CS 170 Homework 11

Due **Monday 11/13/2023, at 10:00 pm (grace period until 11:59pm)**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write "none".

2 Some Sums

Given an array $A = [a_1, a_2, \ldots, a_n]$ of nonnegative integers, consider the following problems:

1 **Partition**: Determine whether there is a subset $S \subseteq [n]$ ($[n] := \{1, 2, \cdots, n\}$) such that $\sum_{i \in S} a_i = \sum_{j \in ([n] \setminus S)} a_j$. In other words, determine whether there is a way to partition A into two disjoint subsets such that the sum of the elements in each subset equal.

2 **Subset Sum**: Given some integer k, determine whether there is a subset $S \subseteq [n]$ such that $\sum_{i \in S} a_i = k$. In other words, determine whether there is a subset of A such that the sum of its elements is k.

3 **Knapsack**: Given some set of items each with weight w_i and value v_i, and fixed numbers W and V, determine whether there is some subset $S \subseteq [n]$ such that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} v_i \geq V$.

For each of the following clearly describe your reduction and justify its correctness.

(a) Find a linear time reduction from Subset Sum to Partition.

(b) Find a linear time reduction from Subset Sum to Knapsack.

Solution:

(a) Suppose we are given some A with target sum t. Let s be the sum of all elements in A. If $s - 2t \geq 0$, generate a new set $A' = A \cup \{s - 2t\}$. If A' can be partitioned, then there is a subset of A that sums to t.

We know that the two sets in our partition must each sum to $s - t$ since the sum of all elements will be $2s - 2t$. One of these sets, must contain the element $s - 2t$. Thus the remaining elements in this set sum to t.

If $s - 2t \leq 0$, generate a new set $A' = A \cup \{2t - s\}$. If A' can be partitioned, then there is a subset of A that sums to t.

We know that the two sets in our partition must each sum to t since the sum of all elements will be $2t$. The set that does not contain $\{2t - s\}$ will be our solution to subset sum.

(b) Suppose we are given some set A with target sum t. For each element i of the set, create an item with weight i and value k. Let $V = t$ and $W = t$. We know Knapsack will determine if there is a combination of items with sum of weights $\leq t$ and values

$\geq t$. Because the weights and values are the same, we know (Sum of chosen weights) = (Sum of chosen values) = t. And since each weight/value pair is exactly the value of one of the original elements of A, we know that there will be a solution to our Knapsack problem iff there is one for our subset sum problem.

3 Coding Questions: Reduction to Integer LP

For this week's coding questions, we'll walk through reducing the **Set Cover** problem to an **Integer Linear Program** and see how reductions can be used in practice. There are two ways that you can access the notebook and complete the problems:

1. **On Local Machine**: `git clone` (or if you already cloned it, `git pull`) from the coding homework repo,

   `https://github.com/Berkeley-CS170/cs170-fa23-coding`

   and navigate to the `hw011` folder. Refer to the `README.md` for local setup instructions.

2. **On Datahub**: Click here and navigate to the `hw11` folder if you prefer to complete this question on Berkeley DataHub.

Notes:

- *Submission Instructions:* Please download your completed submission `.zip` file and submit it to the Gradescope assignment titled "Homework 11 Coding Portion".
- *OH/HWP Instructions:* Designated coding course staff will provide conceptual and debugging help during office hours and homework parties.
- *Edstem Instructions:* Conceptual questions are always welcome on the public thread. If you need debugging help first try asking on the public threads. To ensure others can help you, make sure to:

  1. Describe the steps you've taken to debug the issue prior to posting on Ed.
  2. Describe the specific error you're running into.
  3. Include a few small test cases, alongside both the output you expected to receive and your function's actual output.

  If staff tells you to make a private Ed post, make sure to include *all of the above items* plus your full function implementation. If you don't provide them, we will ask you to provide them.

- *Academic Honesty Guideline:* We realize that code for some of the algorithms we ask you to implement may be readily available online, but we strongly encourage you to not directly copy code from these sources. Instead, try to refer to the resources mentioned in the notebook and come up with code yourself. That being said, we **do acknowledge** that there may not be many different ways to code up particular algorithms and that your solution may be similar to other solutions available online.