

NotebookEvaluate[T:\5183\capstone\Computational-Characterstic-Classes\characteristic classes back Automatic];

NotebookEvaluate[T:\5183\capstone\Computational-Characterstic-Classes\characteristic classes back Automatic];

TODD GENUS - Pontryagin

```
twistedGenusToddP[Q_,deg_Integer]/; deg==0:=chernCh[0];
twistedGenusToddP[Q_,deg_Integer]:=Module[{genusTable, chernCharTable},
genusTable = Table[(toddCompl[i]*t^(2*i)/.c -> p), {i, 0, deg/2}];
chernCharTable = Table[(chernChToP[m]*t^(2*m)/.p -> p), {m, 0, 2*deg/2}];
If[Divisible[deg, 2],
Total[Coefficient[Flatten[Outer[Times, genusTable, chernCharTable]], t^(deg)]],
Print[Error: Input degree must be divisible by 2.]
]
]
```

For $[i = 0, i < 17, i += 2, \text{Print}[\text{Subscript}["[\text{Td}(\text{E})\text{ch}(\text{F}_r)]", i], " = ", \text{twistedGenusToddP}[\#/(1 - \text{Exp}[-\#])\&, i]]]$

$$\begin{aligned}
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_0 &= r \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_2 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_4 &= -\frac{rp_1}{12} + \frac{p_1}{2} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_6 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_8 &= \frac{1}{720}r(3p_1^2 - p_2) - \frac{p_1p_1}{24} + \frac{1}{24}(p_1^2 - 2p_2) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= \frac{r(-10p_1^3 + 9p_1p_2 - 2p_3)}{60480} + \frac{(3p_1^2 - p_2)p_1}{1440} - \frac{1}{288}p_1(p_1^2 - 2p_2) + \frac{1}{720}(p_1^3 - 3p_1p_2 + 3p_3) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= \frac{r(21p_1^4 - 34p_1^2p_2 + 5p_2^2 + 13p_1p_3 - 3p_4)}{3628800} + \frac{(-10p_1^3 + 9p_1p_2 - 2p_3)p_1}{120960} + \frac{(3p_1^2 - p_2)(p_1^2 - 2p_2)}{17280} \\
&\quad - \frac{p_1(p_1^3 - 3p_1p_2 + 3p_3)}{8640} + \frac{p_1^4 - 4p_1^2p_2 + 2p_2^2 + 4p_1p_3 - 4p_4}{40320}
\end{aligned}$$

p_1 = 0

For $[i = 0, i < 17, i += 2, \text{Print}[\text{Subscript}["[\text{Td}(\text{E})\text{ch}(\text{F}_r)]", i], " = ", \text{twistedGenusToddP}[\#/(1 - \text{Exp}[-\#])\&, i]]]$

$$\begin{aligned}
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_0 &= r \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_2 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_4 &= \frac{p_1}{2} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_6 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_8 &= -\frac{rp_2}{720} + \frac{1}{24}(p_1^2 - 2p_2) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= -\frac{rp_3}{30240} - \frac{p_2p_1}{1440} + \frac{1}{720}(p_1^3 - 3p_1p_2 + 3p_3) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= \frac{r(5p_2^2 - 3p_4)}{3628800} - \frac{p_3p_1}{60480} - \frac{p_2(p_1^2 - 2p_2)}{17280} + \frac{p_1^4 - 4p_1^2p_2 + 2p_2^2 + 4p_1p_3 - 4p_4}{40320}
\end{aligned}$$

$$\mathbf{p}'_1=0$$

For [$i = 0, i < 17, i+=2$, **Print** [**Subscript**["[Td(E)ch(F_r)]", i], " = ", **twistedGenusToddP**[#/(1 - Exp[-#])&, i]

$$\begin{aligned} [\text{Td(E)ch(F}_r)]_0 &= r \\ [\text{Td(E)ch(F}_r)]_2 &= 0 \\ [\text{Td(E)ch(F}_r)]_4 &= -\frac{rp_1}{12} \\ [\text{Td(E)ch(F}_r)]_6 &= 0 \\ [\text{Td(E)ch(F}_r)]_8 &= \frac{1}{720}r(3p_1^2 - p_2) - \frac{p_2}{12} \\ [\text{Td(E)ch(F}_r)]_{10} &= 0 \\ [\text{Td(E)ch(F}_r)]_{12} &= \frac{r(-10p_1^3+9p_1p_2-2p_3)}{60480} + \frac{p_1p_2}{144} + \frac{p_3}{240} \\ [\text{Td(E)ch(F}_r)]_{14} &= 0 \\ [\text{Td(E)ch(F}_r)]_{16} &= \frac{r(21p_1^4-34p_1^2p_2+5p_2^2+13p_1p_3-3p_4)}{3628800} - \frac{(3p_1^2-p_2)p_2}{8640} - \frac{p_1p_3}{2880} + \frac{2p_2^2-4p_4}{40320} \end{aligned}$$

