Μοριοδότηση 2019

Ενδεικτικές απαντήσεις και από γραπτά μαθητών

Θέμα Α

Α1-β

A2 - γ

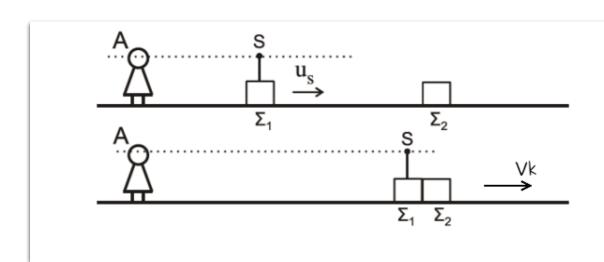
A3 - 00

A4 - y

A5: $\Lambda - \Sigma - \Lambda - \Sigma - \Sigma$

Θέμα Β

B1-(ii)



$$\Sigma \overrightarrow{F}_{\varepsilon\xi} = 0 \Rightarrow A. \Delta. O. \quad \overrightarrow{p}_{\pi\rho\nu} = \overrightarrow{p}_{\mu\varepsilon\tau\alpha}$$

$$m \cdot u_s = (m+m) \cdot V_k \Rightarrow V_k = \frac{m \cdot u_s}{2 \cdot m} \Rightarrow V_k = \frac{u_H}{40}$$













$$f_1 = \frac{u_H}{u_H + u_s} \cdot f_s$$



$$f_2 = \frac{u_H}{u_H + V_k} \cdot f_s$$



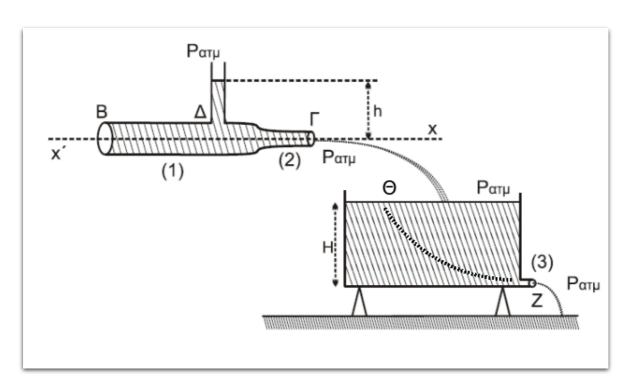
$$\frac{f_1}{f_2} = \frac{u_H + V_k}{u_H + u_S} = \frac{u_H + \frac{u_H}{40}}{u_H + \frac{u_H}{20}} = \frac{\frac{41u_H}{40}}{\frac{21u_H}{20}} = \frac{41}{42}$$



άρα σωστό το ii

B2 - (*iii*)





Όταν σταθεροποιείται το ύψος στο δοχείο

$$\Pi_2 = \Pi_3 \Rightarrow A_2 \cdot v_2 = A_3 \cdot v_3 \stackrel{A_3 = \frac{A_2}{2}}{\Longrightarrow} v_2 = \frac{v_3}{2}$$

1

Εξίσωση Bernoulli για μια ρευματική γραμμή $(\Theta o Z)$

$$P_{\Theta} + \frac{1}{2}\rho \cdot v_{H}^{2} + \rho \cdot g \cdot h = P_{Z} + \frac{1}{2}\rho \cdot v_{3}^{2} \Rightarrow P_{\alpha\tau\mu} + \rho \cdot g \cdot h = P_{\alpha\tau\mu} + \frac{1}{2}\rho \cdot v_{3}^{2}$$
$$v_{3} = \sqrt{2 \cdot g \cdot H}$$



$$\Pi_1 = \Pi_2 \Rightarrow A_1 \cdot \upsilon_1 = A_2 \cdot \upsilon_2 \stackrel{A_1 = 2A_2}{\Longrightarrow} \upsilon_2 = 2\upsilon_1$$

