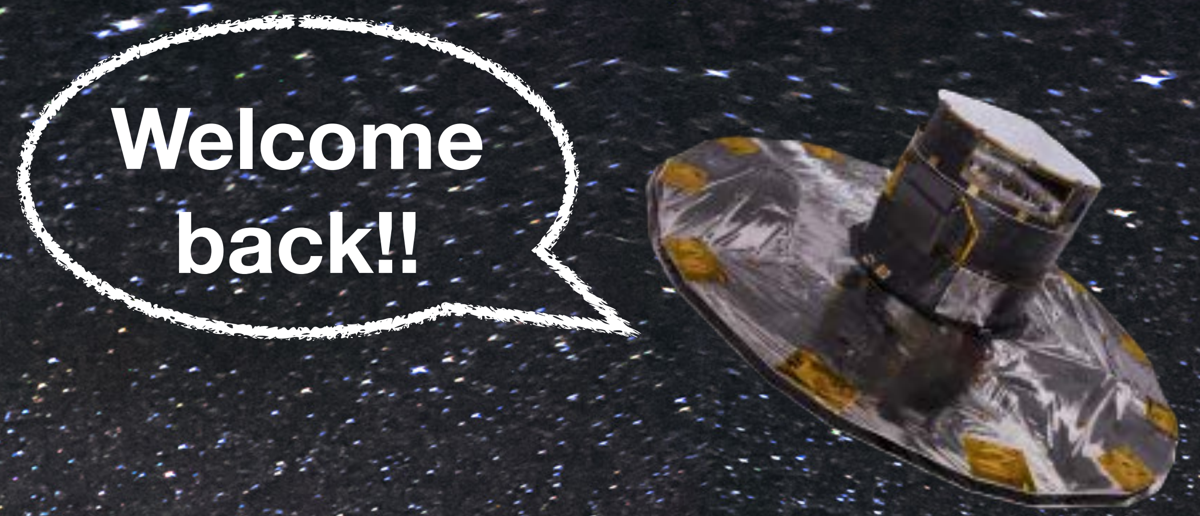
# Galactic Archaeology

## in the era of large surveys

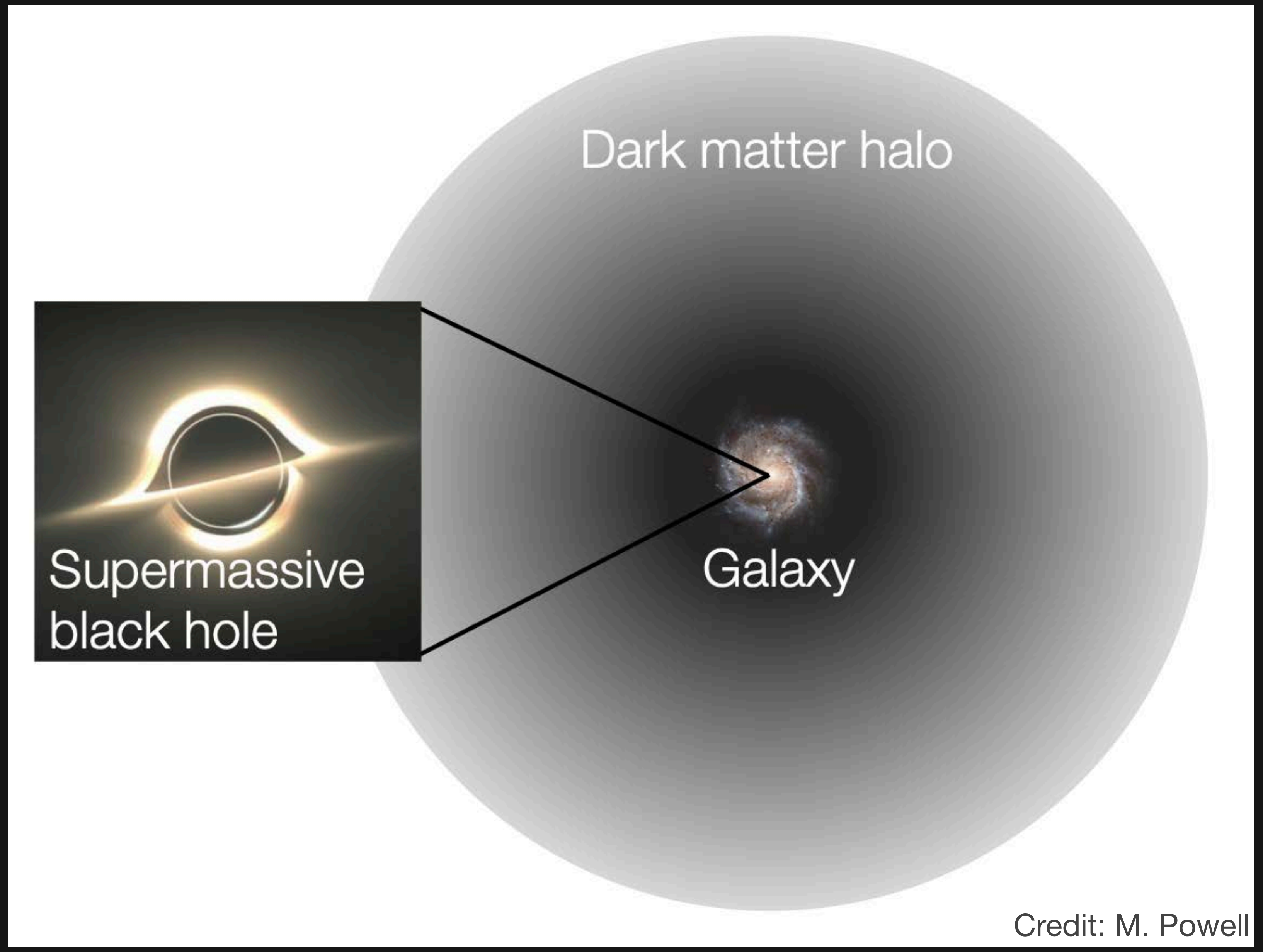


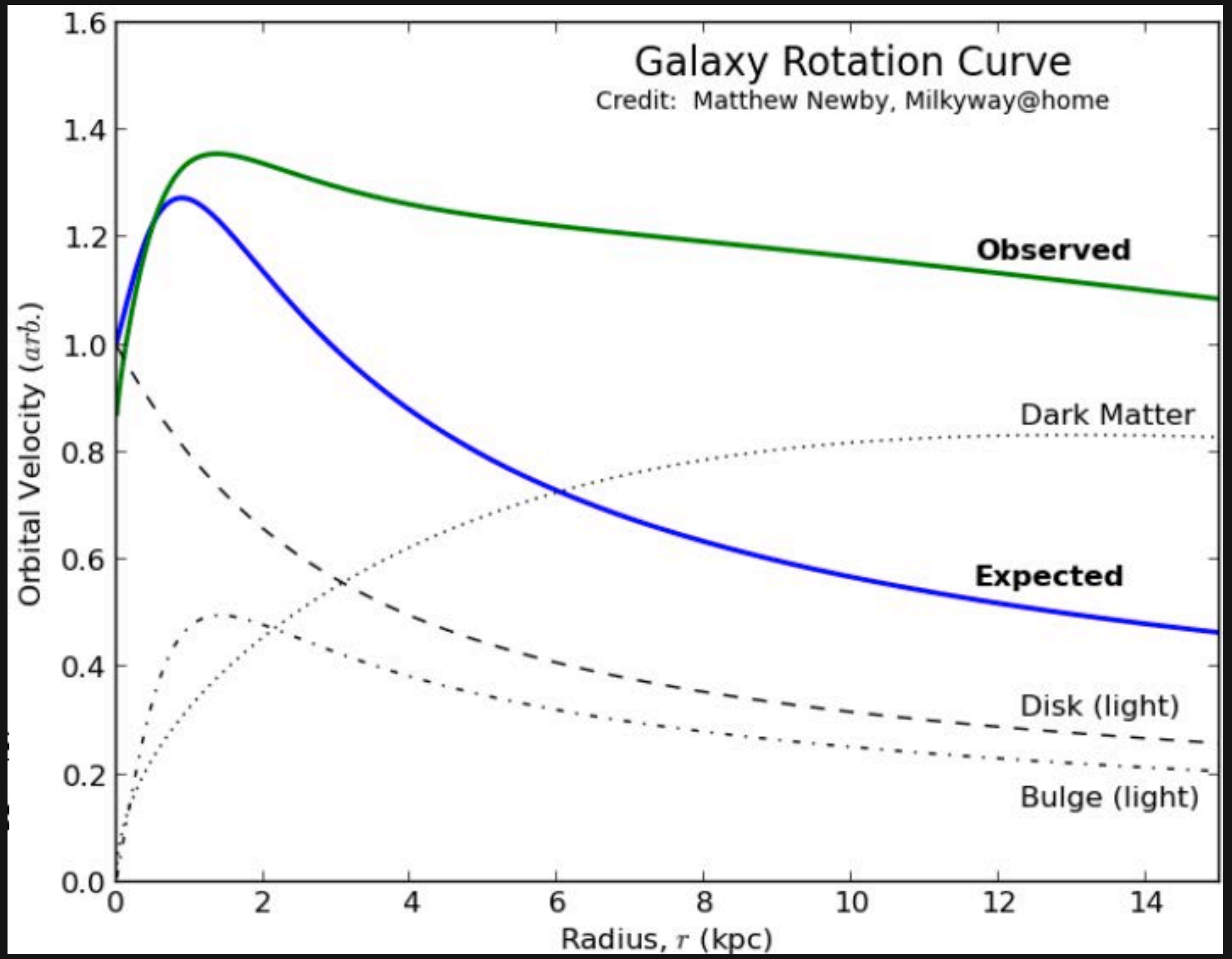



GyuChul Myeong

# Previously…

- **(Dynamical) Overview of our Milky Way**
  - Dark Halo & Rotation curve
    - Most massive component
    - Non-Keplerian rotation curve
    - Power-low density profile
    - Dominates at large radii
- **Various dynamical features**
  - Virial Theorem
    - For a system in dynamical equilibrium
    - System’s kinetic and potential energy are interconnected
    - Simple observations → (rough) mass estimation of the system
  - Kinematics & Dynamics
    - Provides structural understanding of the Galaxy
    - Can retrieve the past state information of the Galaxy!



