Trees

Topical Review Session 05

CS 61B // Spring 2021

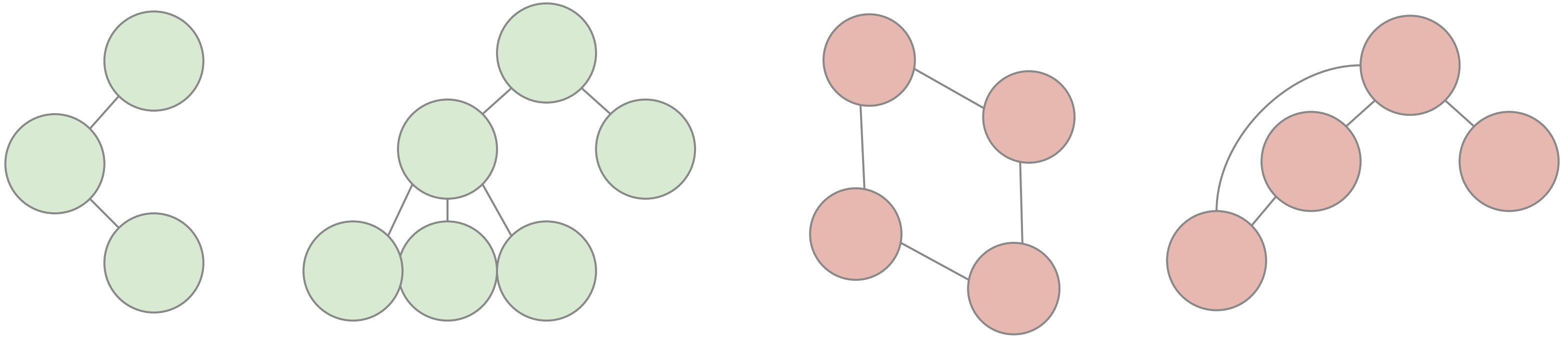
Agenda

Trees

Introduction to Trees

Introduction to Trees

- A **tree** is a special kind of graph with some extra constraints
- All the following rules apply:
    - A tree has N nodes and N-1 edges
    - A tree is a fully connected, acyclic graph
    - If you delete any edge in a tree, it becomes disconnected
    - There is exactly 1 path connecting any two nodes

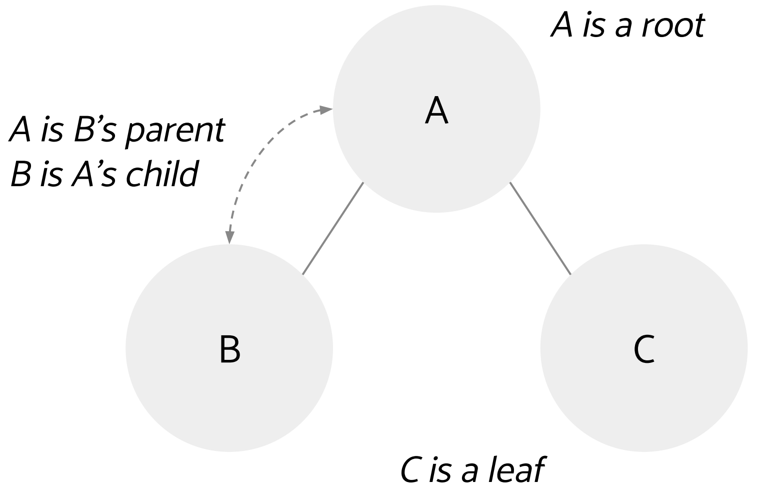


Introduction to Trees

A parent node points towards its child.

A root node is one that has no parent.

A leaf node is one that has no child.



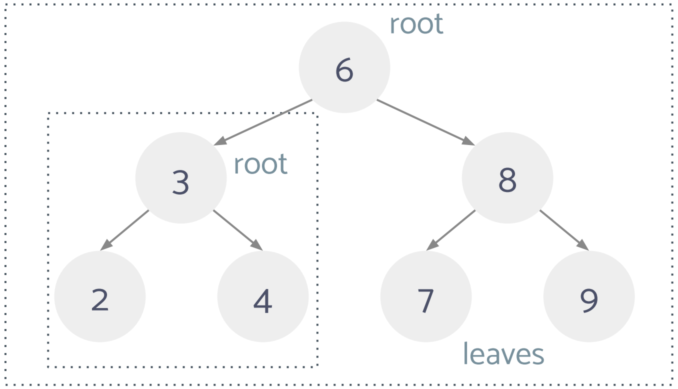
Binary Search Trees

Binary Search Trees

A **Binary Search Tree** (BST) is a data structure that allows us to easily maintain comparable elements.

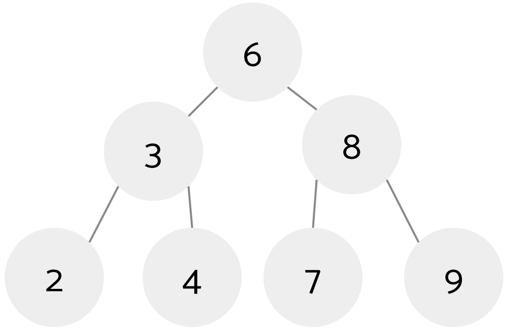
Key Properties:

1. Every node is the root of a smaller sub tree with two children
2. Every node's left child has a value lesser than the parent node `n.left < n`
3. Every node's right child has a value greater than the parent node `n.right > n`



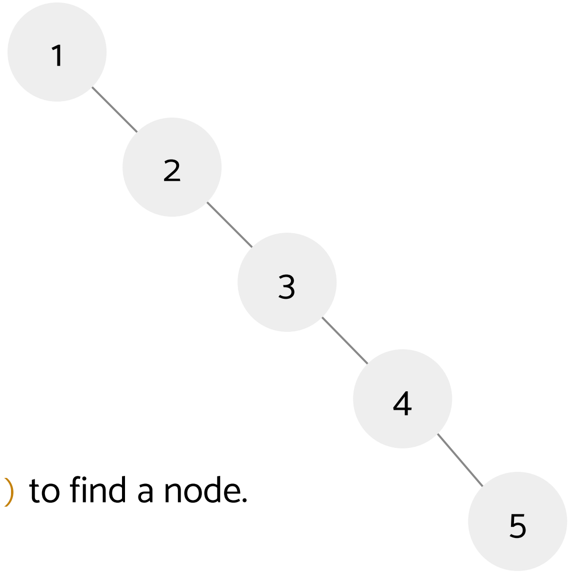
Binary Search Trees

Bushy BSTs are extremely fast because every node has two children.



`Θ(log N)` to find a node.

Spindly BSTs are extremely slow because every node has one child. Think linked list.