Εξίσωση Bernoulli για μια οριζόντια ρευματική γραμμή $(\Delta o \Gamma)$

$$P_{\Delta} + \frac{1}{2}\rho \cdot v_1^2 = P_2 + \frac{1}{2}\rho \cdot v_2^2$$
$$P_{\Delta} = P_{\alpha\tau\mu} + \rho \cdot g \cdot h$$

2

$$P_{\alpha\tau\mu} + \rho \cdot g \cdot h + \frac{1}{2}\rho \cdot \upsilon_1^2 = P_{\alpha\tau\mu} + \frac{1}{2}\rho \cdot \upsilon_2^2 \Rightarrow \rho \cdot g \cdot h = \frac{1}{2} \cdot (\upsilon_2^2 - \upsilon_1^2)$$

$$g \cdot h = \frac{3}{8} \cdot \upsilon_2^2 \stackrel{\upsilon_2 = \frac{\upsilon_3}{2}}{\Longrightarrow} g \cdot h = \frac{3}{8} \cdot \frac{\upsilon_3^2}{4}$$

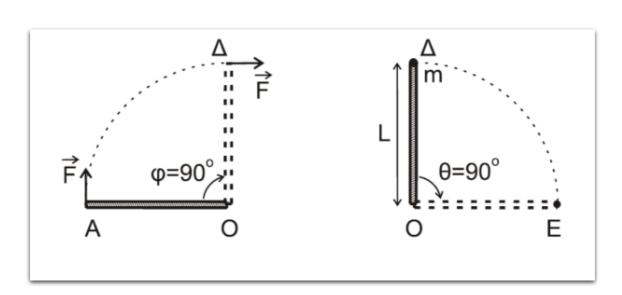
$$\upsilon_3^2 = \frac{32}{3} \cdot g \cdot h \stackrel{\upsilon_3 = \sqrt{2 \cdot g \cdot H}}{\Longrightarrow} 2 \cdot g \cdot H = \frac{32}{3} \cdot g \cdot h \Rightarrow \frac{h}{H} = \frac{3}{16}$$

1)

άρα σωστό το iii

B3 - (ii)





$$\Sigma \tau = I \cdot \alpha_{\gamma \omega \nu} \Rightarrow F \cdot L = \frac{1}{3} \cdot M \cdot L^{2} \cdot \alpha_{\gamma \omega \nu} \Rightarrow \alpha_{\gamma \omega \nu} = \frac{3F}{ML}$$

$$\varphi = \frac{1}{2} \cdot \alpha_{\gamma \omega \nu} \cdot t^{2} \Rightarrow \frac{\pi}{2} = \frac{1}{2} \cdot \alpha_{\gamma \omega \nu} \cdot t_{A\Delta} \Rightarrow t_{A\Delta} = \sqrt{\frac{\pi \cdot M \cdot L}{3F}}$$

$$\omega = \alpha_{\gamma \omega \nu} \cdot t_{A\Delta} = \frac{3F}{ML} \cdot \sqrt{\frac{\pi \cdot M \cdot L}{3F}} \Rightarrow \omega = \sqrt{\frac{3 \cdot Fcdot\pi}{M \cdot L}} \Rightarrow \omega = 3\pi \frac{rad}{s}$$

β)τρόπος

$$\begin{split} \Theta \text{MKE}(\text{A} \to \Delta) \quad & \text{K}_{\Delta} - \text{K}_{\text{A}} = \Sigma W_{\tau} \Rightarrow \frac{1}{2} \cdot \text{I}_{\rho} \cdot \omega^{2} - 0 = \tau_{F} \cdot \theta \\ & \frac{1}{2} \cdot \text{I}_{\rho} \cdot \omega^{2} = F \cdot L \cdot \frac{\pi}{2} \Rightarrow \frac{M}{3} \cdot L^{2} \cdot \omega^{2} = F \cdot L \cdot \pi \\ & \omega = \sqrt{\frac{3 \cdot F \cdot \pi}{M \cdot L}} = \sqrt{9 \cdot \pi^{2}} \\ & \omega = 3\pi \frac{rad}{s} \end{split}$$

