On Portfolio Diversification

Anna Xia

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Abstract

In this paper, we explore different strategies for portfolio diversification and apply them to build optimal portfolios for investors with different levels of risk aversions. The securities we will be evaluating include: Spotify, Exxon, ServiceNow, Blackberry, Berkshire Hathaway Class B, Cardano, Dogecoin, Bitcoin, LEO, and Polkadot.

1 Introduction

Diversification lies at the heart of portfolio theory. The intuition behind it is simple: when an investor divides her money amongst multiple securities, she limits her exposure to firm-specific risk. While there are countless ways to spread risk across independent sources, this paper focuses mainly on diversification across asset classes.

Modern portfolio theory reasons that risk-averse and risk-tolerant investors share the same preferred portfolio of risky assets. Hence, we will build the optimal risky portfolio using mean variance optimization before creating complete portfolios for investors with different risk preferences. To understand why, let's begin with the notion of risk.

1.1 Investor Risk

Every investor has a different tolerance for how much they are willing to risk. This characteristic is called an individual's degree of risk aversion, A, and is measured by the additional returns required for an investor to accept more risk.

$$A = \frac{E(r_C) - r_f}{\sigma_C^2},$$

where $E(r_C)$ is the expected rate of returns on the investor's portfolio C, r_f is the risk-free rate, and σ_C^2 is the variance of the returns on C, a measure of volatility. Accordingly, someone with a high degree of risk aversion will expect a greater risk premium for each unit of risk taken.

First, we assess the chosen securities individually and move onto equal weight portfolios. The portfolios will consist of either the five equities (SPOT, XOM, CRM, BB, BRK-B), the five cryptocurrencies (ADA, DOGE, BTC, LEO, DOT), or all ten securities. I chose these in particular because these are firms that I follow and they are in different sectors so diversification is efficient.

1.2 Returns on Individual Securities

The preliminary step to building any optimal portfolios is understanding its component securities and finding a benchmark for our analysis. Year-long historical data was used to calculated the daily rate of returns on each of the ten securities chosen, the 13 week T-bill, and NASDAQ composite using the following formulas:

$$\text{simple returns} = r_s = \frac{price_t - price_{t-1}}{price_{t-1}}$$

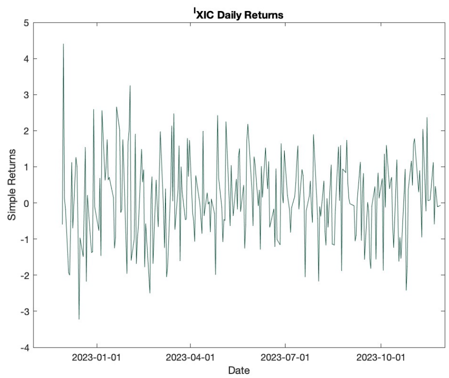
$$\text{log returns} = r_l = ln(\frac{price_t}{price_t})$$

The daily returns were then used in the formula below to calculate the average rate of returns on each security i, denoted $E(r_i)$,

$$E(r_i) = \Sigma_{j=1}^{m} \frac{1}{m} (r_i)_j \ , \tag{1}$$

where $(r_i)_j$ is return on security i on the j-th day and m is the number of days.

The average daily log return of the Treasury bill was calculated to be 0.09%. Since a high risk-free rate skews the optimal portfolio to be less risky, I scaled the number down to 0.07%. The NASDAQ Composite was chosen as the benchmark index because the index covers all the stocks listed on their market, including tech firms unlike the Dow. In particular, he index assigns weights to securities based on their market capitalization, which means the technology sector makes up nearly half the index. Figures (a) and (b) show the daily returns (%) on NASDAQ and a standard statistical breakdown on them.

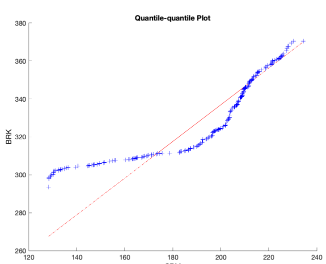


(a) Daily Returns on NASDAQ

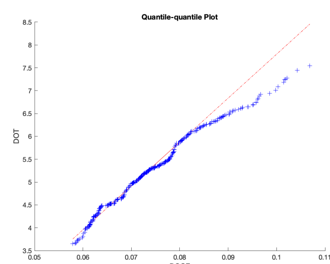
Mean	0.1086
St deviation	1.1919
Variance	1.4206
Skew	0.1510
Kurtosis	3.0990
Coefficient of Variation	10.9744

(b) Stats of NASDAQ Composite

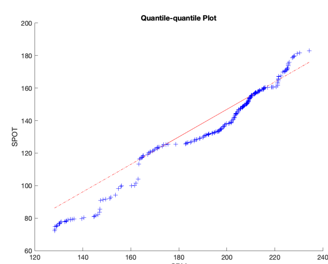
Nearly all of the ten securities I chose have a higher expected rate of return than the index, with ADA leading with 2.412% daily. BRK-B is the only security with expected returns lower than the index, at 0.643%. Notably, BRK-B is also the only security that is less volatile than the index, with a variance of only 0.246%. Additionally, it is interesting to note the distribution of daily returns is heavily skewed right for all but three securities: XOM , BRK-B, and SPOT. From the quantile-quantile plots (c), (d), (e), we also note that the returns from most securities are not normally distributed. The rest of the QQ plots and a table of every asset's statistical measurements can be found in the appendix.



(c) QQ Plot CRM & BRK



(d) QQ Plot DOGE & DOT



(e) QQ Plot CRM & SPOT

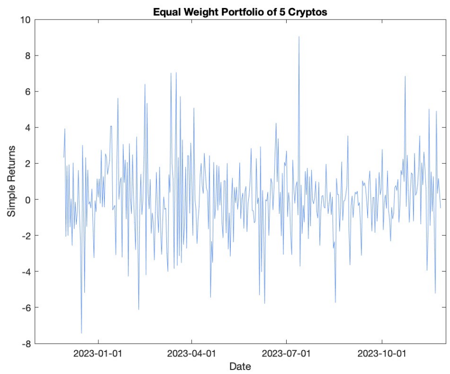
2 Equally Weighted Portfolio

Portfolios where equal amounts of moneys are invested in each security are considered a naive method of diversification. However, it has been observed that equally weighted portfolios (re-balanced monthly) can outperform value-weighted ones. Therefore, it is beneficial to analyze the returns on two equally weighted portfolios of either the five stocks or five cryptocurrencies.