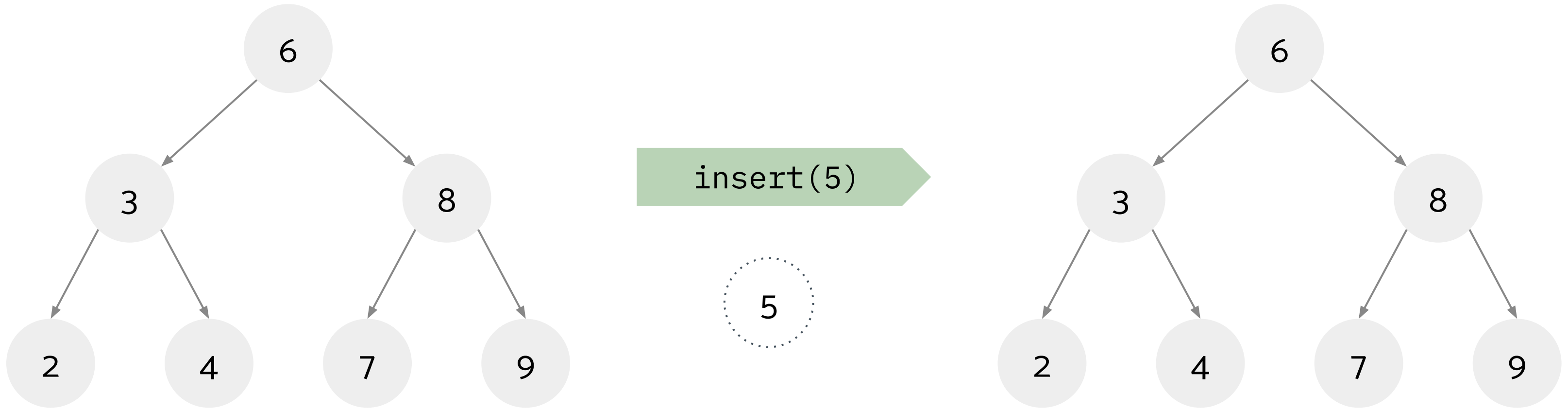


`Θ(N)` to find a node.

In general, the time complexity to find a node is `O(height)`

BST Insertion

New elements are always inserted as leaves.



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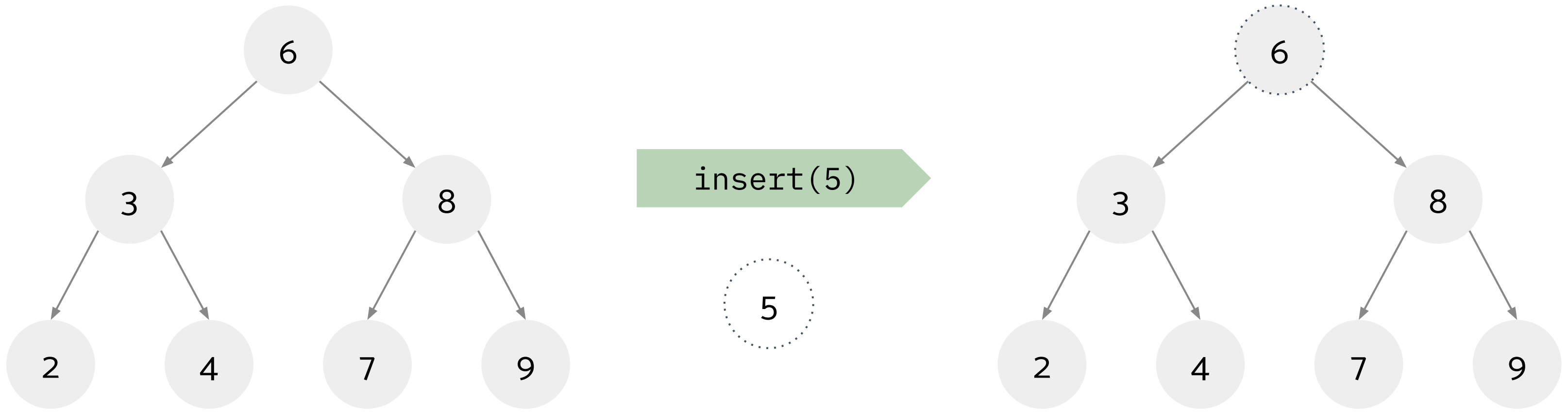

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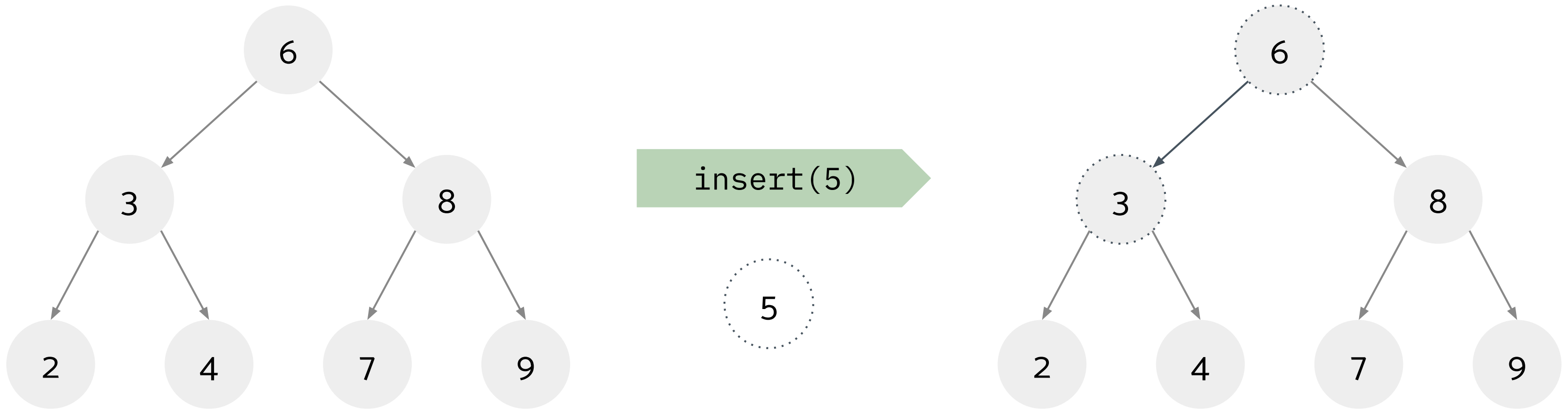