3

$$\Sigma \tau_{\varepsilon\xi} = 0 \Rightarrow A. \Delta. \Sigma \tau \rho_0 \Rightarrow \vec{L}_{\pi\rho\nu} = \vec{L}_{\mu\varepsilon\tau\dot{\alpha}} \Rightarrow I_\rho \cdot \omega = (I_\rho + m \cdot L^2) \cdot \omega_k$$

$$\frac{1}{3} \cdot M \cdot L^2 \cdot \omega = (\frac{1}{3} \cdot M \cdot L^2 + m \cdot L^2) \cdot \omega_k \Rightarrow \omega = 2 \cdot \omega_k \Rightarrow \omega_k = \frac{\omega}{2}$$

2

Ομαλή στροφική κίνηση

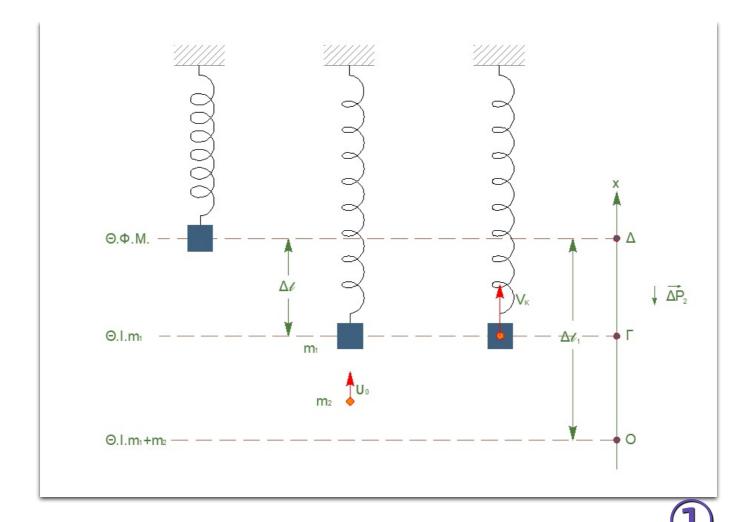
$$\Delta\theta = \omega_k \cdot \Delta t \Rightarrow \Delta t = \frac{\Delta\theta}{\omega_k} = \frac{\Delta\theta}{\frac{\omega}{2}} = \frac{2\Delta\theta}{\omega} = \frac{2 \cdot \frac{\pi}{2}}{\omega} = \frac{\pi}{\omega}$$
$$\Delta t = \frac{1}{3}s$$

2

άρα σωστό το ii

Θέμα Γ

П



$$(\Theta I_{m_1}) \quad \Sigma F = 0 \Rightarrow$$

$$F_{\varepsilon \lambda} = m_1 \cdot g \Rightarrow$$

$$k \cdot \Delta l = m_1 \cdot g \Rightarrow$$

$$k = \frac{m_1 \cdot g}{\Delta l} = 200 \frac{N}{m}$$

$$(\Theta I_{m_1,m_2}) \quad \Sigma F = 0 \Rightarrow$$

$$F'_{\varepsilon \lambda} = (m_1 + m_2) \cdot g \Rightarrow$$

$$k \cdot \Delta l_1 = (m_1 + m_2) \cdot g \Rightarrow$$

$$\Delta l_1 = \frac{(m_1 + m_2) \cdot g}{k} \Rightarrow$$

$$\Delta l_1 = 0.1m$$

Στην ακραία θέση $\upsilon_{\mathrm{ταλ}}=0 \Rightarrow A=0.1m$

1

$$m_2 \cdot u_0 = (m_1 + m_2) \cdot V_k \Rightarrow V_k = \frac{m \cdot u_s}{2 \cdot m} \Rightarrow V_k = \frac{u_H}{40}$$

$$A\Delta E_{\tau\alpha\lambda}(\Gamma \to \Delta)$$

$$K_{\Gamma} + U_{\text{ταλ}\Gamma} = K_{\Delta} + U_{\text{ταλ}\Delta} \Rightarrow \frac{1}{2} \cdot (m_1 + m_2) \cdot V_k^2 + \frac{1}{2} \cdot D \cdot (\Delta l_1 - \Delta l)^2 = 0 + \frac{1}{2} \cdot D \cdot A^2$$

