OQ-HAZARD/NHLIB Quality Assurance Tests

September 10, 2012

0.1 Theoretical background

Assuming seismicity following a *Poissonian* temporal occurrence model, the calculation of the *probability of (at least one) exceedance* of a ground motion value x in a time span T, for an intensity measure IM, can be computed by first calculating the annual rate of exceedance:

$$\lambda(IM \geq x) = \sum_{i=1}^{N_{sources}} \lambda_i(m \geq M_{min}) \int_{M_{min}}^{M^i_{max}} \int_{r=0}^{\infty} P(IM \geq x|m,r) f_i(m) f_i(r) dr dm \tag{1}$$

where:

- $N_{sources}$: total number of sources in the source model
- $\lambda_i(m \geq M_{min})$: annual rate of ruptures in the i-th source with magnitude greater than or equal to M_{min}
- M^i_{max}: maximum magnitude in the i-th source
- $P(IM \geq x|m,r)$: conditional probability for an intensity measure IM to exceede an intensity measure level (x), given magnitude value (m) and distance (r)
- $f_i(m)$: probability density function for magnitude, for the i-th source
- $f_r(r)$: probability density function for distance, for the i-th source

and then computing the Poissonian probability of at least one exceedance using the equation:

$$P(IM \geq x) = 1 - e^{-\lambda(IM \geq x)T} \tag{2}$$

Assuming ground motion distribution to follow (in log scale) a truncated normal distribution, we can compute $P(IM \geq x|m,r)$ as:

$$P(IM \geq x|m,r) = 1 - \frac{\Phi(\frac{ln(x) - \overline{ln(IM;m,r)}}{\sigma}) - \Phi(-n_{trunc})}{\Phi(n_{trunc}) - \Phi(-n_{trunc})} \tag{3}$$

In the following tests, a truncation level of 2 (that is $n_{trunc} = 2$) will be used. In this case, rounding to 4 digits, $\Phi(2) = 0.9772$, $\Phi(-2) = 0.0228$, and $\Phi(2) - \Phi(-2) = 0.9545$. So we can rewrite equation 3 as:

$$P(IM \geq x|m,r) = 1 - \frac{\Phi(\frac{ln(x) - \overline{ln(IM;m,r)}}{\sigma}) - 0.0228}{0.9545} \tag{4}$$

In case a truncation level of 0 is considered, we can write $P(IM \geq x|m,r)$ as:

$$P(IM \geq x|m,r) = H(\overline{ln(IM;m,r)} - ln(x)) \tag{5}$$

where H is the Heaviside (or step) function. That is $P(IM \geq x|m,r) = 1$ if $\overline{ln(IM;m,r)} \geq ln(x)$ and 0 otherwise.

0.2 GMPE

The GMPE utilized for the definition of the QA tests is Sadigh et al 1997, for PGA on rock sites, for strike slip events (rake = 0) with magnitude less or equal to 6.5. The mean of the logarithm of PGA is predicted by the following equation:

$$\overline{ln(IM;m,r)} = -0.624 + m - 2.1ln(r + exp(1.29649 + 0.25m)) \tag{6}$$

where r is the closest distance to the rupture. The standard deviation is given by:

$$\sigma = 1.39 - 0.14m \tag{7}$$

0.3 Magnitude Scaling Relationship

The magnitude scaling relationship utilized in the QA tests for calculating median rupture area from magnitude value is the one defined in the PEER tests, that is:

$$A = 10^{m-4} \tag{8}$$

where A is the rupture area (in squared km).

0.4 Test 1 - Hazard curve calculation with single point source, single magnitude MFD, and with ground motion truncation level = 2.0

NOTE*: This test is meant to exercise the hazard curve calculator (both classical and event-based) with a point source*

Let's consider a source model consisting of a single point source, generating a single rupture of magnitude 4, with aspect ratio (length/width) equal to 1, vertical (dip=90.0), and with an annual occurrence rate ($\lambda(m = 4)$) equal to 1. This means that the probability density function for magnitude is a dirac delta function centered at m = 4, that is:

$$f(m) = \delta(m - 4.0) \tag{9}$$

Let's further assume that the rupture hypocenter (located at the centroid of the rupture surface) is located at 4 km depth, and let's assume upper seismogenic depth at 3.5 km, and lower seismogenic depth at 4.5 km. Let's assume the site of interest to be right on top of the point source (that is same latitude and longitude). The rupture area (from eq. 8) is 1 squared km, and given that the aspect ratio is 1, the rupture length and width are equal to 1 km. Given that the rupture hypocenter is assumed to be in the centre of the rupture surface, the top of rupture depth is 3.5 km. Given that the site is sitting just right on top of the point source, the closest distance from site to rupture is 3.5 km. Given that there is a single rupture scenario, and this is the only possible distance value, the probability density function for the closest distance to rupture is a dirac delta function centered at r = 3.5, that is:

$$f(r) = \delta(r - 3.5) \tag{10}$$

Substituting equations 0.7 and 10 in equation 1, and considering that there is only one source in the source model, and, assuming $M_{min} = 4$, $\lambda_i(m \geq M_{min}) = \lambda(m = 4) = 1$, equation 1 simplifies to:

$$\lambda(IM \geq x) = P(IM \geq x|4.0, 3.5) \tag{11}$$

The mean of the logarithm of PGA, as predicted by equation 6 is (rounded to 4 digits):

$$\overline{ln(IM; m, r)} = -0.624 + 4.0 - 2.1ln(3.5 + exp(1.29649 + 0.25 * 4.0)) = -2.0802 \tag{12}$$

