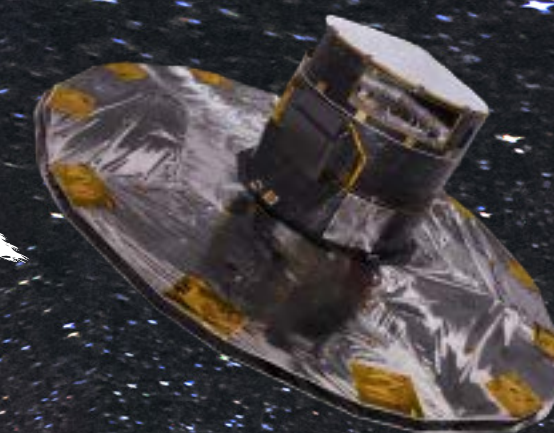




Galactic Archaeology

in the era of large surveys

Welcome
back!



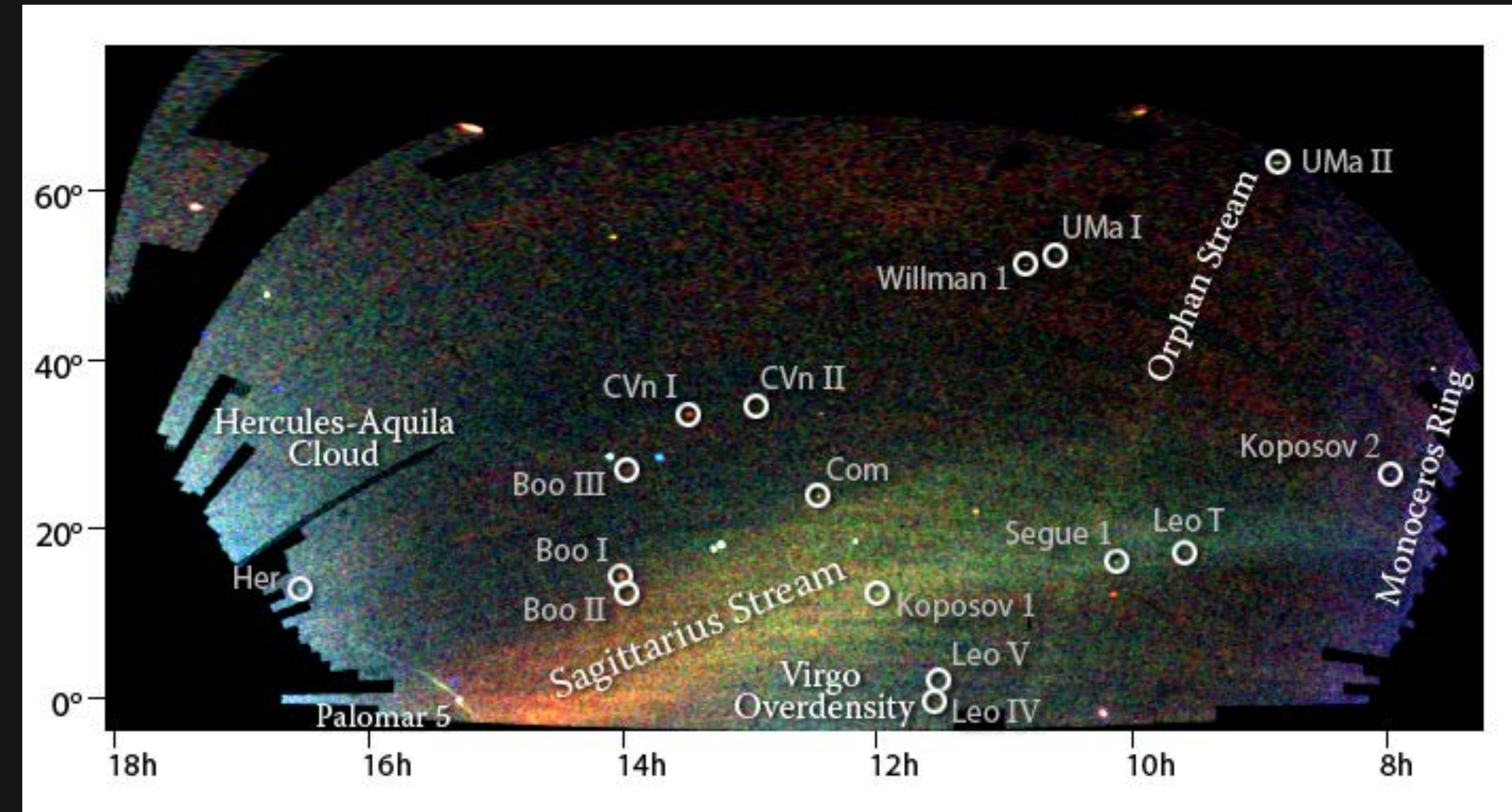
Previously...

- **What we want to know**

- Testing our physical understanding in Galactic scale
- Milky Way is an excellent test site!

