$Notebook Evaluate [T:\5183\capstone\Computational-Characteristic Classes\Characteristic classes\ back Automatic];$

 $Notebook Evaluate [T:\5183\capstone\Computational-Characteristic Classes\Characteristic classes\ back-Automatic];$

TODD GENUS - Pontryagin

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twisted
GenusToddP[Q_,deg_Integer]/; deg ==0:=chernCh[0]; twisted
GenusToddP[Q_, deg_Integer]:=Module[{genusTable, chernCharTable}, genusTable = Table
[(toddCompl[i] * t^{\wedge}(2*i)/.c \rightarrow p), \{i,0,\deg/2\}]; chernCharTable = Table
[(chernChToP[m]*t^{\wedge}(2*m)/.p \rightarrow p), \{m,0,2*\deg/2\}]; If
[Divisible[deg, 2], Total
[Coefficient[Flatten[Outer[Times, genusTable, chernCharTable]], t^{\wedge}(\deg)]], Print
[Error: Input degree must be divisible by 2.]
```

$For[i=0,i<17,i+=2,Print[Subscript["[Td(E)ch(F_r)]",i],"=",twistedGenusToddP[\#/(1-Exp[-\#])\&,i]] + (1-Exp[-\#])\&,i] + (1-$

```
\begin{split} & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_0 = r \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_2 = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_4 = -\frac{rp_1}{12} + \frac{p_1}{2} \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_6 = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_8 = \frac{1}{720}r\left(3p_1^2 - p_2\right) - \frac{p_1p_1}{24} + \frac{1}{24}\left(p_1^2 - 2p_2\right) \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{10} = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{12} = \frac{r\left(-10p_1^3 + 9p_1p_2 - 2p_3\right)}{60480} + \frac{\left(3p_1^2 - p_2\right)p_1}{1440} - \frac{1}{288}p_1\left(p_1^2 - 2p_2\right) + \frac{1}{720}\left(p_1^3 - 3p_1p_2 + 3p_3\right) \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{14} = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{16} = \frac{r\left(21p_1^4 - 34p_1^2p_2 + 5p_2^2 + 13p_1p_3 - 3p_4\right)}{3628800} + \frac{\left(-10p_1^3 + 9p_1p_2 - 2p_3\right)p_1}{120960} + \frac{\left(3p_1^2 - p_2\right)\left(p_1^2 - 2p_2\right)}{17280} - \frac{p_1\left(p_1^3 - 3p_1p_2 + 3p_3\right)}{8640} + \frac{p_1^4 - 4p_1^2p_2 + 2p_2^2 + 4p_1p_3 - 4p_4}{40320} \end{split}
```

 $p_{-} 1 = 0$

$\label{eq:formula} \text{For} \ [i=0, i<17, i+=2, \text{Print} \ [\text{Subscript}[\text{``[Td(E)ch(F_r)]''}, i], \text{``=''}, \\ \text{twistedGenusToddP}[\#/(1-\text{Exp}[-\#])\&, i] = \text{``, twistedGenusToddP}[\#/(1-\text{Exp}[-\#])\&, i] = \text{``, twistedGenusToddP}[\#/$

```
\begin{split} & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_0 = r \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_2 = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_4 = \frac{p_1}{2} \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_6 = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_6 = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_8 = -\frac{rp_2}{720} + \frac{1}{24}\left(p_1^2 - 2p_2\right) \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_{10} = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_{12} = -\frac{rp_3}{30240} - \frac{p_2p_1}{1440} + \frac{1}{720}\left(p_1^3 - 3p_1p_2 + 3p_3\right) \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_{14} = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F.r)]_{16} = \frac{r\left(5p_2^2 - 3p_4\right)}{3628800} - \frac{p_3p_1}{60480} - \frac{p_2\left(p_1^2 - 2p_2\right)}{17280} + \frac{p_1^4 - 4p_1^2p_2 + 2p_2^2 + 4p_1p_3 - 4p_4}{40320} \end{split}
```

$p'_{-1}=0$

 $For [i=0, i<17, i+=2, Print [Subscript["[Td(E)ch(F_r)]", i], "=", twistedGenusToddP[\#/(1-Exp[-\#])\&, i], "=",$

```
\begin{split} &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_0 = r \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_2 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_4 = -\frac{rp_1}{12} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_6 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_8 = \frac{1}{720}r\left(3p_1^2 - p_2\right) - \frac{p_2}{12} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{10} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{12} = \frac{r\left(-10p_1^3 + 9p_1p_2 - 2p_3\right)}{60480} + \frac{p_1p_2}{144} + \frac{p_3}{240} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{14} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{16} = \frac{r\left(21p_1^4 - 34p_1^2p_2 + 5p_2^2 + 13p_1p_3 - 3p_4\right)}{3628800} - \frac{\left(3p_1^2 - p_2\right)p_2}{8640} - \frac{p_1p_3}{2880} + \frac{2p_2^2 - 4p_4}{40320} \\ \end{split}
```

$p_1 = p'_1 = 0$

For $[i=0,i<17,i+=2,Print [Subscript["[Td(E)ch(F_r)]",i],"=",twistedGenusToddP[\#/(1-Exp[-\#])\&,i]]$

