Μοριοδότηση 2018

Ενδεικτικές απαντήσεις και από γραπτά μαθητών

Θέμα Α

A1 - γ

A2 - δ

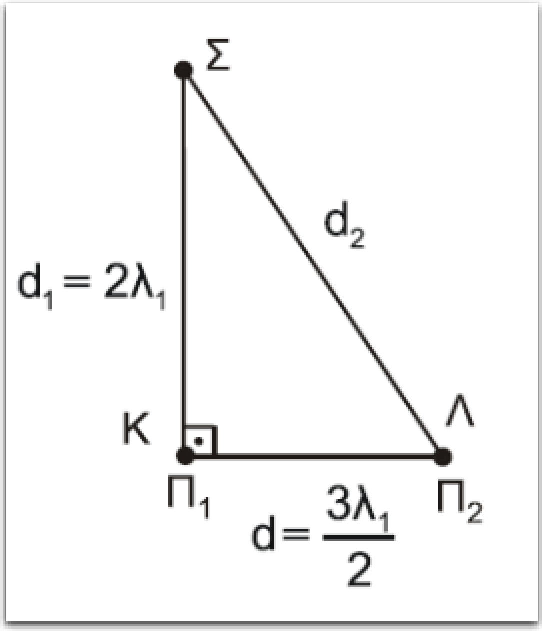
A3 - α

A4 - δ

A5: $\Lambda - \Sigma - \Lambda - \Sigma - \Lambda$

Θέμα Β

B1-(i)



$$d_2 = \sqrt{d_1^2 + d^2} = \sqrt{4 \cdot \lambda_1^2 + \frac{9}{4} \cdot \lambda_1^2} = \frac{5 \cdot \lambda_1}{2}$$

Ίδιο υλικό

$$\upsilon_\delta = \lambda_1 \cdot f_1 = \lambda_2 \cdot f_2 \overset{f_2 = 2 \cdot f_1}{\Longrightarrow} \lambda_2 = \frac{\lambda_1}{2}$$

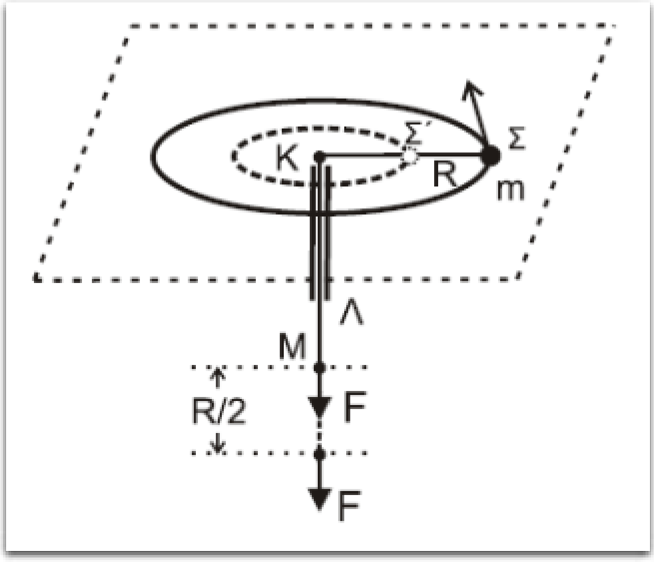
$$\alpha)\underline{\tau\rho\acute{o}\pi o\varsigma}$$

$$|A_\Sigma| = |2A\cdot\sigma\upsilon\nu\frac{2\pi(d_1-d_2)}{2\lambda_2}| = |2A\cdot\sigma\upsilon\nu\frac{\pi(\frac{3\lambda_1}{2}-\frac{5\lambda_1}{2})}{\frac{\lambda_1}{2}}| = |2\mathrm{A}|$$

$\beta)$ <u>*τρόπος*</u>

$$\left.\begin{aligned} d_1-d_2=\tfrac{3\lambda_1}{2}-\tfrac{5\lambda_1}{2}=-\lambda_1=-2\lambda_2 \\ d_1-d_2=N\cdot\lambda_2 \end{aligned}\right\}\mathrm{N}=-2 \quad \varepsilon\nu\iota\sigma\chi\upsilon\sigma\eta$$

B2 - $(\iota\iota\iota)$



$$m: \quad \Sigma\vec{\tau}_{\varepsilon\xi(\mathrm{K})}=\vec{0}\Rightarrow\Delta\vec{L}=\vec{0}\Rightarrow\vec{L_1}=\vec{L_2}$$

