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004-059-195

CS161 Homework 5

1. Use truth tables (worlds) to show that the following pairs of sentences are equivalent:

   - $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$

P	**Q**	**¬P**	**¬Q**	**P⇒ ¬Q**	**Q⇒ ¬P**
True	True	False	False	False	False
True	False	False	True	True	True
False	True	True	False	True	True
False	False	True	True	True	True

Therefore, the statements $P \Rightarrow \neg Q$, $Q \Rightarrow \neg P$ are equivalent.

   - $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$

P	**Q**	**¬P**	**¬Q**	**P ⇔ ¬Q**	**P ∧ ¬Q**	**¬P ∧ Q**	**((P ∧ ¬Q) ∨ (¬P ∧ Q))**
True	True	False	False	False	False	False	False
True	False	False	True	True	True	False	True
False	True	True	False	True	False	True	True
False	False	True	True	False	False	False	False

Therefore, the statements $P \Leftrightarrow \neg Q$, $((P \wedge \neg Q) \vee (\neg P \wedge Q))$ are equivalent.

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2. Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither. Justify your answer using truth tables (worlds). We'll let S = Smoke, F = Fire, and H = Heat.

- (Smoke ⇒ Fire) ⇒ (¬Smoke ⇒ ¬Fire)

S	**F**	**¬S**	**¬F**	**S ⇒ F**	**¬S ⇒ ¬F**	**(S ⇒ F) ⇒ (¬S ⇒ ¬F)**
True	True	False	False	True	True	True
True	False	False	True	False	True	True
False	True	True	False	True	False	False
False	False	True	True	True	True	True

This sentence is not valid, since there is some world that returns false. However, it is not unsatisfiable either. Therefore, it is **satisfiable (neither)**.

- (Smoke ⇒ Fire) ⇒ ((Smoke ∨ Heat) ⇒ Fire)

S	**H**	**F**	**S ⇒ F**	**(S ∨ H) ⇒ F**	**(S ⇒ F) ⇒ ((S ∨ H) ⇒ F)**
True	True	True	True	True	True
True	True	False	False	False	True
True	False	True	True	True	True
True	False	False	False	False	True
False	True	True	True	True	True
False	True	False	True	False	False
False	False	True	True	True	True
False	False	False	True	True	True

Similarly, this sentence is not valid nor unsatisfiable; it is **satisfiable (neither)**.

- ((Smoke ∧ Heat) ⇒ Fire) ⇔ ((Smoke ⇒ Fire) ∨ (Heat ⇒ Fire))

S	**H**	**F**	**(S ∧ H) ⇒ F**	**S ⇒ F**	**H ⇒ F**	**(S ⇒ F) ∨ (H ⇒ F)**	**((S ∧ H) ⇒ F) ⇔ ((S ⇒ F) ∨ (H ⇒ F))**
True	True	True	True	True	True	True	True
True	True	False	False	False	False	False	True
True	False	True	True	True	True	True	True
True	False	True	True	False	True	True	True
False	True	True	True	True	True	True	True
False	True	True	True	True	False	True	True
False	False	True	True	True	True	True	True
False	False	True	True	True	True	True	True

This sentence is both satisfiable and **valid** – all possible worlds return true.

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3. Consider the following: *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

   Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

   a. Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).

      Let Y = Mythical, M = Mortal, A = Mammal, H = Horned, and G = Magical.

Mythical $\Rightarrow \neg$ Mortal	$Y \Rightarrow \neg M$	(1)
$\neg$ Mythical $\Rightarrow$ Mortal $\wedge$ Mammal	$\neg Y \Rightarrow M \wedge A$	(2)
($\neg$ Mortal $\vee$ Mammal) $\Rightarrow$ Horned	$(\neg M \vee A) \Rightarrow H$	(3)
Horned $\Rightarrow$ Magical	$H \Rightarrow G$	(4)

   b. Convert the knowledge base into CNF.

KB

1. $\neg Y \vee \neg M$
2. $(Y \vee M) \wedge (Y \vee A)$
3. $(H \vee M) \wedge (H \vee \neg A)$
4. $\neg H \vee G$

   c. Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned? (α_1: mythical, α_2: magical, α_3: horned)

It may be easier to look at the sentences in a different form:

Sentence 2 is equivalent to $Y \vee (M \wedge A)$
Sentence 3 is equivalent to $H \vee (M \wedge \neg A)$

4.	$\neg M \vee (M \wedge A)$	1,2
5.	$(\neg M \vee A)$	4 (negation)
6.	**H**	3,5
7.	**G**	4,6

This is as far as we can go with derivations. We can prove that the unicorn is horned using the knowledge base, so we also know that it is magical (since horned implies magical). No other resolutions can be made to prove whether or not the unicorn is mythical.

Therefore, the unicorn is horned (α_3) and magical (α_2), but nothing can be said about it being mythical (α_1).