$$
\begin{array}{rcl}
\Pi & ::= & \overline{\textsf{defmodtype}\ X\ \textsf{do}\ \overline{P}\ \overline{D}\ \textsf{end}}\ \textsf{defmodule}\ \textit{Main}\ \textsf{do}\ \overline{B}\ \textsf{end} \\
N & ::= & x \\
 & | & X \\
S & ::= & \textsf{\$behaviour}\ X \\
P & ::= & \textsf{\$param}\ x \\
B & ::= & \textsf{defmodule}\ X\ \textsf{do}\ \overline{P}\ \overline{S}\ \overline{B}\ \textsf{end} \\
 & | & x = v \\
 & | & \textsf{\$type}\ x = t \\
 & | & \textsf{\$opaque}\ x = t \\
E & ::= & v \\
 & | & x \\
 & | & \textsf{let}\ N = E; E \\
 & | & E(\overline{E,}) \\
 & | & \%\{\overline{\ell = E}\} \\
 & | & E.\ell \\
 & | & (E \in t)?E : E \\
 & | & \color{red}{\overline{X\ \left[\overline{x = t}\right]}\ .x} \\
 & | & \color{red}{\overline{X\ \left[\overline{x = t}\right]}\ .X\ \left[\overline{x = t}\right]} \\
v & ::= & c \\
 & | & \%\{\overline{\ell = v}\} \\
 & | & \$ \wedge \overline{t \to t}\ \textsf{fn}\ \overline{x} \to E \\
 & | & \color{red}{\$ \cap \overline{\left(\overline{N : T}\right) \to T}\ \textsf{fn}\ \overline{N} \to E} \\
T & ::= & t \\
 & | & \left(\overline{N : T}\right) \to T \\
 & | & \star \\
 & | & \{\overline{S}; \overline{D}\} \\
t & ::= & \textsf{int} \\
 & | & t \to t \\
 & | & \%\{\overline{f}\} \\
 & | & t \vee t \\
 & | & t \wedge t \\
 & | & \neg t \\
 & | & \alpha \\
 & | & \mathbb{O} \\
 & | & \overline{X\ \left[\overline{x = t}\right]}\ .x \\
D & ::= & \textsf{\$module}\ X : T \\
 & | & \textsf{\$callback}\ x : \bigcap \overline{T} \\
 & | & \textsf{\$opaque}\ x \\
 & | & \textsf{\$type}\ x = t
\end{array}
$$

Figure 1: Syntax of the surface language

$$
\begin{array}{lll}
\tau & ::= & t \\
 & | & \star \\
 & | & \left(\overline{N : \tau}\right) \to \tau \\
 & | & \mathsf{like}\left(\overline{X\left[\overline{x=t}\right].}X\left[\overline{x=t}\right]\right) \\
 & | & \cap\overline{\tau} \\
 & | & X\left[\overline{x=t}\right]
\end{array}
$$

Figure 2: Syntax of surface module types

$$
\begin{array}{ccccc}
\$\mathsf{module}\, X : T & \cap & \$\mathsf{module}\, X : T' & = & \$\mathsf{module}\, X : T \cap T' \\
\$\mathsf{callback}\, X : \bigcap \overline{T} & \cap & \$\mathsf{callback}\, X : \bigcap \overline{T'} & = & \$\mathsf{callback}\, X : \bigcap \overline{T}\, \overline{T'} \\
\$\mathsf{type}\, x = t & \cap & \$\mathsf{type}\, x = t' & = & \$\mathsf{type}\, x = t \wedge t' \\
\$\mathsf{opaque}\, x & \cap & \$\mathsf{opaque}\, x & = & \$\mathsf{opaque}\, x \\
\$\mathsf{opaque}\, x & \cap & \$\mathsf{type}\, x = t & = & \$\mathsf{type}\, x = t \\
\$\mathsf{type}\, x = t & \cap & \$\mathsf{opaque}\, x & = & \$\mathsf{type}\, x = t \\
D & \cap & D' & = & \epsilon
\end{array}
$$