Η τάση του νήματος διέρχεται από τον άξονα περιστροφής

$\alpha)$ <u>*τρόπος*</u>

$$\text{Άρα}\quad m\cdot\upsilon_1\cdot R=m\cdot\upsilon_2\cdot\frac{R}{2}\Rightarrow\upsilon_2=2\upsilon_1$$

$$\Theta\mathrm{MKE}_m(\Sigma\to\Sigma')\quad \mathrm{K}_{\Sigma'}-\mathrm{K}_\Sigma=W_F\Rightarrow\frac{1}{2}\cdot m\cdot\upsilon_2^2-\frac{1}{2}\cdot m\cdot\upsilon_1^2=W_F$$

$$\left.\begin{aligned} W_F=\tfrac{3}{2}\cdot m\cdot\upsilon_1^2 \\ \upsilon_1=\omega\cdot R \end{aligned}\right\}W_F=\frac{3}{2}\cdot m\cdot\omega^2\cdot R^2$$

$\beta)$ <u>*τρόπος*</u>

$$I_1\cdot\omega=\mathrm{I}_2\cdot\omega'\Rightarrow m\cdot R^2\cdot\omega=m\cdot\frac{R^2}{4}\cdot\omega'\Rightarrow\omega'=4\omega$$

$$\Theta\mathrm{MKE}_m(\Sigma\to\Sigma')\quad \mathrm{K}_{\Sigma'}-\mathrm{K}_\Sigma=W_F\Rightarrow\frac{1}{2}\cdot\mathrm{I}_2\cdot\omega'^2-\frac{1}{2}\cdot\mathrm{I}_1\cdot\omega^2=W_F$$

$$W_F=\frac{1}{2}m\frac{R^2}{4}16\omega^2-\frac{1}{2}m\cdot R^2\omega^2\Rightarrow W_F=\frac{3}{2}\cdot m\cdot\omega^2\cdot R^2$$

B3 - (ι)

Εξίσωση Bernoulli για μια ρευματική γραμμή $(\Gamma \rightarrow \Delta)$

$$P_\Gamma + \frac{1}{2}\rho \cdot \upsilon_\Gamma^2 = P_\Delta + \frac{1}{2}\rho \cdot \upsilon_\Delta^2 + \rho \cdot g \cdot h$$

Εξίσωση συνέχειας $(\Gamma \rightarrow \Delta)$

$$\Pi_\Gamma = \Pi_\Delta \Rightarrow A_\Gamma \cdot \upsilon_\Gamma = A_\Delta \cdot \upsilon_\Delta \xRightarrow{A_\Gamma = 2A_\Delta} \upsilon_\Delta = 2\upsilon_\Gamma$$

Οριζόντια βολή $(\Delta \rightarrow K)$

$$\left.\begin{array}{l} h = \frac{1}{2}g \cdot t^2 \\ \\ 4h = \upsilon_\Delta \cdot t \end{array}\right\} 4h = \upsilon_\Delta \cdot \sqrt{\frac{2h}{g}} \Rightarrow \upsilon_\Delta^2 = 8g \cdot h \xRightarrow{\upsilon_\Delta = 2\upsilon_\Gamma} 4\upsilon_\Gamma^2 = 8g \cdot h \Rightarrow \upsilon_\Gamma^2 = 2g \cdot h$$