$$2V_k^2 + 200 \cdot 0.05^2 = 200 \cdot 0.01 \Rightarrow V_k^2 + 0.25 = 1 \Rightarrow V_k = \sqrt{0.75} \Rightarrow |V_k| = 0.5\sqrt{3} \frac{m}{s}, V_k > 0$$



$$K_2 = \frac{1}{2} \cdot m_2 \cdot u_o^2 \Rightarrow K_2 = 1.5J$$



L3

$$\Delta \vec{p_2} = \vec{p}_{\tau \varepsilon \lambda} - \vec{p}_{\alpha \rho \chi} \Rightarrow \Delta p_2 = m_2 \cdot V_k - m_2 \cdot u_o \Rightarrow \Delta p_2 = 0.5\sqrt{3} - \sqrt{3} \Rightarrow \Delta p_2 = -0.5\sqrt{3}$$



$$|\Delta \vec{p_2}| = 0.5\sqrt{3}kg \cdot \frac{m}{s}$$



$$\Delta p_2 = -0.5\sqrt{3}$$

Το πρόσημο δηλώνει την κατεύθυνση του διανύσματος. (Βλέπε σχήμα)



Г4

$$D = k = (m_1 + m_2) \cdot \omega^2 \Rightarrow \omega = \sqrt{\frac{k}{m_1 + m_2}} \Rightarrow \omega = 10 \frac{rad}{s}$$



$$t_o = 0, \quad y = +\frac{A}{2}, \quad v > 0$$

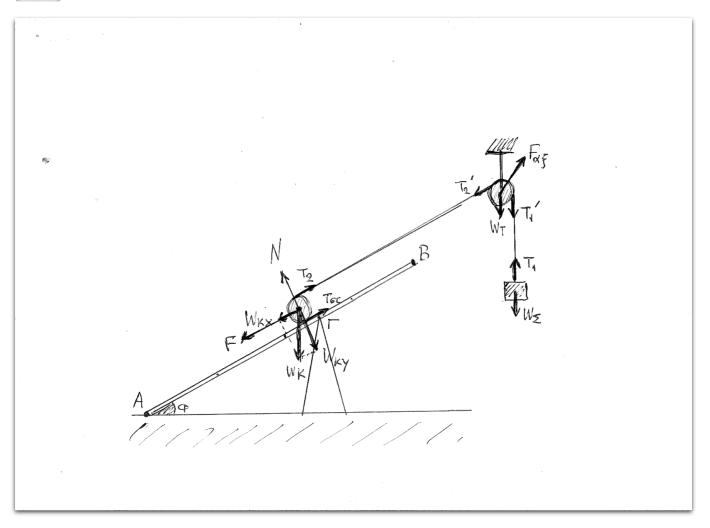
$$y = A \cdot \eta \mu(\omega t + \varphi_0) \Rightarrow \frac{A}{2} = A \cdot \eta \mu \varphi_0$$

$$\eta\mu\phi_o=\frac{1}{2}\Rightarrow\eta\mu\phi_o=\eta\mu\frac{\pi}{6}\Rightarrow\phi_o=\left\{\begin{array}{ll} 2k\pi+\frac{\pi}{6}, & k=0\Rightarrow\phi_o=\frac{\pi}{6} & \text{sun}\phi_o>0\\ 2k\pi+\frac{5\pi}{6}, & k=0\Rightarrow\phi_o=\frac{5\pi}{6} & \text{sun}\phi_o<0, \text{aportitetan} \end{array}\right.$$



$$y = 0.1 \cdot \eta \mu (10t + \frac{\pi}{6}), \quad S.I.$$

Θέμα Δ



 $\Delta 1$

$$M_{\Sigma}$$
, ισορροπία, $\Rightarrow \Sigma F = 0 \Rightarrow T_1 = M_{\Sigma} \cdot g \Rightarrow T_1 = 20N$



$$M_T$$
, ισορροπία, $\Rightarrow \Sigma \tau = 0 \Rightarrow T_1 \cdot R_T = T_1 \cdot R_T \Rightarrow T_2 = 20N$



