

# Regression II

Data Sciences Institute  
Applying Statistical Concepts

## Acknowledgements

- Slides created by Julia Gallucci under the supervision of Rohan Alexander.
- Content adapted from: A First Introduction (Python Edition) Tiffany Timbers, Trevor Campbell, Melissa Lee, Joel Ostblom, Lindsey Heagy  
<https://python.datasciencebook.ca/index.html>

## Learning objectives

- Explain the linear regression algorithm and contrast it with KNN regression.
- Fit simple and multivariable linear regression models on training data using Python.
- Evaluate the linear regression models on test data.
- Explain the impact of outliers and multicollinearity on linear regression.

## Simple linear regression

$$y = mx + b$$

- **Predictor Variable ( $x$ ):**
  - The factor you're using to make the prediction (independent variable)
- **Response Variable ( $y$ ):**
  - The outcome or result you're trying to predict (dependent variable)
- **Slope of the line ( $m$ ):**
  - How much  $y$  changes when  $x$  changes.
- **Y-intercept ( $b$ ):**
  - The starting value of  $y$  when  $x$  is 0.

- KNN regression has limitations: poor prediction beyond the training data range and slower performance with larger datasets.
- Linear regression addresses these limitations: better prediction range and faster performance with larger datasets.
- Linear regression is widely used due to its interpretable mathematical equation linking predictors and response variables.
- Simple linear regression involves one predictor and one response variable and **predicts by creating a straight line of best fit through the training data.**

## Animation of linear regression

[![w:750 Watch on YouTube] (<https://img.youtube.com/vi/SWTeWu1rerK/0.jpg>)]

(<https://youtu.be/SWTeWu1rerK>)

<https://www.youtube.com/watch?v=SWTeWu1rerK>

## Example dataset

932 real estate transactions in Sacramento, California is the dataset we will be using, specifically for predicting whether the size of a house in Sacramento can be used to predict its sale price.

- *Key features:*
  - 932 observations (rows)
  - predictor of interest (sqft; house size, in livable square feet)
  - response variable of interest (house sale price, in USD)

- To decide whether the \$350,000 asking price for the 2,000 square-foot house is fair, we can use our existing data to predict its likely sale price. However, since there are no exact observations for a 2,000 square-foot house in our dataset, we need a method to estimate the price.
- Using simple linear regression, we use the data we can draw a straight line of best fit through our existing data points.

 (./images/simple\_reg.png)



- The equation for the straight line in simple linear regression is:

$$\text{house sale price} = \beta_0 + \beta_1 \text{house size}$$

where:

$\beta_0$  is the vertical intercept (price when house size is 0).

$\beta_1$  is the slope (rate of price increase as house size increases).

- Finding the line of best fit involves determining coefficients  $\beta_0$  and  $\beta_1$  that define the line.
  - $\beta_0$  represents the "base price,"
  - $\beta_1$  is the price increase per square foot.

- Once we have the coefficients, we can use the equation to evaluate the predicted sale price given the value we have for the predictor variable—here 2,000 square feet.
- Linear regression can predict extreme values (e.g., 6 million or -2,000 sq. ft.), but these predictions are unreliable.
- Make predictions within the original data range; extrapolate only when logically justified.