and the corresponding standard deviation (as from eq. 7) is:

$$\sigma = 1.39 - 0.14 * 4.0 = 0.83 \tag{13}$$

The rates of exceedance for PGA levels equal to 0.1, 0.4, 0.6 are:

$$\begin{aligned}
\lambda(PGA \geq 0.1) &= 1 - \frac{\Phi(\frac{ln(0.1)+2.0802}{0.83}) - 0.0228}{0.9545} = 0.6107 \\
\lambda(PGA \geq 0.4) &= 1 - \frac{\Phi(\frac{ln(0.4)+2.0802}{0.83}) - 0.0228}{0.9545} = 0.0605 \\
\lambda(PGA \geq 0.6) &= 1 - \frac{\Phi(\frac{ln(0.6)+2.0802}{0.83}) - 0.0228}{0.9545} = 0.0069
\end{aligned} \tag{14}$$

The corresponding probabilities of exceedance in a time period of 1 year ($T = 1.0$) are:

$$
\begin{aligned}
P(PGA \geq 0.1) = 1 - exp(-0.6107 * 1.0) = 0.4570 \\
P(PGA \geq 0.4) = 1 - exp(-0.0605 * 1.0) = 0.0587 \\
P(PGA \geq 0.6) = 1 - exp(-0.0069 * 1.0) = 0.0069
\end{aligned} \tag{15}
$$

0.5 Test 2 - Hazard curve calculation with single point source, truncated GR MFD, and ground motion truncation level = 0

NOTE: *This test is meant to exercise the hazard curve calculator (both classical and event-based) with a point source having a Gutenberg Richter magnitude frequency distribution*

This test assumes a source model consisting of a single point source, with a magnitude frequency distribution (MFD) defined as a truncated Gutenbergh-Richter with the following parameters: $a = 2.0$, $b = 1.0$, $M_{min} = 4.0$, $M_{max} = 7.0$. The point source is assumed to generate ruptures at a single hypocentral depth of 0.5 km, with an aspect ratio of 1. The upper seismogenic depth is set to 0.0 (the lower seismogenic depth can be set to an arbitrary value, clearly larger than the hypocentral depth).

Starting from the minimum magnitude all ruptures generated by the source will reach the earth surface. Indeed, for magnitude $M = 4$, rupture area is 1 squared km (from equation 8) and assuming aspect ratio of 1 , rupture width is 1, and having set rupture hypocenter to 0.5 km, the rupture will reach the surface. Assuming site of interest to be right on top the source location, for all ruptures the closest distance to the site of interest is 0.0 km. This means that the probability density function for the closest distance to rupture is a dirac delta function centered at r = 0.0, that is:

$$f(r) = \delta(r) \tag{16}$$

The probability density function for a truncated GR can be written as:

$$f(m) = \frac{b\,ln(10)10^{-b(m-M_{min})}}{1 - 10^{-b(M_{max}-M_{min})}} \tag{17}$$

Assuming a truncation level equal to 0, the probability of the logarithm of PGA exceeding a level x can be computed using equation 5.

Substituting equations 16, 17, and 5 in equation 1, and considering that the source model consists of only one source, we can write:

$$
\begin{aligned}
\lambda(PGA \geq x) = \lambda(m \geq M_{min}) \int_{M_{min}}^{M_{max}} H(-0.624 + m - 2.1(1.29649 + 0.25m) - ln(x)) \\
\frac{b\,ln(10)10^{-b(m-M_{min})}}{1 - 10^{-b(M_{max}-M_{min})}} dm = \\
\lambda(m \geq M_{min}) \frac{b\,ln(10)}{1 - 10^{-b(M_{max}-M_{min})}} \int_{M_{min}}^{M_{max}} H(0.475m - 3.346629 - ln(x))10^{-b(m-M_{min})} dm = \\
\lambda(m \geq M_{min}) \frac{b\,ln(10)}{1 - 10^{-b(M_{max}-M_{min})}} \left[-\frac{10^{-b(m-M_{min})}}{b\,ln(10)} \right]_{max\{M_{min}, \frac{3.346629+ln(x)}{0.475}\}}^{M_{max}}
\end{aligned} \tag{18}
$$

if $M_{max} \leq \frac{3.346629+ln(x)}{0.475}$, otherwise the result of the equation is 0.

if $M_{min} \geq \frac{3.346629+ln(x)}{0.475}$, we can simplify equation 18 as:

$$\lambda(PGA \geq x) = \frac{\lambda(m \geq M_{min})}{1 - 10^{-b(M_{max}-M_{min})}} (1 - 10^{-b(M_{max}-M_{min})}) \tag{19}$$

if $M_{min} \leq \frac{3.346629+ln(x)}{0.475}$, we can write equation 18 as:

$$\lambda(PGA \geq x) = \frac{\lambda(m \geq M_{min})}{1 - 10^{-b(M_{max}-M_{min})}} (10^{-b(\frac{3.346629+ln(x)}{0.475} - M_{min})} - 10^{-b(M_{max}-M_{min})}) \tag{20}$$

