

CS 170

# Efficient Algorithms and Intractable Problems

## Lecture 15 Linear Programming

Nika Haghtalab and John Wright

EECS, UC Berkeley

# Announcements

## Homeworks:

- HW7 is due later this evening (Tues evening).
- HW8 will be released later today, and due the following Wed
  - Why more time than usual?! Because it's more interesting (aka challenging!) ☺
  - **Don't wait until the last minute (HWP on Friday and Monday only)**

Midterm 1 regrade requests are open and **will close next Monday!**

→ Submit a regrade request in case we made a grading error.

# Our Journey so far

We have learned o much this semester already!

- Divide and Conquer
- Graph Algorithms
- Greedy Algorithms
- Dynamic Programming

**This and the next few lectures:**  
Linear Programming, Applications, Implications:

**Today:**

- Definitions
- Use cases
- Some Algorithms

# Linear Programs

# An Example

A bakery produces donuts and cakes.

Each have some profit and require some ingredients.

Item	Profit	Flour (200 total)	Sugar (300 total)	Milk (500 total)
------	--------	----------------------	----------------------	---------------------



\$5            2            2            7



\$25            5            9            12

How many units of each should the bakery produce to maximize its profit?

# An Example

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\$5            2            2            7



\$25          5            9            12

## Decision variables

# donuts:  $x$  and # cakes:  $y$

**My objective**  
 $\max 5x + 25y$

## Constraints

$$x \geq 0, y \geq 0$$

$$2x + 5y \leq 200$$

$$2x + 9y \leq 300$$

$$7x + 12y \leq 500$$

How many units of each should the bakery produce to maximize its profit?

# Constrained Optimization

Many algorithm design problems we studied so far:

1) Solution has to meet some constraints:

→ MST: set of edges connecting all vertices of a graph

→ Longest increasing subseq: is a subsequence and monotonically increasing.

→ Interval scheduling: Intervals don't conflict

2) Subject to the above, we optimize an objective:

→ MST: Sum of edge weights

→ Longest increasing subseq: size of subsequence

→ Interval scheduling: Number of intervals

**Decision variables**

**My objective**

**Constraints**

# Linear Programming (LP)

A type of constrained optimization, where

- The objective function is a minimization (or a maximization) of **linear function of the decision variables**.
- Every constraints is a **linear function of the decision variables**.

**The bakery problem is a linear program!**

## Decision variables

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**The bakery problem is a linear program!**

Algorithms for linear programs:

- Goal: an algorithm that solves any LP efficiently.
- So, if you can write a problem as an LP, you can solve it efficiently.

## Decision variables

# donuts:  $x$  and # cakes:  $y$

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# Integer or Fractional Decision Variables?

For linear Programs:

- Decision parameters are real values (belong to  $\mathbb{R}$ )

**CANNOT have integer constraints in LPs.**

Integer Linear Programs (ILP):

- A variant of linear programs, where decision variables can be integer.
- It is not an LP!

There are no algorithms that can solve any ILP in polynomial time.

But there are algorithms that can solve any LP in polynomial time!

**Decision variables**

# donuts:  $x$  and # cakes:  $y$

**My objective**  
 $\max 5x + 25y$

**Constraints**

$$x \geq 0, y \geq 0$$

$$2x + 5y \leq 200$$

$$2x + 9y \leq 300$$

$$7x + 12y \leq 500$$

~~$x$  and  $y$  are integers~~

# LP Standard Form

**Definition:** A linear program is in standard form if you write it as follows

*n* variables  $x_1, \dots, x_n$

$$\max c_1 x_1 + c_2 x_2 + \dots + c_n x_n$$

Subject to  $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$

*m* constraints  
+  $a_{22}x_2 + \dots + a_{2n}x_n \leq b_2$

$\vdots$   
 $a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$

and  $x_1 \geq 0, \dots, x_n \geq 0$

Non-negative variables in  $\mathbb{R}$



$$\max \underline{\mathbf{c}^T \mathbf{x}}$$

Subject to  $\mathbf{Ax} \leq \mathbf{b}$

and  $\mathbf{x} \geq \mathbf{0}$

$\mathbf{A}$ :  $m \times n$  matrix

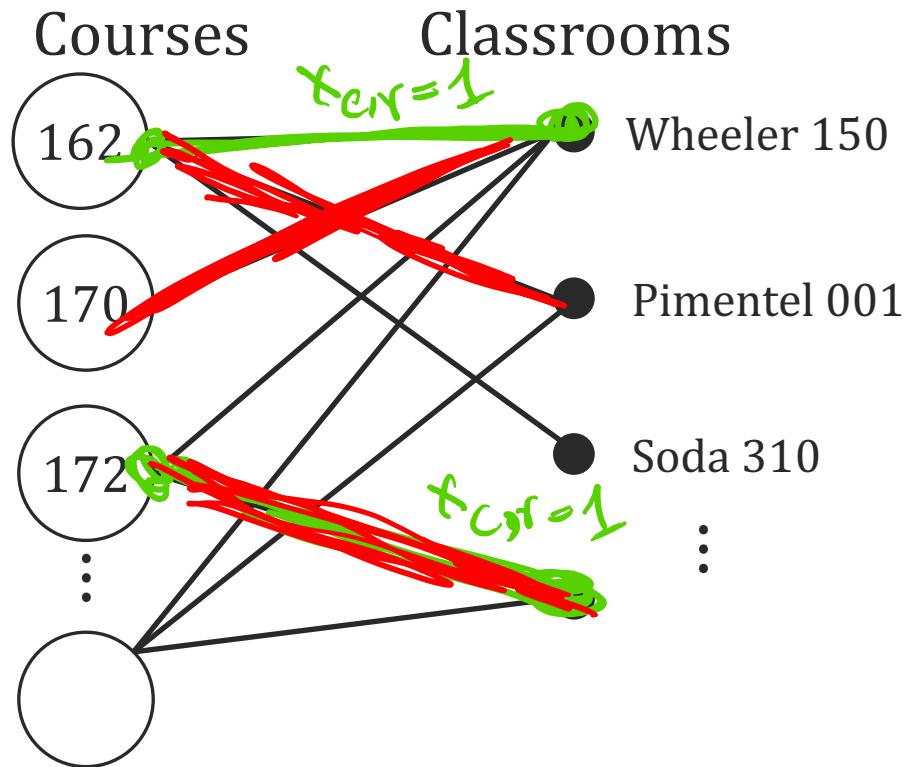
$\underline{\mathbf{c}, \mathbf{x}}$ :  $n$ -dim column vectors

