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MC Simulation using the Risk Solver Platform

Net Revenues Example

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The purpose of this document is to highlight the usage of RSP for performing Monte-Carlo simulations. It covers several topics from setting the parameters of the distributions of interest, to introducing correlations and performing simulation parameter analysis.

Contents

Monte Carlo Simulation: A Guide for RSP

Introduction

This document illustrates how to use the Risk Solver Platform® Excel© Add-in (hereafter, RSP) to perform several tasks within a Monte-Carlo simulation context. The tasks covered are:

- Entering the distributional characteristics of random variables
- Identifying the target function that will be tracked during the simulation
- Calculating simulation-based statistics
- Editing simulation options (seed, number of iterations, and so on)
- Identify how changes in one or more parameters affect the simulation-based statistics (sensitivity analysis)
- Creating and reviewing simulation parameter reports

The analysis is based on the Net Revenue example covered in class. The following section describes the main characteristics of this example.

Net revenues for a new product

The analysis is based on a new hypothetical product that will be introduced in the market on the following months. The management has provided estimates on the variables that drive their net revenues, such as price, quantity sold and costs (both fixed and variable). Their estimates pertain to the first month that the product will be on the market. The exact guidelines that management provided are listed below:

- **Price (P)**: The Company is a "price taker", i.e. they don't have control over the final price of the product. Upper management believes that the price will be in the range of \$8 to \$11 per unit, with a price of \$10 been the most likely. *This information translates into a triangular distribution; it will be used in RSP later on.*
- **Quantity sold (Q)**: Management believes that the first month's sales will most likely be close to 1,500 units. However, given that this is the first month in the life of the product, they do expect some variation (especially on the downside) and the final sales might end up as low as 500 or as high as 2,000 units. *This information translates into a triangular distribution; it will be used in the Risk Solver Platform later on.*
- **Total Fixed cost (FC)** is set at \$20,000 for the first month; this is unrelated with the quantity sold (sunk cost).
- **Variable cost per unit (VC_{unit})** is a function of the quantity produced (and eventually sold) in a given month. The company has performed a regression analysis using inputs from other similar companies that operate in the same sector. They have determined that companies of their size exhibit diminishing returns to scale, meaning that the vari-

able cost per unit (or the marginal cost) is an increasing function of the quantity produced. The estimated relationship between variable cost per unit and quantity sold is the following:

$$VC_{unit} = 1 + 0.004 \times q + \sqrt{0.8}e, \text{ where } e \sim N(0,1).$$

The term $\sqrt{0.8}e$ translates to a normally distributed random variable with 0 mean and standard deviation[1] *of 0.8. This information will be used in the RSP later on.*

- **Net revenues (NR)** are defined as follows:

$$NR = P \times Q - VC_{unit} \times Q - FC \Rightarrow NR = Q(P - VC_{unit}) - FC$$

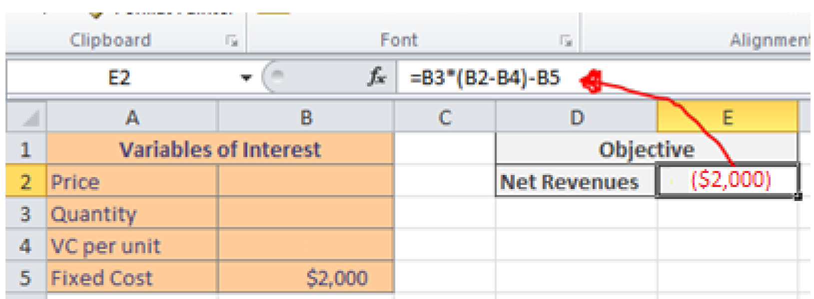
Setting up the Excel worksheet for the MC simulation

This section describes how to setup the problem in Risk Solver Platform. It illustrates the steps you follow to enter all distributional assumptions, identify the target simulation variable, define simulation statistics and so on.

Initial setup of the spreadsheet

Start by creating a new worksheet that has the following information:

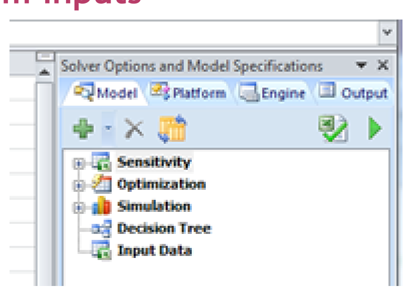
- Place-holders for all variables of interest (price, quantity, variable cost per unit, fixed cost and net revenues).
- Set the value of fixed cost to $2,000; this value is non-random and will not change during the simulations.
- In the Net Revenues cell, enter the formula described earlier for NR. Your worksheet and the formula used should be similar to the adjacent screenshot.



After setting up the initial spreadsheet, proceed to create the random variables in the model.

Enter the distributional assumptions for all random inputs

Make sure that the "Solver Options and Model Specifications" panel is visible in the right hand side of your worksheet, similar to the adjacent screenshot.

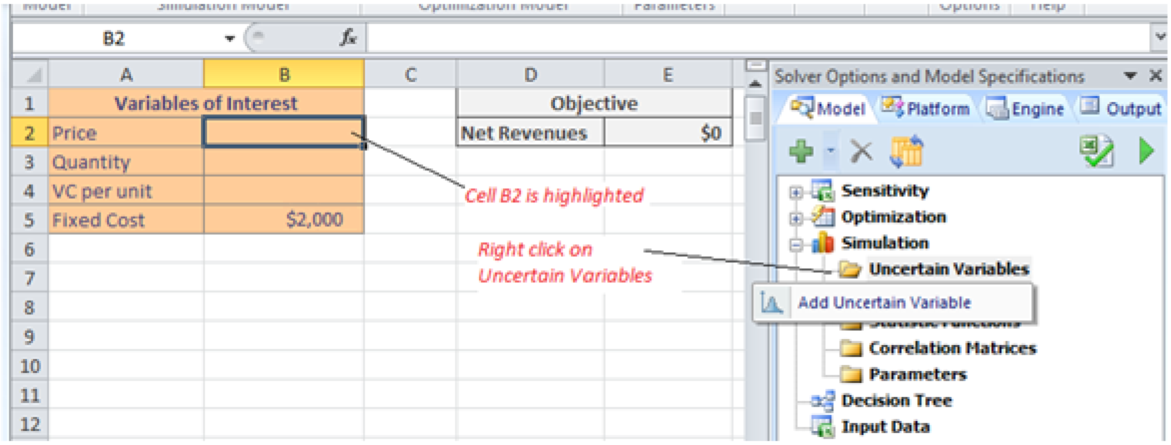


If the panel is not visible, navigate to the "Risk Solver Platform" ribbon and click on the very first icon, named "Model". Once the panel is visible, proceed by setting the dis-