$$\mathbf{p}_1=\mathbf{p}'_1 = 0$$

For [$i = 0, i < 17, i+=2$, **Print** [**Subscript**["[Td(E)ch(F_r)]", i], " = ", **twistedGenusToddP**[#/(1 - Exp[-#])&, i]

$$\begin{aligned} [\text{Td(E)ch(F}_r)]_0 &= r \\ [\text{Td(E)ch(F}_r)]_2 &= 0 \\ [\text{Td(E)ch(F}_r)]_4 &= 0 \\ [\text{Td(E)ch(F}_r)]_6 &= 0 \\ [\text{Td(E)ch(F}_r)]_8 &= -\frac{rp_2}{720} - \frac{p_2}{12} \\ [\text{Td(E)ch(F}_r)]_{10} &= 0 \\ [\text{Td(E)ch(F}_r)]_{12} &= -\frac{rp_3}{30240} + \frac{p_3}{240} \\ [\text{Td(E)ch(F}_r)]_{14} &= 0 \\ [\text{Td(E)ch(F}_r)]_{16} &= \frac{r(5p_2^2-3p_4)}{3628800} + \frac{p_2p_2}{8640} + \frac{2p_2^2-4p_4}{40320} \end{aligned}$$

$$\mathbf{p}_2 = 0$$

For [$i = 0, i < 17, i+=2$, **Print** [**Subscript**["[Td(E)ch(F_r)]", i], " = ", **twistedGenusToddP**[#/(1 - Exp[-#])&, i]

$$\begin{aligned} [\text{Td(E)ch(F}_r)]_0 &= r \\ [\text{Td(E)ch(F}_r)]_2 &= 0 \\ [\text{Td(E)ch(F}_r)]_4 &= -\frac{rp_1}{12} + \frac{p_1}{2} \\ [\text{Td(E)ch(F}_r)]_6 &= 0 \\ [\text{Td(E)ch(F}_r)]_8 &= \frac{rp_1^2}{240} - \frac{p_1p_1}{24} + \frac{1}{24}(p_1^2 - 2p_2) \\ [\text{Td(E)ch(F}_r)]_{10} &= 0 \\ [\text{Td(E)ch(F}_r)]_{12} &= \frac{r(-10p_1^3-2p_3)}{60480} + \frac{1}{480}p_1^2p_1 - \frac{1}{288}p_1(p_1^2 - 2p_2) + \frac{1}{720}(p_1^3 - 3p_1p_2 + 3p_3) \\ [\text{Td(E)ch(F}_r)]_{14} &= 0 \\ [\text{Td(E)ch(F}_r)]_{16} &= \frac{r(21p_1^4+13p_1p_3-3p_4)}{3628800} + \frac{(-10p_1^3-2p_3)p_1}{120960} + \frac{p_1^2(p_1^2-2p_2)}{5760} - \frac{p_1(p_1^3-3p_1p_2+3p_3)}{8640} + \frac{p_1^4-4p_1^2p_2+2p_2^2+4p_1p_3-4p_4}{40320} \end{aligned}$$

$$\mathbf{p}'_2=0$$

For [$i = 0, i < 17, i+=2$, **Print** [**Subscript**["Td(E)ch(F_r)"], i], " = ", **twistedGenusToddP**[#/(1 - Exp[-#])&, i]

$$\begin{aligned} [\text{Td(E)ch(F}_r)]_0 &= r \\ [\text{Td(E)ch(F}_r)]_2 &= 0 \\ [\text{Td(E)ch(F}_r)]_4 &= -\frac{rp_1}{12} + \frac{p_1}{2} \\ [\text{Td(E)ch(F}_r)]_6 &= 0 \\ [\text{Td(E)ch(F}_r)]_8 &= \frac{1}{720}r(3p_1^2 - p_2) - \frac{p_1p_1}{24} + \frac{p_1^2}{24} \\ [\text{Td(E)ch(F}_r)]_{10} &= 0 \\ [\text{Td(E)ch(F}_r)]_{12} &= \frac{r(-10p_1^3+9p_1p_2-2p_3)}{60480} + \frac{(3p_1^2-p_2)p_1}{1440} - \frac{1}{288}p_1p_1^2 + \frac{1}{720}(p_1^3 + 3p_3) \\ [\text{Td(E)ch(F}_r)]_{14} &= 0 \\ [\text{Td(E)ch(F}_r)]_{16} &= \frac{r(21p_1^4-34p_1^2p_2+5p_2^2+13p_1p_3-3p_4)}{3628800} + \frac{(-10p_1^3+9p_1p_2-2p_3)p_1}{120960} + \frac{(3p_1^2-p_2)p_1^2}{17280} - \\ &\quad \frac{p_1(p_1^3+3p_3)}{8640} + \frac{p_1^4+4p_1p_3-4p_4}{40320} \end{aligned}$$