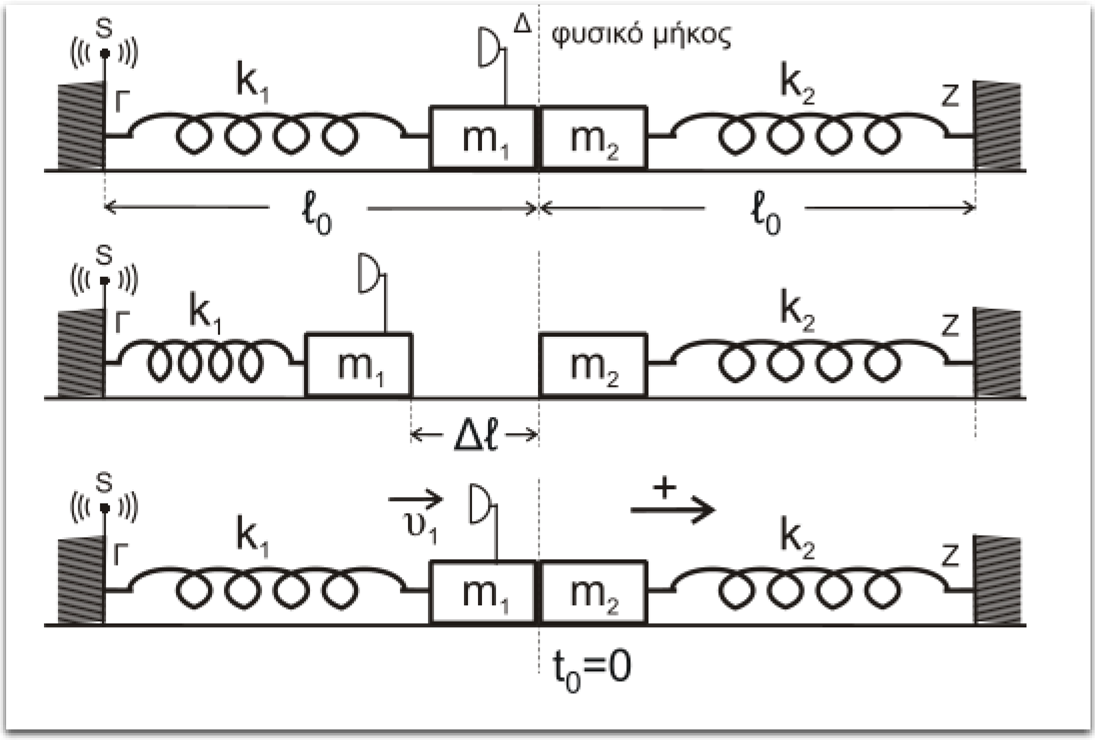
$$g \cdot h = \frac{\upsilon_\Gamma^2}{2}$$

Άρα η εξίσωση Bernoulli γράφεται

$$P_\Gamma - P_\Delta = \frac{1}{2}\rho \cdot \upsilon_\Delta^2 + \rho \cdot g \cdot h - \frac{1}{2}\rho \cdot \upsilon_\Gamma^2 = \frac{1}{2}\rho \cdot 4\upsilon_\Gamma^2 + \rho \cdot \frac{\upsilon_\Gamma^2}{2} - \frac{1}{2}\rho \cdot \upsilon_\Gamma^2 = 2\rho \cdot \upsilon_\Gamma^2$$

Θέμα Γ

Γ1



$$k_1 = k_2 = k$$

$$m_1 = m_2 = m$$

$$\Delta l = 0.4m = A_1$$

$$K_1 - m, \quad \text{AAT} : D_1 = k_1 = m_1 \cdot \omega_1^2 \Rightarrow \omega_1 = \sqrt{\frac{k}{m}} = 5\frac{rad}{sec}$$

$$\upsilon_{max1} = \omega_1 \cdot A_1 = \sqrt{\frac{k}{m}} \cdot \Delta l = 2\frac{m}{sec}$$