- **(Dynamical) Overview of our Milky Way**

- Stellar Disc(s) & Spiral arms
 - Thin (younger) & Thick (older) Discs: Rotationally supported
 - Spiral arms: Very young & hot stars, HII gas; Density wave...?
- Bar & Bulge
 - Bar: Rotates with Pattern Speed; Funnels gas to the inner Galaxy
 - Boxy/Peanut Bulge: Dynamical origin from Bar
- Stellar Halo & Streams
 - Made of (very) old stars; Very low density; Velocity dispersion supported
 - Full of Streams resulted from tidal disruption of various satellite bodies in the halo
- (Dark Halo & Rotation curve) — we will cover it today



Lecture 8

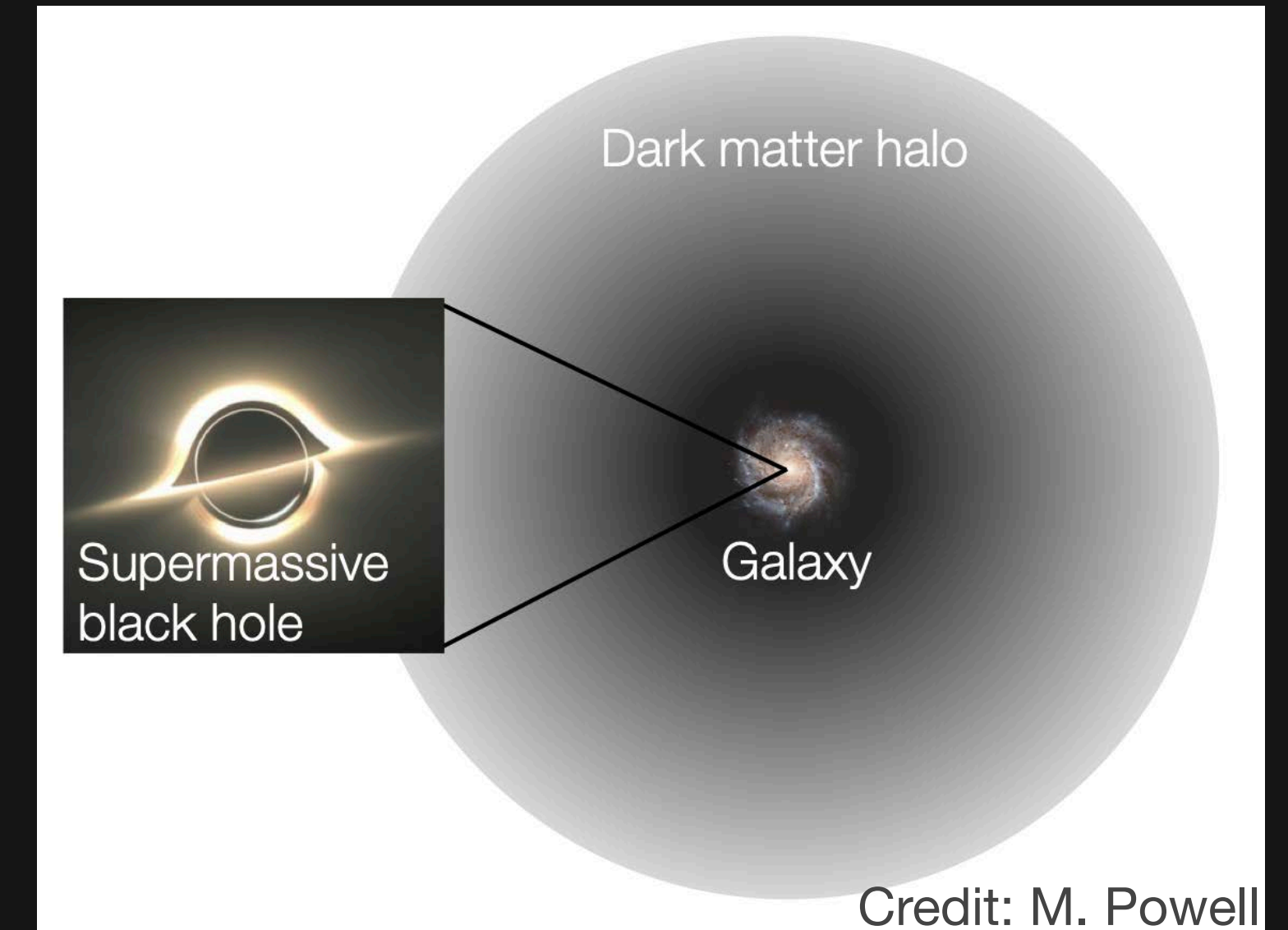
- **(Dynamical) Overview of our Milky Way**
 - Dark Halo & Rotation curve
- **Various dynamical features**
 - Virial Theorem
 - Kinematics & Dynamics
 - Stellar Orbit
 - Integrals-of-motion (Actions)



Dark Halo

- **Dark Halo**

- The first “pocket” for galaxy building
- The most dominating component of the Galaxy ($\sim 10^{12} M_{\odot}$)
- Usually described by power-law density profile
- Evidences (of Dark halo or dark matter in general):
 - Galactic Rotation curve: not Keplerian
 - Stellar stream: sensitive to the mass distribution (and also the potential dark sub halo!)
 - Gravitational lensing: implies the existence of invisible mass
 - X-ray gas in galaxy clusters: too hot to be explained by the gravity of the visible matter alone
 - Cosmic Microwave Background fluctuations: also require a substantial non-baryonic dark matter
 - Large-scale structure formation and evolution needs a substantial amount of mass