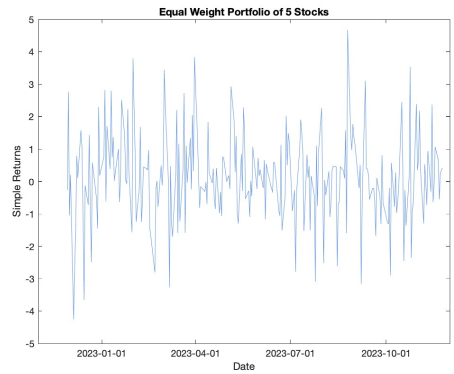
The returns on a portfolio P are the weighted averages of the returns on its component securities.

$$r_P = \sum_{i=1}^{n} w_i r_i$$

for assets i = 1,..,n in P with w_i being the proportion of the portfolio allocated to asset i. These two portfolios are equal weighted ones so $w_i = 0$ for all i. These returns were then plugged into equation



(f) Daily Returns on Equal Weight Crypto Portfolio



(g) Daily Returns on Equal Weight Stock Portfolio

(2) to calculate the average rate of returns and into the following formula to calculate variance:

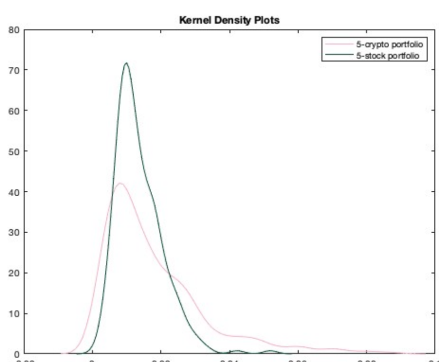
$$\sigma_{r_i}^2 = \frac{\Sigma_{i=1}^{m}((r_i)_j - \overline{r_i})^2}{m-1} ,$$

where $\sigma_{r_i}^2$ is the variance on the returns of security i and $\overline{r_i} = E(r_i)$ Figures (f) and (g) depict the returns on the cryptocurrency and stock portfolio respectively. From first glance, the equally weighted portfolio of cryptos appears to have higher rates of returns than the stock portfolio. This is supported by the kernel density curves in figure (i) that represents the estimated distribution curve of the returns on either portfolios. The curves show that the center (mean) of the distribution of returns on the crypto portfolio is higher than that of the stock portfolio.

A complete table of statistical figures on the returns of both portfolios is found in figure (h). The mean expected rate of returns for both portfolios are higher than that of the index. However, the returns on both portfolios are much more volatile than that of the NASDAQ. Regardless, these equal weighted portfolios are much less volatile than portfolios invested in any single security. In this sense, equally distributing investments between these five stocks or five cryptos benefits the investor greatly even in this example.

	5 Crypto	5 Stock
Mean	0.0921	0.1125
St deviation	2.1768	1.3784
Variance	4.7386	1.8999
Skew	0.1770	0.0959
Kurtosis	4.6765	3.8317
Coefficient of Variance	23.6357	12.2477

(h) Statistics on Equal Weight Portfolios



(i) Daily Returns on Equal Weight Stock Portfolio

3 Optimal Risky Portfolio

Modern portfolio theory assumes investors are risk-averse and aims to construct a portfolio that produces the greatest returns for a given risk level that the investor is willing to tolerate. Harry Markowitz pioneered the field when he recognized variance was often nonlinear and extrapolated how proper diversification of a portfolio could allow investors to reduce risk while maintaining the same expectations for returns. Hence, modern portfolio theory is also commonly known as mean-variance optimization, which we will use to create an optimal risky portfolio.

When we quantify the risk and reward that comes with investing in an asset using standard deviation and mean, we are using statistical measures that describe the probability distribution of the returns on the asset. Consequently, it makes sense to frame Markowitz's optimization problem in a probability theory perspective.

The expected value of a random variable, commonly known as the mean, is defined as the weighted average of all of its possible outcomes. It represents the location of the center of the distribution of a variable or the average outcome over a long run. The expected value of a random [discrete] variable X can be calculated as follows:

$$E(X) = \Sigma x P(x) \ , \tag{2}$$

where x is one possible outcome of X and $P(x)$ is the likelihood of x occurring.

The linearity of expected values follows from its definition. Equation (2) used earlier to find the average returns on a portfolio can be derived from this property. The expected rate of return on a portfolio, $E(r_P)$, is a weighted sum of the expected returns on its component securities.

$$E(r_P) = E(w_1 \times r_1 + ... + w_n \times r_n) = w_1 \times E(r_i) + ... + w_n \times E(r_n) \ ,$$

where $w_1, ..., w_n$ are the weights of each security in the portfolio.

On the other hand, variance measures the spread of a dataset. Mathematically, variance is defined to be the expected value of the difference between actual values of the random variable and its mean:

$$\sigma_X^2 = E[(X - E(X))^2] = E[X^2] - E[X]^2 \ ,$$

where $E(X)$ is the mean of X. The squared term in the definition of variance conveys the non-linearity of variance that Markowitz had observed.

Using the formula above and the linearity of expectation, the variance of two random variables X and Y can be calculated as follows:

$$\sigma_{X+Y}^2 = E[(X+Y)^2] - E[X+Y]^2 = \sigma_X^2 + \sigma_Y^2 + 2Cov(X, Y) \ ,$$

where the term $Cov(X, Y) = E[XY] - E[X]E[Y]$ is the covariance of X and Y, a statistical measure of how two random variables are related.

There are two important observations to make regarding covariance. Firstly, covariance between two variables contributes to the variance of the portfolio so investors benefit from diversification as long as the assets are not perfectly correlated Additionally, the covariance of a random variable and itself is equal to its variance. This quality allows us to compute a neat formula for calculating the variance of a sum of random variables $X_1, ..., X_n$:

$$\sigma_{X_1,...,X_n}^2 = \Sigma_{r=1}^n \Sigma_{s=1}^n \sigma_{rs}^2 \ ,$$

where σ_{rs}^2 is the covariance of X_r and X_s

The previous equation allows us to find the formula for the variance of the returns on a portfolio, σ_P^2

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij} \tag{3}$$

Now, it is evident that portfolio diversification is an optimization practice; we are looking for the portfolio weights adding up to 1 that generate the investor's target return with minimal volatility. The Markowitz's portfolio problem is summarized by the following equations:

$$\min_w w^T \Sigma w \tag{4}$$

$$\text{where } \ \Sigma_{i=1}^n w_i = 1 \ \text{ and } \ \mu^T w = \mu_t \ , \tag{5}$$

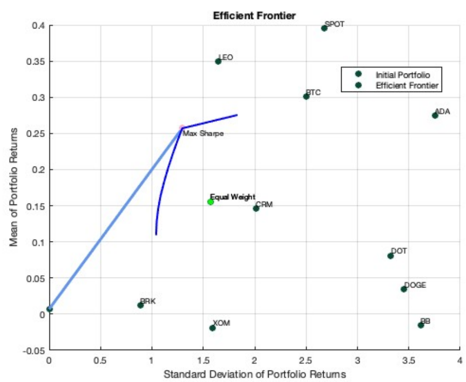
where w is a vector of weights, Σ is the covariance matrix of the returns on each asset, and μ_t is the investor's target rate of return.