The rate of exceedance for PGA level equal to 0.1 is (considering that $(3.346629 + ln(0.1))/0.475 \approx 2.2...$ and therefore using equation 19):

$$\lambda(PGA \geq 0.1) = \frac{10^{-2}}{1-10^{-3}}(1-10^{-3}) = 10^{-2} \tag{21}$$

and the corresponding probability of exceedance in a time period of 1 year is (rounding to 5 digits):

$$P(PGA \geq 0.1) = 1 - exp(-10^{-2} * 1.0) = 0.00995 \tag{22}$$

The rate of exceedance for PGA level equal to 0.4 is (considering that $(3.346629 + ln(0.4))/0.475 = 5.11650$ and therefore using equation 20):

$$\lambda(PGA \geq 0.4) = \frac{10^{-2}}{1-10^{-3}}(10^{-(5.11650-4.0)} - 10^{-3}) = 0.00076 \tag{23}$$

and the corresponding probability of exceedance in a time period of 1 year is (rounding to 5 digits):

$$P(PGA \geq 0.4) = 1 - exp(-0.00076 * 1.0) = 0.00076 \tag{24}$$

The rate of exceedance for PGA level equal to 0.6 is (considering that $(3.346629 + ln(0.6))/0.475 = 5.97011$ and therefore using equation 20):

$$\lambda(PGA \geq 0.6) = \frac{10^{-2}}{1-10^{-3}}(10^{-(5.97011-4.0)} - 10^{-3}) = 9.7 \cdot 10^{-5} \tag{25}$$

and the corresponding probability of exceedance in a time period of 1 year is (rounding to 5 digits):

$$P(PGA \geq 0.6) = 1 - exp(-9.7 \cdot 10^{-5} * 1.0) = 9.7 \cdot 10^{-5} \tag{26}$$

The rate of exceedance for PGA level equal to 1.0 is 0, considering that $(3.346629 + ln(1.0))/0.475 = 7.045531$ and therefore greater than the considered maximum magnitude: and the corresponding probability of exceedance in a time period of 1 year is also 0:

$$P(PGA \geq 1.0) = 1 - exp(-0 * 1.0) = 0 \tag{27}$$

0.6 Test 3 - Hazard curve calculation with single area source, single magnitude MFD, and ground motion truncation level = 0

***NOTE**: This test is meant to exercise the hazard curve calculator (both classical and event-based) with an area source*

This test assumes a source model consisting of a single area source, whose polygon defines a circle of 5 km radius. The MFD contains a single magnitude ($M = 4$) associated to an annual rate of exceedance ($\lambda(m = 4.0)$) equal to 1. The hypocentral depth of the area source is set to 0.5 and the upper seismogenic depth to 0 km. Aspect ratio is assumed equal to 1. Under this conditions all ruptures reach the surface. The lower seismogenic depth can be set to a value greater or equal then 1.0 km. In this case all ruptures have a square shape and have uniform length of 1 km.

With these assumptions, the probability density function for magnitude is a dirac delta function as given in equation 0.7.

Assuming a site located in the centre of the area source, and indicating with l the rupture length and with R the radius of the area source, the probability density function for the closest distance to the rupture is given by:

$$f(r) = \begin{cases} \frac{2r}{R^2} + \frac{l}{2R^2} & \text{if } r \leq R - \frac{l}{2} \\ \frac{1}{R} & \text{if } R - \frac{l}{2} < r \leq R \\ 0 & \text{if } r > R \end{cases} \tag{28}$$

Assuming a truncation level equal to 0, we can write equation 1 (considering a single source and $\lambda(m \geq M_{min}) = \lambda(m = 4.0) = 1$) as:

$$\lambda(IM \geq x) = \int_0^{R-\frac{l}{2}} \left[\frac{2r}{R^2} + \frac{l}{2R^2}\right] H(-0.624 + 4.0 - 2.1ln(r + exp(1.29649 + 0.25 \cdot 4.0)) - ln(x))dr + \frac{1}{R}\int_{R-\frac{l}{2}}^{R} H(-0.624 + 4.0 - 2.1ln(r + exp(1.29649 + 0.25 \cdot 4.0)) - ln(x))dr \quad (29)$$

The Heaviside function is non-zero if and only if $-0.624+4.0-2.1ln(r+exp(1.29649+0.25\cdot 4.0))-ln(x) \geq 0$, that is $r \leq e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649}$. If $R \leq e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649}$, then we can solve equation 29 as:

$$\lambda(IM \geq x) = \frac{1}{R^2}(R - \frac{l}{2})^2 + \frac{l}{2R^2}(R - \frac{l}{2}) + \frac{l}{2R} \quad (30)$$

if $R - \frac{l}{2} < e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649} < R$, then we can solve equation 29 as:

$$\lambda(IM \geq x) = \frac{1}{R^2}(R - \frac{l}{2})^2 + \frac{l}{2R^2}(R - \frac{l}{2}) + \frac{1}{R}(e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649} - R + \frac{l}{2}) \quad (31)$$

if $0 < e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649} < R - \frac{l}{2}$, then we can solve equation 29 as:

$$\lambda(IM \geq x) = \frac{1}{R^2}(e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649})^2 + \frac{l}{2R^2}(e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649}) \quad (32)$$