```
\begin{split} &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_0 = r \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_2 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_4 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_6 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_8 = -\frac{rp_2}{720} - \frac{p_2}{12} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{10} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{12} = -\frac{rp_3}{30240} + \frac{p_3}{240} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{14} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{16} = \frac{r(5p_2^2 - 3p_4)}{3628800} + \frac{p_2p_2}{8640} + \frac{2p_2^2 - 4p_4}{40320} \end{split}
```

$p_{-}2 = 0$

 $\label{eq:formula} \text{For } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right[\text{``[Td(E)ch(F_r)]''}, i \right], \text{``} = \text{''}, \\ \text{twistedGenusToddP} \left[\#/(1-\text{Exp}[-\#]) \&, i \right] = \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i+=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<17, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i<27, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i=2, \text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i=2, \text{Print} \left[\text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i=2, \text{Print} \left[\text{Print} \left[\text{Subscript} \right] + \text{''}, \\ \text{Total } [i=0, i=2, \text{Print} \left[\text{Print}$

$$\begin{split} &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_0 = r \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_2 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_4 = -\frac{rp_1}{12} + \frac{p_1}{2} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_6 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_6 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_8 = \frac{rp_1^2}{240} - \frac{p_1p_1}{24} + \frac{1}{24}\left(p_1^2 - 2p_2\right) \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{10} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{12} = \frac{r\left(-10p_1^3 - 2p_3\right)}{60480} + \frac{1}{480}p_1^2p_1 - \frac{1}{288}p_1\left(p_1^2 - 2p_2\right) + \frac{1}{720}\left(p_1^3 - 3p_1p_2 + 3p_3\right) \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{14} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{16} = \frac{r\left(21p_1^4 + 13p_1p_3 - 3p_4\right)}{3628800} + \frac{\left(-10p_1^3 - 2p_3\right)p_1}{120960} + \frac{p_1^2\left(p_1^2 - 2p_2\right)}{5760} - \frac{p_1\left(p_1^3 - 3p_1p_2 + 3p_3\right)}{8640} + \frac{p_1^4 - 4p_1^2p_2 + 2p_2^2 + 4p_1p_3 - 4p_4}{40320} \end{split}$$

p'_2=0

 $For [i=0, i<17, i+=2, Print [Subscript["[Td(E)ch(F_r)]", i], "=", twistedGenusToddP[\#/(1-Exp[-\#])\&, i], "=",$

$$\begin{split} & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_0 = r \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_2 = 0 \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_4 = -\frac{rp_1}{12} + \frac{p_1}{2} \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_6 = 0 \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_6 = 0 \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_8 = \frac{1}{720}r\left(3p_1^2 - p_2\right) - \frac{p_1p_1}{24} + \frac{p_1^2}{24} \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_{10} = 0 \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_{12} = \frac{r\left(-10p_1^3 + 9p_1p_2 - 2p_3\right)}{60480} + \frac{\left(3p_1^2 - p_2\right)p_1}{1440} - \frac{1}{288}p_1p_1^2 + \frac{1}{720}\left(p_1^3 + 3p_3\right) \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_{14} = 0 \\ & [\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F}.\mathsf{r})]_{16} = \frac{r\left(21p_1^4 - 34p_1^2p_2 + 5p_2^2 + 13p_1p_3 - 3p_4\right)}{3628800} + \frac{\left(-10p_1^3 + 9p_1p_2 - 2p_3\right)p_1}{120960} + \frac{\left(3p_1^2 - p_2\right)p_1^2}{17280} - \frac{p_1\left(p_1^3 + 3p_3\right)}{8640} + \frac{p_1^4 + 4p_1p_3 - 4p_4}{40320} \end{split}$$

p_2=p'_2=0

 $\text{For } [i=0, i<17, i+=2, \text{Print } [\text{Subscript}[\text{``}[\text{Td}(\text{E})\text{ch}(\text{F_r})]\text{''}, i], \text{``} = \text{''}, \text{twistedGenusToddP}[\#/(1-\text{Exp}[-\#])\&, i] \text{''} = \text{''}, \text{''} =$

```
\begin{split} &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_0 = r \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_2 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_4 = -\frac{rp_1}{12} + \frac{p_1}{2} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_6 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_6 = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_8 = \frac{rp_1^2}{240} - \frac{p_1p_1}{24} + \frac{p_1^2}{24} \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{10} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{12} = \frac{r(-10p_1^3 - 2p_3)}{60480} + \frac{1}{480}p_1^2\mathbf{p}_1 - \frac{1}{288}p_1\mathbf{p}_1^2 + \frac{1}{720}\left(\mathbf{p}_1^3 + 3\mathbf{p}_3\right) \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{14} = 0 \\ &[\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F.r})]_{16} = \frac{r(21p_1^4 + 13p_1p_3 - 3p_4)}{3628800} + \frac{(-10p_1^3 - 2p_3)\mathbf{p}_1}{120960} + \frac{p_1^2\mathbf{p}_1^2}{5760} - \frac{p_1(\mathbf{p}_1^3 + 3\mathbf{p}_3)}{8640} + \frac{p_1^4 + 4\mathbf{p}_1\mathbf{p}_3 - 4\mathbf{p}_4}{40320} \end{split}
```

$p_{-}1=p'_{-}1=p_{-}2=0$

For [i=0,i<17,i+=2,Print] [Subscript] "[Td(E)ch(F_r)]", i], "=", twistedGenusToddP[#/(1 - Exp[-#])&, i]

$$\begin{split} & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_0 = r \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_2 = 0 \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_4 = 0 \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_6 = 0 \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_8 = -\frac{\mathbf{P}_2}{12} \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_{10} = 0 \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_{12} = -\frac{rp_3}{30240} + \frac{\mathbf{P}_3}{240} \\ & \left[\mathrm{Td}(\mathbf{E}) \mathrm{ch}(\mathbf{F}.\mathbf{r}) \right]_{14} = 0 \end{split}$$

