$$
\begin{array}{rcl}
\Pi & ::= & \overline{\mathsf{defmodtype}\ X\ \mathsf{do}\ \overline{P}\ \overline{D}\ \mathsf{end}}\ \mathsf{defmodule}\ \mathit{Main}\ \mathsf{do}\ \overline{B}\ \mathsf{end} \\
N & ::= & x \\
 & | & X \\
S & ::= & \mathsf{\$behaviour}\ X \\
P & ::= & \mathsf{\$param}\ x \\
B & ::= & \mathsf{defmodule}\ X\ \mathsf{do}\ \overline{P}\ \overline{S}\ \overline{B}\ \mathsf{end} \\
 & | & x = v \\
 & | & \mathsf{\$type}\ x = t \\
 & | & \mathsf{\$opaque}\ x = t \\
E & ::= & v \\
 & | & x \\
 & | & E(\overline{E}, E) \\
 & | & \%\{\overline{\ell = E}\} \\
 & | & E.\ell \\
 & | & (E \in t)?E : E \\
 & | & {\color{red}\overline{X\ [\overline{x = t}]}\ .x} \\
 & | & {\color{red}\overline{X\ [\overline{x = t}]}\ .X\ [\overline{x = t}]} \\
v & ::= & c \\
 & | & \%\{\overline{\ell = v}\} \\
 & | & \$ \wedge \overline{t \to t}\ \mathsf{fn}\ \overline{x} \to E \\
 & | & {\color{red}\$ \cap \overline{(\overline{N : T}) \to T}\ \mathsf{fn}\ \overline{N} \to E} \\
T & ::= & t \\
 & | & (\overline{N : T}) \to T \\
 & | & \star \\
 & | & \{\overline{S}; \overline{D}\} \\
t & ::= & \mathsf{int} \\
 & | & t \to t \\
 & | & \%\{\overline{f}\} \\
 & | & t \vee t \\
 & | & t \wedge t \\
 & | & \neg t \\
 & | & \alpha \\
 & | & \mathbb{O} \\
 & | & \overline{X\ [\overline{x = t}]}\ .x \\
D & ::= & \mathsf{\$module}\ X : T \\
 & | & \mathsf{\$callback}\ x : \bigcap \overline{T} \\
 & | & \mathsf{\$opaque}\ x \\
 & | & \mathsf{\$type}\ x = t
\end{array}
$$

Figure 1: Syntax of the surface language

$$
\begin{array}{rcl}
\tau & ::= & t \\
 & | & \star \\
 & | & (\overline{N : \tau}) \to \tau \\
 & | & \mathsf{like}\left(\overline{X\ [\overline{x = t}]}\ .X\ [\overline{x = t}]\right) \\
 & | & \cap \overline{\tau} \\
 & | & X\ [\overline{x = t}]
\end{array}
$$

Figure 2: Syntax of surface module types

$$
\begin{array}{ccccc}
\text{\$module}\ X : T & \cap & \text{\$module}\ X : T' & = & \text{\$module}\ X : T \cap T' \\
\text{\$callback}\ X : \bigcap \overline{T} & \cap & \text{\$callback}\ X : \bigcap \overline{T'} & = & \text{\$callback}\ X : \bigcap \overline{T}\,\overline{T'} \\
\text{\$type}\ x = t & \cap & \text{\$type}\ x = t' & = & \text{\$type}\ x = t \wedge t' \\
\text{\$opaque}\ x & \cap & \text{\$opaque}\ x & = & \text{\$opaque}\ x \\
\text{\$opaque}\ x & \cap & \text{\$type}\ x = t & = & \text{\$type}\ x = t \\
\text{\$type}\ x = t & \cap & \text{\$opaque}\ x & = & \text{\$type}\ x = t \\
D & \cap & D' & = & \epsilon
\end{array}
$$

