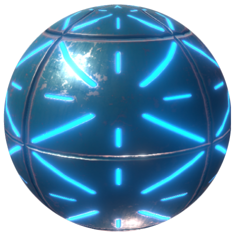

β-boost

Simulating Special Relativity in Unity

Final project report
Models and Simulation DD1354

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Source code: *Github repository*
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Grade aim: **A**

Stockholm, Sweden
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ABSTRACT

This project seeks to bridge the gap between theoretical knowledge and physical intuition of relativistic effects, in particular the length contraction phenomenon. It implements a three-dimensional, virtual simulation built in the Unity game engine, allowing users to interactively explore the physics of special relativity in real-time. Inspired by the concept of natural units, the speed of light in the simulation is artificially reduced, thereby visualizing relativistic effects at velocities more familiar to everyday life. The simulation calculates velocity based parameters, β and γ, to construct a Lorentz transformation matrix at each time step. The matrix is then applied to the vertices of static environment meshes via CPU computations at runtime. The resulting visualization illustrates the connection between the 4-vector formalism of Minkowski spacetime and the observable geometric distortions of the environment as predicted by special relativity, in particular length contraction.

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1. INTRODUCTION

An observer traveling at velocities close to the speed of light will experience exotic phenomena that constitute a unique and unintuitive part of physics. At these extreme velocities, the fabric of spacetime appears distorted from the observer's perspective, causing them to perceive, among other things, how distances become shorter and time intervals appear longer compared to measurements made in a rest frame.

The effects were initially predicted by Albert Einstein who modeled the behavior in his third *annus mirabilis*[1] paper *On the Electrodynamics of Moving Bodies* (1905)[2], later known as the *special theory of relativity*. The theory rewrote the laws imposed by Newtonian mechanics for high velocities and revolutionized our perception of space and time.

While the effects of special relativity are present for every object in the universe at all velocities, they are only pronounced at extremely high speeds relative to the speed of light, c. Moreover, they are modeled mathematically as Lorentz transformations within Minkowski spacetime, an inherently abstract modification of a 4-dimensional manifold. This renders the effects virtually imperceptible in everyday life, posing an educational challenge for students accustomed to our relatively slow, 3-dimensional reality. The mathematical abstraction can leave a gap between theoretical knowledge and intuitive understanding.

This project addresses this gap through a real-time, interactive simulation, implemented in the 3D game engine Unity. The aim is to visually demonstrate the geometric consequences of Lorentz transformations, primarily length contraction, by artificially reducing the speed of light. This provides students and users with an intuitive feel for how these transformations manifest visually as relativistic effects. The initial project specification, outlining the goals and preliminary plan, can be found in Appendix B.

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2. BACKGROUND

Below is a theoretical background on the underpinnings of the special theory of relativity. It provides the fundamental mathematical framework required to understand the simulation, together with examples of how relativistic effects manifest.

The mathematical formalism employed in this project is primarily derived from Wolfgang Rindler's *Introduction to Special Relativity* [3]. All mathematical derivations in this report follow from this book, except where explicitly indicated otherwise.

2.1 POSTULATES OF SPECIAL RELATIVITY

Special relativity is built on two fundamental postulates:

Postulate 1 (Principle of Relativity)**.** *The laws of physics take the same form in all inertial frames of reference.*

Postulate 2 (Invariance of the Speed of Light)**.** *As measured in any inertial frame of reference, light is always propagated in empty space with a definite velocity c that is independent of the state of motion of the emitting body. Or: the speed of light in free space has the same value c in all inertial frames of reference.*

The mathematical structure of special relativity, particularly the Lorentz transformations, follows logically from these two postulates, along with additional assumptions such as the homogeneity and isotropy of space and time [4].

2.2 MINKOWSKI SPACETIME

The calculations of special relativity pertinent to the simulation concern linear transformations between different inertial frames of reference. In these inertial frames, often denoted as S and S', each instance in space and time is a unique point known an *event*, represented as a four-dimensional vector (4-vector): $X = (ct, x, y, z)$. These vectors exist on a 4-dimensional flat manifold known as *Minkowski space*, constructed from three spatial dimensions plus time.

When comparing events between different inertial frames, one frame S is often designated as the *rest frame* relative to the objects being observed, while the other frame S' moves with constant velocity relative to S. It is possible to define a spacetime interval Δs^2 between two events, X_1 and X_2, which is invariant under transformations between inertial frames. Using the metric signature $(+,-,-,-)$, the interval is

$$\Delta s^2 = (c\,\Delta t)^2 - \Delta x^2 - \Delta y^2 - \Delta z^2, \tag{1}$$

where $\Delta t = t_2 - t_1$, $\Delta x = x_2 - x - 1$ etc. The invariance of this interval is a direct consequence of the postulates.

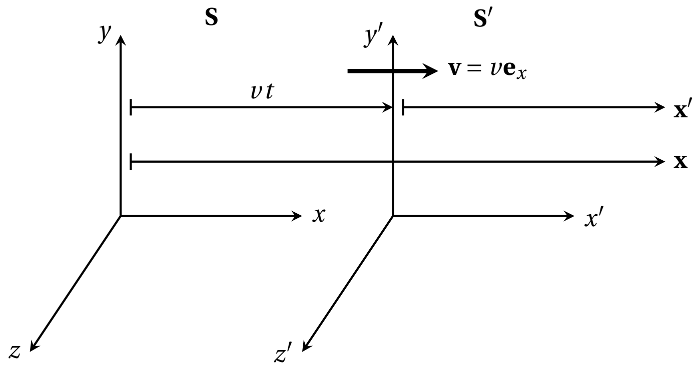

Fig. 1. Schematic representation of two inertial frames, S at rest and S' moving with constant velocity $\vec{v}$ along the positive x-axis. The diagram illustrates relationships between coordinates.

