$$e^{x} = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + o(x^3)$$

$$= 1 + x + \frac{x^2}{2} + \boxed{o(x^2)}$$

$$f(x) = \left(\sum_{k=0}^{n} \frac{f^{(k)}(x_0)}{k!} \cdot (x - x_0)^k \right) + o\left((x - x_0)^n\right)$$

$$o(x^2) \supseteq \left\{ x^3, x^4, x^5, \ldots \right\}$$

$$\lim_{x \to 0} \frac{e^{x^2} - 1 - \log(1 + x\sin(x))}{\sqrt{1 + 2x^4} - 1}$$

$$e^{x} = 1 + x + \frac{x^2}{2} + o(x^2) \qquad \log(1+x) = x - \frac{x^2}{2} + o(x^2)$$

$$\sin(x) = x - \frac{x^3}{6} + o(x^3) \qquad (1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + o(x^2)$$

$$e^{x^2} = 1 + x^2 + \frac{x^4}{2} + o(x^4)$$

$$x \cdot o(x^3) \in o(x^4)$$

$$\log(1 + x\sin(x)) = \log\left(1 + x\left(x - \frac{x^3}{6} + o(x^3)\right)\right) =$$

$$= \log\left(1 + \underbrace{\left(x^2 - \frac{x^4}{6} + o(x^4)\right)}_{f(x)}\right) = (\star)$$

$$f(x) \xrightarrow{x \to 0} 0 \Rightarrow \log(1 + f(x)) = f(x) - \frac{[f(x)]^2}{2} + o\left([f(x)]^2\right)$$

$$(*) = x^2 - \frac{x^4}{6} + o(x^4) - \frac{\left(x^2 - \frac{x^4}{6} + o(x^4)\right)^2}{2} + \underbrace{o\left(\left(x^2 - \frac{x^4}{6} + o(x^4)\right)^2\right)}_{= o(x^4)} =$$

$$= x^2 - \frac{x^4}{6} + o(x^4) - \frac{x^4}{2} + o(x^4) = x^2 - \frac{2}{3}x^4 + o(x^4)$$

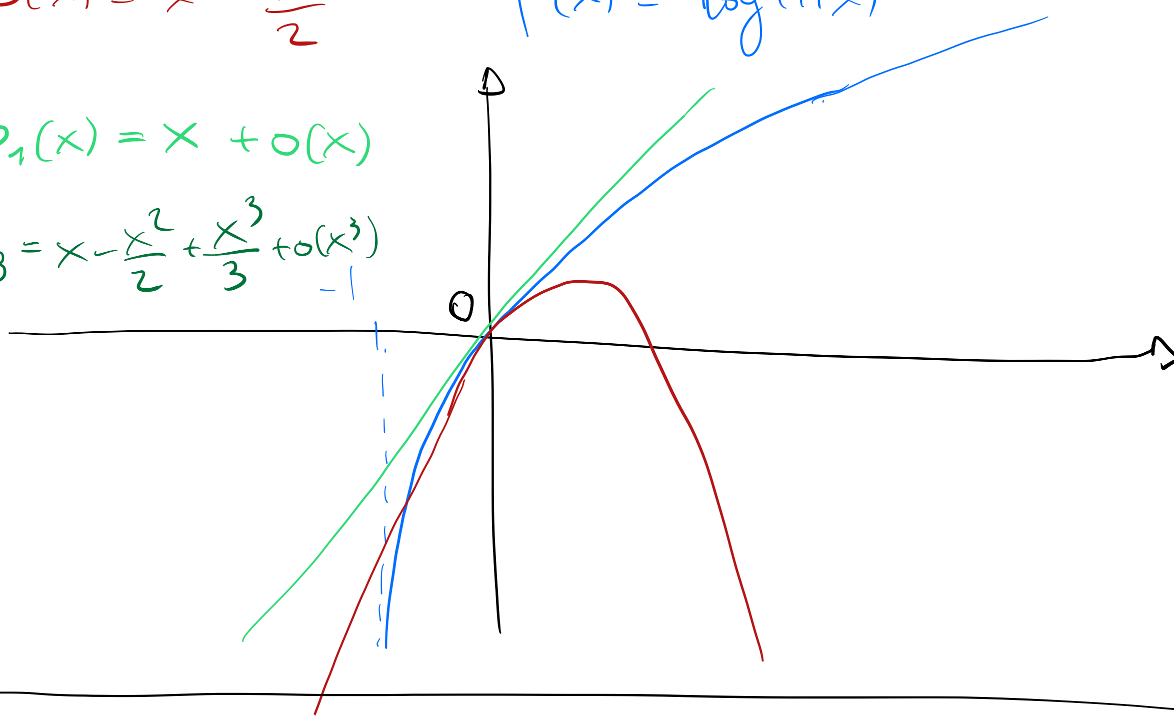
$x \to +\infty$ $\qquad \log\left(1 + \frac{1}{x}\right) = \frac{1}{x} - \frac{\left(\frac{1}{x}\right)^2}{2} + o\left(\left(\frac{1}{x}\right)^2\right)$

$$\log(1+x) = x - \frac{x^2}{2} + o(x^2)$$

$P(x) = x - \frac{x^2}{2}$ $\qquad f(x) = \log(1+x)$

$P_1(x) = x + o(x)$

$P_3 = x - \frac{x^2}{2} + \frac{x^3}{3} + o(x^3)$



$$\sqrt{1 + 2x^4} = (1+2x^4)^{\frac{1}{2}} = 1 + \frac{1}{2}\cdot(2x^4) + o(x^4)$$

$$\Rightarrow \lim_{x \to 0} \frac{e^{x^2} - 1 - \log(1 + x\sin(x))}{\sqrt{1+2x^4} - 1} =$$

