

## Experiment No: 01

### TIME RESPONSE OF SECOND ORDER SYSTEM

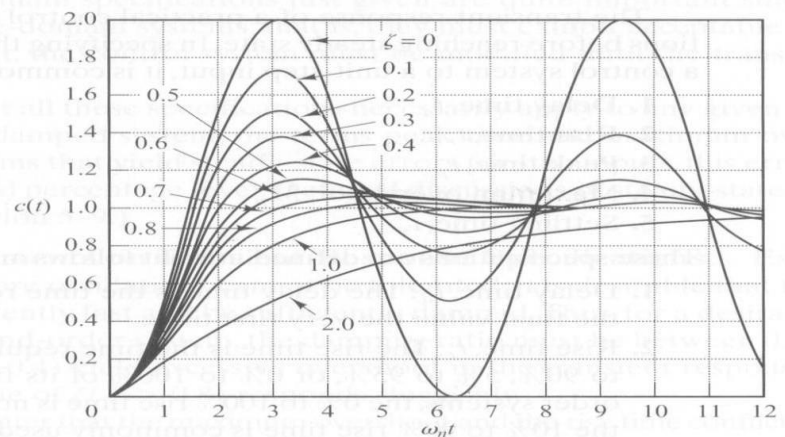
**Aim:** To obtain time response of a second order system in case of under damped, over damped and critically damped systems.

#### Theory:

The time response of control system consists of two parts. Transient response and steady state response.  $C(t) = C_{tr}(t) + C_{ss}(t)$ . Most of the control systems use time as its independent variable. Analysis of response means to see the variation of output with respect to time. The output of the system takes some finite time to reach to its final value. Every system has a tendency to oppose the oscillatory behavior of the system which is called damping. The damping is measured by a factor called damping ratio of the system. If the damping is very high then there will not be any oscillations in the output. The output is purely exponential. Such system is called an over damped system.

If the damping is less compared to over damped case then the system is called a critically damped system. If the damping is very less then the system is called under damped system. With no damping system is undamped.

$1 < \xi < \infty$  --- Over damped system.  
 $\xi = 1$  --- Critically damped system.  
 $0 < \xi < 1$  --- Under damped system.  
 $\xi = 0$  --- Undamped system.



**Time domain specifications:** Delay time  $T_d = (1 + 0.7\xi) / \omega_n$

Rise Time  $T_r = (\pi - \theta) / \omega_d$

Peak overshoot time =  $T_p = \pi / \omega_d$

%  $M_p = \exp(-\pi\xi/\sqrt{1-\xi^2})$

Settling Time  $T_s = 4/\xi\omega_n$  (2% tolerance)

**Apparatus Required:** PC loaded with MATLAB

#### Procedure:

Open the MATLAB command window.

- 1) Click on file-new-M file to open the MATLAB editor window.
- 2) In the given MATLAB editor window enter the program to obtain the step response.

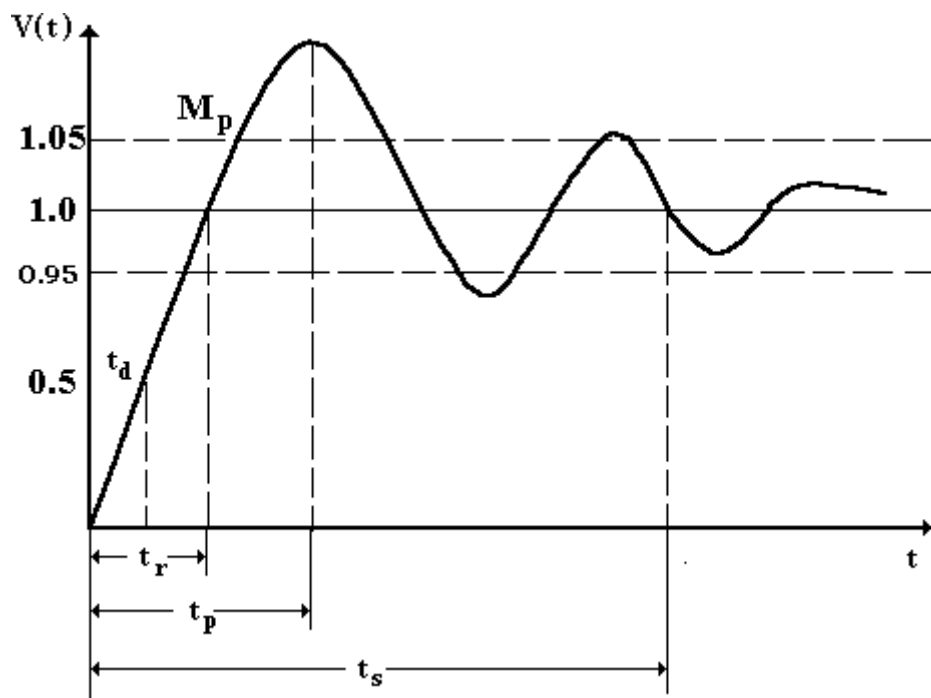
```
% step response of a second order system
wn = input('enter the natural frequency');
xi = input('enter damping ratio');
num = [wn*wn];
den = [1 2*xi*wn wn*wn];
sys = tf(num, den)
```

---

step(sys);

- 3) Save the file in work directory.
- 4) Run the program and enter the respective value for natural frequency, damping ratio and time
- 5) The graphs displayed are according to the above values
- 6) The values of  $\omega_d$ ,  $t_d$ ,  $\theta$ ,  $t_r$ ,  $t_s$ ,  $M_p$  can be obtained by
  - a) Right click on the figure window and select grid to get grids on the curve.
  - b) Right click on the figure window and select characteristics and enable peak response, settling time & rise settling
  - c) Repeat the steps 5,6,7 for different values of  $\xi$ .

**Graph:**



**Tabular Columns :**

Time Domain Specifications	From MATLAB	By Calculation
Rise Time		
Peak Time		
Settling Time		
Max. Overshoot		

**1b) Evaluation of the effect of additional poles and zeros on time response of second order system.**

MATLAB program to evaluate the effect of additional poles and zeros on time response of second order system. This program uses the command `zpk`.

For the second order system the poles are  $-10+30i$  and  $-10-30i$

The program given below gives the time response of 2nd order system

---

```

Z=[];
P=[-10+30i -10-30i];
K=1000;
sys=zpk(z,p,k)
t=[0:0.001:1];
step(sys,t);
grid

```

For the second order system, if we add a pole it changes to third order.

To study the effect of additional poles,

Location of poles	Effect on time response
-1	
-10	
-100	

To study the effect of additional zeros,

Location of zeros	Effect on time response
-1	
-10	
-100	

### 1c) Effect of loop gain of a negative feedback system on stability

The following program is used to study the effect of loop gain of a negative feedback system on stability. The value of gain k is varied and different step responses are obtained.

```

clc
z=[]
p=[-0.5+i -0.5-i -1];
k1=1;
k2=2;
k3=3;
sys1=zpk(z,p,k1)
sys2=zpk(z,p,k2)
sys3=zpk(z,p,k3)
t=[0:0.01:20];
[y1,t]=step(sys1,t)
[y2,t]=step(sys2,t)
[y3,t]=step(sys3,t)
plot(t,y1,t,y2,t,y3)
legend('k=1', 'k=2', 'k=3')
grid

```

**Result:**

## Experiment No.2

### RC LEAD COMPENSATING NETWORK

**Aim:** To design a passive RC lead compensating network for the given specifications and to obtain its frequency response.

**Apparatus Required:** Resistors, capacitors, wires, multimeter, and phase- frequency meter.

#### **Theory:**

If a sinusoidal input is applied to the input of a network and steady state output has a phase lead, then network is called lead compensator/network. Lead compensator has a zero at  $s = 1/T$  and a pole at  $s = 1/\alpha T$  with zero closer to the origin than pole. This compensator speeds up the transient response and increases the margin of stability of a system. It also helps to increase the system error constant through to a limited extent. These compensators are used when fast dynamic response is required.

#### **Effect of Phase Lead Compensation**

1. The velocity constant  $K_v$  increases.
2. The slope of the magnitude plot reduces at the gain crossover frequency so that relative stability improves & error decrease due to error is directly proportional to the slope.
3. Phase margin increases.
4. Response become faster.

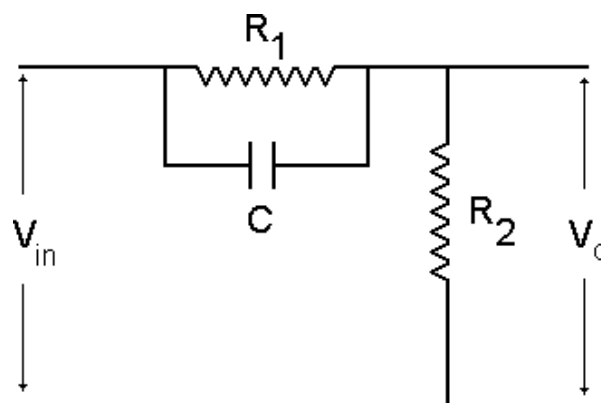
#### **Advantages of Phase Lead Compensation**

1. Due to the presence of phase lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lead compensation maximum overshoot of the system decreases.

#### **Disadvantages of Phase Lead Compensation**

1. Steady state error is not improved.

#### **Circuit Diagram:**



Lead Network

#### **Derivation of transfer function:**

Write the above circuit in Laplace form.

