Homework 3

Advanced Data Analysis

In [ ]:
```
import numpy as np
import matplotlib
#matplotlib.use('TkAgg')
import matplotlib.pyplot as plt

np.random.seed(1)

def generate_sample(x_min=-3., x_max=3., sample_size=10):
    x = np.linspace(x_min, x_max, num=sample_size)
    y = x + np.random.normal(loc=0., scale=.2, size=sample_size)
    y[-1] = y[-2] = y[1] = -4 # outliers
    return x, y

def build_design_matrix(x):
    phi = np.empty(x.shape + (2,))
    phi[:, 0] = 1.
    phi[:, 1] = x
    return phi
```

Define Huber and Tukey reweighted iterative regularized regression

In [ ]:
```
def iterative_reweighted_least_squares_huber(x, y, eta=1., n_iter=1000):
    phi = build_design_matrix(x)
    # initialize theta using the solution of regularized ridge regression
    theta = theta_prev = np.linalg.solve(
    phi.T.dot(phi) + 1e-4 * np.identity(phi.shape[1]), phi.T.dot(y))
    for _ in range(n_iter):
        r = np.abs(phi.dot(theta_prev) - y)
        w = np.diag(np.where(r > eta, eta / r, 1.))
        phit_w_phi = phi.T.dot(w).dot(phi)
        phit_w_y = phi.T.dot(w).dot(y)
        theta = np.linalg.solve(phit_w_phi, phit_w_y)
        if np.linalg.norm(theta - theta_prev) < 1e-3: break
        theta_prev = theta
    return theta

def iterative_reweighted_least_squares_tukey(x, y, eta=1., n_iter=1000):
    phi = build_design_matrix(x)
    # initialize theta using the solution of regularized ridge regression
    theta = theta_prev = np.linalg.solve(
    phi.T.dot(phi) + 1e-4 * np.identity(phi.shape[1]), phi.T.dot(y))
    for _ in range(n_iter):
        r = np.abs(phi.dot(theta_prev) - y)
        w = np.diag(np.where(r > eta, 0, (1-(r/eta)**2)**2))
        phit_w_phi = phi.T.dot(w).dot(phi)
        phit_w_y = phi.T.dot(w).dot(y)
        theta = np.linalg.solve(phit_w_phi, phit_w_y)
        if np.linalg.norm(theta - theta_prev) < 1e-3: break
        theta_prev = theta
    return theta
```

```
def predict(x, theta):
    phi = build_design_matrix(x)
    return phi.dot(theta)
```