3.1 Markowitz's Optimization

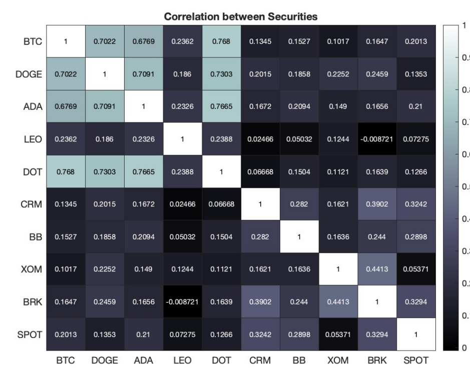
To determine the best composition of risky assets for our portfolios, I conducted mean variance optimization using Matlab on three portfolios: one with all securities and two with five equities or five cryptos. I set a lower constraint on each security such that they must make up at least 10% or 5% of the portfolio. Figures (j), (l) and (m) show the each investment opportunity sets: all the possible combinations of the portfolio's component securities. Each of the points plotted represent a possible portfolio and its respective Sharpe Ratio. The Sharpe ratio, S, quantifies the extra expected return investors can gain per unit of risk taken.

$$S = \frac{\text{portfolio risk premium}}{\text{standard deviation of portfolio excess return}} = \frac{E(r_P) - r_f}{\sigma_P}$$

While investors may have varying degrees of risk aversion, all investors would choose to take less risk if given the chance to do so while maintaining the same returns. As a result, the "efficient" portfolios with the highest reward-to-risk ratio are the ones in the northwestern region of the quadrant, marked by the darker blue curve. On these efficient frontiers, the pink points mark the portfolio with compositions that produces a higher Sharpe ratio than any other in the same opportunity set. We take the Max-Sharpe ratio portfolio to be our optimal risky assets portfolio.

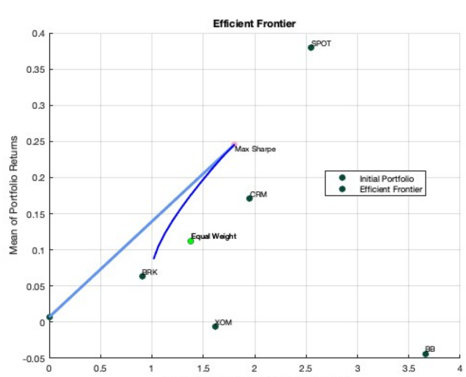


(j) Opportunity Set of 5 Stock, 5 Crypto Portfolio

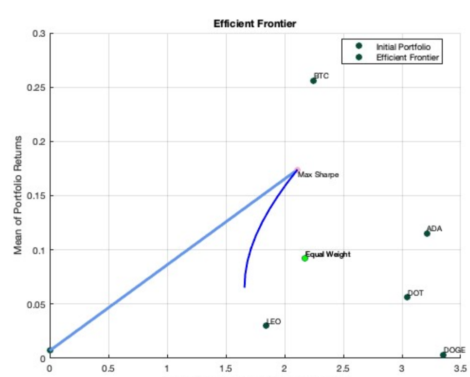


(k) Correlation Matrix of 10 Assets

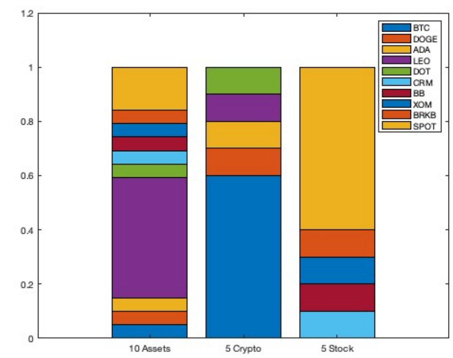
It is interesting to note that while the returns on the cryptocurrency securities are all much higher than those of the five stock securities, our diversified portfolio including all ten assets has a higher max Sharpe Ratio than the portfolio with just five cryptocurrencies. The reason is clear when we look at figure (k), a heat map of the correlation matrix of the assets. The heat map shows that the cryptocurrencies are more strongly correlated to each other than any of the other securities. Thus, it benefits the investor to also invest in the equities.



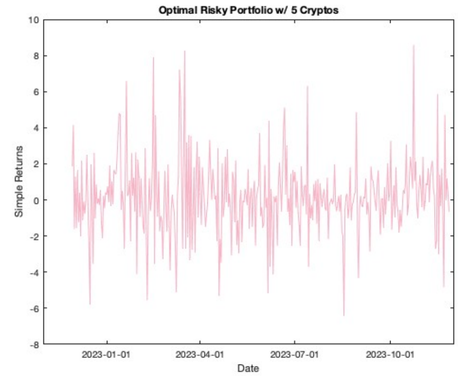
(l) Opportunity Set of 5 Stock Portfolio



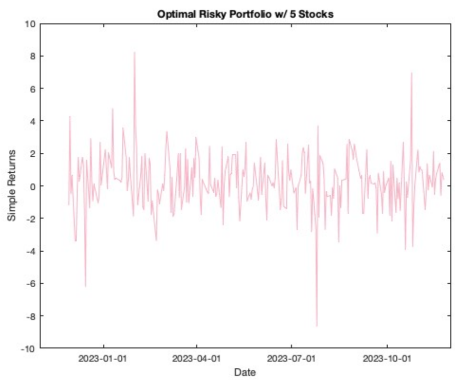
(m) Opportunity Set of 5 Crypto Portfolio



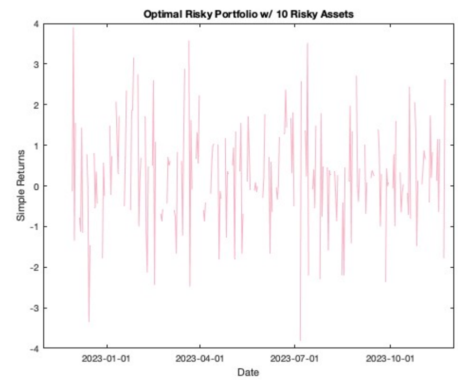
(n) Compositions of Optimal Risky Portfolio



(o) Simple Returns on 5 Crypto Risky Portfolio



(p) Simple Returns for 5 Stock Risky Portfolio



(q) Simple Returns for 10 Asset Risky Portfolio

4 Complete Portfolios

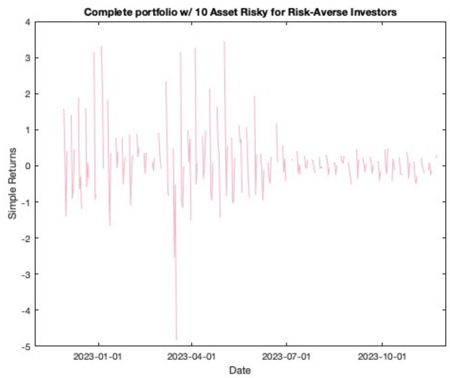
The next step is to consider the risk-free asset in the portfolio. When we do so, the efficient frontier expands to the Capital Allocation Line, which is the light blue line drawn from its x-intercept at the risk-free rate to a point on the risky portfolio frontier. Every portfolio on the CAL has the same Sharpe ratio.

Modern portfolio theory reasons that investors will always choose the portfolio with the max Sharpe Ratio as it offers the highest reward-to-risk ratio. The difference in the portfolios of the investors with different risk tolerances is the proportion invested in the optimal risky portfolio versus the risk free asset. Conveniently, all the different combinations of possible risky portfolios with the max Sharpe ratio lie on the CAL. Therefore, investor's degree of risk aversion dictates which exact portfolio composition is suitable.