$V_i(s) = (Z_1 + Z_2) * I(s)$  (Where  $I(s)$  is the current in the circuit and  $Z_1 = (R_1 // C)$  and  $Z_2 = R_2$ )

$V_o(s) = Z_2 * I(s)$

$V_o(s)/V_i(s) = Z_2 / (Z_1 + Z_2)$

After simplification,

$$G_c(S) = \frac{(S + 1/T)}{(S + 1/\alpha T)} \quad \text{where } T = R_1 C \text{ and } \alpha = R_2 / (R_1 + R_2)$$

### **Design Equations:**

**Specifications for the design:**  $\Phi_m = \dots\dots\dots$  at  $f_m = \dots\dots\dots$

1.  $\sin \Phi_m = \frac{(1 - \alpha)}{(1 + \alpha)} \quad \alpha < 1$
2.  $\alpha = R_2 / (R_1 + R_2)$
3.  $\omega_m = \frac{1}{T \sqrt{\alpha}}$
4.  $T = R_1 C$

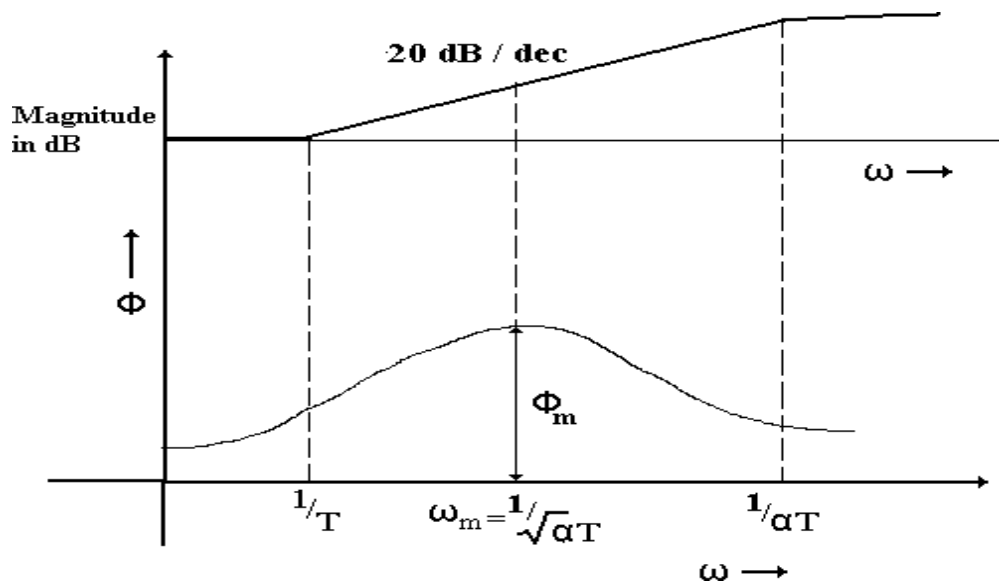
### **Procedure:**

1. Derive the transfer function for the Lead network given above.
2. For the given specification, ie for given  $\Phi_m$  at given  $F_m$ , calculations of  $R_1$ ,  $R_2$  and  $C$ . are done.
3. Connections are made as per the Lead circuit diagram by the selecting the values found in the above step.
4. Switch ON the mains supply and apply sinusoidal wave by selecting suitable amplitude.
5. The frequency of the signal is varied in steps and at each step note down the corresponding magnitude of output and phase angle.
6. Draw the frequency response plot and hence find the transfer function & compare it with the design.

### **Tabular Column:**

Input voltage $V_s = \dots\dots\dots$ V(volts)			
Frequency (Hz)	output $V_o$ (volts)	$\angle$ (degree) indicated	Gain (dB)

**Typical Lead Characteristics:**



**Result:**

### Experiment No.3

#### RC LAG COMPENSATING NETWORK

**Aim:** To design a passive RC lag compensating network for the given specifications and to obtain its frequency response.

**Apparatus Required:** Resistors, capacitors, wires, multimeter, and phase- frequency meter.

**Theory:** If a sinusoidal input is applied to the input of a network and steady state output has a phase lag, then network is called lag compensator/network. Lag compensator has a pole at  $s = 1/\beta T$  and a pole at  $s = 1/T$  with pole closer to the origin than zero. This compensator improves the steady state behavior of the system while nearly preserving its transient response. These compensators are used when low steady state error is required.

#### Effect of Phase Lag Compensation

1. Gain crossover frequency increases.
2. Bandwidth decreases.
3. Phase margin will be increase.
4. Response will be slower before due to decreasing bandwidth, the rise time and the settling time become larger.

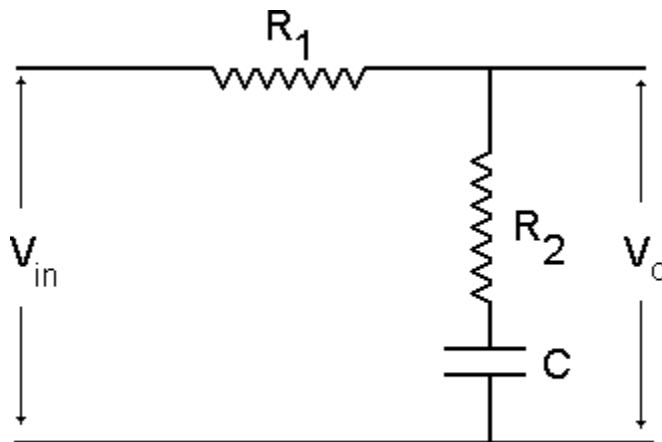
#### Advantages of Phase Lag Compensation

1. Phase lag network allows low frequencies and high frequencies are attenuated.
2. Due to the presence of phase lag compensation the steady state accuracy increases.

#### Disadvantages of Phase Lag Compensation

1. Due to the presence of phase lag compensation the speed of the system decreases.

#### Circuit Diagram:



Lag Network

#### Derivation of transfer function:

Write the above circuit in Laplace form.

$V_i(s) = (Z_1 + Z_2) * I(s)$  (where  $I(s)$  is the current in the circuit and  $Z_1 = R_1$  and  $Z_2 = (R_2 + 1/CS)$ )

$$V_o(s) = Z_2 * I(s)$$

$$V_o(s)/V_i(s) = Z_2 / (Z_1 + Z_2)$$

After simplification,

$$G_C(S) = \frac{(S + 1/T)}{(S + 1/\beta T)} \quad \text{where } \beta = (R_1 + R_2) / R_2 \text{ and } T = R_2 C$$

### **Design Equations:**

**Specifications for the design:**  $\Phi_m = \dots\dots\dots$  at  $f_m = \dots\dots\dots$

To find maximum lag angle:

1.  $\sin \Phi_m = \frac{(\beta - 1)}{(\beta + 1)}$        $\beta > 1$
2.  $\beta = (R_1 + R_2) / R_2$
3.  $\omega_m = \frac{1}{T\sqrt{\beta}}$
4.  $T = R_2 C$

### **Procedure:**

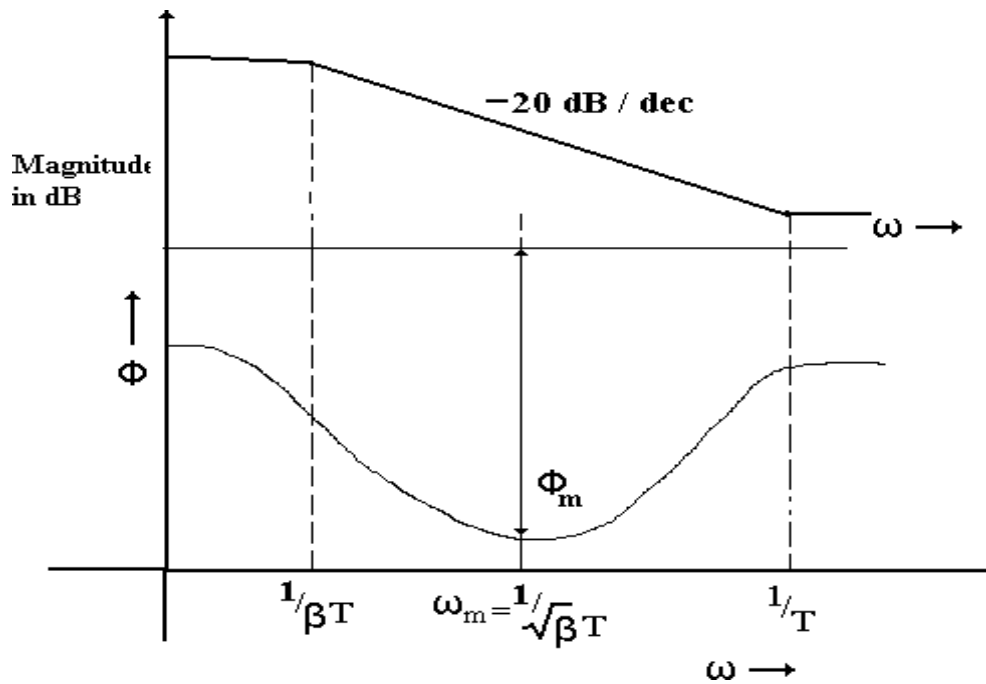
1. Derive the transfer function for the Lag network given above.
2. For the given specifications, ie for given  $\Phi_m$  at given  $f_m$ , calculations of  $R_1$ ,  $R_2$  and  $C$  are done.
3. Connections are made as per the Lag circuit diagram by the selecting the values found in the above step.
4. Switch ON the mains supply and apply sinusoidal wave by selecting suitable amplitude.
5. The frequency of the signal is varied in steps and at each step note down the corresponding magnitude of output and phase angle.
6. Draw the frequency response plot and hence find the transfer function & compare it with the design.

### **Tabular Column:**

Input voltage $V_s = \dots\dots\dots$ V( volts)			
Frequency (Hz)	$V_o$ (volts) (rms)	$\Phi$ (degree) indicated	Gain(dB) $20 \log(V_o/V_s)$



### Typical Lag Characteristics:



### Result:

### Viva questions:

1. What is the need for compensation?
2. What is meant by compensation?
3. What is lag compensation? Write the frequency response of it.
4. What is the importance of lag network?

## Experiment No.4

### LAG -LEAD NETWORKS

**Aim:** Experiment to draw the frequency response of a given lead-lag compensating network.

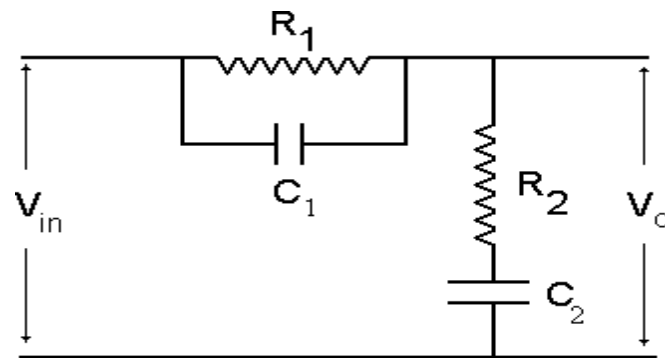
**Apparatus Required:** Resistors – 10k – 2 nos, capacitors – 0.1 $\mu$ F – 2nos, wires, multimeter, and phase- frequency meter.

