$$\int \frac{1}{x^{\alpha}+x^{\beta}}\,dx$$

$$\int_0^{+\infty} \frac{1}{x^{\beta}(x^{\alpha}+3)}\,dx \quad , \alpha, \beta \in \mathbb{R}$$

<u>In 0</u>: $\frac{1}{x^{\alpha}+3} \leq \frac{1}{3}$

Teo confronto: $\frac{1}{x^{\beta}(x^{\alpha}+3)} \leq \frac{1}{3x^{\beta}}$

Se $\beta < 1$, l'integrale converge in 0.

Teo confronto asintotico

$$\frac{1}{x^{\beta}(x^{\alpha}+3)} \overset{x\to 0^+}{\sim} \begin{cases} \frac{1}{3x^{\beta}} & \text{se } \alpha > 0 \\ \frac{1}{4x^{\beta}} & \text{se } \alpha = 0 \\ \frac{1}{x^{\alpha+\beta}} & \text{se } \alpha < 0 \end{cases}$$

$\boxed{\alpha < 0}$

$$\frac{1}{x^{\beta}\left(\frac{1}{x^{-\alpha}}+3\right)} = \frac{x^{-\alpha}}{x^{\beta}(1+3x^{-\alpha})} = \frac{x^{-\alpha}}{x^{\beta}+3x^{\beta}x^{-\alpha}} \overset{x\to 0^+}{\sim} \frac{x^{-\alpha}}{x^{\beta}}$$

<u>Conv</u>: $(\alpha \geq 0 \wedge \beta < 1) \vee (\alpha < 0 \wedge \alpha + \beta < 1)$

$$\sum_{n=1}^{+\infty} \frac{n^2+n^3}{n^4+n^5} \sim \sum_{n=1}^{+\infty} \frac{n^3}{n^5}$$

$$\sum_{n=1}^{+\infty} \frac{1}{n^2+n^3} \sim \sum_{n=1}^{+\infty} \frac{1}{n^3}$$

Teo del confronto asintotico

$$\frac{1}{n^2+n^3} \overset{n\to+\infty}{\sim} \frac{1}{n^3}$$

$$\int_0^{+\infty} f_\alpha(x)\,dx$$

In 0 : l'int. conv. $\Leftrightarrow$ $\alpha \in A$

A $+\infty$: " " $\Leftrightarrow$ $\alpha \in B$

$\Rightarrow \int_0^{+\infty} f_\alpha(x)\,dx$ converge $\Leftrightarrow$ $\alpha \in A \cap B$

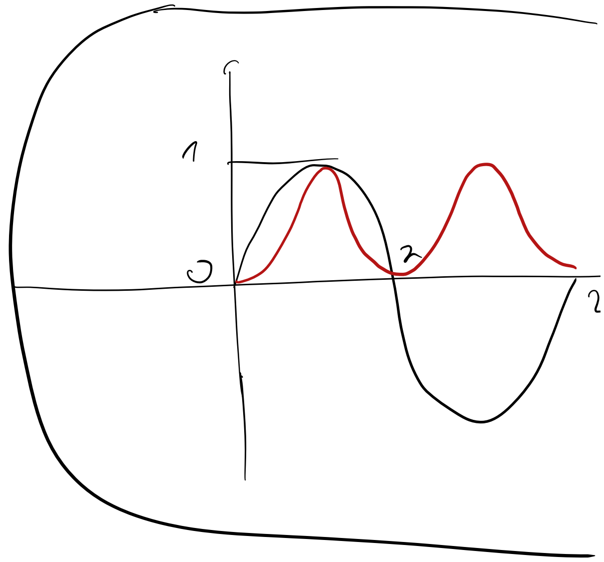
$$I = \int_0^{+\infty} \frac{\sin^2\left(\frac{1}{t^2}\right)}{t^{\alpha}(1+t)}\, dt \quad , \ \alpha \in \mathbb{R}$$

<u>In 0</u>:

$$\sin^2\left(\frac{1}{t^2}\right) \leq 1$$

$$\frac{\sin^2\left(\frac{1}{t^2}\right)}{t^{\alpha}(1+t)} \leq \frac{1}{t^{\alpha}}$$

$$\frac{1}{1+t} \leq 1$$



Per il teo del confronto:

in 0 l'integrale converge se $\alpha < 1$

<u>A $+\infty$</u>:

$$\sin\left(\frac{1}{t}\right) \underset{t\to+\infty}{\sim} \frac{1}{t} \quad \left(\sin(x) \underset{x\to 0}{\sim} x\right)$$

$$\frac{\sin^2\left(\frac{1}{t^2}\right)}{\underbrace{t^{\alpha}(1+t)}_{= t^{\alpha} + t^{\alpha+1}}} \underset{t\to+\infty}{\sim} \frac{\frac{1}{t^4}}{t^{\alpha+1}} = \frac{1}{t^{\alpha+5}}$$