```
[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_{16} = -\frac{rp_4}{1209600} + \frac{2\mathrm{p}_2^2 - 4\mathrm{p}_4}{40320}
p_1=p'_1=p'_2=0
For [i=0,i<17,i+=2,Print [Subscript["[Td(E)ch(F_r)]",i],"=",twistedGenusToddP[\#/(1-Exp[-\#])\&,i]]
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{r})]_{0} = r
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_2 = 0
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_{4} = 0
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_{6} = 0
  [Td(E)ch(F_x)]_8 = -\frac{rp_2}{720}

[Td(E)ch(F_x)]_{10} = 0
\begin{aligned} & \left[ \text{Td(E)ch(F.r)} \right]_{12}^{10} = -\frac{rp_3}{30240} + \frac{p_3}{240} \\ & \left[ \text{Td(E)ch(F.r)} \right]_{14} = 0 \end{aligned}
[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_{16} = \frac{r(5p_2^2 - 3p_4)}{3628800} - \frac{p_4}{10080}
p_1=p'_1=p_2=p'_2=0
\label{eq:formula} \text{For} \ [i=0, i<17, i+=2, \text{Print} \ [\text{Subscript}[\ \text{``[Td(E)ch(F_r)]''}, i], \ \text{``=''}, \\ \text{twistedGenusToddP}[\#/(1-\text{Exp}[-\#])\&, i] \ \text{``=''}, \\ \text{twistedGenusToddP}[\#/(1-\text{Exp}[-\#])\&, i] \ \text{``=''}, \\ \text{Total Print}[\ \text{Print} \ \text{Prin
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_0 = r
  \left[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})\right]_{2} = 0
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\!\mathrm{r})]_{4} = 0
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_{6} = 0
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_{8} = 0
  \left[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}.\mathrm{r})\right]_{10} = 0
  [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}_{-}\mathrm{r})]_{12}^{13} = -\frac{rp_3}{30240} + \frac{p_3}{240}
  \begin{aligned} & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_{14} = 0 \\ & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_{16} = -\frac{rp_4}{1209600} - \frac{p_4}{10080} \end{aligned}
TODD GENUS - Chern
NotebookEvaluate[T:\\5183\\capstone\\Computational-Characteristic-Classes\\characteristic classes backgro
Automatic];
               twistedGenusTodd[Q_-, deg\_Integer]/; deg == 0 := chernCh[0];
twistedGenusTodd[Q_, deg_Integer]:=Module[{genusTable, chernCharTable},
genus Table = Table [(genus [Q, i] * t^{\wedge}(2 * i)/.c \rightarrow c), \{i, 0, \deg/2\}];
chernCharTable = Table[(chernCh[m] * t^{(2*m)}/.c \rightarrow c), {m, 0, 2 * deg/2}];
If[Divisible[deg, 2],
Total[Coefficient[Flatten[Outer[Times, genusTable, chernCharTable]], t^{\wedge}(deg)]].
Print[Error: Input degree must be divisible by 2.]
```

 $For[i = 0, i < 17, i + 2, Print[Subscript["[Td(E)ch(F_r)]", i], " = ", twistedGenusTodd[\#/(1 - Exp[-\#])\&, i]]]$

```
\begin{split} & [Td(E) \text{ch}(F, \mathbf{r})]_0 = r \\ & [Td(E) \text{ch}(F, \mathbf{r})]_2 = \frac{rc_1}{12} + c_1 \\ & [Td(E) \text{ch}(F, \mathbf{r})]_2 = \frac{rc_1}{12} + c_1 \\ & [Td(E) \text{ch}(F, \mathbf{r})]_3 = \frac{1}{12} r (c_1^2 + c_2) + \frac{c_1c_1}{12} (c_1^2 + c_2) + \frac{1}{14} (c_1^2 - 2c_2) + \frac{1}{6} (c_1^3 - 3c_1c_2 + 3c_3) \\ & [Td(E) \text{ch}(F, \mathbf{r})]_6 = \frac{1}{24} rc_1c_2 + \frac{1}{12} (c_1^2 + c_2) + c_1c_3 - c_4) + \frac{1}{24} c_1c_2c_1 + \frac{1}{24} (c_1^2 + c_2) (c_1^2 - 2c_2) + \frac{1}{12} c_1 (c_1^3 - 3c_1c_2 + 3c_3) \\ & [Td(E) \text{ch}(F, \mathbf{r})]_8 = \frac{r}{720} r (-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4) + \frac{1}{24} c_1c_2c_1 + \frac{1}{24} (c_1^2 + c_2) (c_1^2 - 2c_2) + \frac{1}{12} c_1 (c_1^3 - 3c_1c_2 + 3c_3) + \frac{1}{24} (c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) \\ & [Td(E) \text{ch}(F, \mathbf{r})]_{10} = \frac{r}{(-c_1^3c_2+3c_1c_2^2+c_1^2c_3-c_1c_4)} + \frac{1}{720} (-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4) c_1 + \frac{1}{48} c_1c_2 (c_1^2 - 2c_2) + \frac{1}{72} (c_1^2 + c_2) (c_1^3 - 3c_1c_2 + 3c_3) + \frac{1}{48} c_1 (c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) + \frac{1}{120} (c_1^5 - 5c_1^3c_2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5) \\ & [Td(E) \text{ch}(F, \mathbf{r})]_{12} = \frac{r}{(2c_1^6 - 2c_1^4c_2 + 11c_1^2c_2^2 + 10c_3^2 + 5c_1^2c_3 + 1c_1c_2c_2 - c_3^2 - 5c_1^2c_4 - 9c_2c_4 - 2c_1c_5 + 2c_6) + \frac{1}{144} c_1c_2 + c_1^2c_2^2 + 1c_2^2 + 2c_2^2 + 4c_1c_3 - 4c_4) + \frac{1}{120} c_1^2c_1^2 - 3c_1^2c_2^2 + 3c_1^2c_3 + 3c_1^2c_2^2 + 3c_1^2c_3 + 3c_1^2c_2 + 3c_3^2 + c_1^2c_3 - 3c_1c_2 + 3c_3) + \frac{1}{144} c_1^2c_1^2c_1^2 - 3c_1^2c_2^2 + 3c_1^2c_3^2 + 3c_1^2c_2^2 + 3c_1^2c_3^2 + 3c_1^2c_
```

 $c_{-1} = 0$

For $[i=0, i<17, i+=2, Print [Subscript["[Td(E)ch(F_r)]", i], "=", twistedGenusTodd[#/(1 - Exp[-#])&, i]/$