$$\mathbf{p}_2=\mathbf{p}'_2=0$$

For [$i = 0, i < 17, i+=2$, **Print** [**Subscript**["Td(E)ch(F_r)"], i], " = ", **twistedGenusToddP**[#/(1 - Exp[-#])&, i]

$$\begin{aligned} [\text{Td(E)ch(F}_r)]_0 &= r \\ [\text{Td(E)ch(F}_r)]_2 &= 0 \\ [\text{Td(E)ch(F}_r)]_4 &= -\frac{rp_1}{12} + \frac{p_1}{2} \\ [\text{Td(E)ch(F}_r)]_6 &= 0 \\ [\text{Td(E)ch(F}_r)]_8 &= \frac{rp_1^2}{240} - \frac{p_1p_1}{24} + \frac{p_1^2}{24} \\ [\text{Td(E)ch(F}_r)]_{10} &= 0 \\ [\text{Td(E)ch(F}_r)]_{12} &= \frac{r(-10p_1^3-2p_3)}{60480} + \frac{1}{480}p_1^2p_1 - \frac{1}{288}p_1p_1^2 + \frac{1}{720}(p_1^3 + 3p_3) \\ [\text{Td(E)ch(F}_r)]_{14} &= 0 \\ [\text{Td(E)ch(F}_r)]_{16} &= \frac{r(21p_1^4+13p_1p_3-3p_4)}{3628800} + \frac{(-10p_1^3-2p_3)p_1}{120960} + \frac{p_1^2p_1^2}{5760} - \frac{p_1(p_1^3+3p_3)}{8640} + \\ &\quad \frac{p_1^4+4p_1p_3-4p_4}{40320} \end{aligned}$$

$$\mathbf{p}_1=\mathbf{p}'_1=\mathbf{p}_2=0$$

For [$i = 0, i < 17, i+=2$, **Print** [**Subscript**["Td(E)ch(F_r)"], i], " = ", **twistedGenusToddP**[#/(1 - Exp[-#])&, i]

$$\begin{aligned} [\text{Td(E)ch(F}_r)]_0 &= r \\ [\text{Td(E)ch(F}_r)]_2 &= 0 \\ [\text{Td(E)ch(F}_r)]_4 &= 0 \\ [\text{Td(E)ch(F}_r)]_6 &= 0 \\ [\text{Td(E)ch(F}_r)]_8 &= -\frac{p_2}{12} \\ [\text{Td(E)ch(F}_r)]_{10} &= 0 \\ [\text{Td(E)ch(F}_r)]_{12} &= -\frac{rp_3}{30240} + \frac{p_3}{240} \\ [\text{Td(E)ch(F}_r)]_{14} &= 0 \end{aligned}$$

$$[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} = -\frac{rp_4}{1209600} + \frac{2p_2^2 - 4p_4}{40320}$$

$$\mathbf{p_1=p'_1=p'_2=0}$$

For [$i = 0, i < 17, i += 2$, **Print** [**Subscript**["**Td(E)ch(F_r)**", i], " = ", **twistedGenusToddP**[$\#/(1 - \text{Exp}[-\#])\&, i]$]

$$\begin{aligned} [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_0 &= r \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_2 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_4 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_6 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_8 &= -\frac{rp_2}{720} \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= -\frac{rp_3}{30240} + \frac{p_3}{240} \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= \frac{r(5p_2^2 - 3p_4)}{3628800} - \frac{p_4}{10080} \end{aligned}$$

$$\mathbf{p_1=p'_1=p_2=p'_2=0}$$

For [$i = 0, i < 17, i += 2$, **Print** [**Subscript**["**Td(E)ch(F_r)**", i], " = ", **twistedGenusToddP**[$\#/(1 - \text{Exp}[-\#])\&, i]$]

$$\begin{aligned} [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_0 &= r \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_2 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_4 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_6 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_8 &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= -\frac{rp_3}{30240} + \frac{p_3}{240} \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= 0 \\ [\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= -\frac{rp_4}{1209600} - \frac{p_4}{10080} \end{aligned}$$

TODD GENUS - Chern

NotebookEvaluate[T:\5183\capstone\Computational-Characterstic-Classes\characteristic classes backgro
Automatic];

```
twistedGenusTodd[Q_.,deg_Integer]/; deg==0:=chernCh[0];
twistedGenusTodd[Q_.,deg_Integer]:=Module[{genusTable, chernCharTable},
genusTable = Table[(genus[Q,i] * t^(2 * i)/.c -> c), {i, 0, deg/2}];
chernCharTable = Table[(chernCh[m] * t^(2 * m)/.c -> c), {m, 0, 2 * deg/2}];
If[Divisible[deg, 2],
Total[Coefficient[Flatten[Outer[Times, genusTable, chernCharTable]], t^(deg)]],
Print[Error: Input degree must be divisible by 2.]
]
]
```