Figure 3: Component-wise intersection

ElEnv-Empty
$$\frac{}{\Sigma; \Gamma \vdash \epsilon}$$

ElEnv-Expr
$$\frac{\Sigma; \Gamma \vdash t : \star}{\Sigma, \Gamma \vdash x : t, \Gamma}$$

ElEnv-Type
$$\frac{\Sigma; \Gamma \vdash t : \star}{\Sigma; \Gamma \vdash x = t, \Gamma}$$

ModEnv-ModuleType
$$\frac{\Sigma; \Gamma, \overline{x : \star} \vdash \left\{\overline{D}\right\}}{\Sigma; \Gamma \vdash X = \overline{x} \mapsto \overline{D}, \Sigma}$$

ModEnv-Empty
$$\frac{}{\Sigma; \Gamma \vdash \epsilon}$$

ModEnv-Module
$$\frac{\Sigma; \Gamma, \overline{x : \star} \vdash \left\{; \overline{B}\right\}}{\Sigma; \Gamma \vdash X = \overline{x} \mapsto \overline{B}, \Sigma}$$

Figure 4: Formation rules for environments

EqPath-Empty
$$\frac{}{\Sigma; \Gamma \vdash \epsilon \cong \epsilon}$$

EqPath-Add
$$\frac{\Sigma; \Gamma \vdash P_1 \cong P_2 \qquad \forall i. \Sigma, \Gamma \vdash t_i \cong t_i'}{\Sigma; \Gamma \vdash P_1.X\left[\overline{x_i = t_i}\right] \cong P_2.X\left[\overline{x_i = t_i'}\right]}$$

Figure 5: Rules for path equivalence

WF-Field

$$\frac{\Sigma;\Gamma \vdash P : \{\ldots, \$\mathsf{type}\, x = t, \ldots\} \cup \{\ldots, \$\mathsf{opaque}\, x, \ldots\}}{\Sigma;\Gamma \vdash P.x}$$

WF-Star

$$\frac{\Sigma;\Gamma \vdash}{\Sigma;\Gamma \vdash \star}$$

WF-Function

$$\frac{\Sigma;\Gamma \vdash \qquad \forall 0 \le j \le n. \Sigma;\Gamma, \overline{N_i : T_i}^{i=1,\ldots,j} \vdash T_{j+1}}{\Sigma;\Gamma \vdash \left(\overline{N_i : T_i}^{i=1,\ldots,n}\right) \to T_{n+1}}$$

WF-ModuleTypeNames

$$\frac{\begin{array}{c} \forall S_i \in \overline{S}.\Sigma(S_i) = \overline{x : \star}^i \to \overline{D}^i \\ \{\overline{D}\} \preccurlyeq \left\{\overline{\overline{x : [=\_]}^i\ \overline{D}^i}\right\} \qquad \forall i \neq j.\, \mathsf{dom}\left(\overline{D}^i\right) \# \mathsf{dom}\left(\overline{D}^j\right) \qquad \Sigma;\Gamma \vdash \{\overline{D}\} \end{array}}{\Sigma;\Gamma \vdash \{\overline{S};\overline{D}\}}\ (S \neq \epsilon)$$

WF-ModuleTypeModule

$$\frac{\Sigma;\Gamma \vdash T \qquad \Sigma;\Gamma, X : T \vdash \{\overline{D}\}}{\Sigma;\Gamma \vdash \{(\$\mathsf{module}\, X : T)\overline{D}\}}$$

WF-ModuleTypeOpaque

$$\frac{\Sigma;\Gamma, x : \star \vdash \{\overline{D}\}}{\Sigma;\Gamma \vdash \{(\$\mathsf{opaque}\, x)\overline{D}\}}$$