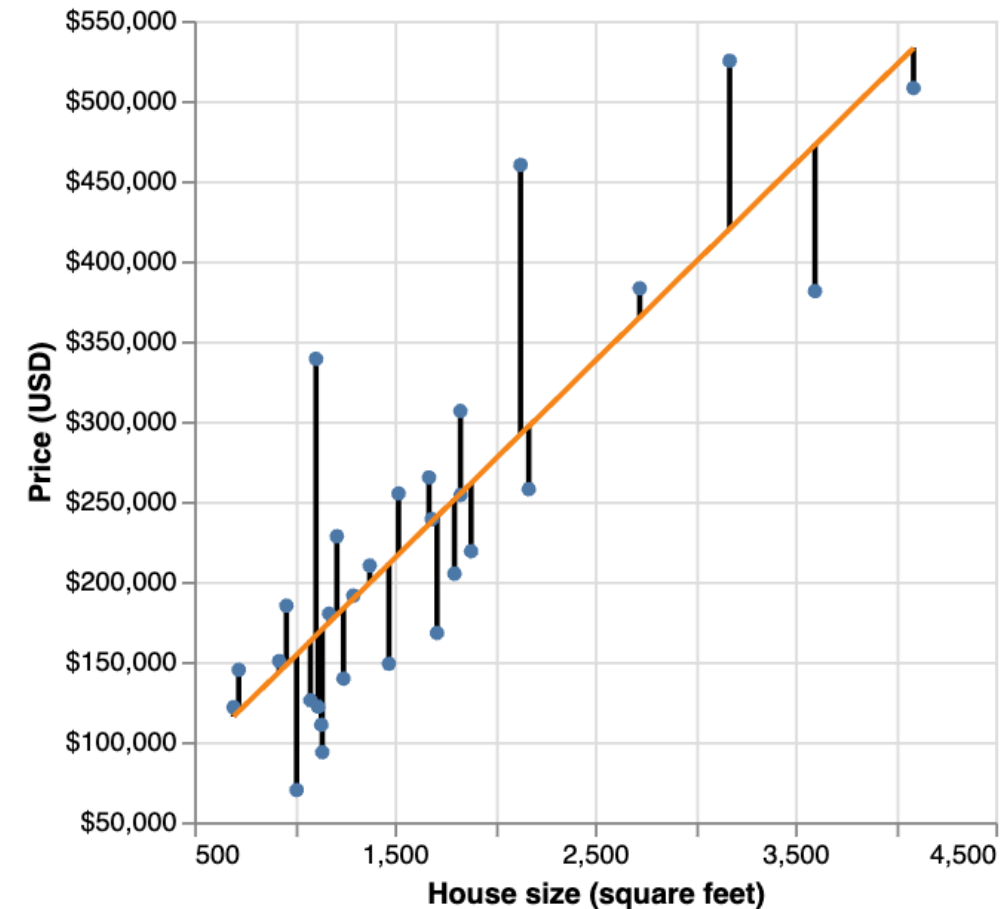
![bg right:45% w:600] (./images/predict\_simple.png)

## Choosing the line of best fit

Many different lines could be drawn through the data points, how do we choose the line of best fit?

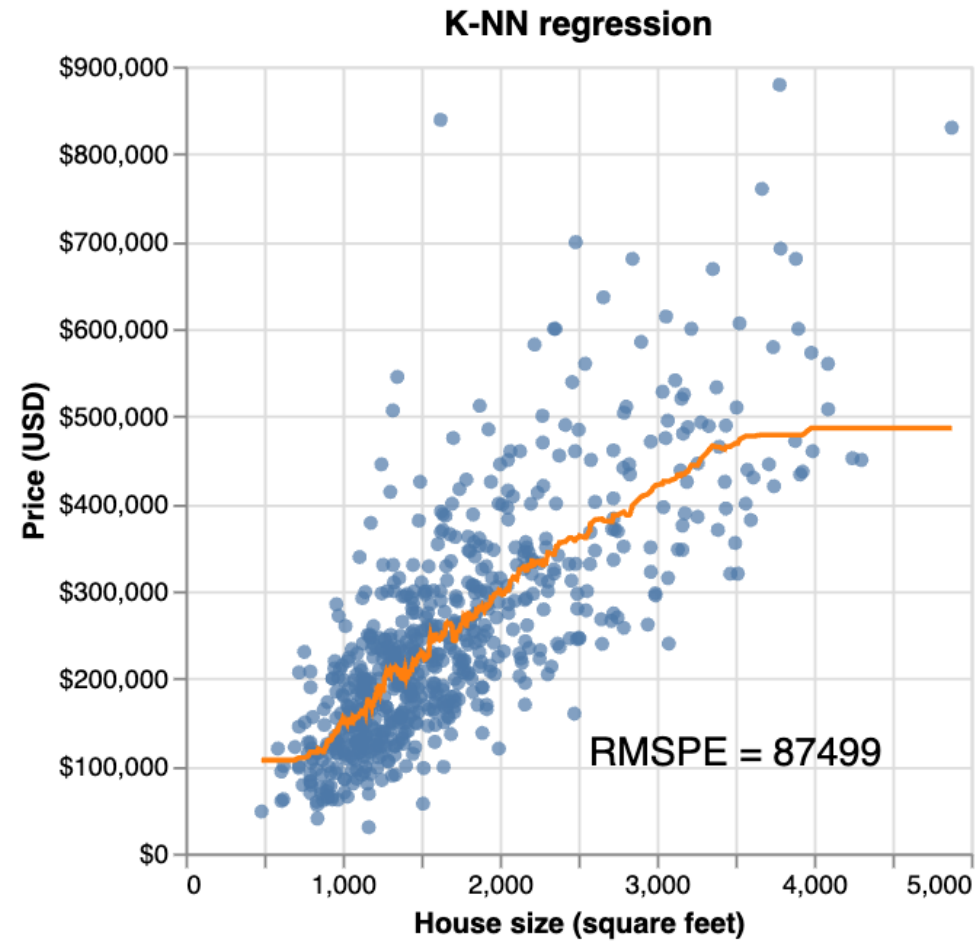
 [bg right:60% w:700]

