Project Specification

```
course:     DD1354
assignment: lab 4 - Project Specification
author:     Axel Hammarskiöld Spendrup
id:         axelhs
email:      axelhs@kth.se
date:       20.02.25
grade aim:  A
```

Relativistic Reality

> A physically accurate special relativity simulator

1. Background

The $3+1-$ dimensional Minkowski spacetime is intimidating in notation yet remarkably easy to intuit. With good reason too, since we live in such a spacetime. As a 3-dimensional being, it is easy to visualize a 2 dimensional observer traversing time as the additional third dimension. Yet, the human brain often struggles to grasp is how the velocity of the observer affects the spacetime itself: the effects of special relativity.

This project aims to allow anyone to experience the effects of special relativity, well below the extreme velocities required in reality. By constructing a special relativity simulation within the context of a computer game, the player of the game will get a better intuition of some notable relativistic effects such as length contraction, Lorentz boosts, coordinate transformations and more. The project aims to attain the grade A.

The game will be built as an interactive 3D environment using Unity. The simulation scripts will be built on the rigorous mathematics of Lorentz transformations in Minkowski space that underpins special relativistic theory. By using natural units, (normalizing the speed of light to 1: $c=1$), the relativistic effects will be prominent at velocities more familiar to us.

2. Problem Definition

The simulation will build on the Lorentz transformation in Minkowski space. The mathematic formalism was initially developed by Albert Einstein. In lieu of Einstein's original work, most of the mathematics are derived from another pioneer in the field: Wolfgang Rindler and his book *Introduction to Special Relativity*. The application of the theory to computer simulation has previously been tested by MIT Game Lab. Their paper *Visualizing relativity: The OpenRelativity project* provides insight into some techniques and limitations.

At the core of the simulation will be the Lorentz transformation matrix. The full transformation matrix allows complete transformation of all coordinates. This is required since only applying the length contraction formula

$$L_{||} = \frac{L_{0,||}}{\gamma}, \quad L_{\perp} = L_{0,\perp}$$

only yields length contracted objects but not distances. The Matrix is defined as

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1+\frac{\gamma^2}{1+\gamma}\beta_x^2 & \frac{\gamma^2}{1+\gamma}\beta_x\beta_y & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z \\ -\gamma\beta_y & \frac{\gamma^2}{1+\gamma}\beta_x\beta_y & 1+\frac{\gamma^2}{1+\gamma}\beta_y^2 & \frac{\gamma^2}{1+\gamma}\beta_y\beta_z \\ -\gamma\beta_z & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z & \frac{\gamma^2}{1+\gamma}\beta_y\beta_z & 1+\frac{\gamma^2}{1+\gamma}\beta_z^2 \end{bmatrix},$$

where $\gamma = \frac{1}{\sqrt{1+\beta^2}}$ and $\beta = \frac{v}{c}$. By restricting the player to only move relativistically in the $xz-$ plane, the Λ matrix takes a simpler form. This allows the Lorentz transformation between a stationary frame S and a moving frame S' as

$$X' = \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta_x & 0 & -\gamma\beta_z \\ -\gamma\beta_x & 1+\frac{\gamma^2}{1+\gamma}\beta_x^2 & 0 & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z \\ 0 & 0 & 1 & 0 \\ -\gamma\beta_z & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z & 0 & 1+\frac{\gamma^2}{1+\gamma}\beta_z^2 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \Lambda X$$

Simulating movement at relativistic velocities would require colossal maps that are computationally expensive to render. To circumvent this problem, the simulation will use natural units $(c = 1)$ which permits smaller maps. This comes with the added benefit that the relativistic effects will become prominent in a context that is more familiar to everyday situations.

A possible movement mechanic that will be investigated is that of a rolling ball. This type of movement can emphasize the concept of relativistic momentum conservation, another important property of special relativity.

