$$
\begin{array}{lll}
\Pi & ::= & \overline{\text{defmodtype } X \text{ do } \overline{P}\,\overline{D} \text{ end}} \text{ defmodule } \textit{Main} \text{ do } \overline{B} \text{ end} \\
N & ::= & x \\
  & | & X \\
S & ::= & \$\text{behaviour } X \\
P & ::= & \$\text{param } x \\
B & ::= & \text{defmodule } X \text{ do } \overline{P}\,\overline{S}\,\overline{B} \text{ end} \\
  & | & x = v \\
  & | & \$\text{type } x = t \\
  & | & \$\text{opaque } x = t \\
E & ::= & v \\
  & | & x \\
  & | & E(\overline{E}, E) \\
  & | & \%\{\overline{\ell = E}\} \\
  & | & E.\ell \\
  & | & (E \in t)?E : E \\
  & | & \overline{X\,\boxed{x = t}}.x \\
  & | & \overline{X\,\boxed{x = t}}.X\,\boxed{x = t} \\
v & ::= & c \\
  & | & \%\{\overline{\ell = v}\} \\
  & | & \$ \wedge \overline{t \to t}\,\text{fn}\,\overline{x} \to E \\
  & | & \$ \cap \overline{(\overline{N:T}) \to T}\,\text{fn}\,\overline{N} \to E \\
T & ::= & t \\
  & | & (\overline{N:T}) \to T \\
  & | & \star \\
  & | & \{\overline{S}; \overline{D}\} \\
t & ::= & \text{int} \\
  & | & t \to t \\
  & | & \%\{\overline{f}\} \\
  & | & t \vee t \\
  & | & t \wedge t \\
  & | & \neg t \\
  & | & \alpha \\
  & | & \mathbb{O} \\
  & | & \overline{X\,\boxed{x = t}}.x \\
D & ::= & \$\text{module } X : T \\
  & | & \$\text{callback } x : \bigcap \overline{T} \\
  & | & \$\text{opaque } x \\
  & | & \$\text{type } x = t
\end{array}
$$

Figure 1: Syntax of the surface language

$$
\begin{array}{lll}
\tau & ::= & t \\
  & | & \star \\
  & | & \overline{(\overline{N:\tau}) \to \tau} \\
  & | & \text{like}\left(\overline{X\,\boxed{x = t}}.X\,\boxed{x = t}\right) \\
  & | & \cap \overline{\tau} \\
  & | & X\,\boxed{\overline{x = t}}
\end{array}
$$

Figure 2: Syntax of surface module types

$$
\begin{aligned}
\$\text{module } X : T &\quad \cap \quad \$\text{module } X : T' &&= \$\text{module } X : T \cap T' \\
\$\text{callback } X : \bigcap \overline{T} &\quad \cap \quad \$\text{callback } X : \bigcap \overline{T'} &&= \$\text{callback } X : \bigcap \overline{T}\,\overline{T'} \\
\$\text{type } x = t &\quad \cap \quad \$\text{type } x = t' &&= \$\text{type } x = t \wedge t' \\
\$\text{opaque } x &\quad \cap \quad \$\text{opaque } x &&= \$\text{opaque } x \\
\$\text{opaque } x &\quad \cap \quad \$\text{type } x = t &&= \$\text{type } x = t \\
\$\text{type } x = t &\quad \cap \quad \$\text{opaque } x &&= \$\text{type } x = t \\
D &\quad \cap \quad D' &&= \epsilon
\end{aligned}
$$

Figure 3: Component-wise intersection

ELENV-EMPTY
$$\frac{}{\Sigma; \Gamma \vdash \epsilon}$$

ELENV-EXPR
$$\frac{\Sigma; \Gamma \vdash t : \star}{\Sigma, \Gamma \vdash x : t, \Gamma}$$

ELENV-TYPE
$$\frac{\Sigma; \Gamma \vdash t : \star}{\Sigma; \Gamma \vdash x = t, \Gamma}$$

MODENV-MODULETYPE
$$\frac{\Sigma; \Gamma, \overline{x : \star} \vdash \{\overline{D}\}}{\Sigma; \Gamma \vdash X = \overline{x} \mapsto \overline{D}, \Sigma}$$