The rate of exceedance of PGA value equal to 0.1 is (considering that $R = 5 < e^{\frac{3.376-ln(0.1)}{2.1}} - e^{2.29649} = 5.00145$, and therefore using equation 30) :

$$\lambda(IM \geq 0.1) = \frac{1}{5^2}(5 - 0.5)^2 + \frac{1}{2 \cdot 5^2}(5 - 0.5) + \frac{1}{2 \cdot 5} = 1.0 \quad (33)$$

and the corresponding probability of exceedance in a time span of 1 year is:

$$P(IM \geq 0.1) = 1 - exp(-1 \cdot 1) = 0.63212 \quad (34)$$

The rate of exceedance of PGA value equal to 0.12 is (considering that $e^{\frac{3.376-ln(0.12)}{2.1}} - e^{2.29649} = 3.75902 < R - \frac{l}{2} = 4.5$ and therefore using equation 32):

$$\lambda(IM \geq 0.12) = \frac{1}{5^2}(e^{\frac{3.376-ln(0.12)}{2.1}} - e^{2.29649})^2 + \frac{1}{2 \cdot 5^2}(e^{\frac{3.376-ln(0.12)}{2.1}} - e^{2.29649}) = 0.640389 \quad (35)$$

and the corresponding probability of exceedance in a time span of 1 year is:

$$P(IM \geq 0.12) = 1 - exp(-0.640389 \cdot 1) = 0.47291 \quad (36)$$

The rate of exceedance of PGA value equal to 0.2 is (considering that $e^{\frac{3.376-ln(0.2)}{2.1}} - e^{2.29649} = 0.80123 < R - \frac{l}{2} = 4.5$ and therefore using equation 32):

$$\lambda(IM \geq 0.2) = \frac{1}{5^2}(e^{\frac{3.376-ln(0.2)}{2.1}} - e^{2.29649})^2 + \frac{1}{2 \cdot 5^2}(e^{\frac{3.376-ln(0.2)}{2.1}} - e^{2.29649}) = 0.04170 \quad (37)$$

and the corresponding probability of exceedance in a time span of 1 year is:

$$P(IM \geq 0.2) = 1 - exp(-0.04170 \cdot 1) = 0.04084 \quad (38)$$

0.7 Test 4 - Hazard curve calculation with single simple fault source, single magnitude MFD, and ground motion truncation level = 0

***NOTE**: This test is meant to exercise the hazard curve calculator (both classical and event-based) with a simple fault source*

This test assumes a source model consisting of a single simple fault source, characterized by a single magnitude MFD ($M = 4$) with annual occurrence rate ($\lambda(m = 4)$) equal to 1. The fault source has an upper seismogenic depth of 0 km, and a lower seismogenic depth of 1 km. Dip is 90 degrees. Aspect ratio is assumed 1.

Under these assumptions the probability density function for magnitude is given by equation , while the probability density function for the closest distance to the rupture (assuming the site to be in the middle point of the fault trace) is:

$$f(r) = \begin{cases} \frac{l}{L-l}\delta(r) & \text{if } r = 0 \\ \frac{2}{L-l} & \text{if } 0 < r \le \frac{L}{2} - l \end{cases} \tag{39}$$

where l is the rupture length, and L is the fault length.

Assuming truncation level equal to 0, we can compute the rate of exceedance of an intensity measure level as:

$$\lambda(PGA \ge x) = \frac{l}{L-l} + \frac{2}{L-l} min(\frac{L}{2} - l, e^{\frac{3.376 - ln(x)}{2.1}} - e^{2.29649}) \tag{40}$$

Assuming L = 10, and given that the rupture length $l = 1$ (because aspect ratio is 1), for x = 0.1, the rate of exceedance is, considering that $\frac{L}{2} - l = 4 < e^{\frac{3.376 - ln(0.1)}{2.1}} - e^{2.29649} = 5.00145$:

$$\lambda(PGA \ge 0.1) = \frac{}{1/9} + \frac{2}{9}4 = 1 \tag{41}$$

and therefore the probability of exceedance in 1 year is:

$$P(PGA \ge 0.1) = 1 - exp(-1 \cdot 1) = 0.63212 \tag{42}$$

The rate of exceedance for x = 0.12 ($\frac{L}{2} - l = 4 > e^{\frac{3.376 - ln(0.12)}{2.1}} - e^{2.29649} = 3.7590$) is:

$$\lambda(PGA \ge 0.12) = \frac{1}{9} + \frac{2}{9}3.7590 = 0.9464 \tag{43}$$

and the corresponding probability of exceedance in 1 year is:

$$P(PGA \ge 0.12) = 1 - exp(-0.9464 \cdot 1) = 0.61186 \tag{44}$$

The rate of exceedance for x = 0.2 ($\frac{L}{2} - l = 4 > e^{\frac{3.376 - ln(0.2)}{2.1}} - e^{2.29649} = 0.801227$) is:

$$\lambda(PGA \ge 0.2) = \frac{1}{9} + \frac{2}{9}0.801227 = 0.28916 \tag{45}$$

and the corresponding probability of exceedance in 1 year is:

$$P(PGA \ge 0.2) = 1 - exp(-0.28916 \cdot 1) = 0.25110 \tag{46}$$

0.8 Test 5 - Hazard curve calculation with single complex fault source, single magnitude MFD, and ground motion truncation level = 0

***NOTE**: This test is meant to exercise the hazard curve calculator (both classical and event-based) with a complex fault source*