Galactic Rotation curve

- **Rotation curve**

- _ For a simplistic case, assume a spherical (symmetric) mass distribution, $M(r) = \int 4 \pi r^2 \rho(r) dr$

- _ For an object to stay in a **circular orbit** (e.g., v_R and $v_z \ll v_\phi$), the **centripetal** and **gravitational forces need to be balanced**

$$\Rightarrow \frac{m v(r)^2}{r} = \frac{G M(r) m}{r^2}, \text{ } M(r) \text{ is mass inclosed in radius } r$$

$$\Rightarrow v_{\text{circ}}(r) = \sqrt{\frac{G M(r)}{r}}$$

- _ Now, close to the Galactic centre, **assuming** $\rho(r) \sim \text{const.}$ (homogeneous),

$$\Rightarrow M(r) = \frac{4}{3} \pi r^3 \rho, \text{ and,}$$

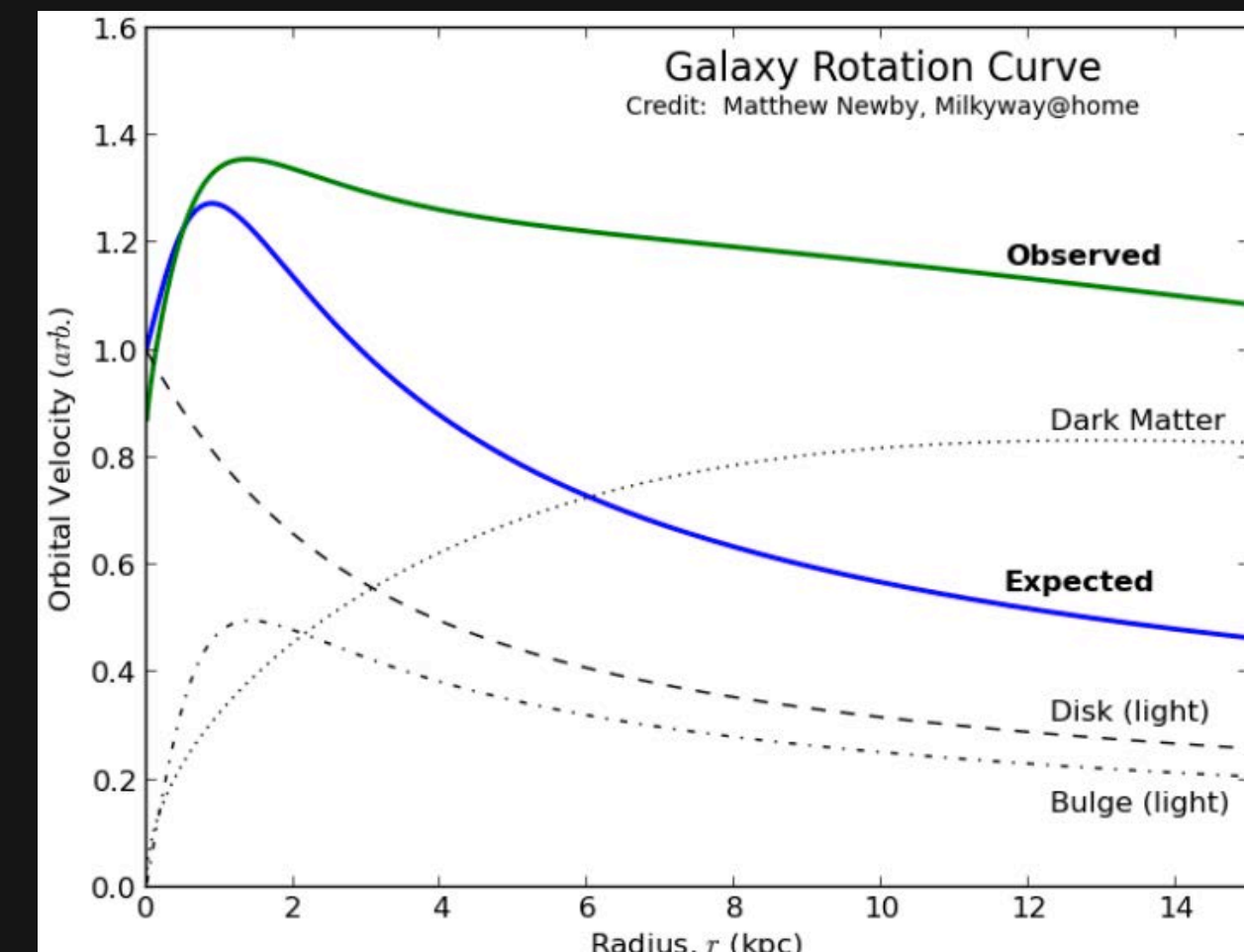
$$\Rightarrow v_{\text{circ}}(r) = r \sqrt{\frac{4}{3} \pi G \rho}, \text{ so } v_{\text{circ}}(r) \propto r \text{ (solid body rotation) ... roughly right but only for the very inner region of the Galaxy...}$$

- _ Imagine that we are on a circular **orbit enclosing some mass distribution**. Then (in practice), $M(r)$ can be approximated as a point mass M located at the centre ($r = 0$).