# Lecture 8

- **Various dynamical features**
  - Stellar Orbit
  - Integrals-of-motion (Actions)
  - Stellar Stream
- ***Gaia* mission (if time allows)**
  - Brief overview
  - Astrometry

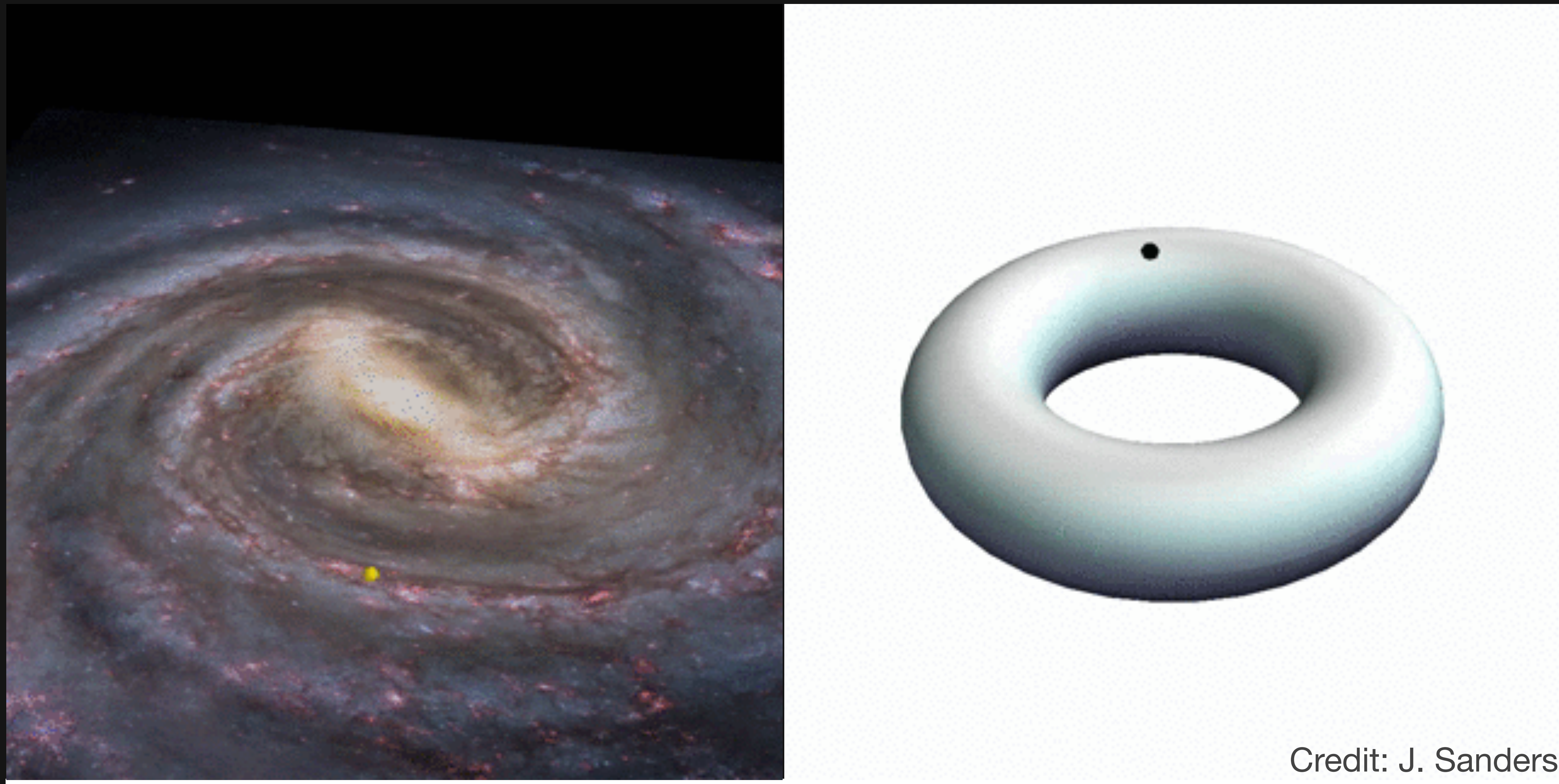
# Stellar Orbit

- **What is Star's "Orbit"?**
  - A trajectory of a star in motion through space as a function of time
  - Follows Hamiltonian mechanics! (i.e., defined by the position and momentum)
- **Hamiltonian mechanics**
  - Describe the state of a physical system & its evolution over time using canonical coordinates.
  - Canonical coordinates: pairs of position, $\mathbf{q}$, and momentum, $\mathbf{p}$. The pair, $(\mathbf{p}, \mathbf{q})$ is also called "phase space" coordinate
  - $H(p, q)$ is the total energy of the system comprising kinetic, $T$, and potential, $V$, energy:

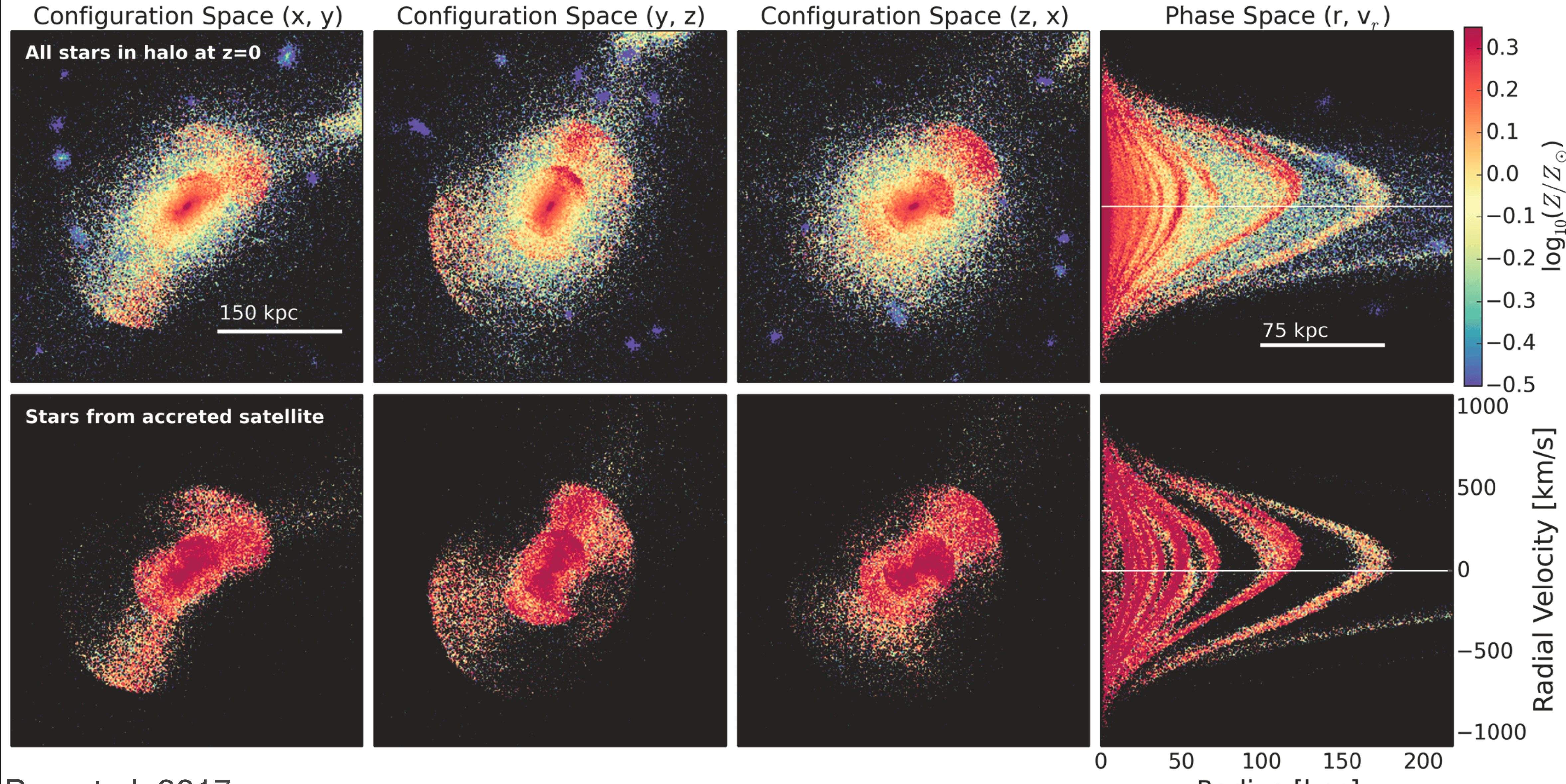
$$\mathscr{H} = T + V, \quad T = \frac{p^2}{2m}, \quad V = V(q)$$