$\underline{\mathbf{b}}$ :  $m$ -dim column vectors

Variables:  $\sum x_{c,r} = 0,1$   $1 \Rightarrow \text{course } c \text{ assigned to room } r$

# Another Example: Classroom Allocation

We have courses and classrooms in which they could fit. Each classroom can fit one course at most. Given Graph  $G = (V, E)$ , where course  $c$  can be assigned to classroom  $r$  iff  $(c, r) \in E$ .



Goal: Maximize the number of courses allocated a classroom.

Decision variable:

$$x_{c,r}$$

for each  $c$  course  
 $r$  room

$$(c, r) \in E.$$

Constraints:

$$0 \leq x_{c,r} \leq 1 \quad \forall c, r$$

for all rooms  $r$ :

$$\sum_{c : (c,r) \in E} x_{c,r} \leq 1$$

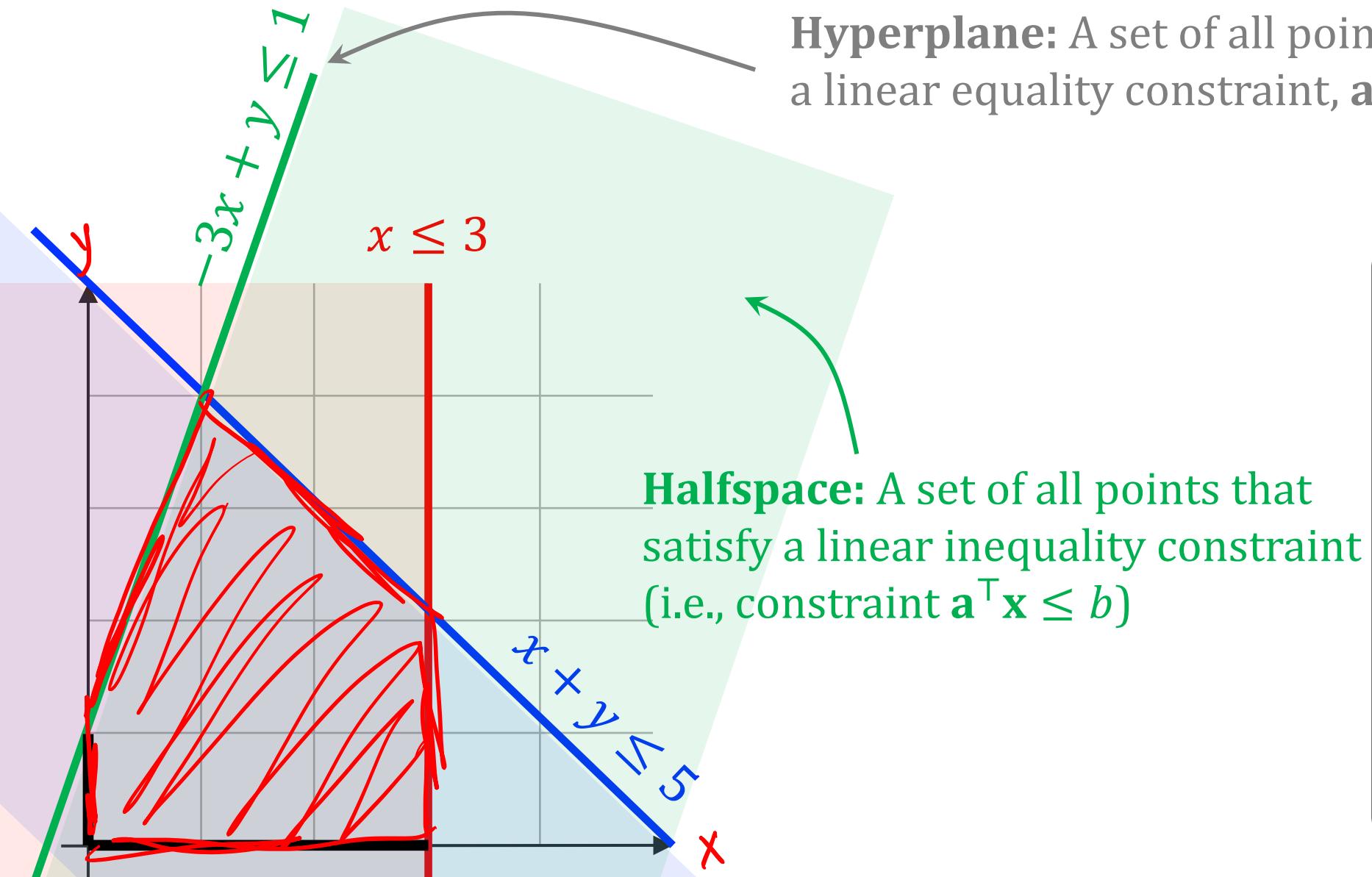
for all courses  $c$ :