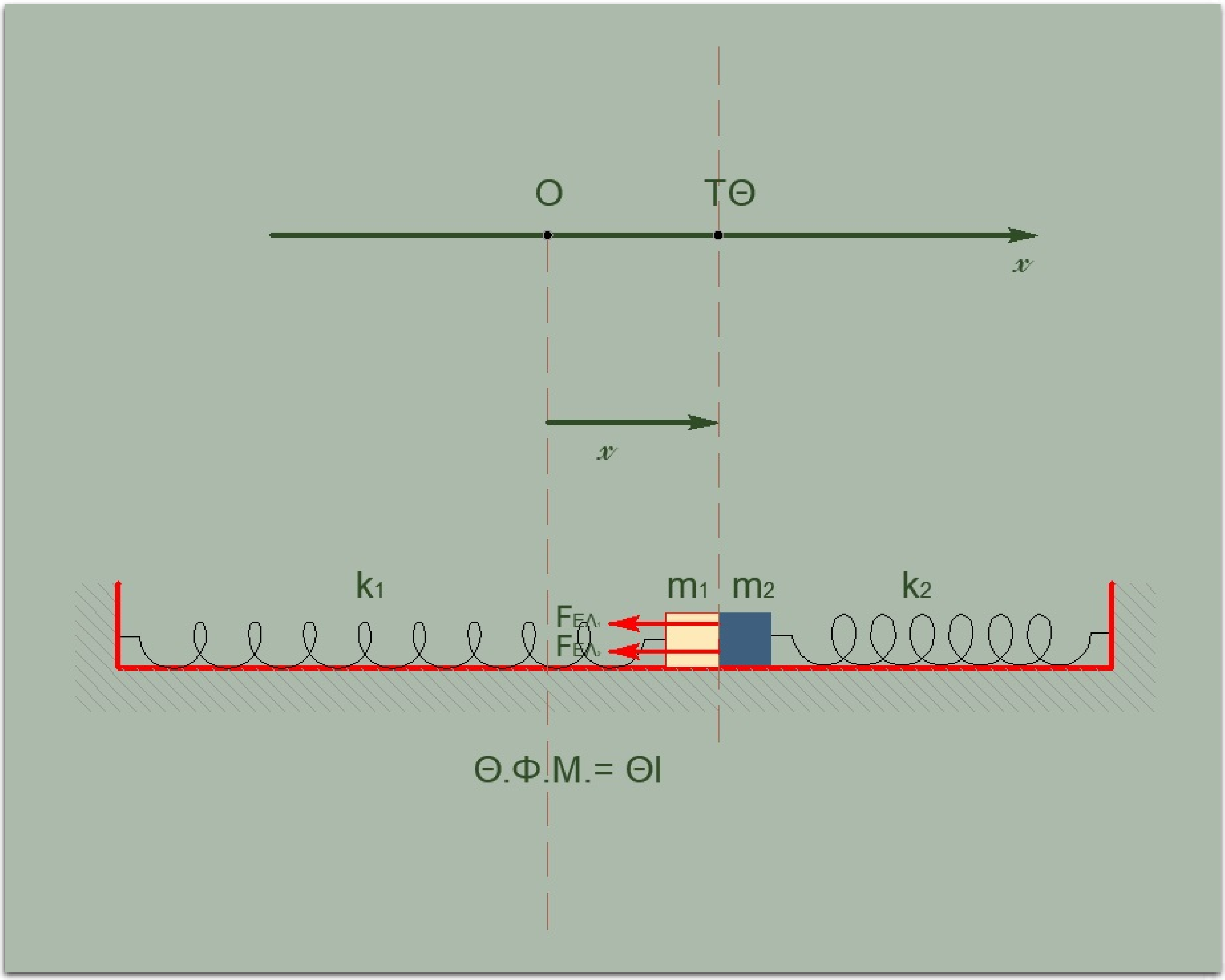
$$f_1 = \frac{\upsilon_{\eta\chi} - \upsilon_{max1}}{\upsilon_{\eta\chi}} \cdot f_s$$

$$\text{A}\Delta\text{O} \quad m_1, m_2 \quad (\Theta.\text{I.}) \quad m_1 \cdot \upsilon_{max1} = (m_1 + m_2) \cdot V \Rightarrow V = 1\frac{m}{sec}$$

$$f_2 = \frac{\upsilon_{\eta\chi} - V}{\upsilon_{\eta\chi}} \cdot f_s$$

$$\frac{f_1}{f_2} = \frac{\upsilon_{\eta\chi} - \upsilon_{max1}}{\upsilon_{\eta\chi} - V} = \frac{338}{339}$$

Γ2



$$(m_1 + m_2):$$

Στη θέση Θ.Φ.Μ. $\Sigma F = 0$ άρα αυτή είναι και Θ.Ι.

$$\mathrm{T.\,\Theta.}: \Sigma F = -F_{\mathrm{E\Lambda 1}} - F_{\mathrm{E\Lambda 2}} = -k_1 \cdot x - k_2 \cdot x = -(2k)x$$

Για να εκτελεί ένα σώμα ΑΑΤ πρέπει να ισχύει

$$\Sigma F = -D \cdot x, D = 2k = (m_1 + m_2)\omega^2 \Rightarrow \omega = \sqrt{\frac{2k}{2m}} = \sqrt{\frac{k}{m}} = 5\frac{rad}{sec}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5} sec$$

$$\mathrm{\Theta.\,I.}: V = \upsilon_{max} \overset{\upsilon_{max}=\omega \cdot A}{\Longrightarrow} 1 = 5 \cdot A \Rightarrow A = 0.2m$$

Γ3

$$\left.\begin{array}{r} f_{\Delta\mathrm{EKTH}} = f_s \\ f_{\Delta\mathrm{EKTH}} = \frac{\upsilon_{\eta\chi} \pm \upsilon_{\Sigma\Upsilon\Sigma}}{\upsilon_{\eta\chi}} \cdot f_s \end{array}\right\} \upsilon_{\Sigma\Upsilon\Sigma} = 0$$

