

Motivation and Big-O

Salaar Liaqat

Data Sciences Institute, UofT

Outline

- Motivation
- Time Complexity: Introduction to Big-O Notation
- Average, Best, and Worst Case
- Space Complexity

About Me

- About the instructor

Learning Objectives

- Assess options and choices around fundamental algorithms and data structures using Big-O notation.
- Develop comfort with recursive functions.
- Decide on appropriate data structures
- Take a client-led problem and translate it into an optimization problem.
- Identify why code is running slowly and know how to improve its performance.

Section 1

Motivating Code Demos

What are Algorithms and Data Structures

- An **algorithm** is a procedure to solve a problem
 - ▶ Sort a data observations from smallest to largest
 - ▶ Find the nearest neighbor to a data point
 - ▶ How fast is each algorithm?
- A **data structure** is a concrete method to store some data.
 - ▶ A pandas data set is a good way to store observations with many features.
 - ▶ How much space does the data structure need? How long does it take to access each observation?

```
import numpy as np
import timeit
import random
```

Loop Versus Vectorized Operations

```
size = 10**4
```

```
# Using Python lists
```

```
list_a = list(range(size))
```

```
list_b = list(range(size))
```

```
# Using NumPy arrays
```

```
array_a = np.arange(size)
```

```
array_b = np.arange(size)
```

```
# Timing for list addition
```

```
list_time = timeit.timeit(lambda:
```

```
    [a + b for a, b in zip(list_a, list_b)], number=1)
```

```
# Timing for vectorized array addition
```

```
array_time = timeit.timeit(lambda:
```

```
    array_a + array_b, number = 1)
```

Loop Versus Vectorized Operations

```
print(f"List Addition: {list_time:.6f} seconds")  
print(f"Vectorized Addition: {array_time:.6f} seconds")
```

List Addition: 0.000350 seconds

Vectorized Addition: 0.000182 seconds

- Why was the NumPy vectorized operation much faster?
- How can we describe how much faster the vectorized operation is?
- This is useful in many iterative algorithms, such as gradient descent.

Search in List Versus Set

```
# Python list
list_time = timeit.timeit(lambda:
    -1 in list_a, number = 1)

# Python set
set_a = set(range(size))
set_time = timeit.timeit(lambda:
    -1 in set_a, number = 1)
```

Search in List Versus Set

```
print(f"List Search: {list_time:.6f} seconds")  
print(f"Set Search: {set_time:.6f} seconds")
```

List Search: 0.000037 seconds

Set Search: 0.000000 seconds

- Why was the set search much faster?
- How can we describe how much faster the vectorized operation is?
- What are the pros and cons of choosing each data structure?

Selection Sort Versus Python Sort

For context, selection sort is a naive sorting algorithm, while Python implements Tim Sort for the default search function.

```
def selection_sort(arr):  
    n = len(arr)  
  
    for i in range(n):  
        min_index = i  
        for j in range(i+1, n):  
            if arr[j] < arr[min_index]:  
                min_index = j  
  
        arr[i], arr[min_index] = arr[min_index], arr[i]
```

Selection Sort Versus Python Sort

```
random.shuffle(list_a)
rand_list = list_a.copy()

sel_time = timeit.timeit(lambda:
    selection_sort(rand_list.copy()), number = 1)

py_time = timeit.timeit(lambda:
    sorted(rand_list.copy()), number = 1)
```

Selection Sort Versus Python Sort

```
print(f"Selection sort: {sel_time:.6f} seconds")  
print(f"Tim sort: {py_time:.6f} seconds")
```

Selection sort: 1.600249 seconds

Tim sort: 0.000793 seconds

- Why was selection sort much slower than Tim sort (not in detail)?
- If we double the size of the list, how much slower will the code be in each case?

Section 2

Time Complexity: Introduction to Big-O Notation

An example

- Imagine you are writing an algorithm to search for a landing position for a rocket. You want it to be simple (to avoid bugs) and fast (since you only have 10 seconds to find a site).¹
- It takes 1 millisecond to check each element. You decide to test a simple search and binary search on 100 elements (more on these methods later).
 - ▶ Simple search takes 100ms. Binary search takes 7ms.
- Then you test binary search with 1 billion elements and it takes 32ms.
 - ▶ Binary search is about 15 times faster than simple search, because simple search took 100 ms with 100 elements, and binary search took 7 ms. So simple search will take $30 \times 15 = 450$ ms with 1 billion elements.
- Since that is within your threshold, you decide to go with simple search.
Is this correct?

¹Example from Grokking Algorithms

A practical example

- Definitely wrong!!
- The run time of different algorithms can grow at different rates.
- Big-O tells us how run time increases as the list size increases.