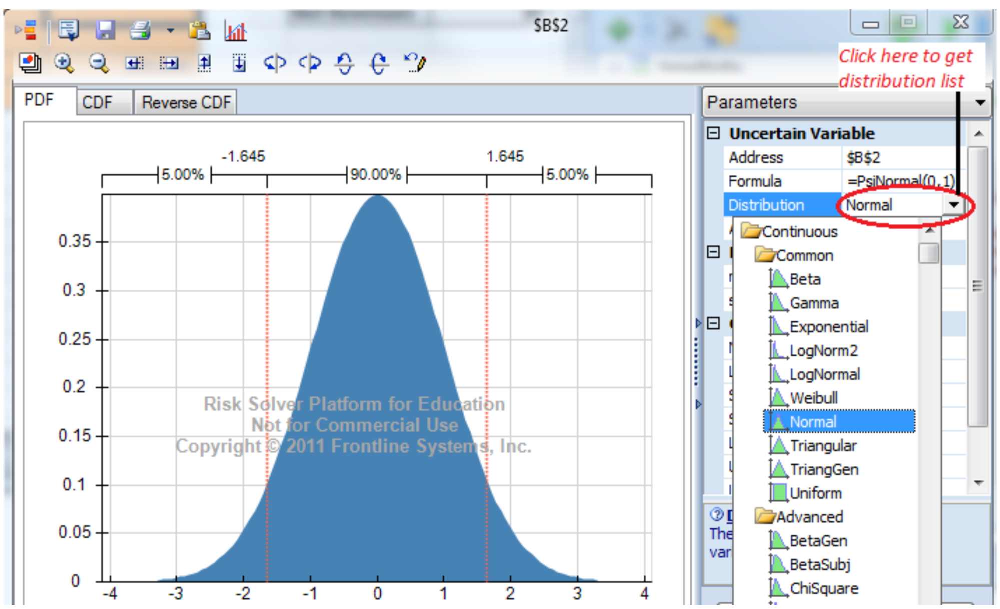
[1] *The value of the standard error is the one produced by the regression analysis the company performed.*

tributional assumptions for “Price”.

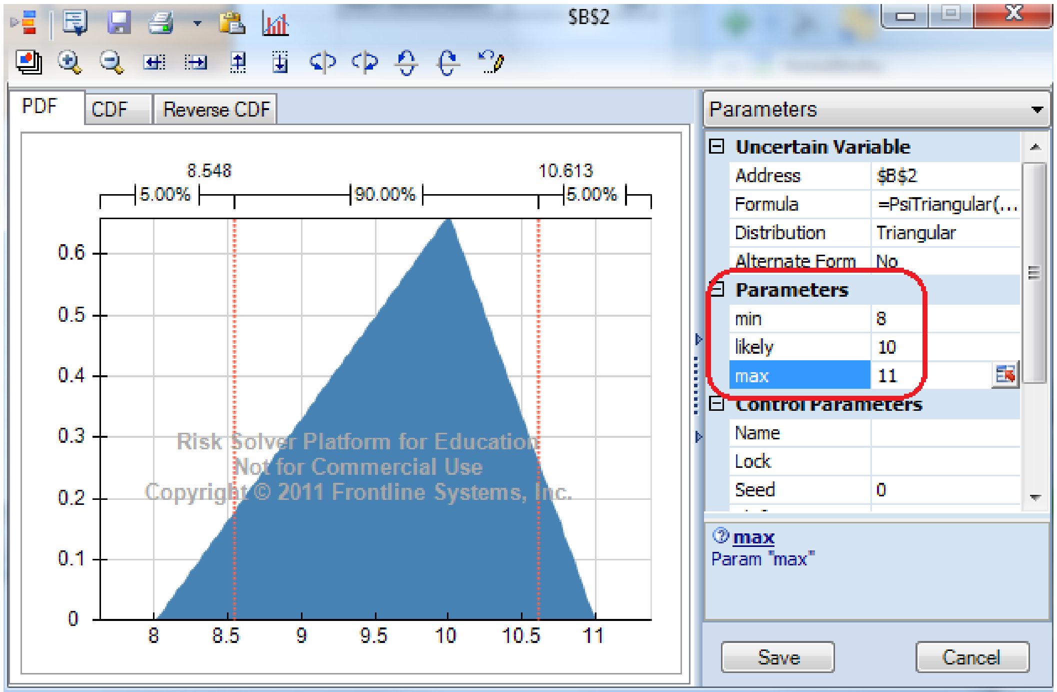
- Highlight cell B2; then, navigate on the model specifications panel, click on the “+” sign next to **Simulation**, and then right click on the **Uncertain Variables.**



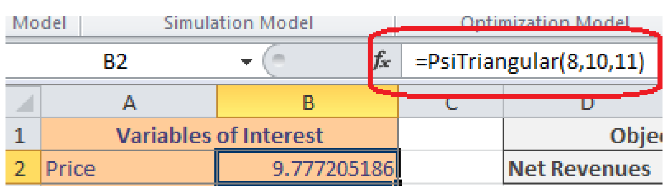
- Click on **Add Uncertain Variable.** This will bring up a new menu where you select the type of distribution to use for cell B2, along with its parameters. By default, the solver assumes a normal distribution. You can see the list of all available distributions by clicking on the word **Normal**, in the row labeled **Distribution**; this will bring up a drop-down menu with the list of all available distributions.



- From the drop-down list select **Triangular**. This will update the window and it will display the triangular PDF, along with its parameters. Set the **min**, **likely** and **max** parameters to the values that were given earlier (see screenshot below) and then hit the **Save** button.



The properties of the random variable "Price" have now been defined. You can verify their successful creation by highlighting cell B2 and looking on the formula that the solver created (see below):



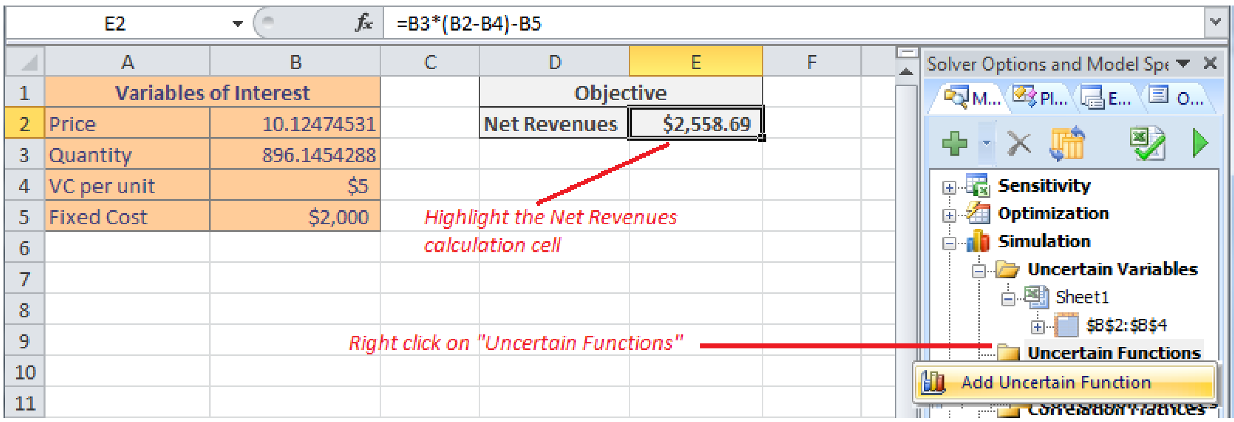
- You can repeat this procedure to define the properties of all random inputs in this model; *in each case, make sure that you set the appropriate parameters for each distribution*. Note that you can also enter the formulas directly to Excel. For example, you can set the properties of Quantity by entering the following in cell B3: =PsiTriangular(500, 1500, 2000).
- After you define the properties of all random inputs, the Excel worksheet should contain the following formulas in each cell:

1	Variables of Interest		Objective	
2	Price	=PsiTriangular(8,10,11)	Net Revenues	=B3*(B2-B4)-B5
3	Quantity	=PsiTriangular(500,1500,2000)		
4	VC per unit	=1+0.04*B3+PsiNormal(0,SQRT(0.8))		
5	Fixed Cost	2000		

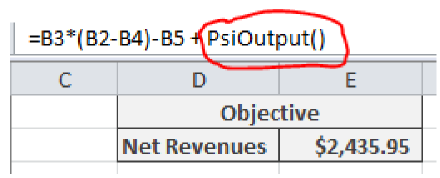
Identify the target variable (or uncertain function)

You will now identify the uncertain function of interest. This is the function that you will track through the simulations. In this example, the uncertain function is the Net Revenues and should be defined in RSP as follows:

- Highlight cell E2 that contains the Net Revenues formula; then, navigate on the model specifications panel, click on the "+" sign next to **Simulation**, and then right click on the **Uncertain Functions.**



- Click on **Add Uncertain Function**. This will automatically mark cell E2 as an uncertain function. You can verify that by looking into the Excel formula for the specific cell. Notice the addition of the term PsiOutput(); this is how RSP "marks" a cell as an uncertain function.



Note: Alternatively, you can designate a cell as an uncertain function by simply typing the "+PsiOutput()" in the cell yourself.

Calculate the statistics of interest

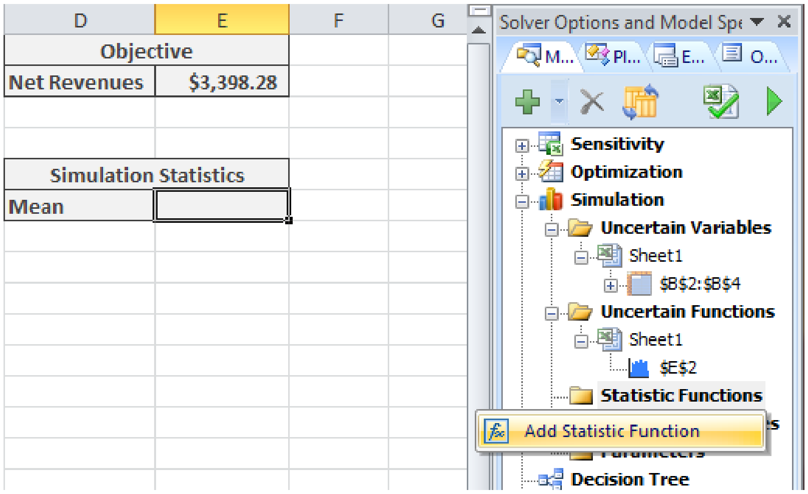
You proceed by identifying the key statistical measures that you want to calculate from the simulated results. For example, what is the mean Net Revenues across the simulations? What is their standard deviation? What are the 1st and 3rd quartiles?

The RSP software has a long list of statistical functions that can be calculate in a simulation. The naming convention for all of them, to avoid clashes with the standard Excel functions, is that they use the prefix "Psi", for example PsiMean for the mean across simulations, PsiStdDev for the standard deviation and so on.

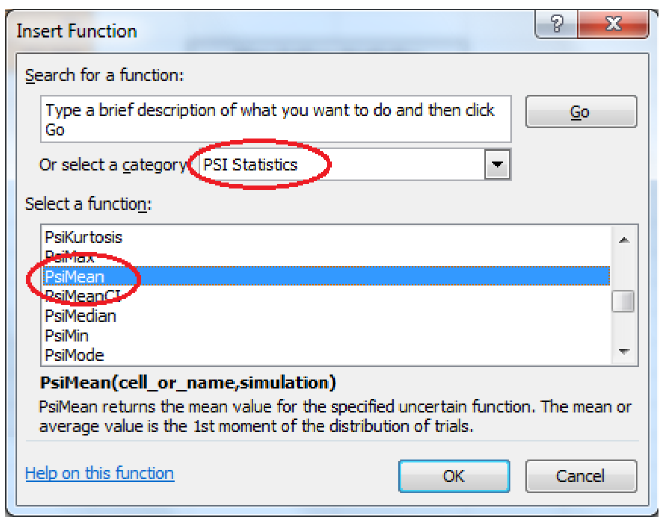
Start by defining the mean across simulations:

- Highlight the cell where the statistic of interest will be placed (in this example, E6); then, navigate on the model specifications panel, click on the "+" sign next to **Simula-**

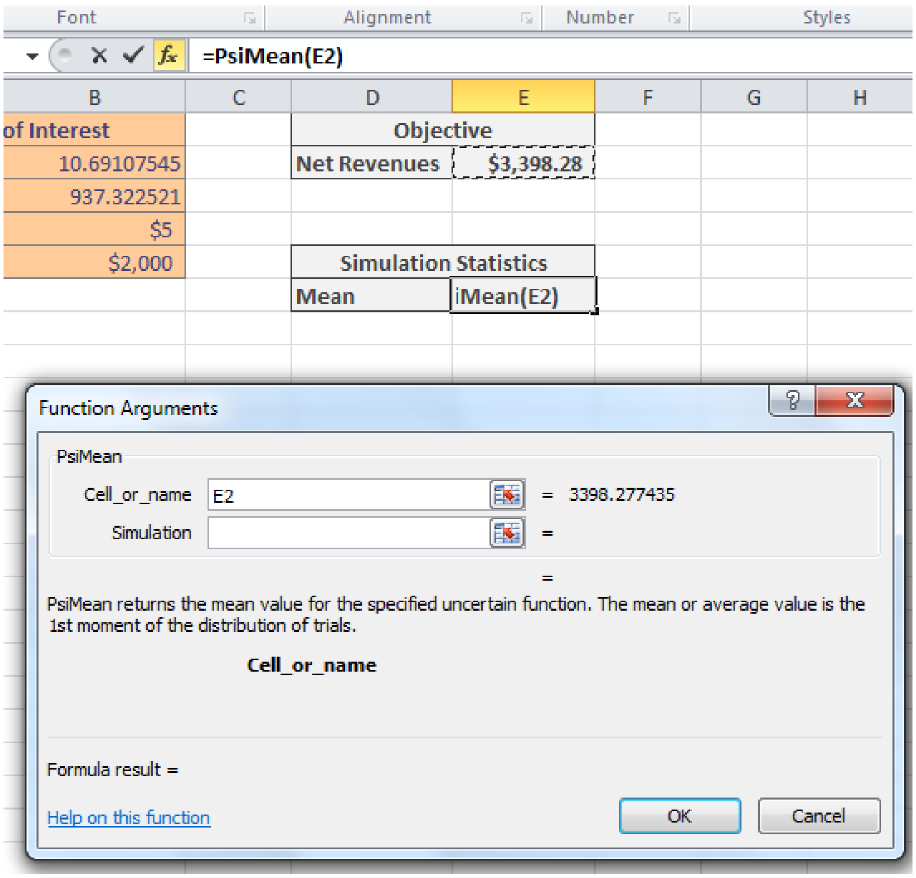
tion, and then right click on the **Statistic Functions.**



