Solutions for Active Portfolio Management

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Chapter 2

Problem 2.1. In December 1992, Sears had a predicted beta of 1.05 with respect to the S&P 500 index. If the S&P 500 Index subsequently underperformed Treasury bills by 5.0 percent, what would be the expected excess return to sears?

Solution. The excess return on the market (relative to the risk free asset Treasury bills) is -5%. Hence, the excess return to Sears is

$$\begin{aligned} r_{Sears} &= \beta_{Sears} r_M \\ &= 1.05 \times -5.0\% \\ &= -5.25\% \end{aligned}$$

Problem 2.2. If the long-term expected excess return to the S&P 500 Index is 7 percent per year, what is the expected excess return to Sears.

Solution. Using the same line of reasoning as above, we have

$$\begin{aligned} r_{Sears} &= \beta_{Sears} r_M \\ &= 1.05 \times 7.0\% \\ &= 7.35\% \end{aligned}$$

Problem 2.3. Assume that residual returns are uncorrelated across stocks. Stock A has a beta of 1.15 and a volatility of 35 percent. Stock B has a beta of 0.95 and a volatility of 33 percent. If the market volatility is 20 percent, what is the correlation of stock A with stock B? Which stock has higher residual volatility?

Solution. The variance of a portfolio P is given by eq. (2.4) as

$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \omega_P^2$$

where ω_P^2 is the residual variance and σ_M^2 is the market variance. The correlation of stock A with stock B is given by

$$\text{Corr}\{r_A, r_B\} = \frac{\text{Cov}\{r_A, r_B\}}{\text{Std}\{r_A\}\,\text{Std}\{r_B\}}$$

So we just need the covariance of stocks A and B. We can write

$$\text{Cov}\{r_A, r_B\} = \beta_A \beta_B \sigma_M^2 + \omega_{A,B}$$

where the cross terms have been omitted since the residual volatility is uncorrelated from the market volatility. We can also set the residual covariance ($\omega_{A,B}$) to zero since we are assuming that the residual returns are uncorrelated across stocks. Hence the correlation of stock A with stock B is

$$
\begin{aligned}
\operatorname{Corr}\{r_A, r_B\} &= \frac{\operatorname{Cov}\{r_A, r_B\}}{\operatorname{Std}\{r_A\}\operatorname{Std}\{r_B\}} \\
&= \frac{\beta_A \beta_B \sigma_M^2}{\sigma_A \sigma_B} \\
&= \frac{1.15 \times 0.95 \times (20\%)^2}{35\% \times 33\%} \\
&= 0.3784
\end{aligned}
$$

We can determine the residual volatility of stock P from

$$
\omega_P = \sqrt{\sigma_P^2 - \beta_P^2 \sigma_M^2}
$$

Hence,

$$
\begin{aligned}
\omega_A &= \sqrt{\sigma_A^2 - \beta_A^2 \sigma_M^2} \\
&= \sqrt{(35\%)^2 - 1.15^2 \times (20\%)^2} \\
&= 26.38\%
\end{aligned}
$$

$$
\begin{aligned}
\omega_B &= \sqrt{\sigma_B^2 - \beta_B^2 \sigma_M^2} \\
&= \sqrt{(33\%)^2 - 0.95^2 \times (20\%)^2} \\
&= 26.98\%
\end{aligned}
$$

so portfolio B has higher residual volatility.

Problem 2.4. What set of expected returns would lead us to invest 100 percent in GE stock?

Solution. According to the CAPM, investing in anything other than the market portfolio involves taking on excess risk. Hence, investing 100 percent in GE stock would expose us to unnecessary risk. In order to minimize risk, we should simply invest in the market portfolio. If we didn't care about risk, we would invest 100 percent in GE whenever the expected returns on the market are positive since GE has a historical beta of 1.3 (table 2.1), which is the highest beta of the MMI stocks in table 2.1.

Problem 2.5. According to the CAPM, what is the expected residual return of an active manager?

Solution. The CAPM states that the expected residual return on all stocks is zero.

Chapter 3

Problem 3.1. If GE has an annual risk of 27.4 percent, what is the volatility of monthly GE returns?

Solution. From eq (3.6) we have

$$\sigma_{annual} = \sqrt{12} \times \sigma_{monthly}$$

Hence,

$$\begin{aligned} \sigma^{GE}_{monthly} &= \frac{27.4\%}{\sqrt{12}} \\ &= 7.91\% \end{aligned}$$

Problem 3.2. Stock A has 25 percent risk stock B has 50 percent risk, an their returns are 50 percent correlated. What fully invested portfolio of A and B has minimum total risk? (*Hint* try solving graphically (e.g. in Excel), if you cannot determine the answer mathematically.)

Solution. The risk of the portfolio will be (see Eq. (3.1))

$$\sigma_P = \sqrt{(f_A\sigma_A)^2 + ((1-f_A)\sigma_B)^2 + 2f_A\sigma_A(1-f_A)\sigma_B\rho_{AB}}$$

where ρ_{AB} (=50%) is the correlation between A and B and f_A is the fraction of the portfolio invested in A. The fully invested constraint, $f_A + f_B = 1$ leads to the $1 - f_A$ term in front of σ_B. To minimize the total risk, we solve

$$\frac{\partial \sigma_P}{\partial f_A} = 0$$

for f_A. We have

$$\frac{\partial \sigma_P}{\partial f_A} = \frac{1}{2}\frac{2f_A\sigma_A^2 - 2(1-f_A)\sigma_B^2 + (2-4f_A)\sigma_A\sigma_B\rho_{AB}}{\sqrt{(f_A\sigma_A)^2 + ((1-f_A)\sigma_B)^2 + 2f_A\sigma_A(1-f_A)\sigma_B\rho_{AB}}} \tag{1}$$

Setting the numerator to zero, we have

$$\begin{aligned} 0 &= 2f_A\sigma_A^2 - 2(1-f_A)\sigma_B^2 + (2-4f_A)\sigma_A\sigma_B\rho_{AB} \\ &= 2f_A(\sigma_A^2 + \sigma_B^2) - 4f_A\sigma_A\sigma_B\rho_{AB} - 2\sigma_B^2 + 2\sigma_A\sigma_B\rho_{AB} \\ 2\sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB} &= f_A(2\sigma_A^2 + 2\sigma_B^2 - 4\sigma_A\sigma_B\rho_{AB}) \\ f_A &= \frac{2\sigma_B^2 - 2\sigma_A\sigma_B\rho_{AB}}{2\sigma_A^2 + 2\sigma_B^2 - 4\sigma_A\sigma_B\rho_{AB}} \end{aligned}$$

Plugging in the values, we have

$$\begin{aligned} f_A &= \frac{2(0.25) - 2(0.5)(0.25)(0.5)}{2(0.0625) + 2(0.25) - 4(0.5)(0.25)(0.5)} \\ &= \frac{0.375}{0.375} \qquad = 1 \end{aligned}$$

Hence, the portfolio with minimum risk will hold 100% stock A.

