

$$|\sin(x)| \leq |x|$$

$$\lim_{x \rightarrow 0} x \cdot \sin(x) =$$

$$|\sin(x)| \leq 1$$

$$|\sin(x)| \leq |x|$$

↓

$$0 \leq |x \sin(x)| \leq |x^2| \quad \begin{matrix} \text{per il teo dei} \\ \text{cavaliere} \end{matrix} \Rightarrow \lim_{x \rightarrow 0} x \sin(x) = 0$$

$$\lim_{x \rightarrow 0} x \sin(x) = 0$$

$$-|x^2| \leq x \sin(x) \leq |x^2|$$

$$\bullet |f(x)| \leq g(x) \Leftrightarrow -g(x) \leq f(x) \leq g(x)$$

• Si $f(x)$ tale che $\lim_{x \rightarrow 0} f(x) = 0$. E si dice

infinitesima di ordine $\alpha > 0$ se

$$\lim_{x \rightarrow 0} \frac{f(x)}{x^\alpha} = l \neq 0, l \in \mathbb{R}$$

① Trova ordine di infinitesimo di $f(x) = \sin(x)$ in 0.

$$\boxed{\alpha > 0} \quad \lim_{x \rightarrow 0} \frac{\sin(x)}{x^\alpha} = \lim_{x \rightarrow 0} \frac{x + o(x)}{x^\alpha} = \begin{cases} 0 & 0 < \alpha < 1 \\ 1 & \alpha = 1 \\ \text{non esiste} & \alpha > 1 \end{cases}$$

Se $0 < \alpha < 1$: $\frac{x}{x^\alpha} = x^{\frac{1-\alpha}{\alpha}} \xrightarrow[x \rightarrow 0]{} 0^{>0}$

Se $\alpha = 1$: $\frac{x}{x} = 1$

Se $\alpha > 1$: $\frac{x}{x^\alpha} = \frac{1}{x^{\alpha-1}} \xrightarrow{x \rightarrow 0} \text{non esiste}$

$\Rightarrow \sin(x)$ ha ordine di infinitesimo 1 in 0.

② $f(x) = \ln(\cos(x))$

$$\lim_{x \rightarrow 0} \frac{\ln(\cos(x))}{x^\alpha} = \lim_{x \rightarrow 0} \frac{\ln(1 - \frac{x^2}{2} + o(x^2))}{x^\alpha} = (\star)$$

$$\begin{aligned} \ln(1+x) &= x + o(x) \\ \ln(1 - \frac{x^2}{2} + o(x^2)) &= -\frac{x^2}{2} + o(x^2) + o\left(\frac{x^2}{2} + o(x^2)\right) = o(x^2) \\ &= -\frac{x^2}{2} + o(x^2) \end{aligned}$$

$$(\star) = \lim_{x \rightarrow 0} \frac{-\frac{x^2}{2} + o(x^2)}{x^\alpha} = \begin{cases} 0 & \text{se } 0 < \alpha < 2 \\ -\frac{1}{2} & \text{se } \alpha = 2 \\ \text{non esiste} & \text{se } \alpha > 2 \end{cases}$$

$\Rightarrow f(x) = \ln(\cos(x))$ ha ordine di infinitesimo 2 in $x \rightarrow 0$.

$$\lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh^2(x) + x^{5/2} \log(x)} =$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

+ $o(x)$

$$e^{x-2x^2} = 1 + x - 2x^2 + o(x-2x^2) \quad \text{??}$$

\otimes

$$= 1 + x - 2x^2 + o(x^2)$$

$\text{NO!} \quad \text{!!}$

$$f(x) \in o(x-2x^2) \quad ! \Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{x-2x^2} = 0$$

$$\Rightarrow o(x-2x^2) = o(x) \quad \text{!!!}$$

$$e^{x-2x^2} = 1 + x - 2x^2 + o(x) = 1 + x + o(x)$$

$$e^x = 1 + x + \frac{x^2}{2} + o(x^2)$$

$$e^{x-2x^2} = 1 + x - 2x^2 + \frac{(x-2x^2)^2}{2} + o((x-2x^2)^2) =$$

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$$= 1 + x - 2x^2 + \frac{x^2 - 4x^3 + 4x^4}{2} + o(x^2 - 4x^3 + 4x^4) =$$