Per il teo del confronto asintotico, l'int. conv.

a $+\infty$ $\iff$ $\alpha + 5 > 1 \iff \alpha > -4$

$$\Rightarrow \int_0^{+\infty} \frac{\sin^2\left(\frac{1}{t^2}\right)}{t^{2}(1+t)}\, dt \text{ converge } \iff -4 < \alpha < 1$$

$$t^t(1+t) = t^t\cdot t + t^t = t^{t+1} + t^t \overset{t\to+\infty}{\sim} t^{t+1}$$

$$\lim_{t\to+\infty} \frac{t^t(1+t)}{t^{t+1}} = \lim_{t\to+\infty} \frac{\cancel{t^t}(1+t)}{\cancel{t^t}\cdot t} = \lim_{t\to+\infty}\frac{1+t}{t} = 1$$

P.C.

$$\begin{cases} y' = \underbrace{y}_{g(y)}\,\underbrace{\sinh^2(t)}_{f(t)} \\ y(\underbrace{0}_{t_0}) = \underbrace{1}_{y_0} \end{cases}$$

- Se $g(y_0) = 0$, allora $y(t) = y_0$ è sol del P.C.

$g(1) = 0$!! , $g(y) = y$, $g(1) = 1 \neq 0$

$\Rightarrow$ P.C. non ammette sol costanti

- $y' = y\sinh^2(t) \rightsquigarrow \dfrac{y'}{y} = \sinh^2(t)$

$$\int_{1 \,\leftarrow\, y_0}^{y} \frac{1}{u}\,du = \int_{0 \,\leftarrow\, t_0}^{t} \sinh^2(z)\,dz \rightsquigarrow \text{soluzione}$$

$\int \sin^2(t)\,dt =$ per parti

$$\int \frac{1}{y}\,dy = \int \sinh^2(t)\,dt$$

$$\ln|y| = \int \left(\frac{e^{t}-e^{-t}}{2}\right)^2 dt$$

$$\ln|y| = \int \frac{e^{2t}-2+e^{-2t}}{4}\, dt$$

$$\ln|y| = \frac{1}{4}\left(\frac{e^{2t}}{2} - 2t - \frac{e^{-2t}}{2}\right) + c$$

$$\ln|y| = \frac{1}{4}\left(\sinh(2t) - 2t\right) + c$$

Per trovare c, $y_0 = 1$, $t_0 = 0$

$$\ln|1| = \frac{1}{4}\left(\sinh(0) - 2\cdot 0\right) + c \iff c = 0$$

$$\ln|y| = \frac{1}{4}\left(\sinh(2t) - 2t\right)$$

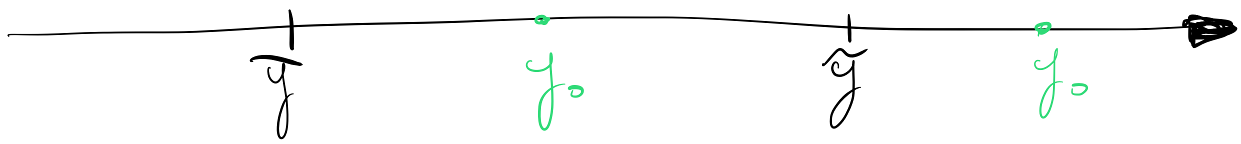
tolgo il valore assoluto perché $y(t) > 0$

$$\ln(y) = \frac{1}{4}\left(\sinh(2t) - 2t\right)$$

$$y(t) = e^{\frac{1}{4}\left(\sinh(2t) - 2t\right)}$$

$$\int_1^y \frac{1}{u}\, du = \int_0^t \sinh^2(z)\, dz$$

$$\ln|y| = \frac{1}{4}\left(\sinh(2t) - 2t\right) \qquad y > 0$$



$$y' = y \cdot f(t)$$

$$y' = \underbrace{(y^2+4)}_{g(y)} \cdot \underbrace{e^t \cos(t) \cdot t^3}_{f(t)}$$

$$y' = y \cdot t^2 + t^2$$

$$y' = (y+1)\,t^2$$

$$y' = a(x)\,y + b(x)$$ eq. lin. primo ordine

$$y(x) = e^{A(x)}\left(c + \int b(x)\, e^{-A(x)}\,dx\right)$$

$$A(x) = \int a(x)\,dx$$

Studio funzione:

$$f(x) = \log\left(\frac{x-7}{5-x}\right)$$

- Dominio

$$\begin{cases} 5-x \neq 0 \\ \frac{x-7}{5-x} > 0 \end{cases} \quad \begin{cases} x \neq 5 \\ 5 < x < 7 \end{cases}$$

$$D(f) = \{ x \mid x \in \mathbb{R} \text{ e } 5 < x < 7 \}$$

- Simmetrie: PARI/DISPARI

No!