Problem 3.3. What is the risk of an equal-weighted portfolio consisting of five stocks, each with 35 percent volatility and a 50 percent correlation with all other stocks? How does that increase as the portfolio increases to 20 stocks or 100 stocks?

Solution. From eq. (3.4) we have

$$\sigma_P = \sigma\sqrt{\frac{1+\rho(N-1)}{N}}$$

Hence, for 5, 20, 100 and an infinite number of stocks, we have

$$\begin{aligned}
\sigma_P^{N=5} &= 27.1\% \\
\sigma_P^{N=20} &= 25.4\% \\
\sigma_P^{N=100} &= 24.9\% \\
\sigma_P^{N=100} &= 24.7\%
\end{aligned}$$

Problem 3.4. How do structural risk models help in estimating asset betas? How do these betas differ from those estimated from a 60-month beta regression?

Solution. The beta of asset M is defined relative to the benchmark as

$$\beta_M = \frac{\sigma_{M,B}}{\sigma_B^2}$$

Structural risk models allow us to predict the risk of and correlations between stocks from which it is straightforward to calculate asset betas. In this chapter, the authors highlight the pros of using structural risk models and the cons of using regressions from historical data. The pros of structural risk models are

- The size of the problem can be greatly reduced. Instead of dealing with individual stocks and correlations between them, we deal only with factors and correlations between the factors. The stocks can then be projected onto the lower dimensional space of the factors.
- The use of factors allows the actual stocks to change. We only need the exposures of the stocks to the factors

The cons of historical regressions are:

- Dividends, splits, and mergers are hard to account for
- There is selection bias as failed companies are omitted
- In general, the number of observations must be greater than the number of stocks. Hence, for a 60 month beta regression, the observations would have to be daily or weekly, while the forecast would likely be quarterly or yearly.
- These models will take much longer to analyze since there are many more stocks than there are risk factors for the factor models

Chapter 4

Problem 4.1. Assume a risk-free rate of 6 percent, a benchmark expected excess return of 6.5 percent, and a long range benchmark expected excess return of 6 percent. Given that McDonald's has a beta of 1.07 and an expected total return of 15 percent, separate its expected return into (a) time premium (b) risk premium (c) exceptional benchmark return (d) alpha (e) consensus expected return (f) expected excess return (g) exceptional expected return. (h) What is the sum of the consensus expected return and the exceptional expected return?

Solution. From eq (4.7) we have the total expected return for stock n broken into components as

$$E\{R_n\} = 1 + i_F + \beta_n \mu_B + \beta_n \Delta f_B + \alpha_n$$

For McDonald's stock:

(a) The time premium is just the risk free rate, $i_F = 6\%$.

(b) The risk premium is $\beta_{McDonald's}\mu_B = 1.07 \cdot 6\% = 6.42\%$ where μ_B is the long range expected excess return of the benchmark.

(c) The exceptional benchmark return is $\beta_{McDonald's}\Delta f_B = 1.07 \cdot (6.5 - 6)\% = 0.535\%$ where Δf_B is the difference between the (immediate) expected excess return of the benchmark and the long run expected excess return of the benchmark

(d) Alpha can be found by solving the above equation and plugging in all of the values. We have

$$\begin{aligned}
\alpha_{McDonald's} &= E\{R_{McDonald's}\} - 1 - i_F - \beta_n \mu_B - \beta_n \Delta f_B \\
&= 1.15 - 1 - 0.06 - 0.0642 - 0.00535 \\
&= 0.02045
\end{aligned}$$

(e) The consensus expected return is just $\beta_{McDonald's \cdot \mu_B} = 6.42\%$

(f) The expected excess return is $f_{McDonald's} = E\{R_{McDonald's}\} - 1 - i_F = 1.15 - 1 - 0.06 = 0.09$ or 9 percent

(g) The exceptional expected return is $f_{McDonald's} - \beta_{McDonald's}\mu_B = 0.09 - 0.0642 = 0.0258$ or 2.58 percent.

(h) The sum of the consensus expected return and the exceptional return is $6.42\% + 2.58\% = 9\%$ which is the expected excess return.

Problem 4.2. Suppose the benchmark is not the market, and the CAPM holds. How will the CAPM expected returns split into the categories suggested in this chapter?

Solution. The CAPM expected returns of stock n are equal to $\beta_n^M \mu_M$ where μ_M is the expected excess return of the market. Using eq (4.7), we can set

$$E\{R_n\} = 1 + i_F + \beta_n^M \mu_M = 1 + i_F + \beta_n^B \mu_B + \beta_n^B \Delta f_B + \alpha_n$$

The time premium will still be equal to i_F. The risk premium will be $\beta_n^B \mu_B = \beta_n^M \mu_M - \beta_n^B \Delta f_B - \alpha_n$. The exceptional benchmark return will be $\beta_n^B \Delta f_B = \beta_n^M \mu_M - \beta_n^B \mu_B - \alpha_n$. Alpha will be $\alpha_n = \beta_n^M \mu_M - \beta_n^B \mu_B - \beta_n^B \Delta f_B$. The consensus expected return will be $\beta_n^B \mu_B = \beta_n^M \mu_M - \beta_n^B \Delta f_B - \alpha_n$. The expected excess return will just be $\beta_n^M \mu_M = \beta_n^B \mu_B + \beta_n^B \Delta f_B + \alpha_n$. The exceptional expected return will be $\beta_n^M \mu_M - \beta_n^B \mu_B = \beta_n^B \Delta f_B + \alpha_n$. Here the superscripts indicate the market (M) and benchmark (B). For example β_n^B is the beta of stock n with respect to the benchmark while β_n^M is the beta with respect to the market.

Problem 4.3. Given a benchmark risk of 20 percent and a portfolio risk of 21 percent, and assuming a portfolio beta of 1, what is the portfolio's residual risk? What is its active risk? How does this compare to the difference between the portfolio risk and the benchmark risk?

Solution. From Eq. (3.13), the variance of a stock n is given by

$$\sigma_n^2 = \beta_n^2 \sigma_B^2 + \omega_n^2$$

so the variance of the portfolio $\boldsymbol{h_P}$ is given by

$$\begin{aligned} \boldsymbol{h_P^T} \cdot \boldsymbol{\sigma^2} &= \boldsymbol{h_P^T} \cdot \boldsymbol{\beta^2} \sigma_B^2 + \boldsymbol{h_P^T} \cdot \boldsymbol{\omega^2} \\ \sigma_P^2 &= \beta_P^2 \sigma_B^2 + \omega_P^2 \end{aligned}$$

where $\boldsymbol{\beta^2}$ is the vector of stock betas squared and $\boldsymbol{\omega^2}$ is the vector of stock residual returns squared. To find the portfolio's residual risk, we can solve for ω_P as.

$$\begin{aligned} \omega_P &= \sqrt{\sigma_P^2 - \beta_P^2 \sigma_B^2} \\ &= \sqrt{(21\%)^2 - 1^2 \times (20\%)^2} \\ &= 6.40\% \end{aligned}$$

The active risk is given by eq (4.20) as

$$\psi_P = \sqrt{\omega_P^2 + \beta_{PA}^2 \cdot \sigma_B^2}$$

The active beta is given by $\beta_{PA} = \beta_P - \beta_B = 1 - 1 = 0$. Hence, the active risk is equal to the residual risk at 6.40%. The active risk is 6.4 percent compared to the difference of risk between the portfolio and the benchmark of 1 percent. The active risk is much larger than the simple difference in portfolio and benchmark risks.

