# Esercizio

$$\frac{\ln n!}{n} \xrightarrow[n\to+\infty]{} +\infty$$

**Soluzione:**

$$\ln n! = \sum_{k=1}^{n} \lg k = \sum_{k=1}^{n} \left(k \lg k - (k-1)\lg k\right) =$$

$$= \sum_{k=1}^{n} k \lg k - \sum_{k=1}^{n} (k-1)\lg k =$$

$$= n \lg n + \sum_{k=1}^{n-1} k \lg k - \sum_{\tilde{k}=0}^{n-1} \tilde{k} \lg\left(\tilde{k}+1\right)$$

"adesso ritorno a $k$"

$$= n \lg n + \sum_{k=1}^{n-1} k \left(\lg k - \lg(k+1)\right) =$$

$$= n \lg n - \sum_{k=1}^{n-1} k \lg\left(\frac{k+1}{k}\right) = n \lg n - \boxed{\sum_{k=1}^{n-1} \lg\left(1+\frac{1}{k}\right)^{k}}$$

$$0 \leq \frac{1}{n} \sum_{k=1}^{n-1} \lg\left(1+\frac{1}{k}\right)^k \leq \frac{(n-1)}{n} \lg\left(1+\frac{1}{n-1}\right)^{n-1} = \underbrace{\left(1-\frac{1}{n}\right)}_{\to 1} \underbrace{\lg\left(1+\frac{1}{n-1}\right)^{n-1}}_{\to \lg e = 1}$$

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Ora

$$\frac{\lg n!}{n} = \lg n - \underbrace{\frac{1}{n}\left(\sum_{k=1}^{n-1} \lg\left(1+\frac{1}{k}\right)^k\right)}_{(*)}$$

N.B. Si ha che $(*)$ è limitata. Così il limite cercato sarà $+\infty$ (essendo prevalente $\lg n$).

Infatti $\lg \underbrace{\left(1+\frac{1}{n-1}\right)^{n-1}}_{\xrightarrow[n\to+\infty]{} e} \longrightarrow 1$.