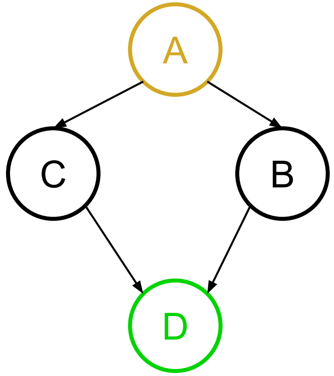
Exam Prep Discussion 9

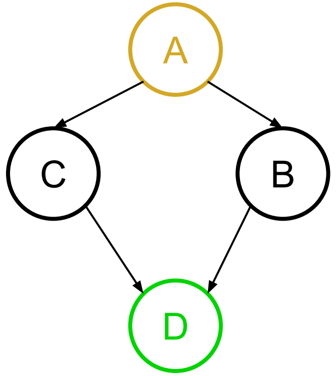
Sampling



- Instead of calculating probabilities directly, we can calculate them by sampling
- Want to estimate the true probability of our query: $P(A = a_q | D = d_q) := P(A = +a | D = +d)$
- Samples are of the form: (a, b, c, d)

Sampling

- Want to estimate the true probability of our query: $P(A = a_q | D = d_q) := P(A = +a | D = +d)$
- Samples are of the form: (a, b, c, d)



PRIOR SAMPLING

1) Sample parents
2) Sample each child with conditional probability as seen in CPT
3) True probability = $\frac{\text{\# of samples matching query}}{\text{total \# of samples}} = \frac{\text{\# of samples with } a = a_q, d = d_q}{\text{total \# of samples}}$

$\boxed{(+a, +b, +c, +d)}$

$(+a, +b, +c, -d)$

$\boxed{(+a, -b, -c, +d)}$

$(-a, +b, -c, +d)$

½

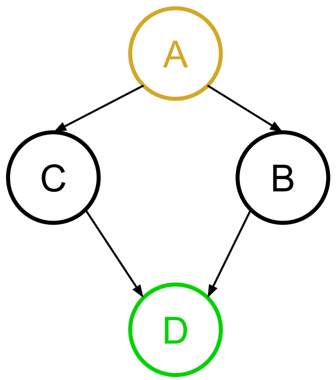
REJECTION SAMPLING

1) Sample parents

2) Sample each child with conditional probability as seen in CPT

3) Reject/throw out all samples that don’t follow evidence

4) True probability = $\dfrac{\text{\# of samples matching } a = a_q, d = d_q}{\text{\# of samples matching } d = d_q}$



$(+a, +b, +c, +d)$

$(+a, +b, +c, -d)$

$(+a, -b, -c, +d)$

$(-a, +b, -c, +d)$

2⁄3

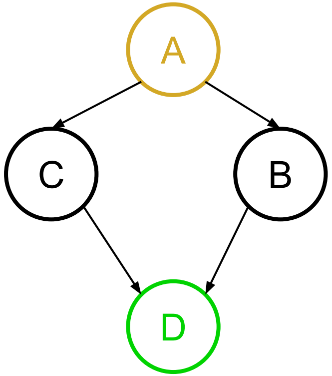
LIKELIHOOD WEIGHTING

1) Fix evidence variables

2) Sample all other variables according to CPTs

3) Calculate weights of samples: $\prod_{i=1}^{m} P(X_i|Parents(X_i))$

4) True probability = $\dfrac{\text{total weight of samples matching } a = a_q}{\text{total weight of samples}}$

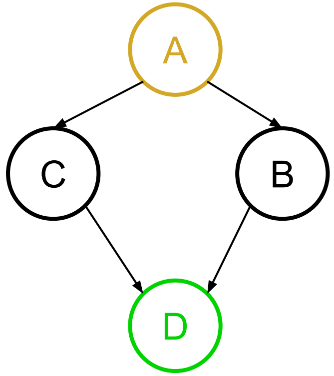


$\boxed{(+a, +b, +c, +d)}$

$\boxed{(+a, -b, -c, +d)}$

$(-a, +b, -c, +d)$

GIBBS SAMPLING



1) Fix evidence variables

2) Initialize other variables randomly (no CPTs)

3) Repeatedly sample a NON-evidence variable X to resample, given other variables and CPTs

4) Probability of X = $P(X|MarkovBlanket(X)) = \frac{\prod(\text{CPT entries with } X = x)}{\sum_z \prod(\text{CPT entries with } X = z)}$

5) With enough samples, the above probability for each variable will converge to the correct distribution

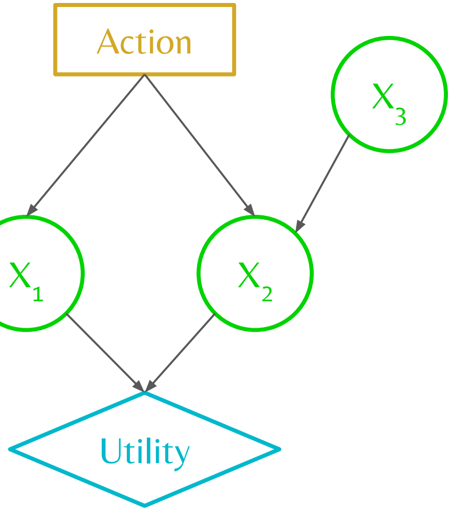
$(+a, +b, +c, +d)$

$(+a, +b, +c, -d)$

$(+a, -b, -c, +d)$

$(-a, +b, -c, +d)$

Decision Networks



- Used to model utilities, actions, and probabilities of different choices
- Expected utility of an action A = a (given evidence E = e):

$$EU(A = a|E = e) = \sum_r Utility(r) \cdot P(Result(a) = r|E = e)$$

- Maximum expected utility WITHOUT evidence:

$$MEU(\emptyset) = \max_a EU(A = a) = \max_a \sum_r Utility(r) \cdot P(Result(a) = r)$$

- Maximum expected utility WITH evidence:

$$MEU(E = e) = \max_a EU(A = a|E = e) = \max_a \sum_r Utility(r) \cdot P(Result(a) = r|E = e)$$

- Value of perfect information knowing evidence E:

$$VPI(E = e) = MEU(E = e) - MEU(\emptyset)$$

- Value of perfect information knowing additional E’, given we know E:

$$VPI(E' = e'|E = e) = MEU(E' = e', E = e) - MEU(E = e)$$