$$\sum_{r : (c,r) \in E} x_{c,r} \leq 1$$

objective:

$$\max \sum_{(c,r) \in E} x_{c,r}$$

# Geometric View of LPs



**My objective**  
 $\max x + 2y$

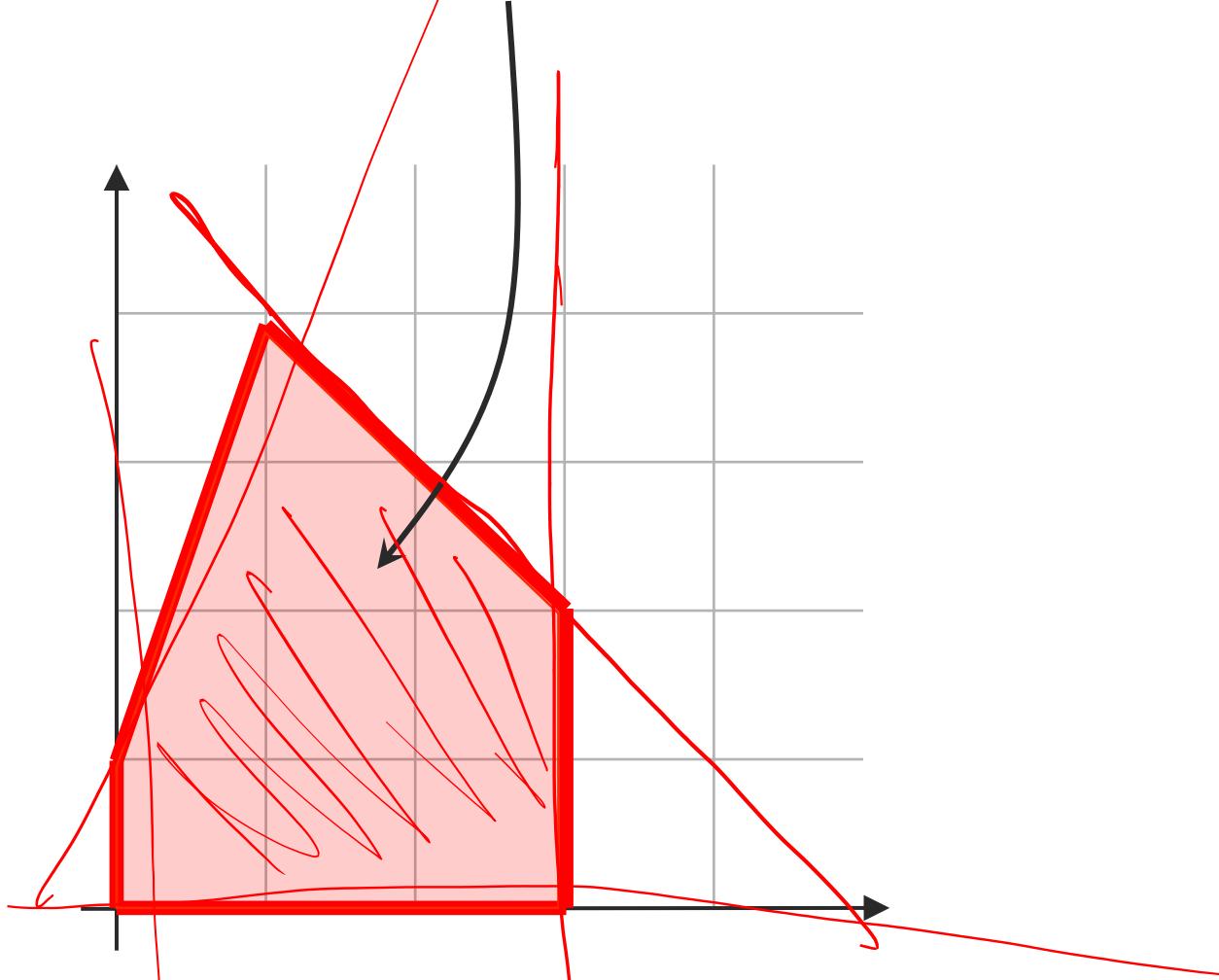
**Constraints**

$$\begin{aligned}x &\leq 3 \\x + y &\leq 5 \\-3x + y &\leq 1 \\x &\geq 0, y \geq 0\end{aligned}$$