This test assumes a complex fault source composed of top and bottom edges. Both the top and bottom edges have the same length, and same latitudes and longitudes, but they are shifted in depth. Both the two edges consist of two line segments, one horizontal (L_1), and a second , of length L_2, inclined of an angle α with respect to the earth surface. The shift in depth between the top and bottom edges is of 1 km.
The source is associated to a single magnitude MFD, defining a single magnitude value equal to 4, and with corresponding annual rate $\lambda(m = 4)$, equal to 1. The aspect ratio is assumed 1. The rupture area is 1 squared km (as predicted by equation 8). Ruptures floating along the first section have a square shape, but as soon as they start propagating along the inclined section, they start acquiring the shape of a parallelogram. This means that while on the first section the rupture length is 1 km, on the second section the rupture length is $l = \frac{A}{w \cdot sin(90-\alpha)}$, where A is the rupture area, w is the fault width (that is the shift between top and bottom edges), and α is the inclination of the second section with respect to the horizontal direction.
The probability density function for magnitude is given by equation 0.7. Assuming a site located at the beginning of the first segment (and assuming the first segment of the top edge starting at the earth surface), the probability density function for the closest distance to the rupture is:

$$f(r) = \begin{cases} \frac{1}{L_1+L_2-l} & \text{if } 0 \le r \le L_1 \\ \frac{r}{(L_1+L_2-l)\sqrt{L_1^2 cos^2\alpha - L_1^2 + r^2}} & \text{if } L_1 < r \le r_{max} \end{cases} \tag{47}$$

where:

$$r_{max} = \sqrt{(L_2 - l)^2 + L_1^2 + 2L_1(L_2 - l)cos\alpha} \tag{48}$$

Assuming $L_1 = 3.0$ km, $L_2 = 3.0$ km, $\alpha = 30.0$ degrees, and $w = 1$, the maximum closest distance to the rupture is 4.68973.
Assuming truncation level = 0, the rate of exceedance of a PGA level x is (from equation 1):

$$\lambda(PGA \ge x) = \int_0^{min(r_{max}, e^{\frac{3.376-ln(x)}{2.1}} - e^{2.29649})} f(r)dr \tag{49}$$

Considering x = 0.1, $r_{max} = 4.68973 < e^{\frac{3.376-ln(0.1)}{2.1}} - e^{2.29649} = 5.00145$, and therefore:

$$\begin{aligned} \lambda(PGA \ge 0.1) = \frac{L_1}{L_1+L_2-l} + \frac{1}{L_1+L_2-l}\left[\sqrt{L_1^2cos^2\alpha - L_1^2 + r^2}\right]_{L_1}^{r_{max}} = \\ \frac{L_1}{L_1+L_2-l} + \frac{1}{L_1+L_2-l}\left[\sqrt{L_1^2cos^2\alpha - L_1^2 + (L_2-l)^2 + L_1^2 + 2L_1(L_2-l)cos\alpha} - L_1cos\alpha\right] = \\ \frac{L_1}{L_1+L_2-l} + \frac{1}{L_1+L_2-l}\left[\sqrt{(L_1cos\alpha + (L_2-l))^2} - L_1cos\alpha\right] = \\ \frac{L_1}{L_1+L_2-l} + \frac{1}{L_1+L_2-l}\left[L_1cos\alpha + L_2 - l - L_1cos\alpha\right] = 1 \end{aligned} \tag{50}$$

The corresponding probability of exceedance in 1 year is therefore:

$$P(PGA \ge 0.1) = 1 - exp(-1 \cdot 1) = 0.632120 \tag{51}$$

Considering x = 0.12, $r_{max} = 4.68973 > e^{\frac{3.376-ln(0.12)}{2.1}} - e^{2.29649} = 3.7590 > L_1 = 3.0$, and therefore:

$$\begin{aligned} \lambda(PGA \ge 0.12) = \frac{L_1}{L_1+L_2-l} + \frac{1}{L_1+L_2-l}\left[\sqrt{L_1^2cos^2\alpha - L_1^2 + 3.7590^2} - L_1cos\alpha\right] = \\ \frac{3}{3+3-\frac{2}{\sqrt{3}}} + \frac{1}{3+3-\frac{2}{\sqrt{3}}}\left[\sqrt{9 \cdot \frac{3}{4} - 9 + 3.7590^2} - \frac{3\sqrt{3}}{2}\right] = 0.79431 \end{aligned} \tag{52}$$

The corresponding probability of exceedance in 1 year is therefore:

$$P(PGA \ge 0.12) = 1 - exp(-0.79431 \cdot 1) = 0.54811 \tag{53}$$

Considering x = 0.2, $r_{max} = 4.68973 > e^{\frac{3.376 - ln(0.2)}{2.1}} - e^{2.29649} = 0.801227 < L_1 = 3.0$, and therefore:

$$\lambda(PGA \geq 0.2) = \frac{0.801227}{L_1 + L_2 - l} = \frac{0.801227}{3 + 3 - \frac{2}{\sqrt{3}}} = 0.16536 \tag{54}$$

The corresponding probability of exceedance in 1 year is therefore:

$$P(PGA \geq 0.2) = 1 - exp(-0.16536 \cdot 1) = 0.15241 \tag{55}$$

0.9 Test 6 - Hazard curve calculation with source model consisting of multiple sources

NOTE: *This test is meant to exercise the parallelization strategy of the hazard curve calculator (both classical and event-based). Given that the source model consists of multiple sources, a task for each source can be defined, and therefore the task creation and aggregation of the results can be tested.*

This test assumes a source model consisting of 2 seismic sources, as described in tests 4 and 5, that is a simple fault source and a complex fault source. For the simple fault source as described in test 4, the rates of exceedance for PGA levels of 0.1, 0.12 and 0.2 are 1.0, 0.9464, 0.28916, respectively. For the complex fault source as described in test 6, the rate of exceedance for the same PGA levels are: 1.0, 0.79431, 0.16536.