```
\begin{split} &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_0 = r \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_4 = \frac{rc_2}{12} + \frac{1}{2}\left(c_1^2 - 2c_2\right) \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_8 = \frac{1}{720}r\left(3c_2^2 - c_4\right) + \frac{1}{24}c_2\left(c_1^2 - 2c_2\right) + \frac{1}{24}\left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right) \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_{12} = \frac{r\left(10c_2^3 - c_3^2 - 9c_2c_4 + 2c_6\right)}{60480} + \frac{\left(3c_2^2 - c_4\right)\left(c_1^2 - 2c_2\right)}{1440} + \frac{1}{288}c_2\left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right) + \\ &\frac{1}{720}\left(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6\right) \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_{16} = \frac{r\left(21c_2^4 - 8c_2c_3^2 - 34c_2^2c_4 + 5c_4^2 + 3c_3c_5 + 13c_2c_6 - 3c_8\right)}{3628800} + \frac{\left(10c_3^3 - c_3^2 - 9c_2c_4 + 2c_6\right)\left(c_1^2 - 2c_2\right)}{120960} + \\ &\frac{\left(3c_2^2 - c_4\right)\left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right)}{17280} + \frac{c_2\left(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_3^2 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6\right)}{8640} + \\ &\frac{17280}{600}c_1^2 - \frac{17280}{600}c_2^2 - \frac{17280}{600}c_3^2 - \frac{1728
```

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 [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}\,\mathbf{r})]_{20} = \frac{r \left(90c_2^5 - 81c_2^2c_3^2 - 219c_2^3c_4 + 21c_3^2c_4 + 87c_2c_4^2 + 53c_2c_3c_5 - 5c_5^2 + 109c_2^2c_6 - 32c_4c_6 - 10c_3c_7 - 43c_2c_8 + 10c_{10}\right)}{479001600} + \frac{\left(21c_2^4 - 8c_2c_3^2 - 34c_2^2c_4 + 5c_4^2 + 3c_3c_5 + 13c_2c_6 - 3c_8\right)\left(c_1^2 - 2c_2\right)}{7257600} + \frac{\left(10c_2^3 - c_3^2 - 9c_2c_4 + 2c_6\right)\left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right)}{1451520} + \frac{\left(3c_2^2 - c_4\right)\left(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6\right)}{1628800} + \frac{c_2\left(c_1^8 - 8c_1^6c_2 + 20c_1^4c_2^2 - 16c_1^2c_2^3 + 2c_2^4 + 8c_1^5c_3 - 32c_1^3c_2c_3 + 2c_2^4 + 6c_2c_3^2 + 3c_2^2c_3^2 + 2c_2^4 + 3c_2^2c_3^2 + 2c_2^2 + 3c_2^2c_3^2 +
```

$c'_{-1}=0$

$\text{For} \ [i=0, i<17, i+=2, \text{Print} \ [\text{Subscript}[\text{``[Td(E)ch(F_r)]''}, i], \text{``=''}, \\ \text{twistedGenusTodd}[\#/(1-\text{Exp}[-\#])\&, i]/(1-\text{Exp}[-\#])\&, i)/(1-\text{Exp}[-\#])\&, i)/(1-\text{Exp}[-$

```
\begin{split} &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_0 = r \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_4 = \frac{1}{12}r\left(c_1^2 + c_2\right) - c_2 \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_8 = \frac{1}{720}r\left(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4\right) - \frac{1}{12}\left(c_1^2 + c_2\right)c_2 + \frac{c_1c_3}{4} + \frac{1}{24}\left(2c_2^2 - 4c_4\right) \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_{12} = \frac{r\left(2c_1^6 - 12c_1^4c_2 + 11c_1^2c_2^2 + 10c_3^2 + 5c_1^3c_3 + 11c_1c_2c_3 - c_3^2 - 5c_1^2c_4 - 9c_2c_4 - 2c_1c_5 + 2c_6\right)}{60480} - \frac{1}{720}\left(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4\right)c_2 + \frac{1}{48}c_1c_2c_3 + \frac{60480}{288}\left(c_1^2 + c_2\right)\left(2c_2^2 - 4c_4\right) + \frac{1}{240}c_1\left(-5c_2c_3 + 5c_5\right) + \frac{1}{720}\left(-2c_2^3 + 3c_3^2 + 6c_2c_4 - 6c_6\right) \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_{16} = \frac{r\left(-3c_1^8 + 24c_1^6c_2 - 50c_1^4c_2^2 + 8c_1^2c_3^3 + 21c_2^4 - 14c_1^5c_3 + 26c_1^3c_2c_3 + 50c_1c_2^2c_3 + 3c_1^2c_3^2 - 8c_2c_3^2 + 14c_1^4c_4 - 19c_1^2c_2c_4 - 34c_2^2c_4 - 13c_2^2c_3 + 3c_1^2c_3^2 + 3c_1^2c_3^2 - 8c_2c_3^2 + 14c_1^4c_4 - 19c_1^2c_2c_4 - 34c_2^2c_4 - 13c_1^2c_3^2 + 3c_1^2c_3^2 + 3c_1^2c_3^2 - 3c_1^2c_3^2 + 3c_1^2c_3^2 - 3c_1^2c_4 - 3c_1^2c_3^2 - 3c_1^2c_4^2 - 3c_1^2c_4
```