  - Hamilton's equations (from Euler-Lagrangian equation) govern the time evolution of the canonical coordinates:

$$\frac{\mathrm{d}\mathbf{q}}{\mathrm{d}t} = \frac{\partial \mathscr{H}}{\partial \mathbf{p}}, \quad \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -\frac{\partial \mathscr{H}}{\partial \mathbf{q}}$$



Credit: J. Sanders




Pop et al. 2017

# Stellar Orbit

- **(some) Orbital parameters**
  - Energy: total sum of kinetic and potential energy for the orbit; $E_{tot} < 0$ (bound), $E_{tot} > 0$ (unbound)

    $$E_{tot} = E_k + E_p$$

  - Angular Momentum: cross product of the position and momentum vectors

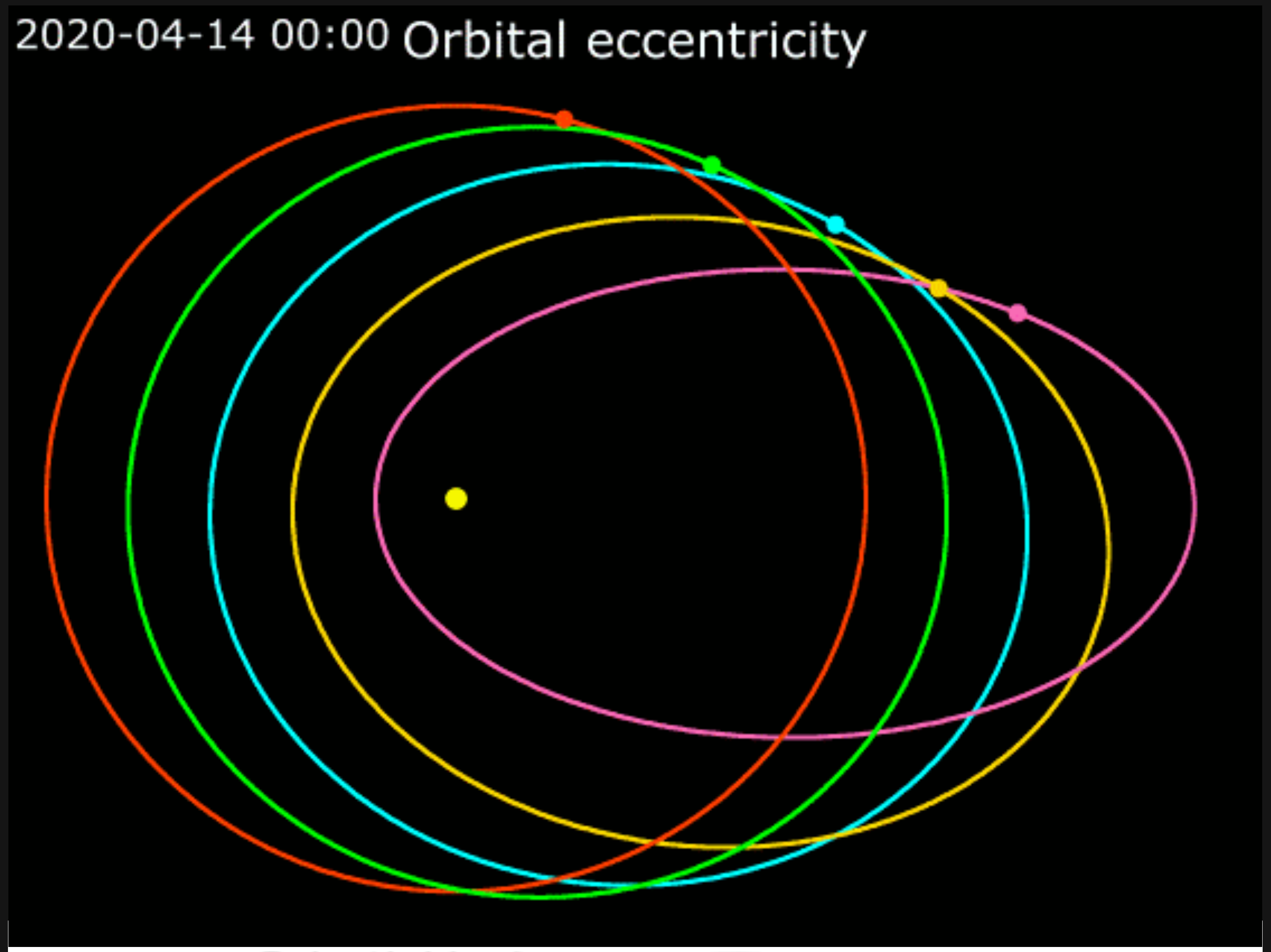
    $$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

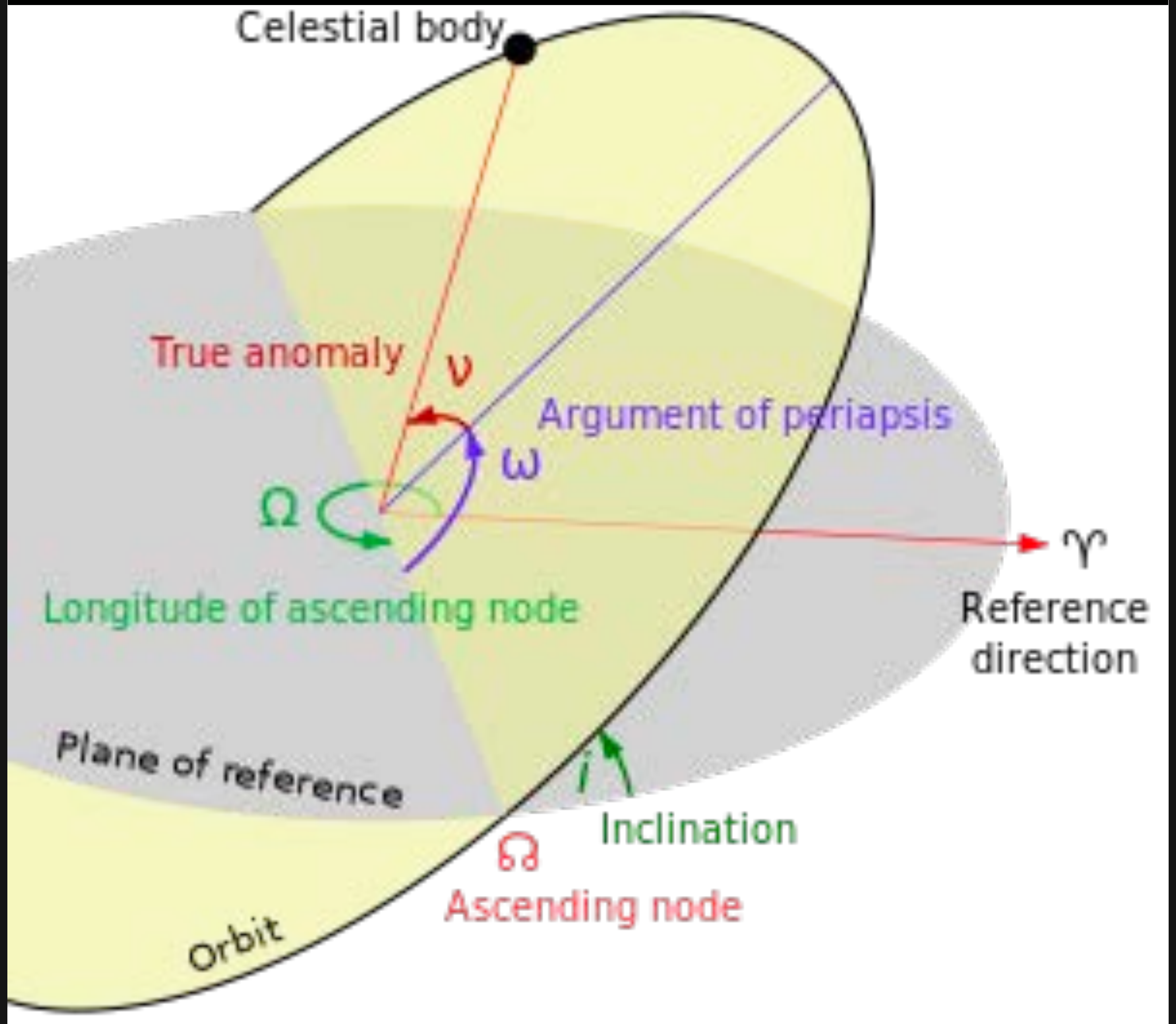
  - Eccentricity: parameter of how much the orbit deviates from a perfect circle; $0$ (circular) → $1$ (elliptic)

