx =$$

$$\cosh(x) = \frac{e^x+e^{-x}}{2}$$

$$\sinh(x) = \frac{e^x-e^{-x}}{2}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

$$= \int \frac{1}{\frac{e^x+e^{-x}}{2}}\,dx =$$

$$= \int \frac{2}{e^x+\frac{1}{e^x}}\,dx = \int \frac{2}{\frac{e^{2x}+1}{e^x}}\,dx =$$

$$= \int \frac{2e^x}{e^{2x}+1}\,dx =$$

$t = e^x,\ x = \ln(t),\ dx = \frac{1}{t}dt$

$f(x) = e^{2x}+1$

$f'(x) = 2e^{2x}$

$e^{2x} = (e^x)^2$

$$= \int \frac{2t}{t^2+1}\cdot\frac{1}{t}\,dt = \int \frac{2}{t^2+1}\,dt = 2\arctan(t)+c$$

$$= 2\arctan(e^x)+c$$

$$\int f'(x)\cdot g(f(x))\,dx = G(f(x)) + c$$

$$g(x) = G'(x) \iff G \text{ è primitiva di } g$$

$$f(x) = e^x \quad , \quad g(x) = \frac{1}{x^2+1} \quad , \quad G(x) = \arctan(x)$$

$$\int e^x \cdot \frac{1}{(e^x)^2+1}\,dx = \int \frac{e^x}{e^{2x}+1}\,dx =$$

$$= \arctan(e^x) + c$$

$$\int e^{\sqrt{x+2}}\,dx \overset{\text{No}}{=} e^{\sqrt{x+2}} + c \quad \text{NO!}$$

$$t = \sqrt{x+2} \qquad t^2 = x+2 \quad , x = t^2 - 2 \quad , dx = 2t\,dt$$

$$\int e^t\, 2t\,dt = 2\int \underset{g}{t}\,\underset{f'}{e^t}\,dt \overset{\text{per parti}}{=}$$

PER PARTI

$$\int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\cdot g'(x)\,dx$$

$$= 2\left[e^t\cdot t - \int e^t\cdot 1\,dt\right] = 2te^t - 2e^t + c =$$

$$= 2e^t(t-1) + c = 2\left(\sqrt{x+2} - 1\right)e^{\sqrt{x+2}} + c$$

$$\int \underset{f'}{1} \cdot \underset{g}{\arctan(x)}\, dx \overset{\text{per parti}}{=} x\arctan(x) - \frac{1}{2}\int \frac{2x}{1+x^2}\, dx =$$

$$= x\arctan(x) - \frac{1}{2}\ln(1+x^2) + c$$

$$\frac{1}{2}\int \underset{f'}{2x}\, \underset{g}{\cos}\, (\underset{f}{x^2})\, dx = \frac{1}{2}\sin(x^2) + c$$

$$\int \sin(x)\, dx = -\cos(x) + c$$

$$\frac{1}{2}\int 2x\sin(x^2)\, dx = -\frac{1}{2}\cos(x^2) + c$$

$$\int x^3\sin(x^2)\, dx = \int \underbrace{x^2}_{g} \cdot \underbrace{x\sin(x^2)}_{f'}\, dx = \quad \Rightarrow f(x) = -\frac{1}{2}\cos(x^2)$$

$$= x^2 \cdot \left(-\frac{1}{2}\cos(x^2)\right) - \int 2x \cdot \left(-\frac{1}{2}\cos(x^2)\right) dx =$$

$$= -\frac{1}{2}x^2\cos(x^2) + \int x\cos(x^2)\, dx = -\frac{1}{2}x^2\cos(x^2) + \frac{1}{2}\sin(x^2) + c$$

$$\int \underset{f'}{x^3}\, \underset{g}{\sin}(x^2)\, dx = \frac{x^4}{4}\sin(x^2) - \int \frac{x^4}{4} \cdot 2x\cos(x^2)\, dx =$$

$$= \frac{x^4}{4}\sin(x^2) - \int \frac{x^5}{2}\cos(x^2)\, dx$$

$$\int \underset{\substack{\uparrow \\ f'}}{e^x} \underset{\substack{\uparrow \\ g}}{\cos(x)}\,dx = e^x\cos(x) - \int e^x(-\sin(x))\,dx =$$

$$= e^x\cos(x) + \int \underset{\substack{\uparrow \\ f'}}{e^x} \underset{\substack{\uparrow \\ g}}{\sin(x)}\,dx =$$

$$= e^x\cos(x) + e^x\sin(x) - \int e^x\cos(x)\,dx$$

$$I = \int e^x\cos(x)\,dx$$

$$I = e^x(\cos(x)+\sin(x)) - I$$

$$2I = e^x(\cos(x)+\sin(x))$$

$$\Rightarrow \int e^x\cos(x)\,dx = \frac{e^x(\cos(x)+\sin(x))}{2} + c$$

$$\int x\ln(x^{\frac{2}{3}})\,dx = \frac{2}{3}\int \underset{\substack{\uparrow \\ f'}}{x} \underset{\substack{\uparrow \\ g}}{\ln(x)}\,dx =$$

$$= \frac{2}{3}\left[\frac{x^2}{2}\ln(x) - \int \frac{x^2}{2}\cdot\frac{1}{x}\,dx\right] =$$

$$= \frac{2}{3}\left[\frac{x^2}{2}\ln(x) - \frac{1}{2}\int x\,dx\right] = \frac{x^2}{3}\ln(x) - \frac{x^2}{6} + c$$

$$\int \underset{f'}{1} \cdot \underset{g}{\ln(x)}\, dx = x\ln(x) - \int x \cdot \frac{1}{x}\, dx = x\ln(x) - \int 1\,dx =$$

$$= x\ln(x) - x + c = x(\ln(x) - 1) + c$$

$$-\int -\sin(x)\cos^8(x)\, dx = \frac{\cos^9(x)}{9} + c$$

$$\int f'(x)\,[f(x)]^n\, dx = \frac{[f(x)]^{n+1}}{n+1} + c$$

$$\int \underset{f'}{\sin(x)}\, \underset{g}{\cos^8(x)}\, dx =$$

$$= -\cos(x)\cdot\cos^8(x) - \int -\cos(x)\cdot 8\cos^7(x)\cdot(-\sin(x))\, dx =$$

$$= -\cos^9(x) - 8\underbrace{\int \sin(x)\cos^8(x)\, dx}_{I}$$

$$I = -\cos^9(x) - 8I \leadsto 9I = -\cos^9(x) \Rightarrow I = -\frac{\cos^9(x)}{9} + c$$

$$D\left(\cos^8(x)\right) = 8\cos^7(x)\cdot(-\sin(x))$$

$$D\left([f(x)]^n\right) = n\,[f(x)]^{n-1}\cdot f'(x)$$