**Theory:** Lag lead compensator is a combination of a lag compensator and lead compensator. The lag section has one real pole and one real zero with pole to the right of zero. The lead section also has one real pole and one zero but zero is to the right of the pole. When both steady state and transient response require improvement, a lag lead compensator is required.

#### Advantages of Phase Lag-Lead Compensation

1. Due to the presence of phase lag-lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
2. Due to the presence of phase lag-lead network accuracy is improved.

#### Circuit Diagram:



Lag-Lead Network

#### Procedure:

1. Derive the transfer function for the lag lead network given above.
2. Connections are made as per the Lag lead circuit diagram by selecting the proper values.
3. Switch ON the mains supply and apply sinusoidal wave by selecting suitable amplitude.
4. The frequency of the signal is varied in steps and at each step note down the corresponding magnitude of output and phase angle.
5. Draw the frequency response plot and hence find the transfer function & compare it with the design.

#### Derivation of transfer function:

Write the above circuit in Laplace form.

$V_i(s) = (Z_1 + Z_2) * I(s)$  (Where  $I(s)$  is the current in the circuit and  $Z_1 = (R_1 // C_1 S)$  and

$Z_2 = (R_2 + 1/C_2 S)$

$V_o(s) = Z_2 * I(s)$

$V_o(s)/V_i(s) = Z_2 / (Z_1 + Z_2)$

After simplification,

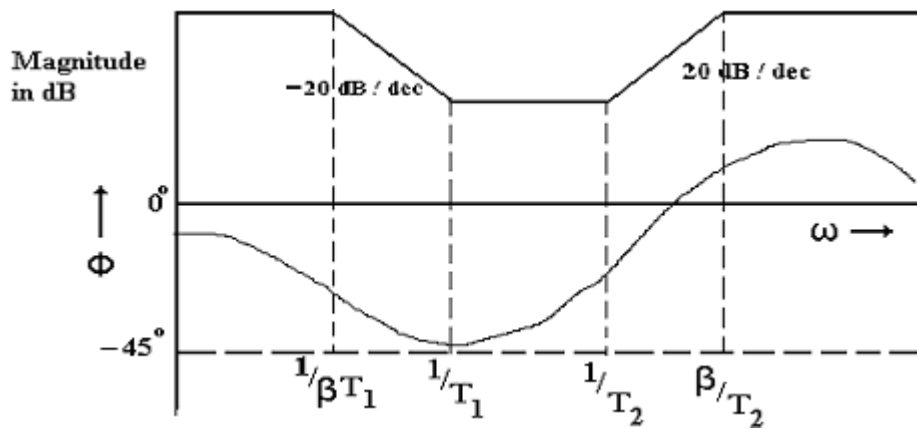
$$G_C(S) = \frac{(S + 1/T_1)(S + 1/T_2)}{(S + 1/\beta T_2)(S + \beta/T_1)}$$

where  $T_1 = R_1 C_1$ ,  $T_2 = R_2 C_2$ ,  $R_1 C_1 + R_2 C_2 + R_1 C_2 = 1/\beta T_2 + \beta/T_1$

**Tabular Column:**

$V_S = \dots\dots\dots$ V(volts)			
Frequency (Hz)	$V_O$ (rms)	$\phi$ (degree) indicated	Gain(dB) $20 \log(V_O/V_S)$

**Typical Characteristics:**



**Result:**

**Viva questions:**

1. What is lag lead Compensation? Write the frequency response of it.
2. What is the importance of lag lead network?

## Experiment No.5

### STUDY OF P, P-I, P-I-D CONTROLLERS

**Aim:** To study the effect of P, PI, PD and PID controller on the step response of a feedback control system.

#### **Theory:**

PID controllers are commercially successful and widely used controllers in Industries. For example, in a typical paper mill there may be about 1500 Controllers and out of these 90% would be PID controllers. The PID controller consists of proportional controller, integral controller and derivative controller. Depending upon the application on or more combinations of the controllers are used.(ex: in a liquid control system where we want zero steady state error, a PI controller can be used and in a temperature control system where we do not want zero steady state error, a simple P controller can be used.

The equation of the PID controller in time domain is given by,

$$m(t) = K_P e(t) + K_i / T_i \int e(t) dt + K_d T_d de(t) / dt$$

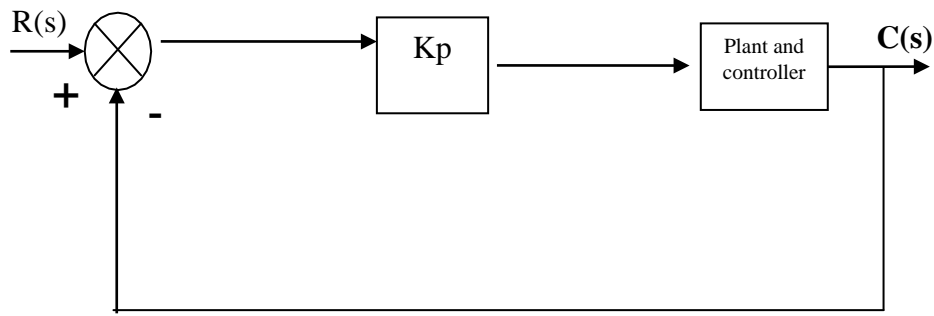
where  $K_P$  is a proportional gain  $T_i$  is the integral reset time and  $T_d$  is the derivative time of the PID controller,  $m(t)$  is the output of the controller and  $e(t)$  is the error signal given by  $e(t) = r(t) - c(t)$ .

#### **The characteristics of P, I, and D controllers**

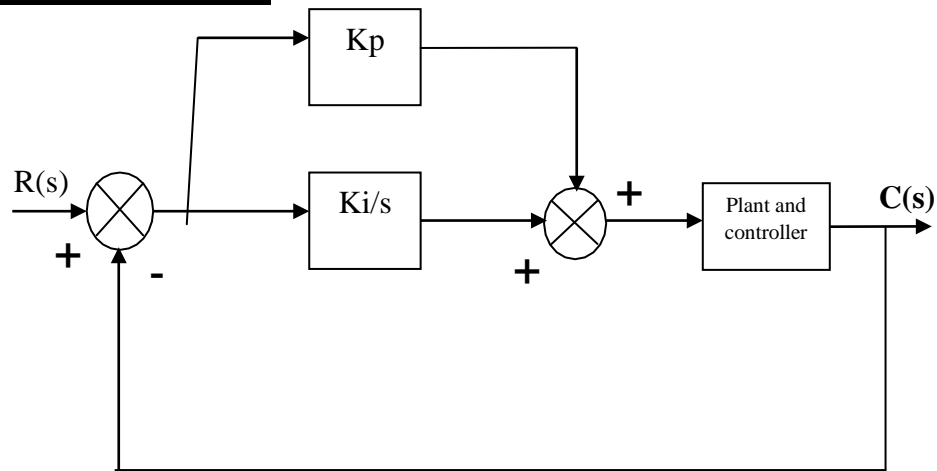
A proportional controller ( $K_P$ ) will have the effect of reducing the rise time and will reduce ,but never eliminate, the steady-state error. An integral control ( $K_i$ ) will have the effect of eliminating the steady-state error, but it may make the transient response worse. A derivative control ( $K_d$ ) will have the effect of increasing the stability of the system, reducing the overshoot, and improving the transient response. Effects of each of controllers  $K_P$ ,  $K_d$ , and  $K_i$  on a closed-loop system are summarized in the table shown below.

Parameter	Rise time	Overshoot	Settling time	Steady-state error	Stability <sup>[14]</sup>
$K_P$	Decrease	Increase	Small change	Decrease	Degrade
$K_i$	Decrease	Increase	Increase	Eliminate	Degrade
$K_d$	Minor change	Decrease	Decrease	No effect	Improve if $K_d$ small

### **PROPORTIONAL (P) CONTROLLER:**



### **PI CONTROLLER:**

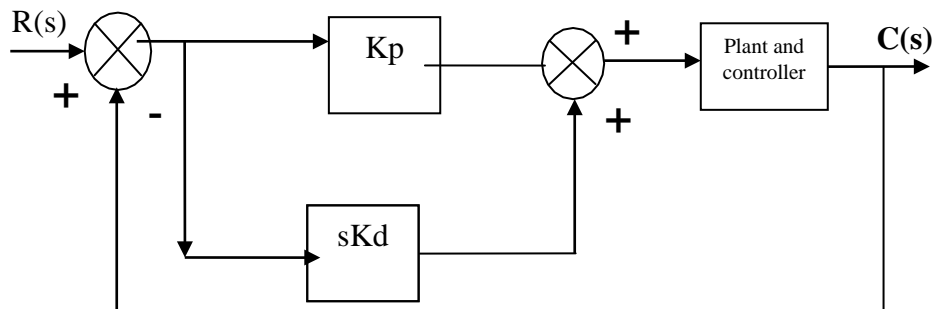


From the block diagram

$$C(s)/R(s) = (s+Ki) \omega_n^2 / s^3 + 2\xi \omega_n s^2 + \omega_n^2 s + Ki \omega_n^2$$

The characteristic equation is third order, so system also becomes third order reducing SS error to zero

### **PD CONTROLLER:**



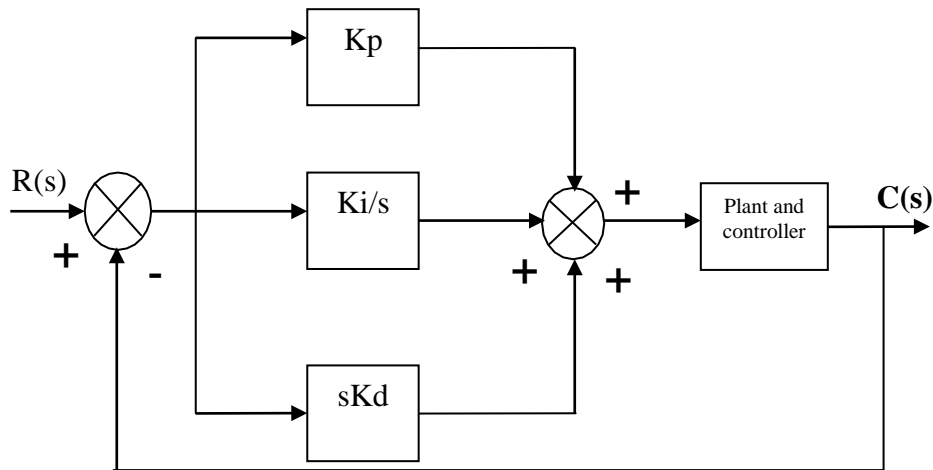
From the block diagram

$$C(s)/R(s) = \omega_n^2 (1+sTd) / S^2 + (2\xi \omega_n + \omega_n^2 Td)S + \omega_n^2$$

Comparing with  $S^2 + 2\xi \omega_n S + \omega_n^2$ , damping ratio increases reducing the peak overshoot in the response

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## **PID CONTROLLER**



In PID controller, the error signal is given by  
 $E_a(s) = K_p E(s) + sT_d E(s) + K_i/s E(s)$

### **Procedure:**

1. The connections are made as in the diagrams.
2. DC supply from the kit is given.
3. The values of  $k_p$ ,  $K_d$ ,  $K_i$  are adjusted and the waveforms are observed on the CRO.