MODENV-EMPTY
$$\frac{}{\Sigma; \Gamma \vdash \epsilon}$$

MODENV-MODULE
$$\frac{\Sigma; \Gamma, \overline{x : \star} \vdash \{; \overline{B}\}}{\Sigma; \Gamma \vdash X = \overline{x} \mapsto \overline{B}, \Sigma}$$

Figure 4: Formation rules for environments

EQPATH-EMPTY
$$\frac{}{\Sigma; \Gamma \vdash \epsilon \cong \epsilon}$$

EQPATH-ADD
$$\frac{\Sigma; \Gamma \vdash P_1 \cong P_2 \qquad \forall i.\Sigma, \Gamma \vdash t_i \cong t_i'}{\Sigma; \Gamma \vdash P_1.X \left[\overline{x_i = t_i}\right] \cong P_2.X \left[\overline{x_i = t_i'}\right]}$$

Figure 5: Rules for path equivalence

Figure 6: Typing rules for declarations

Bind-DefModule

$$\cfrac{\cfrac{}{\Sigma;\Gamma,\overline{P:*}\vdash \overline{B}:\overline{D}} \qquad \cfrac{}{}}{\Sigma;\Gamma, X:\left(\overline{P:\star}\right)\to \left\{\overline{S_i;\overline{D}}\right\}\vdash \overline{B_0}:\overline{D_0} \qquad \forall i.\Sigma(S_i)=\overline{x_i:\star}\to \overline{D_i}}$$

$$\cfrac{\Sigma;\Gamma,\overline{P:*}\vdash \overline{B}:\overline{D} \qquad \Sigma;\Gamma, X:\left(\overline{P:\star}\right)\to \left\{\overline{S_i;\overline{D}}\right\}\vdash \overline{B_0}:\overline{D_0} \qquad \forall i.\Sigma(S_i)=\overline{x_i:\star}\to \overline{D_i} \qquad \forall i\neq j.(\overline{x_i},\mathsf{dom}(D_i)\#\overline{x_j},\mathsf{dom}(D_j)) \qquad \left\{\overline{D}\right\}\preccurlyeq \left\{\overline{\overline{x_i:[=\_]D_i}}\right\}}{\Sigma;\Gamma\vdash \left(\mathsf{defmodule}\ X\ \mathsf{do}\ \overline{PS_iB}\ \mathsf{end}\right)\overline{B_0}:\left(X:\left(\overline{P:\star}\right)\to \left\{\overline{S_i;\overline{D}}\right\}\right)\overline{D_0}}$$

Bind-Type

$$\cfrac{\Sigma;\Gamma\vdash t:\star \qquad \Sigma;\Gamma, x:[=t]\vdash \overline{B}:\overline{D}}{\Sigma;\Gamma\vdash (\$\mathsf{type}\ x=t)\,\overline{B}:(x:[=t])\,\overline{D}}$$

Bind-Opaque

$$\cfrac{\Sigma;\Gamma\vdash t:\star \qquad \Sigma;\Gamma, x:[=t]\vdash \overline{B}:\overline{D}}{\Sigma;\Gamma\vdash (\$\mathsf{opaque}\ x=t)\,\overline{B}:(x:\star)\,\overline{D}}$$

Bind-Empty

$$\cfrac{}{\Sigma;\Gamma\vdash \epsilon:\epsilon}$$

Bind-Value

$$\cfrac{\Sigma;\Gamma\vdash v:\cap \overline{T} \qquad \Sigma;\Gamma, x:\cap \overline{T}\vdash \overline{B}:\overline{D}}{\Sigma;\Gamma\vdash (x=v)\,\overline{B}:\left(x:\cap \overline{T}\right)\overline{D}}$$

Figure 7: Typing rules for bindings

Path-Empty

$$\cfrac{}{\Sigma;\Gamma\vdash \epsilon:\{\epsilon;\Gamma\}}$$

Path-SubModule

$$\cfrac{}{\quad}$$

Figure 8: Well-formdness rules for paths

Type-Variable
$$\frac{\Sigma; \Gamma \vdash P : \left\{ \overline{S}; \overline{D}(x : \cap T)\overline{D'} \right\}}{\Sigma; \Gamma \vdash P.x : \cap T}$$