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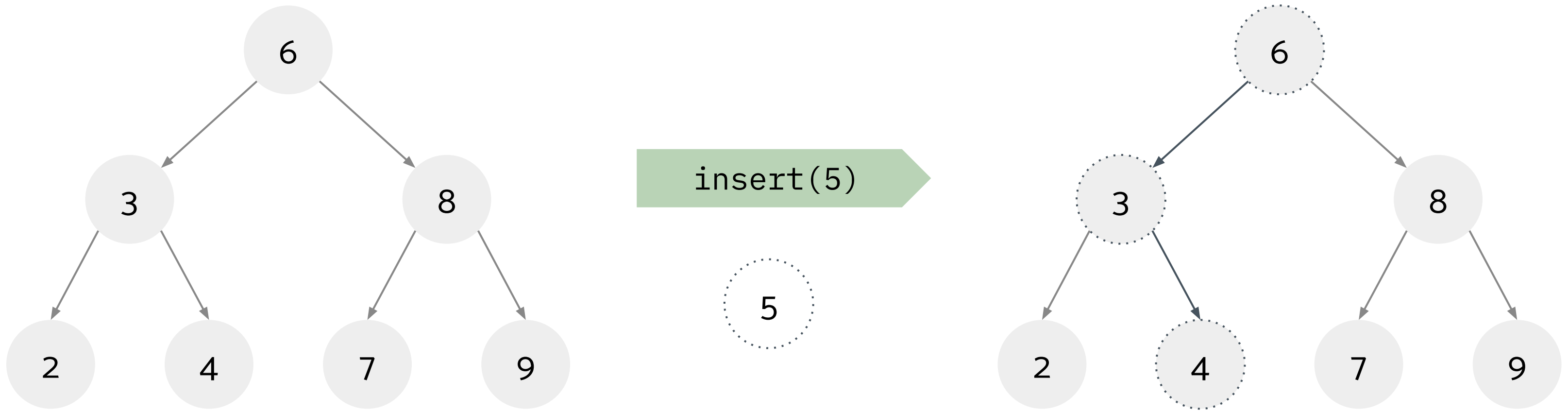
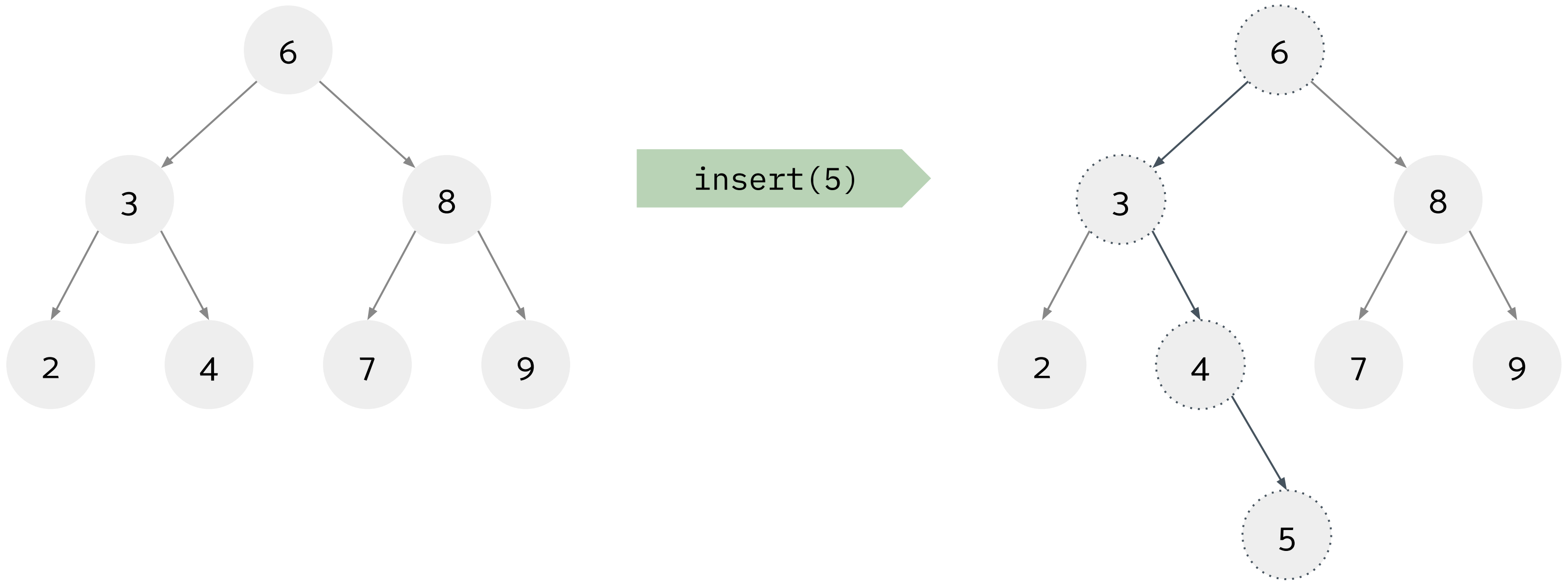


BST Insertion

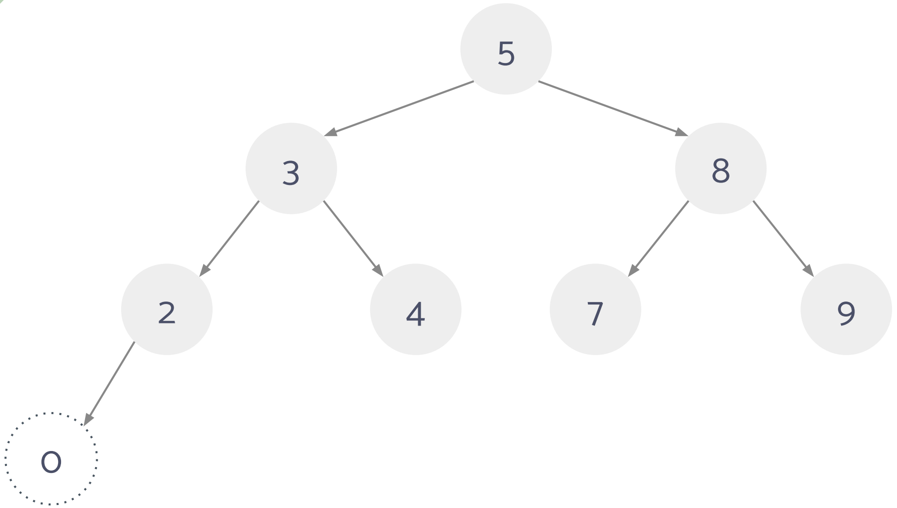
New elements are always inserted as leaves. This takes `O(log N)` time.



BST Deletion: Zero Children 🧍‍♂️

Deleting with no children is the simplest case - the node just disappears.

