# Exam Prep Discussion 12

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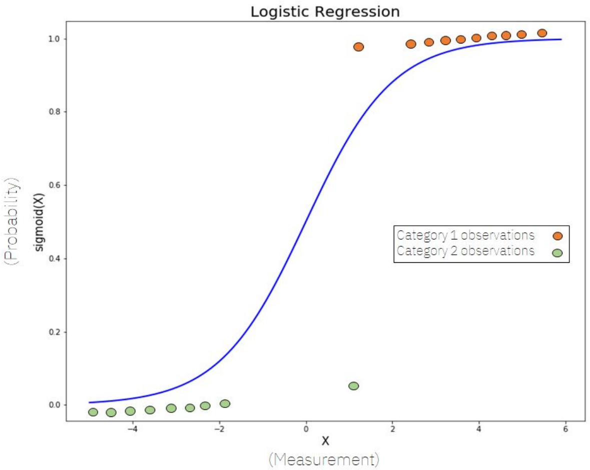
- *k* is a hyperparameter you can choose
  - A smaller *k* adheres to the true observation's distribution
  - A bigger *k* causes a more uniform distribution

# Logistic Regression

- Use $\vec{w}^T\vec{x}$ to obtain a probability of a data point belonging to a class, using the sigmoid function



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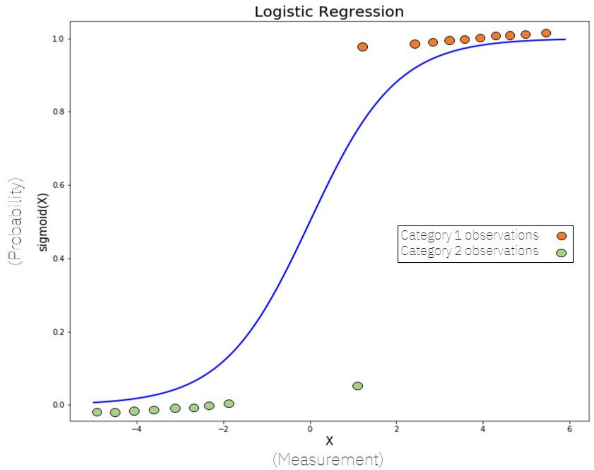

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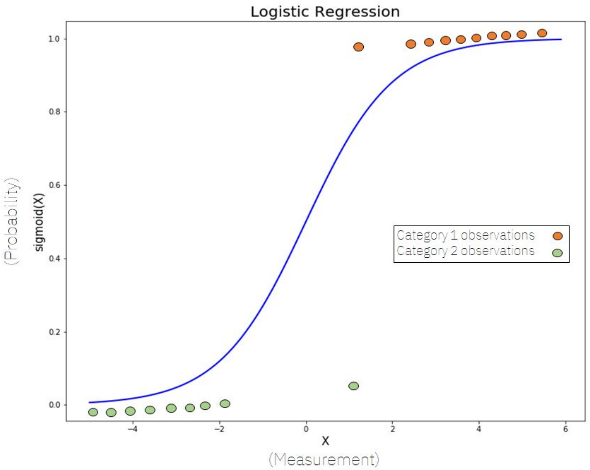

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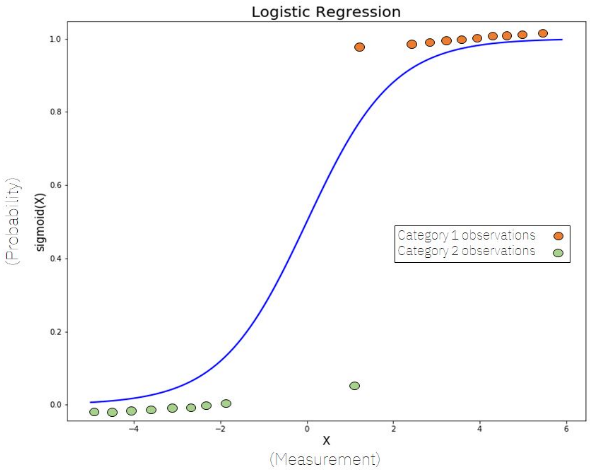
- Goal: Find $\vec{w}$ that maximizes the probability of correct classification for all points

$$\vec{w}^* = \text{argmax}_w \prod_i P(y_i|x_i,w) = \text{argmax}_w \log \prod_i P(y_i|x_i,w)$$



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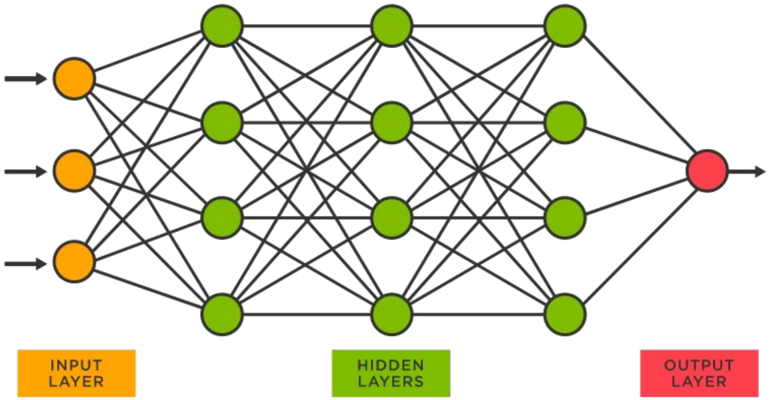
  - Attempts to simulate biological neural networks but does so poorly