For [$i = 0, i < 17, i += 2$, **Print** [**Subscript**["**Td(E)ch(F_r)**", i], " = ", **twistedGenusTodd**[$\#/(1 - \text{Exp}[-\#])\&, i]$]

$$\begin{aligned}
[\text{Td(E)ch(F_r)}]_0 &= r \\
[\text{Td(E)ch(F_r)}]_2 &= \frac{rc_1}{2} + c_1 \\
[\text{Td(E)ch(F_r)}]_4 &= \frac{1}{12}r(c_1^2 + c_2) + \frac{c_1c_1}{2} + \frac{1}{2}(c_1^2 - 2c_2) \\
[\text{Td(E)ch(F_r)}]_6 &= \frac{1}{24}rc_1c_2 + \frac{1}{12}(c_1^2 + c_2)c_1 + \frac{1}{4}c_1(c_1^2 - 2c_2) + \frac{1}{6}(c_1^3 - 3c_1c_2 + 3c_3) \\
[\text{Td(E)ch(F_r)}]_8 &= \frac{1}{720}r(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4) + \frac{1}{24}c_1c_2c_1 + \frac{1}{24}(c_1^2 + c_2)(c_1^2 - 2c_2) + \\
&\quad \frac{1}{12}c_1(c_1^3 - 3c_1c_2 + 3c_3) + \frac{1}{24}(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) \\
[\text{Td(E)ch(F_r)}]_{10} &= \frac{r(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)}{1440} + \frac{1}{720}(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4)c_1 + \\
&\quad \frac{1}{48}c_1c_2(c_1^2 - 2c_2) + \frac{1}{72}(c_1^2 + c_2)(c_1^3 - 3c_1c_2 + 3c_3) + \frac{1}{48}c_1(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) + \\
&\quad \frac{1}{120}(c_1^5 - 5c_1^3c_2 + 5c_1c_2^2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5) \\
[\text{Td(E)ch(F_r)}]_{12} &= \frac{r(2c_1^6 - 12c_1^4c_2 + 11c_1^2c_2^2 + 10c_2^3 + 5c_1^3c_3 + 11c_1c_2c_3 - c_3^2 - 5c_1^2c_4 - 9c_2c_4 - 2c_1c_5 + 2c_6)}{60480} + \\
&\quad \frac{(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)c_1}{1440} + \frac{(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4)(c_1^2 - 2c_2)}{1440} + \frac{1}{144}c_1c_2(c_1^3 - 3c_1c_2 + 3c_3) + \\
&\quad \frac{1}{288}(c_1^2 + c_2)(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) + \frac{1}{240}c_1(c_1^5 - 5c_1^3c_2 + 5c_1c_2^2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5) + \\
&\quad \frac{1}{720}(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6) \\
[\text{Td(E)ch(F_r)}]_{14} &= \frac{r(2c_1^5c_2 - 10c_1^3c_2^2 + 10c_1c_2^3 - 2c_1^4c_3 + 11c_1^2c_2c_3 - c_1c_2^2 + 2c_1^3c_4 - 9c_1c_2c_4 - 2c_1^2c_5 + 2c_1c_6)}{120960} + \\
&\quad \frac{(2c_1^6 - 12c_1^4c_2 + 11c_1^2c_2^2 + 10c_2^3 + 5c_1^3c_3 + 11c_1c_2c_3 - c_3^2 - 5c_1^2c_4 - 9c_2c_4 - 2c_1c_5 + 2c_6)c_1}{60480} + \frac{(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)(c_1^2 - 2c_2)}{2880} + \\
&\quad \frac{(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4)(c_1^3 - 3c_1c_2 + 3c_3)}{4320} + \frac{1}{576}c_1c_2(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) + \\
&\quad \frac{(c_1^2 + c_2)(c_1^5 - 5c_1^3c_2 + 5c_1c_2^2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5)}{4320} + \frac{c_1(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6)}{1440} + \\
&\quad \frac{c_1^7 - 7c_1^5c_2 + 14c_1^3c_2^2 - 7c_1c_2^3 + 7c_1^4c_3 - 21c_1^2c_2c_3 + 7c_2^3c_3 + 7c_1c_2^2c_4 - 7c_1^3c_4 + 14c_1c_2c_4 - 7c_3c_4 + 7c_1^2c_5 - 7c_2c_5 - 7c_1c_6 + 7c_7}{1440} + \\
[\text{Td(E)ch(F_r)}]_{16} &= \frac{r(-3c_1^8 + 24c_1^6c_2 - 50c_1^4c_2^2 + 8c_1^2c_2^3 + 21c_2^4 - 14c_1^5c_3 + 26c_1^3c_2c_3 + 50c_1c_2^2c_3 + 3c_1^2c_3^2 - 8c_2c_3^2 + 14c_1^4c_4 - 19c_1^2c_2c_4 - 34c_2^2c_4 - 13c_1^3c_5 + 20c_1c_2c_3^2 - 2c_1^4c_3 + 11c_1^2c_2c_3 - c_1c_2^2 + 2c_1^3c_4 - 9c_1c_2c_4 - 2c_1^2c_5 + 2c_1c_6)}{3628800} + \\
&\quad \frac{(-c_1^3c_2 + 3c_1c_2^2 + c_1^2c_3 - c_1c_4)(c_1^3 - 3c_1c_2 + 3c_3)}{120960} + \frac{(-c_1^4 + 4c_1^2c_2 + 3c_2^2 + c_1c_3 - c_4)(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4)}{17280} + \\
&\quad \frac{c_1c_2(c_1^5 - 5c_1^3c_2 + 5c_1c_2^2 + 5c_1^2c_3 - 5c_2c_3 - 5c_1c_4 + 5c_5)}{8640} + \frac{(c_1^2 + c_2)(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6)}{17280} + \\
&\quad \frac{c_1(c_1^7 - 7c_1^5c_2 + 14c_1^3c_2^2 - 7c_1c_2^3 + 7c_1^4c_3 - 21c_1^2c_2c_3 + 7c_2^3c_3 + 7c_1c_2^2c_4 - 7c_3c_4 + 7c_1^2c_5 - 7c_2c_5 - 7c_1c_6 + 7c_7)}{2880} + \frac{8640}{8640} + \\
&\quad \frac{c_1^8 - 8c_1^6c_2 + 20c_1^4c_2^2 - 16c_1^2c_2^3 + 2c_2^4 + 8c_1^5c_3 - 32c_1^3c_2c_3 + 24c_1c_2^2c_3 + 12c_1^2c_3^2 - 8c_2c_3^2 - 8c_1^4c_4 + 24c_1^2c_2c_4 - 8c_2^2c_4 - 16c_1c_3c_4 + 4c_4^2 + 8c_1^3c_5 - 16c_1c_2c_5 + 8c_2^2c_5 - 8c_1^2c_6 + 16c_1c_3c_6 - 8c_3^2c_6}{40320}
\end{aligned}$$