- _ And then from **Kepler's law**, we get, $\Rightarrow v_{\text{circ}}(r) \propto \frac{1}{\sqrt{r}}$

- _ But this is **very different from what we observe!** The gravity from the visible mass is insufficient for the observed rotational velocity!

- _ **More mass is required...** something “invisible”... because we see a **flat galactic rotation curve!**



Galactic Rotation curve

- Rotation curve

– To get the observed “flat” rotation curve, $v_{\text{circ}}(r) = \sqrt{\frac{G M(r)}{r}}$, we expect, $v_{\text{circ}}(r) = \text{const.}$

– This requires $M(r) \propto r$

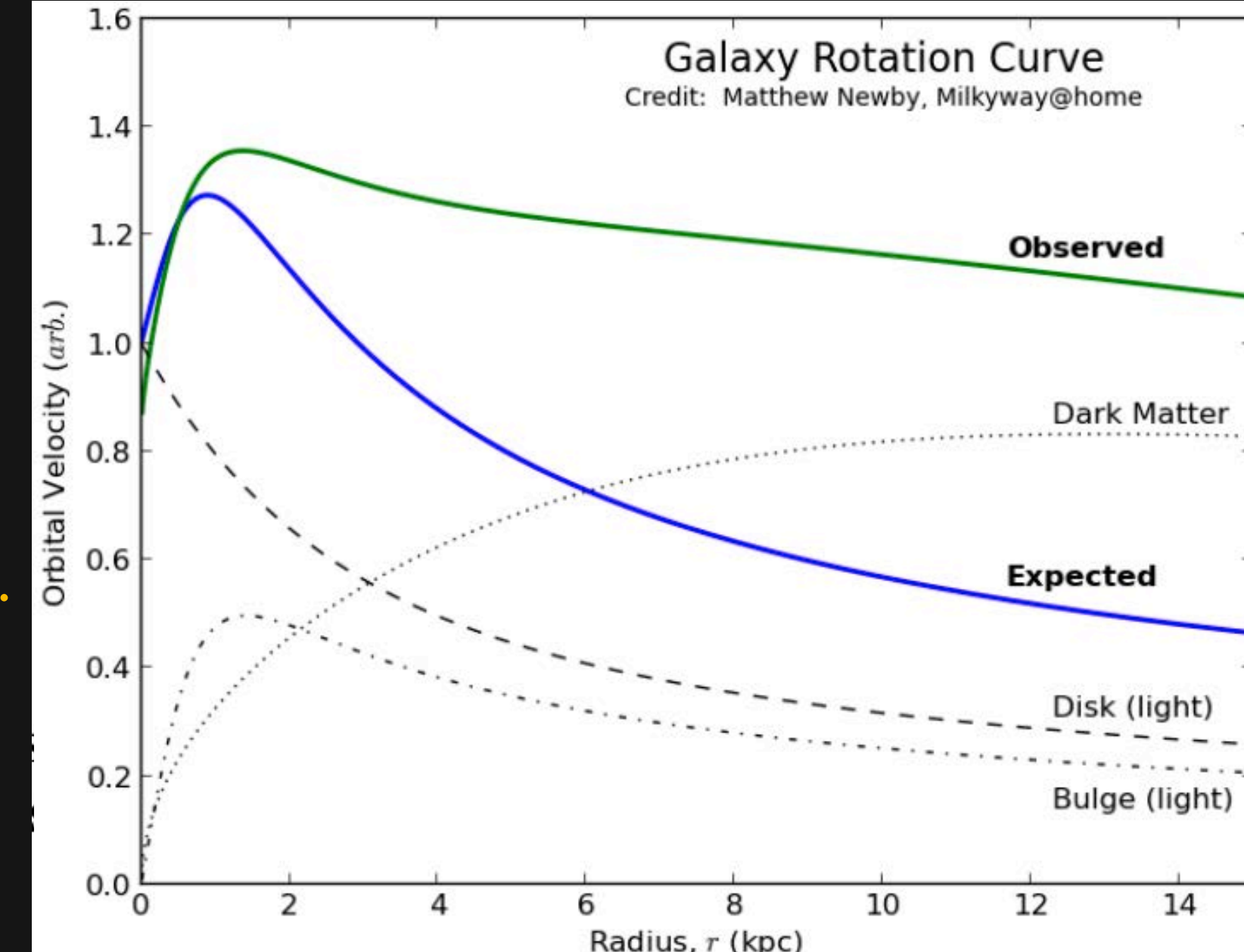
– Consider a power-law density profile, $\rho(r) = \rho_0 \left(\frac{r}{r_0} \right)^{-\alpha}$

$$\Rightarrow v_{\text{circ}}(r) = r^{1-\alpha/2} \sqrt{\frac{4 \pi G \rho_0 r_0^\alpha}{3-\alpha}}, \text{ with } \alpha < 3. \text{ (note that denominator is } 3 - \alpha, \text{ since } M(r) = \int_0^r 4 \pi r'^2 \rho(r') dr')$$

$$\Rightarrow v_{\text{circ}}(r) = \text{const.} \text{ when } \rho(r) \propto r^{-2} \text{ (i.e., } \alpha = 2 \text{, called } \textit{isothermal sphere})$$