    $$e = \frac{r_{max} - r_{min}}{r_{max} + r_{min}}$$

  - Inclination: tilt angle of an orbital plane from the reference (i.e., Galactic) plane; $0°$ (prograde) → $180°$ (retrograde)

    $$i = \arccos \frac{L_z}{|L|}$$



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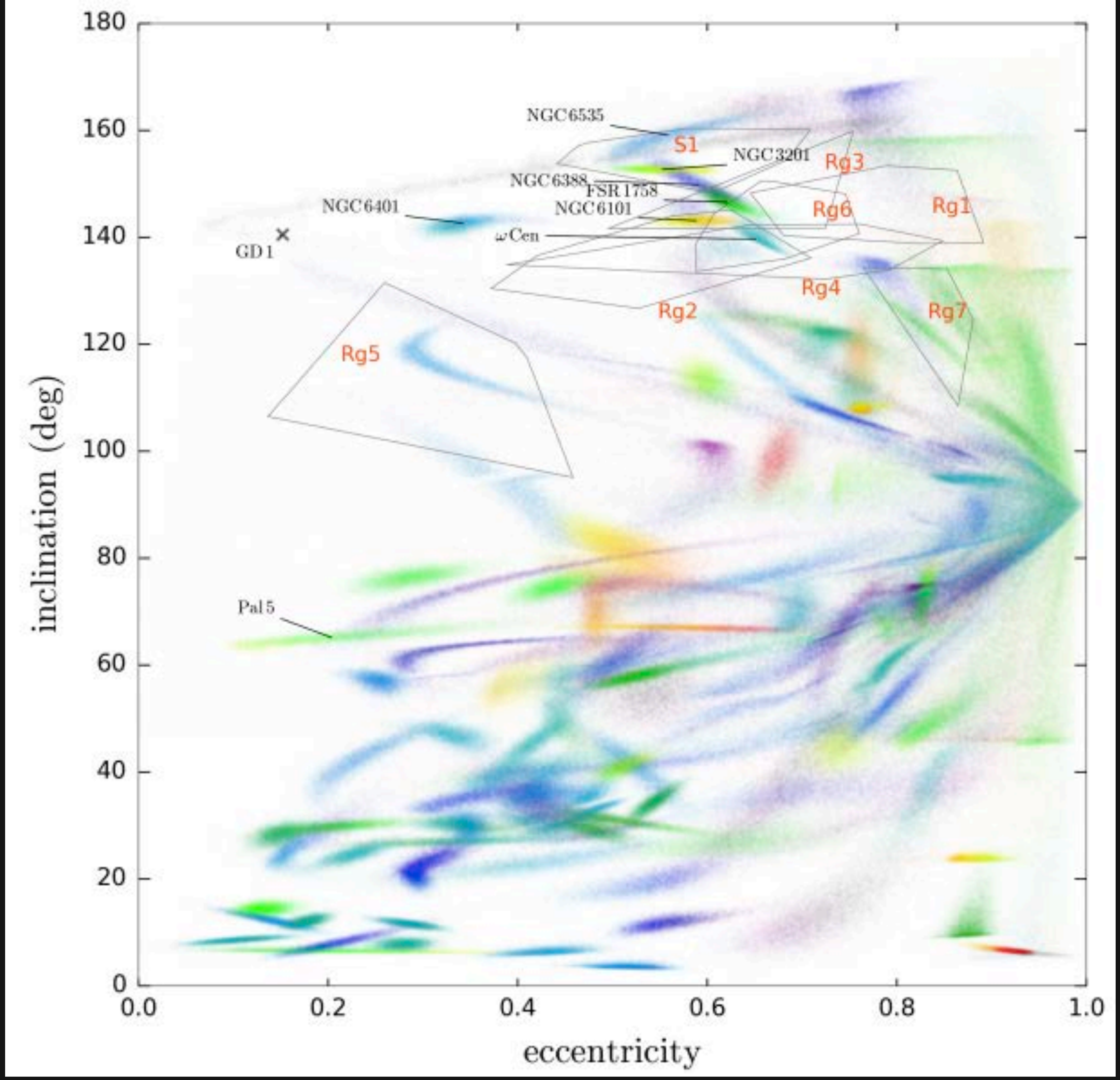


# Action-angle variables

- **Viewing the Milky Way as a "collection of orbits"**
  - action variables provide good & simple characterisation of a (stellar) orbit
  - overall structure as a sum of individual (stellar) orbits
  - actions provide a useful map!
- Recent development of numerical method for "realistic" axisymmetric potentials*
  - orbital integration can be computationally expensive, especially in this era of large datasets
  - fast calculation now available, capable of handling large data
  - approximation but sufficient
  - tact (Sanders & Binney 2006), AGAMA (Vasiliev 2018), galpy (Bovy 2015)

*e.g., a realistic, axisymmetric Milky Way potential from McMillan (2017) comprising disks (stellar + gas), halo, and bulge

# Action-angle variables

Taking special momenta, $\mathrm{J}$, with conjugate coordinates, $\theta$, where $\mathrm{J}$ is constant:

$$\dot{\mathrm{J}}_{\mathrm{i}} = -\frac{\partial \mathrm{H}}{\partial \theta_{\mathrm{i}}} = 0$$

so the Hamiltonian is independent of the coordinates, $\theta$, that,

$$\mathrm{H} = \mathrm{H}(\mathrm{J}, \theta) \Rightarrow \mathrm{H} = \mathrm{H}(\mathrm{J})$$

and so the other Hamiltonian for the $\theta_{\mathrm{i}}$, as functions of time is:

$$\dot{\theta}_{\mathrm{i}} = \frac{\partial \mathrm{H}}{\partial \mathrm{J}_{\mathrm{i}}} = \Omega_{\mathrm{i}}(\mathrm{J})\,, \quad \text{a constant}$$

For a path, where $\mathrm{i}^{\mathrm{th}}$ component of $\theta$ increases by $2\pi$ while $\mathrm{J}$ stays constant:

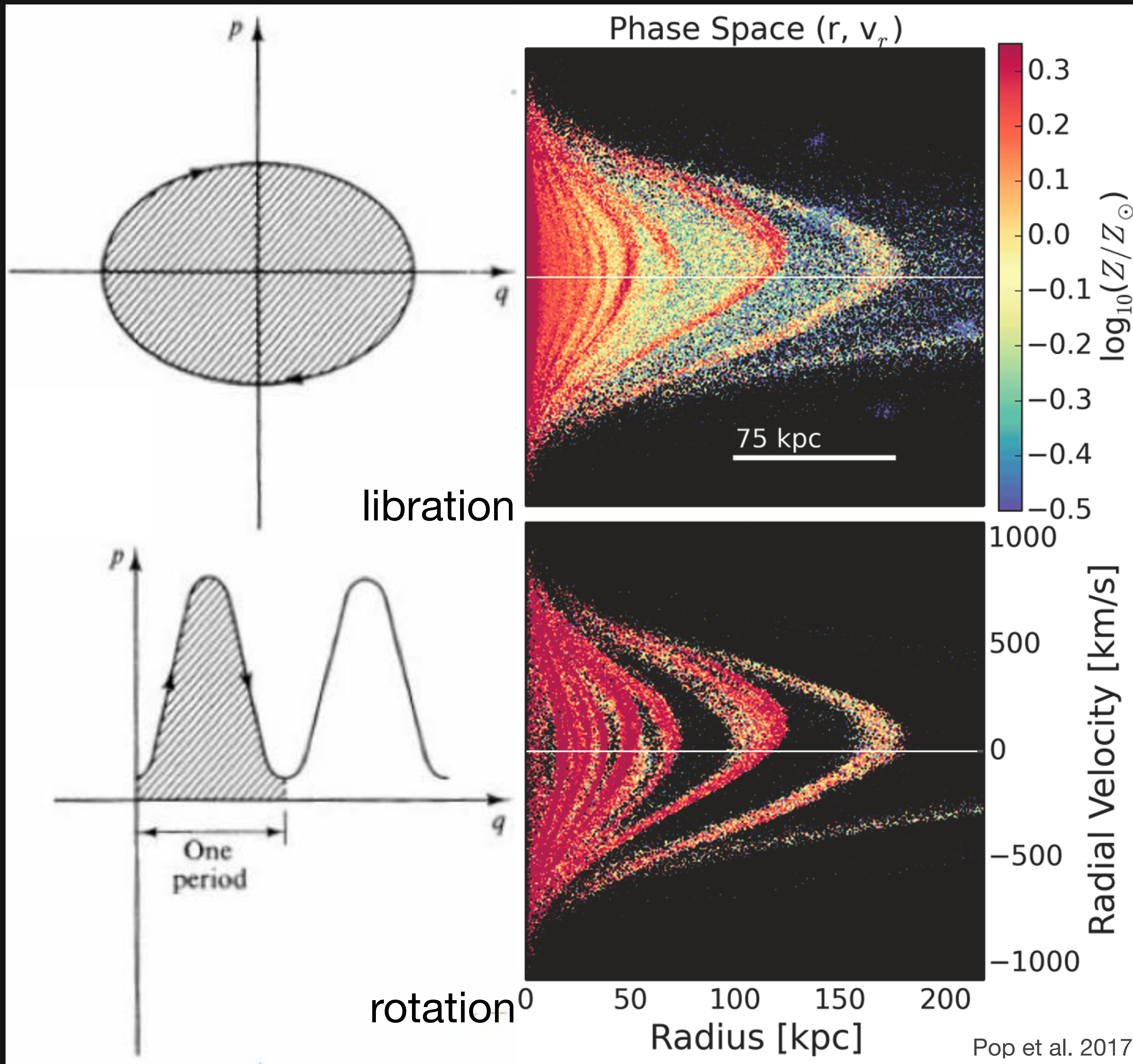
$$\mathrm{J}_{\mathrm{i}} = \frac{1}{2\pi}\int_{\gamma_{\mathrm{i}}} \mathrm{J}\,\mathrm{d}\theta$$