Problem 4.4. Investor A manages total return and risk ($f_P - \lambda_T \cdot \sigma_P^2$) with risk aversion $\lambda_T = 0.0075$. Investor B manages residual risk and return ($\alpha_P - \lambda_R \cdot \omega_P^2$), with risk aversion $\lambda_R = 0.075$ (moderate to aggressive). They each can choose between two portfolios:

$$\begin{aligned} f_1 &= 10\% \\ \sigma_1 &= 20.22\% \\ f_2 &= 16\% \\ \sigma_2 &= 25\% \end{aligned}$$

Both portfolios have $\beta = 1$. Furthermore,

$$\begin{aligned} f_B &= 6\% \\ \sigma_B &= 20\% \end{aligned}$$

Which portfolio will A prefer? Which portfolio will B prefer? (*Hint:* First calculate expected residual return and residual risk for the two portfolios.)

Solution. The residual returns are

$$\begin{aligned} \alpha_1 &= f_1 - \beta_1 f_B \\ &= 10\% - 1 \times 6\% \\ &= 4\% \\ \alpha_2 &= f_2 - \beta_2 f_B \\ &= 16\% - 1 \times 6\% \\ &= 10\% \end{aligned}$$

The residual risks are

$$\begin{aligned}
\omega_1 &= \sqrt{\sigma_1^2 - \beta_1 \sigma_B^2} \\
&= \sqrt{20.22\%^2 - 1 \times 20\%^2} \\
&= 2.975\% \\
\omega_2 &= \sqrt{\sigma_2^2 - \beta_2 \sigma_B^2} \\
&= \sqrt{25\%^2 - 1 \times 20\%^2} \\
&= 15\%
\end{aligned}$$

Investor A will prefer the portfolio with maximum $f_P - \lambda_T \cdot \sigma_P^2$ (highest utility). We have

$$\begin{aligned}
f_1 - \lambda_T \cdot \sigma_1^2 &= 10\% - 0.0075 \times 20.22^2\% \\
&= 10\% - 3.07\% \\
&= 6.93\% \\
f_2 - \lambda_T \cdot \sigma_2^2 &= 16\% - 0.0075 \times 25^2\% \\
&= 16\% - 4.69\% \\
&= 11.31\%
\end{aligned}$$

so that investor A will prefer portfolio 2. On the other hand, investor B while refer the portfolio with maximum $\alpha_P - \lambda_R \cdot \omega_P^2$. We have

$$\begin{aligned}
\alpha 1 - \lambda_R \cdot \omega_1^2 &= 4\% - 0.075 \times 2.975^2\% \\
&= 4\% - 0.65\% \\
&= 3.35\% \\
\alpha 2 - \lambda_R \cdot \omega_2^2 &= 10\% - 0.075 \times 15^2\% \\
&= 10\% - 16.88\% \\
&= -6.88\%
\end{aligned}$$

so that investor B will prefer portfolio 1.

Problem 4.5. Assume that you are a mean/variance investor with total risk aversion of 0.0075. If a portfolio has an expected excess return of 6 percent and risk of 20 percent, what is your *certainty equivalent return*, the certain expected excess return that you would fairly trade for this portfolio.

Solution. The certainty equivalent return would be equal to the utility (see p 121) $f_P - \lambda_T \cdot \sigma_P^2$. We have

$$\begin{aligned}
f_P - \lambda_T \cdot \sigma_P^2 &= 6\% - .0075 \cdot 20^2\% \\
&= 6\% - 3\% \\
&= 3\%
\end{aligned}$$

Chapter 5

Problem 5.1. What is the information ratio of a passive manager?

Solution. Passive managers will just invest in the benchmark and so their residual returns and risk will be zero. The information ratio will therefore be zero by definition.

Problem 5.2. What is the information ratio required to add a risk-adjusted return of 2.5 precent with a moderate risk aversion level of 0.10? What level of active risk would that require?

Solution. We want to find the IR consistent with a rigk-adjusted value added of 2.5% and a risk aversion of 0.10. From eq. (5.12) we have

$$\mathrm{VA}^* = \frac{\mathrm{IR}^2}{4\lambda_R}$$

Which implies

$$\begin{aligned}\mathrm{IR} &= 2\sqrt{\lambda_R \mathrm{VA}^2} \\ &= 2\sqrt{0.1 \times 2.5\%} \\ &= 1\end{aligned}$$

From eq. (5.10) we can then calculate the active risk as

$$\begin{aligned}\omega^* &= \frac{\mathrm{IR}}{2\lambda_R} \\ &= \frac{1}{2 * .1} \\ &= 5\%\end{aligned}$$

Problem 5.3. Starting with the universe of MMI stocks, we make the assumptions

$Q =$ MMI portfolio

$f_q = 6\%$

$B =$ capitalization-weighted MMI portfolio

We calculate (as of January 1995) that

Portfolio	**β with Respect to B**	**β with Respect to Q**	**σ**
B	1.000	0.965	15.50%
Q	1.004	1.000	15.82%
C	0.865	0.831	14.42%

where portfolio C is the minimum-variance (fully invested) portfolio. For each portfolio (Q, B, and C), calculate f, α, ω, SR, and IR.

Solution. For portfolio B we have:

$$\begin{aligned}\alpha_B &= \omega_B = 0 \quad \text{(since B is the benchmark)} \\ f_B &= \beta_{B,Q} f_Q + \alpha_B \\ &= 0.965 \times 6\% - 0 \\ &= 5.79\% \\ \mathrm{SR} &= f_B/\sigma_B \\ &= 5.79\%/15.50\% \\ &= 0.374 \\ \mathrm{IR} &= \alpha_B/\omega_B \\ &= 0 \quad \text{(by definition)}\end{aligned}$$

For portfolio Q we have:

$$\begin{aligned}
f_Q &= 6\% \quad \text{(by definition)} \\
\alpha_Q &= f_Q - \beta_{Q,B} f_B \\
&= 6\% - 1.004 \times 5.79\% \\
&= 0.187\% \\
\omega_Q &= \sqrt{\sigma_Q^2 - \beta_{Q,B}^2 \sigma_B^2} \\
&= \sqrt{(15.82\%)^2 - 1.004^2 \times (15.5\%)^2} \\
&= 2.85\% \\
\text{SR} &= f_Q / \sigma_Q \\
&= 6\% / 15.82\% \\
&= 0.379 \\
\text{IR} &= \alpha_Q / \omega_Q \\
&= 0.187\% / 2.85\% \\
&= 0.066
\end{aligned}$$

