$$f(x) = (1-x)^2 + \log|x+1|$$

$|x+1| > 0 \iff x \neq -1$

Dominio: $D = \{x \in \mathbb{R} \mid x \neq -1\}$

Segno: $(1-x)^2 + \log|x+1| \geq 0$???

Limiti: $\lim_{x \to \pm\infty} f(x) = +\infty$

$\lim_{x \to \pm -1} f(x) = -\infty$

Asintoti obliqui:

$$\boxed{\lim_{x \to \pm\infty} \frac{f(x)}{x} = +\infty}$$

$D([f(x)]^n) = n[f(x)]^{n-1} \cdot f'(x)$

Derivata: $f'(x) = 2(1-x)\cdot(-1) + \frac{1}{x+1} =$

$= 2x - 2 + \frac{1}{x+1} = \frac{2x^2+2x-2x-2+1}{x+1}$

$= \frac{2x^2-1}{x+1}$

$$D(\log|x|) = \begin{cases} D(\log(x)) & \text{se } x>0 \\ D(\log(-x)) & \text{se } x<0 \end{cases} =$$

$$= \begin{cases} \frac{1}{x} & x>0 \\ \frac{1}{-x}\cdot(-1) & x<0 \end{cases} = \frac{1}{x}$$

$$\frac{2x^2-1}{x+1} \geq 0$$

$N: \quad 2x^2 - 1 \geq 0 \iff x^2 \geq \frac{1}{2} \iff x \leq -\frac{1}{\sqrt{2}} \vee x \geq \frac{1}{\sqrt{2}}$

$D: \quad x+1 > 0 \iff x > -1$

		-1		$-\frac{1}{\sqrt{2}}$		$\frac{1}{\sqrt{2}}$	
N	$+$		$+$	0	$-$	0	$+$
D	$-$	$\times$	$+$		$+$		$+$
	$-$	$\times$	$+$	0	$-$	0	$+$

$$f(x) = (1-x)^2 + \log|x+1|$$

$$f\left(-\frac{1}{\sqrt{2}}\right) = \left(1 + \frac{1}{\sqrt{2}}\right)^2 + \log\left|-\frac{1}{\sqrt{2}} + 1\right| =$$

$$= 1 + \frac{1}{2} + \frac{2}{\sqrt{2}} + \log\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$f\left(\frac{1}{\sqrt{2}}\right) = \left(1 - \frac{1}{\sqrt{2}}\right)^2 + \log\left|\frac{1}{\sqrt{2}} + 1\right| > 0$$

$\rightarrow \log(1) = 0$

$$f'(x) = \frac{2x^2-1}{x+1}$$

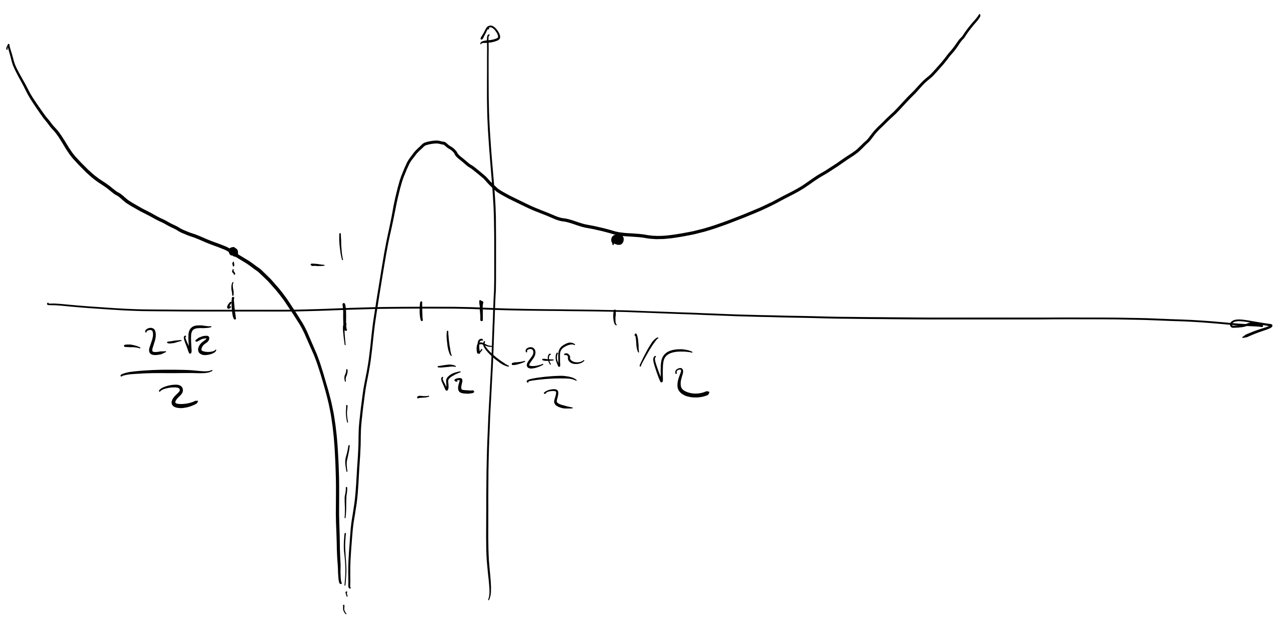
$$f''(x) = \frac{4x(x+1) - (2x^2-1)\cdot 1}{(x+1)^2} =$$