- Simple linear regression finds the line of best fit by minimizing the average squared vertical distance between the line and observed data points.
- This process is equivalent to **minimizing the Root Mean Squared Error (RMSE)**.
- The predictive accuracy of the simple linear regression model is assessed using the Root Mean Squared Prediction Error (RMSPE), similar to KNN regression.



## Comparing simple linear and KNN regression

- **Parameter Tuning:** No need for cross-validation to choose parameters in linear regression.
- **Data Preparation:** No need for standardization (centering and scaling) of the data in linear regression.



Visualization of the simple linear regression model predicting price from house size and the “best” K-NN regression model obtained from the same problem.

	Simple Linear Regression	KNN Regression
<b>Line Shape</b>	Straight line	Flexible, wiggly line
<b>Interpretability</b>	High (defined by intercept and slope)	Low (no clear interpretability due to flexibility)
<b>Fit Quality</b>	May underfit non-linear relationships	Better fit for non-linear relationships
<b>RMSE/RMSPE</b>	Lower on linear relationships	Lower on non-linear relationships
<b>Extrapolation Behavior</b>	Predicts with constant slope, can be inaccurate (e.g., negative prices)	Produces flat predictions at boundaries, may not match reality

## Multivariable linear regression

- Extends simple linear regression to multiple predictors.
- Similar to KNN regression, simply add more predictors to the training data.
- Each predictor variable *may* give us new information to help create our model.

The only difference is the formula:

$$\text{house sale price} = \beta_0 + \beta_1 \text{house size} + \beta_2 \text{number of bedrooms}$$

where:

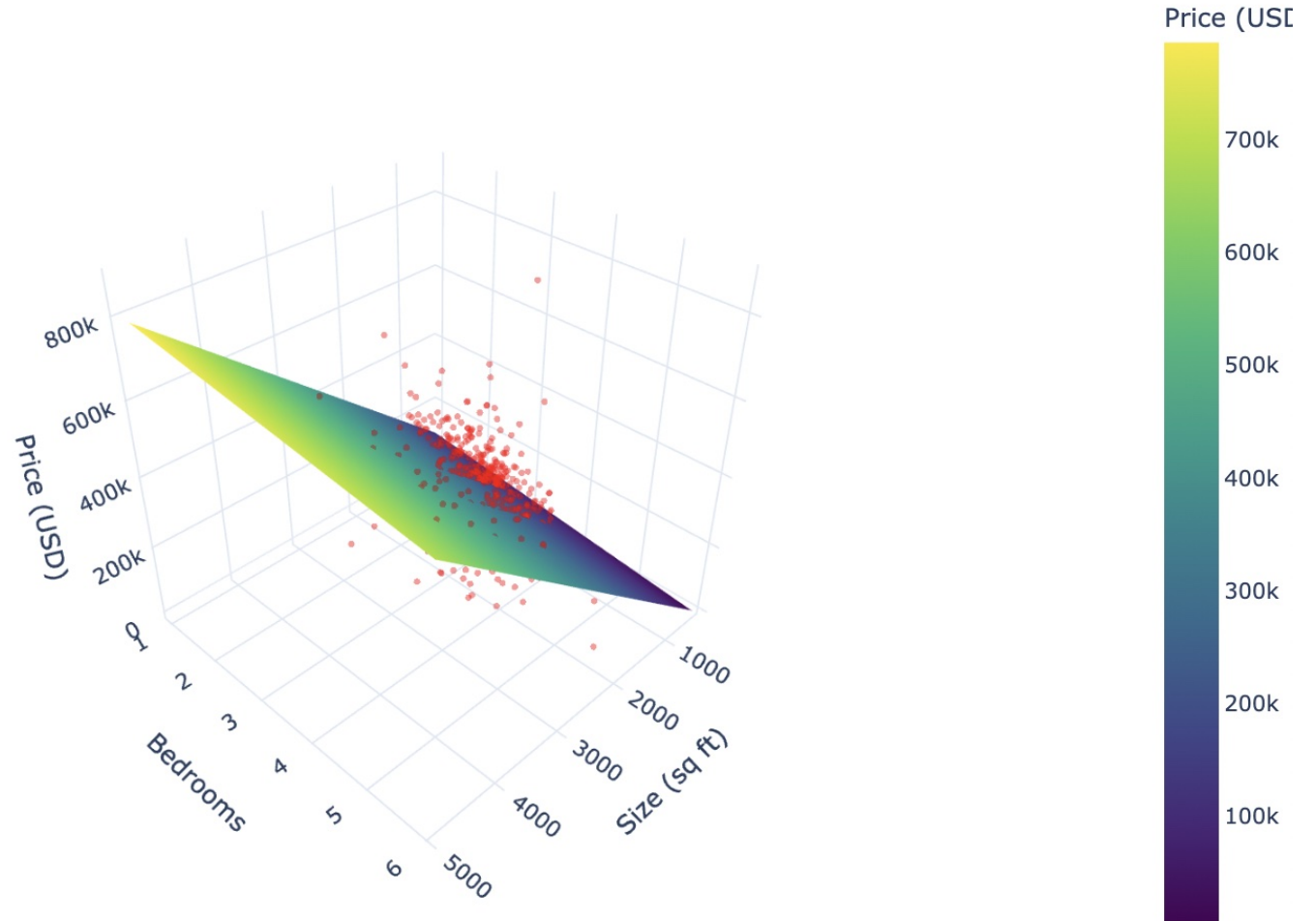
$\beta_0$  is the vertical intercept (price when house size is 0).

$\beta_1$  is the slope (rate of price increase as house size increases).

$\beta_2$  is the slope (rate of price increase as number of bedrooms increases).