`delete(0)`



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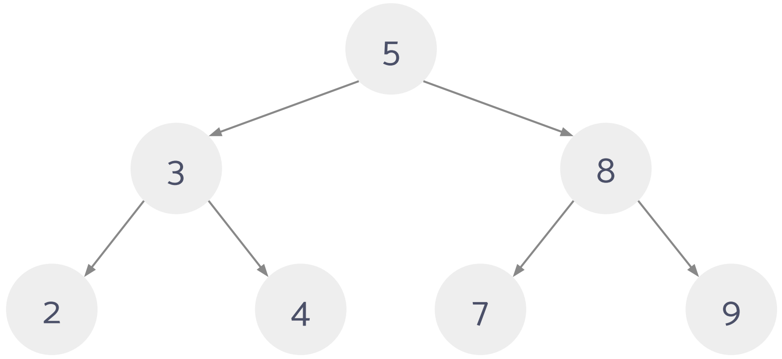
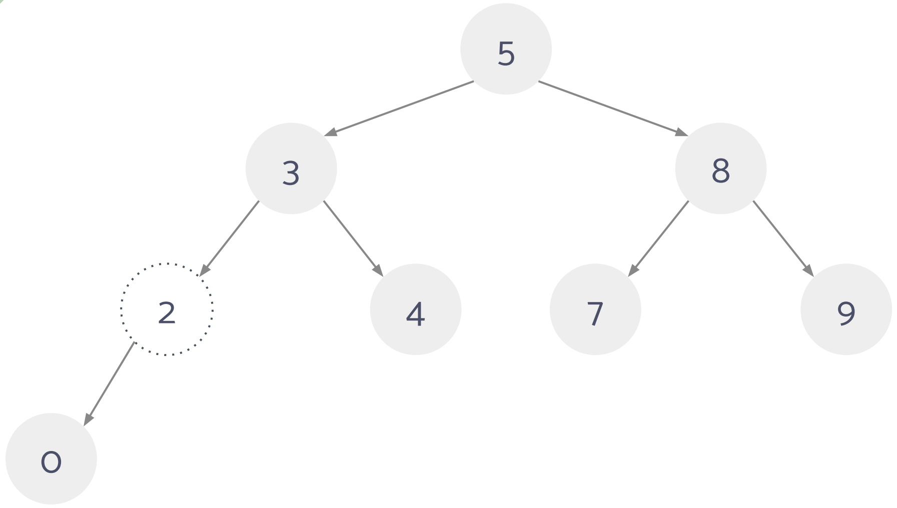


BST Deletion: One Child 👩‍👦

The child can replace the parent that gets deleted.

`delete(2)`



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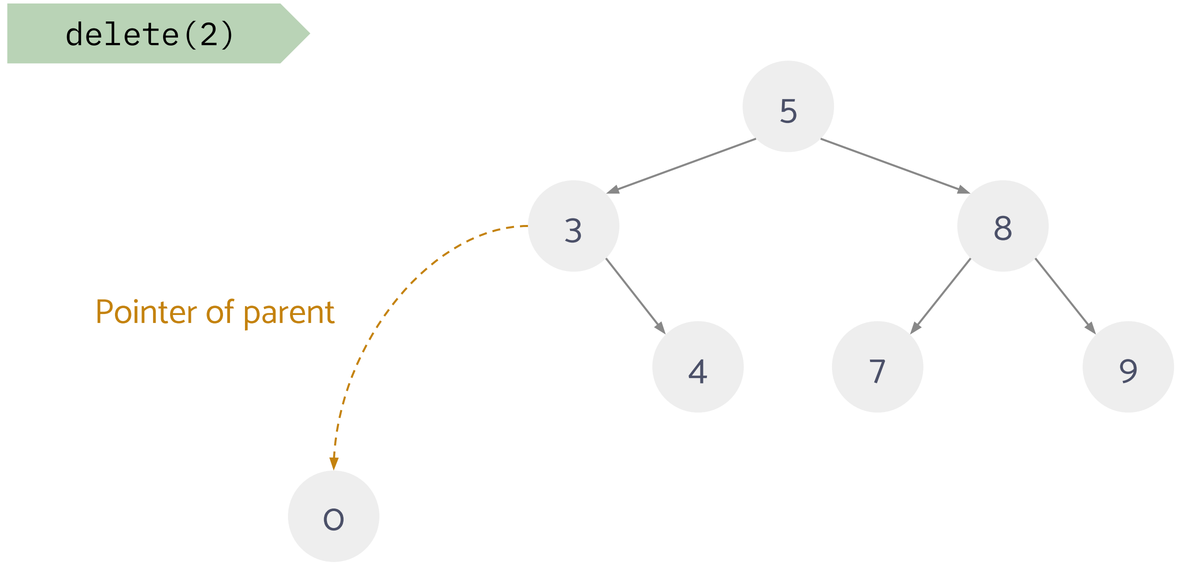

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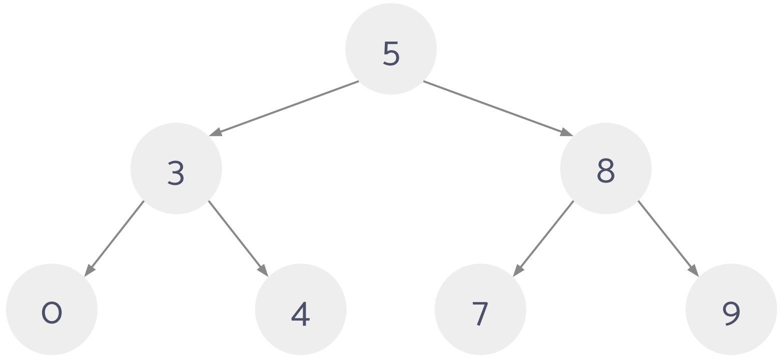
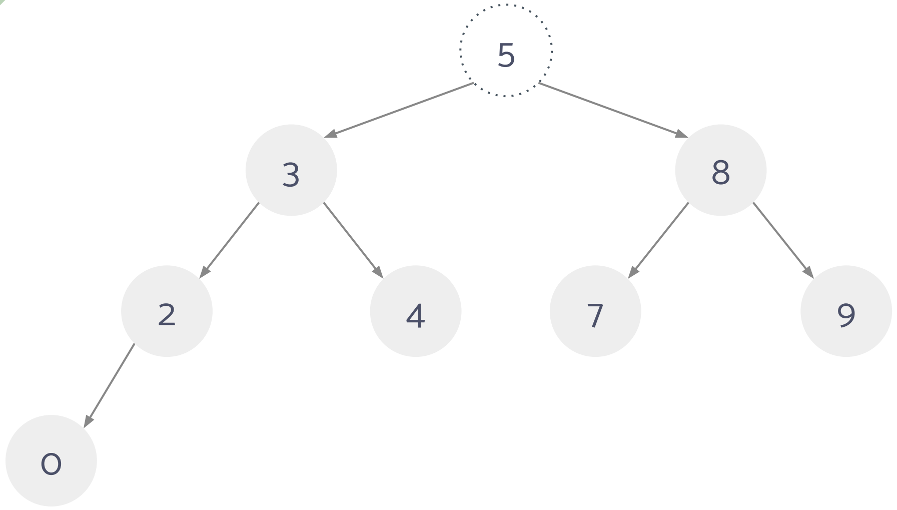


BST Deletion: Two Children 👨‍👧‍👦

We replace the node with either the largest descendent on the left or the smallest descendent on the right.

