Advanced Data Analysis
Homework Week 5

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Homework 5

Given the complimentary slackness conditions of SVM,

$$\alpha_i(y_i w^T x_i - 1 + \xi_i) = 0 \tag{1}$$
$$\beta_i \xi_i = 0 \tag{2}$$

and from the dual problem,

$$y_i w^T x_i - 1 + \xi_i \geq 0 \tag{3}$$
$$\xi_i \geq 0 \tag{4}$$
$$\alpha_i + \beta_i = C \tag{5}$$

To prove:

1) $\alpha_i = 0 \implies y_i w^T x_i \geq 1$

$$\begin{aligned}
&\alpha_i = 0 \\
&\implies \beta_i = C \ (Eq.\ 5) \\
&\implies \xi_i = 0 \ (Eq.\ 2) \\
&\implies y_i w^T x_i - 1 \geq 0 \ (Eq.\ 3) \\
&\implies y_i w^T x_i \geq 1
\end{aligned}$$

2) $0 < \alpha_i < C \implies y_i w^T x_i = 1$

$$
\begin{aligned}
0 < \alpha_i < C & \\
& \implies y_i w^T x_i - 1 + \xi_i = 0 \; (Eq.\ 1) \\
& \implies 0 < C - \beta_i < C \; (Eq.\ 5) \\
& \implies 0 < C - \beta_i < C \\
& \implies C > \beta_i > 0 \\
& \implies \xi_i = 0 \; (Eq.\ 2) \\
& \implies y_i w^T x_i = 1 \; (From\ above)
\end{aligned}
$$

3) $\alpha_i = C \implies y_i w^T x_i \leq 1$

$$
\begin{aligned}
& \alpha_i = C \\
& \implies \beta_i = 0 \; (Eq.\ 5) \\
& \implies y_i w^T x_i - 1 + \xi_i = 0 \; (Eq.\ 1) \\
& \implies y_i w^T x_i = 1 - \xi_i \\
& \implies y_i w^T x_i \leq 1 \; (Eq.\ 4)
\end{aligned}
$$

4) $y_i w^T x_i > 1 \implies \alpha_i = 0$

$$
\begin{aligned}
& y_i w^T x_i - 1 > 0 \; Given \\
& \implies y_i w^T x_i - 1 + \xi_1 > 0 \; (Eq.\ 4) \\
& \implies \alpha_i = 0 \; (Eq.\ 1)
\end{aligned}
$$

5) $y_i w^T x_i < 1 \implies \alpha_i = C$

$$
\begin{aligned}
& y_i w^T x_i - 1 < 0 \; Given \\
& but,\ y_i w^T x_i - 1 + \xi_i \geq 0 \; (Eq.\ 3) \\
& \implies \xi_i > 0 \; (\xi_i\ can't\ be\ 0) \\
& \implies \beta_i = 0 \; (Eq.\ 2) \\
& \implies \alpha_i = C \; (Eq.\ 5)
\end{aligned}
$$

Thus proven.