Type-Subsumption
$$\frac{\Sigma; \Gamma \vdash T \qquad \Sigma\Gamma \vdash E : \cap\overline{T'} \qquad \Sigma; \Gamma \vdash \cap\overline{T'} \preccurlyeq T}{\Sigma; \Gamma \vdash E : T}$$

Type-BigFunction
$$\frac{\Sigma; \Gamma \vdash \cap\overline{\overline{N : T} \to T'} \quad \Sigma; \Gamma, \overline{N : T} \vdash E : T'}{\Sigma; \Gamma \vdash \$ \cap \overline{\forall \overline{\alpha} \left(\overline{N : T}\right) \to T'}\, \mathsf{fn}\, \overline{N} \to E : \cap\overline{\overline{N : T} \to T'}}$$

Type-Module
$$\frac{\Sigma; \Gamma \vdash P : \left\{ \overline{S_0}; \overline{D_0}(X : \overline{y : \star} \to \left\{ \overline{S}; \overline{D} \right\})\overline{D_1} \right\} \qquad \Sigma; \Gamma \vdash \overline{t} \qquad \overline{x} \simeq \overline{y}}{\Sigma; \Gamma \vdash P.X \left[\overline{x = t}\right] : \left\{ \overline{S}; \overline{D} \right\}}$$

Figure 9: Typing rules for the surface language

Sub-StarRefl
$$\frac{}{\star \preccurlyeq \star}$$

Sub-Elixir
$$\frac{t \preccurlyeq t'}{\Sigma; \Gamma \vdash t \preccurlyeq t'}$$

Sub-Intersection
$$\frac{\exists i \in I, T_i \preccurlyeq T}{\Sigma; \Gamma \vdash \cap_I \overline{T_i} \preccurlyeq T}$$

Sub-ModuleLeft
$$\frac{\Sigma; \Gamma \vdash P : \left\{ \overline{S}; \overline{D}(x : [= t])\overline{D'} \right\} \qquad \Sigma; \Gamma \vdash t \preccurlyeq T}{\Sigma; \Gamma \vdash P.x \preccurlyeq T}$$

Sub-ModuleRight
$$\frac{\Sigma; \Gamma \vdash P : \left\{ \overline{S}; \overline{D}(x : [= t])\overline{D'} \right\} \qquad \Sigma; \Gamma \vdash \cap\overline{T} \preccurlyeq t}{\Sigma; \Gamma \vdash \cap\overline{T} \preccurlyeq P.x}$$

Sub-Opaque
$$\frac{\Sigma; \Gamma \vdash P \cong P' \qquad \Sigma; \Gamma \vdash P : \left\{ \overline{S}; \overline{D}(x : \star)\overline{D'} \right\}}{\Sigma; \Gamma \vdash P.x \preccurlyeq P'.x}$$

Sub-BigFunction
$$\frac{\forall i.\Sigma; \Gamma, x_1 : R_1, \ldots, X_{i-1} : R_{i-1} \vdash T_i \succcurlyeq R_i \qquad \Sigma; \Gamma, \overline{X_i : R_i} \vdash T' \preccurlyeq R'}{\Sigma; \Gamma \vdash (X_i : T_i) \to T' \preccurlyeq (X_i : R_i) \to R'}$$

Figure 10: Subtyping rules $\Sigma; \Gamma \vdash \cap\overline{T} \preccurlyeq T$

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$$\begin{array}{lcccl}
\text{\$module } X : T & \cup & \text{\$module } X : T' & = & \text{\$module } X : T \cup T' \\
\text{\$callback } X : \bigcap \overline{T} & \cup & \text{\$callback } X : \bigcap \overline{T'} & = & ? \\
\text{\$type } x = t & \cup & \text{\$type } x = t' & = & \text{\$type } x = t \cup t' \\
\text{\$opaque } x & \cup & \text{\$opaque } x & = & \text{\$opaque } x \\
\text{\$opaque } x & \cup & \text{\$type } x = t & = & \text{\$opaque } x \\
\text{\$type } x = t & \cup & \text{\$opaque } x & = & \text{\$opaque } x \\
D & \cup & D' & = & \epsilon
\end{array}$$

Figure 11: Component-wise union