CS188 Discussion 8:

Bayes Nets: Exact Inference

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Bayes Nets

- Syntax
  - Directed acyclic graph
  - One node per variable
  - A conditional probability table for each node given its parents
- Independence
  - Given its parents, each node is conditionally independent of its ancestors, aka $P(X_T|X_1 \ldots X_{T-1}) = P(X_T|\text{Parents}(X_T))$
  - $P(X_1, \ldots, X_N) = P(X_1)P(X_2|X_1) \ldots P(X_N|X_1 \ldots X_{N-1})$

    $= \prod_i P(X_i|\text{Parents}(X_i))$

Inference by Enumeration

- Want P(Q|e)
  - Query variables: Q
  - Evidence variables: e
  - Hidden variables: H
- Steps
  - Select entries consistent with evidence where E=e
  - Sum H out: $P(Q, e) = \Sigma_h P(Q,h,e)$
  - Normalize: $P(Q|e) = P(Q, e)/P(e)$
- Con: exponentially many products

		Temperature		
		hot	cold	
Weather	sun	0.45	0.15	0.60
	rain	0.02	0.08	0.10
	fog	0.03	0.27	0.30
	meteor	0.00	0.00	0.00
		0.50	0.50	

P(*W*)

P(*T*)

Variable Elimination

- Want P(Q|e)
- $P(B \mid j, m) = \alpha \sum_{e,a} P(B)\, P(e)\, P(a|B,e)\, P(j|a)\, P(m|a)$ ← Inference by enumeration

  $= \alpha\, P(B) \sum_{e} P(e) \sum_{a} P(a|B,e)\, P(j|a)\, P(m|a)$ ← Variable elimination

- Steps
  - While there are still hidden variables:
    - Pick a hidden variable h
    - P = the product of all factors including h
    - Eliminate (sum out) h from P
  - Join remaining variables (using pointwise product) and normalize

Types of factors

Factors are like the intermediary CPTs

For example, A and J are binary variables

- $\Sigma_{a,j}$ **P(A=a,J=j)**

  = P(a,j) + P(¬a,j) + P(a,¬j) + P(¬a,¬j) = 1
- Σ_{a} **P(A=a, J=j)** = P(a,j) + P(¬a,j) = P(j)
- Σ_{a} **P(A=a|J=j)** = P(¬a|j) + P(a|j) = 1

  ^show by expanding using conditional probability rule

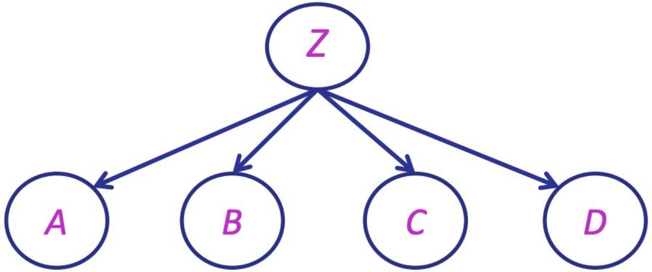
P(A,J)

A \ J	true	false
true	0.09	0.01
false	0.045	0.855

Order of Factors

- Ex. **Query:** P(D), **hidden**: Z, A, B, C
- ✅ Yes: C, B, A, Z ✅

$$P(D) = \Sigma_{z,a,b,c} P(z)P(a|z)P(b|z)P(c|z)P(D|z)$$
$$= \Sigma_z P(z)\Sigma_a P(a|z)\Sigma_b P(b|z)\Sigma_c P(c|z)P(D|z)$$
$$= \Sigma_z P(D, z)$$



- 🚫 No: Z, C, B, A 🚫

$$P(D) = \Sigma_{a,b,c,z} P(z)P(a|z)P(b|z)P(c|z)P(D|z)$$
$$= \Sigma_a \Sigma_b \Sigma_c \Sigma_z P(z)P(a|z)P(b|z)P(c|z)P(D|z)$$
$$= \Sigma_a \Sigma_b \Sigma_c P(a,b,c,D)$$