...and indeed, $M(r) \propto r$

- There are variety of density profiles describing the dark matter halo (e.g., Hernquist, NFW)
- And indeed, the **dark matter dominates at large radii!**
- It also indicates us that dark matter wouldn't be found concentrated in the disc — it would make the velocity dispersion very high! (making “visible” structure, i.e., disc dynamically “hot”)
- Dark matter: non-collisional, not interacting by Electromagnetic nor Strong force, but by Gravity
 - Feels no pressure, no friction, won't dissipate energy! → will reach equilibrium under Virial Theorem! → won't collapse very dense



The Virial Theorem

- **Virtual Theorem**

- For a star bound to a galaxy, we know its total energy is: $\langle T \rangle + \langle U \rangle < 0$, $\langle T \rangle$ and $\langle U \rangle$ are time averaged kinetic and potential energy

- Consider the **moment of inertia** of this system, $I \equiv \sum_i^N m_i \mathbf{x}_i \cdot \mathbf{x}_i = \sum_i^N m_i x_i^2$

$$\Rightarrow \dot{I} = \sum_i^N m_i (\dot{\mathbf{x}}_i \cdot \mathbf{x}_i + \mathbf{x}_i \cdot \dot{\mathbf{x}}_i) = 2 \sum_i^N m_i \dot{\mathbf{x}}_i \cdot \mathbf{x}_i$$

$$\Rightarrow \ddot{I} = 2 \sum_i^N m_i (\ddot{\mathbf{x}}_i \cdot \mathbf{x}_i + \dot{\mathbf{x}}_i \cdot \dot{\mathbf{x}}_i) = 2 \sum_i^N m_i \ddot{\mathbf{x}}_i \cdot \mathbf{x}_i + 2 \sum_i^N m_i \dot{x}_i^2$$

- Now, we know the kinetic energy of the i th particle is $\frac{1}{2} m_i \dot{x}_i^2$. Also, $\sum_i^N m_i \ddot{\mathbf{x}}_i \cdot \mathbf{x}_i$ is related to the gravitational potential energy

$$\Rightarrow \ddot{I} = 2U + 4T$$

- And for the time averaged case

$$\Rightarrow \langle \ddot{I} \rangle = 2 \langle U \rangle + 4 \langle T \rangle$$

- Now, a **system at dynamical equilibrium**: the time-averaged moment of inertia, I , is constant, i.e., the **second derivative is zero!**

$$\Rightarrow 2 \langle T \rangle + \langle U \rangle = 0$$

- If we know the kinetic energy of the system (from the measured r.m.s. velocity dispersion) then we can estimate the system's total mass!

The Virial Theorem

- **Virial Theorem – A simple case**

- Let's imagine the circular orbit case:
- For an object to stay in a **circular orbit**, the **centripetal** and **gravitational forces need to be balanced**