Using equation 1 the rates of exceedance in case of a source model consisting of the two above mentioned sources are: (1+1)=2.0, (0.9464 + 0.79431) = 1.74071, (0.28916+0.16536) = 0.45452. The corresponding probabilities of exceedance in a period of 1 year are:

$$\begin{aligned} P(PGA \geq 0.10) &= 1 - exp(-2 \cdot 1) = 0.86466 \\ P(PGA \geq 0.12) &= 1 - exp(-1.74071 \cdot 1) = 0.82460 \\ P(PGA \geq 0.20) &= 1 - exp(-0.45452 \cdot 1) = 0.36525 \end{aligned} \tag{56}$$

0.10 Test 7 - Hazard curve calculation with logic tree containing multiple source model

NOTE: *This test is meant to exercise the hazard curve calculator (both classical and event-based) when considering a non-trivial logic tree (that is a logic tree with more than one path). The test should check that the correct solution for each path (when using Path Enumeration) is obtained and that the mean hazard curve is correctly computed. This test should also check that Monte Carlo Sampling and Path Enumeration should provide the same mean hazard curve.*

This test assumes a logic tree defining 2 source models. Source model 1 consists of a simple and a complex fault sources, as described in test 6, while source model 2 consists of a single simple fault source as described in test 4.

For PGA levels equal to 0.1, 0.12, 0.2, the probabilities of exceedance from source model 1 are:

$$\begin{aligned} P(PGA \geq 0.10) &= 0.86466 \\ P(PGA \geq 0.12) &= 0.82460 \\ P(PGA \geq 0.20) &= 0.36525 \end{aligned} \tag{57}$$

the probabilities of exceedance from source model 2 are instead:

$$\begin{aligned} P(PGA \geq 0.10) &= 0.63212 \\ P(PGA \geq 0.12) &= 0.61186 \\ P(PGA \geq 0.20) &= 0.25110 \end{aligned} \tag{58}$$

Assuming source model 1 to be assigned to probability value of 0.7 and source model 2 to be assigned to probability value of 0.3, the mean hazard curve is:

$$
\begin{aligned}
P(PGA \geq 0.10) &= 0.7 * 0.86466 + 0.3 * 0.63212 = 0.794898 \\
P(PGA \geq 0.12) &= 0.7 * 0.82460 + 0.3 * 0.61186 = 0.760778 \\
P(PGA \geq 0.20) &= 0.7 * 0.36525 + 0.3 * 0.25110 = 0.331005
\end{aligned}
\tag{59}
$$

0.11 Test 8 - Hazard curve calculation with logic tree containing single source model and a and b Gutenberg Richter absolute uncertainties

NOTE: *This test is meant to exercise the 'uncertaintyType="abGRAbsolute" ' option in the logic tree construction.*

This test assumes a logic tree defining a single source model. The source model contains a single point source defining a Gutenberg Richter magnitude frequency distribution, as defined in test 2. The logic tree defines absolute uncertainties on the Gutenberg Richter a and b values:

$$
\begin{aligned}
probability = 0.2 - a = 2.2, b = 0.8 \\
probability = 0.6 - a = 2.0, b = 1.0 \\
probability = 0.2 - a = 1.8, b = 1.2
\end{aligned}
\tag{60}
$$

For the case $a = 2.2$ and $b = 0.8$, the rates of exceedance for PGA levels of 0.1, 0.4, 0.6, 1.0 are:

$$
\begin{aligned}
\lambda(PGA \geq 0.1) &= \frac{10^{2.2-0.8\cdot4.0}}{1-10^{-0.8(7.0-4.0)}}(1-10^{-0.8(7.0-4.0)}) = 0.1 \\
\lambda(PGA \geq 0.4) &= \frac{10^{2.2-0.8\cdot4.0}}{1-10^{-0.8(7.0-4.0)}}(10^{-0.8(5.11650-4.0)} - 10^{-0.8(7.0-4.0)}) = 0.012439 \\
\lambda(PGA \geq 0.6) &= \frac{10^{2.2-0.8\cdot4.0}}{1-10^{-0.8(7.0-4.0)}}(10^{-0.8(5.97011-4.0)} - 10^{-0.8(7.0-4.0)}) = 0.002265 \\
\lambda(PGA \geq 1.0) &= 0.0
\end{aligned}
\tag{61}
$$

The corresponding probabilities are:

$$
\begin{aligned}
P(PGA \geq 0.1) &= 0.095163 \\
P(PGA \geq 0.4) &= 0.012362 \\
P(PGA \geq 0.6) &= 0.002262 \\
P(PGA \geq 1.0) &= 0.0
\end{aligned}
\tag{62}
$$

For the case $a = 2.0$, $b = 1$, the rates of exceedance, (and the corresponding probabilities) are:

$$
\begin{aligned}
\lambda(PGA \geq 0.1) &= 10^{-2} \\
\lambda(PGA \geq 0.4) &= 0.00076 \\
\lambda(PGA \geq 0.6) &= 9.7 \cdot 10^{-5} \\
\lambda(PGA \geq 1.0) &= 0.0
\end{aligned}
\tag{63}
$$

The corresponding probabilities are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.009950 \\
P(PGA \geq 0.4) &= 0.00076 \\
P(PGA \geq 0.6) &= 9.99995 \cdot 10^{-6} \\
P(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{64}$$