2.3 LORENTZ TRANSFORMATIONS

The Lorentz transformation is the linear transformation relating the spacetime coordinates of an event as measured in two different inertial frames, S and S', moving with constant velocity relative to each other. The transformation can include a spatial rotation and a velocity boost. If the relative motion involves no rotation, the transformation is referred to as a *Lorentz boost*, illustrated schematically in figure 2. A key property of the transformation is *Lorentz invariance*: it preserves the spacetime interval (1) between any two events when calculated in either frame.

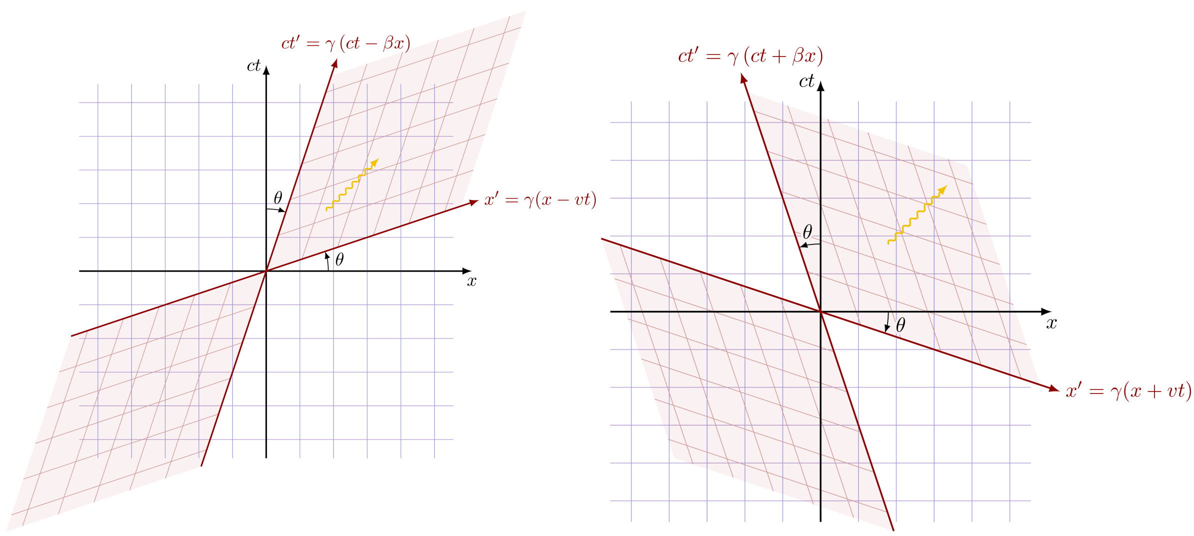

Fig. 2. Left: Spacetime diagram illustrating a Lorentz boost for a frame S' moving in the positive x-direction relative to S. Note that the x' and ct' axes are tilted at an equal angle θ relative to the x and ct' axes. **Right:** The inverse transformation, equivalent to observing S from S', where S moves in the negative x-direction relative to S'. The angle α is given by $\tanh\alpha = \beta = v/c$ [5].

Mathematically, the transformation is represented as the matrix multiplication $X' = \Lambda X$, where X and X' are the 4-vector representations of the same event in frames S and S' respectively, and Λ is the *Lorentz transformation matrix*. For the specific case where S' moves with velocity $\mathbf{v} = v\mathbf{e}_x$ relative to S as in figure 1, the transformation is calculated as

$$X' = \begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} = \Lambda_x X \tag{2}$$

where we have introduced the normalized, dimensionless velocity $\beta = \frac{v}{c}$ and the *Lorentz factor* $\gamma = \frac{1}{\sqrt{1-\beta^2}}$. This factor grows significantly as the velocity v approaches the speed of light c, i.e., $\beta \to 1$, as seen in figure 3, leading to substantial relativistic effects. Equation (2) gives X' in terms of X. The inverse transformation, $X = \Lambda_x^{-1} X'$, is found by considering S' the rest frame such that S travels along the x-axis with velocity $\mathbf{v} = -v\mathbf{e}_x$. Replacing β with $-\beta$ in (2) yields

$$X = \begin{bmatrix} t \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta & 0 & 0 \\ \gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} t' \\ x' \\ y' \\ z' \end{bmatrix} = \Lambda_x^{-1} X' \tag{3}$$

Expanding these gives the familiar coordinate relations

$$\begin{aligned} t' &= \gamma(t - \frac{vx}{c^2}) \qquad & t &= \gamma(t' + \frac{vx'}{c^2}) \\ x' &= \gamma(x - vt) \qquad & x &= \gamma(x' + vt') \\ y' &= y \qquad & y &= y' \\ z' &= z \qquad & z &= z' \end{aligned} \tag{4}$$