$$\Rightarrow \frac{m v^2}{r} = \frac{G M m}{r^2}$$

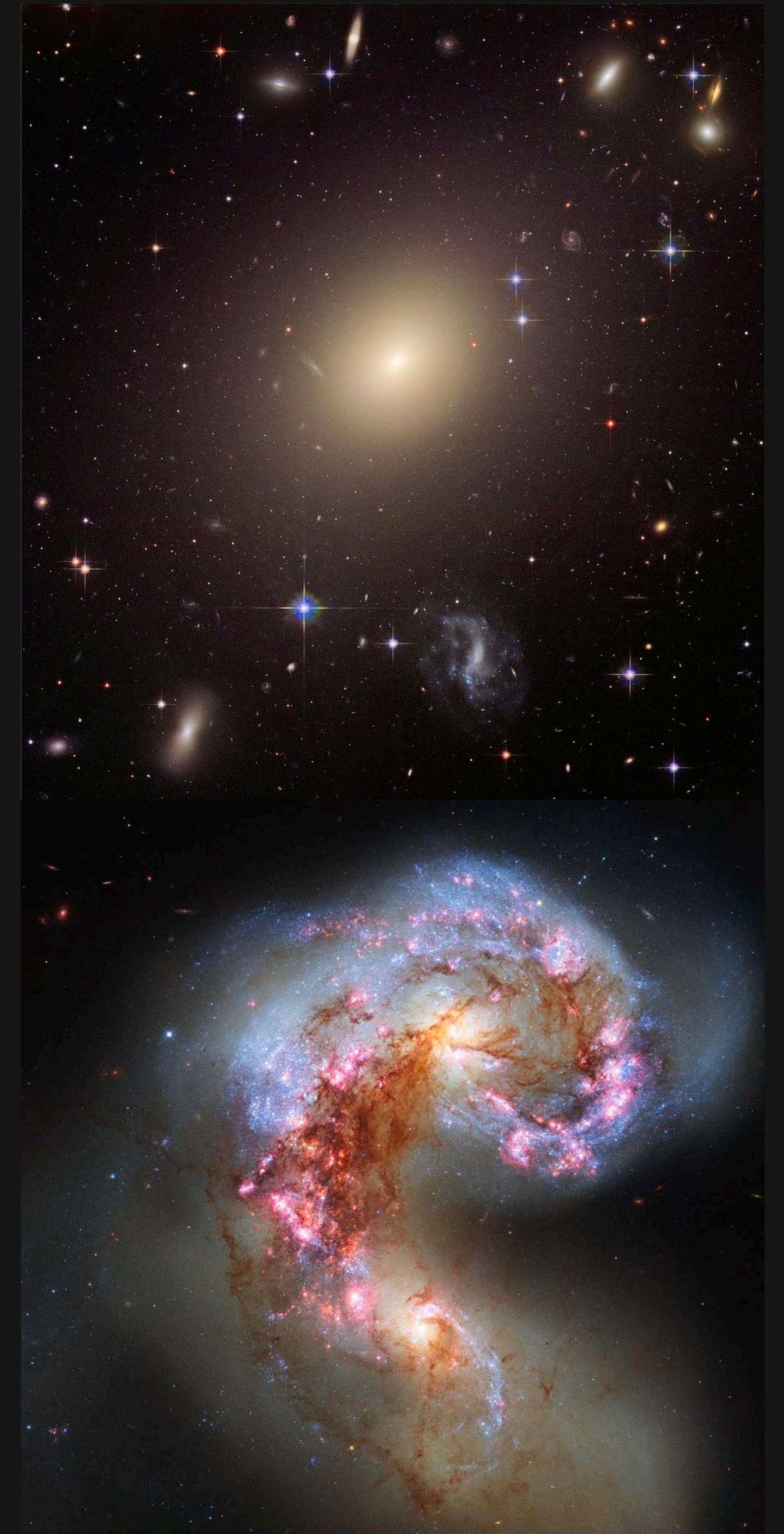
$$\Rightarrow m v^2 = \frac{G M m}{r}$$

- We know the kinetic energy is $\frac{1}{2}m v^2$ and potential energy is $-\frac{G M m}{r}$