$$\mathbf{c_1 = 0}$$

For [$i = 0, i < 17, i += 2$, **Print** [**Subscript**["Td(E)ch(F_r)", i], " $=$ ", **twistedGenusTodd**["#/(1 - Exp[-#])&, i]/

$$\begin{aligned}
[\text{Td(E)ch(F_r)}]_0 &= r \\
[\text{Td(E)ch(F_r)}]_4 &= \frac{rc_2}{12} + \frac{1}{2}(c_1^2 - 2c_2) \\
[\text{Td(E)ch(F_r)}]_8 &= \frac{1}{720}r(3c_2^2 - c_4) + \frac{1}{24}c_2(c_1^2 - 2c_2) + \frac{1}{24}(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) \\
[\text{Td(E)ch(F_r)}]_{12} &= \frac{r(10c_2^3 - c_3^2 - 9c_2c_4 + 2c_6)}{60480} + \frac{(3c_2^2 - c_4)(c_1^2 - 2c_2)}{1440} + \frac{1}{288}c_2(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4) + \\
&\quad \frac{1}{720}(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6) \\
[\text{Td(E)ch(F_r)}]_{16} &= \frac{r(21c_2^4 - 8c_2c_3^2 - 34c_2^2c_4 + 5c_4^2 + 3c_3c_5 + 13c_2c_6 - 3c_8)}{3628800} + \frac{(10c_2^3 - c_3^2 - 9c_2c_4 + 2c_6)(c_1^2 - 2c_2)}{120960} + \\
&\quad \frac{(3c_2^2 - c_4)(c_1^4 - 4c_1^2c_2 + 2c_2^2 + 4c_1c_3 - 4c_4)}{3628800} + \frac{c_2(c_1^6 - 6c_1^4c_2 + 9c_1^2c_2^2 - 2c_2^3 + 6c_1^3c_3 - 12c_1c_2c_3 + 3c_3^2 - 6c_1^2c_4 + 6c_2c_4 + 6c_1c_5 - 6c_6)}{120960} + \\
&\quad \frac{17280}{8640} + \frac{8640}{40320} + \\
&\quad \frac{c_1^8 - 8c_1^6c_2 + 20c_1^4c_2^2 - 16c_1^2c_2^3 + 2c_2^4 + 8c_1^5c_3 - 32c_1^3c_2c_3 + 24c_1c_2^2c_3 + 12c_1^2c_3^2 - 8c_2c_3^2 - 8c_1^4c_4 + 24c_1^2c_2c_4 - 8c_2^2c_4 - 16c_1c_3c_4 + 4c_4^2 + 8c_1^3c_5 - 16c_1c_2c_5 + 8c_2^2c_5 - 8c_1^2c_6 + 16c_1c_3c_6 - 8c_3^2c_6}{40320}
\end{aligned}$$