$$= \frac{4x^2+4x-2x^2+1}{(x+1)^2} = \frac{2x^2+4x+1}{(x+1)^2}$$

$N:\ 2x^2+4x+1 \geq 0$

$\frac{\Delta}{4} = 2^2 - 2 = 2$, $x_{1,2} = \frac{-2 \pm \sqrt{2}}{2}$

	$\frac{-2-\sqrt{2}}{2}$		-1		$\frac{-2+\sqrt{2}}{2}$	
+	0	−	X	−	0	+
∪	F	∩		∩	F	∪



$$\int \frac{1}{1+\sqrt{4x^2+1}}\,dx = \qquad \cosh^2(x) = \sinh^2(x)+1$$

$$2x = \sinh(t) \Rightarrow x = \frac{\sinh(t)}{2}$$

$$\Rightarrow dx = \frac{\cosh(t)}{2}\,dt$$

$$= \int \frac{1}{1+\sqrt{(2x)^2+1}}\,dx =$$

$$t = \operatorname{arcsinh}(2x)$$

$$= \int \frac{1}{1+\sqrt{\sinh^2(t)+1}} \cdot \frac{\cosh(t)}{2}\,dt = \frac{1}{2}\int \frac{\cosh(t)}{1+\sqrt{\cosh^2(t)}}\,dt$$

$$u = \cosh(t)$$

$$t = \operatorname{arccosh}(u)$$

$$= \frac{1}{2}\int \frac{1+1\cosh(t)}{1+\cosh(t)}\,dt = \frac{1}{2}\int 1 - \frac{1}{1+\cosh(t)}\,dt = \quad dt = \frac{1}{\sqrt{u^2-1}}\,du$$

$$\cosh(t) = \frac{e^t+e^{-t}}{2} > 0$$

$$= \frac{1}{2}\int \left(1-\frac{1}{u}\right)\cdot\frac{1}{\sqrt{u^2-1}}\,du = \frac{1}{2}\int \frac{1}{\sqrt{u^2-1}} - \frac{1}{u\sqrt{u^2-1}}\,du$$

$$\star = \frac{1}{2}\int 1 - \frac{1}{1+\cosh(t)}\,dt = \frac{1}{2}\int 1 - \frac{1}{1+\frac{e^t+e^{-t}}{2}}\,dt =$$

$$\cosh(t) = \frac{e^t+e^{-t}}{2}$$

$$= \frac{1}{2}\int 1 - \frac{1}{\frac{2+e^t+\frac{1}{e^t}}{2}}\, dt = \frac{1}{2}\int 1 - \frac{2e^t}{e^{2t}+2e^t+1}\, dt =$$

$u = e^t \quad t = \log(u) \quad dt = \frac{1}{u}\, du$

$$= \frac{1}{2}\int \left(1 - \frac{2u}{u^2+2u+1}\right)\cdot \frac{1}{u}\, du =$$

$$= \frac{1}{2}\int \frac{1}{u} - \frac{2}{(u+1)^2}\, du = \frac{1}{2}\left(\log|u| + \frac{2}{u+1}\right) + c$$

$$= \frac{1}{2}\left(\log|e^t| + \frac{2}{e^t+1}\right) + c = \frac{1}{2}\left(t + \frac{2}{e^t+1}\right) + c =$$

$$= \frac{1}{2}\left(\operatorname{arcsinh}(2x) + \frac{2}{e^{\operatorname{arcsinh}(2x)}+1}\right) + c$$

$$\int \frac{f'(x)}{[f(x)]^n} \overset{\substack{n \neq 1\\ \downarrow}}{=} \frac{1}{1-n}\cdot\frac{1}{[f(x)]^{n-1}} + c$$

$$\int f'(x)\cdot[f(x)]^{-n}\, dx = \frac{[f(x)]^{-n+1}}{-n+1} + c = \frac{1}{1-n}\cdot\frac{1}{[f(x)]^{n-1}}$$

$$\int_0^{+\infty} \frac{|\sinh(x) - \alpha \sin(x)|}{x^2 \beta^x} dx \qquad \alpha \in \mathbb{R} \quad \beta > 0$$