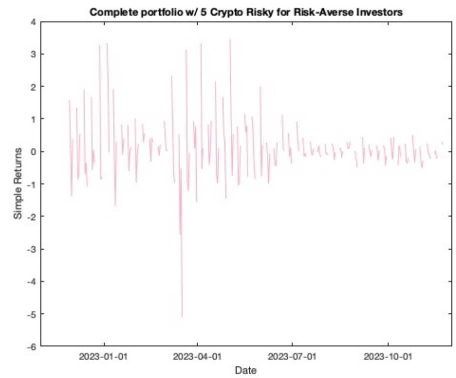
4.1 Risk-Averse Investors

Risk-averse investors have high degrees of risk aversion. I took a degree of risk aversion of 4 and used equation (1) to calculate the optimal weights for the risky and risk-free portions of the portfolio.

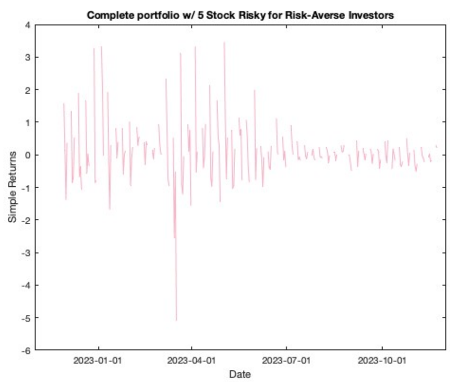
A risk-averse investor would invest either 4.83% of her total portfolio into the ten stock risky asset, or 3.31% of her total investments toward five stock risky portfolio, or 2% toward the five crypto risky portfolio. The remaining amount is invested in treasury bills. Due to the weight of investment in the Treasury bill for all three portfolios, the behavior of the returns are nearly identical. The portfolios come with equivalent expected rates of returns and volatility. They only differ marginally in their skewnesses; the distribution of the returns on the complete portfolio with the five stock risky is slightly more normal. Additionally, its means values tend to peak more than those of the other distributions.



(r) Complete Portfolio w/ 10 Asset Risky



(s) Complete Portfolio w/ 5 Crypto Risky



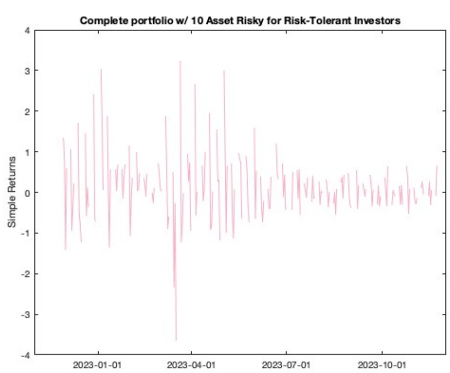
(t) Complete Portfolio w/ 5 Stock Risky

	10 Asset	5 Crypto	5 Stock
Mean	0.1143	0.1157	0.1139
St Dev	0.9196	0.9325	0.9446
Variance	0.8457	0.8696	0.8922
Skew	0.2529	0.1636	0.2686
Kurtosis	9.4652	10.0491	9.4313
Coefficient of Var	8.0463	8.0571	8.2940

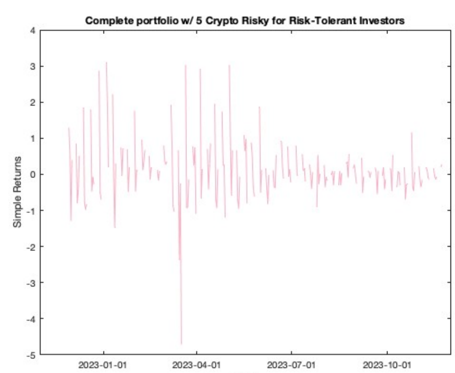
(u) Statistics on Portfolios for Risk-Averse Investor

4.2 Risk-Seeking Investors

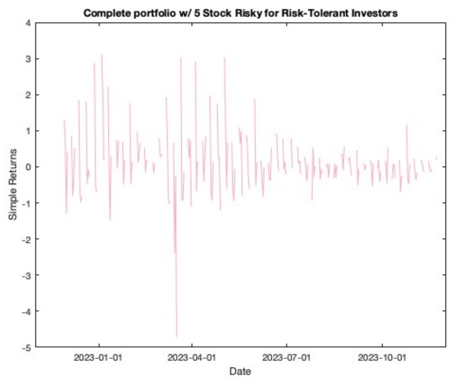
Risk-tolerating investors have low degrees of risk aversion. I used the value 1 as a degree of risk aversion to calculate the expected returns and weights at which a risk-loving investor would invest in the risky portfolio and risk free asset. A risk-loving investor would invest 13.23% of her total portfolio into the five stock risky portfolio or 7% into the five crypto risky portfolio or 19.3% into the portfolio with ten risky assets.



(v) Complete Portfolio w/ 10 Risky Assets



(w) Complete Portfolio w/ 5 Crypto Risky



(x) Complete Portfolio w/ 5 Stock Risky

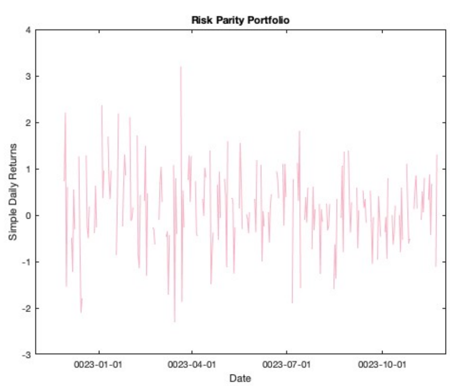
	10 Asset	5 Crypto	5 Stock
Mean	0.13607	0.12928	0.12187
St Dev	0.82295	0.87398	0.90916
Variance	0.67724	0.76383	0.82657
Skew	0.40521	0.11312	0.48362
Kurtosis	7.25140	9.37231	7.70725
Coefficient of Var	6.04781	6.76059	7.45980

(y) Statistics on Portfolios for Risk-Loving Investor

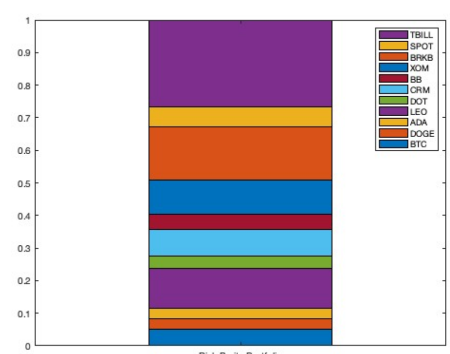
4.3 Risk Parity Portfolio

Risk parity diversification is a strategy that allocates investments so that the each asset contributes an equal amount of risk to the entire portfolio. The strategy leverages how different asset classes behave throughout the economic cycle. For example, equities tend to excel in economic booms while bonds perform well during recessions. This is very different from mean-variance optimization, where we try to reduce the overall volatility instead of balancing it. Risk parity portfolios correspond to risk-neutral investors, whom only seek to avoid large losses.

It is natural to consider a portfolio of all ten securities and the 13 week Treasury bill as the a risk parity portfolio requires multiple asset classes. We use each assets' covariance and variance to determine how risk can be distributed, allocating 9% of risk to each security and 10% to the risk-free asset. A portfolio with the set distribution of risk is generated by the weights depicted in figure (aa).