$\mathrm{J}$, $\theta$ are canonical coordinates, and so for any canonical $\mathrm{p}$, $\mathrm{q}$:

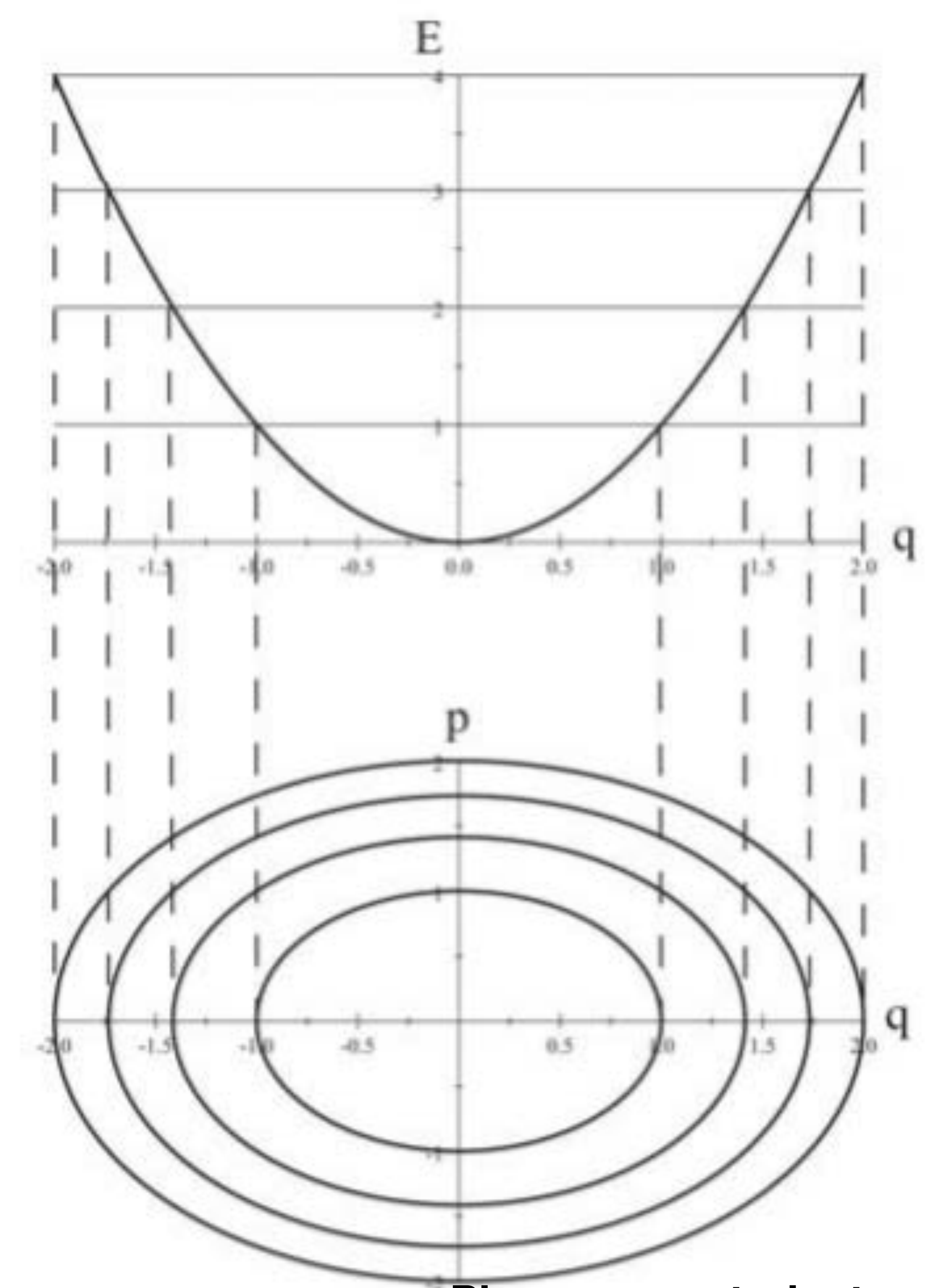
$$\mathrm{J}_{\mathrm{i}} = \frac{1}{2\pi}\int_{\gamma_{\mathrm{i}}} \mathrm{p}\,\mathrm{dq}$$

e.g., for a loop orbit in an axisymmetric potential, the azimuthal action, $\mathrm{J}_{\phi}$:

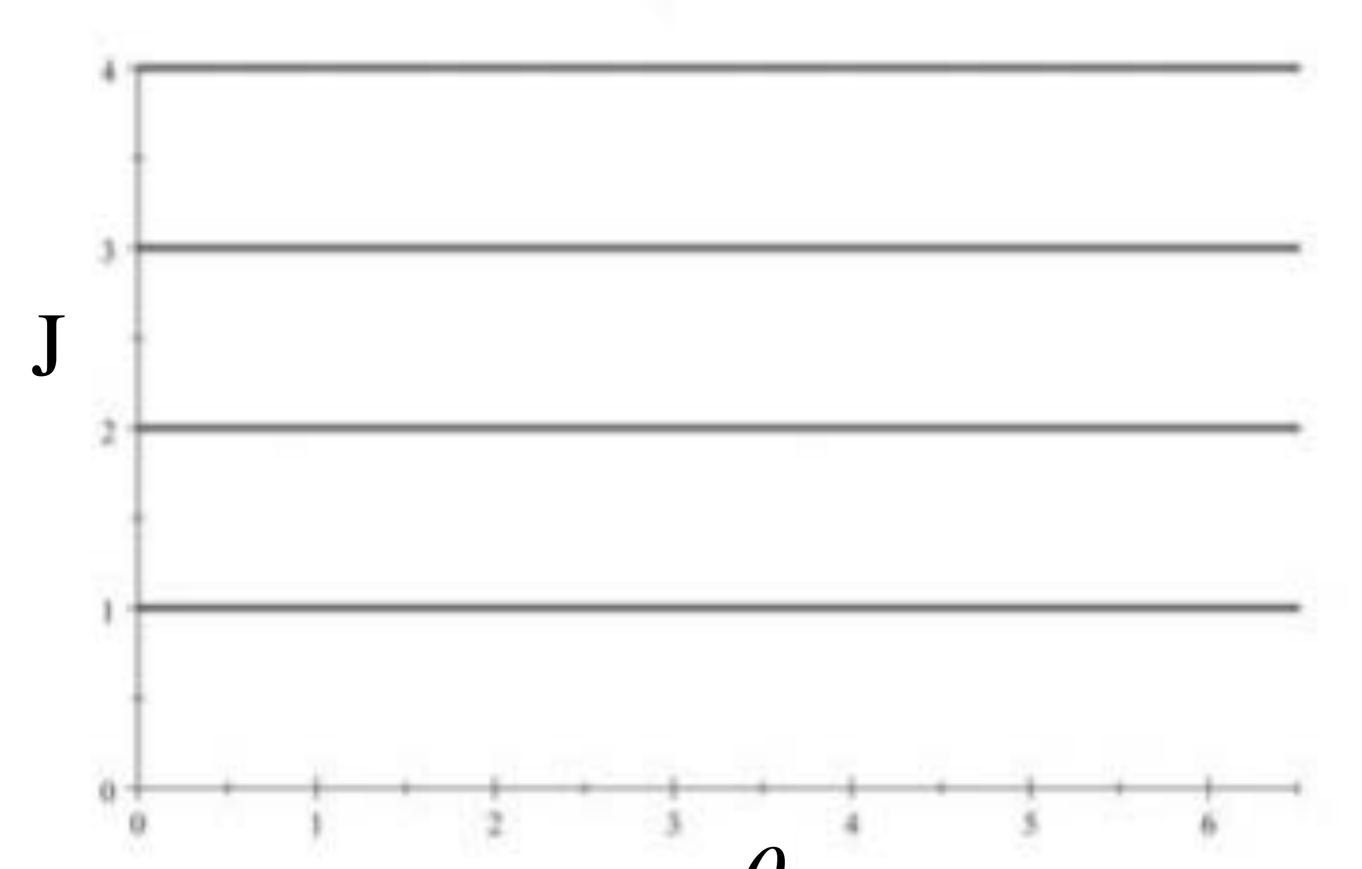
$$\mathrm{J}_{\phi} = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{p}_{\phi}\,\mathrm{d}\phi = \mathrm{L}_{\mathrm{z}}\,, \quad \text{since } \mathrm{p}_{\phi} = \mathrm{R}\,\mathrm{v}_{\phi} = \mathrm{L}_{\mathrm{z}}$$