### **Result:**

### **Viva questions:**

1. What is P-I control?
2. What is P-D control?
3. What is P-I-D control?
4. Why differential control is not used alone?
5. What is the problem with proportional control?

## Experiment No.6 (a)

### AC SERVO MOTOR

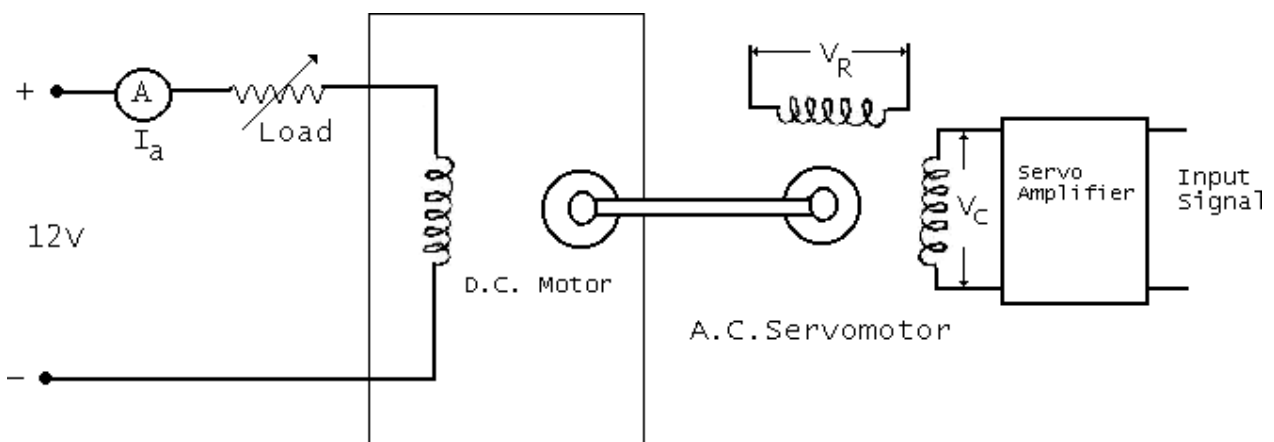
**Aim:** To obtain torque speed characteristics of AC servo motor.

#### **Theory:**

An AC servomotor is basically a two phase induction motor except for certain special design features. The motor of the servo motor is built with high resistance so that its  $X/R$  ratio is small and the torque speed characteristics is linear. For low resistance, the characteristics is nonlinear. Such a characteristics is unacceptable in control systems. The motor construction is usually squirrel cage or drag cup type. The diameter of rotor is kept small in order to reduce inertia to obtain good accelerating characteristics. In servo applications the voltage applied to the two stator windings are seldom balanced. One of the phases known as the reference phase is excited by constant voltage and the other phase is known as the control phase is excited by a voltage of variable magnitude from a servo amplifier and polarity with respect to the voltage supplied to the reference winding. For low power applications AC servo motors are preferred because they are light weight, rugged construction.

**Apparatus Required:** AC servo motor speed torque unit, multimeter.

#### **Circuit Diagram:**



#### **AC servomotor specifications:**

Reference winding voltage: 230v AC

Control winding voltage: 230v AC

Rated power = 50 watts

Moment of Inertia ( J ) =  $0.7 \text{ gm/cm}^2$

Friction coefficient B = 0.021

Speed: 2000rpm

#### **Procedure:**

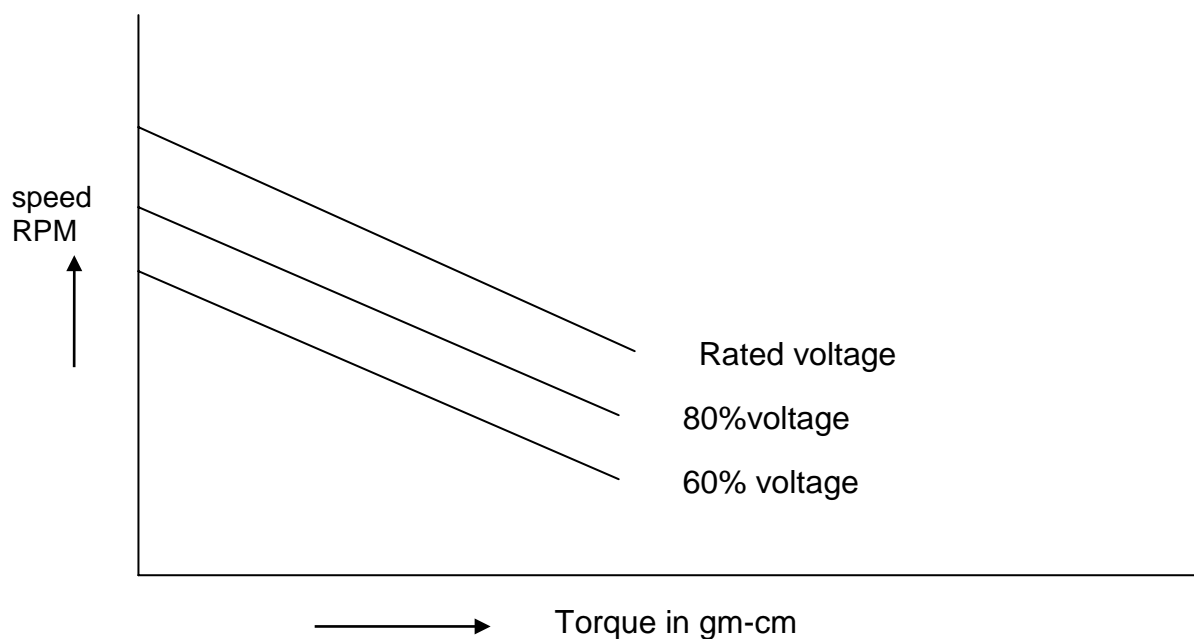
1. Connections are made as per circuit diagram.
2. The load switch is in OFF position, so that DC machine is not connected to auxiliary power supply (12V). AC servomotor switch is in OFF position.
3. Ensure speed control and load control pot is in minimum position.

4. Switch ON the mains and also AC servomotor. The AC servomotor starts rotating and speed will be indicated by meter in front panel.
5. The reference winding voltage can be measured.
6. Now switch ON the load control switch and start loading the servomotor.
7. Note down corresponding values of  $I_a$  &  $E_b$  (& speed).
8. Now new control winding voltage is set by varying position of speed control switch. Again the machine is loaded &  $I_a, E_b$  are noted down.
9. Speed –Torque characteristics are plotted.

**Tabular column:**

Current(mA)	$E_b$ (v)	Speed (rpm)	Power(watts)	Torque(gm-cm)

**Graph:**



**Calculation:**

$$\text{Power} = E_b \times I_a \text{ Watts}$$

$$\text{Torque} = (E_b \times I_a \times 60 \times 1.0196 \times 10^5) / 2 \pi \times N \text{ gm-cm}$$

**Viva questions:**

1. What are the applications of AC servo motor?
2. How is AC servo motor different from normal AC motor?
3. What is the working principle of AC servo motor?



## **Experiment No.6 (b)**

### **DC SERVO MOTOR**

**Aim:** To plot Torque -Speed characteristics of a DC servomotor.

#### **Theory :**

The mechanism in which the control variable is adjusted by the error served by comparing output and input is called servomechanism. Any quantity e.g. voltage, speed, temperature, position, torque be controlled by providing appropriate feedback. The motor which respond to the error signal abruptly and actuate the load quickly are called servo motors. These are specifically designed and built primarily for use in feedback control systems as output actuators. The power rating can vary from a fraction of a watt up to a few hundred watts. They have high speed response which requires low rotor inertia. These motors are therefore smaller in diameter and longer in length. DC servo motors are used in high power applications. Some DC motors with relatively small power rating are used in instruments and computer related instruments. Other applications are CNC machines, robot systems, radars, machine tools, etc. Most important among the characteristics of the DC servo motor is the maximum acceleration obtainable. The operation of this motor is same as normal DC motor.