|

$$= 1 + x - 2x^2 + \frac{1}{2}x^2 + o(x^2) = 1 + x - \frac{3}{2}x^2 + o(x^2)$$

$$\sinh^2(x) = \left[\sinh(x) \right]^2 = \left[x + o(x) \right]^2 = x^2 + 2x o(x) + [o(x)]^2 =$$

$$= x^2 + o(x^2)$$

$$x^2 o(x^3) = o(x^5)$$

$$\lim_{\substack{x \rightarrow 0^+ \\ x \rightarrow \infty}} \frac{e^{x - 2x^2} - 1 - x}{\sinh^2(x) + x^{3/2} \log(x)} = \lim_{\substack{x \rightarrow 0^+ \\ x \rightarrow \infty}} \frac{1 + x - \frac{3}{2}x^2 - 1 - x}{x^2 + o(x^2) + x^{3/2} \log(x)} = \star$$

$$x^{3/2} \log(x) \in o(x^2) ?$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{x^{3/2} \log(x)}{x^2} = \lim_{x \rightarrow 0^+} x^{3/2} \log(x) =$$

$$= \lim_{y \rightarrow +\infty} \left(\frac{1}{y}\right)^{3/2} \log\left(\frac{1}{y}\right) = \lim_{y \rightarrow \infty} \frac{-\log(y)}{y^{3/2}} = 0$$

$$\lim_{x \rightarrow 0^+} x^2 \log(x) = 0 \quad \forall \alpha > 0$$

$$\Rightarrow x^{3/2} \log(x) \in o(x^2)$$

$$\star = \lim_{x \rightarrow 0^+} \frac{-\frac{3}{2}x^2 + o(x^2)}{x^2 + o(x^2)} = \lim_{x \rightarrow 0^+} \frac{x^2 \left(-\frac{3}{2} + \frac{o(x^2)}{x^2}\right)}{x^2 \left(1 + \frac{o(x^2)}{x^2}\right)} =$$

$$= -\frac{3}{2}$$

$$\lim_{x \rightarrow +\infty} 3^{\frac{x+1}{\sqrt{x^2+1}}} = \lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(\frac{3^{x+1}}{3^{\sqrt{x^2+1}}} - 1 \right) =$$

$$= \lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(3^{\frac{x+1-\sqrt{x^2+1}}{\sqrt{x^2+1}}} - 1 \right)$$

$$x+1 - \sqrt{x^2+1} = (x+1 - \sqrt{x^2+1}) \cdot \frac{x+1 + \sqrt{x^2+1}}{x+1 + \sqrt{x^2+1}} =$$

$(a+b)(a-b)=a^2-b^2$

$$= \frac{(x+1)^2 - (x^2+1)}{x+1 + \sqrt{x^2+1}} =$$

$$= \frac{x^2 + 2x + 1 - x^2 - 1}{x+1 + \sqrt{x^2 \left(1 + \frac{1}{x^2}\right)}} =$$

$$= \frac{2x}{x+1 + |x| \sqrt{1 + \frac{1}{x^2}}} = \frac{2x}{x+1 + x \sqrt{1 + \frac{1}{x^2}}} \quad \begin{matrix} x \rightarrow +\infty \\ \downarrow \end{matrix}$$

$$= \frac{2x}{x \left(1 + \frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}\right)} = \frac{2}{1 + \frac{1}{x} + \sqrt{1 + \frac{1}{x^2}}}$$

$$\lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(3^{\frac{1+x-\sqrt{x^2+1}}{\sqrt{x^2+1}}} - 1 \right) = \lim_{x \rightarrow +\infty} 3^{\sqrt{x^2+1}} \left(3^{\frac{2}{1+\frac{1}{x}+\sqrt{1+\frac{1}{x^2}}}} - 1 \right)$$

$\frac{2}{2} = 1$

$\downarrow +\infty$

$\downarrow 3 - 1 = 2$

$$= +\infty$$