# Action-angle variables



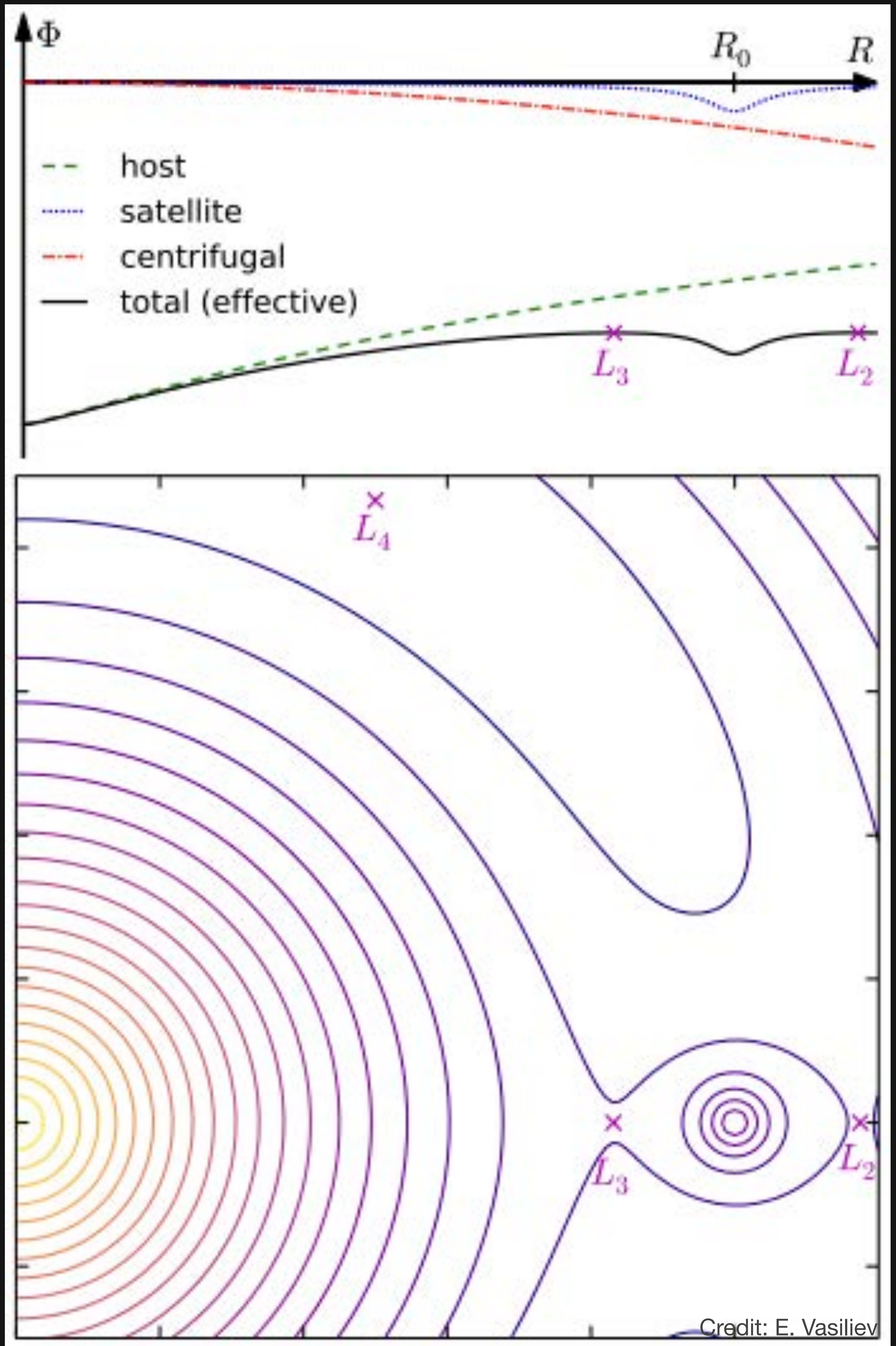

Phase space trajectory




Action-angle

# Stellar Stream

- **(gravitationally) Torn & (phase) Mixed**
  - Origin: surfing in a tidal field
    - Imagine, a satellite system (e.g., star cluster, satellite galaxy) containing stars within its own gravitational potential, $\Phi_s$, is orbiting in the host Galaxy's gravitational potential, $\Phi_h$ . For individual stars, $\Phi = \Phi_h + \Phi_s$
    - The effective potential should include the centrifugal term: $\Phi_{\rm eff}(\mathbf{x}) = \Phi(\mathbf{x}) - \frac{1}{2}|\Omega \times \mathbf{x}|^2$
  - Lagrangian points: There are "saddle points" where the gradient, $\frac{\partial \Phi_{\rm eff}}{\partial x} = 0$
  - Tidal radius (Jacobi radius, Hill sphere…): gravitational "boundary" of the satellite system
    - $R_t \approx R_0 (\frac{M_s}{3\,M_h})^{\frac{1}{3}}$ (assuming point mass systems…)
    - Stars outside of the radius (a sphere!) are no longer bound to there home
    - Will evolve across the orbits! $R_0$ & $M_h$ depends on the orbital radius (e.g., apocentre vs pericentre → where would the tidal disruption be violent the most?)
    - Will evolve over the time! $M_s$ will decrease as the satellite system is losing mass → eventually will dissolve/disrupted completely!
    - Two Lagrangian points are the main exit points (why are there only two? → think about the centrifugal force)