It is apparent that only the spatial coordinate parallel to $\mathbf{v}$ is affected, along with time. Expressing the position vector as $\mathbf{x} = \mathbf{x}_\perp + \mathbf{x}_\parallel$, i.e., parallel and perpendicular to $\mathbf{v}$, the transformation can be expressed for a boost in a general direction as

$$t' = \gamma\left(t - \frac{\mathbf{v}\cdot\mathbf{x}}{c^2}\right) \tag{5}$$

$$\mathbf{x}'_\parallel = \gamma(\mathbf{x}_\parallel - \mathbf{v}t) \tag{6}$$

$$\mathbf{x}'_\perp = \mathbf{x}_\perp \tag{7}$$

The full transformation matrix $\Lambda(\mathbf{v})$ for a velocity boost in an arbitrary direction $\mathbf{v} = (v_x, v_y, v_z)$ with $\beta = |\mathbf{v}|/c$ and $\boldsymbol{\beta} = \mathbf{v}/c = (\beta_x, \beta_y, \beta_z)$ is given by

$$\Lambda(\mathbf{v}) = \begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1 + (\gamma-1)\frac{\beta_x^2}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_y}{\beta^2} & (\gamma-1)\frac{\beta_x\beta_z}{\beta^2} \\ -\gamma\beta_y & (\gamma-1)\frac{\beta_y\beta_x}{\beta^2} & 1 + (\gamma-1)\frac{\beta_y^2}{\beta^2} & (\gamma-1)\frac{\beta_y\beta_z}{\beta^2} \\ -\gamma\beta_z & (\gamma-1)\frac{\beta_z\beta_x}{\beta^2} & (\gamma-1)\frac{\beta_z\beta_y}{\beta^2} & 1 + (\gamma-1)\frac{\beta_z^2}{\beta^2} \end{bmatrix}. \tag{8}$$

Note that the term $\frac{\gamma-1}{\beta^2}$ is well-defined even as $\beta \to 0$, where it approaches $1/2$. This is the matrix implemented in the simulation.

2.4 LENGTH CONTRACTION

A remarkable consequence of the Lorentz transformation is that observers in relative motion disagree on the length of objects measured parallel to the direction of motion. Consider a rod at rest in frame S, aligned along the x-axis, with endpoints at x_A and x_B. Its length in its rest frame, the *proper length*, is $L_0 = x_B - x_A$. To determine the length L' of the rod as measured by an observer in S' moving with velocity $v\mathbf{e}_x$ relative to S, the observer must measure the positions of the endpoints, x'_A and x'_B, *simultaneously* in their own frame, S'. This gives the length of the rod in S' is $L' = x'_B - x'_A$. Let this simultaneous measurement occur at time $t' = 0$ in S'.

From the inverse transformation for time in (4), $t = \gamma(t' + \frac{vx'}{c^2})$. Setting $t' = 0$, the times in frame S corresponding to the measurements are $t_A = \gamma v x'_A$ and $t_B = \gamma v x'_B$. Note that these are generally different times in S.

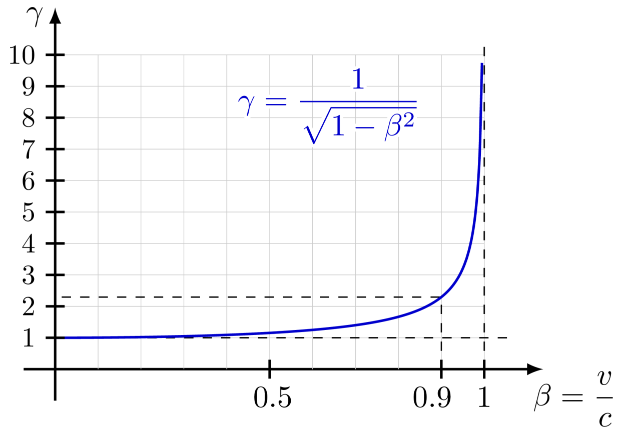

Fig. 3. The Lorentz factor γ as a function of relative speed $\beta = v/c$. Note the rapid increase as β approaches 1 [5].

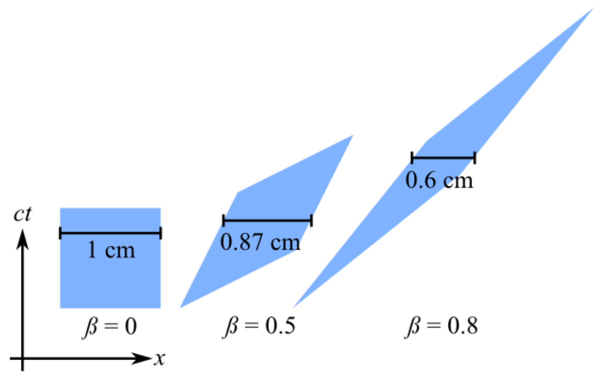

Fig. 4. Conceptual illustration of how length contraction, applied only along the direction of motion, can lead to skewing of objects viewed from a moving frame [6].

