

$$\bullet f(x) = |x|$$

Non esiste derivata di $f(x)$ in $x_0=0$

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'_+(x_0) := \lim_{\substack{x \rightarrow x_0^+ \\ x > x_0}} \frac{f(x) - f(x_0)}{x - x_0}$$

$$f'_-(x_0) := \lim_{\substack{x \rightarrow x_0^- \\ x < x_0}} \frac{f(x) - f(x_0)}{x - x_0}$$

Se $\exists f'(x_0)$ allora $f'_+(x_0) = f'_-(x_0)$

$$\lim_{x \rightarrow 0} \frac{|x| - |0|}{x - 0} = \lim_{x \rightarrow 0} \frac{|x|}{x} \text{ non esiste}$$

$$\begin{aligned} \lim_{x \rightarrow 0^+} \frac{|x|}{x} &= \lim_{x \rightarrow 0^+} \frac{\cancel{x}^{>0^+}}{\cancel{x}^{>0^+}} = 1 \\ \lim_{x \rightarrow 0^-} \frac{|x|}{x} &= \lim_{x \rightarrow 0^-} \frac{-\cancel{x}^{<0^-}}{\cancel{x}^{<0^-}} = -1 \end{aligned} \quad \left. \begin{array}{l} f'_+(x_0) \neq f'_-(x_0) \end{array} \right\}$$

$$\Rightarrow \cancel{\exists f'(x_0)}$$

$$\lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh(x^2) + x^{7/3} \log(x)} =$$

$$e^{x-2x^2} = 1 + (x-2x^2) + o(x-2x^2)$$

$$o(x-2x^2) = o(x)$$

$$f(x) \in o(x-2x^2)$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{x-2x^2} = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \left[\frac{f(x)}{x} \right] \cdot \frac{1}{(1-2x)} = 0$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{f(x)}{x} = 0$$

$$\Rightarrow f(x) \in o(x)$$

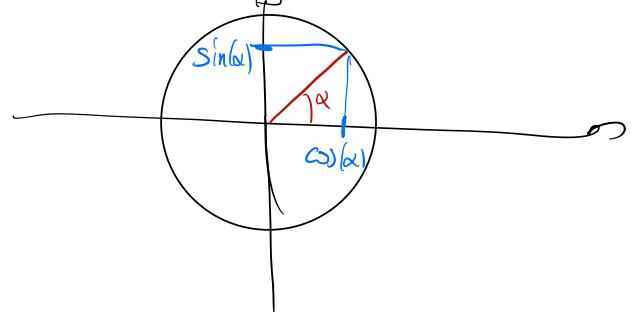
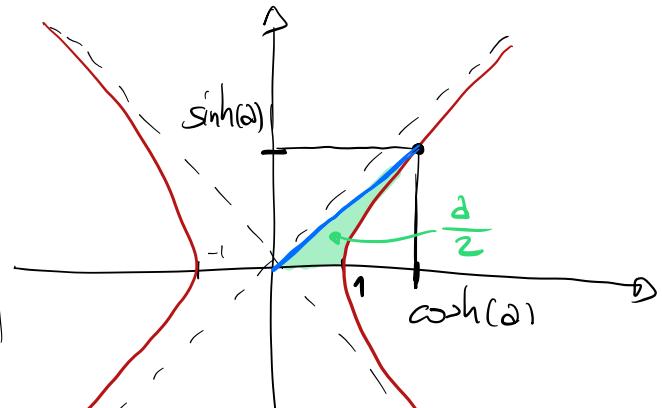
In generale,

$$o(x^m + x^{m+1} + \dots) = o(x^m)$$

$$\begin{aligned} & e^{x-2x^2} \\ \Rightarrow & e^x = 1 + (x-2x^2) + o(x-2x^2) = \\ & = 1 + x - 2x^2 + o(x) \\ & = 1 + x + o(x) \end{aligned}$$

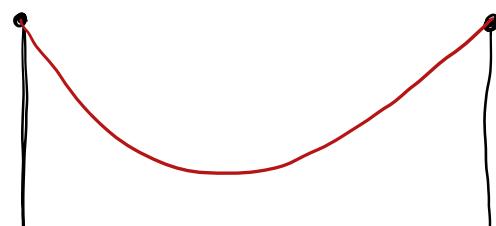
$$\cosh^2(x) - \sinh^2(x) = 1$$

$$x^2 - y^2 = 1$$



$$f(x) = 2 \cdot \cosh\left(\frac{x}{2}\right), \quad x > 0$$

si chiama catenaria



$$e^y = 1 + y + \frac{y^2}{2} + o(y^2)$$

Svilupperemo fino al secondo ordine:

$$\begin{aligned}
 e^{x-2x^2} &= 1 + (x-2x^2) + \frac{(x-2x^2)^2}{2} + o((x-2x^2)^2) = \\
 &= 1 + x - 2x^2 + \frac{x^2 - 4x^3 + 4x^4}{2} + o(x^2 - 4x^3 + 4x^4) = \\
 &= 1 + x - 2x^2 + \frac{x^2}{2} - 2x^3 + 2x^4 + o(x^2) \\
 &= 1 + x - \frac{3}{2}x^2 + o(x^2)
 \end{aligned}$$