(z) Risk Parity Portfolio



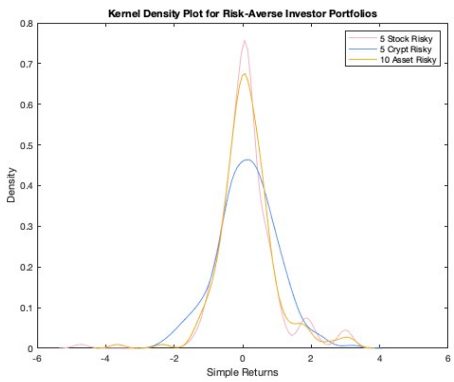
(aa) Risk Parity Portfolio Composition

Mean	0.1345
St deviation	0.8832
Variance	0.7801
Skew	0.0350
Kurtosis	3.5767
Coefficient of Variation	6.5645

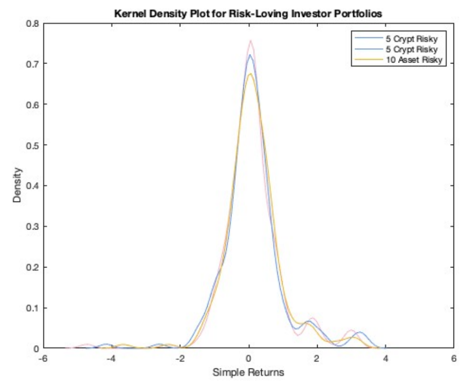
(ab) Statistics of Risk Parity Portfolio

5 Analysis

Now we can the decide which portfolio is best for the risk-averse and risk-loving investor. Observe the kernel density plot of the portfolios for the risk-averse investor in figure (ac). The returns on all three portfolios follow a very normal distribution. This is due to diversification but mainly because of the composition of the portfolio. Each of these three portfolios plotted allocate over 95 % of total investments into the treasury bill. Looking at table (u), it is apparent that a risk-averse investor would choose the complete portfolio containing all 10 risky assets because it has the lowest variance (0.8457%) with a comparable level of expected returns (0.1143%).



(ac) Kernel Densities for Risk-Averse Portfolios



(ad) Kernel Densities for Risk-Loving Portfolios

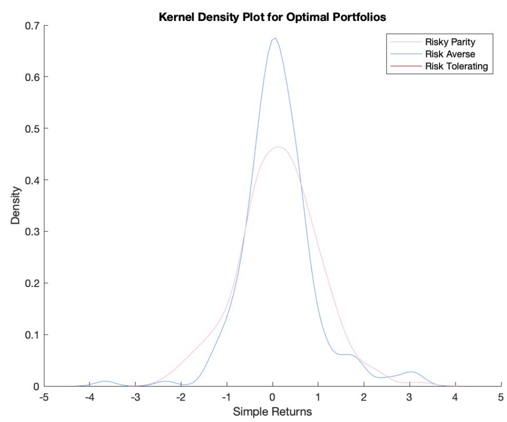


Figure 2: Kernel Density for Best Portfolios

The optimal portfolios for the risk-loving investor are also very normally distributed, with slightly more right skew. Surprisingly, the portfolios for the risk-loving investor have higher expected rates of returns and lower variance than all of the portfolios for the risk-averse investor. Nevertheless, this investor faces a similar scenario where all her optimal portfolios contain mostly of risk free assets over risky assets. This may be attributed to the current state of the economy as the risk-free rate is very high, meaning the marginally higher returns from investing in risky assets might not be worth the risk. From the complete portfolios, the risk-loving investor is likely to favor the one with 10 assets because it has the highest Sharpe ratio. Additionally, it has the lowest variance of the three at 0.67724 % and a reasonable rate of return of 0.1143 %.

How do these portfolios compare to the risk parity portfolio? It is hard to say. One cool observation to make is that the 10-risky asset portfolios for risk-averse and risk-tolerating investors have the same kernel density distribution. Despite, the 10-risky asset portfolio for our risk-loving investor is clearly preferred over the the one for our risk-averse investor because of its higher returns and lower risk. In fact, the 10-risky asset portfolio has a higher expected rate of return and lower variance than our risk parity portfolio as well. Though this may sound surprising at first, we know from our risk parity portfolio composition that much of it is invested in treasury bills, just like our risk-loving investor's portfolio. This requires further comparison as changing the constraints could very well affect the completely change the composition of the optimal portfolio. For the future, I would like to explore methods of diversifying portfolios using hierarchical equal weight contribution.

	Risk Parity	10-Asset Risk Averse	10-Asset Risk Loving
Mean	0.1345	0.1143	0.13607
St deviation	0.8832	0.9196	0.82295
Variance	0.7801	0.8457	0.67724
Skew	0.0350	0.2529	0.40521
Kurtosis	3.5767	9.4652	7.25140
Coefficient of Variation	6.5645	8.0463	6.04781

(a) Statistics of Risk Parity Portfolio

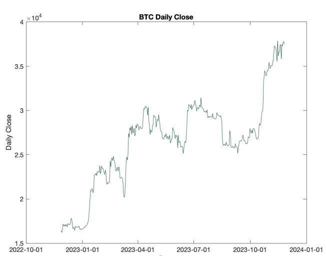
References

Report Hurst, B., Johnson, B., Ooi, Y. H. (2010, Fall). Industry report - Understanding Risk Parity. AQR Capital Management

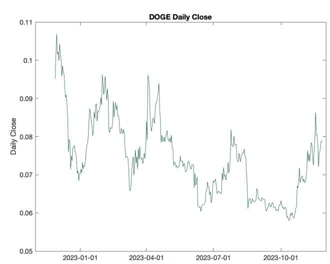
Bodie, Zvi. Essentials of Investment. MCGRAW-HILL EDUCATION, 2018.