- Segno: $\log\left(\frac{x-7}{5-x}\right) \geq 0$

$$\Leftrightarrow \frac{x-7}{5-x} \geq 1 \Leftrightarrow \frac{x-7}{5-x} - 1 \geq 0$$

$$\Leftrightarrow \frac{2x-12}{5-x} \geq 0 \Leftrightarrow \frac{2(x-6)}{5-x} \geq 0 \Leftrightarrow 5 < x \leq 6$$

$f(x) > 0 \Leftrightarrow 5 < x < 6$ | $f(x) < 0 \Leftrightarrow 6 < x < 7$

$f(x) = 0 \Leftrightarrow x = 6$

- **Limiti** (agli estremi del dominio):

$$\lim_{x\to 5^+} \log\left(\frac{\overset{\to -2}{x-7}}{\underset{\to 0^-}{5-x}}\right) = +\infty \quad \Rightarrow \quad x=5 \text{ è asintoto verticale}$$

$$\lim_{x\to 7^-} \log\left(\underbrace{\frac{\overset{\to 0^-}{x-7}}{\underset{\to -2}{5-x}}}_{\to 0^+}\right) = -\infty \quad \Rightarrow \quad x=7 \text{ è asintoto verticale}$$

- Crescenza/decrescenza di $f(x) = \log\left(\frac{x-7}{5-x}\right)$

$$f'(x) = \frac{1}{\frac{x-7}{5-x}} \cdot \frac{5-x+(x-7)}{(5-x)^2} =$$

$$= \frac{-2}{(x-7)(5-x)}$$

$$f'(x) \geq 0 \quad \Leftrightarrow \quad \frac{-2}{(x-7)(5-x)} \geq 0$$

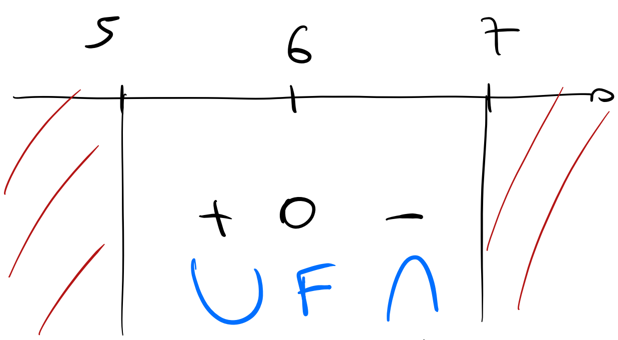
		5		7	
N	$-$		$-$		$-$
$(x-7)$	$-$		$-$	0	$+$
$(5-x)$	$+$	0	$-$		$-$
	$-$	$\times$	$-$	$\times$	$-$