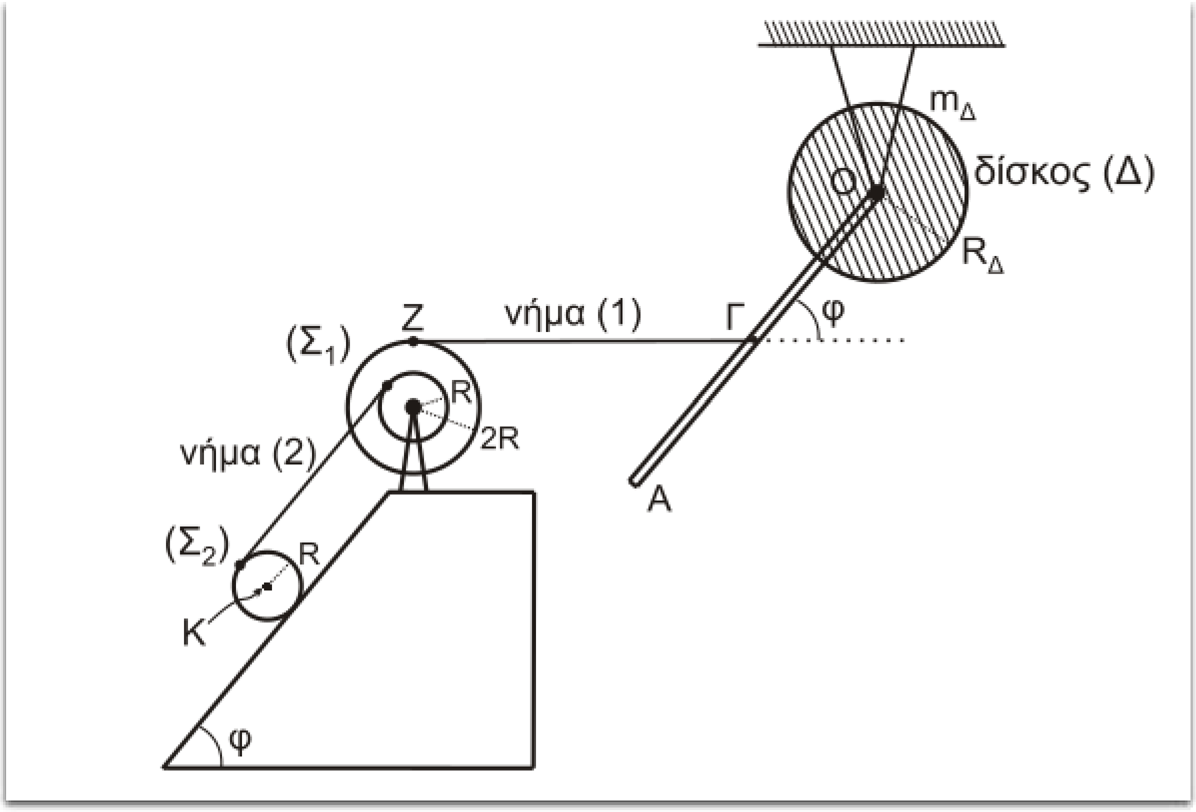
Για πρώτη φορά, δηλαδή ακραία θέση, οπότε

$$\Delta t = \frac{T}{4} = \frac{\pi}{10} sec$$

Γ4

$$|\frac{dp}{dt}|_{m_1+m_2(max)} = \Sigma F_{max} = D \cdot A \overset{D=100}{\Longrightarrow} \Sigma F_{max} = 20N, \quad ή \quad \frac{kg \cdot m}{sec^2}$$

Θέμα Δ



Ράβδος (ρ)

$$M = 8kg$$

$$l = 3m$$

Δίσκος (Δ)

$$m_\Delta = 4kg$$

$$R_\Delta = \frac{\sqrt{2}}{2}m$$

Τροχαλία (τροχ)