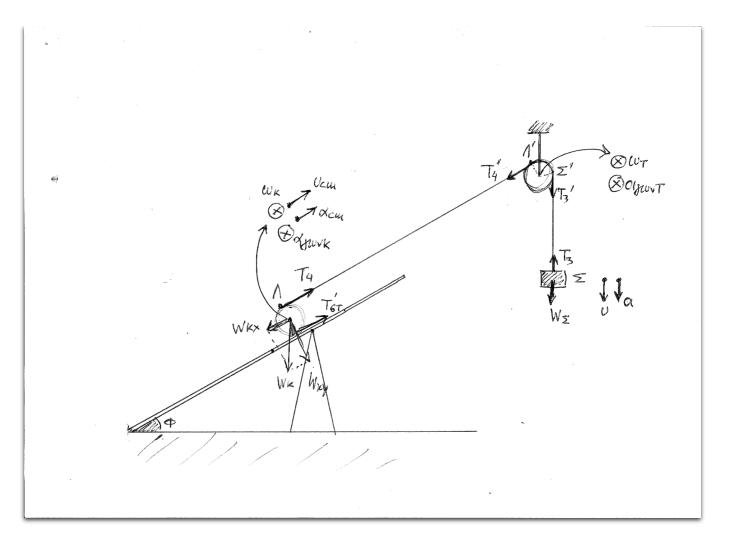
α)τρόπος

$$M_K$$
, ισορροπία, $\Rightarrow \Sigma \tau_{(K)} = 0 \Rightarrow T_2 \cdot R_K = T_{\sigma \tau} \cdot R_K \Rightarrow T_2 = T_{\sigma \tau}$
 $\Sigma F = 0 \Rightarrow T_2 + T_{\sigma \tau} = F + M_K \cdot g \cdot \eta \mu \phi \Rightarrow 2T_2 = F + 10 \Rightarrow F = 30N$



β)τρόπος

$$\mathit{M}_{\mathit{K}}, \quad \mathsf{isorropopia}, \Rightarrow \mathsf{St}_{(\Delta)} = 0 \Rightarrow \mathit{T}_2 \cdot 2 \cdot \mathit{R}_{\mathit{K}} = (\mathit{F} + \mathsf{M}_{\mathsf{K}} \cdot \mathit{g} \cdot \mathsf{\eta} \mathsf{\mu} \varphi) \cdot \mathit{R}_{\mathit{K}} \Rightarrow 40 = \mathit{F} + 10 \Rightarrow \mathit{F} = 30\mathit{N}$$



$$M_{\Sigma}$$
: *MET*АФОРІКН

$$\Sigma F = \mathbf{M}_{\Sigma} \cdot g - T_3 = M_{\Sigma} \cdot \alpha_{\Sigma} \quad (1)$$



 M_T : Σ TPO Φ IKH

$$\Sigma \tau = I_T \cdot \alpha_{\gamma \omega \nu_T} \Rightarrow T_3 \cdot R_T - T_4 \cdot R_T = \frac{1}{2} \cdot M_T \cdot R_T^2 \cdot \alpha_{\gamma \omega \nu_T} \Rightarrow T_3 - T_4 = \frac{1}{2} \cdot M_T \cdot R_T \cdot \alpha_{\gamma \omega \nu_T}$$
(2)



νήμα αβαρές, μη εκτατό

$$\alpha_{\Sigma} = \alpha'_{\Sigma} = \alpha_{\gamma \rho} = \alpha_{\gamma \omega \nu_{T}} \cdot R_{T}$$
 (3)



$$(2)\Lambda(3) \Rightarrow T_3 - T_4 = \alpha_{\Sigma}$$
 (4)

 $M_{\rm K}: META\Phi OPIKH$

$$\Sigma F = M_K \cdot \alpha_{cm}$$

$$T_4 + T_{\sigma\tau} - \mathrm{M_K} \cdot g \cdot \eta \mu \phi = \mathrm{M_K} \cdot \alpha_{cm} \Rightarrow T_4 + T_{\sigma\tau} - 10 = 2 \cdot \alpha_{cm} \quad (5)$$