For portfolio C we have:

$$\begin{aligned}
f_C &= \beta_{C,B} f_B / \beta_{Q,B} \quad \text{(see Eq 2A.38)} \\
&= 0.865 \times 5.79\% / 1.004 \\
&= 4.99\% \\
\alpha_C &= f_C - \beta_{C,B} f_B \\
&= 4.99\% - 0.865 \times 5.79\% \\
&= -0.018\% \\
\omega_C &= \sqrt{\sigma_C^2 - \beta_{C,B}^2 \sigma_B^2} \\
&= \sqrt{(14.42\%)^2 - 0.865^2 \times (15.50\%)^2} \\
&= 5.31\% \\
\text{SR} &= f_C / \sigma_C \\
&= 4.99\% / 14.42\% \\
&= 0.346 \\
\text{IR} &= \alpha_C / \omega_C \\
&= -0.018\% / 5.31\% \\
&= -0.0034\%
\end{aligned}$$

Problem 5.4. You have a residual risk aversion of $\lambda_R = 0.12$ and an information ratio of $\text{IR} = 0.60$. What is your optimal level of residual risk? What is your optimal value added?

Solution. From eq. (5.10), the optimal residual risk is:

$$\begin{aligned}
\omega^* &= \frac{\text{IR}}{2\lambda_R} \\
&= \frac{0.60}{2 \times 0.12} \\
&= 2.5\%
\end{aligned}$$

The optimal value added is (see eq. (5.12)):

$$
\begin{aligned}
\mathrm{VA}^* &= \frac{\mathrm{IR}^2}{4\lambda_R} \\
&= \frac{0.60^2}{4 \times 0.12} \\
&= 0.75\%
\end{aligned}
$$

Problem 5.5. Oops. In fact, your information ratio is really only IR = 0.30. How much value added have you lost by setting your residual risk level according to Problem 4 instead of at its correct optimal level?

Solution. The optimal residual risk for IR = 0.30 is

$$
\begin{aligned}
\omega^* &= \frac{\mathrm{IR}}{2\lambda_R} \\
&= \frac{0.30}{2 \times 0.12} \\
&= 1.25\%
\end{aligned}
$$

The value added using the optimal residual risk is

$$
\begin{aligned}
\mathrm{VA}^* &= \frac{\mathrm{IR}^2}{4\lambda_R} \\
&= \frac{0.30^2}{4 \times 0.12} \\
&= 0.1875\%
\end{aligned}
$$

The value added using the residual risk from problem 4 is (see eq. (5.9)

$$
\begin{aligned}
\mathrm{VA}[\omega] &= \omega \cdot \mathrm{IR} - \lambda_R \cdot \omega^2 \\
&= 2.5\% \times 0.30 - 0.12 \times (2.5\%)^2 \\
&= 0
\end{aligned}
$$

Hence, by using the non-optimal residual risk from problem 4, we loose 0.1875% value added.

Problem 5.6. You are an active manager with an information ratio of IR = 0.50 (top quartile) an a target level of residual risk of 4 percent. What residual risk aversion should lead to that level of risk?

Solution. From eq. (5.11) we have

$$
\begin{aligned}
\lambda_R &= \frac{\mathrm{IR}}{2\omega^*} \\
&= \frac{0.50}{2 \times 4\%} \\
&= 0.0625/\%
\end{aligned}
$$

Chapter 6

Problem 6.1. Manager A is a stock picker. He follows 250 companies, making new forecasts each quarter. His forecasts are 2 percent correlated with subsequent residual returns. Manager B engages in tacit asset allocation, timing four equity styles (value, growth, large, small) every quarter. (a) What must Manager B's skill level be to match Manager A's information ratio? (b) What information ratio could a sponsor achieve by employing both managers, assuming that Manager B has a skill level of 8 percent?

Solution.

(a) Manager A has an information ratio of

$$\begin{aligned} \mathrm{IR} &= \mathrm{IC}\sqrt{\mathrm{BR}} \\ &= 0.02 \times \sqrt{1000} \\ &= 0.632 \end{aligned}$$

For manager B to have an information ratio of 0.632, his information coefficient would need to be

$$\begin{aligned} \mathrm{IC} &= \mathrm{IR}/\sqrt{\mathrm{BR}} \\ &= 0.632/\sqrt{16} \\ &= 0.158 \end{aligned}$$

So manager B's forecasts would need to be 16% correlated with the subsequent residual returns.

(b) A sponsor could achieve an information ratio of

$$\begin{aligned} \mathrm{IR} &= \sqrt{\mathrm{IR_A}^2 + \mathrm{IR_B}2} \\ &= \sqrt{0.632^2 + (0.08 * \sqrt{16})^2} \\ &= 0.71 \end{aligned}$$

if manager A and B's forecasts are independent and if manager B has an information coefficient of 0.08 (a skill of 8%), giving him an IC of 0.32.

Problem 6.2. A stock picker follows 500 stocks and updates his alphas every month. He has an IC = 0.05 and an IR = 1.0. (a) How many bets does he make per year? (b) How many independent bets does he make per year? (c) What does this tell you about his alphas?

Solution.

(a) The stock picker makes $500 \times 12 = 6000$ bets per year.

(b) The stock pickers breadth is

$$\begin{aligned} \mathrm{BR} &= \mathrm{IR}^2/\mathrm{IC}^2 \\ &= (1/.05)^2 \\ &= 400 \end{aligned}$$

so he makes 400 independent bets per year.

(c) Since the number of bets he makes per year is not equal to the number of independent bets he makes per year, his alphas are not independent.

Problem 6.3. In the example involving residual returns θ_n composed of 300 elements $\theta_{n,j}$, an investment manager must choose between three research programs:

(a) Follow 200 stocks each quarter and accurately forecast $\theta_{n,12}$ and $\theta_{n,15}$

(b) Follow 200 stocks each quarter and accurately forecast $\theta_{n,5}$ and $\theta_{n,105}$

(c) Follow 100 stocks each quarter and accurately forecast $\theta_{n,5}$, $\theta_{n,12}$, and $\theta_{n,105}$

Compare the three programs, all assumed to be equally costly. Which would be most effective (highest value added)?

Solution. For (a) and (b) there are 800 pieces of information each year, while for (c) there are only 400 pieces of information each year. Furthermore, the residual return of stock n is given by $\theta_n = \sum_{j=1}^{300} \theta_{n,j}$.

