CS 188 Discussion 9:

Bayes Net Sampling, Decision Networks, VPI

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Slides based on Sashrika + Joy

Administrivia

- Project 4 due on Mon, Nov 6
- Homework is due on Tuesdays
- We have office hours pretty much all day every weekday (12-7), come to Soda 341B! (my hours are 1-3 PM on Mondays)
- Discussion slides are on Ed
- **No discussion last week - Bayes Nets discussion worksheet + solns posted**

Today’s Topics

- Bayes Net Sampling
  - Prior Sampling
  - Rejection Sampling
  - Likelihood Weighting
  - Gibbs Sampling
- Decision Networks
- Value of Perfect Information (VPI)

Sampling

- Want some probability, for example P(A) or P(A | +b)
- Calculating exact probability (for example, with variable elimination) can be horribly expensive
- Take random samples and infer the probability with reasonable accuracy

Prior Sampling

- Algorithm:
  - Generate samples

    - For i=1, 2, …, n (in topological order)
      - Sample X_i from $P(X_i \mid parents(X_i))$
    - Return $(x_1, x_2, \ldots, x_n)$

  - Discard samples inconsistent with evidence
  - Calculate probability of the query
- Pros:
  - Easy!
- Cons:
  - Requires a large number of samples, especially when we condition on unlikely scenarios

Rejection Sampling   Worksheet Q1a

- Algorithm:
  - Generate samples

    - Input: evidence $e_1,..,e_k$
    - For i=1, 2, …, n
      - Sample X_i from $P(X_i \mid parents(X_i))$
      - If x_i not consistent with evidence
        - Reject: Return, and no sample is generated in this cycle
    - Return $(x_1, x_2, \ldots, x_n)$

  - Calculate probability of the query
- Pros:
  - Bad samples take less time to create, so more efficient than Prior Sampling
- Cons:
  - Still requires a large number of samples, especially when we condition on unlikely scenarios

Likelihood Weighting

Worksheet Q1b-d

- Idea: Set all your variables to agree with evidence, then sample
  - **Weight** all samples by the probability of the evidence given the sampled variables

- Pros:
  - Never generate a bad sample!
- Cons:
  - Evidence influences the choice of downstream variables, but not upstream ones. Samples could have low weight!

- Input: evidence $e_1,..,e_k$
- $w = 1.0$
- for i=1, 2, …, n
  - if X_i is an evidence variable
    - x_i = observed value$_i$ for X_i
    - Set $w = w * P(x_i \mid parents(X_i))$
  - else
    - Sample x_i from $P(X_i \mid parents(X_i))$
- return $(x_1, x_2, \ldots, x_n)$, w

Gibbs Sampling **Worksheet Q1e**

- A bit less intuitive
- Algorithm:
  - Step 1: Fix all evidence variables
  - Step 2: Assign all other variables randomly
  - Keep looping through non-evidence variables:
    - Resample this variable ***X*** from **P(X | all other variables)**
    - New state of variables with the new ***X*** is new sample
  - As the number of iterations grows large, the samples converge to coming from the right distribution

- Pros:
  - Both upstream and downstream variables condition on evidence

Decision Networks

- MDP Sampling
- **Decision Networks**
- Value of Perfect Information (VPI)

Decision Networks

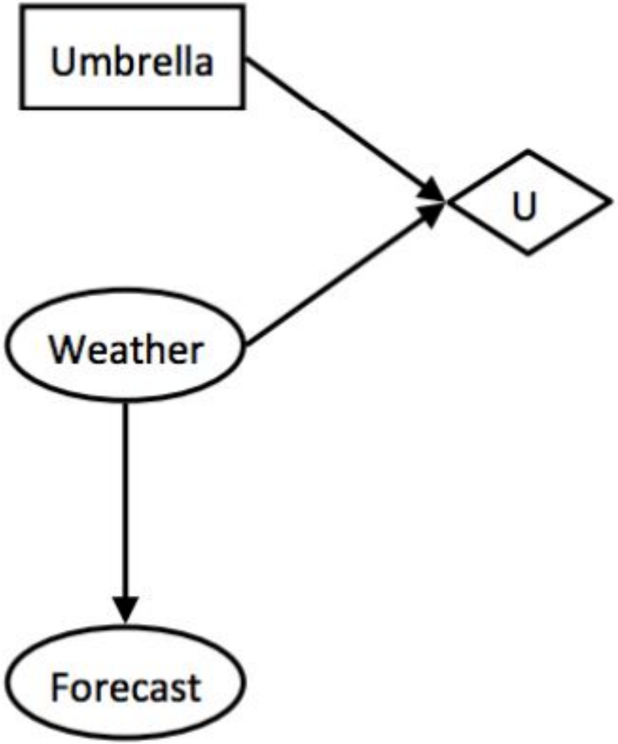
Chance nodes: associated with a probability (like Bayes nets), ovals

- Ex: 80% chance of rain forecasted

Action nodes: represent choice from a number of actions, rectangles

- Ex: do I take my umbrella?

Utility nodes: combination of action + chance, utility determined from parents' values, diamonds



Utility

- Expected utility (EU) of taking action a given evidence e with n chance nodes

$$EU(a|e) = \sum_{x_1,\ldots,x_n} P(x_1,\ldots,x_n|e)U(a,x_1,\ldots,x_n)$$

- Maximum expected utility (MEU): choose the action that maximizes EU

$$MEU(E=e) = \max_a EU(A=a|E=e)$$

Value of Perfect Information (VPI)

- MDP Sampling
- Decision Networks
- **Value of Perfect Information (VPI)**

VPI

- Observing evidence may have a cost (time, money) → let's quantify how much it's worth
  - Current max utility with evidence e is

$$MEU(e) = \max_a \sum_s P(s|e)U(s,a)$$

  - If we observed some evidence e', MEU becomes

$$MEU(e,e') = \max_a \sum_s P(s|e,e')U(s,a)$$

  - But we don't know what e' is → replace with random variable E'. Represent new MEU with expected value of MEU (our best estimate)

$$MEU(e,E') = \sum_{e'} P(e'|e)MEU(e,e')$$

  - How much is MEU expected to increase? Find the difference!

$$\boxed{VPI(E'|e) = MEU(e,E') - MEU(e)}$$

VPI Properties Worksheet Q2

- Nonnegativity
  - Observing new evidence cannot hurt you

$$\forall E', e\; VPI(E'|e) \geq 0$$

- Nonadditivity
  - The VPI of observing two pieces of evidence isn't necessarily the sum of their individual VPI'

$$VPI(E_j, E_k|e) \neq VPI(E_j|e) + VPI(E_k|e) \text{ in general}$$

- Order Independence
  - Order of observing evidence doesn't matter since we only take an action after observing new evidence

$$VPI(E_j, E_k|e) = VPI(E_j|e) + VPI(E_k|e, E_j) = VPI(E_k|e) + VPI(E_j|e, E_k)$$

Rest of the Worksheet

Thank you for attending!

Attendance link:

- https://tinyurl.com/cs188fa23

Discussion No: 9

Remember my name is Kenny

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