`delete(5)`



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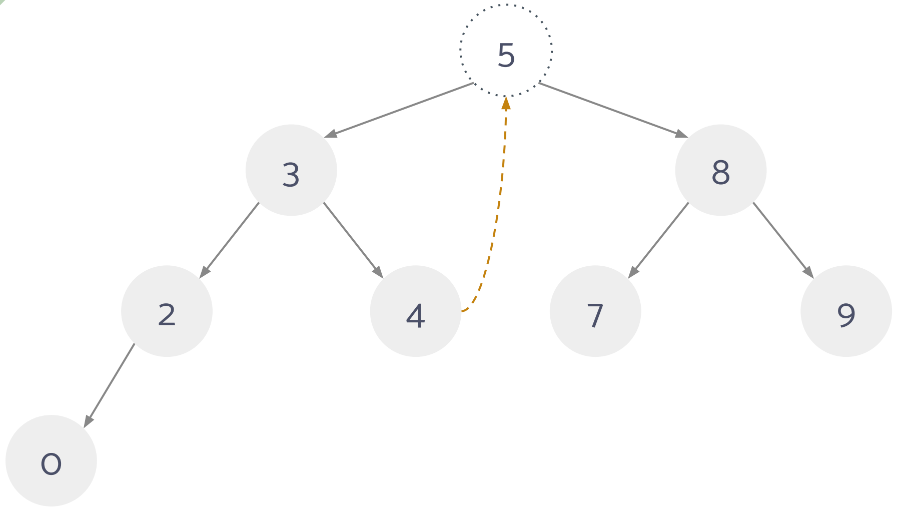

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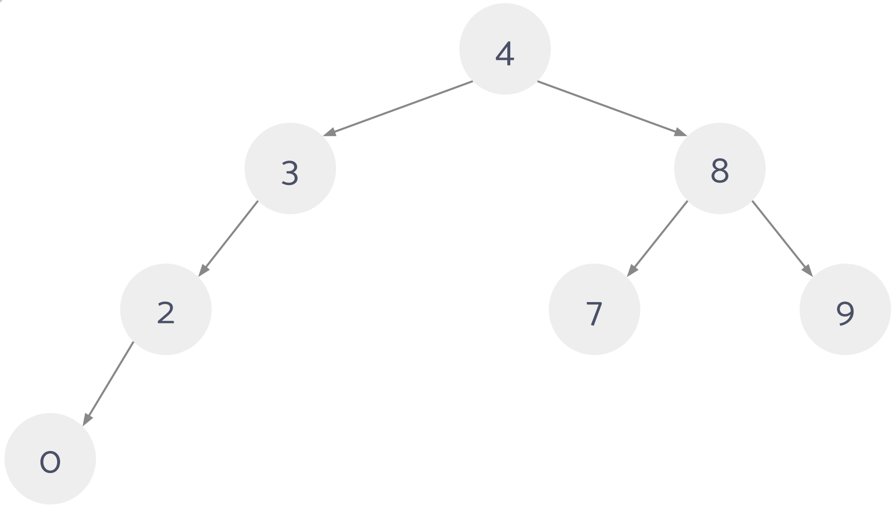

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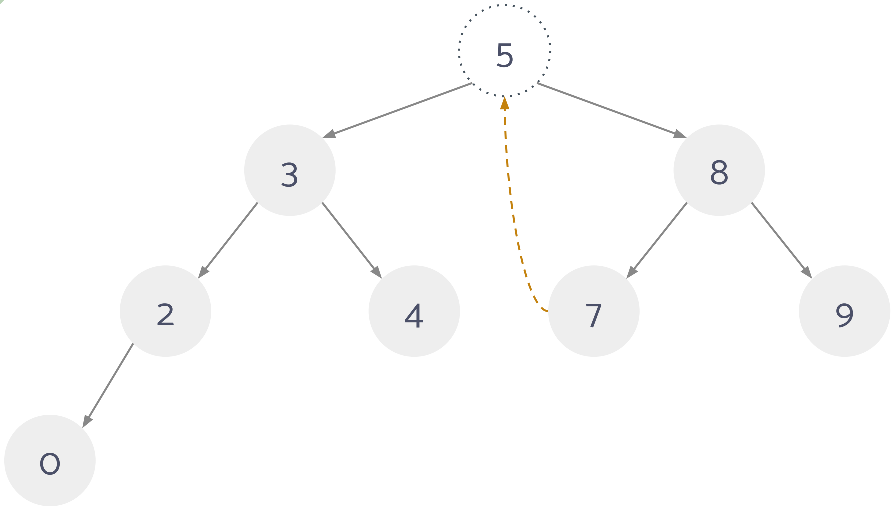

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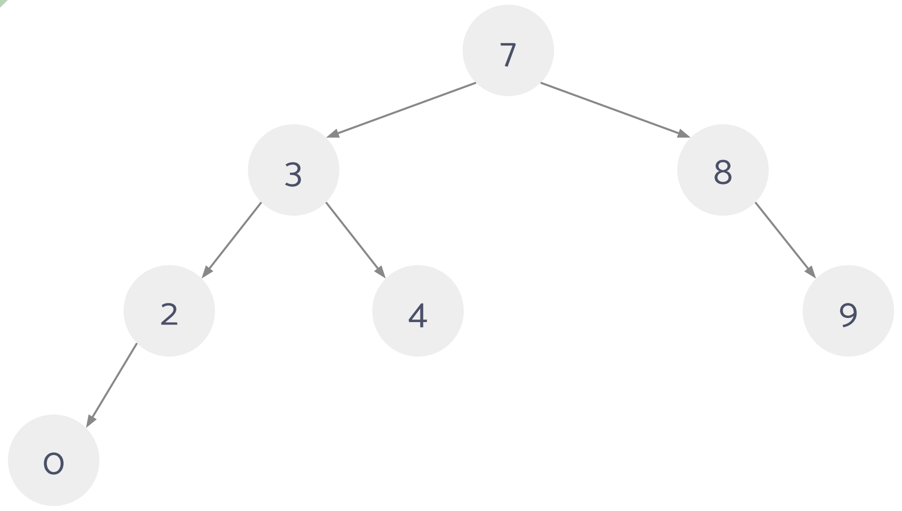