**Apparatus Required:** DC servo motor speed torque unit, multimeter.

#### **DC servomotor specifications:**

Type : permanent magnet type

Voltage : 24v DC

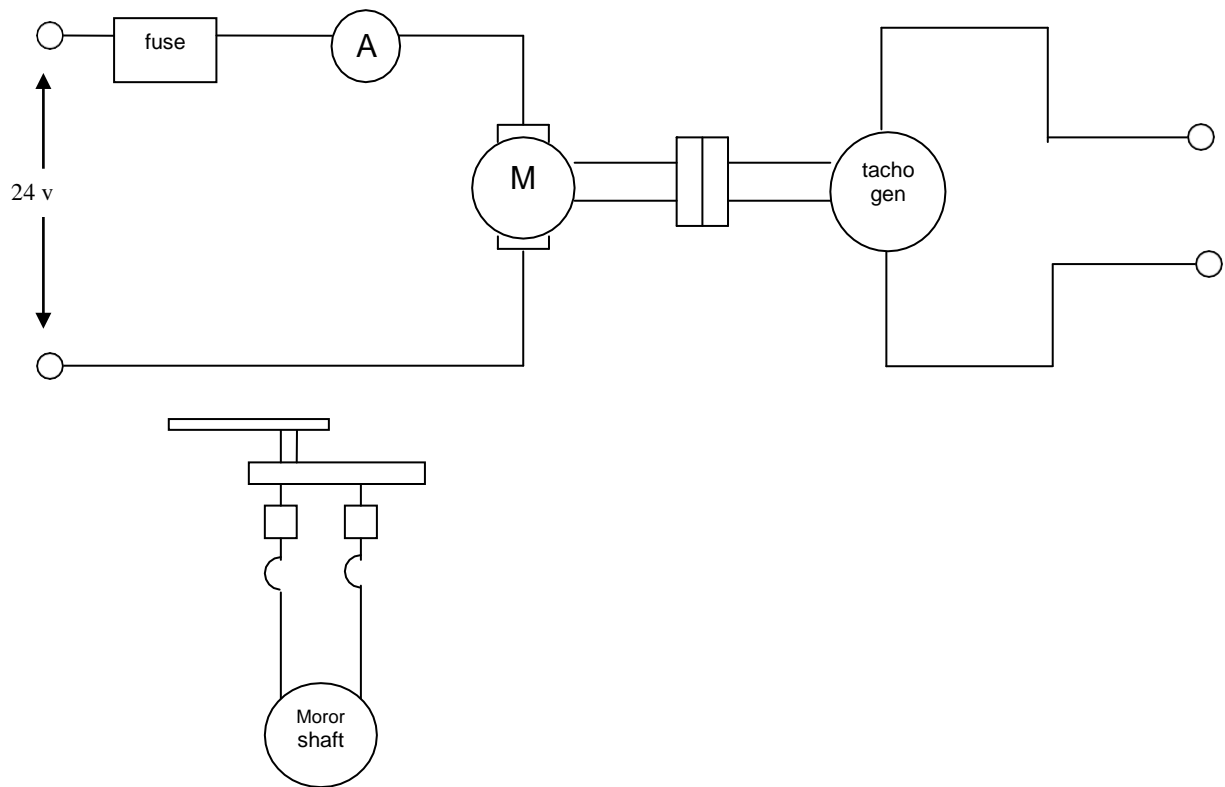
Torque : 400gm-cm

Speed : 4400 RPM

#### **Procedure:**

1. The connections are made as per circuit diagram.
2. All the front panel controls are carefully studied. Adjust zero reading in both the balances.
3. Keep the servomotor switch at off position, spring balances at minimum position.
4. Switch on the mains supply to the unit and both the r.p.m meter and ammeter shows zero reading.
5. Switch on DC servomotor. The motor starts rotating and the speed is indicated by r.p.m meter. Now vary the speed potentiometer to maximum position and check the DC voltage and it will be around 24V.
6. At this voltage note down speed and current.
7. Slowly the motor is loaded by adjusting the wheel and note down current, speed,  $w_1$  and  $w_2$  spring balance readings for every 25 gms or 50 gms of load. Enter the reading in tabular column. The motor is loaded upto 200 gms, since the torque is 400 gms and radius of brake drum is 2 cms.
8. Now release the load and repeat for different percentage of rated voltage.
9. The readings are tabulated and speed Vs torque is plotted.

**Circuit Diagram:**



**Tabular Column:**

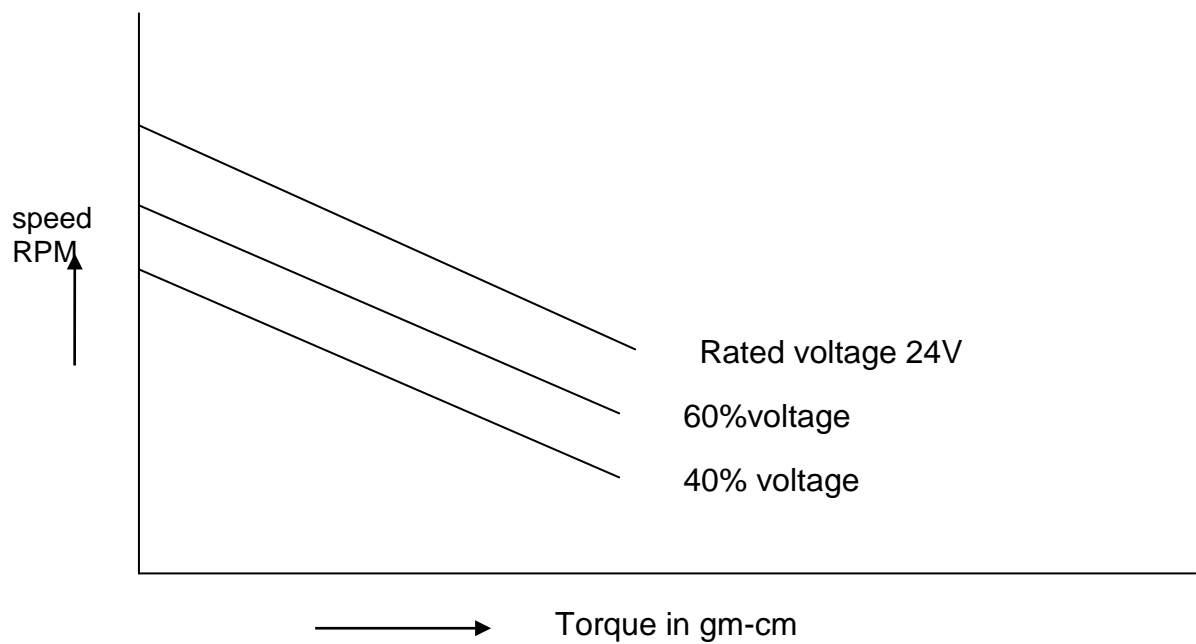
**At Rated voltage = 24V**

Speed(rpm)	$W_1$ gm	$W_2$ gm	$W=(W_1 - W_2)$ gm	Torque gm-cm

**Calculation:**

Torque =  $W \times R$  gm-cms and  $R$ =radius of the shaft,  $W=W_1-W_2$ .

**Graph:**



**Result:**

**Viva questions:**

1. What is meant by servomechanism?
2. How servomotor is different from ordinary DC motor?
3. What is meant by speed control?
4. What are the applications of DC Servo motor?

## Experiment No.7

### **FREQUENCY RESPONSE OF A SECOND ORDER SYSTEM**

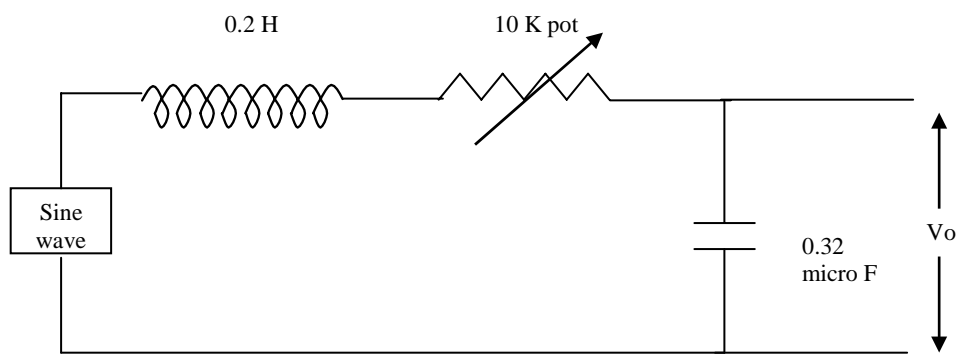
**Aim:** To determine frequency response of a second order system and evaluation of Frequency domain specifications.

#### **Theory:**

The frequency response of a system or a component is normally performed by keeping the amplitude A fixed and determining B and  $\Phi$  for a suitable range of frequencies where steady state output may be represented as  $c(t) = B \sin(\omega t + \Phi)$ . The ease and Accuracy of measurements are some of the advantages of the frequency response method. Without the knowledge of transfer function, the frequency response of stable open loop system can be obtained experimentally or the systems with very large time constants, the frequency response test is cumbersome to perform. We can use the data obtained from measurements on the physical system without deriving its mathematical model. Nyquist, bode, Nichols etc are some of the frequency response methods. For difficult cases, such as conditionally stable systems, Nyquist Plot is probably the only method to analyse stability.

**Apparatus Required:** Second order system study unit. Function generator, wires, multimeter, CRO

#### **Circuit Diagram:**



#### **Procedure:**

1. Connections are made as per the circuit diagram.
2. A sinusoidal signal with amplitude of 1V is applied to the circuit.
3. The frequency is varied in steps and at each step frequency, phase angle, output is noted down.
4. A frequency response characteristic is plotted.
5. From the graph note down  $M_R$ ,  $\omega_R$ ,  $\omega_C$ .

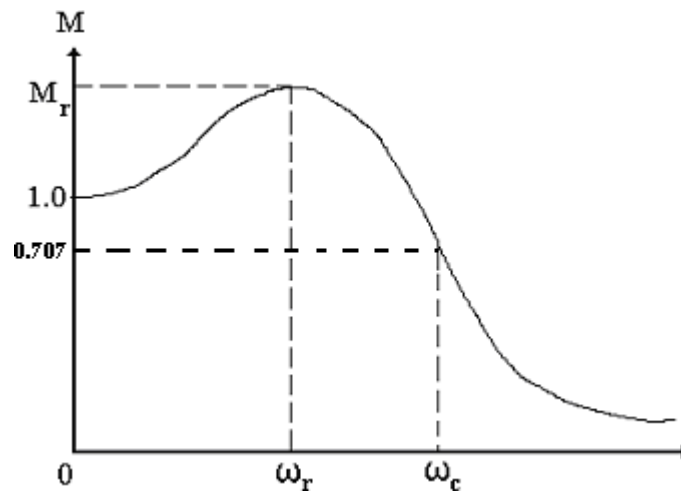
#### **Frequency domain specifications:**

$$\omega_r = \omega_n \sqrt{1 - 2\zeta^2}$$

$$M_R = 1/2\zeta \sqrt{1 - \zeta^2}$$

$$\omega_d = \omega_n \sqrt{1 - \zeta^2}$$

**Graph:**



**Result:**

**Viva questions:**

1. State the advantages and limitations of frequency domain approach.
2. Define bandwidth.

## Experiment No.8

### DC POSITION CONTROL

**Aim:** To simulate a DC position control system and hence to find the step response using MATLAB.

**Theory:**

DC position control system which we study here is used control the position of shaft, by use of potentiometer error detector. The error is to be amplified by the amplifier and must be given to the armature controlled motor whose shaft position will get controlled as per the control signal. The motor shaft is coupled to the load through gearing arrangement with ratio  $N_1/N_2$ . Position control systems have innumerable applications namely machine tool position Control, constant tension control of sheet rolls in paper mills, control of sheet metal thickness in hot rolling mills, radar tracking system, missile guidance systems, inertial guidance, In armature controlled DC servomotor the excitation of the field winding is kept constant and torque is varied by varying the applied voltage connected to the armature. Here the servo system is used to position the load shaft in which the driving motor is geared to the load to be varied.