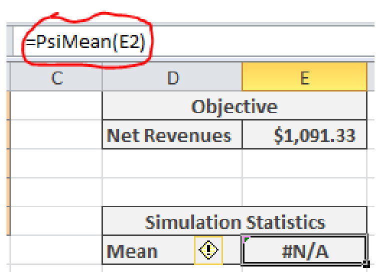
- Click on **Add Statistic Function.** This will bring up the standard Excel menu for inserting a function into a cell. Make sure that you select the category **"PSI Statistics"**; from the list of available functions, highlight the **PsiMean** (see screenshot below).



- Click the **OK** button. A window with the list of arguments for the function PsiMean will come up. You set the argument **Cell_or_name** to E2, which is the cell that holds the uncertain function that you track in the simulation (i.e. the Net Revenues). You can leave the **Simulation** argument empty (this is used only for parameter analysis). See the following screenshot for an example



- Click the **OK** button. The PsiMean is now entered in cell E6; the screenshot below shows the exact formula entered on cell E6.



There are two things worth noticing here:

1. The value in cell E6 is #N/A; this is expected since the simulation hasn't run yet and the statistic of interest cannot be calculated.
2. Instead of following all steps above, you can simply enter the formula "=PsiMean(E2)" directly in cell E6. All RSP formulas are entered in the same manner as the standard Excel formulas; therefore, the PsiMean can be entered directly and you don't need to follow all steps above.

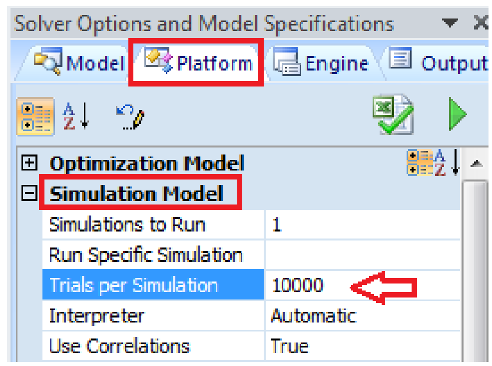
- You can repeat this procedure to define all simulation statistics of interest, such as the standard deviation, quartiles, minimum, maximum and so on. After you define the simulation statistics of interest, the Excel worksheet should contain the following formulas on simulation statistics:

	D	E
5	Simulation Statistics	
6	Mean	=PsiMean(E2)
7	Std. Dev	=PsiStdDev(E2)
8	Min	=PsiMin(E2)
9	1st Quartile	=PsiPercentile(E2,0.25)
10	Median	=PsiMedian(E2)
11	3rd Quartile	=PsiPercentile(E2)
12	Maximum	=PsiMax(E2)

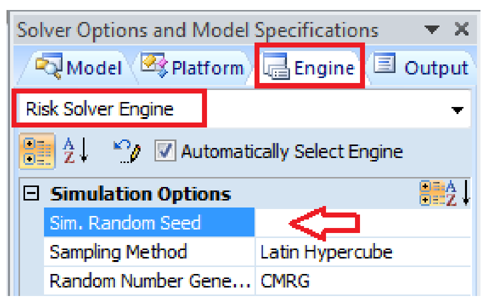
Run the simulation

After performing all steps above, you are ready to perform the simulation analysis. You begin by selecting the number of simulations and random seed (if needed).

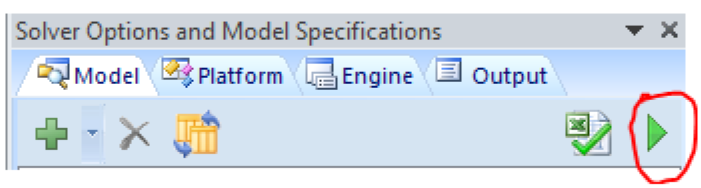
- Set trials per simulation: Navigate to the model specifications panel, click on the **Platform** tab, look in the **Simulations** section and set the **Trials per Simulation** to 10000 (default value is 1000).



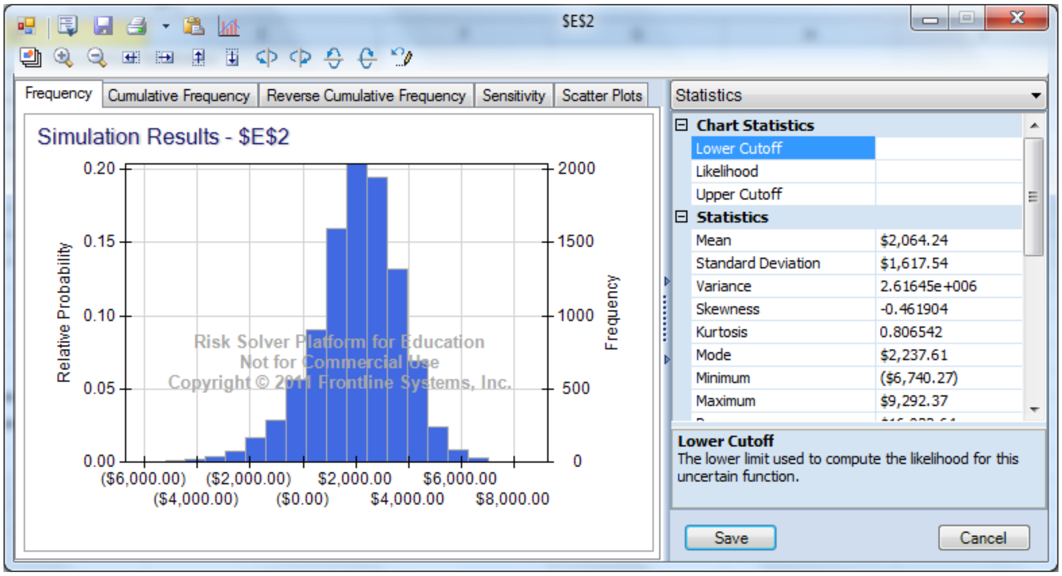
- Set the random seed (optional): Navigate to the model specification panel, click on the **Engine** tab and make sure that you select **Risk Solver Engine** from the drop down menu of available engines. Look into the **Simulations Options** section and set the **Sim. Random Seed** to any integer.



- You can now start the simulation by clicking on the green arrow in the model specification window.



- After the successful run of the simulation, RSP will open a new window where it shows the PDF, CDF, inverse CDF, descriptive statistics, percentiles and so on for the variable of interest (Net Revenues). The results will look similar to the following:



- The simulation statistics displayed in the window above will be the same with the ones you have entered in the Excel worksheet. However, the statistics showing up in the simulation Results window will disappear once the window is closed; the ones that have been entered in the worksheet will remain there. The simulation statistics calculated/displayed in the worksheet are the following (you can verify that these are the same with the ones printed in the Simulation Results window displayed above).