WF-ModuleTypeType

$$\frac{\Sigma;\Gamma \vdash t \qquad \Sigma;\Gamma, x : [= t] \vdash \{\overline{D}\}}{\Sigma;\Gamma \vdash \{(\$\mathsf{type}\, x = t)\overline{D}\}}$$

WF-ModuleTypeCallback

$$\frac{\overline{\Sigma;\Gamma \vdash T} \qquad \Sigma;\Gamma, x : \bigcap \overline{T} \vdash \{\overline{D}\}}{\Sigma;\Gamma \vdash \left\{(\$\mathsf{callback}\, x : \bigcap \overline{T})\overline{D}\right\}}$$

Figure 6: Well-formedness rules for types $\boxed{\Sigma;\Gamma \vdash T}$

Bind-DefModule

$$\frac{\Sigma;\Gamma, \overline{P : \star} \vdash \overline{B} : \overline{D} \qquad \Sigma;\Gamma, X : (\overline{P : \star}) \to \{\overline{S};\overline{D}\} \vdash \overline{B_0} : \overline{D_0}}{\Sigma;\Gamma \vdash (\mathsf{defmodule}\, X\, \mathsf{do}\, \overline{PSB}\, \mathsf{end})\, \overline{B_0} : \left(X : (\overline{P : \star}) \to \{\overline{S};\overline{D}\}\right) \overline{D_0}}$$

Bind-Type

$$\frac{\Sigma;\Gamma \vdash t : \star \qquad \Sigma;\Gamma, x : [= t] \vdash \overline{B} : \overline{D}}{\Sigma;\Gamma \vdash (\$\mathsf{type}\, x = t)\, \overline{B} : (x : [= t])\, \overline{D}}$$

Bind-Opaque

$$\frac{\Sigma;\Gamma \vdash t : \star \qquad \Sigma;\Gamma, x : [= t] \vdash \overline{B} : \overline{D}}{\Sigma;\Gamma \vdash (\$\mathsf{opaque}\, x = t)\, \overline{B} : (x : \star)\, \overline{D}}$$

Bind-Empty

$$\frac{}{\Sigma;\Gamma \vdash \epsilon : \epsilon}$$

Bind-Value

$$\frac{\Sigma;\Gamma \vdash v : \cap \overline{T} \qquad \Sigma;\Gamma, x : \cap \overline{T} \vdash \overline{B} : \overline{D}}{\Sigma;\Gamma \vdash (x = v)\, \overline{B} : \left(x : \cap \overline{T}\right) \overline{D}}$$

Figure 7: Typing rules for bindings $\boxed{\Sigma;\Gamma \vdash \overline{B} : \overline{D}}$

Path-Empty

$$\frac{}{\Sigma; \Gamma \vdash \epsilon : \{\epsilon; \Gamma\}}$$

Path-SubModule

$$\frac{\overline{D} = \ldots, \mathsf{defmodule}\, X\, \mathsf{do}\, \overline{\overline{P}\overline{S}\overline{B}}\, \mathsf{end}, \ldots \qquad \overset{\Sigma; \Gamma \vdash P : \left\{\_; \overline{D}\right\}}{\overline{P} = \overline{\mathsf{\$param}\, x}} \qquad \Sigma; \Gamma \vdash \overline{t}}{\Sigma; \Gamma \vdash P.X \left[\overline{x = t}\right] : \left\{\overline{S}; \overline{\mathsf{\$type}\, x = t}\, \overline{B}\right\}}$$

Figure 8: Well-formdness rules for paths

Type-Variable

$$\frac{\Sigma; \Gamma \vdash P : \left\{\overline{S}; \overline{D}(x : \cap T)\overline{D'}\right\}}{\Sigma; \Gamma \vdash P.x : \cap T}$$

Type-Subsumption

$$\frac{\Sigma; \Gamma \vdash T \qquad \Sigma\Gamma \vdash E : \cap\overline{T'} \qquad \Sigma; \Gamma \vdash \cap\overline{T'} \preccurlyeq T}{\Sigma; \Gamma \vdash E : T}$$