Asymptotics of BST

Bushy BST - Best Case

Search: `Θ(log N)`

Insertion: `Θ(log N)`

Deletion: `Θ(log N)`

☺️

Spindly BST - Worst Case

Search: `Θ(N)`

Insertion: `Θ(N)`

Deletion: `Θ(N)`

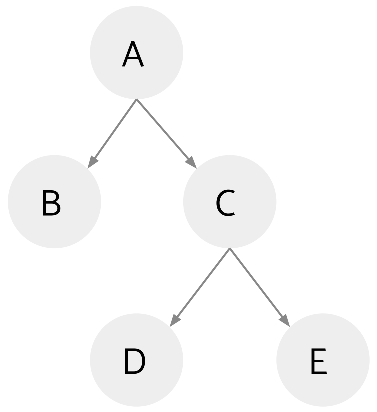
☹️

Tree Traversals

Breadth First Search

Breadth first search means we visit the nodes of a tree level by level, also known as level order traversals.

BFS is usually done using a queue.



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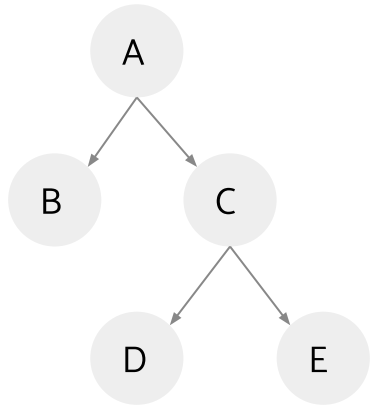


A B C D E

Depth First Search

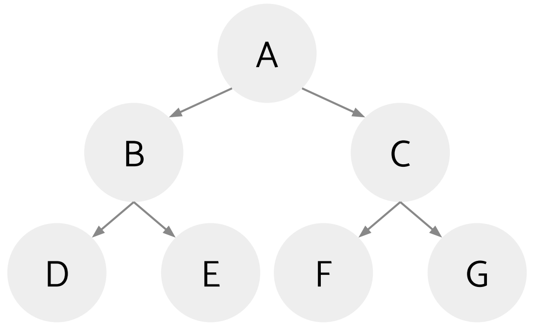
Three types of DFS: **Preorder**, **Inorder**, **Postorder**. Overall, we traverse deeper nodes before nodes that are closer to the root. Usually, we can implement this recursively or iteratively.

DFS is usually done iteratively using a stack.

Depth First Search: Preorder

In **preorder traversal**, we visit a node, then traverse its children.

```
preOrder(BSTNode x) {
    if (x == null)
        return;
    print(x.key)
    preOrder(x.left)
    preOrder(x.right)
}
```

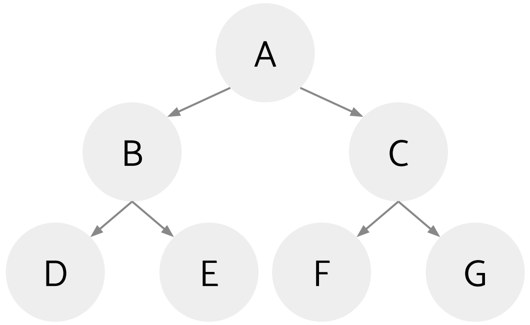


A B D E C F G

Depth First Search: Inorder

In **inorder**, we traverse left child, visit, then traverse right child.

```
inOrder(BSTNode x) {
    if (x == null)
       return;
    inOrder(x.left)
    print(x.key)
    inOrder(x.right)
}
```

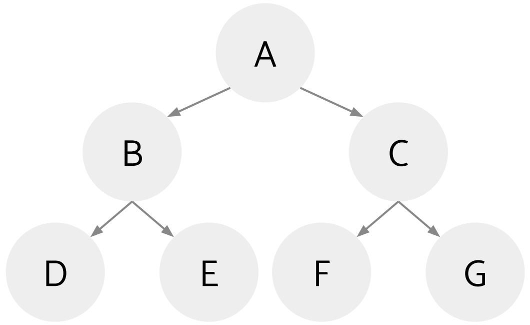


D B E A F C G

Depth First Search: Postorder

In **postorder**, we traverse left, traverse right, then visit

```
postOrder(BSTNode x) {
    if (x == null)
       return;
    postOrder(x.left)
    postOrder(x.right)
    print(x.key)
}
```

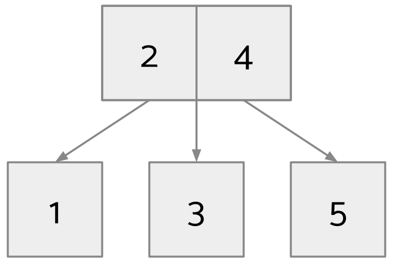


D E B F G C A

B-Trees

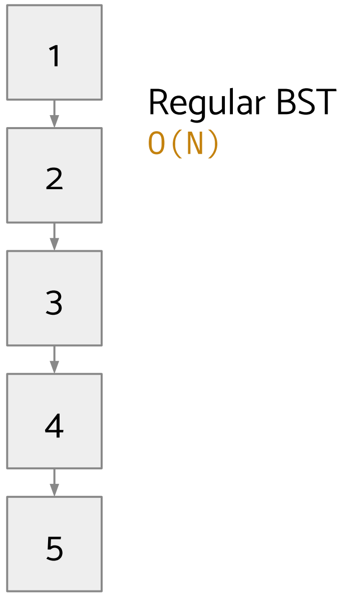
2-3 Trees

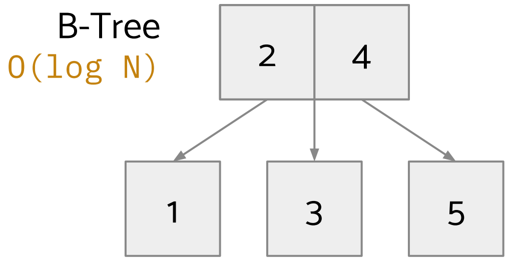
- **2-3 Trees** are a modified representation of BSTs, so they still follow the left child smaller than parent, right child larger than parent rule.
- A node may contain up to 2 values
- Non-leaf nodes can have 2 or 3 children



Why B-Trees?