Figure 3: Component-wise intersection

$$\textsc{ElEnv-Empty}\quad \frac{}{\Sigma; \Gamma \vdash \epsilon} \qquad \textsc{ElEnv-Expr}\quad \frac{\Sigma; \Gamma \vdash t : \star}{\Sigma, \Gamma \vdash x : t, \Gamma} \qquad \textsc{ElEnv-Type}\quad \frac{\Sigma; \Gamma \vdash t : \star}{\Sigma; \Gamma \vdash x = t, \Gamma}$$

$$\textsc{ModEnv-ModuleType}\quad \frac{\Sigma; \Gamma, \overline{x : \star} \vdash \left\{\overline{D}\right\}}{\Sigma; \Gamma \vdash X = \overline{x} \mapsto \overline{D}, \Sigma} \qquad \textsc{ModEnv-Empty}\quad \frac{}{\Sigma; \Gamma \vdash \epsilon} \qquad \textsc{ModEnv-Module}\quad \frac{\Sigma; \Gamma, \overline{x : \star} \vdash \left\{; \overline{B}\right\}}{\Sigma; \Gamma \vdash X = \overline{x} \mapsto \overline{B}, \Sigma}$$

Figure 4: Formation rules for environments

$$\textsc{EqPath-Empty}\quad \frac{}{\Sigma; \Gamma \vdash \epsilon \cong \epsilon} \qquad \textsc{EqPath-Add}\quad \frac{\Sigma; \Gamma \vdash P_1 \cong P_2 \qquad \forall i. \Sigma, \Gamma \vdash t_i \cong t'_i}{\Sigma; \Gamma \vdash P_1.X \left[\overline{x_i = t_i}\right] \cong P_2.X \left[\overline{x_i = t'_i}\right]}$$

Figure 5: Rules for path equivalence

Figure 6: Typing rules for declarations

Bind-DefModule

$$\frac{\begin{array}{c}\Sigma; \Gamma, \overline{P : *} \vdash \overline{B} : \overline{D} \\ \Sigma; \Gamma, X : \left(\overline{P : \star}\right) \to \left\{\overline{S_i}; \overline{D}\right\} \vdash \overline{B_0} : \overline{D_0} \qquad \forall i. \Sigma(S_i) = \overline{x_i : \star} \to \overline{D_i} \\ \forall i \neq j. (\overline{x_i}, \mathsf{dom}(D_i) \# \overline{x_j}, \mathsf{dom}(D_j)) \qquad \left\{\overline{D}\right\} \preccurlyeq \left\{\overline{\overline{x_i : [= \_]}\overline{D_i}}\right\}\end{array}}{\Sigma; \Gamma \vdash \left(\mathsf{defmodule}\, X\, \mathsf{do}\, \overline{P S_i B}\, \mathsf{end}\right) \overline{B_0} : \left(X : \left(\overline{P : \star}\right) \to \left\{\overline{S_i}; \overline{D}\right\}\right) \overline{D_0}}$$

Bind-Type

$$\frac{\Sigma; \Gamma \vdash t : \star \qquad \Sigma; \Gamma, x : [= t] \vdash \overline{B} : \overline{D}}{\Sigma; \Gamma \vdash (\$\mathsf{type}\, x = t)\, \overline{B} : (x : [= t])\, \overline{D}}$$

Bind-Opaque

$$\frac{\Sigma; \Gamma \vdash t : \star \qquad \Sigma; \Gamma, x : [= t] \vdash \overline{B} : \overline{D}}{\Sigma; \Gamma \vdash (\$\mathsf{opaque}\, x = t)\, \overline{B} : (x : \star)\, \overline{D}}$$

Bind-Empty

$$\frac{}{\Sigma; \Gamma \vdash \epsilon : \epsilon}$$

Bind-Value

$$\frac{\Sigma; \Gamma \vdash v : \cap \overline{T} \qquad \Sigma; \Gamma, x : \cap \overline{T} \vdash \overline{B} : \overline{D}}{\Sigma; \Gamma \vdash (x = v)\, \overline{B} : \left(x : \cap \overline{T}\right) \overline{D}}$$