$$R = 0.2m$$

$$I_{\tau\rho o\chi} = 1.95kg \cdot m^2$$

Κύλινδρος

$$m = 30kg$$

$$R = 0.2m$$

$$\eta\mu\varphi = 0.8$$

$$\sigma\upsilon\nu\varphi = 0.6$$

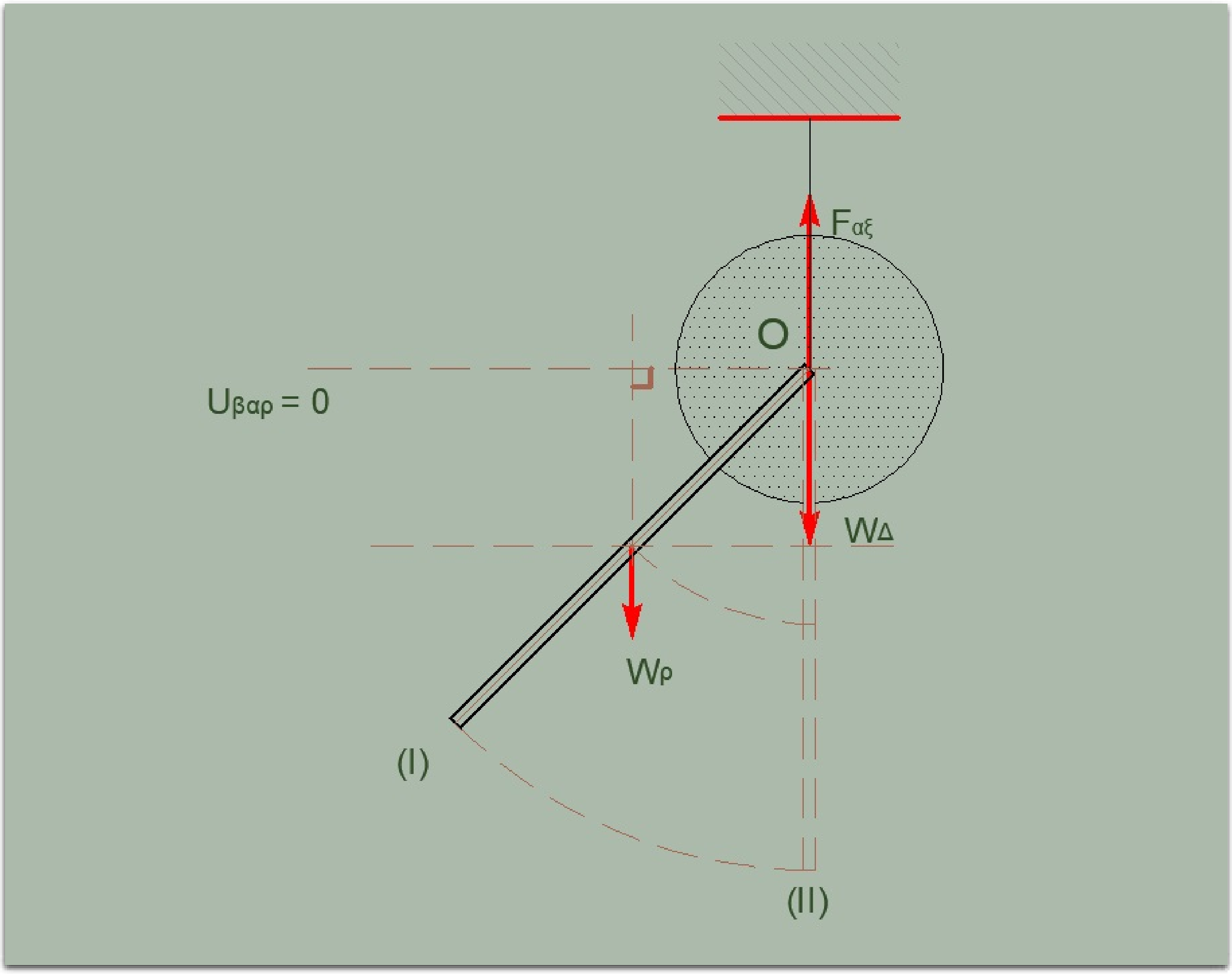
$$g = 10\frac{m}{sec^2}$$

Δ1

$$I_{\rho-\Delta} = (\frac{1}{12}\cdot M\cdot l^2 + M\frac{l^2}{4}) + \frac{1}{12}\cdot m_\Delta \cdot R_\Delta^2 = 25 kg\cdot m^2$$

Δ2

$$|\frac{dL}{dt}|_{\rho-\Delta} = \Sigma\tau_{(0)} = W_\rho \cdot \frac{l}{2}\cdot \sigma\upsilon\nu\varphi = 72\frac{kg\cdot m^2}{sec^2} \quad \text{ή} \quad N\cdot m$$