A Appendix

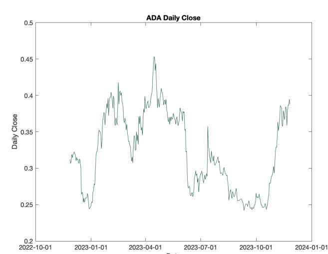
A.1 Daily Price Plots



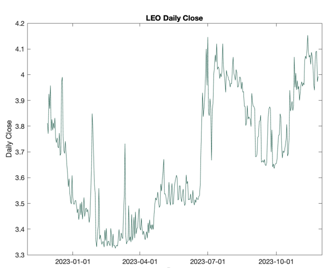
(b) BTC Daily Close Prices



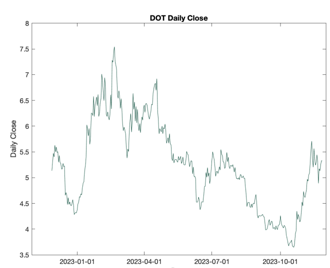
(c) DOGE Daily Close Prices



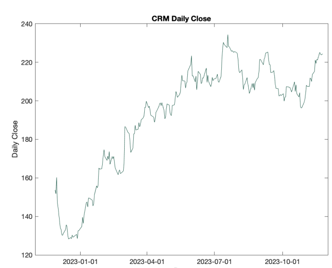
(d) ADA Daily Close Prices



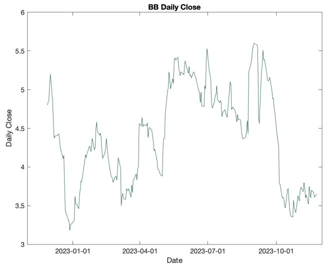
(e) LEO Daily Close Prices



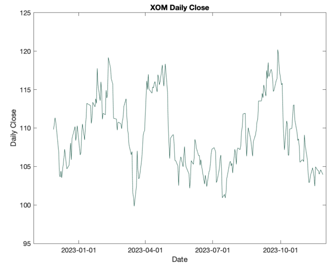
(f) DOT Daily Close Prices



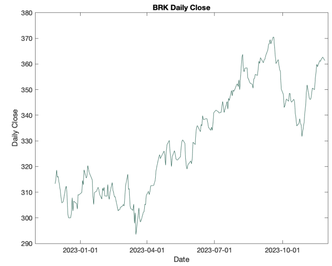
(g) CRM Daily Close Prices



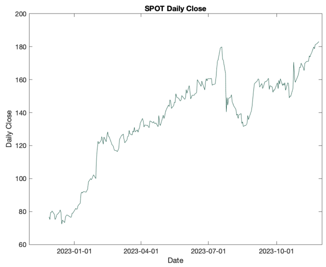
(h) BB Daily Close Prices



(i) XOM Daily Close Prices

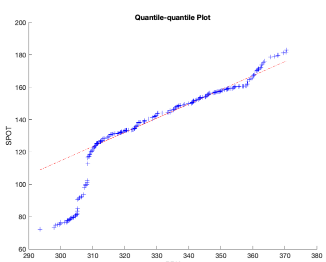


(j) BRK-B Daily Close Prices

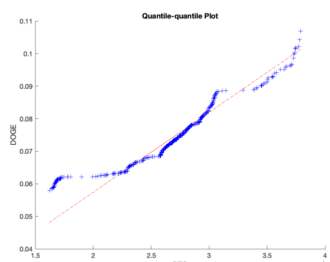


(k) SPOT Daily Close Prices

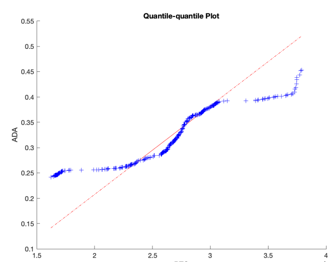
A.2 Quantile-Quantile Plots



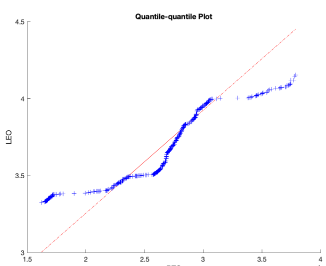
(l) QQ Plot BRK-B & SPOT



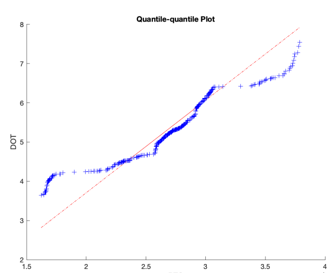
(m) QQ Plot BTC & DOGE



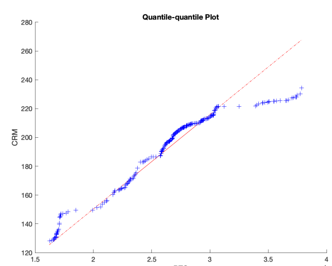
(n) QQ Plot BTC & ADA



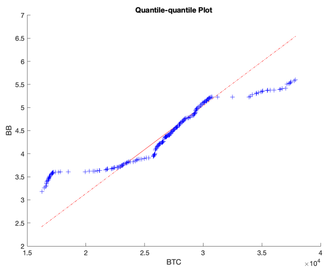
(o) QQ Plot BTC & LEO



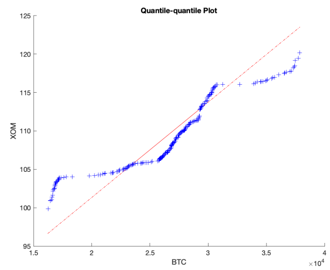
(p) QQ Plot BTC & DOT



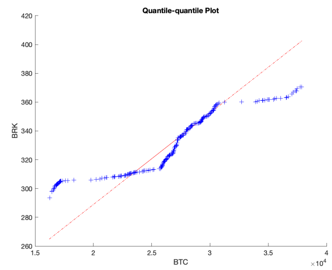
(q) QQ Plot BTC & CRM



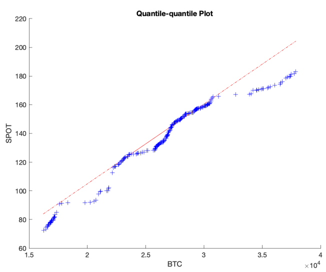
(r) QQ Plot BTC & BB



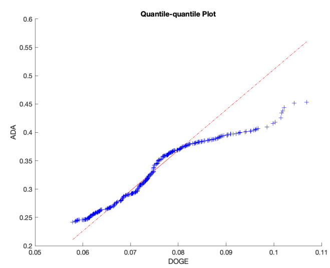
(s) QQ Plot BTC & XOM



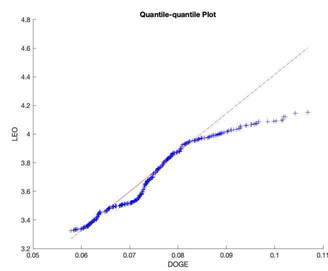
(t) QQ Plot BTC & BRK-B



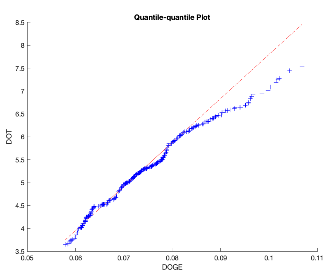
(u) QQ Plot BTC & SPOT



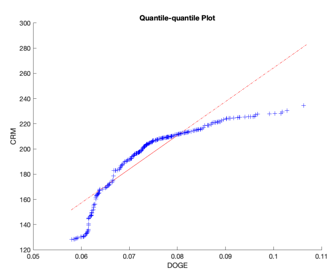
(v) QQ Plot DOGE & ADA



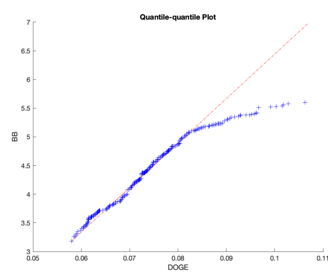
(w) QQ Plot DOGE & LEO



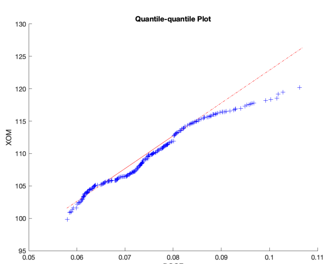
(x) QQ Plot DOGE & DOT



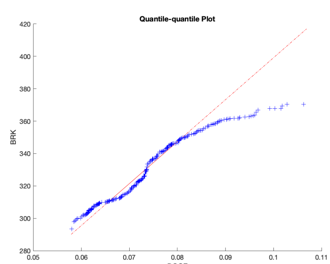
(y) QQ Plot DOGE & CRM



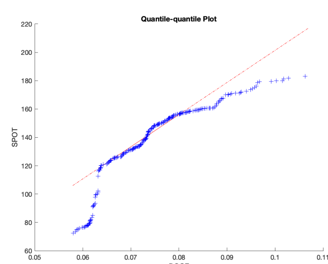
(z) QQ Plot DOGE & BB



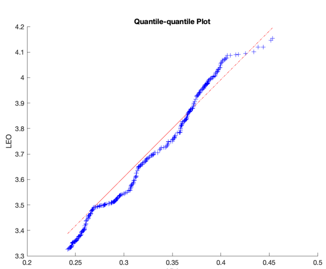
(aa) QQ Plot DOGE & XOM



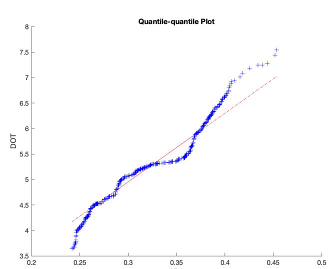
(ab) QQ Plot DOGE & BRK-B



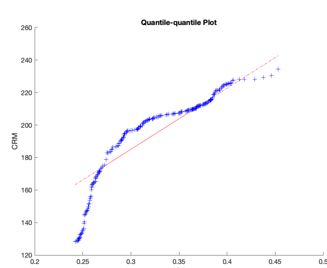
(ac) QQ Plot DOGE & SPOT



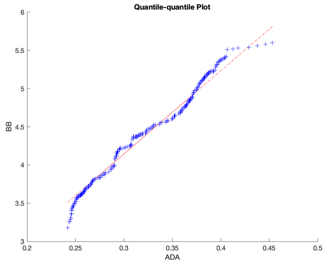
(ad) QQ Plot ADA & LEO



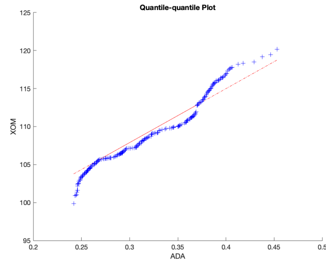
(ae) QQ Plot ADA & DOT



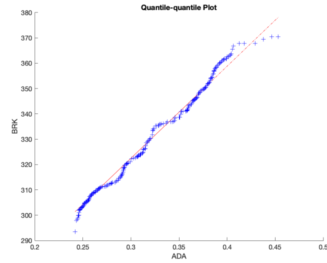
(af) QQ Plot ADA & CRM



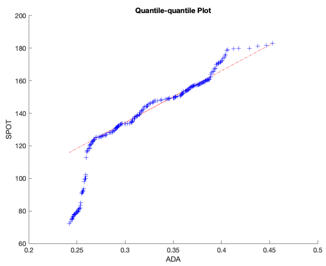
(ag) QQ Plot ADA & BB



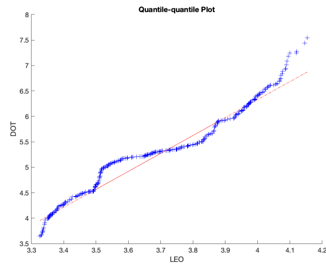
(ah) QQ Plot ADA & XOM



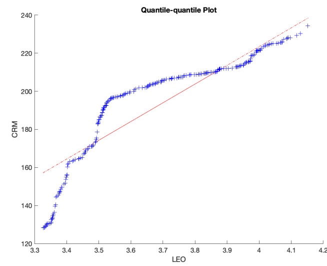
(ai) QQ Plot ADA & BRK-B



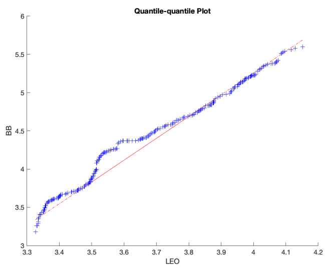
(aj) QQ Plot ADA & SPOT



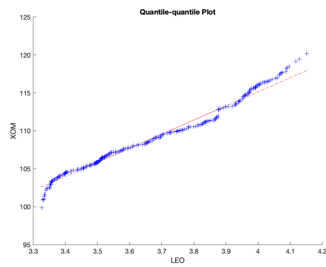
(ak) QQ Plot LEO & DOT



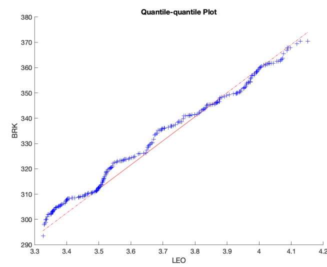
(al) QQ Plot LEO & CRM



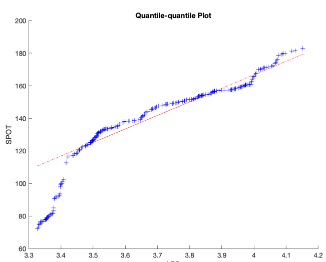
(am) QQ Plot LEO & BB



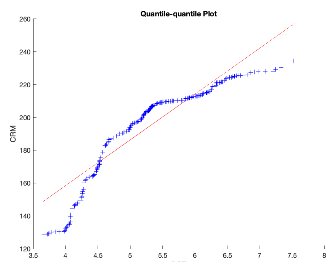
(an) QQ Plot LEO & XOM



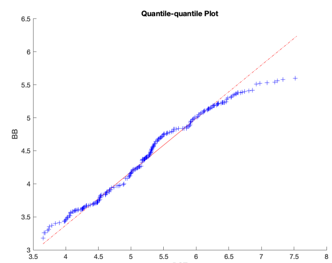
(ao) QQ Plot LEO & BRK-B



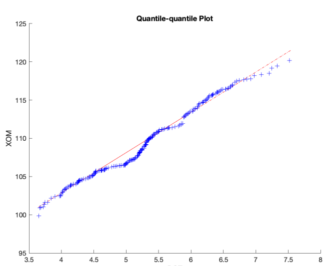
(ap) QQ Plot LEO & SPOT



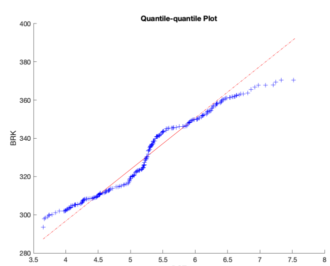
(aq) QQ Plot DOT & CRM



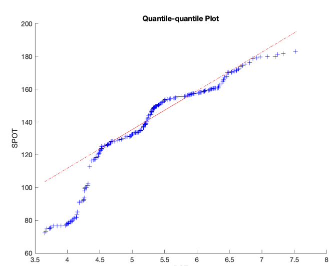