**Apparatus required:** PC loaded with MATLAB

Formation of mathematical model:

$K_p$  = gain of the potentiometer of error detector.

$K_a$  = gain of the amplifier.

$K_b$  = back emf constant.

$K_t$  = torque constant.

$J$  = equivalent moment of inertia of motor and load referred to motor.

$B$  = equivalent viscous friction of motor and load referred to motor

For potentiometer error detector

$$E(s) = K_p[R(s) - C(s)]$$

For amplifier,

$$E_a(s) = K_a E(s)$$

For armature controlled DC motor,

$i_f$  = constant as flux  $\Phi$  is constant.

$$T = K_t i_a.$$

$e_b \propto \theta$  (angular displacement of the DC motor shaft)

$$e_b = K_b d\theta/dt$$

For the armature circuit,

$$e_b = e_b + L_a di_a/dt + i_a R_a$$

Taking Laplace transforms,

$$E_b(s) = K_b s\theta(s)$$

$$E_a(s) = E_b(s) + I_a(s) [R_a + sL_a]$$

$$T = k_t I_a(s)$$

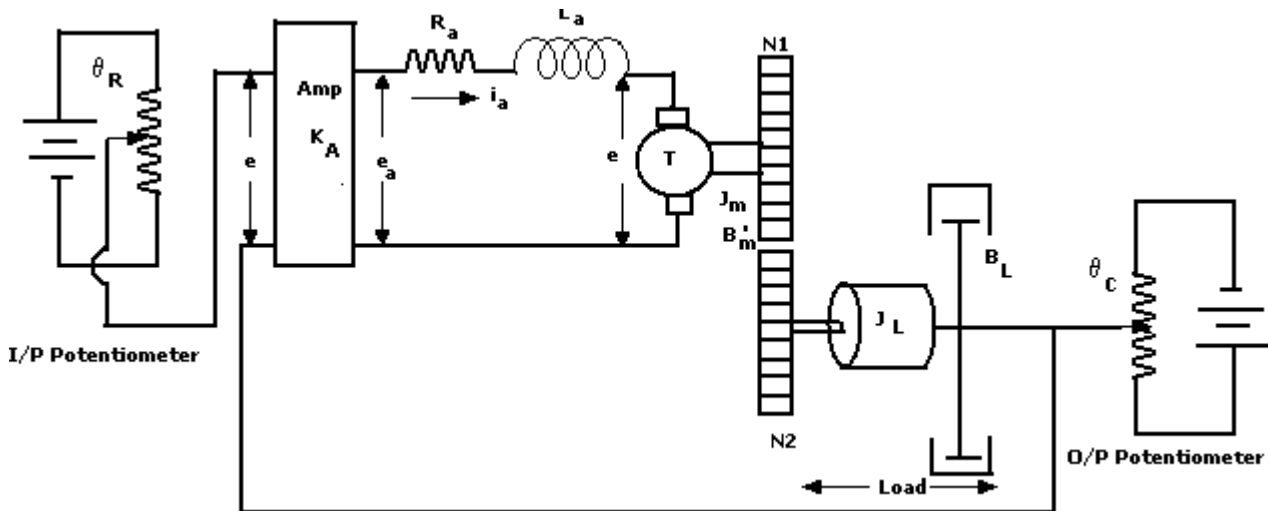
Now, torque is utilized to drive load = shaft of motor.

$$T = k_t I_a(s) = J \frac{d^2\theta}{dt^2} + f \frac{d\theta}{dt}$$

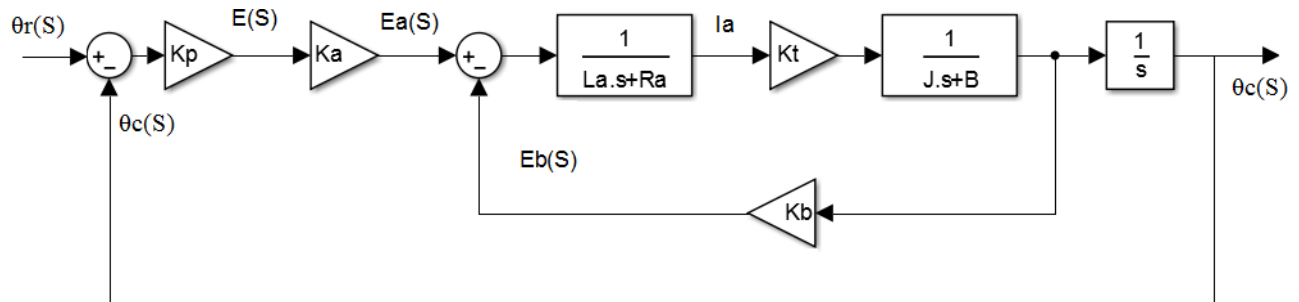
After simplification,

$$\theta(s)/E_a(s) = \frac{K_t}{s[J_s + f][R_a + sL_a] + K_t K_b s}$$

### Circuit Diagram:



### Block Diagram:

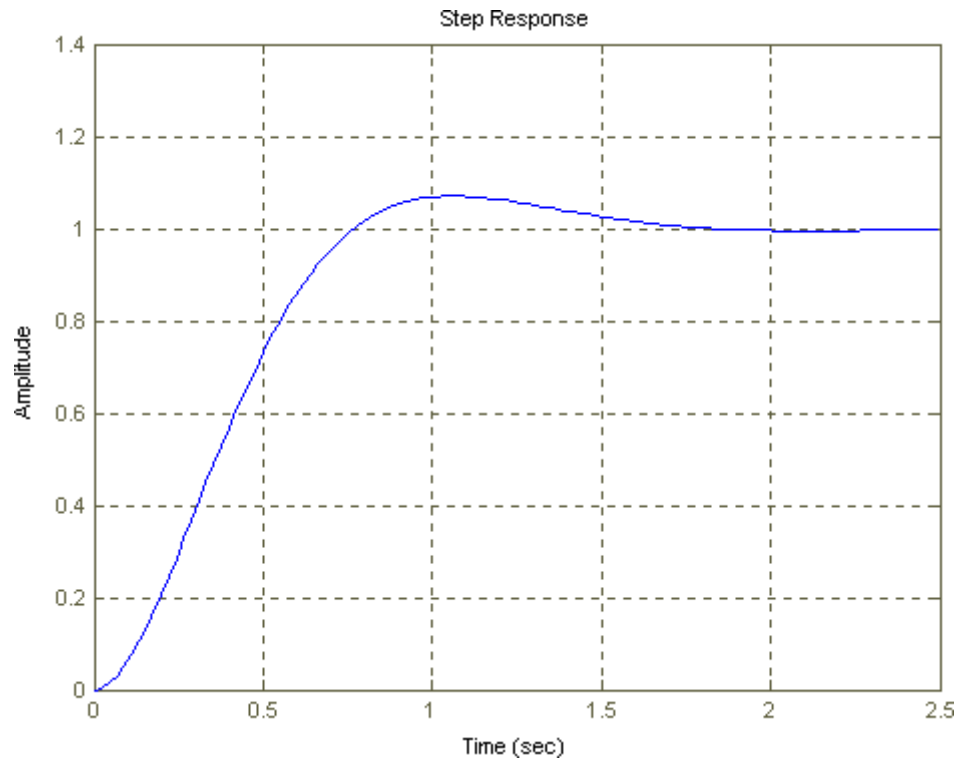


### Procedure:

1. Develop the complete block diagram of the given schematic diagram of position control servomechanism using simulink.
2.  $K_p$  is the variable and set it to some suitable value to simulate over, under damped conditions of the system.
3. From the response curve obtained determine the time domain specifications.

**Typical example:**

$k_P = (24/\pi)$ ,  $k_A = 10$ ,  $k_T = 6 \cdot 10^{-5}$  Nm/amps,  $J = 5.4 \cdot 10^{-5}$  kg-m<sup>2</sup>,  $B = 4 \cdot 10^{-4}$  Nm rad/sec  
 $R_a = 0.1$  ohms,  $L_a = 0.001$  H.

**Result:****Viva questions:**

1. What do you mean by DC position control?
2. What are the applications of DC servo mechanism



## Experiment No.9

### BODE PLOT

**AIM:** Obtain the phase margin and gain margin for a given transfer function by drawing bode plots and verify the same using MATLAB.

#### **Theory:**

One of the most useful representation of transfer function is a logarithmic plot which consists of two graphs, one giving the logarithm of  $[G(j\omega)]$  and the other phase angle of  $G(j\omega)$  both plotted against frequency in logarithmic scale. These plots are called bode plots. The main advantage of using bode diagram is that the multiplication of magnitudes can be converted into addition. Bode plots are a good alternative to the Nyquist plots.