# Feasible Region

$\mathbb{R}^n$  space

**Feasible Region:** The set of all points that satisfy all constraints.



**My objective**  
 $\max x + 2y$

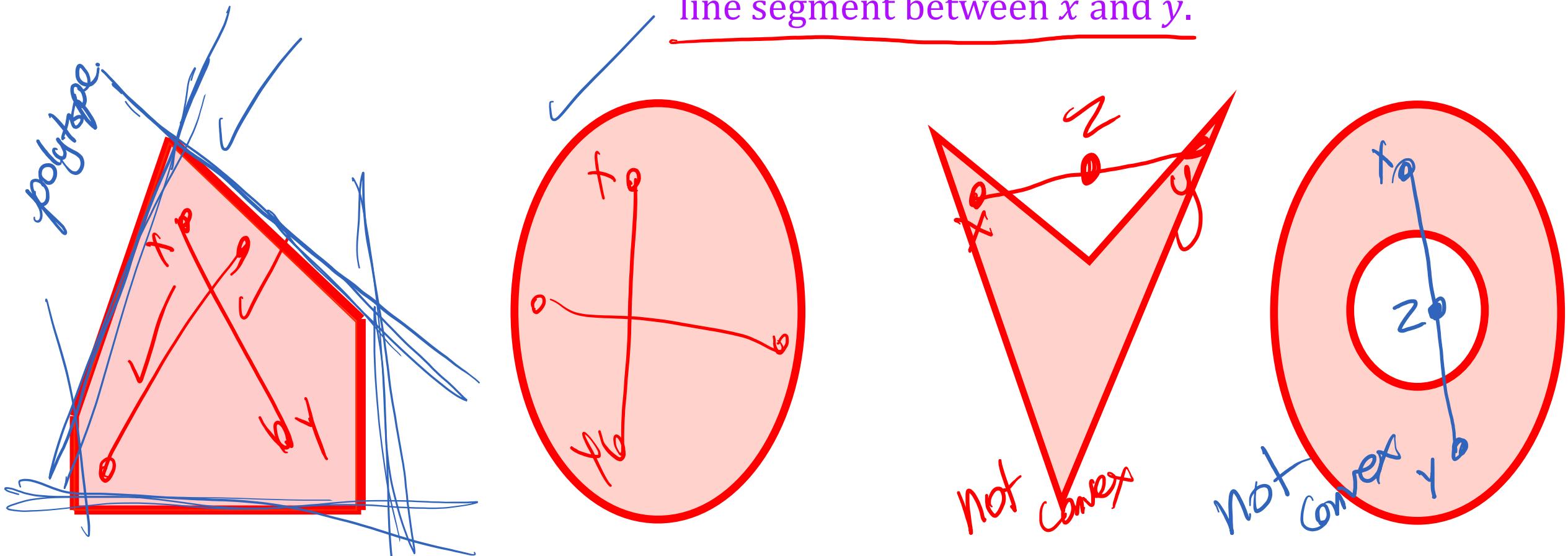
**Constraints**

$$x \leq 3$$
$$x + y \leq 5$$
$$-3x + y \leq 1$$
$$x \geq 0, y \geq 0$$

# Feasible Region and Convexity

**Definition:** A set of points  $S \subseteq \mathbb{R}^n$  is convex, if for any two points  $x, y \in S$ , all points  $z = \alpha x + (1 - \alpha)y$  for  $\alpha \in [0,1]$  also satisfy  $z \in S$

line segment between  $x$  and  $y$ .



# Feasible Region and Convexity

Show this is convex.

Claim: In any linear program, the feasible region is a convex set.

Sketch: feasible solution  $\iff \vec{x}$

$\vec{x}$  meets every constraint  
 $a_i^T \vec{x} \leq b_i$  for  $i \in [m]$ .

If  $x, y$  meet constraint  $i$ :

$$\begin{cases} a_i^T x \leq b_i \\ a_i^T y \leq b_i \end{cases} \Rightarrow$$

any  $z = \alpha x + (1-\alpha)y$   
 $\alpha \in [0, 1]$

also meets the constraint

$$a_i^T z = a_i^T (\alpha x + (1-\alpha)y) \stackrel{?}{\leq} b_i$$

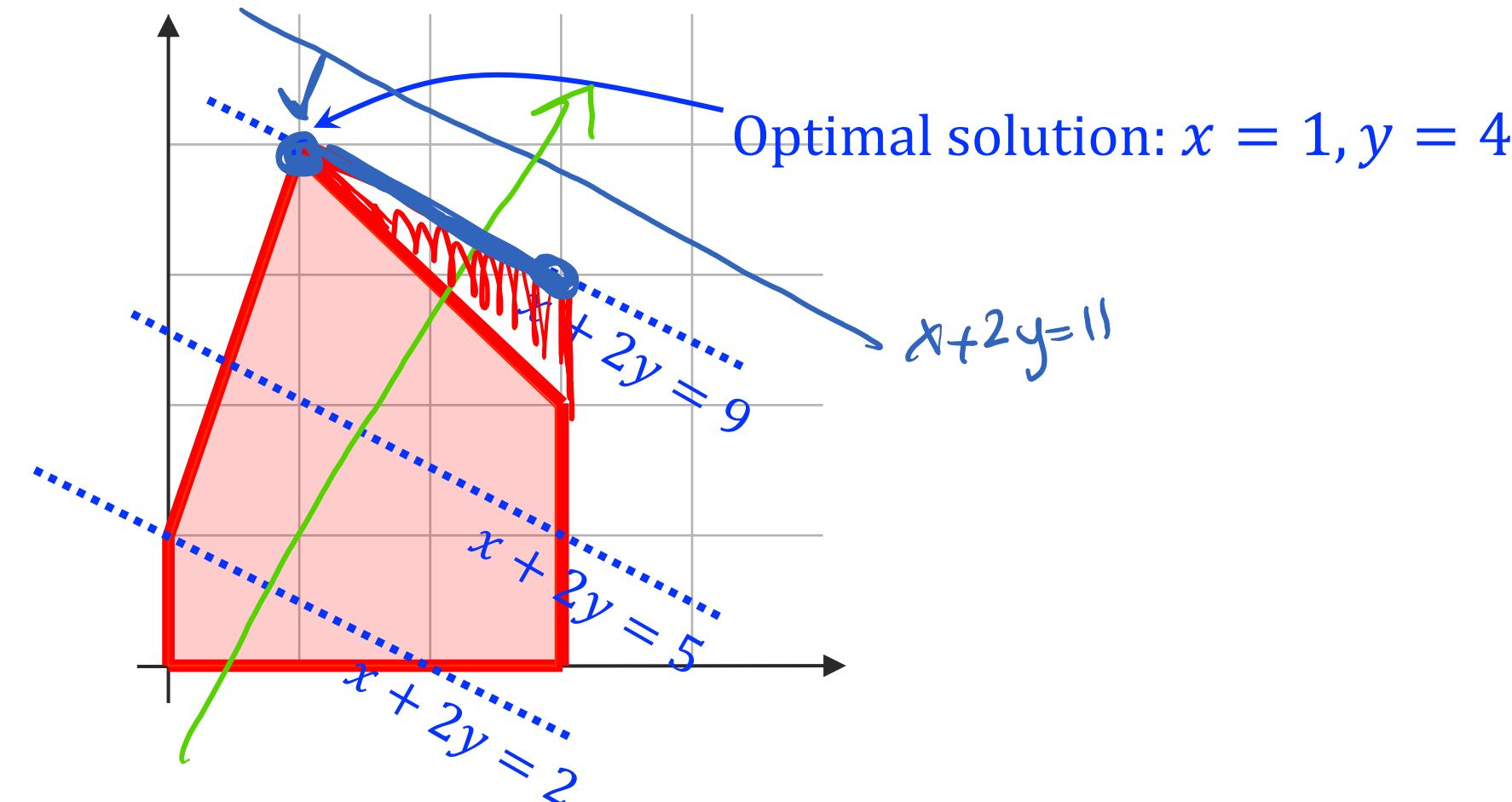
$\stackrel{?}{\leq}$   $\leq b_i$   $\leq b_i$