Now use the transformation for x from (4) with simultaneous measurement in S' implying $t' = 0$ such that

$$x_A = \gamma\, x'_A,$$
$$x_B = \gamma\, x'_B.$$

Therefore, the proper length of the rod in the rest frame is $L_0 = x_B - x_A = \gamma\,(x'_B - x'_A) = \gamma\, L'$. Rearranging this expression gives the length measured in the boosted frame S' as

$$L' = \frac{L_0}{\gamma} = L_0\sqrt{1-\beta^2} \tag{9}$$

Since $\gamma \geq 1$, the length L' is always less than or equal to the proper length L_0. This is the phenomena known as *length contraction*. The contraction only occurs along the direction of motion while dimensions perpendicular to $\mathbf{v}$ remains unchanged according to (7).

As visualized by figure 3, when $v \to c$, $\beta \to 1 \implies \gamma \to \infty$, implying that the observed length L' is considerably reduced at relativistic velocities and approaches zero as $v \to c$. For boosts in arbitrary directions, the directional dependence of (6) indicates that length contraction along the parallel component leads to skewed geometry as suggested in figure 4. This somewhat unintuitive effect is a characteristic feature of special relativity and is the primary phenomenon this project aims to visualize.

2.5 TIME DILATION

Another distinguishing relativistic phenomena is time dilation. Analogous to length contraction, observers in relative motion also disagree on the duration of time intervals. It is often summarized by the adage "moving clocks run slower".

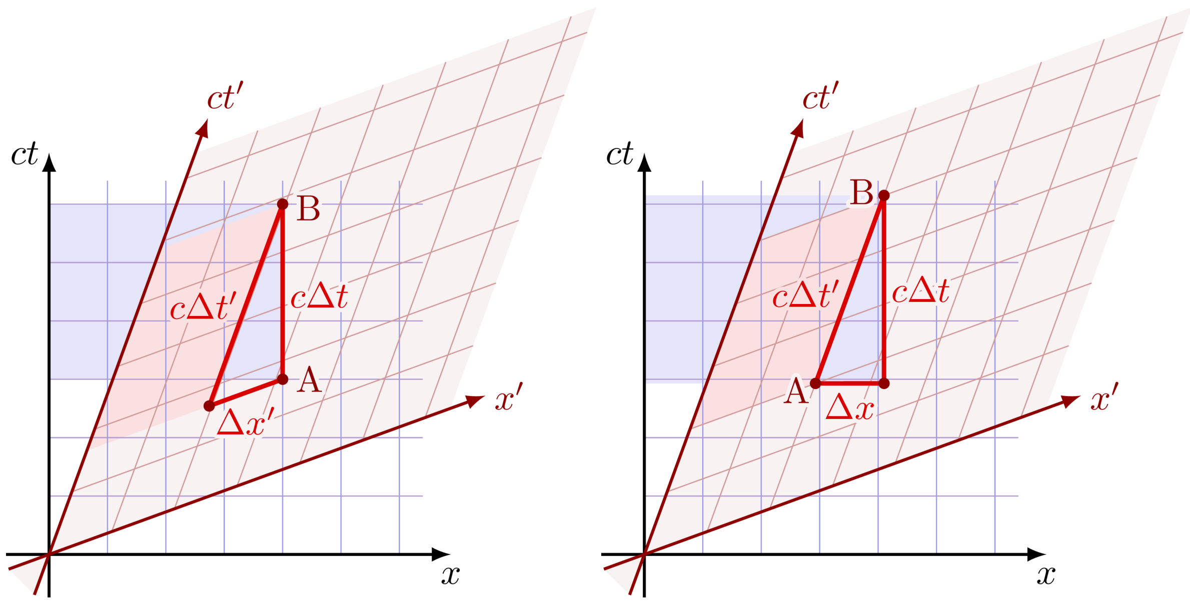

Fig. 5. Left: Events A and B occur at the same location in the rest frame S separated by the proper time Δt, signified by a vertical parallel to the ct axis. In the boosted frame S', A and B occur at different locations and are separated by a longer time interval $\Delta t' = \gamma \Delta t$. **Right:** A and B occur at the same location in the boosted frame S'. In S the events occur at different locations separated by a longer time interval $\Delta t = \gamma \Delta t'$ [5].

Consider a clock at rest at a fixed position x' in frame S'. It ticks at times t'_A and t'_B, marking a time interval $\Delta t' = t'_B - t'_A$ in its own rest frame known as the *proper time* of the clock. An observer in frame S, relative to whom the clock is moving with velocity $v\mathbf{e}_x$, measures these ticks occurring at times t_A and t_B in their frame.

Using the time transformation $t = \gamma(t' + \frac{vx'}{c^2})$ from (4), we have:

$$t_A = \gamma(t'_A + vx')$$
$$t_B = \gamma(t'_B + vx')$$

The time interval measured in frame S is $\Delta t = t_B - t_A$. Subtracting the two equations gives

$$\Delta t = \gamma(t'_B - t'_A) = \gamma \Delta t'. \tag{10}$$

Since $\gamma \geq 1$, the time interval Δt measured in the frame where the clock is moving is always greater than or equal to the proper time interval $\Delta t'$ measured in the clock's rest frame.

While fundamental, simulating time dilation visually poses significant implementation challenges. Given the focus on geometric effects and the CPU-based transformation approach, implementing robust time dilation was deemed outside the scope of this project.

2.6 RELATIVITY OF SIMULTANEITY

The similarity between length contraction and time dilation stems from a fundamental principle that is critical to relativistic theory. As demonstrated in these effects, different frames with relative motion cannot agree that two spatially separated events happen at the same time. In other words, simultaneity is not absolute. This is known as the *relativity of simultaneity* [3].