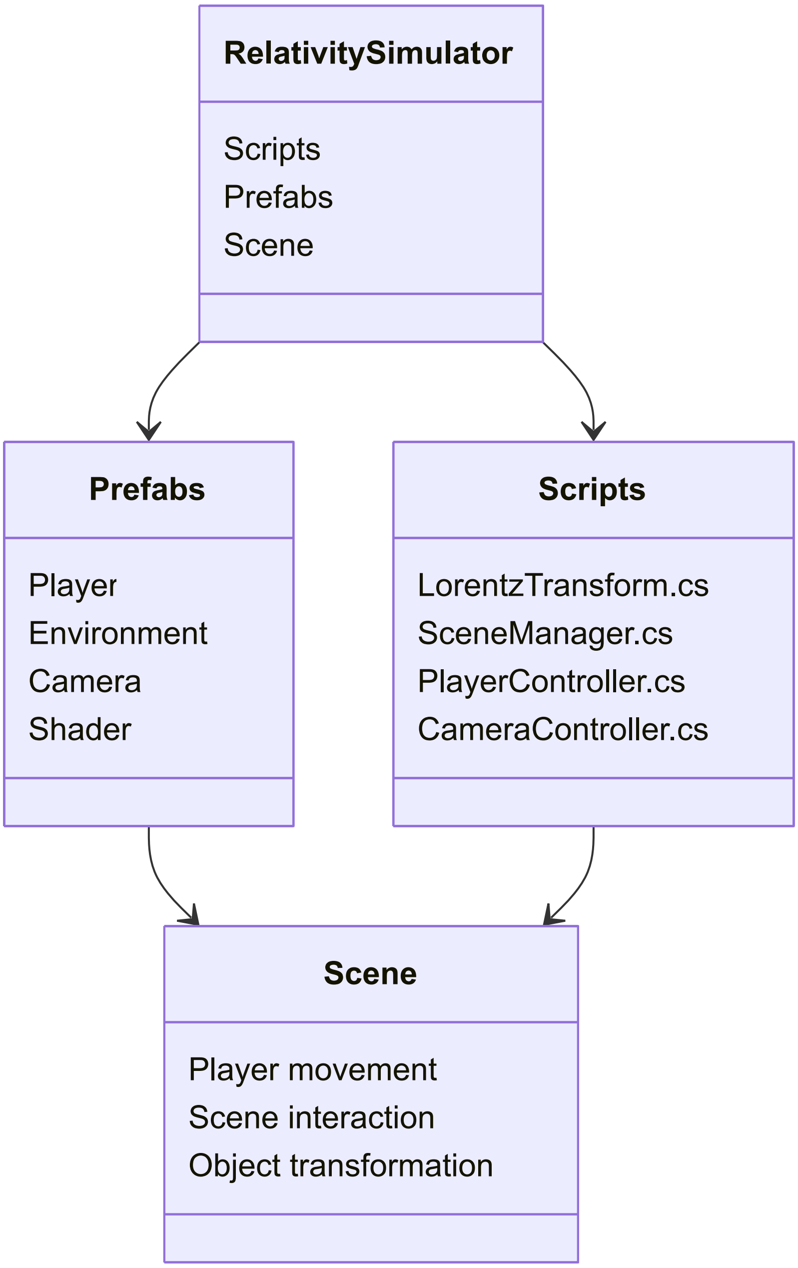
Limitations and constraints

The following limitations and constraints have previously been identified by the OpenRelativity project. Since this project will have fewer features than those offered by the OpenRelativity library, it is possible to assume that some constraints might be circumvented.

- The speed of light lowered to allow relativistic effects to occur at human scale.
- The player must never reach or exceed the speed of light.
- All objects in the scene must either be stationary of have a constant velocity.
- Gravity is not modeled in the framework of special relativity.
- Shadow and lighting must be permanent.
- The player's movement is assumed to be constant at each time step.
- The Doppler shift is constrained to limit wavelengths to the visible spectrum.

3. Implementation

The project is inspired by MIT Game Lab's OpenRelativity library and games such A Slower Speed of Light and Velocity Raptor. The simulation will be built in Unity 3D. This will permit the use of existing models and platforms for character control, prefabricated 3D environments and fundamental physics simulation to allow increased focus on the relativistic simulation. A rudimentary architecture is as follows:



The scripts above will control the following

- *LorentzTransform.cs*

  Handles the computation of the Lorentz transformation. Outputs the transformation matrix Λ and the $\gamma-$ factor.
- *SceneManager.cs*

  Manages the relativistic effects on the environment by applying the Lorentz transformation to the scene based on player velocity.
- *PlayerController.cs*

  A third person player controller built by Unity and modified to handle player movement, player input, camera tracking, collision detection, gravity and more.

- *CameraController.cs*
  Applies any relativistic effects that are unique to the camera, such as aberration and Doppler shift. Also handles any visual effects emphasizing when relativistic effects take place.

Inspiration for the level construction and obstacle design comes from the online resource *The Level Design Book*. Prefabs, shaders, materials and other scene-specific resources are available at the Unity AssetStore.

4. Demonstration

The ideal version of the final product will be a 3D virtual environment where a character is controlled from a third person view. In this environment, the speed of light is normalized to allow the character to move through the environment close to the speed of light. As they do, the high velocity will induce relativistic effects on the environment such as length contraction, time dilation and relativistic Doppler shift.

The player will be able allowed some input of how these effects are portrayed, either by toggling their implementation or by alternating the frame of reference. While beyond the scope of this project, it would be interesting to design an obstacle or a puzzle that requires relativity to solve.

The main focus of this project will be to model length contraction of static objects in the environment. If the mathematics are readily implementable to all aspects of the physics engine, more advanced effects, such as time dilation, Doppler shift, aberration and relativistic addition of velocities will be investigated. The character movement mechanic will be built to simulate relativistic conservation of momentum.

5. References

- Zachary W. Sherin, Ryan Cheu, Philip Tan, Gerd Kortemeyer; *Visualizing relativity: The OpenRelativity project.* Am. J. Phys. 1 May 2016; 84 (5): 369–374. https://doi.org/10.1119/1.4938057
- Wolfgang Rindler; *Introduction to Special Relativity*. Clarendon Press, 1991; ISBN 9780198539520

- MIT Game Lab, *A Slower Speed of Light*.
  https://gamelab.mit.edu/games/a-slower-speed-of-light/
- MIT Game Lab, *OpenRelativity*.
  https://gamelab.mit.edu/research/openrelativity/
- TestTube Games, *Velocity Raptor*.
  https://testtubegames.com/velocityraptor.html
- Robert Yang, Andrew Yoder, et. al.; *The Level Design Book.*
  https://book.leveldesignbook.com/appendix/resources
- Unity 3D, *Starter Assets - Third Person*
  https://assetstore.unity.com/packages/essentials/starter-assets-thirdperson-updates-in-new-charactercontroller-pa-196526?srsltid=AfmBOooKVuJbBXXMiHGurCJa8d09U8uOp4EBKVPuBFpS43a-HsvRpJo2