$$\begin{aligned}
& \frac{(-c_1^4 + 4c_1^2 c_2 + 3c_2^2 + c_1 c_3 - c_4)(c_1^3 + 3c_3)}{4320} + \frac{1}{576} c_1 c_2 (c_1^4 + 4c_1 c_3 - 4c_4) + \frac{(c_1^2 + c_2)(c_1^5 + 5c_1^2 c_3 - 5c_1 c_4 + 5c_5)}{1440} + \\
& \frac{c_1(c_1^6 + 6c_1^3 c_3 + 3c_3^2 - 6c_1^2 c_4 + 6c_1 c_5 - 6c_6)}{1440} + \frac{c_1^7 + 7c_1^4 c_3 + 7c_1 c_3^2 - 7c_1^3 c_4 - 7c_3 c_4 + 7c_1^2 c_5 - 7c_1 c_6 + 7c_7}{5040} \\
\text{[Td(E)ch(F_r)]}_{16} = & \frac{r(-3c_1^8 + 24c_1^6 c_2 - 50c_1^4 c_2^2 + 8c_1^2 c_2^3 + 21c_2^4 - 14c_1^5 c_3 + 26c_1^3 c_2 c_3 + 50c_1 c_2^2 c_3 + 3c_1^2 c_3^2 - 8c_2 c_3^2 + 14c_1^4 c_4 - 19c_1^2 c_2 c_4 - 34c_2^2 c_4 - 13c_1^3 c_4^2 - 13c_1 c_2 c_3^2 - 10c_1^2 c_2^2 c_3 + 10c_1 c_2^3 - 2c_1^4 c_3 + 11c_1^2 c_2 c_3 - c_1 c_3^2 + 2c_1^3 c_4 - 9c_1 c_2 c_4 - 2c_1^2 c_5 + 2c_1 c_6) c_1}{3628800} + \frac{(2c_1^6 - 12c_1^4 c_2 + 11c_1^2 c_2^2 + 10c_2^3 + 5c_1^3 c_3 + 11c_1 c_2 c_3 - c_3^2 - 5c_1^2 c_4 + 5c_1 c_5 - 5c_1^2 c_6 + 5c_1 c_7) c_1}{120960} \\
& \frac{(-c_1^3 c_2 + 3c_1 c_2^2 + c_1^2 c_3 - c_1 c_4)(c_1^3 + 3c_3)}{8640} + \frac{(-c_1^4 + 4c_1^2 c_2 + 3c_2^2 + c_1 c_3 - c_4)(c_1^4 + 4c_1 c_3 - 4c_4)}{17280} + \frac{c_1 c_2 (c_1^5 + 5c_1^2 c_3 - 5c_1 c_4 + 5c_5)}{2880} + \\
& \frac{(c_1^2 + c_2)(c_1^6 + 6c_1^3 c_3 + 3c_3^2 - 6c_1^2 c_4 + 6c_1 c_5 - 6c_6)}{8640} + \frac{c_1(c_1^7 + 7c_1^4 c_3 + 7c_1 c_3^2 - 7c_1^3 c_4 - 7c_3 c_4 + 7c_1^2 c_5 - 7c_1 c_6 + 7c_7)}{10080} + \\
& \frac{c_1^8 + 8c_1^5 c_3 + 12c_1^2 c_3^2 - 8c_1^4 c_4 - 16c_1 c_3 c_4 + 4c_4^2 + 8c_1^3 c_5 + 8c_3 c_5 - 8c_1^2 c_6 + 8c_1 c_7 - 8c_8}{40320}
\end{aligned}$$

$$\mathbf{c_2} = \mathbf{c'}_2 = 0$$

For $[i = 0, i < 17, i += 2, \text{Print} [\text{Subscript}["[\text{Td(E)ch(F_r)}]", i], " = ", \text{twistedGenusTodd}[\#/(1 - \text{Exp}[-\#])\&, i]/$