(a) Here, $\theta_{n,12}$ and $\theta_{n,15}$ are perfectly correlated with θ_n while all others are uncorrelated. Hence, we have $\mathrm{STD}\{\theta_n\} = 17.32$ (see p 152) and $\mathrm{STD}\{\theta_{n,12} + \theta_{n,15}\} = \sqrt{(0-1)^2 + (0-1)^2} = \sqrt{2}$ since the mean of each $\theta_{n,j}$ is zero and the standard deviation is 1. Furthermore, the covariance between our predictions and the actual return will be 2 since $\theta_{n,12}$ and $\theta_{n,15}$ are forecast perfectly. The IC is then given by the correlation between the forecasts and the residual return as

$$\begin{aligned}
\mathrm{IC} &= \frac{\mathrm{Cov}\{\theta_n, \theta_{n,12} + \theta_{n,15}\}}{\mathrm{STD}\{\theta_n\} \times \mathrm{STD}\{\theta_{n,12} + \theta_{n,15}\}} \\
&= 2/(17.32 \times \sqrt{2}) \\
&= 0.0817
\end{aligned}$$

The information ratio is then given by

$$\begin{aligned}
\mathrm{IR} &= \mathrm{IC}\sqrt{\mathrm{BR}} \\
&= 0.0817 \times \sqrt{800} \\
&= 2.31
\end{aligned}$$

(b) This research program will have the same IR as (a). The only difference are the elements that are forecast accurately, but the number of correct forecasts does not change

(c) Using the same reasoning as in (a), we find that

$$\begin{aligned}
\mathrm{IC} &= \frac{\mathrm{Cov}\{\theta_n, \theta_{n,5} + \theta_{n,12} + \theta_{n,105}\}}{\mathrm{STD}\{\theta_n\} \times \mathrm{STD}\{\theta_{n,5} + \theta_{n,12} + \theta_{n,105}\}} \\
&= \frac{3}{\sqrt{300}\sqrt{3}} \\
&= 0.1
\end{aligned}$$

so

$$\begin{aligned}
\mathrm{IR} &= 0.1 \times \sqrt{400} \\
&= 2
\end{aligned}$$

Hence, even though the skill of research program (c) would be better, there aren't enough bets made for the IR to be better than research programs (a) or (b). (a) and (b) will be the most effective strategies and should have the highest value added since $\mathrm{VA}^2 \propto \mathrm{IR}^2$.

Chapter7

Problem 7.1. According to the APT, what are the expected values of the u_n in Eq. (7.1)? What is the corresponding relationship for the CAPM?

Solution. According to the APT, the expected excess return is

$$\begin{aligned} f_n &= E\{r_n\} \\ &= E\left\{\sum_{k=1}^{K} X_{n,k} \cdot b_k + u_n\right\} \\ &= \sum_{k=1}^{K} X_{n,k} \cdot m_k \end{aligned}$$

where b_k is the factor return for factor k and m_k is the factor forecast for factor k. Hence it seems that the expected value of u_n is zero. This is in line with the CAPM which is a one factor APT model where the factor is the stock's beta:

$$\begin{aligned} f_n &= E\{r_n\} \\ &= E\left\{\beta_n r_M + \theta_n\right\} \\ &= \beta_n f_m \end{aligned}$$

where the expected residual return θ_n is zero.

Problem 7.2. Work by Fama and French, and others, over the past decade has identified size and book-to-price ratios as two critical factors determining expected returns. How would you build an APT model based on those two factors? Would the model require additional factors?

Solution. I would use one of the structural models presented in this chapter, and it seems like Structural Model 3 would be the most appropriate. The process might look something like:

1. Take a broad universe of stocks. For each year of historical returns, calculate the size and book to price ratio of each stock. The size will likely need to be standardized (since it is extensive), but I think the book to price ratio will be fine as is, since it is a ratio (it is intensive). This will determine the factor exposures
2. Regress the yearly returns against the size and book to price ratio of the stocks from step 1 and look for statistically significant correlations. This will give estimates for the factor returns.
3. Estimate (or calculate) the factor exposures for each stock for the current year we are trying to forecast. From these factor exposures and the historical returns, we can forecast the expected returns for the upcoming year

The model should not *require* any additional factors, but they might be useful for building better forecasts.

Problem 7.3. In the example shown in Table 7.2, most of the CAPM forecasts exceed the APT forecasts. Why? Are APT forecasts required to match CAPM forecasts of average?

Solution. The CAPM forecasts exceed the APT forecast because of the factor forecasts. It could be the other way around depending on the factor forecasts. The forecasts can be anything (there don't seem to be any hard and fast constraints) and so they are not required to match the CAPM forecasts on average.

Problem 7.4. In an earnings-to-price tilt fund, the portfolio holdings consist (approximately) of the benchmark plus a multiple c times the earnings-to-price factor portfolio (which has unit exposure to earnings-to-price and zero exposure all other factors). Thus, the tilt fund manager has an active exposure c to earnings-to-price. If the manager uses a constant multiple c over time, what does that imply about the manager's factor forecasts for earnings-to-price?

Solution. If a manager uses a constant c over time, that implies that his forecasts for earnings-to-price are not changing. However, the exposures to earnings to price will be changing leading to changes in the stock forecasts.

Problem 7.5. You have built an APT model based on industry, growth, bond beta, size, and return on equity (ROE). This month your factor forecasts are

Heavy electrical industry	6.0%
Growth	2.0%
Bond beta	-1.0%
Size	-0.5%
ROE	1.0%

These forecasts lead to a benchmark expected excess return of 6.0 percent. Given the following data for GE,

Industry	Heavy electrical
Growth	-0.24
Bond beta	0.13
Size	1.56
ROE	0.15
Beta	1.10

what is its alpha according to your model

Solution. We can calculate the expected excess return as

$$f_{GE} = \sum_k X_k b_k$$

where the factor returns b_k are given in the first table and the factor exposures, X_k are given in the second table. Hence we have

$$\begin{aligned} f_{GE} &= 1 \times 6\% + (-0.24) \times 2.0\% + 0.13 \times (-1.0\%) + 1.56 \times (-0.5\%) + 0.15 \times (1.0\%) \\ &= 4.76\% \end{aligned}$$

Hence, alpha is given by

$$\begin{aligned} \alpha_{GE} &= f_{GE} - \beta_{GE} \times f_M \\ &= 4.76\% - 1.1 \times 6\% \\ &= -1.84\% \end{aligned}$$

Chapter8

Problem 8.1. In the simple stock example described in the text, value a European call option on the stock with a strike price of 50, maturing at the end of the 1-month period. The option cash flows at the end of the period are Max$\{0,p(t,s)-50\}$, where $p(t,s)$ is the stock price at time t in state s.

Solution. In the stock example in the text, a stock is currently valued at 50 and in 1 month will be worth either 49 ($p_{down} = 49$) or 53 ($p_{up} = 53$) with equal probability($\pi_{up} = \pi_{down} = 0.5$). The risk free interest rate, i_F over the year is 6 percent so that the return after 1 month is given by $R_F = (1+i_F)^{1/12}) = 1.00487$. Furthermore, the valuation multiples are $\nu_{up} = 0.62$ and $\nu_{down} = 1.38$. We can use equations 8.8 and 8.9 to value the stock as

$$\begin{aligned} p_0 &= \frac{\pi_{up}\nu_{up}p_{up} + \pi_{down}\nu_{down}p_{down}}{R_F} \\ &= \frac{0.5 \times 0.62 \times 53 + 0.5 \times 1.38 \times 49}{1.00487} \\ &= 50 \end{aligned}$$

The value of the option can be calculated similarly by replacing the stock price at the end of the period with the value of the option at the end of the period. The value of the option is either 0 or 3, if the stock went down or up respectively. Hence, the current value of the option is

$$\begin{aligned} p_0 &= \frac{\pi_{up}\nu_{up}c_{up} + \pi_{down}\nu_{down}c_{down}}{R_F} \\ &= \frac{0.5 \times 0.62 \times 3 + 0.5 \times 1.38 \times 0}{1.00487} \\ &= 0.93 \end{aligned}$$

Problem 8.2. Compare Eq. (8.16) to the CAPM result for expected returns, to relate ν to r_Q. Impose the requirement that $E\{\nu\} = 1$ to determine ν exactly as a function of r_Q.