 M_K : Σ TPO Φ IKH

$$\Sigma \tau = \mathbf{I}_{K} \cdot \alpha_{\gamma \omega \nu_{K}} \Rightarrow T_{4} \cdot R_{K} - T_{\sigma \tau} \cdot R_{K} = \frac{1}{2} \cdot M_{K} \cdot R_{K}^{2} \cdot \alpha_{\gamma \omega \nu_{K}} \Rightarrow T_{4} - T_{\sigma \tau} = \frac{1}{2} \cdot M_{K} \cdot R_{K} \cdot \alpha_{\gamma \omega \nu_{K}}$$

$$K. X. O. \upsilon_{A} = 0 \Rightarrow \upsilon_{cm} = \omega \cdot R_{K} \Rightarrow \alpha_{cm} = \alpha_{\gamma \omega \nu_{K}} \cdot R_{K}$$

$$T_4 - T_{\sigma\tau} = \frac{1}{2} \cdot M_{\rm K} \cdot R_{\rm K} \cdot \alpha_{\gamma\omega\nu_{\rm K}} \Rightarrow T_4 - T_6 = \alpha_{cm}$$
 (6)

(1)

νήμα αβαρές, μη εκτατό

$$\alpha_{\Lambda} = \alpha_{\Lambda}' = \alpha_{\gamma\rho} = \alpha_{\gamma\omega\nu_{T}} \cdot R_{T}$$

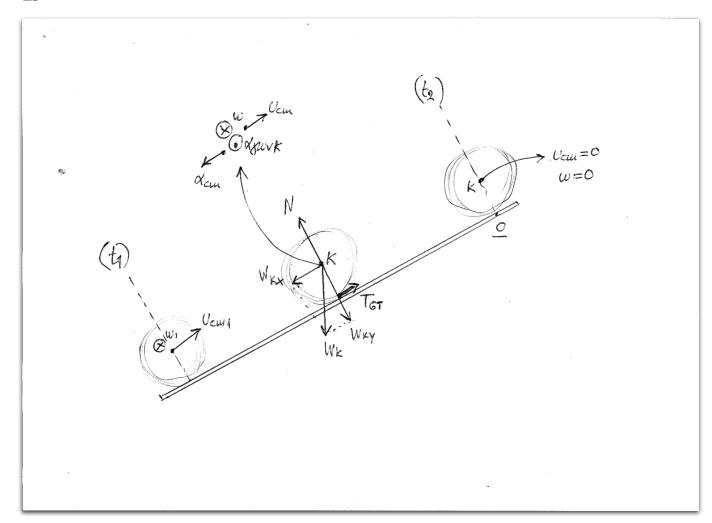
$$\upsilon_{\Lambda} = \upsilon_{cm} + \omega \cdot R_{K} \Rightarrow \upsilon_{\Lambda} = 2 \cdot \upsilon_{cm} \Rightarrow \alpha_{\Lambda} = 2 \cdot \alpha_{cm}$$
$$2 \cdot \alpha_{cm} = \alpha_{\gamma \omega \nu_{T}} \cdot R_{T} \Rightarrow 2 \cdot \alpha_{cm} = \alpha_{\Sigma} \quad (7)$$

1

Λύση του συστήματος $\alpha_{\Sigma}=4rac{m}{s^2}$

1

Δ3



$$v_{cm1} = \alpha_{cm} \cdot t_1 \Rightarrow v_{cm1} = 1 \frac{m}{s}$$

 $M_{\rm K}:$ METАФОРІКН

$$\Sigma F = M_K \cdot \alpha_{cm}$$

 $M_{K} \cdot g \cdot \eta \mu \phi - T_{\sigma \tau} = M_{K} \cdot \alpha_{cm} \Rightarrow 10 - T_{\sigma \tau} = 2\alpha_{cm}$, επιβραδονόμενη