D	E
Simulation Statistics	
Mean	2064.240146
Std. Dev	1617.544603
Min	-6740.269533
1st Quartile	1145.568063
Median	2179.101101
3rd Quartile	-6740.269533
Maximum	9292.365564

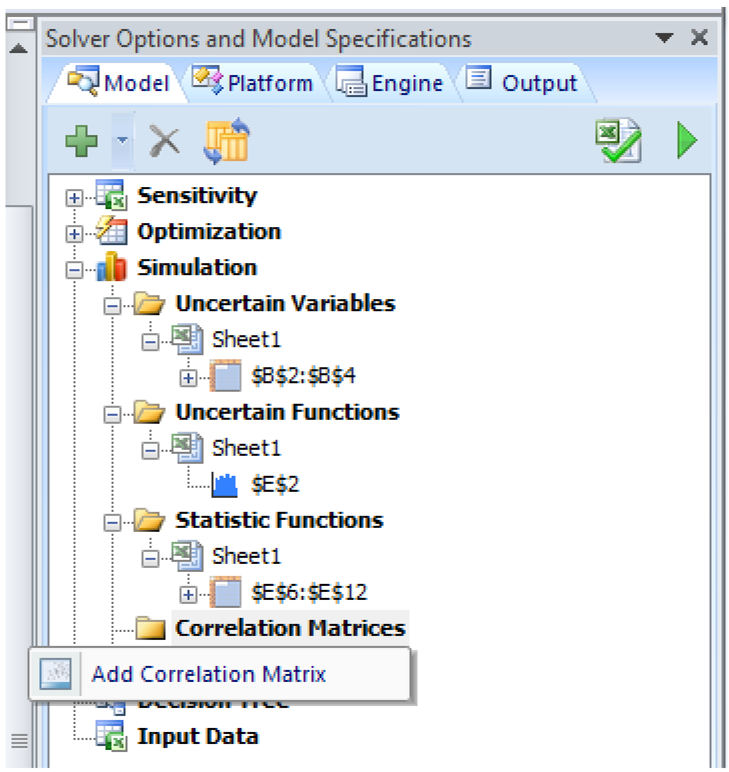
Monte-Carlo Simulation in RSP: Additional Options

This section describes how you can use the RSP software to perform additional tasks, such as creating correlated variables, performing parameter analysis and so on.

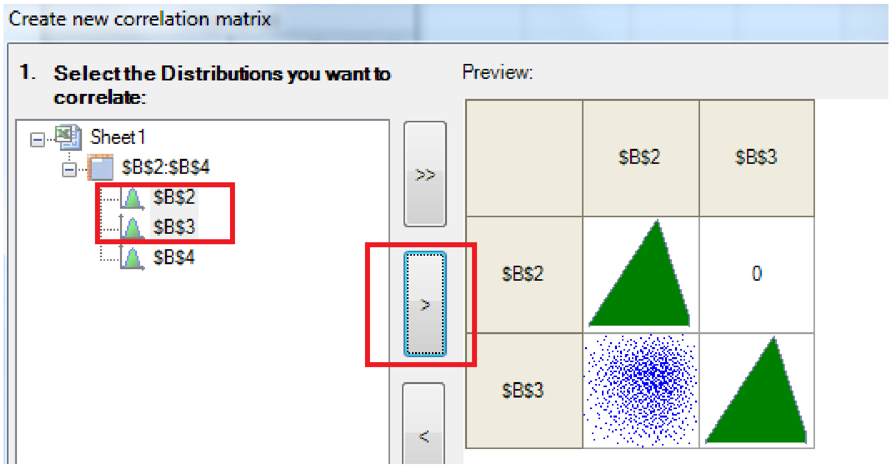
Simulating correlated variables

It is quite common that the random variables of interest are correlated. For example one might suggest that **Price (P)** and **Quantity Sold (Q)** are correlated, due to the demand elasticity for the product. For example, P and Q might have a negative correlation of -0.6. This section describes how to add this correlation between the two variables of interest (you can also correlate more than two variables at once, using the same logic described in this example).

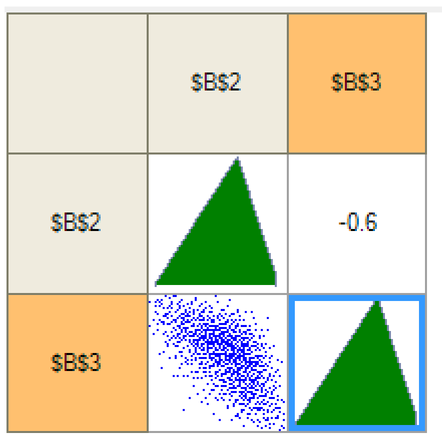
- Navigate to the model specification panel, click on the "+" sign next to **Simulation** and then right click on **Correlation Matrices.**



- Click on **Add Correlation Matrix**. This will bring up a new window where you can select which variables to correlate and also edit the correlation value. First, select the two variables (cells) that hold the variables to be correlated; in this example, **$B$2** and **$B$3**. Click on the single right arrow to move these variables to the right panel of the window. You should get something similar to the screenshot below.



- By default, the two variables have a correlation of 0; you can double click the 0 value and edit it. Double-click and set it equal to -0.6. The right panel should now look as follows



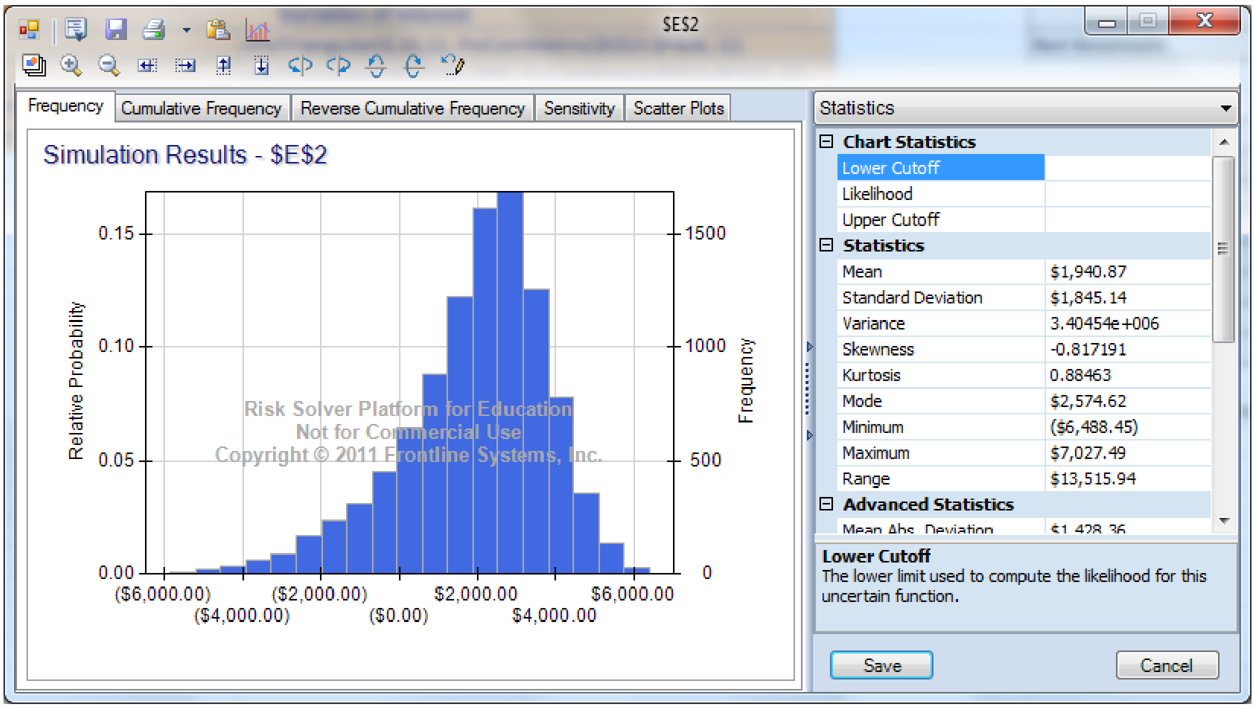
- Click on the **Save** button. You will now be prompted to select a cell where the correlation matrix will be entered. Click on cell D14 (or any other cell where the adjacent cells are empty). The correlation matrix has now been entered in the worksheet (see below).