Solution. Equation 8.16 says that the expected return is given by

$$\mathrm{E}\{R\} = 1 + i_F - \mathrm{Cov}\{\nu, R\}$$

By comparing to the expected return according to the CAPM

$$\mathrm{E}\{R\} = 1 + i_F + \beta f_Q$$

we find that

$$\begin{aligned} \mathrm{Cov}\{\nu, R\} &= -\beta f_Q \\ &= -\frac{\mathrm{Cov}\{r_Q, R\}}{\sigma_Q^2} f_Q \end{aligned}$$

Using the definition of covariance,

$$\begin{aligned} \mathrm{E}\{\nu \cdot R\} - \mathrm{E}\{\nu\}\mathrm{E}\{R\} &= -\frac{\mathrm{E}\{r_Q \cdot R\} - \mathrm{E}\{r_Q\}\mathrm{E}\{R\}}{\sigma_Q^2} f_Q \\ \mathrm{E}\{\nu \cdot R\} &= \mathrm{E}\{\nu\}\mathrm{E}\{R\} - \frac{\mathrm{E}\{r_Q \cdot R\} - f_Q\mathrm{E}\{R\}}{\sigma_Q^2} f_Q \\ \mathrm{E}\{\nu \cdot R\} &= \mathrm{E}\left\{R\left[\mathrm{E}\{\nu\} + \frac{f_Q}{\sigma_Q^2}(f_Q - r_Q)\right]\right\} \end{aligned}$$

Which implies

$$\nu = 1 + \frac{f_Q}{\sigma_Q^2}(f_Q - r_Q)$$

after imposing the condition that $\mathrm{E}\{\nu\} = 1$.

Problem 8.3. Using the simple stock example in the text, (a) price an instrument which pays \$1 in state 1 [$\mathrm{cf}(t,1) = 1$] and -\$1 in state 2 [$\mathrm{cf}(t,2) = -1$]. (b) What is the expected return to this asset? (c) What is its beta with respect to the stock? (d) How does this relate to the breakdown of Eq. (8.7)?

Solution.

(a) State 1 corresponds to when the stock is down and state two corresponds to when the stock is up. Using the same procedure and valuation multiples as in problem 1, the price is

$$\begin{aligned} p_0 &= \frac{0.5}{1.00487}(\times 1.38 \times 1 - 0.62 \times 1) \\ &= 0.378 \end{aligned}$$

(b) The expected return is

$$\begin{aligned} \mathrm{E}\{\mathrm{R}\} &= 0.5 \times 1 + 0.5 \times -1 \\ &= 0 \end{aligned}$$

(c) The beta of the asset (A) with respect to the stock (S) is

$$\begin{aligned} \beta &= \frac{\mathrm{Cov}\{A, S\}}{\sigma_S^2} \\ &= \frac{(49-51) \times (1-0)/2 + (53-51) \times (-1-0)/2}{(49-51)^2/2 + (53-51)^2/2} \\ &= \frac{-2}{4} \\ &= -1/2 \end{aligned}$$

(d) According to equation 8.7

$$\begin{aligned} p_0 &= \frac{\mathrm{E}\{\mathrm{cf}\}}{1 + i_F + \beta f_S} \\ &= 0 \end{aligned}$$

Because the expected value of the asset is zero, the price will always be zero, an equation (8.7) will therefore not be able to properly value the stock, regardless of the value of the discount rate in the denominator.

Problem 8.4. You believe that stock X is 25 percent undervalued, and that it will take 3.1 years for half of this misvaluation to disappear. What is your forecast for the alpha of stock X over the next year?

Solution. We want to use equation (8.22) but first we have to define κ and γ. κ is given as 0.25 and γ can be found from $\tau = -0.69/\ln\{\gamma\}$ where $\tau = 3.1$ years is the misvaluation half life. Hence, $\gamma = \exp(-0.69/3.1) = 0.80$. Plugging these values into equation 8.22, we find

$$\begin{aligned} \alpha &= (1 + i_F) \cdot \left[\frac{\kappa \cdot (1-\gamma)}{1 + \kappa \cdot \gamma}\right] \\ &= (1.06) \cdot \left[\frac{0.25 \times (1-0.8)}{1 + 0.25 \times 0.8}\right] \\ &= 0.044 \end{aligned}$$

Chapter9

Problem 9.1. According to Modigliani and Miller (and ignoring tax effects), how would the value of a firm change if (a) it borrowed money to repurchase outstanding common stock, greatly increasing its leverage? (b) What if it changed its payout ratio?

Solution. Modigliani and Miller demonstrated that (1) dividend policy only influences the scheduling of cash flows received by the share holder and that it does not affect the total value of the payments and (2) a firms financing policy does not affect the total value of the firm. Hence, the value of the firm will not be affected in either case (a) or (b). The value of a firm comes from its profitable activities and not dividend and financing policies.

Problem 9.2. Discuss the problem of growth forecasts in the context of the constant-growth dividend discount model [Eq. (9.5)]. How would you reconcile the growth forecasts with the implied growth forecasts for AT&T in Tables 9.1 and 9.2?

Solution. The authors mention that the raw growth forecasts can be unrealistic due to bias. Since the dividend discount model depends sensitively on the growth forecasts, it is important to have good forecasts. The implied growth rates, which assume the asset is fairly priced, can help adjust the growth forecasts to more realistic value. Fore instance, the raw growth forecast for AT&T in table 9.2 is -19.21% and the implied growth rate, given in table 9.1, is 6.26%. Using equation (9.22) results in a more modest AT&T growth forecast of 2.21%. Hence, the implied growth rates can be used to correct unrealistic growth forecasts.

Problem 9.3. Stock X has a beta of 1.20 and pays no dividend. If the risk-free rate is 6 percent and the expected excess market return is 6 percent, what is stock X's implied growth rate?

Solution. Using equation (9.20), the implied growth rate is given by

$$\begin{aligned} g_X^* &= (i_F + \beta_X \cdot f_B) - \frac{d_X}{p_X} \\ &= 6\% + 1.2 \times 6\% - 0 \\ &= 13.2\% \end{aligned}$$

Problem 9.4. You are a manager who believes that book-to-price (B/P), earnings to price (E/P), and beta are the three variables that determine stock value. Given monthly B/P, E/P, and beta values for 500 stocks, how could you implement your strategy (a) using comparative valuation? (b) using returns-based analysis?

Solution.