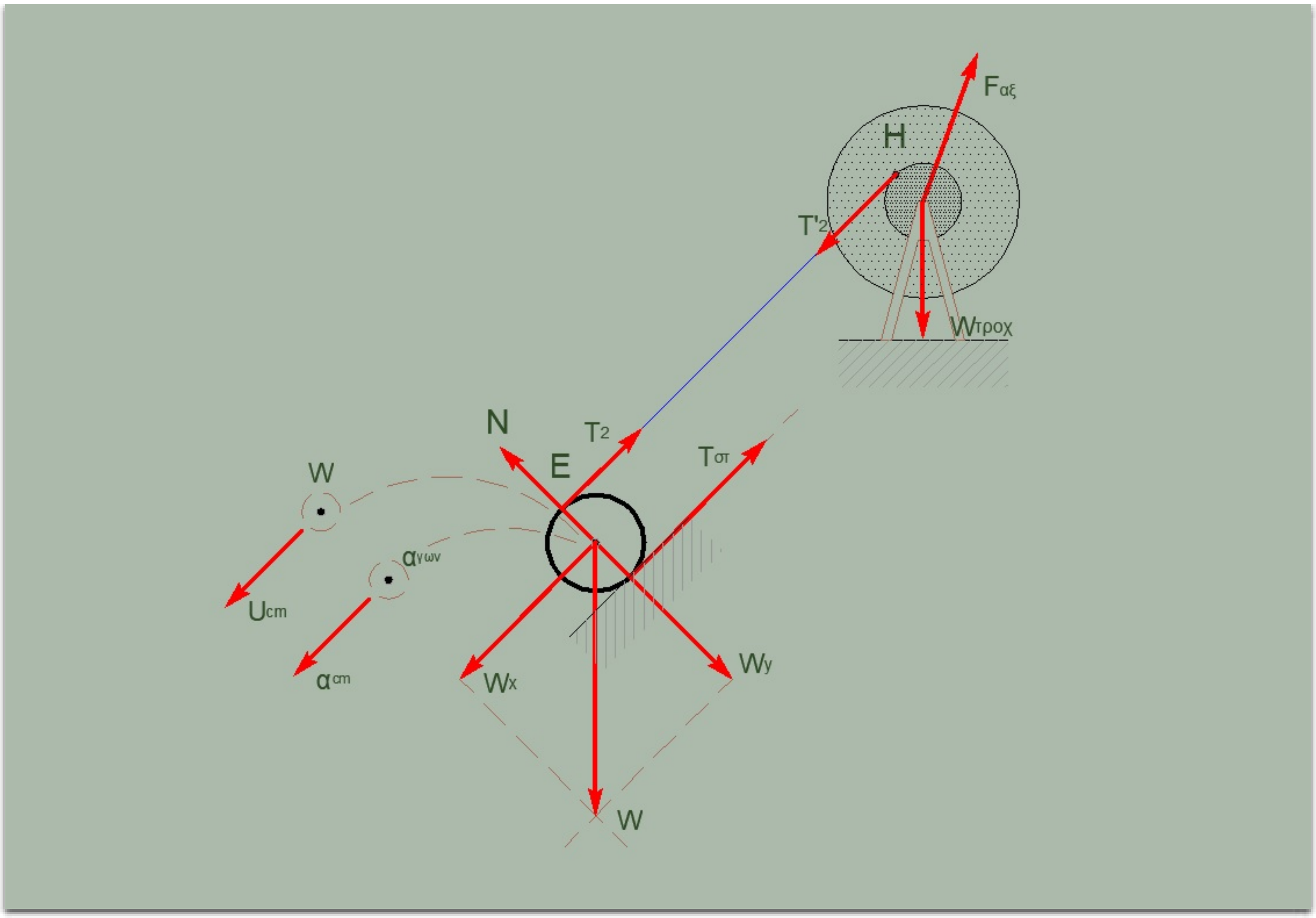
Δ3

$$\mathrm{A\Delta ME}_{\rho-\Delta}(\mathrm{I}\to II): K_I + U_1 = K_{II} + U_{II}$$

$$0 + (-M\cdot g\cdot \frac{l}{2}\cdot \eta\mu\varphi + U_{\beta\alpha\rho(\Delta)(\mathrm{I})}) = \mathrm{K_{II}} + (-M\cdot g\cdot \frac{l}{2} + U_{\beta\alpha\rho(\Delta)(\mathrm{II})})$$

$$K_{II} = M\cdot g\cdot \frac{l}{2}\cdot(1-\eta\mu\varphi) \Rightarrow K_{II} = 24J$$

Δ4



νήμα(2) αβαρές, μη εκτατό $(\mathrm{T_2} = \mathrm{T_2'})$

ΚΧΟ:

$$\upsilon_{\mathrm{B}} = 2 \cdot \upsilon_{cm} = 2 \cdot \omega \cdot R \Rightarrow \alpha_{\mathrm{B}} = 2 \cdot \alpha_{cm} = 2 \cdot \alpha_{\gamma\omega\nu} \cdot R$$

$$\upsilon_{\mathrm{B}} = \upsilon_{H} = \omega_{\tau\rho o\chi} \cdot R \Rightarrow \alpha_{\mathrm{B}} = \alpha_{\mathrm{H}} = \alpha_{\gamma\omega\nu_{\tau\rho o\chi}} \cdot R$$

m: ΜΕΤ.