	C	D	E	F
13				
14		Correlations	$B$2	$B$3
15		$B$2	1	-0.6
16		$B$3	-0.6	1
17				

- RSP has also updated the formulas for simulating **P** and **Q**; the new formulas are highlighted below. Note that the term PsiCorrMatrix has been added automatically in both of them; this instructs RSP to generate correlated variables.

	A	B
1	Variables of Interest	
2	Price	=PsiTriangular(8,10,11, PsiCorrMatrix($E$15:$F$16, 1))
3	Quantity	=PsiTriangular(500,1500,2000, PsiCorrMatrix($E$15:$F$16, 2))
4	VC per unit	=1+0.004*B3+PsiNormal(0,SQRT(0.8))
5	Fixed Cost	2000

You can see the effect of the correlation by running again the simulation. The results are displayed below:



Notice that introducing the correlation affects the skewness of the distribution (negatively-skewed). The mean revenues are lower, their standard deviation is higher and the maximum has been affected more than the minimum value (skewness effect). Overall, introducing a negative correlation between price and quantity has a negative effect on net revenues.

Performing Simulation Parameter Analysis

The purpose of parameter analysis is to investigate how the simulated distribution and its statistics will change if you alter some simulation parameters. The purpose of parameter analysis is to answer questions such as:

- What will happen if the constant of the **VC per unit (VC)** changes?
- What if the slope of the **VC per unit (VC)** changes?
- Which of the two measures has a bigger effect on the simulated results?
- Are all statistical measures are affected the same, or, say, the mean of Net Revenues is less sensitive to parameter changes than its standard deviation?

The way that parameter analysis works is rather simple: It runs the simulation multiple times and each time it alters the parameter(s) of interest. It then records how the changes in the parameter alter the key statistics of the distribution.

In this example, you will alter the constant and the slope of the **VC_{unit}** equation. To do that, you first create a new worksheet that is more appropriate for simulation parameter analysis. You do that as follows:

- Copy all data from the existing Excel sheet to a new one (for example, from **Sheet1** to **Sheet2**). Make sure that you copy all formulas, correlation matrices, etc.
- Update the newly created worksheet as follows:
    - Add a cell labeled "VC per unit (constant)"; assign it the value of 1.
    - Add a cell labeled "VC per unit (slope)"; assign it the value of 0.004.
    - Change the formula in the existing "VC per unit" cell so that it refers to the two cells created above.

The updated formulas in the new worksheet should look similar to the following (altered cells are in **bold**):

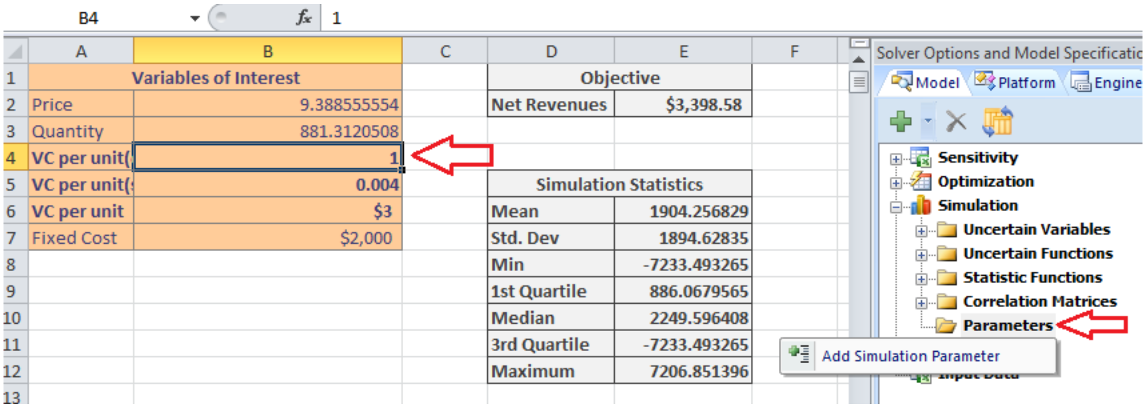
	A	B
1	**Variables of Interest**	
2	Price	=PsiTriangular(8,10,11, PsiCorrMatrix($E$15:$F$16, 1))
3	Quantity	=PsiTriangular(500,1500,2000, PsiCorrMatrix($E$15:$F$16, 2))
4	**VC per unit(constant)**	**1**
5	**VC per unit(slope)**	**0.004**
6	**VC per unit**	**=B4+B5*B3+PsiNormal(0,SQRT(0.8))**
7	Fixed Cost	2000

You now proceed to define the simulation parameters and their respective changes.

