

$\Pi$	::=	$\overline{\text{defmodtype } X \text{ do } \overline{P} \overline{D} \text{ end } \$\text{defmodule } Main \text{ do } \overline{B} \text{ end}}$
$I$	::=	$x$
		$X$
$S$	::=	$\$behaviour X$
		$\$param x = t$
$P$	::=	$\$param x$
$B$	::=	$\$defmodule X \text{ do } \overline{P} \overline{S} \overline{B} \text{ end}$
		$x = v$
		$\$type x = t$
		$\$opaque x = t$
$E$	::=	$v$
		$x$
		$E(\overline{E},)$
		$\% \{ \overline{l} = \overline{E} \}$
		$E.l$
		$(E \in t)?E : E$
		$\overline{X[x = t]}.x$
		$\overline{X[x = t]}.X[x = t]$
$v$	::=	$c$
		$\% \{ \overline{l} = \overline{v} \}$
		$\$ \wedge \overline{t} \rightarrow \overline{t} \text{ fn } \overline{x} \rightarrow E$
		$\$ \cap (\overline{I} : \overline{T}) \rightarrow \overline{T} \text{ fn } \overline{I} \rightarrow E$
$T$	::=	$t$
		$(\overline{I} : \overline{T}) \rightarrow T$
		$\star$
		$M$
$M$	::=	$X[x = t]$
		$\{ \overline{D} \}$
		$M \cap M$
$t$	::=	$\text{int}$
		$t \rightarrow t$
		$\% \{ \overline{f} \}$
		$t \vee t$
		$t \wedge t$
		$\neg t$
		$\alpha$
		$\mathbb{O}$
		$\overline{X[x = t]}.x$
$D$	::=	$\$module X : T$
		$\$callback x : \bigcap \overline{T}$
		$\$opaque x$
		$\$type x = t$

Figure 1: Syntax of the surface language

$$\begin{array}{lll}
\$module\ X : T & \cap & \$module\ X : T' \\
\$callback\ X : \bigcap \overline{T} & \cap & \$callback\ X : \bigcap \overline{T'} \\
\$type\ x = t & \cap & \$type\ x = t' \\
\$opaque\ x & \cap & \$opaque\ x \\
\$opaque\ x & \cap & \$type\ x = t \\
\$type\ x = t & \cap & \$opaque\ x \\
D & \cap & D' \\
\epsilon & & \epsilon
\end{array}$$

Figure 2: Component-wise intersection

$$\begin{array}{lll}
\$module\ X : T & \cup & \$module\ X : T' \\
\$callback\ X : \bigcap \overline{T} & \cup & \$callback\ X : \bigcap \overline{T'} \\
\$type\ x = t & \cup & \$type\ x = t' \\
\$opaque\ x & \cup & \$opaque\ x \\
\$opaque\ x & \cup & \$type\ x = t \\
\$type\ x = t & \cup & \$opaque\ x \\
D & \cup & D' \\
\epsilon & & \epsilon
\end{array}$$

Figure 3: Component-wise union

$$\begin{array}{lll}
\text{ElEnv-Empty} & \text{ElEnv-Expr} & \text{ElEnv-Type} \\
\frac{}{\Sigma, \Gamma \vdash \epsilon} & \frac{\Sigma, \Gamma \vdash t : \star}{\Sigma, \Gamma \vdash x : t, \Gamma} & \frac{\Sigma, \Gamma \vdash t : \star}{\Sigma, \Gamma \vdash x = t, \Gamma} \\
\text{ModEnv-ModuleType} & \text{ModEnv-Empty} & \text{ModEnv-Module} \\
\frac{\Sigma, \Gamma \vdash \{\$type\ x = \alpha; \overline{D}\}}{\Sigma, \Gamma \vdash X = \overline{x} \mapsto \overline{D}, \Sigma} & \frac{}{\Sigma, \Gamma \vdash \epsilon} & \frac{\Sigma, \Gamma \vdash \{\$type\ x = \alpha; \overline{B}\}}{\Sigma, \Gamma \vdash X = \overline{x} \mapsto \overline{B}, \Sigma}
\end{array}$$

Figure 4: Formation rules for environments

$$\begin{array}{l}
\text{STRUCT-COM} \\
\frac{}{\Sigma, \Gamma \vdash \text{struct}(M \cap M') = \text{struct}(M' \cap M)} \\
\text{STRUCT-ASSOC} \\
\frac{}{\Sigma, \Gamma \vdash \text{struct}(M \cap (M' \cap M'')) = \text{struct}((M \cap M') \cap M'')} \\
\text{STRUCT-DECLARATION} \\
\frac{\Sigma, \Gamma \vdash \overline{D}}{\Sigma, \Gamma \vdash \text{struct}(\{\overline{D}\}) = \{\overline{D}\}} \\
\text{STRUCT-MODULETYPE} \\
\frac{X = (\overline{x_i} \mapsto \overline{D}) \in \Sigma \quad \forall i. \Sigma, (x_1 = t_1, \dots, x_i = t_i, \Gamma) \vdash t_{i+1} : \star}{\Sigma, \Gamma \vdash \text{struct}(X[\overline{x_i} = t_i]) = \{\$type\ x_i = t_i; \overline{D}\}}
\end{array}$$

Figure 5: Erasure of name subtyping

Figure 6: Typing rules for declarations

Figure 7: Typing rules for the surface language