For the case $a = 1.8$ and $b = 1.2$, the rates of exceedance are:

$$\begin{aligned}
\lambda(PGA \geq 0.1) &= \frac{10^{1.8-1.2 \cdot 4.0}}{1 - 10^{-1.2(7.0-4.0)}}(1 - 10^{-1.2(7.0-4.0)}) = 0.001 \\
\lambda(PGA \geq 0.4) &= \frac{10^{1.8-1.2 \cdot 4.0}}{1 - 10^{-1.2(7.0-4.0)}}(10^{-1.2(5.11650-4.0)} - 10^{-1.2(7.0-4.0)}) = 4.5490 \cdot 10^{-5} \\
\lambda(PGA \geq 0.6) &= \frac{10^{1.8-1.2 \cdot 4.0}}{1 - 10^{-1.2(7.0-4.0)}}(10^{-1.2(5.97011-4.0)} - 10^{-1.2(7.0-4.0)}) = 4.07366 \cdot 10^{-6} \\
\lambda(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{65}$$

The corresponding probabilities are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.0009995 \\
P(PGA \geq 0.4) &= 4.5489 \cdot 10^{-5} \\
P(PGA \geq 0.6) &= 4.07365 \cdot 10^{-6} \\
P(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{66}$$

0.12 Test 9 - Hazard curve calculation with logic tree containing single source model and absolute uncertainties on Gutenberg Richter Maximum Magnitude

NOTE*: This test is meant to exercise the 'uncertaintyType="maxMagGRAbsolute" ' option in the logic tree construction*

This test assumes a logic tree defining a single source model. The source model contains a single point source defining a Gutenberg Richter magnitude frequency distribution, as defined in test 2. The logic tree defines absolute uncertainties on the Gutenberg Richter maximum magnitude:

$$\begin{aligned}
probability = 0.5 - Mmax = 7.0 \\
probability = 0.5 - Mmax = 7.5
\end{aligned} \tag{67}$$

For the case $Mmax = 7.0$, the probabilities of exceedance are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.00995 \\
P(PGA \geq 0.4) &= 0.00076 \\
P(PGA \geq 0.6) &= 9.7 \cdot 10^{-5} \\
P(PGA \geq 1.0) &= 0
\end{aligned} \tag{68}$$

For the case $Mmax = 7.5$, the rates of exceedance are:

$$\begin{aligned}
\lambda(PGA \geq 0.1) &= \frac{10^{-2}}{1-10^{-3.5}}(1-10^{-3.5}) = 0.01 \\
\lambda(PGA \geq 0.4) &= \frac{10^{-2}}{1-10^{-3.5}}(10^{-(5.11650-4.0)} - 10^{-3.5}) = 0.000762 \\
\lambda(PGA \geq 0.6) &= \frac{10^{-2}}{1-10^{-3.5}}(10^{-(5.97011-4.0)} - 10^{-3.5}) = 0.000104 \\
\lambda(PGA \geq 1.0) &= \frac{10^{-2}}{1-10^{-3.5}}(10^{-(7.5-4.0)} - 10^{-3.5}) = 0.0
\end{aligned} \tag{69}$$

The corresponding probabilities of exceedance in 1 year are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.00995 \\
P(PGA \geq 0.4) &= 0.00076 \\
P(PGA \geq 0.6) &= 0.000104 \\
P(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{70}$$

0.13 Test 10 - Hazard curve calculation with logic tree containing source model and relative uncertainties on Gutenberg Richter b value

***NOTE**: This test is meant to exercise the 'uncertaintyType="bGRRelative" ' option in the logic tree construction*

This test assumes a logic tree defining a single source model. The source model contains a single point source defining a Gutenberg Richter magnitude frequency distribution, as defined in test 2. The logic tree defines relative uncertainties on the Gutenberg Richter b value:

$$\begin{aligned}
probability = 0.5 &\quad - \quad \delta b = 0.0 \\
probability = 0.5 &\quad - \quad \delta b = +0.4
\end{aligned} \tag{71}$$

For the case $\delta b = 0.0$, the probabilities of exceedance are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.00995 \\
P(PGA \geq 0.4) &= 0.00076 \\
P(PGA \geq 0.6) &= 9.7 \cdot 10^{-5} \\
P(PGA \geq 1.0) &= 0
\end{aligned} \tag{72}$$

For the case $\delta b = +0.4$, the a value of the Gutenberg Richter magnitude frequency distribution is changed to conserve the total moment rate. The new a value is 4.243. The rates of exceedance are:

$$\begin{aligned}
\lambda(PGA \geq 0.1) &= \frac{10^{4.243-1.4\cdot 4.0}}{1-10^{-1.4(7-4)}}(1-10^{-1.4(7-4)}) = 0.04395 \\
\lambda(PGA \geq 0.4) &= \frac{10^{4.243-1.4\cdot 4.0}}{1-10^{-1.4(7-4)}}(10^{-1.4(5.11650-4.0)} - 10^{-1.4(7-4)}) = 0.0012 \\
\lambda(PGA \geq 0.6) &= \frac{10^{4.243-1.4\cdot 4.0}}{1-10^{-1.4(7-4)}}(10^{-1.4(5.97011-4.0)} - 10^{-1.4(7-4)}) = 7.394 \cdot 10^{-5} \\
\lambda(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{73}$$

The corresponding probabilities of exceedance in 1 year are:

$$\begin{aligned} P(PGA \geq 0.1) &= 0.043 \\ P(PGA \geq 0.4) &= 0.0012 \\ P(PGA \geq 0.6) &= 7.394 \cdot 10^{-5} \\ P(PGA \geq 1.0) &= 0.0 \end{aligned} \tag{74}$$

0.14 Test 11 - Hazard curve calculation with logic tree containing single source model and relative uncertainties on Gutenberg Richter Maximum Magnitude