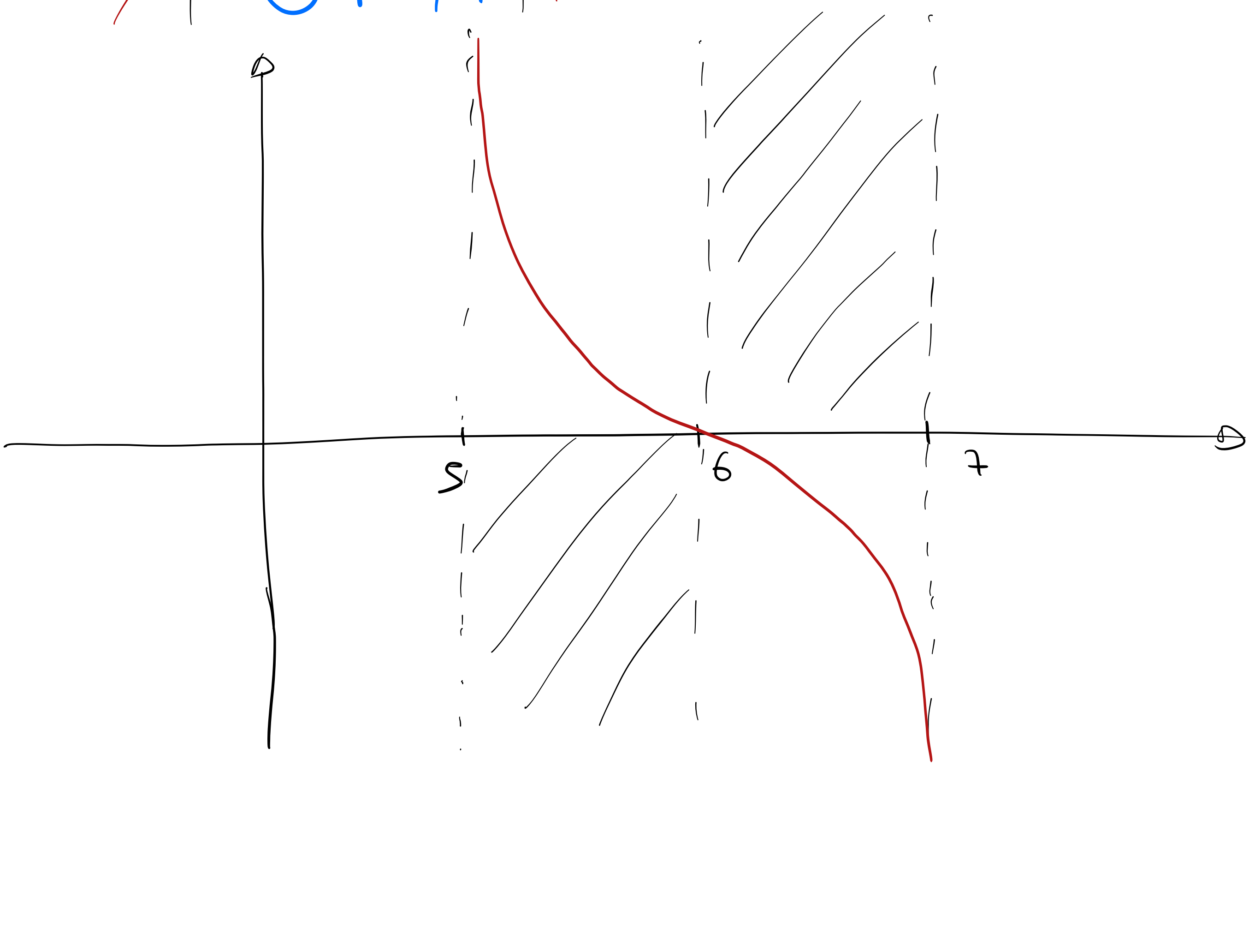
$$\Rightarrow f'(x) < 0 \quad \forall x \in D$$

$$f'(x) = \frac{-2}{(x-7)(5-x)}$$

$$f''(x) = \frac{2\left[(5-x)+(-1)(x-7)\right]}{(x-7)^2(5-x)^2} = \frac{2(12-2x)}{(x-7)^2(5-x)^2} =$$

$$= \frac{4(6-x)}{(x-7)^2(5-x)^2}$$





$$\sum_{k=2}^{+\infty} \frac{1}{k\log(k)}$$

Teo confronto serie <-> integrale

$$\int_2^{+\infty} \underbrace{\frac{1}{x\log(x)}}_{f(x)} dx$$

- $f(x)$ continua
- $f(x)$ positiva
- $f(x)$ monotona decrescente

$$f'(x) = -\frac{\log(x)+1}{(x\log(x))^2} \Rightarrow f'(x) < 0 \quad \text{se } x > 2$$

$$y = \log(x) \downarrow \quad x = e^y \quad dx = e^y dy$$

$$\int_2^{+\infty} \frac{1}{x\log(x)} dx = \lim_{t\to+\infty} \int_2^t \frac{1}{x\log(x)} dx =$$

$$= \lim_{t\to+\infty} \int_{\log(2)}^{\log(t)} \frac{1}{e^y \cdot y} \cdot e^y dy = \lim_{t\to+\infty} \int_{\log(2)}^{\log(t)} \frac{1}{y} dy =$$

$$= \lim_{t\to+\infty} \Big[\log|y|\Big]_2^{\log(t)} = \lim_{t\to+\infty} \log(\log(t)) - \log(\log(2))$$

$$= +\infty$$

$$\Rightarrow \sum_{k=2}^{+\infty} \frac{1}{k\log(k)} \quad \text{diverge}$$

$$\sum_{k=2}^{+\infty} \frac{1}{k^{\alpha} \log^{\beta}(x)} \quad \text{converge} \iff \begin{array}{c} \alpha > 1 \\ \vee \\ \alpha = 1,\ \beta > 1 \end{array}$$

$$\sum_{k=2}^{+\infty} \frac{1}{\log(k) \cdot k}$$

$\log(k) < k^{\alpha}$ vera definitivamente

$$\frac{1}{\log(k)\,k} > \frac{1}{k^{1+\alpha}}$$

$$\frac{a_{k+1}}{a_k} = \frac{\log(k) \cdot k}{\log(k+1) \cdot (k+1)}$$