$$\begin{aligned}
& [\text{Td(E)ch(F_r)}]_0 = r \\
& [\text{Td(E)ch(F_r)}]_2 = \frac{rc_1}{2} + c_1 \\
& [\text{Td(E)ch(F_r)}]_4 = \frac{rc_1^2}{12} + \frac{c_1 c_1}{2} + \frac{c_1^2}{2} \\
& [\text{Td(E)ch(F_r)}]_6 = \frac{1}{12} c_1^2 c_1 + \frac{1}{4} c_1 c_1^2 + \frac{1}{6} (c_1^3 + 3c_3) \\
& [\text{Td(E)ch(F_r)}]_8 = \frac{1}{720} r (-c_1^4 + c_1 c_3 - c_4) + \frac{1}{24} c_1^2 c_1^2 + \frac{1}{12} c_1 (c_1^3 + 3c_3) + \frac{1}{24} (c_1^4 + 4c_1 c_3 - 4c_4) \\
& [\text{Td(E)ch(F_r)}]_{10} = \frac{r(c_1^2 c_3 - c_1 c_4)}{1440} + \frac{1}{720} (-c_1^4 + c_1 c_3 - c_4) c_1 + \frac{1}{72} c_1^2 (c_1^3 + 3c_3) + \\
& \frac{1}{48} c_1 (c_1^4 + 4c_1 c_3 - 4c_4) + \frac{1}{120} (c_1^5 + 5c_1^2 c_3 - 5c_1 c_4 + 5c_5) \\
& [\text{Td(E)ch(F_r)}]_{12} = \frac{r(2c_1^6 + 5c_1^3 c_3 - c_3^2 - 5c_1^2 c_4 - 2c_1 c_5 + 2c_6)}{60480} + \frac{(c_1^2 c_3 - c_1 c_4) c_1}{1440} + \frac{(-c_1^4 + c_1 c_3 - c_4) c_1^2}{1440} + \\
& \frac{1}{288} c_1^2 (c_1^4 + 4c_1 c_3 - 4c_4) + \frac{1}{240} c_1 (c_1^5 + 5c_1^2 c_3 - 5c_1 c_4 + 5c_5) + \frac{1}{720} (c_1^6 + 6c_1^3 c_3 + 3c_3^2 - 6c_1^2 c_4 + 6c_1 c_5 - 6c_6) \\
& [\text{Td(E)ch(F_r)}]_{14} = \frac{r(-2c_1^4 c_3 - c_1 c_3^2 + 2c_1^3 c_4 - 2c_1^2 c_5 + 2c_1 c_6)}{120960} + \frac{(2c_1^6 + 5c_1^3 c_3 - c_3^2 - 5c_1^2 c_4 - 2c_1 c_5 + 2c_6) c_1}{1440} + \\
& \frac{(c_1^2 c_3 - c_1 c_4) c_1^2}{2880} + \frac{(-c_1^4 + c_1 c_3 - c_4)(c_1^3 + 3c_3)}{4320} + \frac{c_1^2 (c_1^5 + 5c_1^2 c_3 - 5c_1 c_4 + 5c_5)}{1440} + \frac{c_1 (c_1^6 + 6c_1^3 c_3 + 3c_3^2 - 6c_1^2 c_4 + 6c_1 c_5 - 6c_6)}{1440} + \\
& \frac{c_1^7 + 7c_1^4 c_3 + 7c_1 c_3^2 - 7c_1^3 c_4 - 7c_3 c_4 + 7c_1^2 c_5 - 7c_1 c_6 + 7c_7}{5040} \\
& [\text{Td(E)ch(F_r)}]_{16} = \frac{r(-3c_1^8 - 14c_1^5 c_3 + 3c_1^2 c_3^2 + 14c_1^4 c_4 - 13c_1 c_3 c_4 + 5c_4^2 - 7c_1^3 c_5 + 3c_3 c_5 + 7c_1^2 c_6 + 3c_1 c_7 - 3c_8)}{3628800} + \\
& \frac{(-2c_1^6 c_3 - c_1 c_3^2 + 2c_1^3 c_4 - 2c_1^2 c_5 + 2c_1 c_6) c_1}{120960} + \frac{(2c_1^6 + 5c_1^3 c_3 - c_3^2 - 5c_1^2 c_4 - 2c_1 c_5 + 2c_6) c_1^2}{120960} + \frac{(c_1^2 c_3 - c_1 c_4)(c_1^3 + 3c_3)}{8640} + \\
& \frac{(-c_1^4 + c_1 c_3 - c_4)(c_1^4 + 4c_1 c_3 - 4c_4)}{17280} + \frac{c_1^2 (c_1^6 + 6c_1^3 c_3 + 3c_3^2 - 6c_1^2 c_4 + 6c_1 c_5 - 6c_6)}{8640} + \frac{c_1 (c_1^7 + 7c_1^4 c_3 + 7c_1 c_3^2 - 7c_1^3 c_4 - 7c_3 c_4 + 7c_1^2 c_5 - 7c_1 c_6 + 7c_7)}{10080} + \\
& \frac{c_1^8 + 8c_1^5 c_3 + 12c_1^2 c_3^2 - 8c_1^4 c_4 - 16c_1 c_3 c_4 + 4c_4^2 + 8c_1^3 c_5 + 8c_3 c_5 - 8c_1^2 c_6 + 8c_1 c_7 - 8c_8}{40320}
\end{aligned}$$