(a) To use comparative valuation, we would regress the companies current price against the three variables to come up with a price equation in the form of (9.43). The error associated with our price function and the actual price would identify misvaluation. It would be wise to check for outliers to make sure that they are not dominating the regression coefficients and skewing the model.

(b) Given monthly attributes (or exposures) for each stock, we can regress an equation in the form of (9.46) to determine the factor returns, $b_k(t)$, during each time period (or for each month). We can then use these factor returns to model future returns given current exposures of each stock to the factors. It might also be useful to look at how the error, or idiosyncratic, terms vary with time. If they are constant in time, this would identify that our model can be improved by choosing appropriate factors.

Problem 9.5. A stock trading with a P/E ratio of 15 has a payout ratio of 0.5 and an expected return of 12 percent. What is its growth rate, according to the constant-growth DDM?

Solution. From equation (9.7), the growth rate is given by

$$g = i_F + f - \frac{d}{p}$$

The dividends are given by

$$d = \kappa \cdot e$$

where κ is the payout ratio and $e(t)$ are the earning. Since the P/E ratio is 15, we can write

$$\begin{aligned}\frac{d}{p} &= \kappa \cdot \frac{e}{p} \\ &= 0.5 \times \frac{1}{15} \\ &= 0.0\bar{3}\end{aligned}$$

Hence, given an expected return of 12 percent, the growth rate is

$$\begin{aligned}g &= 0.12 - 0.0\bar{3} \\ &= 0.08\bar{6}\end{aligned}$$

Chapter10

Problem 10.1. Assume that residual returns are uncorrelated, and that we will use an optimizer to maximize risk-adjusted residual return. Using the data in Table 10.3, what asset will the optimizer choose as the largest positive active holding? How would that change if we had assigned $\alpha = 1$ for buys and $\alpha = -1$ for sells? *Hint:* At optimality, assuming uncorrelated residual returns, the optimal active holdings are

$$h_n = \frac{\alpha_n}{2\lambda_R \omega_n^2}$$

Solution. Using the alpha's from table 10.3 from the refined forecasts, the optimizer will chose the stock that maximizes h_n as the largest positive active holding. This means that only stocks with positive α need to be considered. If we calculate h_a for all stocks with a positive alpha (regardless of tolerance to risk, λ_R) we find that the α/ω_n^2 ratio is largest for AT&T, where it equals 0.056635. Even though AT&T has the smallest α, it also has the smallest ω. If instead of the refined forecasts, buy recommendations were given $\alpha = 1$ and sell recommendations were given $\alpha = -1$, the optimizer would just pick the buy recommendation with the smallest residual volatility, which in this case is AT&T.

Problem 10.2. For the situation described in Problem 1, show that using the forecasting rule of thumb, we assume equal risk for each asset. What happens if we just use $\alpha = 1$ for buys and $\alpha = -1$ for sells?

Solution. The forecasting rule of thumb states

$$\text{Refined forecast} = \text{volatility} \times \text{IC} \times \text{score}$$

If we assign just assign $\alpha = 1$ for buy and $\alpha = -1$ for sell, since the IC are constant and the scores are 1 for buy and 1 for sell, we find that the volatility for each stock is

$$\begin{aligned} \text{volatility} &= 1/(0.09 \times 1) \text{ for buy} \\ \text{volatility} &= -1/(0.09 \times -1) \text{ for sell} \end{aligned}$$

Hence, using $\alpha = 1$ for buys and $\alpha = -1$ for sells assumes equal risk for each asset.

Problem 10.3. Use the basic forecasting formula [Eq. (10.1)] to derive Eq. (10.20), the refined forecast in the case of one asset and two forecasts.

Solution. Since Eq (10.20) is a refined forecast, let us start with the definition of the refined forecast in Eq. (10.2). We have

$$\boldsymbol{\phi} = \text{Cov}\{\boldsymbol{r}, \boldsymbol{g}\} \cdot \text{Var}^{-1}\{\boldsymbol{g}\} \cdot (\boldsymbol{g} - E\{\boldsymbol{g}\})$$

For the case of one asset and two forecasts, the vectors and matrices can be written as

$$\begin{aligned}
\text{Cov}\{\boldsymbol{r}, \boldsymbol{g}\} &= \text{Std}\{r\} \cdot \boldsymbol{\rho_{r,g}} \cdot \text{Std}\{\boldsymbol{g}\} \\
&= \sigma_r \cdot \begin{bmatrix} \text{IC}_{g_1} & \text{IC}_{g_2} \end{bmatrix} \cdot \begin{bmatrix} \text{Std}\{g_1\} & 0 \\ 0 & \text{Std}\{g_2\} \end{bmatrix} \\
\text{Var}^{-1}\{\boldsymbol{g}\} &= \text{Std}\{\boldsymbol{g}\}^{-1} \boldsymbol{\rho}_{\boldsymbol{g_1},\boldsymbol{g_2}}^{-1} \text{Std}\{\boldsymbol{g}\}^{-1} \\
&= \begin{bmatrix} \frac{1}{\text{Std}\{g_1\}} & 0 \\ 0 & \frac{1}{\text{Std}\{g_2\}} \end{bmatrix} \frac{1}{\rho_{g_1 g_1}\rho_{g_2 g_2} - \rho_{g_1 g_2}\rho_{g_2 g_1}} \begin{bmatrix} \rho_{g_2 g_2} & -\rho_{g_2 g_1} \\ -\rho_{g_1 g_2} & \rho_{g_1 g_1} \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\text{Std}\{g_1\}} & 0 \\ 0 & \frac{1}{\text{Std}\{g_2\}} \end{bmatrix} \\
&= \begin{bmatrix} \frac{1}{\text{Std}\{g_1\}} & 0 \\ 0 & \frac{1}{\text{Std}\{g_2\}} \end{bmatrix} \frac{1}{1 - \rho_{g_1 g_2}^2} \begin{bmatrix} 1 & -\rho_{g_2 g_1} \\ -\rho_{g_1 g_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\text{Std}\{g_1\}} & 0 \\ 0 & \frac{1}{\text{Std}\{g_2\}} \end{bmatrix} \\
\boldsymbol{g} - E\{\boldsymbol{g}\} &= \begin{bmatrix} g_1 - m_{g_1} \\ g_2 - m_{g_2} \end{bmatrix}
\end{aligned}$$

Here, we have used the usual definitions of the information coefficient and $g_{1(2)}$ and $m_{g_{1(2)}}$ is the forecast and mean of signal 1 (2) respectively. The ρ represent correlations and σ_r the standard deviation of the return. The inverse variance was calculated using the usual formula to invert a 2 by two matrix, and then simplified using the fact that self correlations (i.e. ρ_{11}) are equal to 1. Now that we have these expressions, we can determine $\boldsymbol{\phi}$ as

$$\boldsymbol{\phi} = \sigma_r \cdot \begin{bmatrix} \mathrm{IC}_{g_1} & \mathrm{IC}_{g_2} \end{bmatrix} \cdot \frac{1}{1-\rho_{g_1 g_2}^2} \begin{bmatrix} 1 & -\rho_{g_2 g_1} \\ -\rho_{g_1 g_2} & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\mathrm{Std}\{g_1\}} & 0 \\ 0 & \frac{1}{\mathrm{Std}\{g_2\}} \end{bmatrix} \cdot \begin{bmatrix} g_1 - m_{g_1} \\ g_2 - m_{g_2} \end{bmatrix}$$