$$\Sigma F_x = m \cdot \alpha_x \Rightarrow W_x - T_{\sigma\tau} - \mathrm{T_2} = m \cdot \alpha_{cm} \quad (1)$$

m: ΣΤΡΟΦ.

$$\Sigma\tau = \mathrm{I} \cdot \alpha_{\gamma\omega\nu} \Rightarrow T_{\sigma\tau} \cdot R - T_2 \cdot R = \frac{1}{2} \cdot m \cdot R^2 \cdot \alpha_{\gamma\omega\nu} \xRightarrow{\alpha_{cm} = \alpha_{\gamma\omega\nu} \cdot R} T_{\sigma\tau} - \mathrm{T_2} = \frac{1}{2} \cdot m \cdot \alpha_{cm} \quad (2)$$

$$(1)\Lambda(2) \Rightarrow W_x - 2T_2 = \frac{3}{2} \cdot m \cdot \alpha_{cm} \quad (3)$$

$$\tau\rho o\chi: \quad \Sigma\tau = \mathrm{I} \cdot \alpha_{\gamma\omega\nu} \Rightarrow T_2' \cdot R = 1.95 \cdot \alpha_{\gamma\omega\nu(\tau\rho o\chi)} \xRightarrow[(3)]{\alpha_{\gamma\omega\nu(\tau\rho o\chi)} = \frac{2\alpha_{cm}}{R}}$$

$$W_x - 2 \cdot \frac{1.95 \cdot \frac{2\alpha_{cm}}{R}}{R} = \frac{3}{2} \cdot m \cdot \alpha_{cm}$$

$$300 \cdot 0.8 - \frac{4 \cdot 1.95 \cdot \alpha_{cm}}{4 \cdot 10^{-2}} = 45 \cdot \alpha_{cm} \Rightarrow \alpha_{cm} = 1\frac{m}{sec^2}$$

$$\alpha_{cm} = \alpha_{\gamma\omega\nu} \cdot R \Rightarrow \alpha_{\gamma\omega\nu} = 5\frac{rad}{sec^2}$$

κύλινδρος:

$$\alpha)\underline{\tau\rho\acute{o}\pi o\varsigma}$$

$$s = \frac{1}{2} \cdot \alpha_{cm} \cdot t^2 \Rightarrow t = 2sec$$

$$\upsilon_{cm} = \alpha_{cm} \cdot t \Rightarrow \upsilon_{cm} = 2\frac{m}{sec}$$

$$\beta)\underline{\tau\rho\acute{o}\pi o\varsigma}$$

$$\Theta\mathrm{MKE}_{O\to S} : K_{\tau\epsilon\lambda} - \mathrm{K}_{\alpha\rho\chi} = \Sigma W \Rightarrow (\frac{1}{2} \cdot m \cdot \upsilon_{cm}^2 + \frac{1}{2} \cdot m \cdot R^2 \cdot \omega^2) - 0 = (\Sigma F_x) \cdot S + (\Sigma\tau) \cdot \Delta\theta$$

$$\Sigma F_x = m \cdot \alpha_{cm}$$

$$\Sigma\tau = \mathrm{I} \cdot \alpha_{\gamma\omega\nu}$$

$$\frac{3}{4} \cdot m \cdot \upsilon_{cm}^2 = m \cdot \alpha_{cm} \cdot s + \frac{1}{2} \cdot m \cdot R^2 \cdot \alpha_{\gamma\omega\nu} \cdot \Delta\theta \xRightarrow[\Delta\theta \cdot R = S]{\alpha_{\gamma\omega\nu} \cdot R = \alpha_{cm}}$$

$$\frac{3}{4} \cdot m \cdot \upsilon_{cm}^2 = m \cdot \alpha_{cm} \cdot s + \frac{1}{2} \cdot m \cdot \alpha_{cm} \cdot s \Rightarrow \frac{3}{4} \cdot m \cdot \upsilon_{cm}^2 = \frac{3}{2} \cdot \alpha_{cm} \cdot S \Rightarrow \upsilon_{cm} = 2\frac{m}{sec}$$

Μπορείτε να εκτυπώσετε τις λύσεις σε μορφή pdf από εδώ