In fact, the feasible region is a "convex polytope": Intersection of halfspaces.

# Geometric view of the optimal solution

Consider the level-sets of the objective function (All points that lead to the same objective value)

$$\rightarrow x + 2y = c \text{ for all } c.$$



**My objective**  
 $\max x + 2y$

**Constraints**

$$x \leq 3$$

$$x + y \leq 5$$

$$-3x + y \leq 1$$

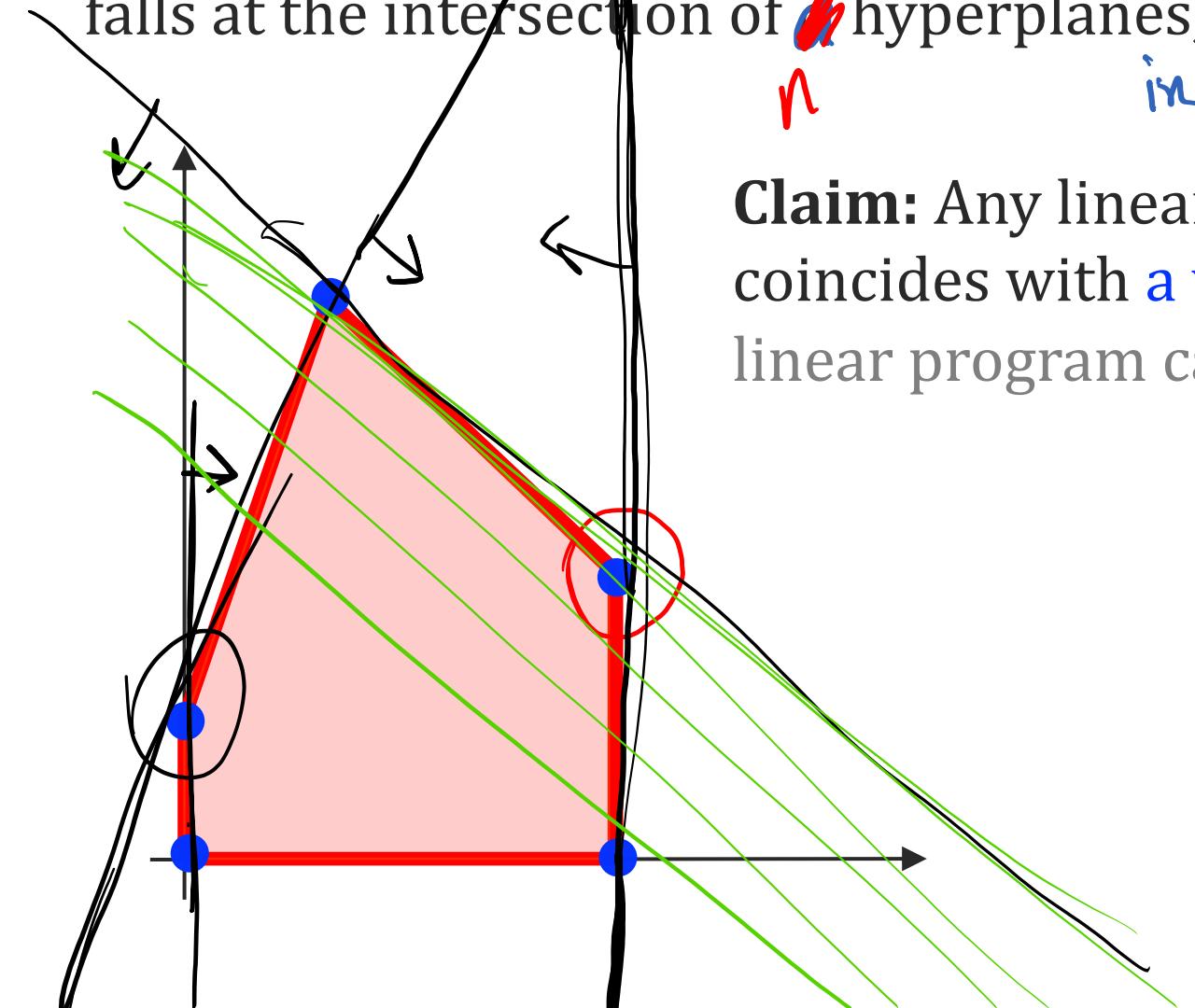
$$x \geq 0, y \geq 0$$

# Optimal solutions and extreme points / vertex -

**Definition:** An vertex (or extreme point) of the feasible region is a point that falls at the intersection of  $\neq$  hyperplanes, each corresponding to a constraint.