Studiare cosa succede per $x \to 0^+$ e per $x \to +\infty$

- In 0^+:

$$|\sinh(x) - \alpha \sin(x)| = |x + o(x) - \alpha(x + o(x))| =$$

$$= |(1-\alpha)x + o(x)| = |1-\alpha| x + o(x)$$

$$\frac{|\sinh(x) - \alpha \sin(x)|}{x^2 \beta^x} \overset{x\to 0^+}{\sim} \frac{|1-\alpha| x}{x^2 \cdot 1} = \frac{|1-\alpha|}{x}$$

$$\lim_{x \to 0^+} \frac{\frac{|\sinh(x) - \alpha \sin(x)|}{x^2 \beta^x}}{\frac{|1-\alpha| x}{x^2}} = 1$$

Se $\alpha \neq 1$: $\displaystyle\int_0^2 \frac{|\sinh(x) - \alpha \sin(x)|}{x^2 \beta^x} dx$

$$\sim \int_0^1 \frac{|1-\alpha|}{x} dx \quad \text{divergente}$$

$$\to \lim_{t \to 0^+} \int_t^1 \frac{|1-\alpha|}{x} dx = \lim_{t \to 0^+} \left[|1-\alpha| \log(1) - |1-\alpha| \log(t) \right]$$

$$\int_0^1 \frac{1}{x^\gamma}\,dx \quad \text{converge} \iff \gamma < 1$$

Se $\alpha = 1$:

$$|\sinh(x) - \sin(x)| = \left| x + \frac{x^3}{6} + o(x^3) - \left(x - \frac{x^3}{6} + o(x^3)\right)\right| =$$

$$= \left| \frac{x^3}{3} + o(x^3) \right| = \frac{x^3}{3} + o(x^3)$$

$$\Rightarrow \frac{|\sinh(x) - \sin(x)|}{x^2 \beta^x} \sim \frac{\frac{x^3}{3}}{x^2} = \frac{x}{3}$$

$$\int_0^1 \frac{x}{3}\,dx \quad \text{é convergente} \quad \left(\frac{1}{x^{-1}} = x\right)$$

$$\Rightarrow \int_0^1 \frac{|\sinh(x) - \sin(x)|}{x^2 \beta^x}\,dx \quad \text{é convergente}$$

Se $\alpha = 1$ l'integrale converge in 0.

A $+\infty$:

$$|\sinh(x) - \alpha \sin(x)| \overset{x\to+\infty}{\sim} \frac{e^x}{2}$$

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\Rightarrow \frac{|\sinh(x) - 2\sin(x)|}{x^2 \beta^x} \sim \frac{\frac{e^x}{2}}{x^2 \beta^x} = \frac{\left(\frac{e}{\beta}\right)^x}{2x^2}$$

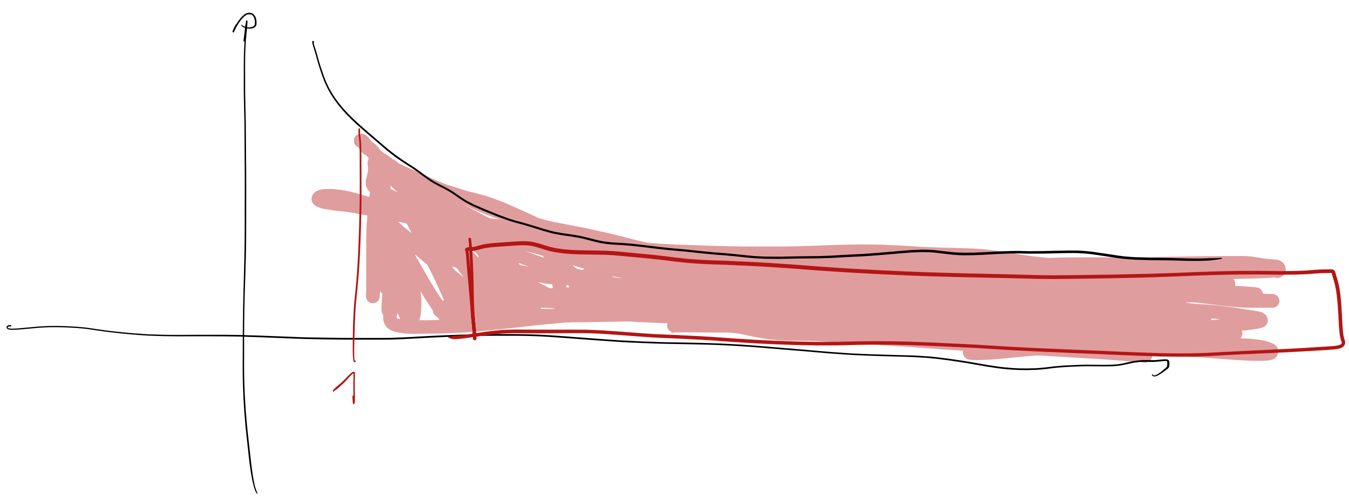
$$f(x) \overset{x \to +\infty}{\sim} g(x) \Leftrightarrow \lim_{x \to +\infty} \frac{f(x)}{g(x)} = 1$$