$$\Rightarrow 2T + U = 0$$

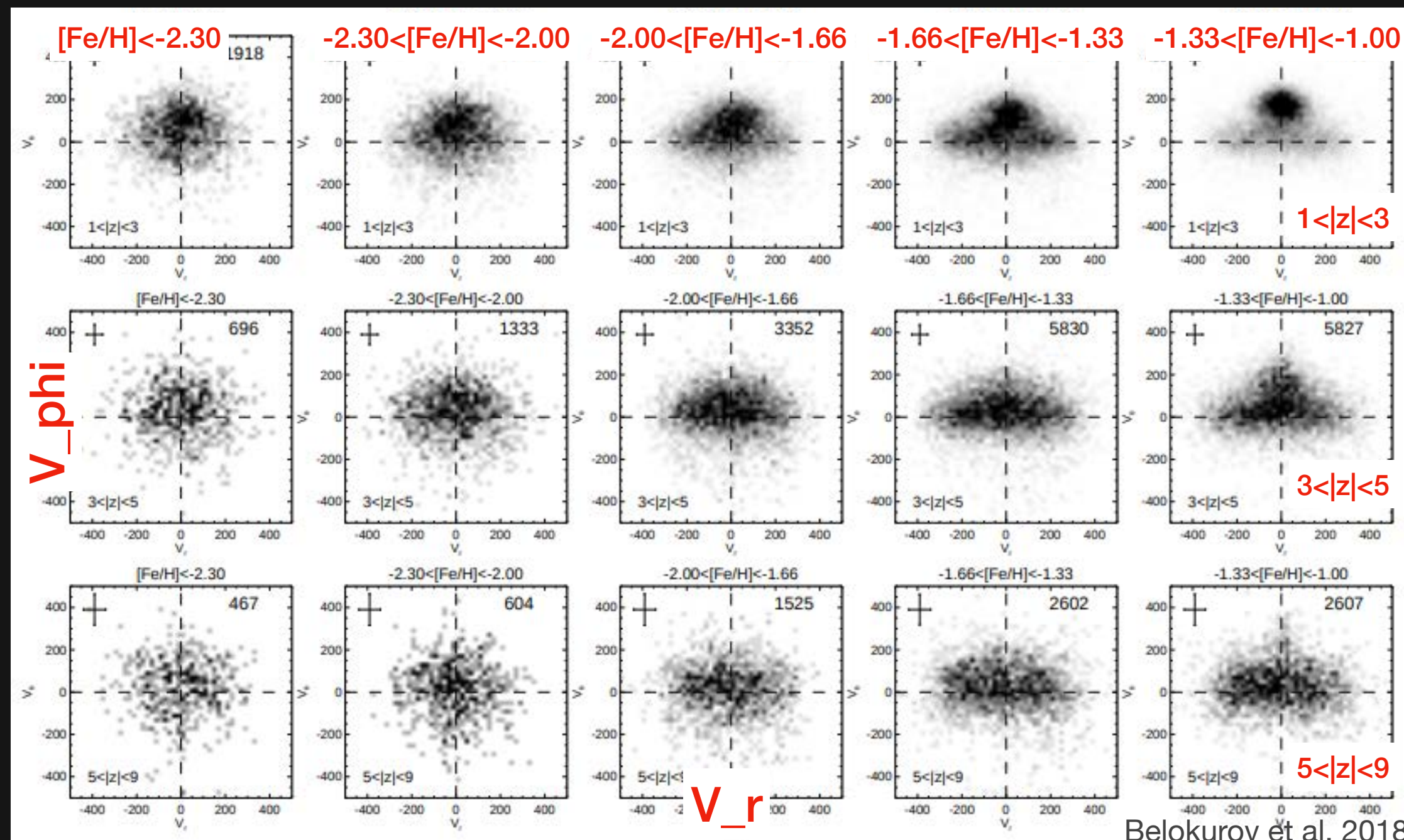
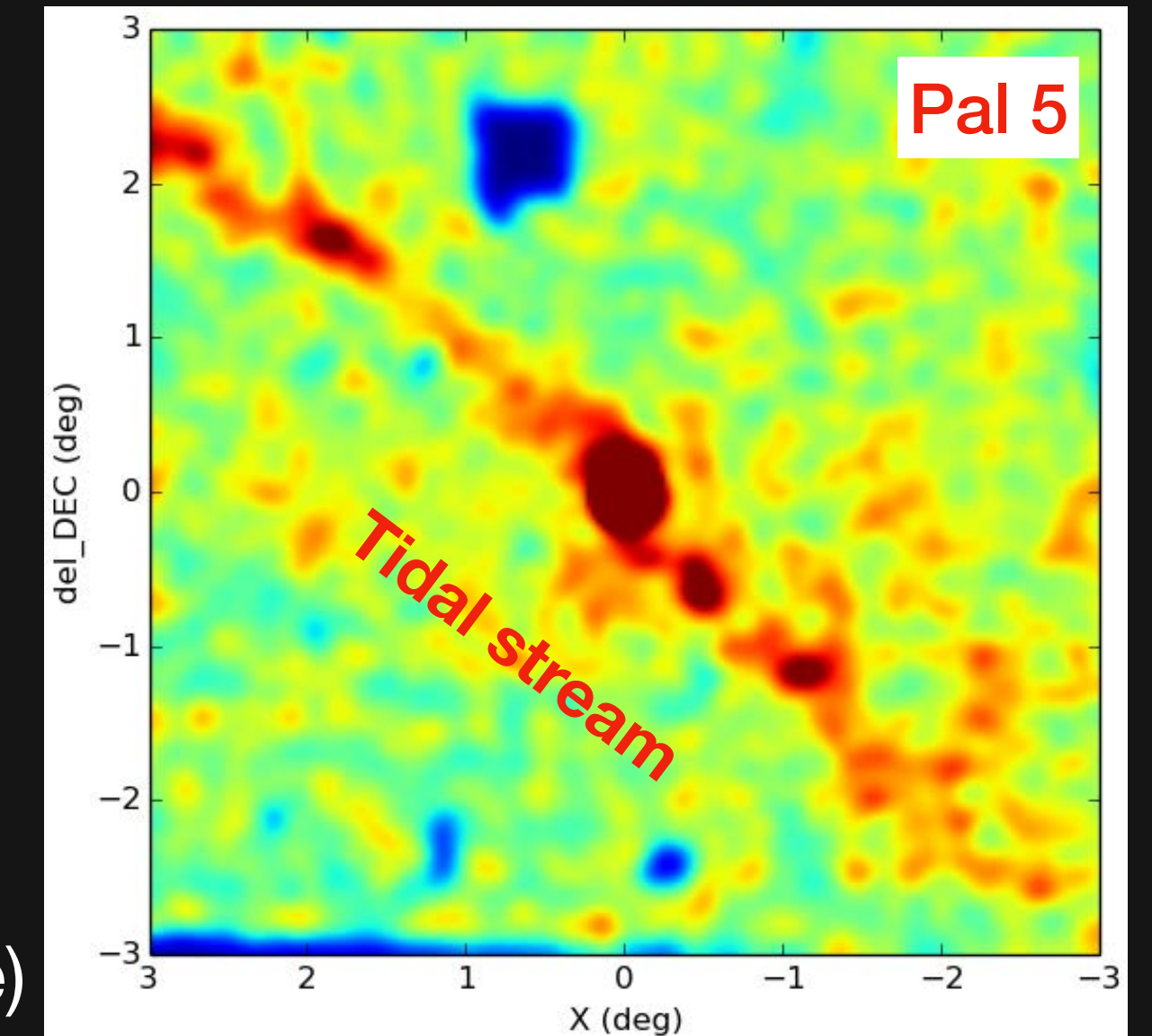
The Virial Theorem

- Virial Theorem **can** be used:
 - Systems of stars at a steady equilibrium state — macroscopic properties do not change over time
 - Elliptical galaxies
 - Evolved globular clusters, e.g. globular clusters
 - Evolved galaxies clusters
- Virial Theorem **cannot** be used:
 - Systems of stars NOT at a steady equilibrium state
 - Merging galaxies
 - Newly formed star clusters
 - Clusters of galaxies that are still forming/still have infalling galaxies
- **Virial theorem** provides easy (...but rough) estimates on various **key properties of the system** (including the total mass)