 M_K : Σ TPO Φ IKH

 $\Sigma \tau = \mathrm{I}_K \cdot \alpha_{\mathsf{YON}_K} \Rightarrow T_{\mathsf{GT}} \cdot R_{\mathsf{K}} = \frac{1}{2} \cdot M_{\mathsf{K}} \cdot R_{\mathsf{K}}^2 \cdot \alpha_{\mathsf{YON}_K} \Rightarrow T_{\mathsf{GT}} = R_{\mathsf{K}} \cdot \alpha_{\mathsf{YON}_K} \Rightarrow T_{\mathsf{GT}} = \alpha_{\mathit{cm}}, \varepsilon \text{πιβραδονόμενη}$



$$10 - \alpha_{cm} = 2 \cdot \alpha_{cm} \Rightarrow \alpha_{cm} = \frac{10}{3} \frac{m}{s^2}$$



$$v_{cm} = v_{cm1} - \alpha_{cm} \cdot \Delta t \Rightarrow 0 = 1 - \frac{10}{3} \cdot \Delta t \Rightarrow \Delta t = 0.3s$$
$$t_2 = t_1 + \Delta t \Rightarrow t_2 = 0.8s$$



 $\Delta 4$

 M_K : МЕТАФОРІКН, $0-t_1$

$$x_{cm1} = \frac{1}{2} \cdot \alpha_{cm} \cdot t_1^2 \Rightarrow x_{cm1} = 0.25m$$



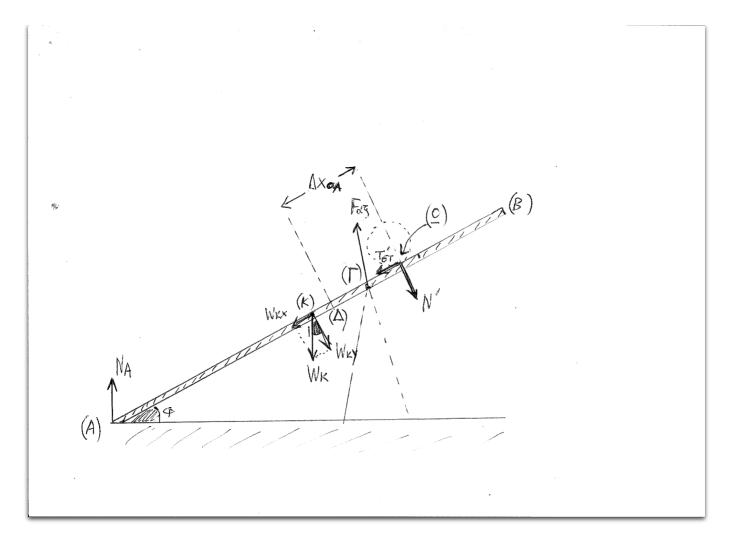
 M_K : МЕТАФОРІКН, $t_1 - t_2$

$$\Delta x_{cm} = v_{cm1} \cdot \Delta t - \frac{1}{2} \cdot \alpha_{cm} \cdot \Delta t^2 \Rightarrow \Delta x_{cm} = 0.15m$$



$$x_{o\lambda} = x_{cm1} + \Delta x_{cm} \Rightarrow x_{o\lambda} = 0.4m$$





$$(\Gamma\Delta) = 0.2m, \quad (K\Gamma) = 0.5m, \quad (\Gamma\Omega) = x_{\rm o\lambda} - (\Gamma\Delta) \Rightarrow (\Gamma\Omega) = 0.2m$$

$$|\tau_{W_p}| = M_p \cdot g \cdot$$
συν $\phi \cdot (K\Gamma) \Rightarrow |\tau_{W_p}| = 5\sqrt{3}N \cdot m$

1

$$|\tau_N'| = M_p \cdot g \cdot$$
συνφ $\cdot (K\Gamma) \Rightarrow |\tau_N'| = 2\sqrt{3}N \cdot m$

1

$$| au_{W_{
ho}}| > | au_N'|$$

άρα η σανίδα δεν ανατρέπεται

1

Μπορείτε να εκτυπώσετε τις λύσεις σε μορφή pdf από εδώ και τα θέματα από εδώ