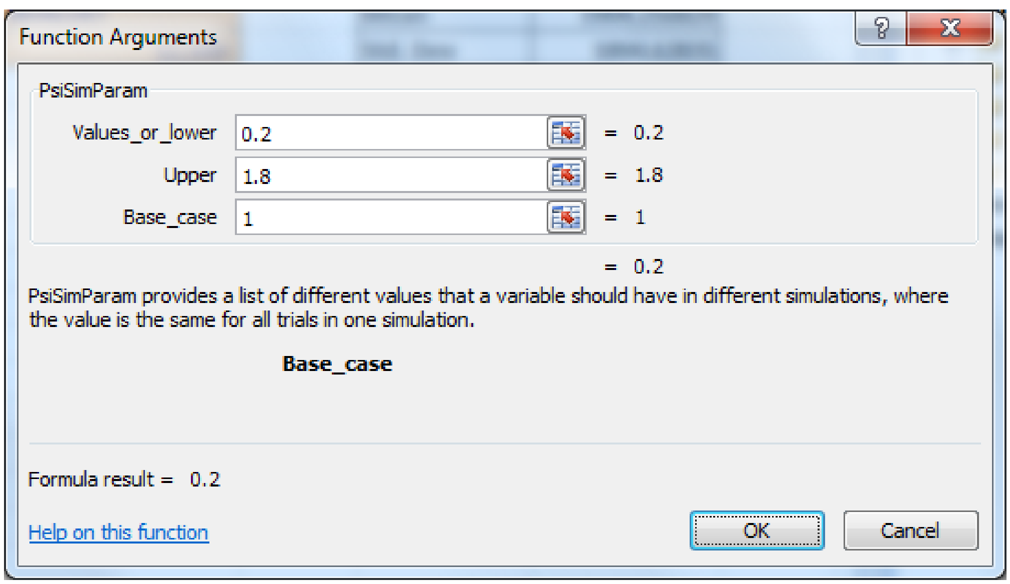
Altering the constant of VC per unit

Consider the case where the constant of VC_{unit} can change by up to 80% from its current value, in both directions; this means that the constant can take values in the range 0.2 to 1.8. You follow these steps:

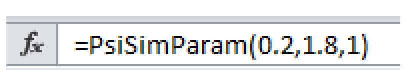
- Highlight the cell that contains the constant's value (in this example, B4); then navigate on the model specifications panel, click on the "+" sign next to **Simulation**, and then right click on **Parameters** (see the following screenshot).



- Click on **Add Simulation Parameter**. This will bring up the Function Argument window where you set the minimum, maximum and base case values for the variable of interest. The minimum value to consider is 0.2. The maximum is 1.8. The base case is 1. Set the values as in the screenshot below and hit **OK**. (*Note: The exact number of values that will be used in the specified range will be set later*)



The constant has now been added as a simulation parameter and can be used on a simulation parameter analysis. Notice that the cell holding the constant value – in this example B4 –has been changed to identify that this cell is now a simulation parameter; instead of a single value, equal to 1, the cell holds the following function:



Note: You can also enter the above formula directly in the cell, without having to follow the step-by-step procedure described above.

Altering the slope of VC per unit

Consider the case where the slope can also change by up to 80%, in both directions. In this case, the slope will have a range from 0.0008 to 0.0072. You will add this simulation parameter directly in the worksheet, without following a step-by-step procedure. This can be done as follows:

- You should entered the function PsiSimParam(Lower,Upper,Base case) directly in the cell. In this example, lower, upper and base case values are 0.0008, 0.0072 and 0.004, respectively. The function for the minimum price should then be **PsiSimParam(0.0008, 0.0072, 0.004).** You enter the above formula in the cell that holds the slope coefficient, in this case B5. The function in the worksheet will look as follows:

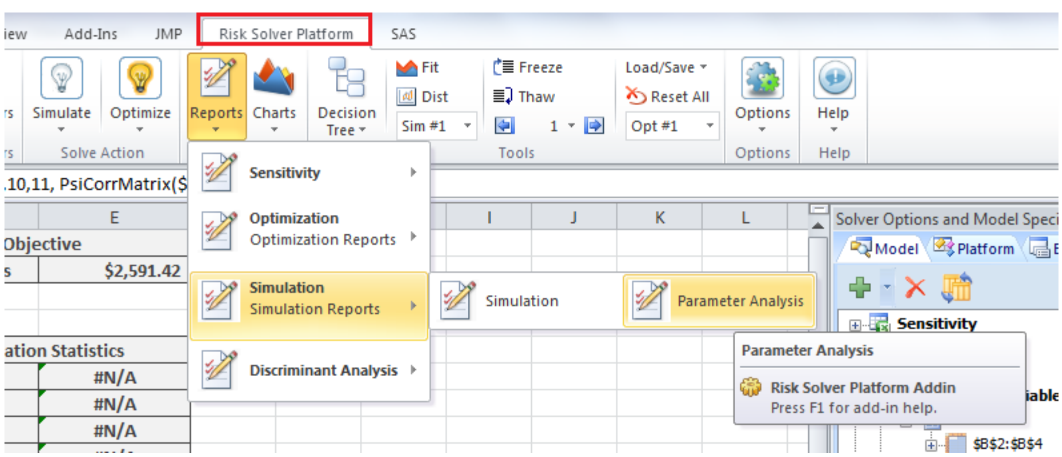
Having identified the simulation parameters for the constant and the slope, you proceed on running the parameter analysis.

Running the simulation parameter report

The simulation parameter analysis can produce either a report, showing the results in a tabular format, or as a graph, showing the results in graphical format. Here you will produce a simulation parameter report; the steps for a simulation parameter graph are very similar.

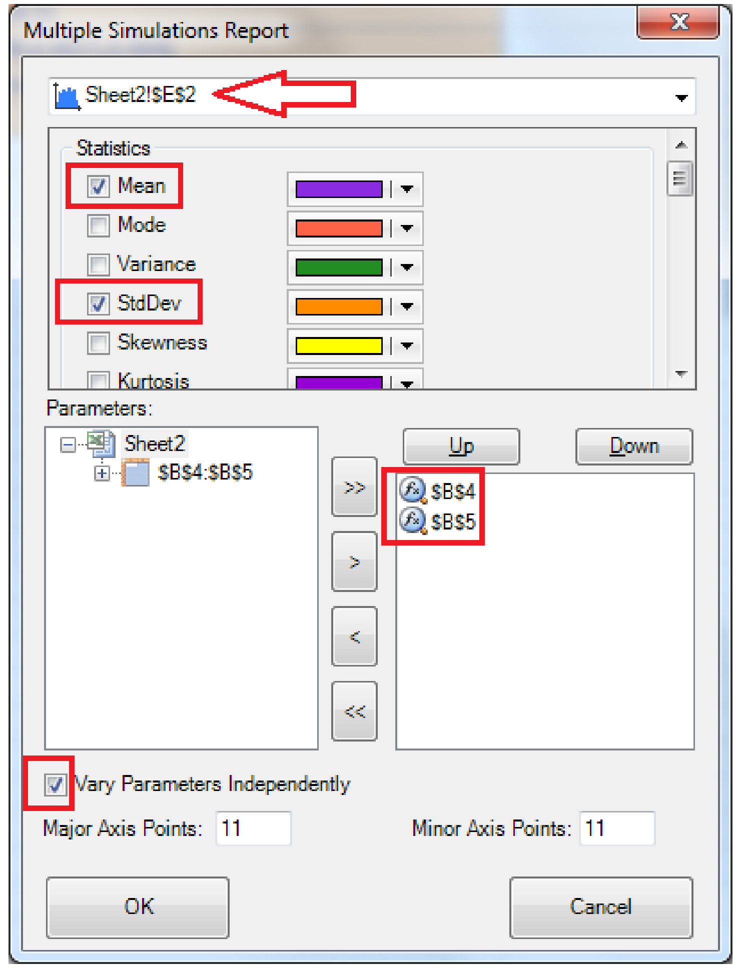
To create the simulation parameter report, follow these steps:

- Navigate to the **Risk Solver Platform** ribbon. Click on the **Reports** button and from the drop-down menu navigate to **Simulation→Parameter Analysis** (see the screenshot below)



*Note: If you prefer to create a simulation parameter chart, click on the **Charts** button and navigate to **Multiple Simulations→Parameter Analysis**. The rest of the steps will follow a similar logic.*

- Click on **Parameter Analysis**. This will bring up the **Multiple Simulations Report** window.