Figure 7: Typing rules for bindings

Path-Empty

$$\frac{}{\Sigma; \Gamma \vdash \epsilon : \{\epsilon; \Gamma\}}$$

Path-SubModule

Figure 8: Well-formdness rules for paths

$$\textsc{Type-Variable}\quad \frac{\Sigma;\Gamma \vdash P : \left\{\overline{S};\overline{D}(x : \cap T)\overline{D'}\right\}}{\Sigma;\Gamma \vdash P.x : \cap T}$$

$$\textsc{Type-Subsumption}\quad \frac{\Sigma;\Gamma \vdash T \qquad \Sigma\Gamma \vdash E : \cap\overline{T'} \qquad \Sigma;\Gamma \vdash \cap\overline{T'} \preccurlyeq T}{\Sigma;\Gamma \vdash E : T}$$

$$\textsc{Type-BigFunction}\quad \frac{\Sigma;\Gamma \vdash \cap\overline{\overline{N : T} \to T'\Sigma;\Gamma, \overline{N : T}} \vdash E : T'}{\Sigma;\Gamma \vdash \$ \cap \overline{\forall\overline{\alpha}\left(\overline{N : T}\right) \to T'}\,\mathsf{fn}\,\overline{N} \to E : \cap\overline{\overline{N : T} \to T'}}$$

$$\textsc{Type-Module}\quad \frac{\Sigma;\Gamma \vdash P : \left\{\overline{S_0};\overline{D_0}(X : \overline{y : \star} \to \left\{\overline{S};\overline{D}\right\})\overline{D_1}\right\} \qquad \Sigma;\Gamma \vdash \overline{t} \qquad \overline{x} \simeq \overline{y}}{\Sigma;\Gamma \vdash P.X\left[\overline{x = t}\right] : \left\{\overline{S};\overline{D}\right\}}$$

Figure 9: Typing rules for the surface language

$$\textsc{Sub-StarRefl}\quad \frac{}{\star \preccurlyeq \star} \qquad\qquad \textsc{Sub-Elixir}\quad \frac{t \preccurlyeq t'}{\Sigma;\Gamma \vdash t \preccurlyeq t'} \qquad\qquad \textsc{Sub-Intersection}\quad \frac{}{\Sigma;\Gamma \vdash \cap\overline{T_i} \preccurlyeq T_i}$$

$$\textsc{Sub-ModuleLeft}\quad \frac{\Sigma;\Gamma \vdash P : \left\{\overline{S};\overline{D}(x : [= t])\overline{D'}\right\} \qquad \Sigma;\Gamma \vdash t \preccurlyeq T}{\Sigma;\Gamma \vdash P.x \preccurlyeq T}$$

$$\textsc{Sub-ModuleRight}\quad \frac{\Sigma;\Gamma \vdash P : \left\{\overline{S};\overline{D}(x : [= t])\overline{D'}\right\} \qquad \Sigma;\Gamma \vdash \cap\overline{T} \preccurlyeq t}{\Sigma;\Gamma \vdash \cap\overline{T} \preccurlyeq P.x}$$

$$\textsc{Sub-Opaque}\quad \frac{\Sigma;\Gamma \vdash P \cong P' \qquad \Sigma;\Gamma \vdash P : \left\{\overline{S};\overline{D}(x : \star)\overline{D'}\right\}}{\Sigma;\Gamma \vdash P.x \preccurlyeq P'.x}$$

$$\textsc{Sub-BigFunction}\quad \frac{\forall i.\Sigma;\Gamma, x_1 : R_1, \ldots, X_{i-1} : R_{i-1} \vdash T_i \succcurlyeq R_i \qquad \Sigma;\Gamma, \overline{X_i : R_i} \vdash T' \preccurlyeq R'}{\Sigma;\Gamma \vdash (X_i : T_i) \to T' \preccurlyeq (X_i : R_i) \to R'}$$

Figure 10: Subtyping rules

$$
\begin{array}{cccc}
\text{\$module}\, X : T & \cup & \text{\$module}\, X : T' & = & \text{\$module}\, X : T \cup T' \\
\text{\$callback}\, X : \bigcap \overline{T} & \cup & \text{\$callback}\, X : \bigcap \overline{T'} & = & ? \\
\text{\$type}\, x = t & \cup & \text{\$type}\, x = t' & = & \text{\$type}\, x = t \cup t' \\
\text{\$opaque}\, x & \cup & \text{\$opaque}\, x & = & \text{\$opaque}\, x \\
\text{\$opaque}\, x & \cup & \text{\$type}\, x = t & = & \text{\$opaque}\, x \\
\text{\$type}\, x = t & \cup & \text{\$opaque}\, x & = & \text{\$opaque}\, x \\
D & \cup & D' & = & \epsilon
\end{array}
$$

Figure 11: Component-wise union