Comparing run times of simple and binary search

Elements	Simple Search	Binary Search
100	100 ms	7 ms
10,000	10 s	14 ms
1,000,000,000	11 days	32 ms

Big-O Notation

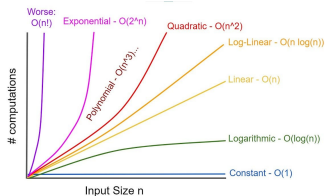
- Big-O tells you how fast an algorithm is in terms of the number of operations, n .
- Simple search needs to take each element, so it will take n operations. The run time in Big-O notation is $O(n)$.
- Binary search needs $\log n$ operations, so the run time in Big-O notation is $O(\log n)$
 - ▶ Note: \log in computer science usually refers to \log base 2.

Big-O is Upper Bound Run Time

- Big-O notation is about the *worst-case* scenario.
 - ▶ If you were conducting linear search through a phone book, even if you were looking for Abe Aberdeen, it is still considered $O(n)$.
- Formally, it characterizes an upper bound on the asymptotic behavior of the run time.
- For example, the function $7n^3 + 30n^2 - 200n + 9$ has highest-order term $7n^3$. The function's growth rate is n^3 because the function grows no faster than n^3 . The Big-O is $O(n^3)$.

Common Big-O Run Times

Here are seven Big-O run times that you'll encounter frequently, sorted from fastest to slowest.



- $O(1)$, known as *constant time*. Ex: addition, division
- $O(\log n)$, known as *logarithmic time*. Ex: binary search
- $O(n)$, known as *linear time*. Ex: Linear search
- $O(n \log n)$. Ex: Tim Sort
- $O(n^2)$, known as *quadratic time*. Ex: Selection sort.
- $O(2^n)$, known as *exponential time*. Ex: Naive recursive solution for n th Fibonacci number
- $O(n!)$, known as *factorial time*. Ex. Traveling salesperson

Determining Time Complexity

Consider the following code. How can you determine the Big-O?

```
def f(n):  
    for i in range(n):  
        for j in range(n):  
            print(i, j)
```

Determining Time Complexity

- With “raw” Python code, you can usually count the number of nested for loops to determine the Big-O
 - ▶ A loop gives $O(n)$
 - ▶ A nested loop gives $O(n^2)$
- It's usually not so simple in Data Science because of packages we use.
- There are many factors affecting the constants in your run time
 - ▶ How complicated is each step? Is it n or $2000n$?
 - ▶ How are your algorithms implemented? Is your programming language fast? Are your libraries fast?
- Implementation issues will be covered later in the course.

Big-O with Two Variables

Consider the following code. What is it's Big-O?

```
def fun(n,m):  
    for i in range(n):  
        for j in range(m):  
            print("Hello")
```

Big-O with Two Variables

- The time complexity here is $O(nm)$.
 - ▶ If $n = m$, then $O(n^2)$.
- All terms should be combined into one Big-O
 - ▶ $O(nm)$ is correct and $O(n)O(m)$ is incorrect.
 - ▶ $O(n + m)$ is correct and $O(n) + O(m)$ is incorrect.
 - ▶ $O(n^2 + mn + m)$ is written as $O(n^2 + nm)$. We can't throw away either term because we don't know which term will dominate.
- Important to think about this when working with datasets.
 - ▶ They have n rows and p columns.
 - ▶ Can you reason how long it will take to fit a decision tree?

Section 3

Best, Average, and Worst Case

Best, Average, and Worst Case

- Big-O deals with worst case.
- If we can develop a notion of an “average input,” then we can devise the average case of an algorithm.
- Best case is useful to think about the constants in your algorithm.
 - ▶ $O(\log n)$ is always faster than $O(n)$ except with very small n .

Section 4

Space Complexity

What is Space Complexity

- Aside from our algorithm taking too long to run, its also an issue if you run out of memory.
 - ▶ Note, memory (RAM), is not the same as disk space.
 - ▶ The computer will load data into memory from the disk
- It will be problematic if you need to load 2 billion observations all at once.
- We can also analyze space complexity with Big-O notation
- Notice that time complexity is usually about the *algorithm*, while space complexity is about the *data structure*.

Examples

- Code that prints hello {your name} will have $O(1)$ space.
- Code that sums a list of size n has $O(n)$ space.
- You have users on Instagram, and you want to store who follows who. The answer depends (why?). The worst case space is $O(n^2)$

Section 5

Recommended Problems and References

Recommended Problems

- Cormen: Chapter 1 exercises
 - ▶ 1.2-1, 1.2-2, 1.2-3
- Bhargava: Chapter 1 exercises
 - ▶ 1.3 to 1.5
- Additional (for the mathematically inclined)
 - ▶ In CS, log is usually base 2, but a strong distinction is not made because *logs of different bases only differ by a constant factor* and constants are dropped in Big-O. Show this is true
 - ▶ Show that exponents of different bases **do not** differ by a constant factor

References

- Bhargava, A. Y. (2016). *Grokking algorithms: An illustrated guide for programmers and other curious people*. Manning. Chapter 1.
- Cormen, T. H. (Ed.). (2009). *Introduction to algorithms* (3rd ed). MIT Press. Chapter 1 and 3.