$c_{-}1=c'_{-}1=0$

$\text{For}\ [i=0,i<17,i+=2,\text{Print}\ [\text{Subscript}[\text{``[Td(E)ch(F_r)]''},i],\text{``=''}, \\ \text{twistedGenusTodd}[\#/(1-\text{Exp}[-\#])\&,i]/(1-\text{Exp}[-\#])\&,i)/$

```
\begin{split} & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_0 = r \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_2 = 0 \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_4 = \frac{rc_2}{12} - c_2 \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_6 = \frac{c_3}{2} \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_6 = \frac{c_3}{2} \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_{10} = \frac{c_2 c_3}{24} + \frac{1}{120} \left( -5 c_2 c_3 + 5 c_5 \right) \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_{10} = \frac{r \left( 10 c_3^3 - c_3^2 - 9 c_2 c_4 + 2 c_6 \right)}{60480} - \frac{1}{720} \left( 3 c_2^2 - c_4 \right) c_2 + \frac{1}{288} c_2 \left( 2 c_2^2 - 4 c_4 \right) + \\ & \frac{1}{720} \left( -2 c_2^3 + 3 c_3^2 + 6 c_2 c_4 - 6 c_6 \right) \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_{14} = \frac{\left( 3 c_2^2 - c_4 \right) c_3}{1440} + \frac{c_2 \left( -5 c_2 c_3 + 5 c_5 \right)}{1440} + \frac{7 c_2^2 c_3 - 7 c_3 c_4 - 7 c_2 c_5 + 7 c_7}{5040} \\ & \left[ \mathrm{Td}(\mathsf{E}) \mathrm{ch}(\mathsf{F.r}) \right]_{16} = \frac{r \left( 21 c_4^4 - 8 c_2 c_3^2 - 34 c_2^2 c_4 + 5 c_4^2 + 3 c_3 c_5 + 13 c_2 c_6 - 3 c_8 \right)}{3628800} - \frac{\left( 10 c_3^3 - c_3^2 - 9 c_2 c_4 + 2 c_6 \right) c_2}{60480} + \\ & \frac{\left( 3 c_2^2 - c_4 \right) \left( 2 c_2^2 - 4 c_4 \right)}{17280} + \frac{c_2 \left( -2 c_3^2 + 3 c_3^2 + 6 c_2 c_4 - 6 c_6 \right)}{8640} + \frac{2 c_4^4 - 8 c_2 c_3^2 - 8 c_2^2 c_4 + 4 c_4^2 + 8 c_3 c_5 + 8 c_2 c_6 - 8 c_8}{40320} \end{split}
```