Consider the illustrations in figure 6. The two events A and B are separated in space and time. In the rest frame S, A happens before B. However, in the boosted frame S', B occurs before A.

2.7 OPTICAL EFFECTS

Relativistic motion does not only affect observed distance and time intervals, it also alters the visual appearance of moving objects. While this project focuses on the geometric effects on spacetime, a more realistic simulation needs to account for this, as demonstrated by OpenRelativity [7], ReSim [8] and others.

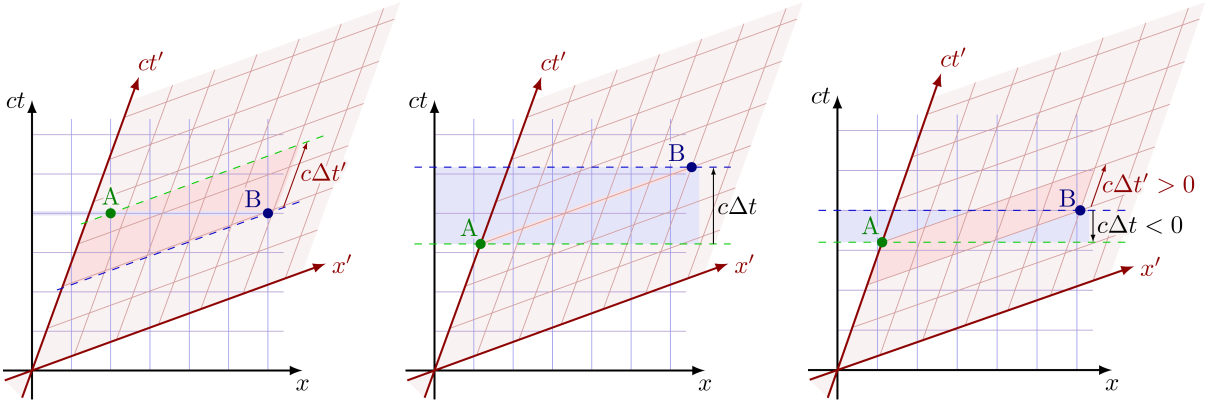

Fig. 6. Visualization of the relativity of simultaneity. **Left:** two spatially separated events A and B are simultaneous in the rest frame. In the boosted frame, B happens before A. **Center:** A and B are simultaneous in the boosted frame but in the rest frame, A happens before B. **Right:** in the rest frame, A happens before B. In the boosted frame, B happens before A [5].

At relativistic velocities, light waves will be subject to similar effects as subliminal waves of classical mechanics, such as Doppler shift and aberration of light. Light rays emitted from a stationary frame will appear to bend and change frequency, wavelength, amplitude angle when observed from a moving frame.

Relativistic aberration of light occurs as an observer moves relative to a light source with relative velocity v. The angle of the incoming light rays θ' will be altered from the outgoing angle θ, causing the observer to perceive the light rays as being emitted from a different direction. This is described by the formula

$$\cos\theta' = \frac{\cos\theta + \beta}{1 + \beta\cos\theta} \Leftrightarrow \sin\theta' = \frac{\sin\theta}{\gamma(1 + \beta\cos\theta)}.$$

When the observer moves towards the source, light rays will concentrate or "tunnel" in the forward direction, illustrated in figure 7. This produces a curving effect of the field of view.

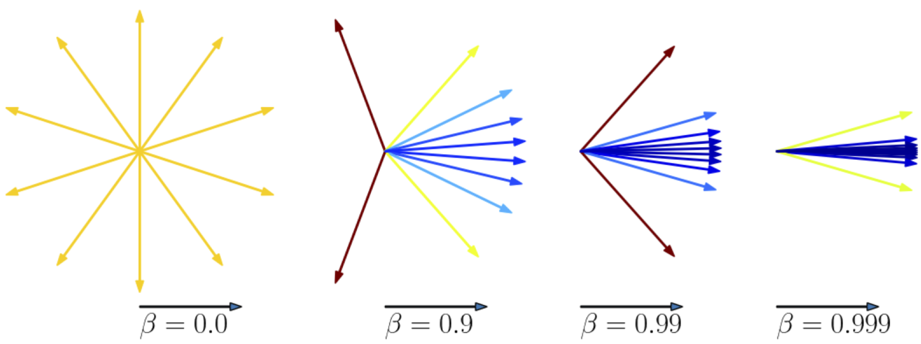

Fig. 7. Relativistic aberration of light [9].

Relativistic Doppler effect is the relativistic generalization of the classical Doppler effect. The relative velocity between observer and source causes the emitted light ray to be received at a different frequency. When the relative motion is purely radial, the observed frequency ν is related to the emitted frequency ν_0 as

$$\frac{\nu_0}{\nu} = \sqrt{\frac{1+\beta}{1-\beta}}. \tag{11}$$

The result of this frequency shift is that the light spectrum of approaching sources will appear blueshifted while receding sources will appear redshifted.

2.8 IMPLICATIONS FOR REAL-TIME SIMULATIONS

Implementing these relativistic effects in a real-time simulation requires computationally expensive Lorentz transformations of each point in Minkowski spacetime for every frame. To address this, simulations must balance theoretical accuracy and performance constraints.