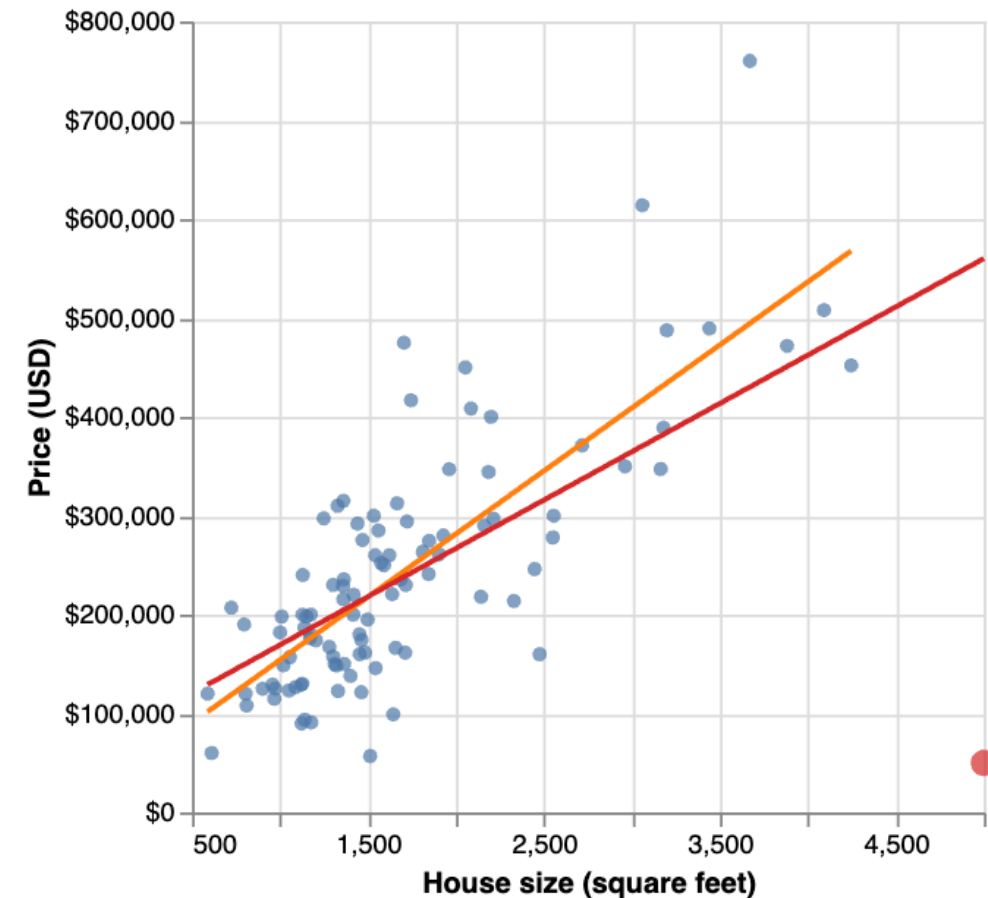


In the case of two predictors, we can plot the predictions made by our linear regression to create a plane of best fit.