Multiplying the last two matrices give us the scores z_{g_1} and z_{g_2}. Multiplying the first two matrices give us

$$\begin{bmatrix} \mathrm{IC}_{g_1} & \mathrm{IC}_{g_2} \end{bmatrix} \cdot \frac{1}{1-\rho_{g_1 g_2}^2} \begin{bmatrix} 1 & -\rho_{g_2 g_1} \\ -\rho_{g_1 g_2} & 1 \end{bmatrix} = \frac{1}{1-\rho_{g_1 g_2}^2} \begin{bmatrix} \mathrm{IC}_{g_1} - \mathrm{IC}_{g_2}\rho_{g_1 g_2} & \mathrm{IC}_{g_2} - \mathrm{IC}_{g_1}\rho_{g_2 g_1} \end{bmatrix}$$

Multiplying the remaining matrices, we find

$$\begin{aligned} \boldsymbol{\phi} &= \sigma_r \frac{1}{1-\rho_{g_1 g_2}^2} \begin{bmatrix} \mathrm{IC}_{g_1} - \mathrm{IC}_{g_2}\rho_{g_1 g_2} & \mathrm{IC}_{g_2} - \mathrm{IC}_{g_1}\rho_{g_2 g_1} \end{bmatrix} \cdot \begin{bmatrix} z_{g_1} \\ z_{g_2} \end{bmatrix} \\ &= \sigma_r \frac{1}{1-\rho_{g_1 g_2}^2} \left(\left[\mathrm{IC}_{g_1} - \mathrm{IC}_{g_2}\rho_{g_1 g_2}\right] z_{g_1} + \left[\mathrm{IC}_{g_2} - \mathrm{IC}_{g_1}\rho_{g_2 g_1}\right] z_{g_2} \right) \\ &= \sigma_r \left(IC^*_{g_1} z_{g_1} + IC^*_{g_1} z_{g_2} \right) \end{aligned}$$

where we have used the definitions of $\mathrm{IC}^*_{g_1}$ and $\mathrm{IC}^*_{g_2}$ from equations (10.21) and (10.22). This completes the derivation of Eq. (10.20).

Problem 10.4. In the case of two forecasts [Eq. (10.20)], (a) what is the variance of the combined forecast? (b) What is its covariance with the return? (c) Verify explicitly that the combination of g and g' in the example leads to an IC of 0.1090. Compare this to the result from Eq. (10.27).

Solution. We can use the matrix expressions from question 10.3 to determine these properties.

(a) The variance of the combined forecasts will be

$$\begin{aligned} \sigma_{\boldsymbol{g}}^2 &= \begin{bmatrix} 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sigma_{g_1}^2 & \sigma_{g_1}\sigma_{g_2}\rho_{1,2} \\ \sigma_{g_1}\sigma_{g_2}\rho_{1,2} & \sigma_{g_2}^2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \sigma_{g_1}^2 + \sigma_{_2}^2 + 2\sigma_{g_1}\sigma_{g_2}\rho_{1,2} \end{aligned}$$

where the σ^2 are the individual variances and $\rho_{1,2}$ the correlation between forecasts.

(b) The covariance with the return will be

$$\begin{aligned} \sigma_{r,\boldsymbol{g}} &= \sigma_r \cdot \begin{bmatrix} \mathrm{IC}_{g_1} & \mathrm{IC}_{g_2} \end{bmatrix} \cdot \begin{bmatrix} \sigma_{g_1} & 0 \\ 0 & \sigma_{g_2} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \sigma_r \left(\mathrm{IC}_{g_1}\sigma_{g_1} + \mathrm{IC}_{g_2}\sigma_{g_2} \right) \end{aligned}$$

(c) The IC of the combined forecasts is equal to its correlation with the return

$$\begin{aligned} \mathrm{IC}_{\boldsymbol{g}} &= \frac{\sigma_{r,g}}{\sigma_r \sigma_g} \\ &= \frac{\mathrm{IC}_{g_1}\sigma_{g_1} + \mathrm{IC}_{g_2}\sigma_{g_2}}{\sqrt{\sigma_{g_1}^2 + \sigma_{_2}^2 + 2\sigma_{g_1}\sigma_{g_2}\rho_{1,2}}} \end{aligned}$$

For the example in the text, $\mathrm{IC}_{g_1} = 0.0833$, $\mathrm{IC}_{g_2} = 0.089$,$\sigma_{g_1} = 4$,$\sigma_{g_2} = 5$, and $\rho_{g_1,g_2} = 1/4$. Plugging these values in, we find

$$\begin{aligned} \mathrm{IC}_{\boldsymbol{g}} &= \frac{0.0833 \times 4 + 0.089 \times 5}{\sqrt{25 + 16 + 2 \times 5 \times 4 \times 1/4}} \\ &= 0.1090 \end{aligned}$$

We can compare this to Eq (10.27), which states

$$
\begin{aligned}
\mathrm{IC}_{\boldsymbol{g}} &= \sqrt{\frac{\mathrm{IC}_{\boldsymbol{g_1}}^2 + \mathrm{IC}_{\boldsymbol{g_2}}^2 - 2\rho_{g_1,g_2}\mathrm{IC}_{g_1}\mathrm{IC}_{g_2}}{1-\rho_{g_1,g_2}^2}} \\
&= \sqrt{\frac{0.0833^2 + 0.089^2 - 2 \times 1/4 \times 0.0833 \times 0.089}{1-1/16}} \\
&= 0.1091
\end{aligned}
$$

The difference is likely due to a rounding error in the calculation of the input terms, most likely the information coefficients.

Problem 10.5. You are using a neural net to forecast returns to one stock. The net inputs include fundamental counting data, analyst's forecasts, and past returns. The net combines these nonlinearly. How would the forecasting rule of thumb change under these circumstances?

Solution. The neural network will take the raw inputs and forecast the returns to the stock directly. Hence, it doesn't seem as if the rule of thumb [equation (10.11)] will apply since the conversion from raw signal to forecast is done behind the scenes. However, it should be straightforward to decompose the forecast of the neural network into the terms in the rule of thumb, since the volatility can be determined and a reasonable IC can be assigned.

Chapter11

Problem 11.1. Signal 1 and Signal 2 have equal IC, and both exhibit signal volatilities proportional to asset volatilities. Do the two signals receive equal weight in the forecast exceptional return?

Solution.

Problem 11.2. What IR would you naïvely expect if you combined strategies A and C in Table 11.3? Why might the observed answer differ from the naïve result?

Solution.

Problem 11.3. How much should you shrink coefficient b, connecting raw signals and realized returns, estimated with $R^2 = 0.05$ after 120 months?

Solution.