(ar) QQ Plot DOT & BB



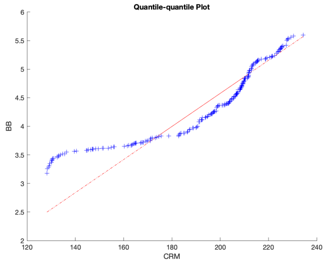
(as) QQ Plot DOT & XOM



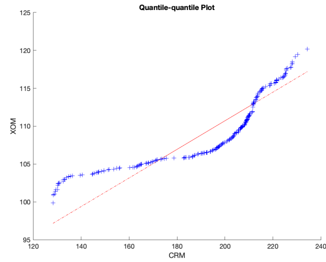
(at) QQ Plot DOT & BRK-B



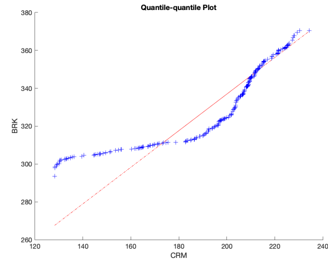
(au) QQ Plot DOT & SPOT



(av) QQ Plot CRM & BB



(aw) QQ Plot CRM & XOM



(ax) QQ Plot CRM & BRK

Table 1: Statistical Values for Daily Returns on Each Risky Asset

	BTC	DOGE	ADA	LEO	DOT	CRM	BB	XOM	BRK-B	SPOT
Mean	0.2514	0.0034	0.1100	0.0298	0.0560	0.1455	-0.0153	-0.0191	0.0120	0.3951
St deviation	2.2469	3.3546	3.2079	1.8432	3.0449	2.0125	3.6145	1.5920	0.8879	2.6820
Variance	5.0487	11.2531	10.2905	3.3973	9.2713	4.0503	13.0645	2.5343	0.7884	7.19305
Skew	0.9321	0.6002	1.0323	-0.3295	0.3414	0.6776	0.3556	-0.0928	-0.3193	-0.2200
Kurtosis	6.6138	8.0664	11.6816	6.9942	5.4420	8.6297	8.9194	3.4466	3.2855	9.8791
Coefficient of Variance	8.9388	988.2884	29.1555	61.8256	54.3585	13.8332	-236.4524	-83.4822	74.2453	6.7882

```
function[start_date] = plot_timesrs(dates,values,plot_title,color)
    figure()
    plot(dates,values,'Color',color)

    xtickformat("yyyy-MM-dd")
    xlabel('Date')
    ylabel('Simple Daily Returns')
    title(plot_title)
    start_date = dates(1);
end

function[netretT] =findnetret(rets,weights)

    Date = rets.Date
    rets = table2array(rets)
    netRets = NaN(size(rets,1),1);
    for i = 1:size(rets,1);
    class(netRets)
    retrow = rets(i,:);

    netRets(i,1) = dot(retrow,weights);

    end
    netretT = timetable(Date,netRets)
end


function[dailyrets, statsT] = porteval(securities)
    % takes in a cell array of strings (names of files of historical
       data)
    % calculates the returns, does standard stat analysis for each
       asset
```

```
    bigtable = combine(securities);
    dailyrets = findReturns(bigtable);
    statsT = statsoftable(dailyrets);


end

function[bigstatsT] = statsoftable(returnT)
    % takes in a table where each col is the daily returns of an asset
    % returns a table of stats for each asset in og table

    %initalize empty table
    bigstatsT = table();
    rets = returnT
    % get names of assets
    names = rets.Properties.VariableNames
    for i = 1:width(rets)
        return_col = rets{:,i};

        meann = mean(return_col,'omitnan');
        stats_i = static(rets{:,i},names{i});
        if i == 1
            bigstatsT = stats_i;

        else
            bigstatsT = join(bigstatsT,stats_i);
        end
    end
end

function[bt] = combine(assets)
    % combines all the securites we care about
    % into one table for easy access

    % initalize empty table
    bt = table;

    A = string(assets);

    % iterate across each asset
    for i = 1:length(A)
        if contains(A{1,i},'-')
            ticker = extractBefore(A{1,i},"-");
        else
            ticker = extractBefore(A{1,i},".");
        end
        bt.(ticker) = {readtimetable(assets{1,i})};
    end

end

function[big_rets] = findReturns(bigTable)
    % takes in a DAYS by n table of n securites & hdata
    % returns TT of simple returns for n securities