$n$  in  $\mathbb{R}^n$

**Claim:** Any linear program has an optimal solution that coincides with a vertex of the feasible region (or else, the linear program can achieve unbounded value).



# One Algorithm for finding all vertices

An algorithm for finding **all vertices** of the feasible region:

Given  **$m$  constraints**

For each subset of  **$n$  constraints**:

1. Solve for  $x^*$  at their intersection  
→ Linear system: replace  $\leq$  with  $=$  and use Gaussian elimination.
2. Check to see if  $x^*$  is in the feasible region  
→ Check all other  $m - n$  constraints are held.  
→ If so, add  $x^*$  to the list of extreme points.

$$\max \mathbf{c}^\top \mathbf{x}$$

$$\text{Subject to } \mathbf{A}\mathbf{x} \leq \mathbf{b}$$

$$\text{and } \mathbf{x} \geq \mathbf{0}$$

**A:**  $m \times n$  matrix

**c, x:**  $n$ -dim column vectors

**b:**  $m$ -dim column vectors

# LP Algorithm 1

Go through all vertices and find the one with highest payoff  $\mathbf{c}^T \mathbf{x}$ .

## Discuss

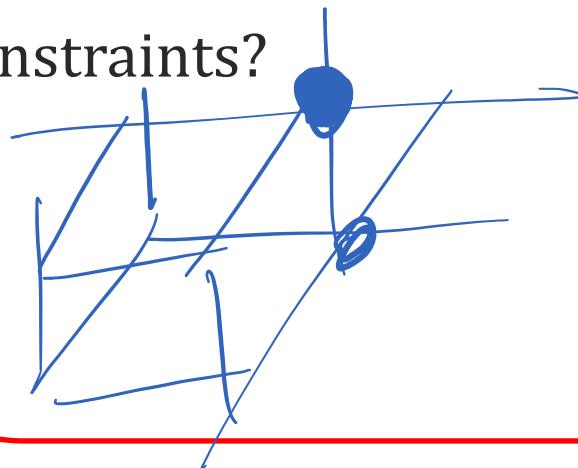
Why would such an algorithm be very slow?

How large can the # of vertices be, given  $m$  constraints and  $n$  variables?

vertex is intersection of  $n$  of the constraints

$$\leq \binom{m}{n} \approx m^n$$

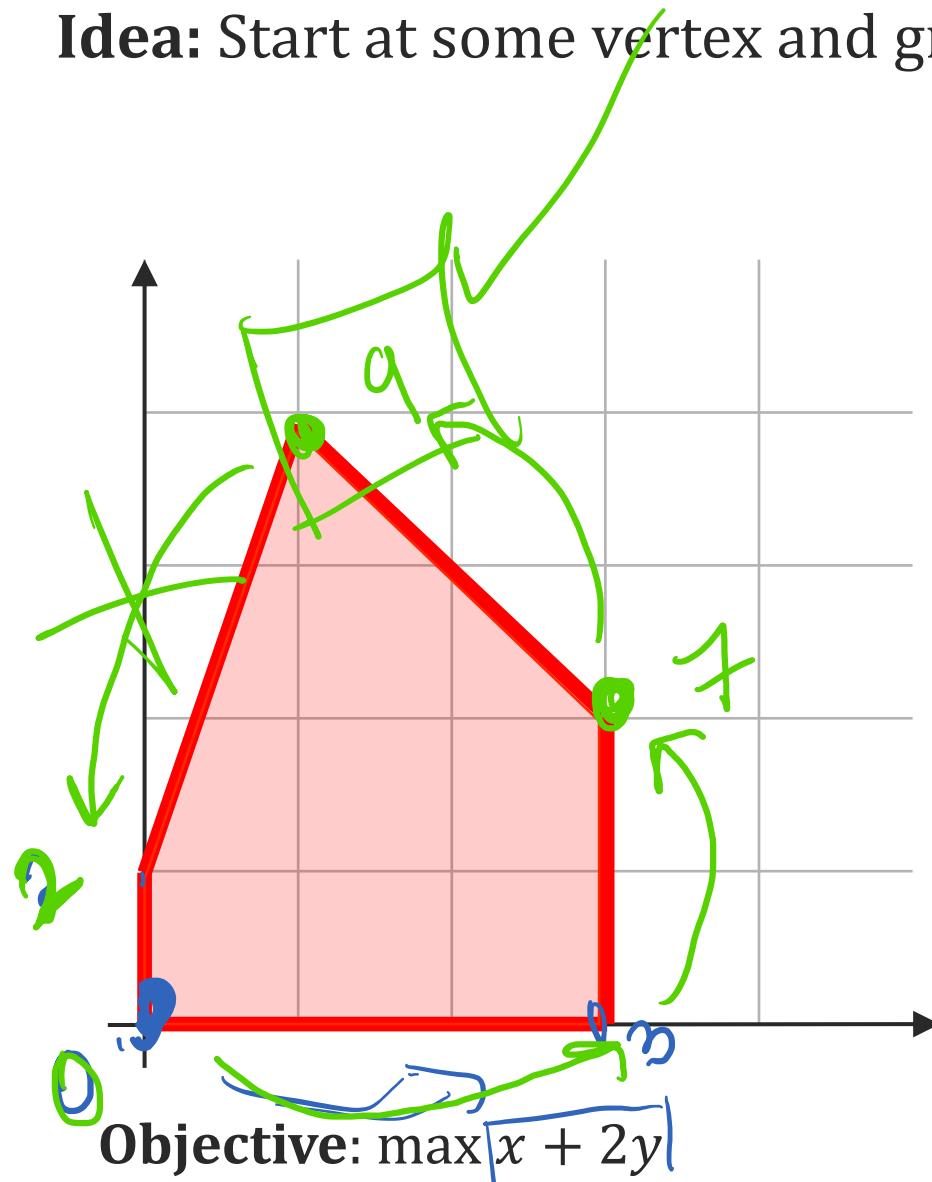
Give a construction of a feasible region with  $2^n$  vertices using  $m = O(n)$  of constraints?



