Computational Complexity

Let's take a look at some familiar problems.

- Diophantine Problem
- Matching Problem
- Vertex Cover Problem
- Graph Isomorphism Problem

We learnt from Computability Course and Algorithm Course that

- Diophantine is undecidable,
- Matching is in **P**,
- Vertex Cover is **NP**-complete, and
- Graph Isomorphism is yet to be classified.

This course is about classifying and comparing problems by the amount of resource necessary to solve them.

We shall get to know some of the main techniques in theoretical investigation.

- Recursion Theoretical Method
- Combinatorial Method
- Algebraic Method
- Probabilistic Method
- ...

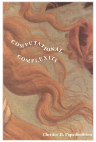
We shall be exposed to many great ideas in Computer Science.

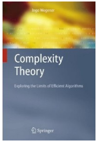
Blum's Speedup Theorem, Borodin-Trakhtenbrot Gap Theorem, **BPP**, Hierarchy Theorem, Savitch Theorem, Stockmeyer-Meyer Theorem, **NC**, Karp Theorem, Cook-Levin Theorem, $\mathbf{PH} \subseteq \mathbf{PSPACE}$, Baker-Gill-Solovay Theorem, Immerman-Szelepcsényi Theorem, $\mathtt{Dist}\mathbf{NP}$, Ladner Theorem, Circuit Complexity, Chandra-Kozen-Stockmeyer Theorem, PCP Theorem, **P**-Completeness, Aleliunas-Karp-Lipton-Lovász-Rackoff Theorem, **PP**, Valiant Theorem, $\sharp\mathbf{P}$, Valiant-Vazirani Theorem, Toda Theorem, Impagliazzo-Levin Theorem, Adleman Theorem, Goldbach-Levin Theorem, NP-Completeness, Zero Knowledge, Yao's Unpredictability Theorem, Lund-Karloff-Fortnow-Nisan Theorem, Yao's Max-Min Theorem, Derandomization, **AM**, Barrier Results, Goldbach-Goldwasser-Micali Theorem, Pseudorandomness, One-Way Function, Nisan-Wigderson Generator, $\mathbf{IP} = \mathbf{PSPACE}$, Hartmanis Conjecture, Hardness Amplification, Hierarchy Theorem, Exponential Conjecture, Sudan's List Decoding, **RP**, Reingold Theorem, Hartmanis-Stearns-Hennie Theorem, Goldwasser-Sipser Theorem, Randomness Extractor, $\mathbf{QIP} = \mathbf{PSPACE}$, Log-Rank Conjecture, Circuit Lower Bound, Levin Theory, Natural Proof, hardness of approximation, communication complexity, **BQP**, Håstad Switching Lemma, Circuit Hierarchy Theorem, ...

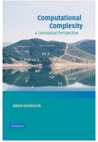
In Part I we discuss efficient computation. In Part II we study hard problems using a combination of ideas that can be summarized as

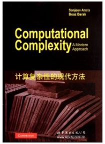
$$\text{“error} + \text{probability} + \text{interaction”}.$$

1. Christos Papadimitriou. Computational Complexity. 1994.
2. Ingo Wegener. Complexity Theory, exploring the limits of efficient algorithms. 2005.
3. Oded Goldreich. Computational Complexity, a conceptual perspective. 2008.
4. Sanjeev Arora, Boaz Barak. Computational Complexity, a modern approach. 2009.









Your final score:

- Attendance (5)
- Homework (20)
- Tests (75)

Test One

Design a universal Turing Machine $\mathbb{U}$ that satisfies the following:

- If $\mathbb{M}_\alpha$ runs in $O(T(n))$ time, then $\mathbb{U}(\alpha, \_)$ runs in $O(T(n) \log T(n))$ time.

You need to write down the complete (executable) program of $\mathbb{U}$ and explain how it works.

Test Two

Prove Ladner Theorem.

Test Three

Let L be decided by a P-time NDTM $\mathbb{N}$. Construct a Cook-Levin reduction from L to $\texttt{SAT}$ that is implicitly logspace computable.

Test Four

Prove Immerman-Szelepcsényi Theorem.

Test Five

1. Prove that reachability problem is in $\mathbf{NC}^2$.
2. Prove that logspace reduction is efficient parallel.
3. Suppose L is $\mathbf{P}$-complete. Prove that $L \in \mathbf{NC}$ iff $\mathbf{P} = \mathbf{NC}$.

Enjoy the course!