 $c_{-} 2 = 0$

```
\begin{split} [Td(E)\operatorname{ch}(F,r)]_0 &= r \\ [Td(E)\operatorname{ch}(F,r)]_2 &= \frac{r_{c_1}}{2} + c_1 \\ [Td(E)\operatorname{ch}(F,r)]_4 &= \frac{r_{c_1}^2}{12} + \frac{1}{2} + \frac{1}{2} \left(c_1^2 - 2c_2\right) \\ [Td(E)\operatorname{ch}(F,r)]_4 &= \frac{r_{c_1}^2}{12} + \frac{1}{4}c_1 \left(c_1^2 - 2c_2\right) + \frac{1}{6} \left(c_1^3 - 3c_1c_2 + 3c_3\right) \\ [Td(E)\operatorname{ch}(F,r)]_6 &= \frac{1}{12}c_1^2c_1 + \frac{1}{4}c_1 \left(c_1^2 - 2c_2\right) + \frac{1}{6} \left(c_1^3 - 3c_1c_2 + 3c_3\right) \\ [Td(E)\operatorname{ch}(F,r)]_8 &= \frac{7}{720}r \left(-c_1^4 + c_1c_3 - c_4\right) + \frac{1}{24}c_1^2 \left(c_1^2 - 2c_2\right) + \frac{1}{12}c_1 \left(c_1^3 - 3c_1c_2 + 3c_3\right) + \\ \frac{1}{24} \left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right) \\ [Td(E)\operatorname{ch}(F,r)]_{10} &= \frac{r(c_1^2c_3-c_1c_4)}{1440} + \frac{1}{720} \left(-c_1^4 + c_1c_3 - c_4\right) c_1 + \frac{1}{72}c_1^2 \left(c_1^3 - 3c_1c_2 + 3c_3\right) + \\ \frac{1}{48}c_1 \left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right) + \frac{1}{120} \left(c_1^5 - 5c_1^3c_2 + 5c_1c_2^2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5\right) \\ [Td(E)\operatorname{ch}(F,r)]_{12} &= \frac{r(2c_1^4 + 5c_1^3c_3 - c_3^2 - 5c_1^2c_4 - 2c_1c_5 + 2c_6)}{60480} + \frac{(c_1^2c_3 - c_1c_4)}{1440} + \frac{(-c_1^4 + c_1c_3 - c_4)(c_1^2 - 2c_2)}{14} + \\ \frac{1}{288}c_1^2 \left(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4\right) + \frac{1}{240}c_1 \left(c_1^5 - 5c_1^3c_2 + 5c_1c_2^2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5\right) + \\ \frac{1}{20} \left(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6\right) \\ [Td(E)\operatorname{ch}(F,r)]_{14} &= \frac{r(-2c_1^4c_3 - c_1c_3^2 + 2c_1^2c_3 - 2c_1^2c_5 + 2c_1c_6}{120960} + \frac{(2c_1^2+5c_1^2c_3 - 5c_1^2c_3 - 5c_1c_4 + 5c_5) + (2c_1^2+5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5) + \\ \frac{(c_1^2c_3 - c_1c_4)(c_1^2 - 2c_2)}{2880} + \frac{(-c_1^4 + c_1c_3 - c_4)(c_1^2 - 3c_1c_2 + 3c_3^2 + 5c_1^2c_4 + 5c_1^2c_5^2 + 5c_1^2c_3 - 5c_1^2c_4 - 2c_1c_5 + 2c_1c_5) + \\ \frac{(c_1^2c_3 - c_1c_4)(c_1^2 - 2c_2)}{2880} + \frac{(-c_1^4 + c_1c_3 - c_4)(c_1^2 - 3c_1c_2 + 3c_3^2 + 5c_1^2c_4 + 5c_1^2c_3 - 5c_1^2c_4 - 2c_1c_5 + 2c_1c_5 + c_1^2c_3 - 5c_1^2c_4 + 2c_1^2c_2 - 2c_1^2c_3 + 7c_1^2c_3 + 7c_1^2c_3 - 7c_1^2c_4 + 4c_1^2c_2^2 - 7c_1^2c_2 + 7c_1^2c_3 + 7c_2^2c_3 + 7c_1^2c_3 - 7c_1^2c_4 + 4c_1^2c_2^2 - 7c_1^2c_2 + 7c_1^2c_3 + 7c_1^
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c'_2=0

$\text{For } [i=0, i<17, i+=2, \text{Print } [\text{Subscript}[\text{``}[\text{Td}(\text{E})\text{ch}(\text{F_r})]\text{''}, i], \text{``} = \text{''}, \\ \text{twistedGenusTodd}[\#/(1-\text{Exp}[-\#])\&, i]/(1-\text{Exp}[-\#])\&, i)/(1-\text{Exp}[-\#])\&, i)/(1-\text{Exp}[-\#]$

$$\begin{split} &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_0 = r \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_2 = \frac{rc_1}{2} + c_1 \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_4 = \frac{1}{12}r\left(c_1^2 + c_2\right) + \frac{c_1c_1}{2} + \frac{c_1^2}{2} \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_6 = \frac{1}{24}rc_1c_2 + \frac{1}{12}\left(c_1^2 + c_2\right)c_1 + \frac{1}{4}c_1c_1^2 + \frac{1}{6}\left(c_1^3 + 3c_3\right) \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_8 = \frac{1}{720}r\left(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4\right) + \frac{1}{24}c_1c_2c_1 + \frac{1}{24}\left(c_1^2 + c_2\right)c_1^2 + \frac{1}{12}c_1\left(c_1^3 + 3c_3\right) + \frac{1}{24}\left(c_1^4 + 4c_1c_3 - 4c_4\right) \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_{10} = \frac{r(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)}{1440} + \frac{1}{720}\left(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4\right)c_1 + \frac{1}{48}c_1c_2c_1^2 + \frac{1}{72}\left(c_1^2 + c_2\right)\left(c_1^3 + 3c_3\right) + \frac{1}{48}c_1\left(c_1^4 + 4c_1c_3 - 4c_4\right) + \frac{1}{120}\left(c_1^5 + 5c_1^2c_3 - 5c_1c_4 + 5c_5\right) \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_{12} = \frac{r(2c_1^6 - 12c_1^4c_2 + 11c_1^2c_2^2 + 10c_2^3 + 5c_1^3c_3 + 11c_1c_2c_3 - c_3^2 - 5c_1^2c_4 - 9c_2c_4 - 2c_1c_5 + 2c_6}{60480} + \frac{(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)c_1}{1440} + \frac{(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4)c_1^2}{1240}c_1\left(c_1^5 + 5c_1^2c_3 - 5c_1c_4 + 5c_5\right) + \frac{1}{720}\left(c_1^6 + 6c_1^3c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_1c_5 - 6c_6\right) \\ &[\operatorname{Td}(\mathsf{E})\operatorname{ch}(\mathsf{F.r})]_{14} = \frac{r(2c_1^5c_2 - 10c_1^3c_2^2 + 10c_1^3c_2^2 + 10c_1^3c_2 - 2c_1^4c_3 + 11c_1^2c_2c_3 - c_1c_3^2 + 2c_1^3c_4 - 9c_1c_2c_4 - 2c_1^2c_5 + 2c_1c_6}) + \frac{(2c_1^6 - 12c_1^4c_2 + 11c_1^2c_2^2 + 10c_2^3 + 5c_1^3c_3 + 11c_1c_2c_3 - c_2^3 - 5c_1^2c_4 - 9c_2c_4 - 2c_1c_5 + 2c_6})c_1}{120960} + \frac{(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)c_1}{2880} + \frac{(-c_1^3c_2 + 3c_1c_2^2 + 2c_1^2c_3 - 2c_1c_4)c_1}{2880} + \frac{(-c_1^3c_2 + 3c_1c_2^2 + 2c_1^2c_3 - 2c_1c_4)c_1}{2880} + \frac{(-c_1^3c_2 + 3c_1c_2^2 + 2c_1^2c_3 -$$

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\frac{\left(-c_1^4+4c_1^2c_2+3c_2^2+c_1c_3-c_4\right)\left(c_1^3+3c_3\right)}{4320} + \frac{1}{576}c_1c_2\left(c_1^4+4c_1c_3-4c_4\right) + \frac{\left(c_1^2+c_2\right)\left(c_1^5+5c_1^2c_3-5c_1c_4+5c_5\right)}{1440} + \frac{c_1\left(c_1^6+6c_1^3c_3+3c_3^2-6c_1^2c_4+6c_1c_5-6c_6\right)}{1440} + \frac{c_1^7+7c_1^4c_3+7c_1c_3^2-7c_1^3c_4-7c_3c_4+7c_1^2c_5-7c_1c_6+7c_7}{5040} + \frac{c_1\left(c_1^6+6c_1^3c_3+3c_3^2-6c_1^2c_4+6c_1c_5-6c_6\right)}{5040} + \frac{c_1\left(c_1^6+3c_1^2c_3+2c_1^2c_3+2c_1^2c_3+3c_1^2c_3-5c_1c_2^2c_3+3c_1^2c_3^2-8c_2c_3^2+14c_1^4c_4-19c_1^2c_2c_4-34c_2^2c_4-13c_1^2c_2^2c_3+3c_1^2c_3^2+2c_1^2c_3^2+3c_1^2c_3^2+2c_1^2c_3+3c_1^2c_3^2+2c_1^2c_3+3c_1^2c_3^2+3c_1^2c_3^2+3c_1^2c_3^2+2c_1^2c_3+11c_1^2c_2c_3-c_1^2c_3^2+2c_1^2c_3+9c_1c_2c_4-2c_1^2c_5+2c_1c_6\right)c_1}{\frac{\left(2c_1^5c_2-10c_1^3c_2^2+10c_1c_3^2-2c_1^4c_3+11c_1^2c_2c_3-c_1c_3^2+2c_1^2c_3-2c_1c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1^2c_3+2c_1
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$c_2=c_2=0$


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\begin{split} &[Td(E)ch(F\,x)]_0 = r \\ &[Td(E)ch(F\,x)]_2 = \frac{rc_1}{2} + c_1 \\ &[Td(E)ch(F\,x)]_4 = \frac{rc_1^2}{12} + \frac{c_1c_1}{2} + \frac{c_1^2}{2} \\ &[Td(E)ch(F\,x)]_6 = \frac{1}{12}c_1^2c_1 + \frac{1}{4}c_1c_1^2 + \frac{1}{6}\left(c_1^3 + 3c_3\right) \\ &[Td(E)ch(F\,x)]_8 = \frac{1}{720}r\left(-c_1^4 + c_1c_3 - c_4\right) + \frac{1}{24}c_1^2c_1^2 + \frac{1}{12}c_1\left(c_1^3 + 3c_3\right) + \frac{1}{24}\left(c_1^4 + 4c_1c_3 - 4c_4\right) \\ &[Td(E)ch(F\,x)]_{10} = \frac{r(c_1^2c_3 - c_1c_4)}{1440} + \frac{1}{720}\left(-c_1^4 + c_1c_3 - c_4\right) c_1 + \frac{1}{72}c_1^2\left(c_1^3 + 3c_3\right) + \frac{1}{48}c_1\left(c_1^4 + 4c_1c_3 - 4c_4\right) + \frac{1}{120}\left(c_1^5 + 5c_1^2c_3 - 5c_1c_4 + 5c_5\right) \\ &[Td(E)ch(F\,x)]_{12} = \frac{r(2c_1^6 + 5c_1^3c_3 - c_3^2 - 5c_1^2c_4 - 2c_1c_5 + 2c_6}{60480} + \frac{(c_1^2c_3 - c_1c_4)c_1}{1440} + \frac{(-c_1^4 + c_1c_3 - c_4)c_1^2}{4140} + \frac{1}{288}c_1^2\left(c_1^4 + 4c_1c_3 - 4c_4\right) + \frac{1}{240}c_1\left(c_1^6 + 5c_1^2c_3 - 5c_1c_4 + 5c_5\right) + \frac{(c_1^6c_3 - c_1c_4)c_1^2}{120}\left(c_1^6 + 6c_1^3c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_1c_5 - 6c_6\right) \\ &[Td(E)ch(F\,x)]_{14} = \frac{r(-2c_1^4c_3 - c_1c_3^2 + 2c_1^2c_3 - 2c_1^2c_5 + 2c_1c_6)}{120960} + \frac{(2c_1^6 - 5c_1^2c_3 - 2c_1^2c_5 + 2c_1c_6)}{60480} + \frac{(2c_1^6 - 5c_1^2c_3 - 2c_1^2c_5 + 2c_1c_6)}{4320} + \frac{(2c_1^6 + 5c_1^2c_3 - 5c_1c_4 + 5c_5)}{1440} + \frac{c_1(c_1^6 + 6c_1^2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_1c_5 - 6c_6)}{60480} + \frac{(2c_1^2c_3 - c_1c_4)c_1^2}{2880} + \frac{(2c_1^4c_3 - c_1c_3^2 + 2c_1^2c_5 + 2c_1c_6 + 7c_7}{1440} + \frac{(2c_1^6 + 5c_1^2c_3 - 2c_1^2c_5 + 2c_1c_6)c_1}{1440} + \frac{(2c_1^6 + 5c_1^2c_3 - 2c_1^2c_5 + 2c_1c_6)c_1}{1440} + \frac{(2c_1^6 + 6c_1^2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_1c_5 - 6c_6)}{1440} + \frac{(2c_1^4c_3 - 2c_1^2c_3 + 2c_1^2c_5 + 2c_1c_6)c_1}{1440} + \frac{(2c_1^6 + 5c_1^2c_3 - 2c_1^2c_5 + 2c_1c_