One technique utilized by many relativity simulation projects is to drastically reduce the speed of light, letting relativistic effects to manifest at speeds that humans are more accustomed to. While this approach is strictly unphysical, it provides a more intuitive context for the user while allowing the simulation environment to be constructed at a more manageable size.

One project that does this is the aptly named *A Slower Speed of Light* [10], a game developed by MIT Game Lab which later developed into the *OpenRelativity* project [7]. The project is a toolkit for incorporating relativistic effects in Unity by using custom shaders to offload intensive calculations from the CPU. By placing heavier graphics operations on the GPU instead, OpenRelativity achieves realistic rendering without significantly sacrificing performance.

Other examples of similar projects relying on GPU shaders include Fons van der Plas' *ReSim* [8], which utilizes C# and OpenGL, and the more recent *special-relativity* by Harry Gifford [11] using WebGL. These methods demonstrate that GPU accelerated customs shaders make it possible to create realistic simulations of relativistic effects while retaining high performance.

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3. PROBLEM

The physical manifestations of special relativity are rarely observed in everyday life, where velocities remain far below the speed of light. The mathematical framework of the theory is elegantly formulated but geometrically abstract. This project seeks to visualize the effects of Lorentz transformation between different frames of reference through an interactive 3D simulation built in Unity. By allowing users to control their velocity as they navigate an environment, the simulation aims to bridge the gap between theory and experience by visualizing relativistic effects to build intuitive understanding. Attain this goal required overcoming several conceptual and computational challenges.

3.1 THE SPEED OF LIGHT

A key obstacle of demonstrating relativistic physics is the immense speed of light: $c \approx 3 \times 10^8$ m/s. At human-scale velocities, relativistic effects remain negligible. Creating a simulation at velocities closer to c would require building and rendering a 3D environment at unfeasible scales.

The solution to this problem is inspired by a common notational trick. To simplify calculation, theoretical physicists often employ *natural units* where the speed of light is normalized to 1. In the context of a simulation, this seemingly arbitrary notational convention functions as an artificial reduction of the speed of light. This is a common technique also employed by the other projects referenced above. Although unphysical, it enables length contraction to occur at ordinary velocities.

3.2 FRAMES OF REFERENCE

From a simulation perspective, enforcing relativistic simultaneity across many moving objects is a complex undertaking. Since the simulation must account for and transform each point in spacetime in relation to the player, an independently moving environment would require the timing of events and updated positions to be calculated with respect to the moving observer at each frame.

To sidestep the full complexity of multi-frame simultaneity, the simulation in this project assumes a single rest frame for all objects in the environment. This is implemented in practice by requiring all environment objects to be static in their shared frame. The moving player, along with the camera, constitutes another, boosted frame. Calculations are then based on the relative velocity and position between environment and player. Since the user follows the player from a third-person perspective, they are observing from the rest frame of the player. As indicated by (3) and figure 2, it is equivalent to transform the player's boosted frame to the rest frame of the environment with velocity $\mathbf{v}$ and to transform the environment's boosted frame traveling at $-\mathbf{v}$ to the player's rest frame.

3.3 PER-VERTEX TRANSFORMATION

To get a proper Lorentz transformation of the scene, all points in the environment need to be

This project takes another approach to vertex transformations that differs from the methods employed by previous projects outlined above. Instead of modifying Unity's render pipeline through custom shaders, all transformations are performed on the CPU. The reason behind this is that shaders are built in another programming language than C. It was deemed too time consuming to learn a new environment. Furthermore, writing the simulation exclusively in C facilitates the application of pen-and-paper methods to a 3D environment.

3.4 TIME-COORDINATE FOR THE ENVIRONMENT

how unity handles spatial transformations (in rest frame of player)
how unity handles time (observed / proper time for player)
which objects to transform and how to get them

3.5 GENERATIVE TOOLS

Brief description of how generative AI was used in this project.

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4. IMPLEMENTATION

4.1 IMPLEMENTATION DETAILS: PER-VERTEX TRANSFORMATION AND TIME COORDINATE

This section now turns from theoretical underpinnings to the practical implementation visible in the project scripts. All cited code excerpts (e.g., from `VertexManager.cs` and `LorentzTransform.cs`) come from the Unity project environment, where we apply the Lorentz transformation to each vertex in real time.

4.1.1 HANDLING THE TIME COMPONENT FOR EACH VERTEX

Physical interpretation and limitations.

4.1.2 PERFORMING THE LORENTZ TRANSFORMATION ON VERTICES

Educational context.

4.2 PLAYER MOVEMENT & VELOCITY COMPUTATION

- `PlayerController.cs` snippet for computing β and γ:

$$\beta = \frac{\|\mathbf{v}\|}{\text{MoveSpeed}}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}.$$

- Show relevant code lines, highlight numerical stability concerns.

4.3 CONSTRUCTING THE LORENTZ MATRIX

- `LorentzTransform.cs`: main loop building the 4×4 boost matrix.

- Fallback to identity if speed $< 10^{-6}$ to avoid division by near-zero.

4.4 FULL VERTEX MANIPULATION

`VertexManager.cs`: storing *rest* world vertices, applying Lorentz transformation matrix each frame.

4.5 COORDINATE OFFSETS & TIME CALCULATION

4.6 LIMITATIONS & CONSTRAINTS

Due to the limited time frame, the scope had to be narrowed considerably. A result of this was that there was not time to learn how to program shaders.