$$\mathbf{c_1} = \mathbf{c'}_1 = \mathbf{c_2} = 0$$

For $[i = 0, i < 17, i += 2, \text{Print} [\text{Subscript}["[\text{Td(E)ch(F_r)}]", i], " = ", \text{twistedGenusTodd}[\#/(1 - \text{Exp}[-\#])\&, i]/$

$$\begin{aligned}
& [\text{Td(E)ch(F_r)}]_0 = r \\
& [\text{Td(E)ch(F_r)}]_2 = 0 \\
& [\text{Td(E)ch(F_r)}]_4 = -c_2 \\
& [\text{Td(E)ch(F_r)}]_6 = \frac{c_3}{2} \\
& [\text{Td(E)ch(F_r)}]_8 = -\frac{rc_4}{720} + \frac{1}{24} (2c_2^2 - 4c_4)
\end{aligned}$$

$$\begin{aligned}
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= \frac{1}{120} (-5c_2c_3 + 5c_5) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= \frac{r(-c_3^2+2c_6)}{60480} + \frac{c_4c_2}{720} + \frac{1}{720} (-2c_2^3 + 3c_3^2 + 6c_2c_4 - 6c_6) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= -\frac{c_4c_3}{1440} + \frac{7c_2^2c_3-7c_3c_4-7c_2c_5+7c_7}{5040} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= \frac{r(5c_4^2+3c_3c_5-3c_8)}{3628800} - \frac{(-c_3^2+2c_6)c_2}{60480} - \frac{c_4(2c_2^2-4c_4)}{17280} + \frac{2c_2^4-8c_2c_3^2-8c_2^2c_4+4c_4^2+8c_3c_5+8c_2c_6-8c_8}{40320}
\end{aligned}$$

$$\mathbf{c_1=c'_1=c'_2=0}$$

For $[i = 0, i < 17, i += 2, \text{Print} [\text{Subscript}["[\text{Td}(\text{E})\text{ch}(\text{F}_r)]", i], " = ", \text{twistedGenusTodd}[\#/(1 - \text{Exp}[-\#])\&, i]/$

$$\begin{aligned}
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_0 &= r \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_2 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_4 &= \frac{rc_2}{12} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_6 &= \frac{c_3}{2} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_8 &= \frac{1}{720} r (3c_2^2 - c_4) - \frac{c_4}{6} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= \frac{c_2c_3}{24} + \frac{c_5}{24} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= \frac{r(10c_2^3-c_3^2-9c_2c_4+2c_6)}{60480} - \frac{c_2c_4}{72} + \frac{1}{720} (3c_3^2 - 6c_6) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= \frac{(3c_2^2-c_4)c_3}{1440} + \frac{c_2c_5}{288} + \frac{-7c_3c_4+7c_7}{5040} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= \frac{r(21c_2^4-8c_2c_3^2-34c_2^2c_4+5c_4^2+3c_3c_5+13c_2c_6-3c_8)}{3628800} - \frac{(3c_2^2-c_4)c_4}{4320} + \frac{c_2(3c_3^2-6c_6)}{8640} + \frac{4c_4^2+8c_3c_5-8c_8}{40320}
\end{aligned}$$

$$\mathbf{c_1=c'_1=c_2=c'_2=0}$$

For $[i = 0, i < 17, i += 2, \text{Print} [\text{Subscript}["[\text{Td}(\text{E})\text{ch}(\text{F}_r)]", i], " = ", \text{twistedGenusTodd}[\#/(1 - \text{Exp}[-\#])\&, i]/$

$$\begin{aligned}
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_0 &= r \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_2 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_4 &= 0 \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_6 &= \frac{c_3}{2} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_8 &= -\frac{rc_4}{720} - \frac{c_4}{6} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{10} &= \frac{c_5}{24} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{12} &= \frac{r(-c_3^2+2c_6)}{60480} + \frac{1}{720} (3c_3^2 - 6c_6) \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{14} &= -\frac{c_4c_3}{1440} + \frac{-7c_3c_4+7c_7}{5040} \\
[\text{Td}(\text{E})\text{ch}(\text{F}_r)]_{16} &= \frac{r(5c_4^2+3c_3c_5-3c_8)}{3628800} + \frac{c_4c_4}{4320} + \frac{4c_4^2+8c_3c_5-8c_8}{40320}
\end{aligned}$$