Type-BigFunction

$$\frac{\overline{\Sigma; \Gamma \vdash \cap\overline{\overline{N : T} \to T'}\Sigma; \Gamma, \overline{N : T} \vdash E : T'}}{\Sigma; \Gamma \vdash \$ \cap \overline{\forall\overline{\alpha}\left(\overline{N : T}\right) \to T'}\, \mathsf{fn}\, \overline{N} \to E : \cap\overline{\overline{\overline{N : T} \to T'}}}$$

Type-Module

$$\frac{\Sigma; \Gamma \vdash P : \left\{\overline{S_0}; \overline{D_0}(X : \overline{y : \star} \to \left\{\overline{S}; \overline{D}\right\})\overline{D_1}\right\} \qquad \Sigma; \Gamma \vdash \overline{t} \qquad \overline{x} \simeq \overline{y}}{\Sigma; \Gamma \vdash P.X \left[\overline{x = t}\right] : \left\{\overline{S}; \overline{D}\right\}}$$

Figure 9: Typing rules for the surface language

$$\frac{}{\star \preccurlyeq \star}\ \textsc{Sub-StarRefl} \qquad \frac{t \preccurlyeq t'}{\Sigma; \Gamma \vdash t \preccurlyeq t'}\ \textsc{Sub-Elixir} \qquad \frac{\exists i \in I, T_i \preccurlyeq T}{\Sigma; \Gamma \vdash \cap_I \overline{T_i} \preccurlyeq T}\ \textsc{Sub-Intersection}$$

$$\frac{\Sigma; \Gamma \vdash P : \left\{\overline{S}; \overline{D}(x : [= t])\overline{D'}\right\} \qquad \Sigma; \Gamma \vdash t \preccurlyeq T}{\Sigma; \Gamma \vdash P.x \preccurlyeq T}\ \textsc{Sub-ModuleLeft}$$

$$\frac{\Sigma; \Gamma \vdash P : \left\{\overline{S}; \overline{D}(x : [= t])\overline{D'}\right\} \qquad \Sigma; \Gamma \vdash \cap\overline{T} \preccurlyeq t}{\Sigma; \Gamma \vdash \cap\overline{T} \preccurlyeq P.x}\ \textsc{Sub-ModuleRight}$$

$$\frac{\Sigma; \Gamma \vdash P \cong P' \qquad \Sigma; \Gamma \vdash P : \left\{\overline{S}; \overline{D}(x : \star)\overline{D'}\right\}}{\Sigma; \Gamma \vdash P.x \preccurlyeq P'.x}\ \textsc{Sub-Opaque}$$

$$\frac{\forall i.\Sigma; \Gamma, x_1 : R_1, \ldots, X_{i-1} : R_{i-1} \vdash T_i \succcurlyeq R_i \qquad \Sigma; \Gamma, \overline{X_i : R_i} \vdash T' \preccurlyeq R'}{\Sigma; \Gamma \vdash (X_i : T_i) \to T' \preccurlyeq (X_i : R_i) \to R'}\ \textsc{Sub-BigFunction}$$

Figure 10: Subtyping rules $\boxed{\Sigma; \Gamma \vdash \cap\overline{T} \preccurlyeq T}$

\$module $X : T$	$\cup$	\$module $X : T'$	=	\$module $X : T \cup T'$
\$callback $X : \bigcap \overline{T}$	$\cup$	\$callback $X : \bigcap \overline{T'}$	=	?
\$type $x = t$	$\cup$	\$type $x = t'$	=	\$type $x = t \cup t'$
\$opaque x	$\cup$	\$opaque x	=	\$opaque x
\$opaque x	$\cup$	\$type $x = t$	=	\$opaque x
\$type $x = t$	$\cup$	\$opaque x	=	\$opaque x
D	$\cup$	D'	=	ϵ

Figure 11: Component-wise union