Exercise problems

Problem 1

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & -5 \\ 1 & 1 & 1 & 1 & -1 \end{pmatrix}.$$

Compute:

1. extreme rays of $\ker(A) \cap \mathbb{R}^5_+$,
2. circuits of A,
3. Hilbert basis of $\ker(A) \cap \mathbb{R}^5_+$,
4. Graver basis of A
5. facets of $\text{cone}(A)$,
6. Hilbert basis of $\text{cone}(A)$,
7. monoid generators for $\text{rowspan}(A) \cap \mathbb{Z}^5_+$,
8. generators for I_A,
9. lexicographic Gröbner basis for I_A,
10. walk to degrevlex Gröbner basis of I_A.

Problem 2

Solve

$$\begin{aligned} x + y + z &\geq 3 \\ 2x + 3y &\equiv 1 \pmod 5 \end{aligned}$$

over $\mathbb{Z}$.

Problem 3

Transform the resulting sets from Problem 2 into Maple input format.

Problem 4

Solve

$$\min\{3x + 4x + 17z + 13u : x + 2y + 12z + 27u = 111111, x, y, z, u \in \mathbb{Z}_+\}.$$