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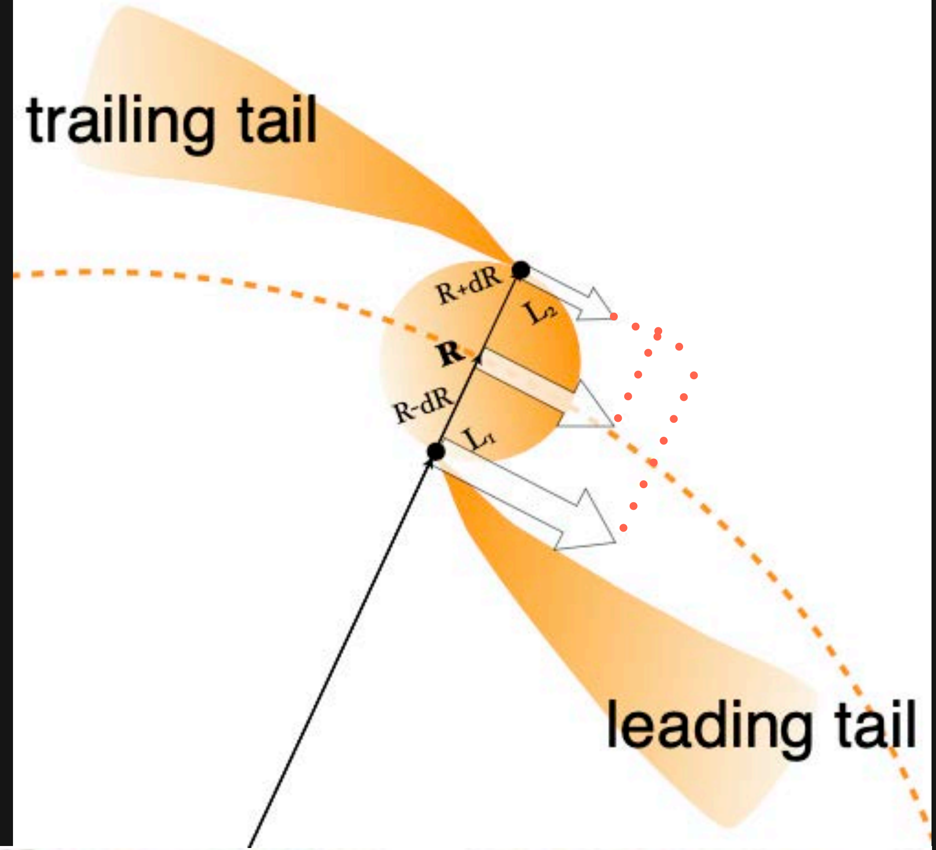
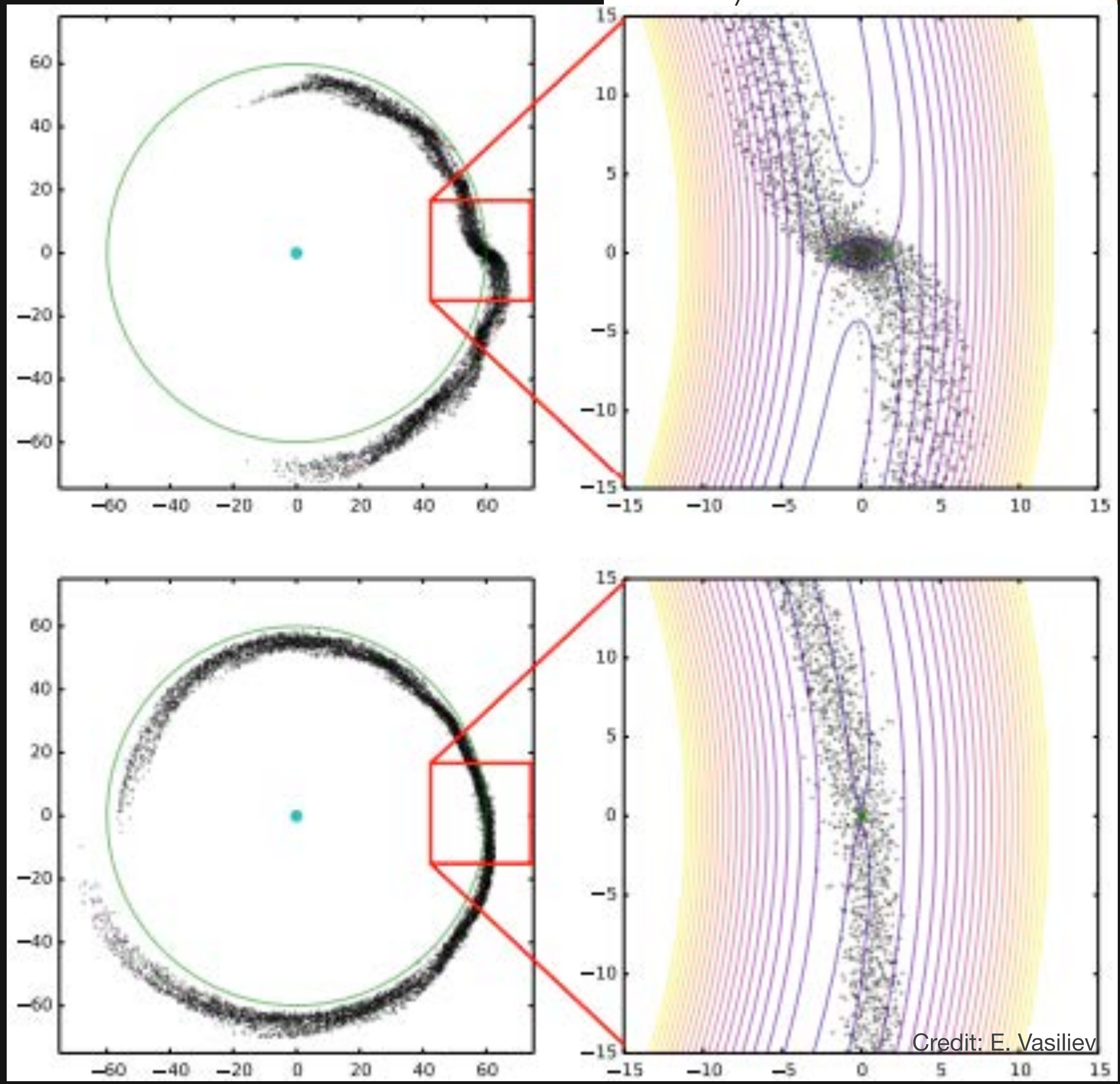
# Stellar Stream




- **Tidal tails**
  - Stars leaving the satellite body via two Lagrangian points → different orbital radii, $R_0 - R_t < R_0 + R_t$ → different orbital period!
  - The stars escaping from the inner/outer Lagrangian point will:
    - → have smaller/larger orbital radius from the Galactic centre
    - → have shorter/longer orbital period (i.e., higher/lower orbital frequency)
    - → complete the orbit faster/slower
    - → spread ahead/behind the satellite body
    - → form the leading/trailing tails (arms) → we have a stream!
  - An example of phase mixing!
  - The stars in a stream do not exactly follow the same orbit → still form a family of orbits with well-defined orbital characteristics reflecting the properties of the progenitor system



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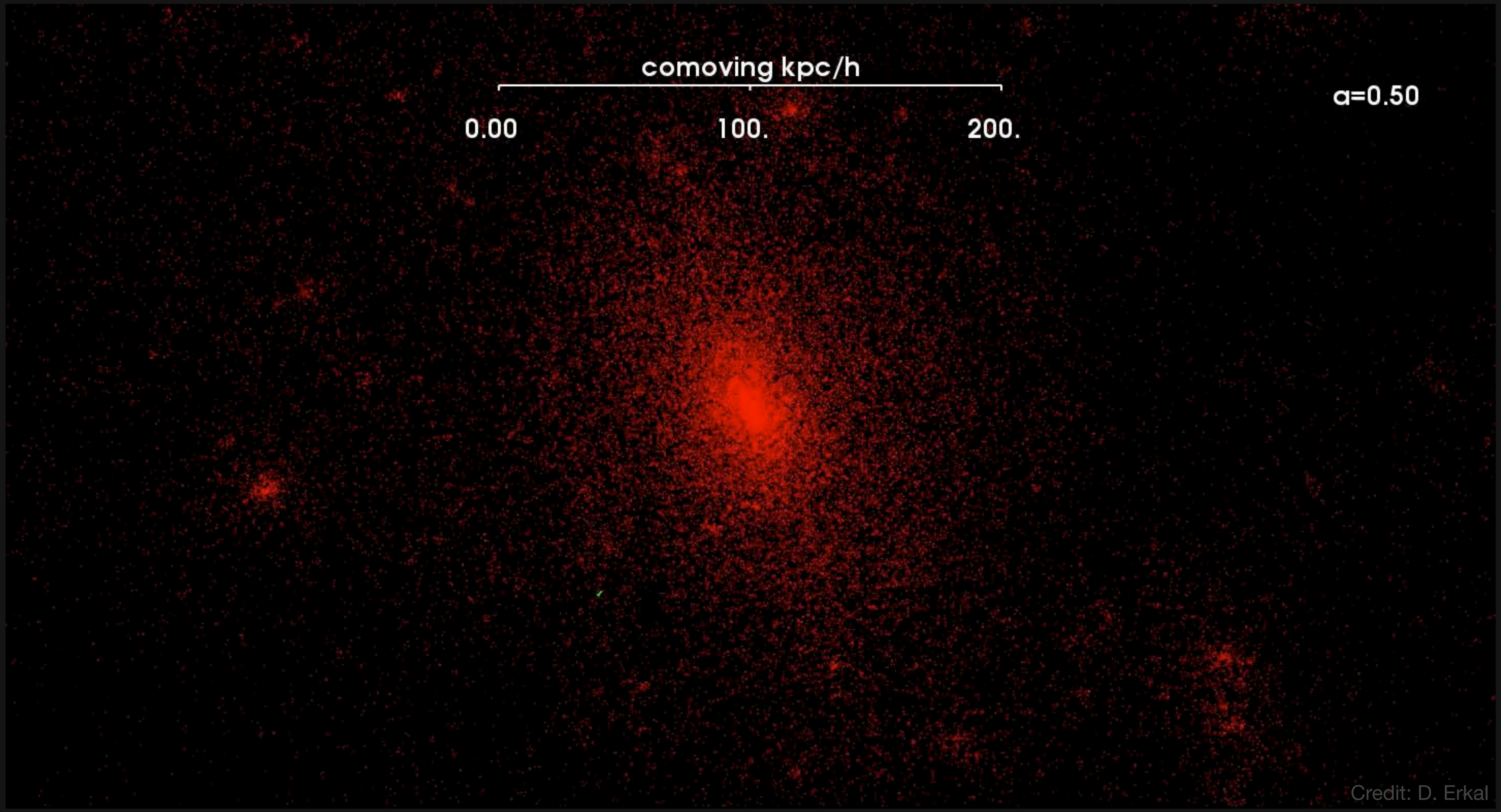
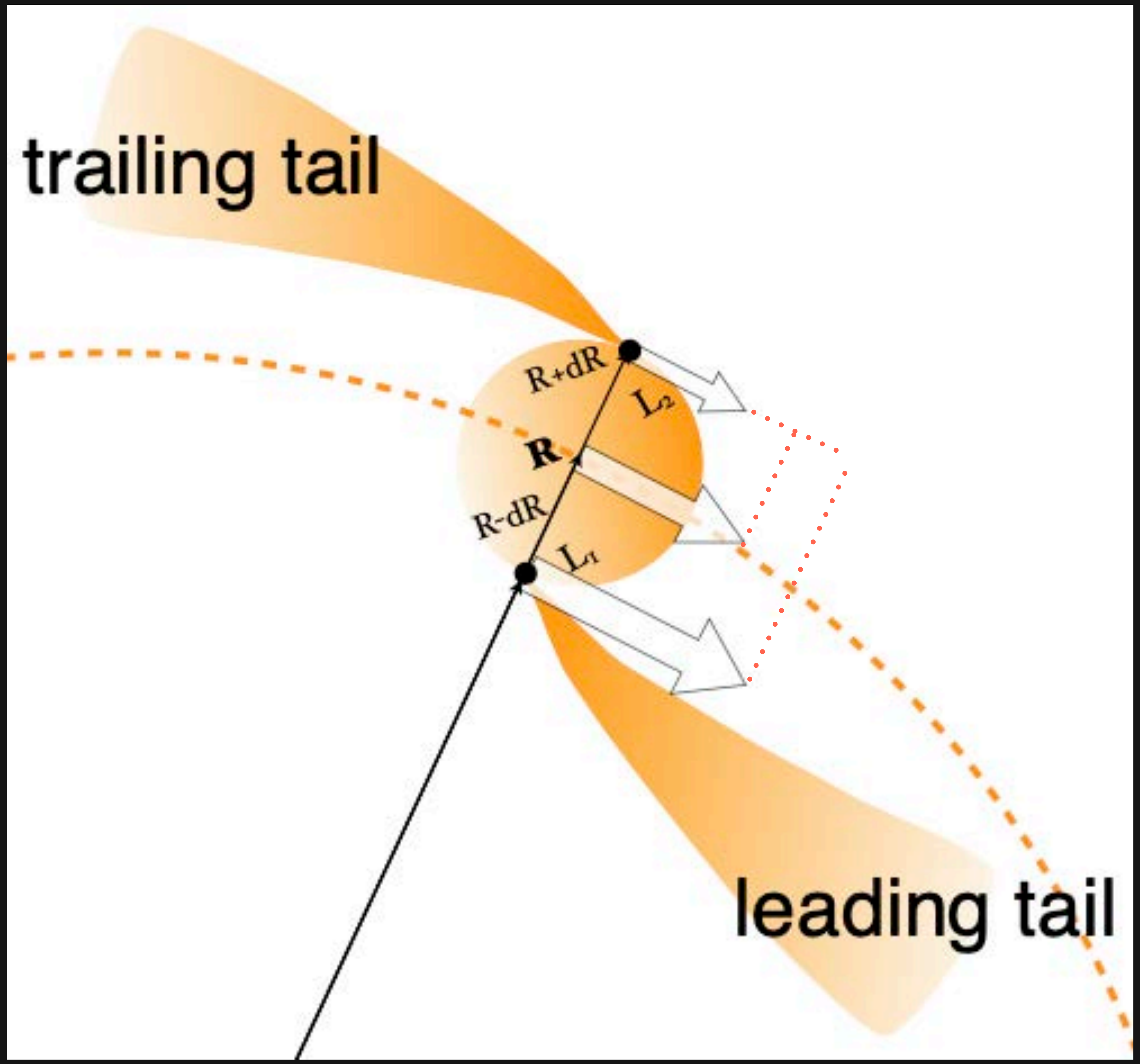
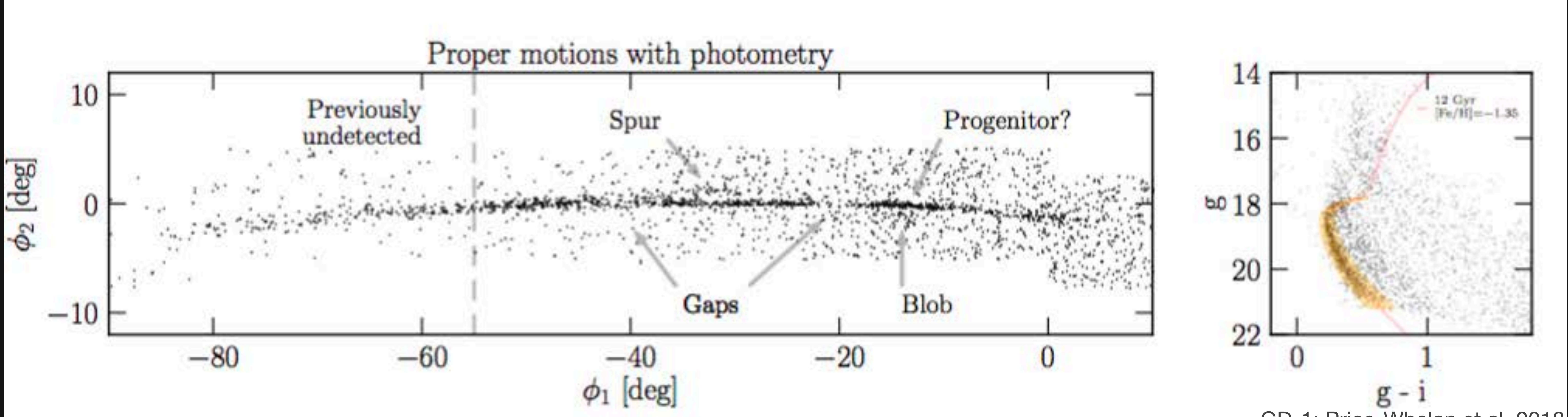