***NOTE**: This test is meant to exercise the 'uncertaintyType="maxMagGRRelative" ' option in the logic tree construction. It also allows to check the calculation of mean and quantile hazard curves.*

This test assumes a logic tree defining a single source model. The source model contains a single point source defining a Gutenberg Richter magnitude frequency distribution, as defined in test 2. The logic tree defines relative uncertainties on the Gutenberg Richter maximum magnitude:

$$\begin{aligned} probability = 0.2 &\quad - \quad \delta M = +0.5 \\ probability = 0.6 &\quad - \quad \delta M = 0.0 \\ probability = 0.2 &\quad - \quad \delta M = -0.5 \end{aligned} \tag{75}$$

For the case $\delta M = +0.5$, the new a value is 1.7438. The corresponding rates of exceedance for PGA levels of 0.1, 0.4, 0.6, 1.0 are:

$$\begin{aligned} \lambda(PGA \geq 0.1) &= \frac{10^{1.7438-1.0\cdot4.0}}{1-10^{-1.0(7.5-4.0)}}(1-10^{-1.0(7.5-4.0)}) = 0.0055 \\ \lambda(PGA \geq 0.4) &= \frac{10^{1.7438-1.0\cdot4.0}}{1-10^{-1.0(7.5-4.0)}}(10^{-1.0(5.11650-4.0)}-10^{-1.0(7.5-4.0)}) = 0.00042 \\ \lambda(PGA \geq 0.6) &= \frac{10^{1.7438-1.0\cdot4.0}}{1-10^{-1.0(7.5-4.0)}}(10^{-1.0(5.97011-4.0)}-10^{-1.0(7.5-4.0)}) = 5.77 \cdot 10^{-5} \\ \lambda(PGA \geq 1.0) &= 0.0 \end{aligned} \tag{76}$$

The corresponding probabilities are:

$$\begin{aligned} P(PGA \geq 0.1) &= 0.0055 \\ P(PGA \geq 0.4) &= 0.00042 \\ P(PGA \geq 0.6) &= 5.77 \cdot 10^{-5} \\ P(PGA \geq 1.0) &= 0.0 \end{aligned} \tag{77}$$

For the case $\delta M = 0.0$, the probabilities of exceedance are equal to the one provided in test 2, that is:

$$\begin{aligned} P(PGA \geq 0.1) &= 0.00995 \\ P(PGA \geq 0.4) &= 0.00076 \\ P(PGA \geq 0.6) &= 9.7 \cdot 10^{-5} \\ P(PGA \geq 1.0) &= 0 \end{aligned} \tag{78}$$

For the case $\delta M = -0.5$, the new a value is 2.261. The corresponding rates of exceedance for PGA levels of 0.1, 0.4, 0.6, 1.0 are:

$$\begin{aligned}
\lambda(PGA \geq 0.1) &= \frac{10^{2.261-1.0\cdot 4.0}}{1-10^{-1.0(6.5-4.0)}}(1-10^{-1.0(6.5-4.0)}) = 0.018 \\
\lambda(PGA \geq 0.4) &= \frac{10^{2.261-1.0\cdot 4.0}}{1-10^{-1.0(6.5-4.0)}}(10^{-1.0(5.11650-4.0)}-10^{-1.0(6.5-4.0)}) = 0.0013 \\
\lambda(PGA \geq 0.6) &= \frac{10^{2.261-1.0\cdot 4.0}}{1-10^{-1.0(6.5-4.0)}}(10^{-1.0(5.97011-4.0)}-10^{-1.0(6.5-4.0)}) = 0.00014 \\
\lambda(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{79}$$

The corresponding probabilities are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.018 \\
P(PGA \geq 0.4) &= 0.0013 \\
P(PGA \geq 0.6) &= 0.00014 \\
P(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{80}$$

The mean probabilies of exceedance are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.2 * 0.0055 + 0.6 * 0.00995 + 0.2 * 0.018 = 0.01067 \\
P(PGA \geq 0.4) &= 0.2 * 0.00042 + 0.6 * 0.00076 + 0.2 * 0.0013 = 0.0008 \\
P(PGA \geq 0.6) &= 0.2 * 5.77 \cdot 10^{-5} + 0.6 * 9.7 \cdot 10^{-5} + 0.2 * 0.00014 = 9.774e-05 \\
P(PGA \geq 1.0) &= 0.2 * 0.0 + 0.6 * 0.0 + 0.2 * 0.0 = 0.0
\end{aligned} \tag{81}$$

The probabilities of exceedance for quantile level 0.1 are:

$$\begin{aligned}
P(PGA \geq 0.1) &= 0.0055 \\
P(PGA \geq 0.4) &= 0.00042 \\
P(PGA \geq 0.6) &= 5.77 \cdot 10^{-5} \\
P(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{82}$$

The probabilities of exceedance for quantile level 0.9 are:

$$\begin{aligned}
P(PGA \geq 0.1) &= \frac{0.018-0.00995}{1.0-0.8}(0.9-0.8)+0.00995 = 0.013975 \\
P(PGA \geq 0.4) &= \frac{0.0013-0.00076}{1.0-0.8}(0.9-0.8)+0.00076 = 0.00103 \\
P(PGA \geq 0.6) &= \frac{0.00014-9.7\cdot 10^{-5}}{1.0-0.8}(0.9-0.8)+9.7\cdot 10^{-5} = 0.0001185 \\
P(PGA \geq 1.0) &= 0.0
\end{aligned} \tag{83}$$