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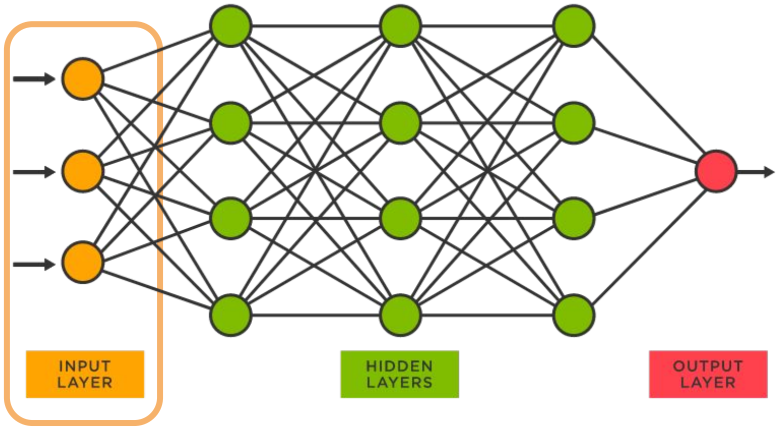

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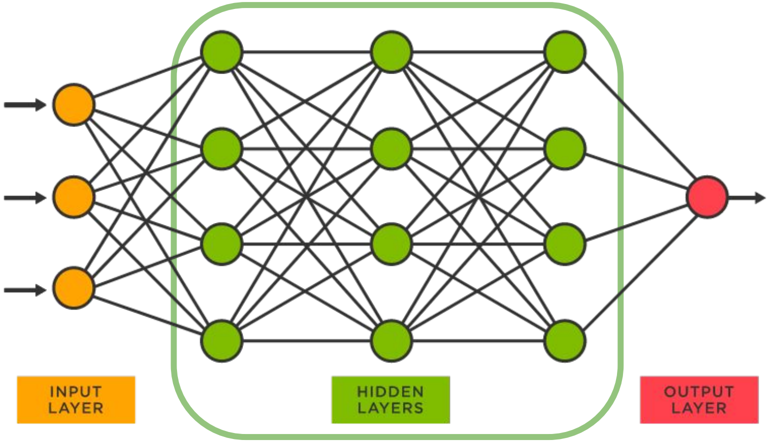

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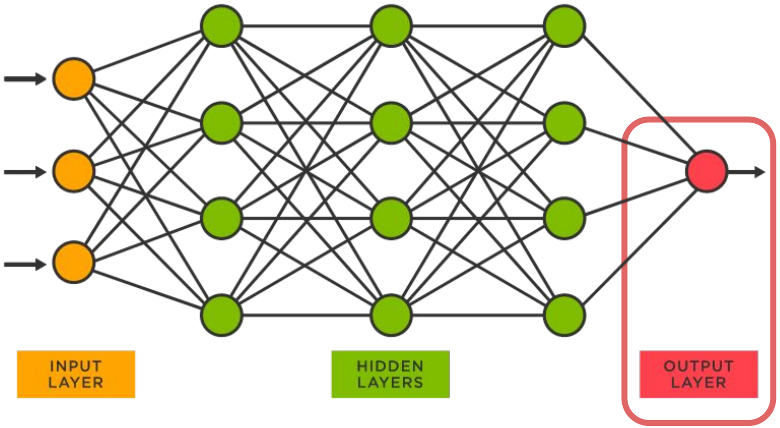

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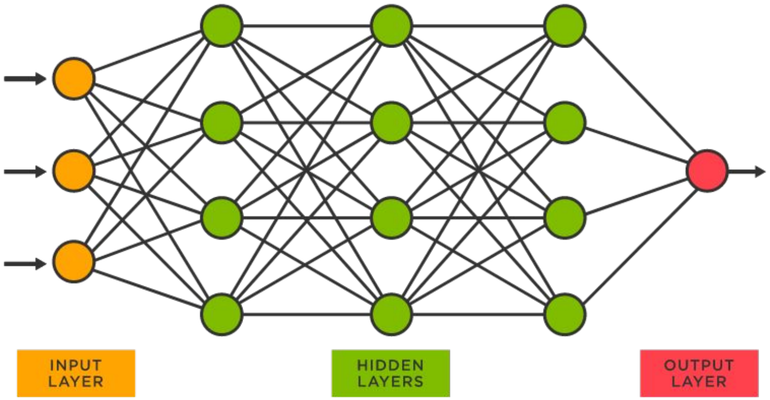


$$\mathrm{softmax}(W^T(\mathrm{act}(W^T(x))))$$

# Activation Functions

- Sigmoid: $\sigma(x) = \dfrac{1}{1 + e^{-x}}$
- Rectified Linear Unit (ReLU): $\mathrm{ReLU}(x) = \max(x, 0)$
- Tanh: $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$
- Softmax: $\mathrm{softmax}(x_i) = \frac{e^{x_i}}{\sum_{j=1}^{d} e^{x_j}}$