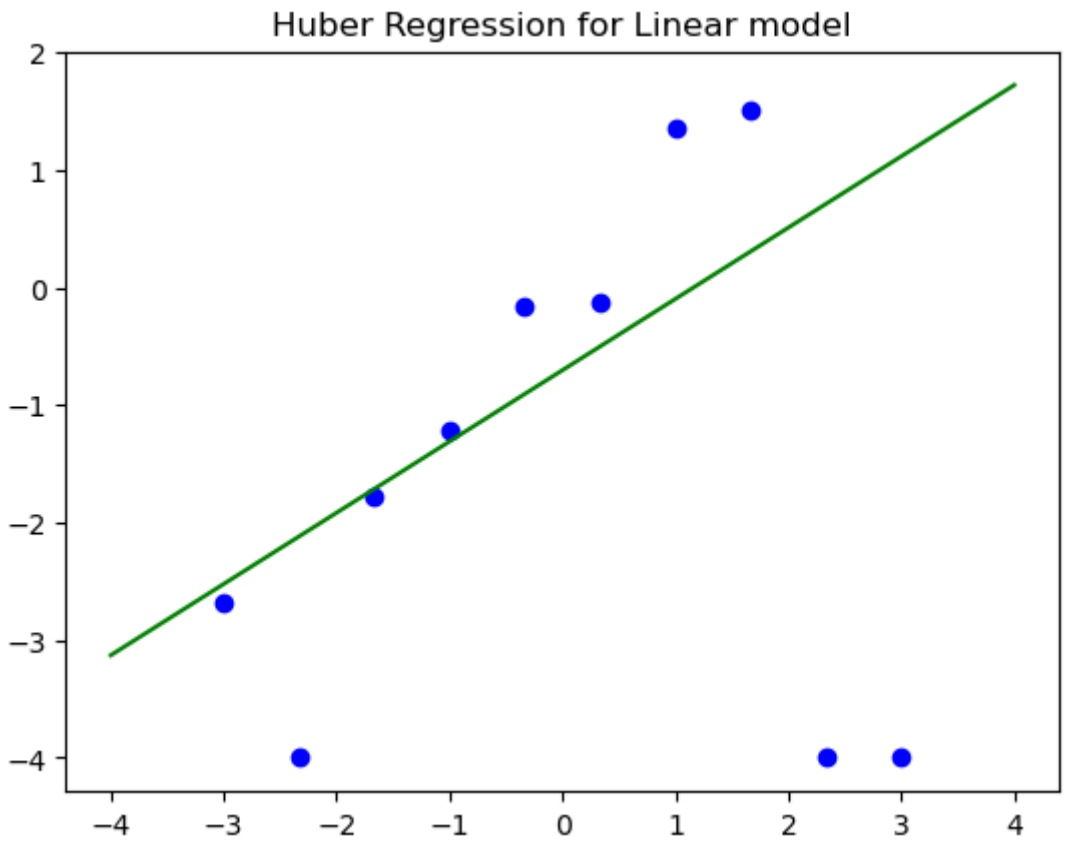
Visualize

In [ ]:
```
def visualize(x, y, theta, x_min=-4., x_max=4., title='plot'):
    X = np.linspace(x_min, x_max, 1000)
    Y = predict(X, theta)
    plt.clf()
    plt.plot(X, Y, color='green')
    plt.scatter(x, y, c='blue', marker='o')
    plt.title(title)
    plt.show()

x, y = generate_sample()
theta1 = iterative_reweighted_least_squares_huber(x, y, eta=1.)
theta2 = iterative_reweighted_least_squares_tukey(x, y, eta=1.)
```

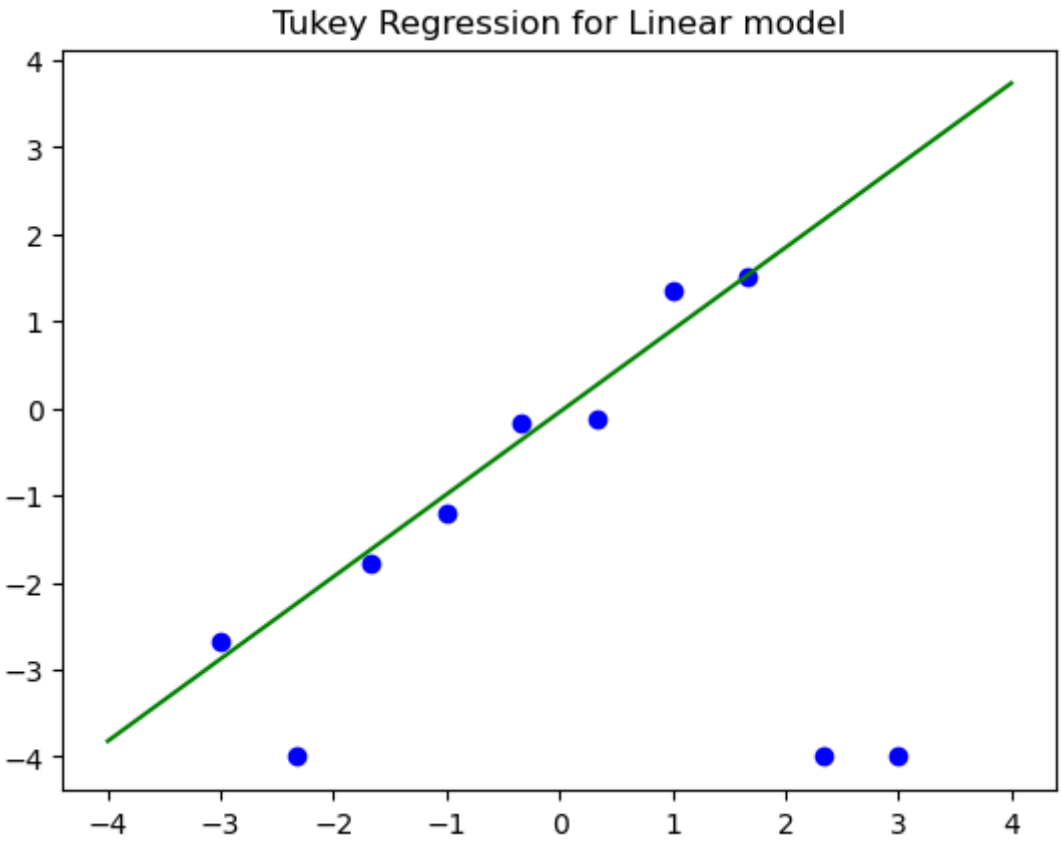
Visualize Huber

In [ ]:
```
visualize(x, y, theta1, title = "Huber Regression for Linear model")
```



Visualize Tukey

In [ ]:
```
visualize(x, y, theta2, title = "Tukey Regression for Linear model")
```


Tukey Regression for Linear model
4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3
4