$0 \leq x_i$      $1 \geq x_i$      $H_i \Rightarrow 2^n$  vertices     $2^n$  constraints.

# LP Algorithm 2: Simplex

**Idea:** Start at some vertex and greedily move to a better neighboring vertex.



1. Start at a vertex  $x^*$
2. Look at all “neighboring” vertices.
3. Find the neighbor  $y^*$  with the best objective value.
4. If  $y^*$  has a better value than  $x^*$ , move to  $y^*$  and repeat. Otherwise return  $x^*$ .

# Simplex: Defining Neighboring Vertices

How do we define “neighboring” vertices in high dimension?

Imagine we have  $n = 3$  variables and  $m = 5$  constraints (call the constraints  $1, 2, \dots, 5$ ).

**Recall:** vertices are defined as the intersection of  $n$  constraints

→ Take vertex at the intersection of constraints  $\{1, 2, 3\}$

→ One way to define neighbors: Swap out one constraint only

→ Neighbors:  $\{1, 2, 4\}, \{1, 2, 5\}, \{1, 4, 3\}, \{1, 5, 3\}, \{4, 2, 3\}, \{5, 2, 3\}$

*vertex at intersection of constraint b2, 4*

**Definition:** Formally, two vertices are **neighbors** if they differ only in **one constraint**.

## Discuss

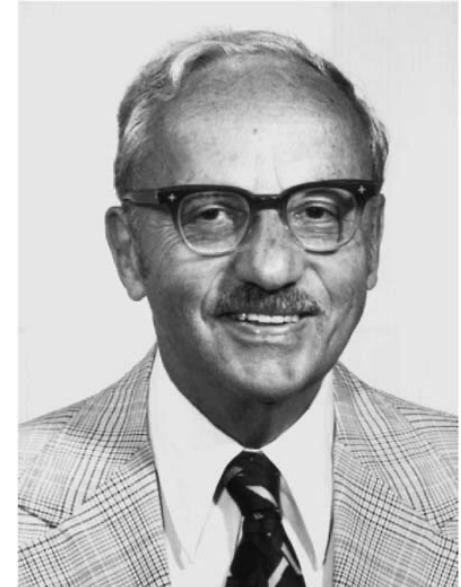
Suppose we have  $n$  variables and  $m$  constraints. What is the largest number of neighbors any vertex can have?

$$\text{neighbors} \leq m \times n.$$

# Simplex: Great in Practice, Slow in the Worst-Case

**Facts about simplex:** Given  $n$  variables and  $m$  constraints

1. In the worst-case, Simplex still runs in time **exponential in  $n$ .**
2. In practice:
  - Simplex runs very fast.
  - Beats more algorithms that are provably polynomial time!
  - Impressive, given that it's 75 years old algorithm!



How to explain the performance of Simplex algorithm in practice?

- Smoothed Analysis framework (20 years ago)
- If a bit of Gaussian noise is added to the constraints, Simplex provably runs in polynomial time!
- Smoothed analysis → Useful theoretical tool for addressing runtime in practice!

George Dantzing  
Berkeley Math Ph.D

# Polynomial time Algorithms for LPs.

**Theorem:** Linear Programs can be solved in Polynomial Time, even in the worst-case.

[Khachian 1979]: Ellipsoid Method

→ Good in theory, kind of slow in practice (way slower than Simplex in practice)

[Karmarkar 1984]: Interior point method

→ Good in theory, reasonably good in practice

Advanced algorithms: we won't give a description or prove these claims in CS170.

**But you can use them to prove that any problem that can be written as an LP can be solved in polynomial time!**

# Linear Programs and the World

## A Soviet Discovery Rocks World of Mathematics

By MALCOLM W. BROWNE

A surprise discovery by an obscure Soviet mathematician has rocked the world of mathematics and computer analysis, and experts have begun exploring its practical applications.

Mathematicians describe the discovery by L.G. Khachian as a method by which computers can find guaranteed solutions to a class of very difficult problems that have hitherto been tackled on a kind of hit-or-miss basis.

Apart from its profound theoretical interest, the discovery may be applicable

in weather prediction, complicated industrial processes, petroleum refining, the scheduling of workers at large factories, secret codes and many other things.

"I have been deluged with calls from virtually every department of government for an interpretation of the significance of this," a leading expert on computer methods, Dr. George B. Dantzig of Stanford University, said in an interview.

The solution of mathematical problems by computer must be broken down into a series of steps. One class of problem sometimes involves so many steps that it

could take billions of years to compute.

The Russian discovery offers a way by which the number of steps in a solution can be dramatically reduced. It also offers the mathematician a way of learning quickly whether a problem has a solution or not, without having to complete the entire immense computation that may be required.

According to the American journal Sci-

Continued on Page A20, Column 3

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"All the News  
That's Fit to Print"

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NEW YORK, WEDNESDAY, NOVEMBER 7, 1979

WEEKEND EDITION

LATE CITY EDITION  
Weather: Mostly sunny, cool today;  
clear, cold tonight. Rainy tomorrow.  
Temperature range: today 43-51;  
tonight 46-56. Details on page A8.