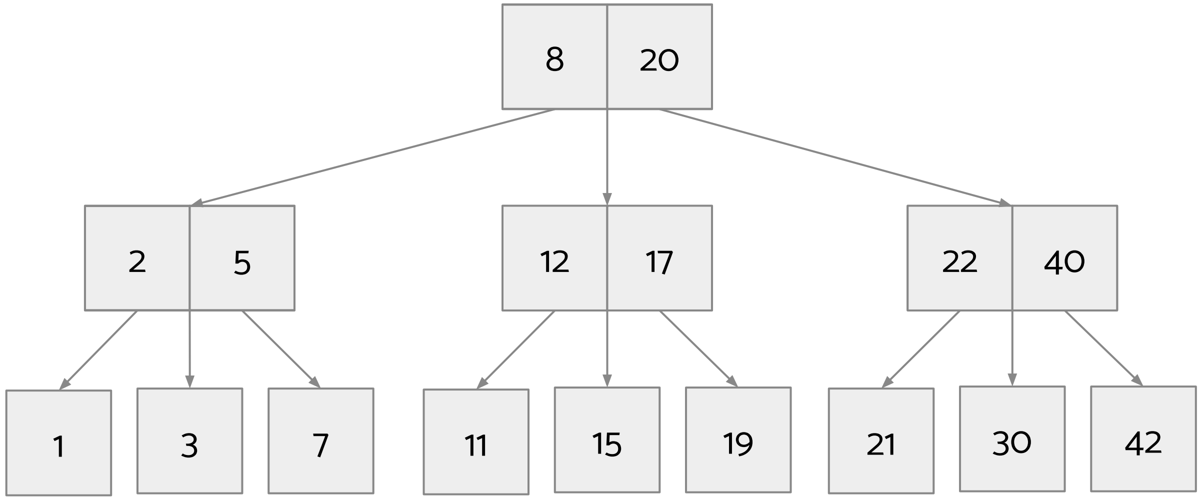
Let’s say we insert [1, 2, 3, 4, 5] in order.





B-Tree Insertion

`insert(25)`



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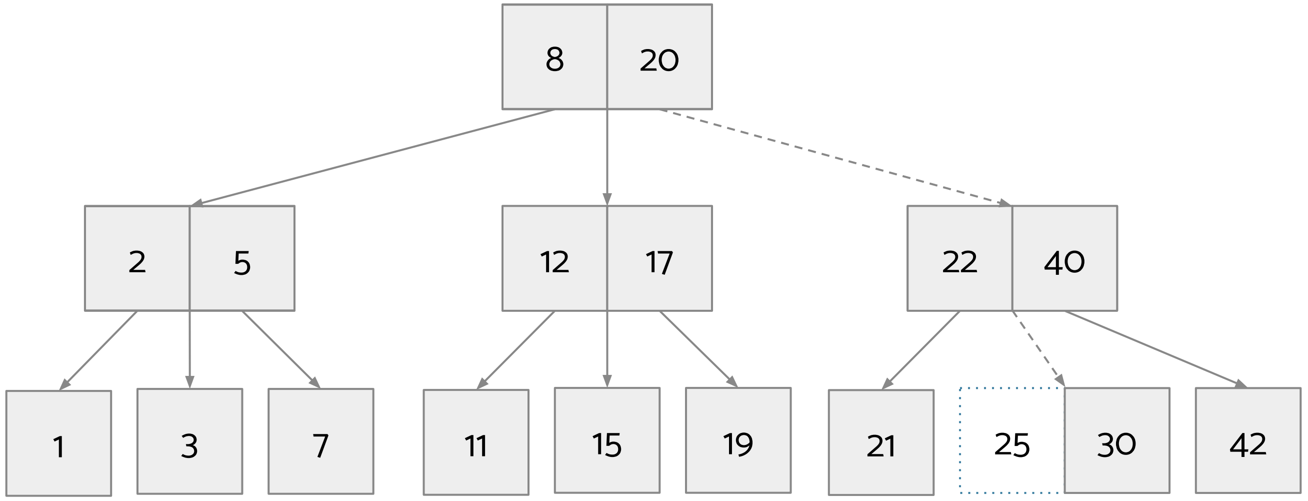
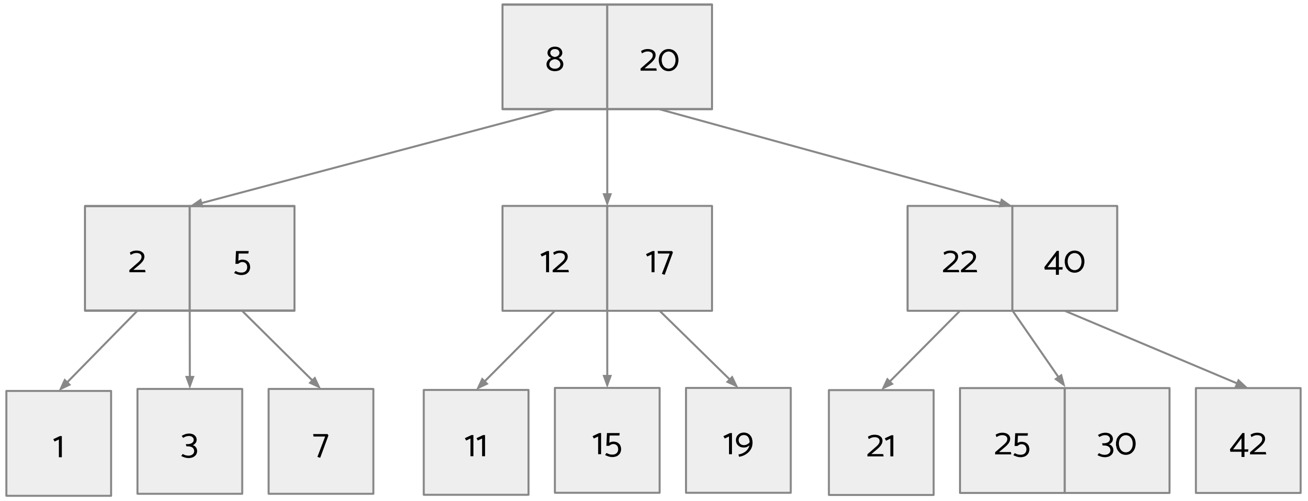


`25` is smaller than `30`, but larger than `22`, so it would be inserted in the leaf node to the left of `30`.
Because the node only has two elements, no additional steps are required.