As a result, there was not adequate time to learn how to

---

5. RESULTS

5.1 VISUALIZATION OF CONTRACTION

- Screenshots or references to the blog: rods, cubes, environment objects.
- Observed contraction along velocity axis matches $\frac{1}{\gamma}$ scaling.

5.2 CONSISTENCY WITH THEORY

5.3 PERFORMANCE AND LIMITATIONS

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6. DISCUSSION

6.1 FUTURE DEVELOPMENT

- Time dilation
- Relativistic Doppler shift
- Relativistic aberration
- Rindler coordinates

6.2 REFLECTING ON DEVELOPMENT

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A SOURCE CODE

B PROJECT SPECIFICATION

Project Specification

```
course:     DD1354
assignment: lab 4 - Project Specification
author:     Axel Hammarskiöld Spendrup
id:         axelhs
email:      axelhs@kth.se
date:       20.02.25
grade aim:  A
```

Relativistic Reality

> A physically accurate special relativity simulator

1. Background

The $3+1-$ dimensional Minkowski spacetime is intimidating in notation yet remarkably easy to intuit. With good reason too, since we live in such a spacetime. As a 3-dimensional being, it is easy to visualize a 2 dimensional observer traversing time as the additional third dimension. Yet, the human brain often struggles to grasp is how the velocity of the observer affects the spacetime itself: the effects of special relativity.

This project aims to allow anyone to experience the effects of special relativity, well below the extreme velocities required in reality. By constructing a special relativity simulation within the context of a computer game, the player of the game will get a better intuition of some notable relativistic effects such as length contraction, Lorentz boosts, coordinate transformations and more. The project aims to attain the grade A.

The game will be built as an interactive 3D environment using Unity. The simulation scripts will be built on the rigorous mathematics of Lorentz transformations in Minkowski space that underpins special relativistic theory. By using natural units, (normalizing the speed of light to 1: $c = 1$), the relativistic effects will be prominent at velocities more familiar to us.

2. Problem Definition

The simulation will build on the Lorentz transformation in Minkowski space. The mathematic formalism was initially developed by Albert Einstein. In lieu of Einstein's original work, most of the mathematics are derived from another pioneer in the field: Wolfgang Rindler and his book *Introduction to Special Relativity*. The application of the theory to computer simulation has previously been tested by MIT Game Lab. Their paper *Visualizing relativity: The OpenRelativity project* provides insight into some techniques and limitations.

At the core of the simulation will be the Lorentz transformation matrix. The full transformation matrix allows complete transformation of all coordinates. This is required since only applying the length contraction formula

$$L_{||} = \frac{L_{0,||}}{\gamma}, \quad L_{\perp} = L_{0,\perp}$$

only yields length contracted objects but not distances. The Matrix is defined as

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta_x & -\gamma\beta_y & -\gamma\beta_z \\ -\gamma\beta_x & 1+\frac{\gamma^2}{1+\gamma}\beta_x^2 & \frac{\gamma^2}{1+\gamma}\beta_x\beta_y & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z \\ -\gamma\beta_y & \frac{\gamma^2}{1+\gamma}\beta_x\beta_y & 1+\frac{\gamma^2}{1+\gamma}\beta_y^2 & \frac{\gamma^2}{1+\gamma}\beta_y\beta_z \\ -\gamma\beta_z & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z & \frac{\gamma^2}{1+\gamma}\beta_y\beta_z & 1+\frac{\gamma^2}{1+\gamma}\beta_z^2 \end{bmatrix},$$

where $\gamma = \frac{1}{\sqrt{1+\beta^2}}$ and $\beta = \frac{v}{c}$. By restricting the player to only move relativistically in the $xz-$ plane, the Λ matrix takes a simpler form. This allows the Lorentz transformation between a stationary frame S and a moving frame S' as

$$X' = \begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta_x & 0 & -\gamma\beta_z \\ -\gamma\beta_x & 1+\frac{\gamma^2}{1+\gamma}\beta_x^2 & 0 & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z \\ 0 & 0 & 1 & 0 \\ -\gamma\beta_z & \frac{\gamma^2}{1+\gamma}\beta_x\beta_z & 0 & 1+\frac{\gamma^2}{1+\gamma}\beta_z^2 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix} = \Lambda X$$

Simulating movement at relativistic velocities would require colossal maps that are computationally expensive to render. To circumvent this problem, the simulation will use natural units $(c = 1)$ which permits smaller maps. This comes with the added benefit that the relativistic effects will become prominent in a context that is more familiar to everyday situations.

A possible movement mechanic that will be investigated is that of a rolling ball. This type of movement can emphasize the concept of relativistic momentum conservation, another important property of special relativity.