Similar Transportation Bond Issues Are Passed in New York and Jersey

Con Ed Takeover Action Fails in Westchester —  
Simon Wins in Bronx

By FRANKLYNN

Democrats approved mass transportation bond issues yesterday in New York and New Jersey, while Republicans stalled a long awaited bond issue in New Jersey and at least two constitutional amendments were rejected.

In Westchester County, voters rejected a proposal county takeover of the Con Edison utility, which had been proposed by the distribution system. [Page B3.]

A committee of public officers, Rep. Robert Stenner, Simon Simon of the Bronx, a Democrat, and Peter F. Cahalan, a Republican, defeated the proposal. All three candidates for County Executive, were elected in landslide.

Simon Getting 50% of Vote

With 80 percent of the vote counted, Mr. Stenner was winning the race, while Assemblyman G. Oliver Koppell, the Liberal candidate, and State Senator Jeanne Shaheen, the independent candidate, each got second place with about 17 percent each. In a potential endorsement of the proposal, however, the Right to Life Party candidate, Jeannine Donahue, was well ahead of the Conservative.

In Sutton County, Mr. Cahalan, who had been elected in 1976, beat State Executive John V. Klein, a Democrat, in the Republican primary on Sept. 11. It was the first time in 10 years that a Democrat had won the nomination.

In Bronx, Democrat Edward R. Gianni, a former Bronx Borough President, was re-elected on State lines, Erie County Executive Edward J. Butkowicz, a Democrat, serving an interim term,

Continued on Page B4, Column 4

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# Linear Programs and the World

## BREAKTHROUGH IN PROBLEM SOLVING

A 28-year-old mathematician at A.T.&T. Bell Laboratories has made a startling theoretical breakthrough in the solving of systems of equations that often grow too vast and complex for the most powerful computers. The discovery, which is to be formally published next month, is already circulating rapidly through the mathematical world. It has also set off a deluge of inquiries from brokerage houses, oil companies and airlines, industries with millions of dollars at

stake in problems known as linear programming. These problems are fiendishly complicated systems, often with thousands of variables. They arise in a variety of commercial and government applications, ranging from allocating time on a communications satellite to routing millions of telephone calls over long distances. Linear programming is particularly useful whenever a limited, expensive resource must be spread most efficiently among competing users. And investment companies use the approach in creating portfolios with the best mix of stocks and bonds.

**By LESLIE MATTISON WERNER**  
A Cuban defector whose politics, according to United States authorities, gave him entry to high-level Government circles, Llorente Menéndez, who before his defection served as chief adviser to the head of the Cuban National Institute of Finance from 1977 to 1980 and as chief adviser to the Minister of Culture from 1980 to 1982, has been granted political asylum in Canada.

He was described by officials in Washington as knowledgeable about his country's financial and economic policies. Mr. Llorente said he was summoned to the Cuban Foreign Ministry and attended meetings with Mr. Castro.

Mr. Llorente said that he first moved to Canada in late 1981, and succeeded in obtaining a United States visa after a friend in the United States helped him gain admittance to the United States.

Mr. Llorente, who is now a citizen of Canada, was granted political asylum in Canada in 1982.

**FOR A LIMITED TIME, HOME OFFICE**  
SALVATION. The Veterans' Aid and Care Fund, a division of the American Legion, has established a new service for veterans.

The service, called "Salvation," is designed to help veterans find temporary shelter, food, clothing and medical care.

For more information, call 1-800-232-ADVT.

**FOR THOSE FAVORING CANNABIS**  
REGISTRATION. The Marijuana Tax Act of 1937, which imposed a tax on the manufacture, sale and distribution of mariju-

juana, was declared unconstitutional by the Supreme Court in 1962.

Since then, the federal government has not enforced the law.

But some states have passed laws requiring registration of marijuana users.

For more information, call 1-800-232-ADVT.

"All the News  
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NEW YORK, MONDAY, NOVEMBER 19, 1984

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SOVIETS HELP TO DELIVER U.S. FAMINE SUPPLIES: In Kembelcha, Ethiopia, wheat from the United States is loaded onto a Soviet helicopter to help relieve emergency. Page A12.

## Cocaine Traffickers Kill 17 in Peru Raid On Antidrug Team

By ALAN COWELL  
Special to The Times

LIMA, Peru, Nov. 18 — A band of cocaine smugglers who breached a jungle campsite and opened fire with automatic weapons, killing at least 17 people, including 12 members of a United Nations-backed program to destroy coca crops, the police said today.

At the center of the attack, which took place early Saturday, were identified as Peruvian employees of the United Nations Organization for Drug Control and Eradication.

President Fernando Belaúnde Terry said three others working in the program were killed, but the police said the total number of dead was 17.

It was the second raid by a police antiterror unit that was trained by the United States about 10 months with assistance from the United Nations, the police said.

The discovery, which is to be voted on this week, would give the United Nations a \$6 million program to help combat coca cultivation, which is spreading rapidly through the mountains of southern Peru.

According to those who were killed, three people were critically wounded and seven others escaped injury by day.

Continued on Page A17, Column 3

The official numbers numbered 27, they said. The police said 17 bodies had been recovered, though they believed there were more than 5,000 people.

"May we will pick up," one of them said, referring to the search for the remaining victims.

Among survivors his support was even higher, with 66 percent saying he was a good man and 25 percent saying he was a bad man, and 25 percent saying they preferred him.

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# Wrap up

We saw an introduction of Linear Programming!

- Definitions
- How to formulate problems as LPs
- Intuition and some algorithms

Takeaway: Anything that can be written as an LP can be solved in polynomial times (in the number of constraints and variables).

**Next lectures:** More about linear programming