Beijing-Dublin International College

SEMESTER II FINAL EXAMINATION – 2018/2019

School of Mathematics and Statistics

BDIC1032J Maths 4 (Advanced Mathematics; Engineering)

HEAD OF SCHOOL: Wenying Wu
MODULE LECTURERS: Yanru Ping, Wenqing Xu

Time Allowed: 90 minutes

Instructions for Candidates

Answer ALL questions. The marks that each question carry is written as shown.

BJUT Student ID: ________________ **UCD Student ID:** ________________

I have read and clearly understand the Examination Rules of both Beijing University of Technology and University College Dublin. I am aware of the Punishment for Violating the Rules of Beijing University of Technology and/or University College Dublin. I hereby promise to abide by the relevant rules and regulations by not giving or receiving any help during the exam. If caught violating the rules, I accept the punishment thereof.

Honesty Pledge： ____ **(Signature)**

Instructions for Invigilators

Non-programmable calculators are permitted. NO dictionaries are permitted.
No rough-work paper is to be provided for candidates.

SECTION A — GAP-FILLING QUESTIONS

This section is worth a total of **60** marks, with each question worth **5** marks.

1. Given $f(x+2y, y-3x) = x^2 + y^2$, we have $f(x, y) =$ ____________________.

2. Determine the limit

$$\lim_{(x,y)\to(0,0)} \frac{\sqrt[3]{xy+1}-1}{xy} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}.$$

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3. Given the function $z = \ln(\sqrt{x} + \sqrt{y})$, compute

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}.$$

4. Suppose an implicit function $z = z(x, y)$ is determined by an equation $F\left(\frac{x}{y}, \frac{z}{x}\right) = 0$, where $F(u, v)$ is a differentiable function. Then

$$dz = \text{\_\_\_\_\_\_\_\_\_\_}.$$

5. Given a function $z = f(x^2y, 3x+2y)$ where $f(u, v)$ is of second order continuous partial derivatives, we have $\frac{\partial z}{\partial x} =$ __________.

6. Let C be the curve determined by the parametric equations

$$\begin{cases} x &= \cos(\frac{\pi t}{2}), \\ y &= \sin(\frac{\pi t}{2}), \\ z &= \arctan t. \end{cases}$$

The equation of the tangent line to the curve C at the point when $t = 1$ is

____________________.

And the equation of the normal plane is given by

____________________.

7. Find the point on the plane $3x + y + z - 2 = 0$ such that it is the closest point from the origin $O(0, 0, 0)$ by using the Lagrange multiplier method. First write down the auxiliary function,

____

and then find the point: ____________________.

8. Given the function $f(x, y) = 2xy + x^2 + 2y^2 - 1$, find its extreme values, ____________________, and classify it as a maximum or a minimum: ____________________.

9. Calculate the iterated integral

$$\int_0^2 dx \int_x^2 e^{-y^2} dy = \underline{\hspace{6cm}}.$$

10. Suppose that $f(x,y)=\sqrt{1-x^2-y^2}-\frac{1}{\pi}\iint\limits_D f(x,y)dxdy$, and $D=\{(x,y)|x^2+y^2\le 1\}$. Evaluate

$$\iint\limits_D f(x,y)dxdy = \underline{\hspace{5cm}}.$$

11. Calculate the iterated integral

$$\int_0^1 dx \int_x^1 \frac{y}{x^2+y^2}dy = \underline{\hspace{5cm}}.$$

12. Find the directions in which the function $f(x,y)=\ln(x^2+y^2)$

(a) increases most rapidly at the point $P(1,1)$: ____________________;

(b) decreases most rapidly at the point $P(1,1)$: ____________________;

(c) has zero directional derivative at the $P(1,1)$: ____________________.

SECTION B — EXTENDED ANSWER QUESTIONS

Write your answers on the **Examination Book** provided.

This section is worth a total of **40** marks, with each question worth **10** marks.

13. Compute the following double integral by converting it to the polar coordinates

$$I=\iint\limits_{D:\ x^2+y^2\le 1}(3x^2+4y^2+y+1)dxdy.$$

(Hint: First sketch the region D. You may also use symmetry to simplify your calculations.)

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14. First express the following triple integral as an iterated integral in spherical coordinate system, and then evaluate the triple integral.

$$I = \iiint_{\Omega} \left(x^2 + y^2 + z^2\right) dv,$$

where $\Omega = \{(x, y, z) | x^2 + y^2 + z^2 \leq 4, \text{and } z \geq 0\}$.

15. Express the following triple integral as an iterated integral in cylindrical coordinates, and then find the result

$$I = \iiint_{\Omega} z dv,$$

where Ω is the region bounded by the paraboloid $z = x^2 + y^2$ and the hemisphere $z = \sqrt{6 - x^2 - y^2}$.

16. Suppose $f(x)$ is a continuous function over $[0, 1]$. Prove

$$\int_0^1 dx \int_0^x f(x) f(y) dy = \frac{1}{2} \left[\int_0^1 f(x) dx \right]^2.$$

Glossary

Auxiliary function	辅助函数
Cartesian form	坐标形式
Cone	锥面
Cylinder	柱面
Cylindrical coordinate	柱坐标
Derivable	可导的
Directional derivative	方向导数
Ellipsoid	椭球面
Extreme value	极值
Gradient	梯度
Iterated integral	累次积分
Lagrange multiplier method	拉格朗日乘数法
Maximum	极大值
Minimum	极小值
Normal line	法线
Normal plane	法平面
Paraboloid	抛物面
Partial derivative	偏导数
Polar form	极坐标形式
Sphere	球面
Spherical coordinate	球坐标
Tangent line	切线
Tangent plane	切平面
Total differential	全微分