    big_closes = table;
    names = bigTable.Properties.VariableNames;
```

```
    for i = 1:width(bigTable)
        name = names{1,i};
        hdata = bigTable.(name){1};

        %extract close col
        date_close = hdata(:, {'Close'});
        % rename close w name of security
        date_close = renamevars(date_close,'Close',name);
        if i == 1
            big_closes = date_close;
        else
            big_closes=synchronize(big_closes,date_close);
        end
    end
    big_rets=tick2ret(big_closes);
end



function[statsT] = static(rets,name)
    avg = mean(rets,'omitnan');
    stdev = std(rets,'omitnan');

    variance = (stdev)^2;
    skeeew = skewness(rets);
    kurt = kurtosis(rets);
    coeff_of_var = stdev / avg;
    Stats = ["Mean"; "StDev";"Variance";"Skew";"Kurtosis";"Coefficient
        of Var"];
    vals = [avg;stdev;variance;skeeew;kurt;coeff_of_var];
    statsT = table(Stats);
    statsT.(name) = vals;
end



function[w_MS, msrisk, msret,covie] = mvopt(return_table)
    global riskfreerate
    symbol = return_table.Properties.VariableNames;
    symbol = symbol';

    % creat Port obj
    p = Portfolio('AssetList',symbol,'RiskFreeRate',riskfreerate);
    hay = table2array(return_table);
    p = estimateAssetMoments(p,hay);


    %eq weight portfolio is set as benchmark
    p = setInitPort(p,1/p.NumAssets);
    [erisk,eret] = estimatePortMoments(p,p.InitPort);
    % set constraints
    p = setDefaultConstraints(p);

    %y0
```

```
    weight = 1/(2 * width(return_table))
    y0 = ones(width(return_table),1)*weight
    p = setBounds(p,y0);
    w_MS = estimateMaxSharpeRatio(p)

    [msrisk, msret] = estimatePortMoments(p,w_MS);
    retArray = table2array(return_table);
    covie = cov(retArray,'omitrows');
    covrrMat= corrcov(covie);
    figure()
    h= heatmap(covrrMat);
    title('Correlation between Securities')
    colormap("bone");
    xdata=symbol;
    h.XDisplayLabels=xdata;
    h.YDisplayLabels=xdata;

    %covrrMat= corrcov(covMat)

    % plot opprtunity set + efficient frontier
    figure()
    clf;
    scatter(sqrt(diag(p.AssetCovar)),p.AssetMean,'MarkerEdgeColor','
       #084b39','MarkerFaceColor','#084b39')
    text(sqrt(diag(p.AssetCovar)), p.AssetMean, p.AssetList, 'Vert','
       bottom', 'Horiz','left', 'FontSize',7)
    hold on
    scatter(erisk,eret,'MarkerEdgeColor','#084b39','MarkerFaceColor','
       #084b39')
    text(erisk,eret, 'Equal Weight', 'Vert','bottom', 'Horiz','left',
       'FontSize',7)
    scatter(0,riskfreerate,'MarkerEdgeColor','#084b39','
       MarkerFaceColor','#084b39')
    text(erisk,eret, 'Equal Weight', 'Vert','bottom', 'Horiz','left',
       'FontSize',7)
    hold on
    plot([0,msrisk],[riskfreerate,msret],"LineWidth",3,"Color",'#6495
       ED')
    hold on
    scatter(msrisk,msret,'MarkerEdgeColor','#f7b4c6','MarkerFaceColor'
       ,'#f7b4c6')
    text(msrisk,msret, 'Max Sharpe', 'Vert','bottom', 'Horiz','left',
       'FontSize',7)
    plotFrontier(p)

end
```