#### Frequency response specifications:

1. Gain cross over frequency  $w_{gc}$  = It is the frequency at which magnitude of  $G(j\omega) H(j\omega)$  is unity ie 1.
2. Phase cross over frequency  $w_{pc}$  = It is the frequency at which phase angle of  $G(j\omega) H(j\omega)$  is  $-180^\circ$
3. Gain margin G.M = It is defined as the margin in gain allowable by which gain can be increased till system reaches on the verge of instability. Mathematically it is defined as the reciprocal of the magnitude of the  $G(j\omega) H(j\omega)$  measured at phase cross over frequency.
4. Phase margin P.M = Amount of additional phase lag which can be introduced in the system till system reaches on the verge of instability. Mathematically it can be defined as  
$$P.M = 180^\circ + \angle G(j\omega)H(j\omega) \text{ at } \omega=w_{gc}$$

#### Procedure:

- 1) Open the MATLAB command window.
- 2) Click on file/new/M file to open the MATLAB editor window. In MATLAB editor window enter the program
- 3) Save the program as .M file.
- 4) Execute the program by selecting run.
- 5) Note down the gain crossover, Phase crossover frequencies, gain margin and phase margin from the plot.
- 6) Also copy the plot.

#### Typical Problem

Transfer function :  $36 / (s^3 + 6s^2 + 11s + 6)$  .

```
num = [36];  
den = [1 6 11 6];  
sys = tf (num , den)  
bode (sys)  
margin(sys)
```

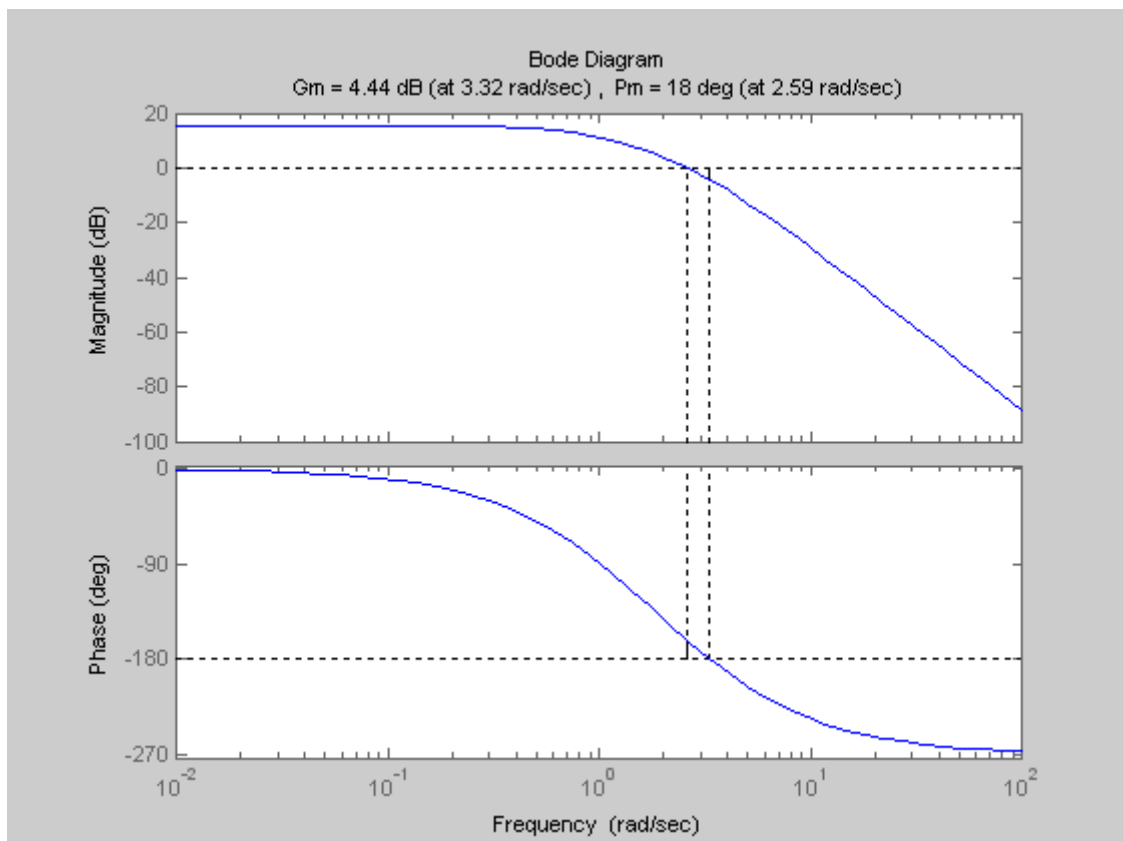
Examples

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Find gain Margin and Phase Margin:

a)  $TF = \frac{0.5}{S(S+1)(S+0.5)}$ .

b)  $TF = \frac{10}{(S+2)(S+3)(S+4)}$ .



**Result:**

**Viva questions:**

1. write MATLAB program to find the value of k for a given phase margin
2. write MATLAB program to find the value of k for a given gain margin

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## Experiment No.10 (a)

### ROOT LOCUS DIAGRAM

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**a)AIM:** To obtain Root locus of a given T. F. and hence finding breakaway point, intersection point on imaginary axis and to draw the Nyquist plot for the given transfer function using MATLAB.

#### **Theory:**

Root locus technique is used to find the roots of the characteristics equation. This technique provides a graphical method of plotting the locus of the roots in the s plane as a given parameter usually gain is varied over the complete range of values. This method brings in to focus the complete dynamic response of the system. By using root locus method the designer can predict the effects location of closed loop poles by varying the gain value or adding open loop poles and/or open loop zeroes. The closed loop poles are the roots of the characteristic equation.

Various terms related to root locus technique that we will use frequently in this article.

1. Characteristic Equation Related to Root Locus Technique :  $1 + G(s)H(s) = 0$  is known as characteristic equation. Now on differentiating the characteristic equation and on equating  $dk/ds$  equals to zero, we can get break away points.
2. Break away Points: Suppose two root loci which start from pole and moves in opposite direction collide with each other such that after collision they start moving in different directions in the symmetrical way. Or the break away points at which multiple roots of the characteristic equation  $1 + G(s)H(s) = 0$  occur. The value of K is maximum at the points where the branches of root loci break away. Break away points may be real, imaginary or complex.
3. Break in Point: Condition of break in to be there on the plot is written below : Root locus must be present between two adjacent zeros on the real axis.
4. Centre of Gravity: It is also known centroid and is defined as the point on the plot from where all the asymptotes start. Mathematically, it is calculated by the difference of summation of poles and zeros in the transfer function when divided by the difference of total number of poles and total number of zeros. Centre of gravity is always real & it is denoted by  $\sigma_A$ . Where N is number of poles & M is number of zeros.

$$\sigma_A = \frac{(\text{Sum of real parts of poles}) - (\text{Sum of real parts of zeros})}{N - M}$$

5. Asymptotes of Root Loci: Asymptote originates from the centre of gravity or centroid and goes to infinity at definite some angle. Asymptotes provide direction to the root locus when they depart break away points.
6. Angle of Asymptotes: Asymptotes makes some angle with the real axis and this angle can be calculated from  $\text{Angle of asymptotes} = \frac{(2p + 1) \times 180}{N - M}$  the given formula, Where  $p = 0, 1, 2, \dots, (N-M-1)$
7. Angle of Arrival or Departure: We calculate angle of departure when there exists complex poles in the system. Angle of departure can be calculated as  $180 - \{(\text{sum of angles to a complex pole from the other poles}) - (\text{sum of angle to a complex pole from the zeros})\}$ .
8. Intersection of Root Locus with the Imaginary Axis : In order to find out the point of intersection root locus with imaginary axis, we have to use Routh Hurwitz criterion. First, we find the auxiliary equation then the corresponding value of K will give the value of the point of intersection.
9. Symmetry of Root Locus: Root locus is symmetric about the x axis or the real axis.

**Program Procedure:**

- 5) copy the plot obtained, note down the breaking point, intersection point.

**Program for root locus:**

Given ;  $G(s)H(s) = 10/S^4 + 8S^3 + 36 S^2 + 80 S$

Program:

```
p = [ 0  0  0  0  10 ];
```

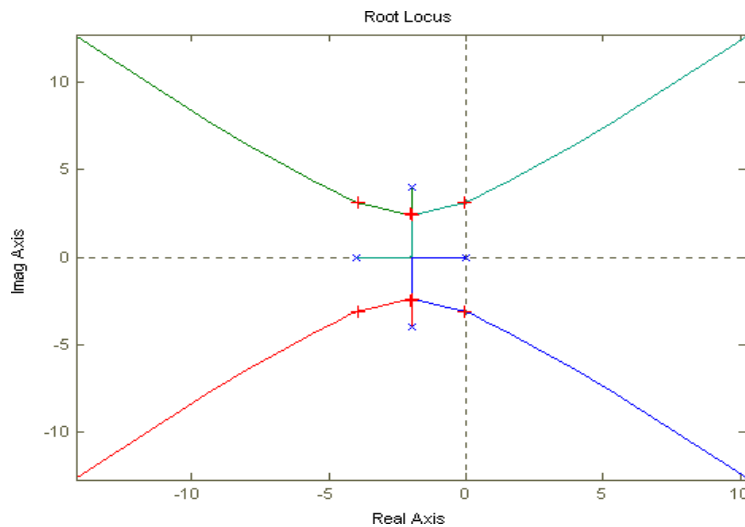
$$q = [1 \ 8 \ 36 \ 80 \ 0];$$
$$\text{sys} = \text{tf}(p,q)$$
$$Z_{pk}(sys)$$
$$\text{rlocus}(\text{sys})$$

Output in the command window:

Transfer function:

36

-----

$$s^3 + 6s^2 + 11s + 6$$


**Result:**

### **Viva questions:**

1. What is the significance of root locus method?
2. What are the rules of construction of root locus?
3. What are the disadvantages of Root Locus Method?
4. What are the advantages of root-locus method?
5. What is meant by Asymptotes and when it comes into picture in root-locus method?
6. What is meant by angle of arrival and angle of departure?
7. What you mean by breakaway and break-in points?
8. When do you expect a breaking point in a root-locus?