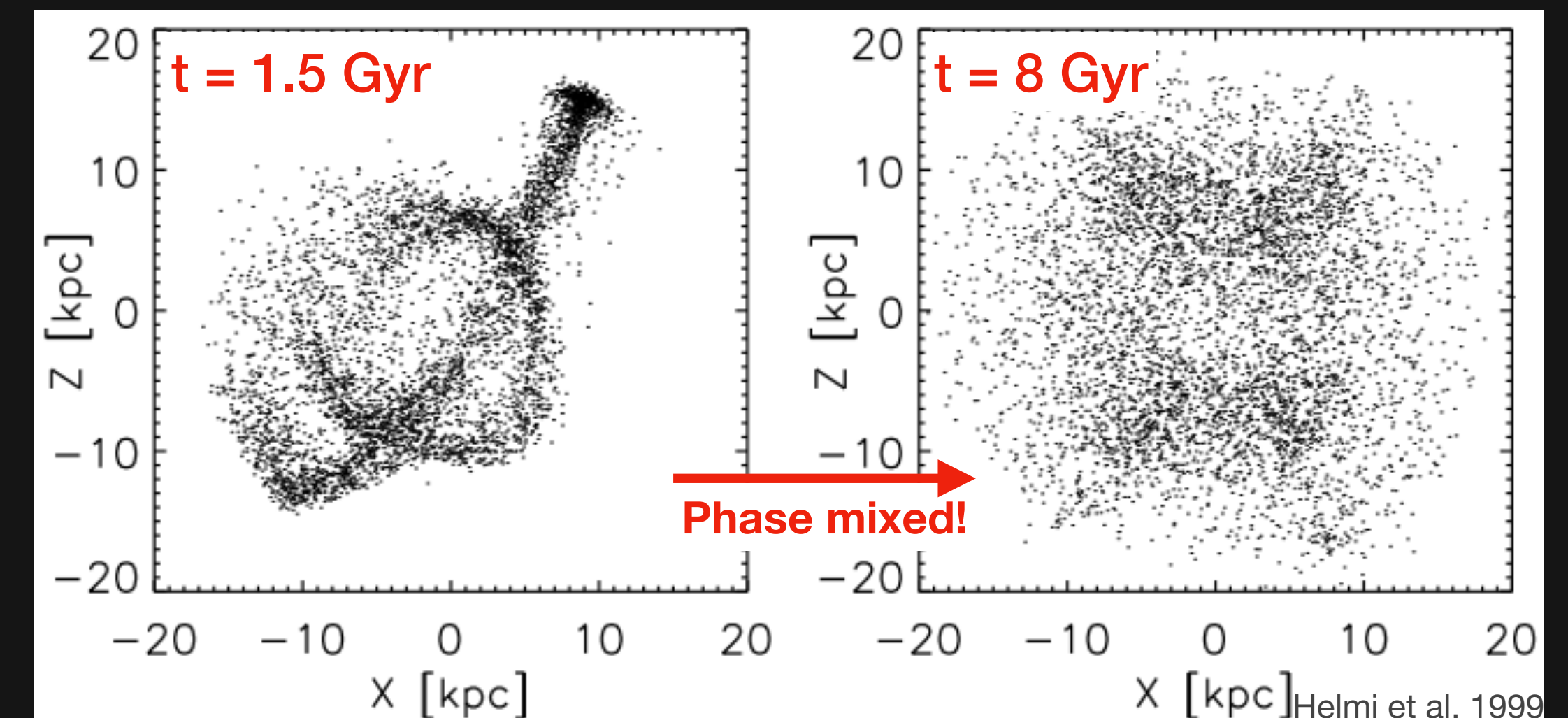


Structural view

- Various perspectives...
 - **Spatial distribution** in configuration space (or even, on the sky)
 - **substructure as a spatial correlation**
 - not-so-smooth footprint, and complex selection function
 - **Current kinematics**: velocity components
 - distribution, mean, and dispersion
 - directly looking for currently **“co-moving” groups** (e.g., clustering in velocity space)



Belokurov et al. 2018



Structural view

- More “dynamic” perspective
 - **Orbital parameters** (e.g., energy, eccentricity, inclination from orbital integration)
 - **Action-angle variables**
 - a set of **canonical coordinates** on phase space
 - for a closed (& periodic) orbit, **action, J , is constant** (i.e., integrals-of-motion)
 - for an axisymmetric potential:
 - **action variables** can **characterise the orbit of an object** (e.g., star) in **radial** (J_R), **azimuthal** ($J_\phi = L_z$), and **vertical** (J_z) **components**
 - what the orbit is
 - **angles** are **conjugate variables** and constantly increase from 0 to 2π
 - where on the orbit
 - **adiabatic invariant**
 - stays approximately constant over the slow change of system (e.g., slow growth of the MW)
 - “**phase mix**” is **not necessarily a problem!**