Se $\frac{e}{\beta} > 1$, allora $\lim_{x \to +\infty} \frac{\left(\frac{e}{\beta}\right)^x}{2x^2} = +\infty$

$\Rightarrow$ Se $\beta < e$, l'integrale diverge a $+\infty$.

$$\int_1^{+\infty} f(x)\,dx \quad , \; f(x) > 0$$

Se $\lim_{x \to +\infty} f(x) \neq 0$, allora l'integrale diverge



Se $\frac{e}{\beta} \le 1$: $\frac{\left(\frac{e}{\beta}\right)^x}{2x^2} \le \frac{1}{2x^2}$

e $\int_1^{+\infty} \frac{1}{2x^2}\,dx$ è convergente

$\int_1^{+\infty} \frac{1}{x^\gamma}\,dx$ converge $\Leftrightarrow \gamma > 1$

$\Rightarrow$ Per il teo del confronto $\int_2^{+\infty} \frac{\left(\frac{e}{\beta}\right)^x}{2x^2}$ converge

se $\beta \ge e$

$\Rightarrow \int_0^{+\infty} \frac{|\sinh(x) - \alpha \sin(x)|}{x^2 \beta^x}\,dx$ converge

se e solo se $\alpha = 1$, $\beta \ge e$

$$\lim_{x \to +\infty} (1)^x = \lim_{x \to +\infty} 1 = 1$$

$$\lim_{x \to +\infty} \left(1 + \frac{1}{x}\right)^x \overset{[1^\infty]}{=} e$$

$$\sum_{n=1}^{+\infty} \left[1 - \cos\left(\frac{1}{\sqrt{n}}\right)\right]^{\alpha}$$

$1 - \cos\left(\frac{1}{\sqrt{n}}\right) \geq 0 \Rightarrow$ La serie è a termini di segno positivo.

$\cos(x) \overset{x \to 0}{=} 1 - \frac{x^2}{2} + o(x^2)$

$\left[1 - \cos\left(\frac{1}{\sqrt{n}}\right)\right]^{\alpha} \sim \frac{1}{2n^{\alpha}}$ $\qquad \cos\left(\frac{1}{\sqrt{n}}\right) \overset{n \to +\infty}{=} 1 - \frac{1}{2}\frac{1}{n} + o\left(\frac{1}{n}\right)$

confronto con la serie armonica gener.

$\sum_{n=1}^{+\infty} \frac{1}{2n^{\alpha}}$ converge $\Leftrightarrow \alpha > 1 \Leftrightarrow \alpha > 1$

$\Rightarrow$ Per il crit. del confronto asintotico:

$\sum_{n=1}^{+\infty} \left[1 - \cos\left(\frac{1}{\sqrt{n}}\right)\right]^{\alpha}$ converge $\Leftrightarrow \alpha > 1$

$$\sum_{n=1}^{+\infty} \frac{1}{n}$$

Se $\sum_{k=1}^{+\infty} a_k$ converge, allora $\lim_{n \to +\infty} \sum_{k=n}^{p_n} = 0$

$$\sum_{k=1}^{+\infty} \frac{1}{k}$$

$$\sum_{k=n}^{2n} \frac{1}{k} = \overset{\geq \frac{1}{2n}}{\frac{1}{n}} + \overset{\geq \frac{1}{2n}}{\frac{1}{n+1}} + \cdots + \overset{\geq \frac{1}{2n}}{\frac{1}{2n}} \geq$$

$$\geq \frac{1}{2n} + \frac{1}{2n} + \cdots + \frac{1}{2n} = \sum_{k=n}^{2n} \frac{1}{2n} =$$

$$= \frac{n+1}{2n} = \frac{1}{2} + \frac{1}{2n} \xrightarrow{n \to +\infty} \frac{1}{2}$$