

# Game Tree Searching by Min / Max Approximation In Review

Nathan Findley

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## Abstract

A new game[1] tree search technique called Min / Max Approximation was introduced by Ronald L. Rivest in 1986. An iterative approach, it is used to determine which leaf should be further expanded so that the root will have a value that approaches the true values of a fully expanded tree. When evaluated in comparison with minimax alpha-beta pruning using iterative deepening over the course of nearly 1,000 trials, Min / Max approximation was seen to be either a strong contender when limiting turns by move count or a slightly weaker contender when limiting turns by time.

## 1 Problem

When designing an algorithm to handle all possible branches of all possible plays within any given game, the immediate limiting factors are time and computation: a perfect view of the entire tree of gameplay for a particular game is not feasible.

## 2 Goal

Min / Max Approximation is attempting to solve this problem. Unlike minimax alpha-beta pruning with iterative deepening which moves down the tree with a uniform depth, Min / Max approximation follows a given branch in a tree because it appears to be beneficial. As a result, different levels of depth are reached in different branches of the partial tree. Essentially one branch is explored until the root node's value is considered better when exploring a different tip. At that point in time, search switches to the seemingly more valuable tip. Time permitting, the depth of the most valuable branch will reach a terminating leaf. As to which tip should be explored, Min / Max approximation is presented as a possible answer.

Penalties are introduced in order to determine which tip to pursue. "Bad" tips of the tree are weighted with a nonnegative penalty that distinguishes them from "good" tips. The overall penalty for a given tip is the sum of all penalties tracing up to the original root of the partial tree. This calculation of a viable tip is what distinguishes this algorithm from minimax alpha-beta: the tip that causes the root to have the lowest penalty is the one that is further explored.

As soon as a tip's weight drops its penalty score below any other tip in the partially expanded tree, that more favorable tip is then explored. According to the author, one of Min / Max approximations strengths is to encourage branches that have secondary lines of play that might also be winnable since the penalty calculation is determined cumulatively rather than just using a child with the highest value.

### 3 Results

Connect four was used to explore whether or not this is a viable algorithm in comparison with minimax alpha-beta pruning. After nearly 1000 trials, the author felt confident that this method could be further explored and may possibly be incorporated with existing techniques at the time. Overall Min / Max Approximation appears more effective than traditional minimax alpha-beta pruning with iterative deepening as long as the number of moves is the limiting resource when comparing the two methods. Unfortunately game rounds are typically constrained by time, preventing this method from immediately being the best option available.

### References

- [1] Leslie Lamport, *TEX: a document preparation system*, Addison Wesley, Massachusetts, 2nd edition, 1994.