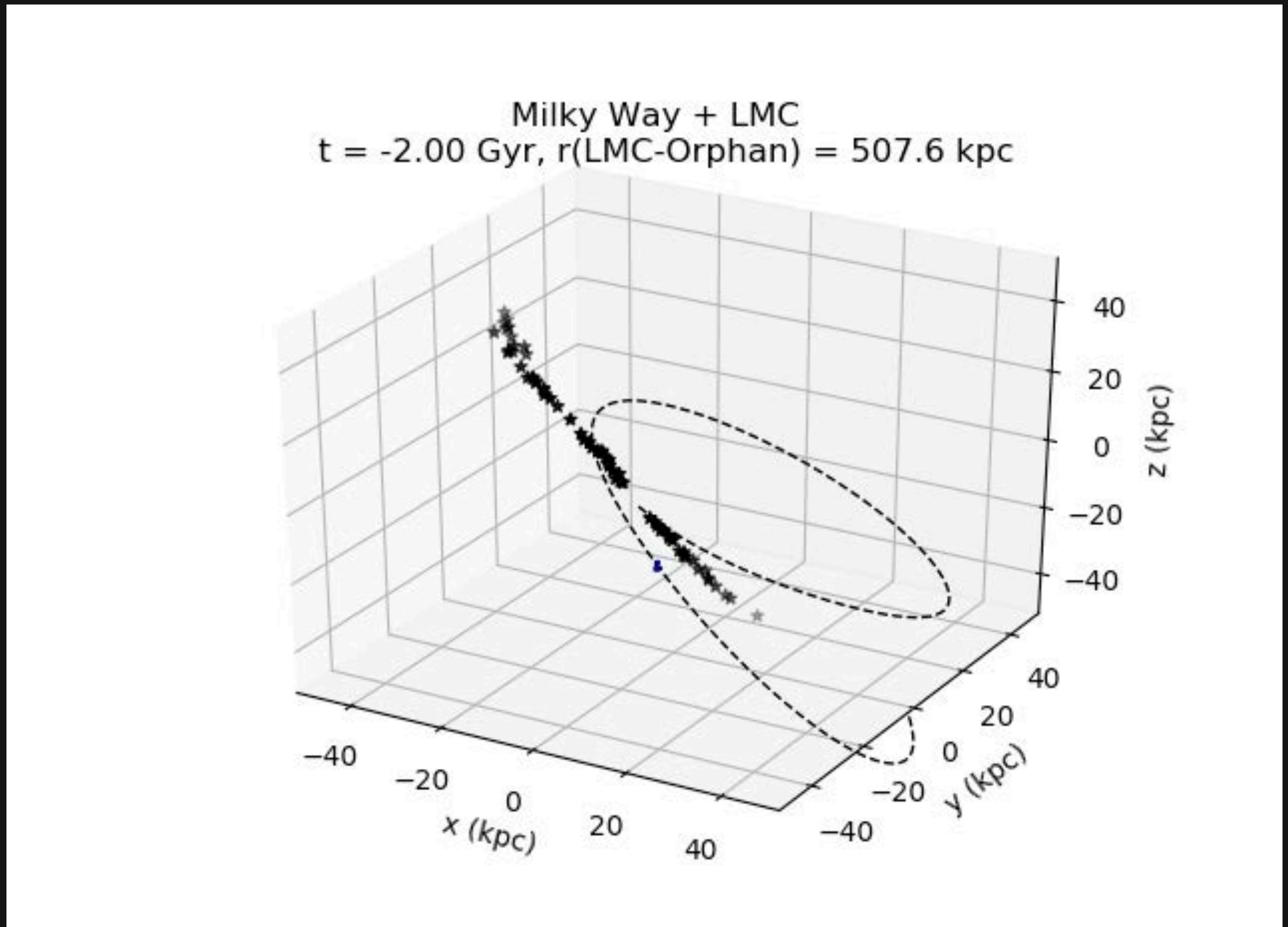



# Stellar Stream

- **Perturbed & modified**
  - Streams' morphology, kinematics are very sensitive to the surrounding environment!
  - Can be perturbed, modified by "encounters"
    - anything with sufficient gravitational impact
    - passing by star clusters, satellite galaxies
    - dark sub haloes…?



GD-1; Price-Whelan et al. 2018



Credit: D. Erkal