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c_1=c'_1=c_2=0


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\begin{split} & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_0 = r \\ & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_2 = 0 \\ & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_4 = -\mathrm{c_2} \\ & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_6 = \frac{\mathrm{c_3}}{2} \\ & [\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F.r})]_8 = -\frac{rc_4}{720} + \frac{1}{24} \left(2\mathrm{c_2^2} - 4\mathrm{c_4}\right) \end{split}
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$$\begin{split} &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}.\mathrm{r})]_{10} = \frac{1}{120} \left(-5\mathrm{c}_2\mathrm{c}_3 + 5\mathrm{c}_5 \right) \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}.\mathrm{r})]_{12} = \frac{r\left(-c_3^2 + 2c_6 \right)}{60480} + \frac{c_4\mathrm{c}_2}{720} + \frac{1}{720} \left(-2\mathrm{c}_2^3 + 3\mathrm{c}_3^2 + 6\mathrm{c}_2\mathrm{c}_4 - 6\mathrm{c}_6 \right) \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}.\mathrm{r})]_{14} = -\frac{c_4\mathrm{c}_3}{1440} + \frac{7c_2^2\mathrm{c}_3 - 7c_3\mathrm{c}_4 - 7c_2\mathrm{c}_5 + 7c_7}{5040} \\ &[\mathrm{Td}(\mathrm{E})\mathrm{ch}(\mathrm{F}.\mathrm{r})]_{16} = \frac{r\left(5c_4^2 + 3c_3c_5 - 3c_8 \right)}{3628800} - \frac{\left(-c_3^2 + 2c_6 \right)\mathrm{c}_2}{60480} - \frac{c_4\left(2c_2^2 - 4\mathrm{c}_4 \right)}{17280} + \frac{2c_2^4 - 8c_2c_3^2 - 8c_2^2\mathrm{c}_4 + 4c_4^2 + 8c_3c_5 + 8c_2c_6 - 8c_8}{40320} \end{split}$$

c_1=c'_1=c'_2=0

$$\begin{split} & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_0 = r \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_2 = 0 \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_4 = \frac{rc_2}{12} \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_6 = \frac{c_3}{2} \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_8 = \frac{1}{720}r\left(3c_2^2 - c_4\right) - \frac{c_4}{6} \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{10} = \frac{c_2c_3}{24} + \frac{c_5}{24} \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{12} = \frac{r\left(10c_2^3 - c_3^2 - 9c_2c_4 + 2c_6\right)}{60480} - \frac{c_2c_4}{72} + \frac{1}{720}\left(3c_3^2 - 6c_6\right) \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{14} = \frac{\left(3c_2^2 - c_4\right)c_3}{1440} + \frac{c_2c_5}{288} + \frac{-7c_3c_4 + 7c_7}{5040} \\ & [\operatorname{Td}(E)\operatorname{ch}(F \cdot x)]_{16} = \frac{r\left(21c_2^4 - 8c_2c_3^2 - 34c_2^2c_4 + 5c_4^2 + 3c_3c_5 + 13c_2c_6 - 3c_8\right)}{3628800} - \frac{\left(3c_2^2 - c_4\right)c_4}{4320} + \frac{c_2\left(3c_3^2 - 6c_6\right)}{8640} + \frac{4c_4^2 + 8c_3c_5 - 8c_8}{40320} \end{split}$$

c_1=c'_1=c_2=c'_2=0

$$\begin{split} & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_0 = r \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_2 = 0 \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_4 = 0 \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_6 = \frac{c_3}{2} \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_8 = -\frac{rc_4}{720} - \frac{c_4}{6} \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{10} = \frac{c_5}{24} \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{12} = \frac{r(-c_3^2 + 2c_6)}{60480} + \frac{1}{720} \left(3c_3^2 - 6c_6\right) \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{14} = -\frac{c_4c_3}{1440} + \frac{-7c_3c_4 + 7c_7}{5040} \\ & [\mathrm{Td}(\mathsf{E})\mathrm{ch}(\mathsf{F}.\mathsf{r})]_{16} = \frac{r\left(5c_4^2 + 3c_3c_5 - 3c_8\right)}{3628800} + \frac{c_4c_4}{4320} + \frac{4c_4^2 + 8c_3c_5 - 8c_8}{40320} \end{split}$$