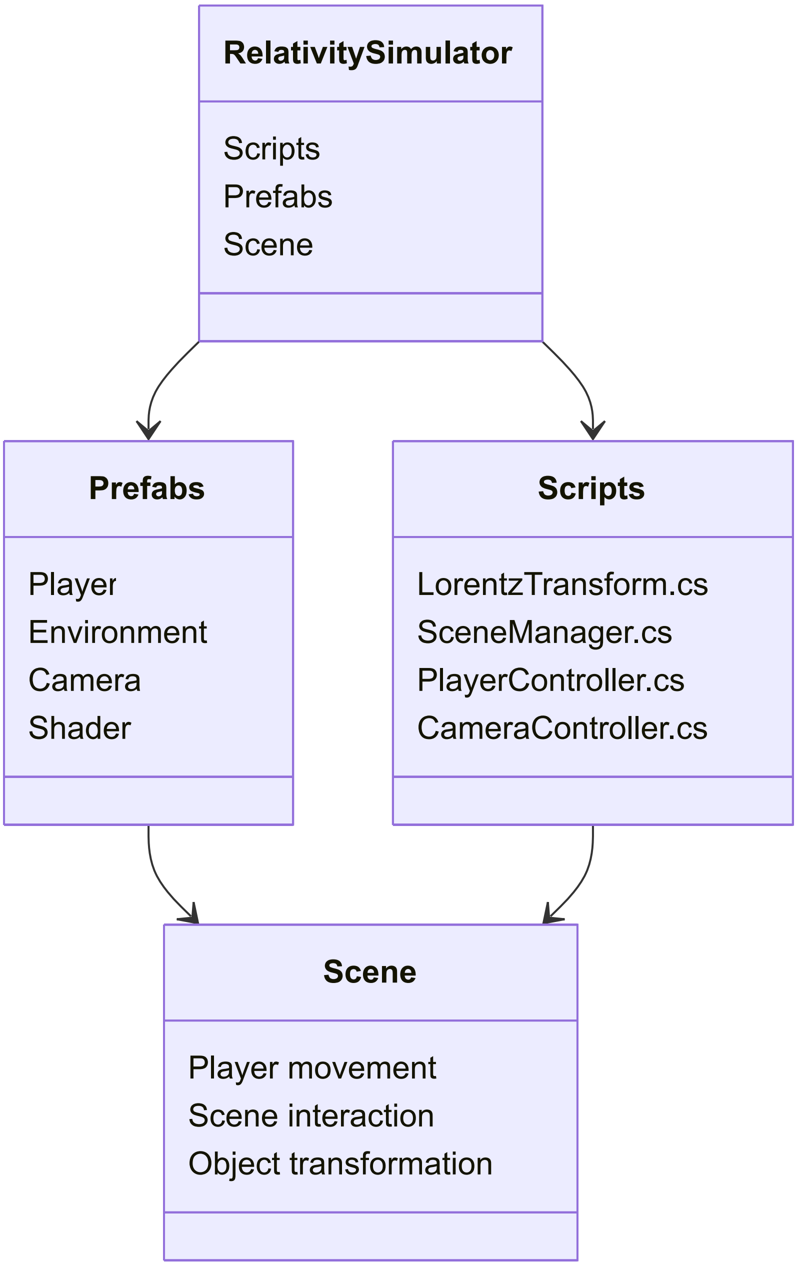
Limitations and constraints

The following limitations and constraints have previously been identified by the OpenRelativity project. Since this project will have fewer features than those offered by the OpenRelativity library, it is possible to assume that some constraints might be circumvented.

- The speed of light lowered to allow relativistic effects to occur at human scale.
- The player must never reach or exceed the speed of light.
- All objects in the scene must either be stationary of have a constant velocity.
- Gravity is not modeled in the framework of special relativity.
- Shadow and lighting must be permanent.
- The player's movement is assumed to be constant at each time step.
- The Doppler shift is constrained to limit wavelengths to the visible spectrum.

3. Implementation

The project is inspired by MIT Game Lab's OpenRelativity library and games such A Slower Speed of Light and Velocity Raptor. The simulation will be built in Unity 3D. This will permit the use of existing models and platforms for character control, prefabricated 3D environments and fundamental physics simulation to allow increased focus on the relativistic simulation. A rudimentary architecture is as follows:



The scripts above will control the following

- *LorentzTransform.cs*

  Handles the computation of the Lorentz transformation. Outputs the transformation matrix Λ and the $\gamma-$ factor.
- *SceneManager.cs*

  Manages the relativistic effects on the environment by applying the Lorentz transformation to the scene based on player velocity.
- *PlayerController.cs*

  A third person player controller built by Unity and modified to handle player movement, player input, camera tracking, collision detection, gravity and more.

- *CameraController.cs*
  Applies any relativistic effects that are unique to the camera, such as aberration and Doppler shift. Also handles any visual effects emphasizing when relativistic effects take place.

Inspiration for the level construction and obstacle design comes from the online resource *The Level Design Book*. Prefabs, shaders, materials and other scene-specific resources are available at the Unity AssetStore.

4. Demonstration

The ideal version of the final product will be a 3D virtual environment where a character is controlled from a third person view. In this environment, the speed of light is normalized to allow the character to move through the environment close to the speed of light. As they do, the high velocity will induce relativistic effects on the environment such as length contraction, time dilation and relativistic Doppler shift.

The player will be able allowed some input of how these effects are portrayed, either by toggling their implementation or by alternating the frame of reference. While beyond the scope of this project, it would be interesting to design an obstacle or a puzzle that requires relativity to solve.

The main focus of this project will be to model length contraction of static objects in the environment. If the mathematics are readily implementable to all aspects of the physics engine, more advanced effects, such as time dilation, Doppler shift, aberration and relativistic addition of velocities will be investigated. The character movement mechanic will be built to simulate relativistic conservation of momentum.

5. References

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