  - ○ Make sure that the Sheet/Cell listed on the top of this window is the target cell you want to analyze (in this example, Sheet2!$E$2).
  - ○ Select the measures that you want to track during the parameter analysis: click on the check-boxes for **Mean** and **StdDev**.
  - ○ In the **Parameters** box, highlight both parameters and hit on the single right arrow to bring them in the right side box (if they are not there by default).
  - ○ Click on **Vary Parameters Independently**.
    - ▪ The **Major Axis Points** and **Minor Axis Points** control the number of values that will be used to cover the range for the simulated parameters. Note that the simulation will run over the Cartesian product of the parameter values; hence, in this example, you will run 11x11=121 simulations, each one on a different combination for constant and slope coefficients.

Once you have verified your selections, hit the **OK** button to run the parameter analysis.

The RSP will perform 121 simulations and create a table with the all results for the two statistical measures you are tracking (Mean and Std. Deviation). The output will be placed in a new Sheet on your workbook, e.g. **Sim Analysis Report.** You can review the parameter analysis output to identify the effect of the changes on the simulation statistics. This is discussed in the next section.

Reviewing the parameter analysis results

You can easily visualize the effect of the parameter changes is by performing a conditional formatting on the output tables.

For the case of the mean net revenues, you can use the built-in Green-Yellow-Red color scale. You use the Green-Yellow-Red because large values on the mean of net revenues are desired, thus they have a green value; as the value for the mean of net revenues drops, the color changes gradually from Green to Red.

Following an analogous case for the standard deviation, you use the built-in Red-Yellow-Green color scale. You use this color scale because large values on the standard deviation are less desired, thus they have a red value; as the value of the standard deviation drops, the color changes gradually from Red to Green.

The color-formatted tables for the mean and standard deviation are displayed below.

Mean : $B$4 by $B$5

$B$4	$B$5										
	0.0008	0.00144	0.00208	0.00272	0.00336	0.004	0.00464	0.00528	0.00592	0.00656	0.0072
0.2	8968.2	7768.2	6568.2	5368.2	4168.2	2968.2	1768.2	568.2	-631.8	-1831.8	-3031.8
0.36	8755.6	7555.6	6355.6	5155.6	3955.6	2755.6	1555.6	355.6	-844.4	-2044.4	-3244.4
0.52	8541.2	7341.2	6141.2	4941.2	3741.2	2541.2	1341.2	141.2	-1058.8	-2258.8	-3458.8
0.68	8331.0	7131.0	5931.0	4731.0	3531.0	2331.0	1131.0	-69.0	-1269.0	-2469.0	-3669.0
0.84	8112.8	6912.8	5712.8	4512.8	3312.8	2112.8	912.8	-287.2	-1487.2	-2687.2	-3887.2
1	7905.2	6705.2	5505.2	4305.2	3105.2	1905.2	705.2	-494.8	-1694.8	-2894.8	-4094.8
1.16	7691.7	6491.7	5291.7	4091.7	2891.7	1691.7	491.7	-708.3	-1908.3	-3108.3	-4308.3
1.32	7476.2	6276.2	5076.2	3876.2	2676.2	1476.2	276.2	-923.8	-2123.8	-3323.8	-4523.8
1.48	7261.5	6061.5	4861.5	3661.5	2461.5	1261.5	61.5	-1138.5	-2338.5	-3538.5	-4738.5
1.64	7049.7	5849.7	4649.7	3449.7	2249.7	1049.7	-150.3	-1350.3	-2550.3	-3750.3	-4950.3
1.8	6837.1	5637.1	4437.1	3237.1	2037.1	837.1	-362.9	-1562.9	-2762.9	-3962.9	-5162.9

StdDev : $B$4 by $B$5

$B$4	$B$5										
	0.0008	0.00144	0.00208	0.00272	0.00336	0.004	0.00464	0.00528	0.00592	0.00656	0.0072
0.2	2164.6	1815.6	1563.3	1458.9	1532.8	1762.8	2098.2	2496.8	2933.0	3392.3	3866.5
0.36	2137.4	1795.4	1554.2	1464.3	1552.3	1792.1	2133.3	2535.1	2973.1	3433.5	3908.3
0.52	2089.4	1755.3	1527.4	1456.7	1564.5	1819.4	2170.1	2577.9	3019.7	3482.6	3959.1
0.68	2067.3	1736.3	1514.1	1451.6	1568.0	1829.5	2184.5	2595.0	3038.6	3502.6	3980.0
0.84	2010.8	1691.9	1489.5	1453.2	1594.2	1872.9	2238.6	2655.4	3102.9	3569.6	4048.8
1	2000.6	1686.1	1490.1	1461.0	1607.9	1890.3	2257.8	2675.6	3123.6	3590.5	4069.9
1.16	1956.1	1647.1	1461.6	1447.9	1610.4	1904.5	2279.9	2702.8	3154.3	3623.6	4104.6
1.32	1918.7	1622.6	1456.4	1465.0	1645.6	1951.1	2332.8	2759.4	3213.0	3683.6	4165.6
1.48	1886.9	1602.9	1454.3	1482.4	1678.4	1993.3	2380.4	2810.0	3265.4	3737.1	4219.7
1.64	1859.2	1588.1	1457.4	1504.4	1714.3	2037.5	2429.1	2861.3	3318.1	3790.7	4273.9
1.8	1824.7	1561.4	1443.3	1505.1	1727.5	2059.1	2456.1	2891.6	3350.8	3825.0	4309.3

In both tables, the changes in the slope coefficient are identified across the columns (i.e. moving from left to right); the changes in the constant are identified across the rows (i.e. from top to bottom). From an inspection of both tables, you can deduce the following:

- Increases in the constant coefficient result to a decrease in the mean of net revenues.
- Increases in the constant coefficient result to increases in the standard deviation.

- Increases in the slope coefficient result to a decrease in the mean of net revenues.
- Increases in the slope coefficient have a nonlinear effect on the standard deviation of net revenues. Initially, they decrease the standard deviation and then they decrease it; in mathematical terms, there is a convex relationship between slope and standard deviation. For the specific case analyzed, a slope coefficient in the range of 0.00208 to 0.00272 seems to minimize the standard deviation.
- For any given level of the slope, changes in the constant have a small relative effect on both the mean and the standard deviation of net revenues.
- For any given level of the constant, changes in the slope have a large relative effect on both the mean and the standard deviation of net revenues.

The above results are mostly expected; when the constant or the slope of the VC per unit increase, the expected net revenues decrease and their variability, as measured by the standard deviation, increases. The only case that is worth more consideration is the fourth bullet above: the convex relationship of the slope and the standard deviation. This seems to be driven by both the correlation coefficient and the actual values of the constant and the slope.