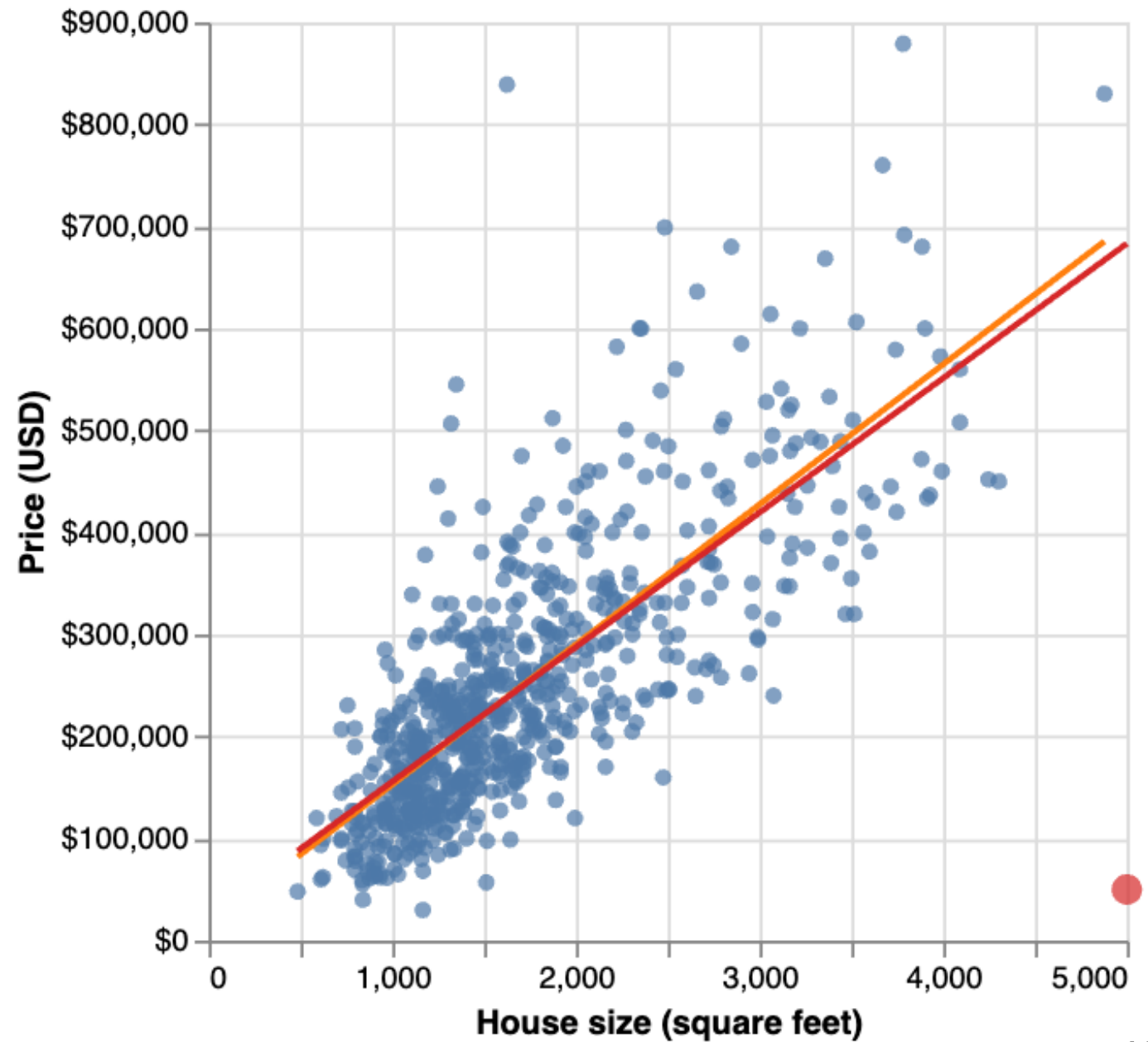


## Outliers

- Data points with unusually high or low vertical distances from the line of best fit.
- Outliers can disproportionately influence the line of best fit.
- Identifying outliers accurately often requires advanced techniques.
- Eg., a single outlier (a 5,000 sq. ft. house sold for \$50,000) dramatically alters the line of best fit, changing it from the original (orange) to a new line (red).

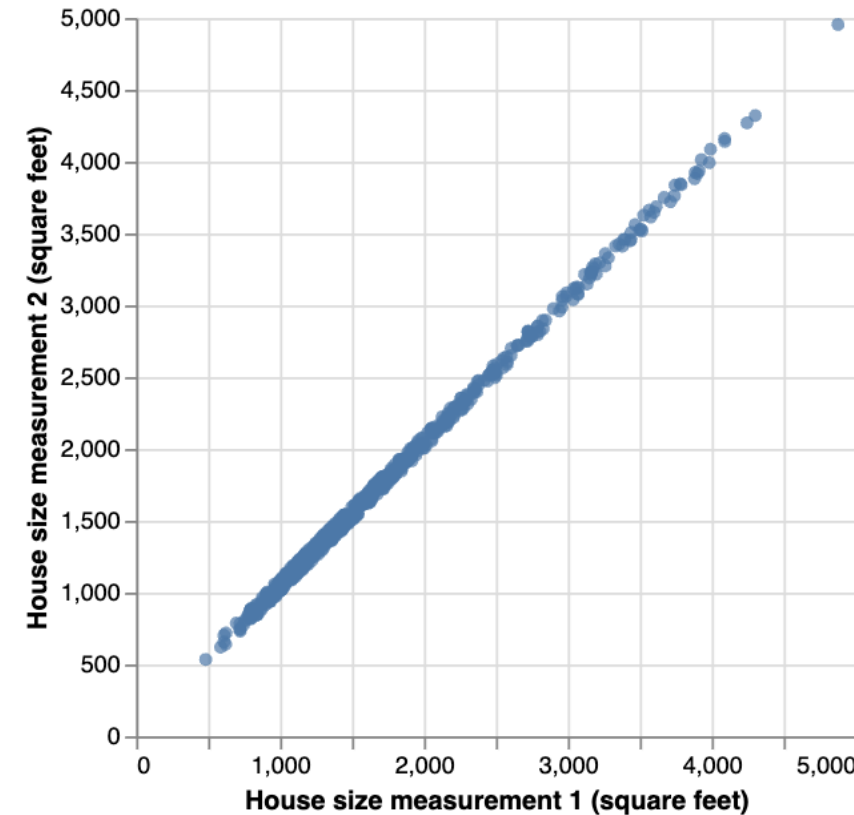


Fortunately, if you have enough data, the inclusion of one or two outliers—as long as their values are not too wild— will typically not have a large effect on the line of best fit.



## Multicollinearity

- Occurs in multivariable linear regression when predictors are strongly linearly related.
- if predictors are highly correlated, the model's coefficients can become very sensitive to slight changes in the data.
- This sensitivity can lead to large variations in the estimated coefficients when using different data splits or subsets.
- Identifying multicollinearity often requires techniques like Variance Inflation Factor that are beyond the scope of this module.



**Putting it all together**

**linear regression with scikit-learn**