$$= \lim_{x \to 0} \frac{1 + x^2 + \frac{x^4}{2} + o(x^4) - 1 - \left(x^2 - \frac{2}{3}x^4 + o(x^4)\right)}{1 + x^4 + o(x^4) - 1} =$$

$$= \lim_{x \to 0} \frac{\frac{x^4}{2} + \frac{2}{3}x^4 + o(x^4)}{x^4 + o(x^4)} =$$

$$= \lim_{x \to 0} \frac{\frac{7}{6}x^4 + o(x^4)}{x^4 + o(x^4)} = \frac{7}{6}$$

$$o(x^4) - o(x^4) = f(x) - g(x) = o(x^4)$$

$$f(x) \in o(x^4) \qquad g(x) \in o(x^4)$$

$$\sin(x) \cdot \sinh(x) =$$

$$\sin(x) = x - \frac{x^3}{6} + o(x^3)$$

$$\sinh(x) = x + \frac{x^3}{6} + o(x^3)$$

$$= (x + o(x))\left(x + \frac{x^3}{6} + o(x^3)\right) =$$

$$= x^2 + o(x^2) + \frac{x^4}{6} + o(x^4) = x^2 + o(x^2)$$

$$\lim_{x \to +\infty} \sqrt{1 + x + x^2} - \log(1 + e^x) =$$

$$= \lim_{x \to +\infty} \sqrt{x^2\left(\frac{1}{x^2} + \frac{1}{x} + 1\right)} - \log\left(e^x\left(\frac{1}{e^x} + 1\right)\right) =$$

$$= \lim_{x \to +\infty} x\sqrt{1 + \frac{1}{x} + \frac{1}{x^2}} - \left(\log(e^x) + \log\left(\frac{1}{e^x} + 1\right)\right) = (\star)$$

$\sqrt{1+x} = 1 + \frac{1}{2}x + o(x)$

$$\sqrt{1+\boxed{\frac{1}{x}+\frac{1}{x^2}}} = 1 + \frac{1}{2}\left(\frac{1}{x}+\frac{1}{x^2}\right) + o\left(\underbrace{\frac{1}{x}}_{\sim x^{-1}} + \underbrace{\frac{1}{x^2}}_{\sim x^{-2}}\right)$$

$x \to +\infty \rightarrow 0$

$$= 1 + \frac{1}{2x} + o\left(\frac{1}{x}\right)$$

$$\frac{1}{e^x} + o\left(\frac{1}{e^x}\right)$$

$$||$$

$$(\star) = \lim_{x\to+\infty} x\left(1+\frac{1}{2x}+o\left(\frac{1}{x}\right)\right) - x - \log\left(\frac{1}{e^x}+1\right) =$$

$\rightarrow \log(1) = 0$

$$= \lim_{x\to+\infty} \cancel{x} + \frac{1}{2} + o(1) - \cancel{x} - \log\left(\frac{1}{e^x}+1\right) = \frac{1}{2}$$

$\rightarrow \infty$

$$\lim_{x\to+\infty} x\sqrt{1+\boxed{\frac{1}{x}+\frac{1}{x^2}}} - x \neq$$

$\rightarrow 0$

$$\neq \lim_{x\to+\infty} x - x = 0 \text{ ! NO!}$$

$$\lim_{x\to+\infty} x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}} - x = \lim_{x\to+\infty} x\left(1+o(1)\right) - x =$$

$$= \lim_{x\to+\infty} o(x) = (\star)$$

$$f(x) \in \underset{+\infty}{o}(x) \iff \lim_{x\to+\infty} \frac{f(x)}{x} = 0$$

$f(x) = x^{\frac{1}{2}}$ $\quad (\star) = +\infty$

$f(x) = \frac{1}{2}$ $\quad (\star) = \frac{1}{2}$

$f(x) = x^{-2}$ $\quad (\star) = 0$

$$\lim_{x \to 0} o_0(x) = 0$$

$$f(x) \in o(x) \iff \lim_{x \to 0} \frac{f(x)}{x} = 0$$

$f(x) = x^2$

$f(x) = x^5$

$f(x) = x^{\frac{3}{2}}$

$$\lim_{x \to 0^+} \frac{x^x - e^x}{\cos(x) - e^x} = (\star)$$

$$x^x = e^{\boxed{x \log(x)} \to 0} \Rightarrow 1 + x\log(x) + o(x\log(x))$$

$$e^{x\log(x)} - e^x = (e^x)^{\log(x)} - e^x$$

$$(\star) = \lim_{x \to 0^+} \frac{1 + x\log(x) + o(x\log(x)) - (1 + x + o(x))}{1 - \frac{x^2}{2} + o(x^2) - (1 + x + o(x))} =$$

$$\overset{!}{=} \lim_{x \to 0^+} \frac{x\log(x) + o(x\log(x)) - x + o(x)}{-x + o(x)}$$

$x\log(x) \in o(x)$??? No!

$$\lim_{x \to 0^+} \frac{x\log(x)}{x} = \lim_{x \to 0^+} \log(x) = -\infty$$

$$x \in o(x\log(x)) \Leftrightarrow \lim_{x \to 0^+} \frac{x}{x\log(x)} = 0$$

$$\overset{!}{=} \lim_{x \to 0^+} \frac{x\log(x) + o(x\log(x))}{-x + o(x)} =$$

$$= \lim_{x \to 0^+} \frac{\cancel{x}\underbrace{\log(x)}_{\to -\infty}\left(1 + \underbrace{\frac{o(x\log(x))}{x\log(x)}}_{\to 0}\right)}{\cancel{x}\left(-1 + \underbrace{\frac{o(x)}{x}}_{\to 0}\right)} = +\infty$$