## Experiment No.10 (b)

### NYQUIST PLOT

#### Program for Nyquist plot:

Given the T.F= $60/(s+1)(s+2)(s+5)$

Programme:

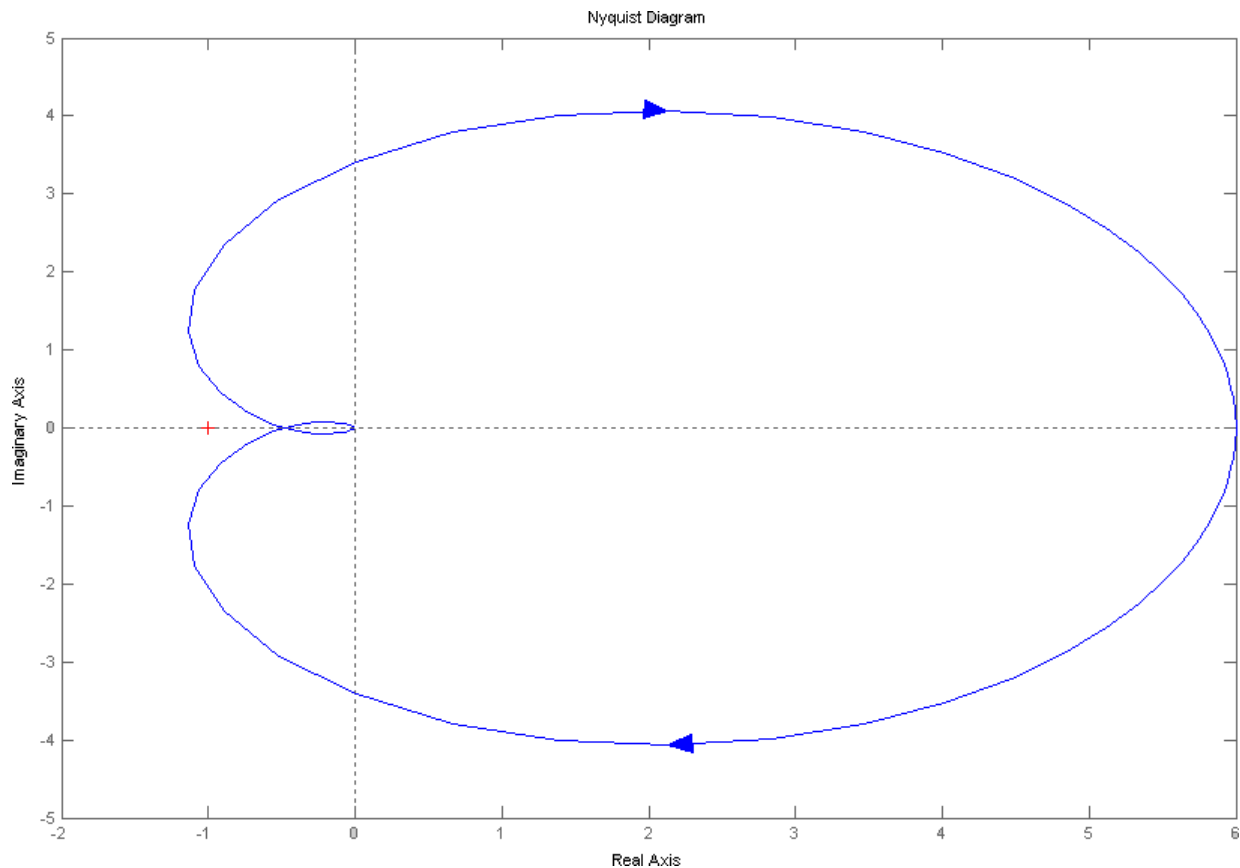
P=[60]

Q=[1 8 17 10]

Sys=tf(P,Q)

Nyquist(sys)

#### Result:



#### Examples

Draw the Nyquist plot for following T.F

1) T.F= $1/s(1+2s)(1+s)$

2) T.F= $(1+4s)/s^2(1+s)(1+2s)$

## Add on Experiment 1:

### **DIGITAL SIMULATION OF LINEAR SYSTEMS**

**AIM:** To digitally simulate the time response characteristics of Linear SISO systems using state variable formulation.

**APPARATU REQUIRED:** A PC with MATLAB package.

#### **THEORY:**

State Variable approach is a more general mathematical representation of a system, which, along with the output, yields information about the state of the system variables at some predetermined points along the flow of signals. It is a direct time-domain approach, which provides a basis for modern control theory and system optimization. SISO (single input single output) linear systems can be easily defined with transfer function analysis. The transfer function approach can be linked easily with the state variable approach.

The state model of a linear-time invariant system is given by the following equations:

$$\dot{X}(t) = A X(t) + B U(t) \text{ State equation}$$

$$Y(t) = C X(t) + D U(t) \text{ Output equation}$$

Where  $A = n \times n$  system matrix,

$B = n \times m$  input matrix,

$C = p \times n$  output matrix and

$D = p \times m$  transmission matrix,

#### **PROGRAMME:**

##### **OPEN LOOP RESPONSE (FIRST ORDER SYSTEM)**

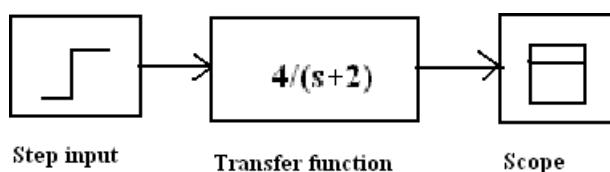
$$T.F = 4/(s+2)$$

##### **Response of system to Step and Impulse input**

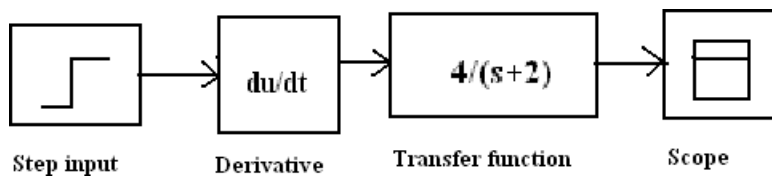
```
n=[4];          n=[4];
d=[1 2];        d=[1 2];
sys=tf(n,d);    sys=tf(n,d);
step(sys)       impulse(sys)
```

#### **SIMULINK**

Step Input Open Loop –I Order



### Impulse Input Open Loop –I Order

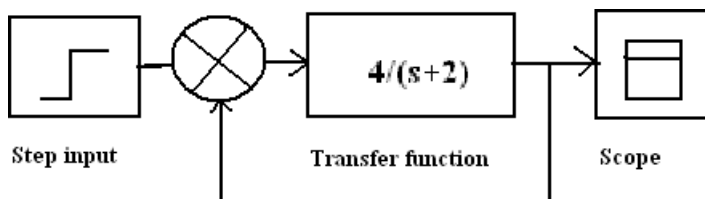


### Close Loop Response

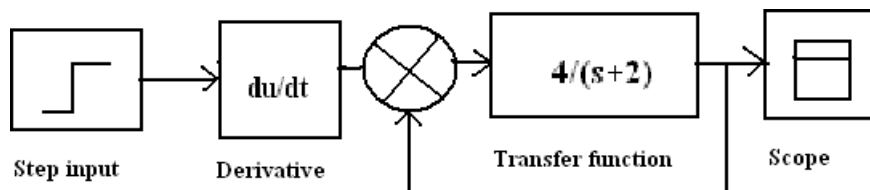
```
n=[4];  
d=[1 2];  
sys=tf(n,d);  
sys=feedback(sys,1,-1)  
step(sys)  
impulse(sys)
```

### SIMULINK

#### Step Input Close Loop –I Order



#### Impulse Input Close Loop –I Order



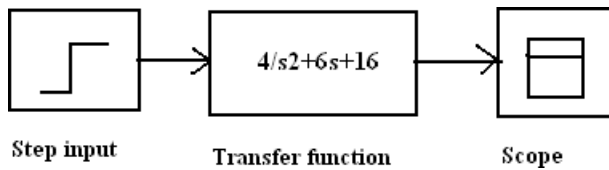
### SECOND ORDER SYSTEM

$$TF = \frac{4}{s^2 + 6s + 16}$$

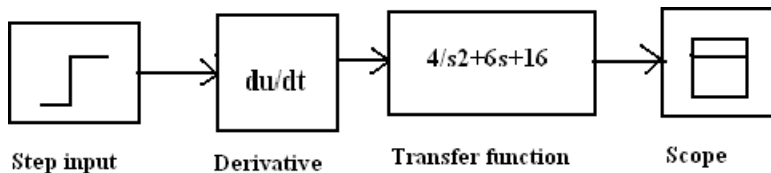
#### Open Loop Response

```
n=[4];  
d=[1 6 16];  
sys=tf(n,d);  
step(sys)  
impulse(sys)
```

Step Input Open Loop –II Order



Impulse Input Open Loop –II Order

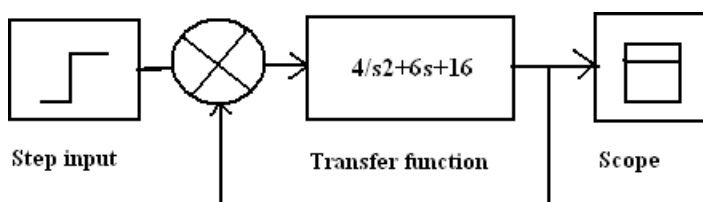


**Close Loop Response**

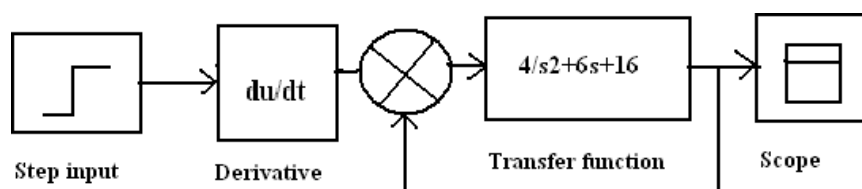
```
n=[4];  
d=[1 6 16];  
sys=tf(n,d);  
sys=feedback(sys,1,-1)  
step(sys)  
impulse(sys)
```

**SIMULINK**

Step Input Close Loop –I Order



Impulse Input Close Loop –II Order





**STATE SPACE EQUATION**

$A = \begin{bmatrix} -6 & -16 \\ 1 & 0 \end{bmatrix};$

$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix};$

$C = \begin{bmatrix} 0 & 4 \end{bmatrix};$

$D = \begin{bmatrix} 0 \end{bmatrix};$

$\text{Sys} = \text{ss}(A, B, C, D);$

$\text{step}(\text{sys})$

$\text{impulse}(\text{sys})$

**RESULT:**