$$\begin{aligned}
 \lim_{x \rightarrow 0^+} \frac{e^{x-2x^2} - 1 - x}{\sinh(x^2) + x^{\frac{7}{3}} \log(x)} &= \sinh(x) = x + o(x) \\
 &= x + \frac{x^3}{6} + o(x^3) \\
 = \lim_{x \rightarrow 0^+} \frac{\left(1 + x - \frac{3}{2}x^2 + o(x^2)\right) - 1 - x}{x^2 + o(x^2) + x^{\frac{7}{3}} \log(x)} &\stackrel{?}{=} \log(1+x) = x - \frac{x^2}{2} + o(x^2) \\
 &\quad \xrightarrow{x \rightarrow 0^+} \log(1+\boxed{x-1}) \\
 x^{\frac{7}{3}} \log(x) &\in o(x^2) \quad ???
 \end{aligned}$$

$$\lim_{x \rightarrow 0^+} \frac{x^{\frac{7}{3}} \log(x)}{x^2} = \lim_{x \rightarrow 0^+} x^{\frac{7}{3}-2} \log(x) = \lim_{x \rightarrow 0^+} x^{\frac{1}{3}} \log(x) = 0$$

$$\Rightarrow x^{\frac{7}{3}} \log(x) \in o(x^2)$$

$$\lim_{x \rightarrow 0^+} x^\beta \log(x) = \lim_{y \rightarrow +\infty} \frac{\log(\frac{1}{y})}{y^\beta} = \lim_{y \rightarrow +\infty} -\frac{\log(y)}{y^\beta} = 0$$

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 degli infiniti

$$\log\left(\frac{a}{b}\right) = \log(a) - \log(b) \quad | \quad \log(a \cdot b) = \log(a) + \log(b)$$

$$\log\left(\frac{1}{b}\right) = \log(1) - \log(b) = -\log(b)$$

$$\log(a^x) = x \cdot \log(a)$$

$$\log\left(\frac{1}{b}\right) = \log(b^{-1}) = -1 \cdot \log(b)$$

$$\begin{aligned}
 & \underset{x \rightarrow 0^+}{\lim} \frac{1 + x - \frac{3}{2}x^2 + o(x^2) - 1 - x}{x^2 + o(x^2) + x^{7/3} \log(x)} = \\
 & \underset{x \rightarrow 0^+}{\lim} \frac{-\frac{3}{2}x^2 + o(x^2)}{x^2 + o(x^2)} = \underset{x \rightarrow 0^+}{\lim} \frac{x^2 \left(-\frac{3}{2} + \frac{o(x^2)}{x^2} \right)}{x^2 \left(1 + \frac{o(x^2)}{x^2} \right)} = \\
 & = -\frac{3}{2}
 \end{aligned}$$

$$\lim_{\substack{x \rightarrow 0^+}} \frac{-\frac{3}{2}x^2 + o(x^2)}{x^2 + o(x^2) + x^2 \log(x)} = \frac{-\frac{3}{2}}{1 + o(1) + \log(1)} = 0$$

$$\bullet f(x) = \sin(x)^{\cos(x)} = e^{\cos(x) \cdot \log(\sin(x))} \quad \boxed{x = e^{x \cdot \log(x)}}$$

$$g(x) = e^x \quad h(x) = \cos(x) \cdot \log(\sin(x))$$

$$g(h(x)) = e^{\cos(x) \cdot \log(\sin(x))} = f(x)$$

$$D(f(x)) = D(g(h(x))) = g'(h(x)) \cdot h'(x) = \textcolor{red}{\bullet}$$

$$g'(x) = e^x$$

$$\begin{aligned} h'(x) &= -\sin(x) \cdot \log(\sin(x)) + \cos(x) \cdot D(\log(\sin(x))) = \\ &= -\sin(x) \log(\sin(x)) + \cos(x) \cdot \frac{1}{\sin(x)} \cdot \cos(x) = \\ &= -\sin(x) \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \end{aligned}$$

$$\begin{aligned} \textcolor{red}{\bullet} f'(x) &= g'(h(x)) \cdot h'(x) = \\ &= e^{\cos(x) \log(\sin(x))} \cdot \left(-\sin(x) \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right) = \\ &= \sin(x)^{\cos(x)} \cdot \left(-\sin(x) \log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right) \end{aligned}$$

$$\bullet D(x^n) = n \cdot x^{n-1}$$

$$\bullet D(f(x))^n = n (f(x))^{n-1} \cdot f'(x)$$

⚠ $D((f(x))^{g(x)}) \stackrel{?}{=} g(x)(f(x))^{g(x)-1} \cdot f'(x) \cdot g'(x)$ ⚠

$$D((f(x))^{g(x)}) = D(e^{g(x)\log(f(x))}) = \dots$$

$$= (f(x))^{g(x)} \left[g'(x)\log(f(x)) + \frac{g(x) \cdot f'(x)}{f(x)} \right]$$

$$D((\sin(x))^{\cos(x)}) = \sin(x)^{\cos(x)} \left[-\sin(x)\log(\sin(x)) + \frac{\cos^2(x)}{\sin(x)} \right]$$