B-Tree Insertion

`insert(27)`



B-Tree Insertion

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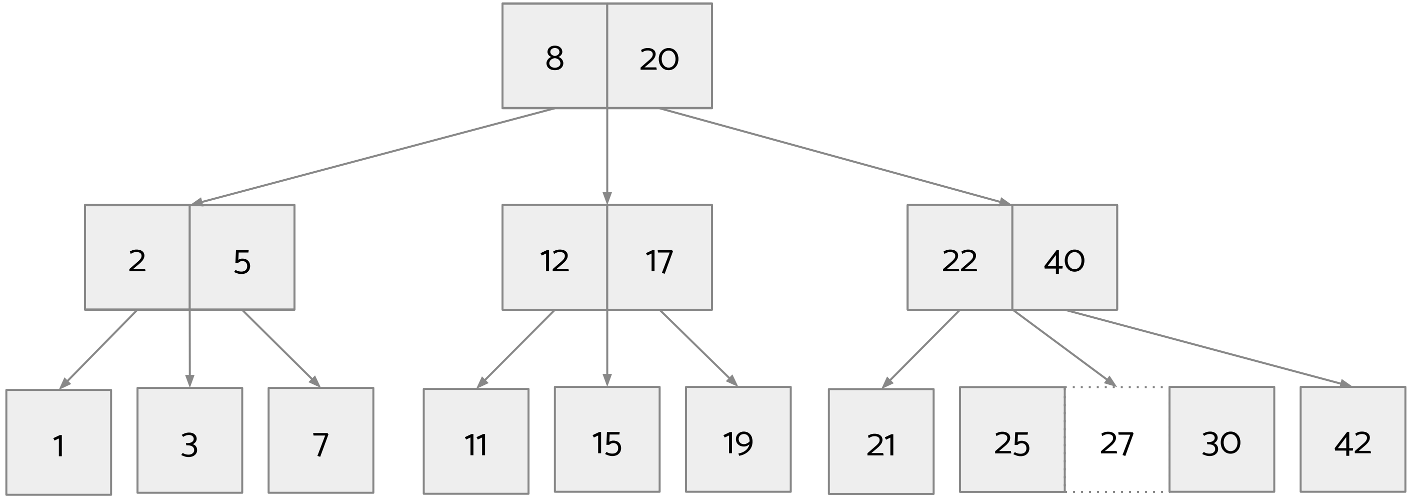
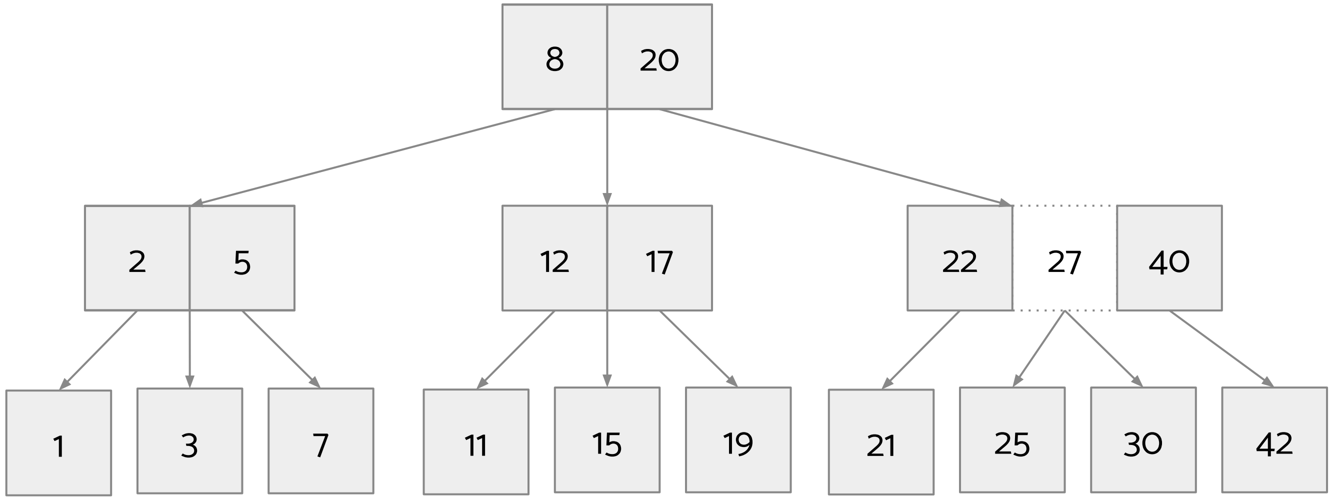


`27` goes in between `25` and `30`, but now we have a node with three elements, so we have to push the middle element (27) up.

B-Tree Insertion

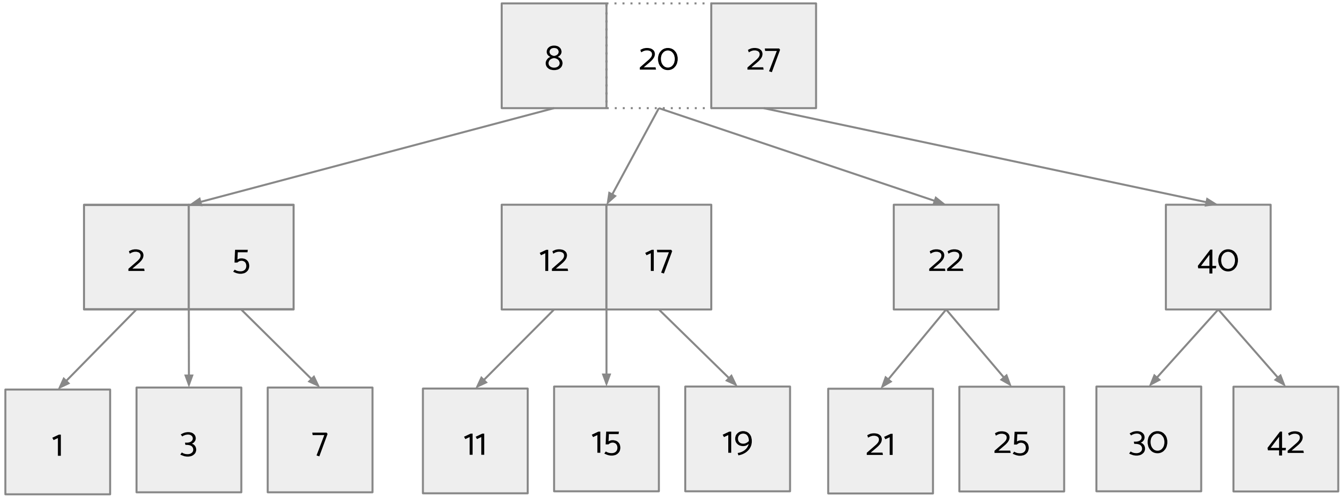
`insert(27)`



Now the parent node of `22`, `27`, and `40` also has three elements, so we push the middle element (`27`) up again. Note how the `25` and the `30` were split up to maintain the correct relationship between the parent node and its left/right children.

B-Tree Insertion

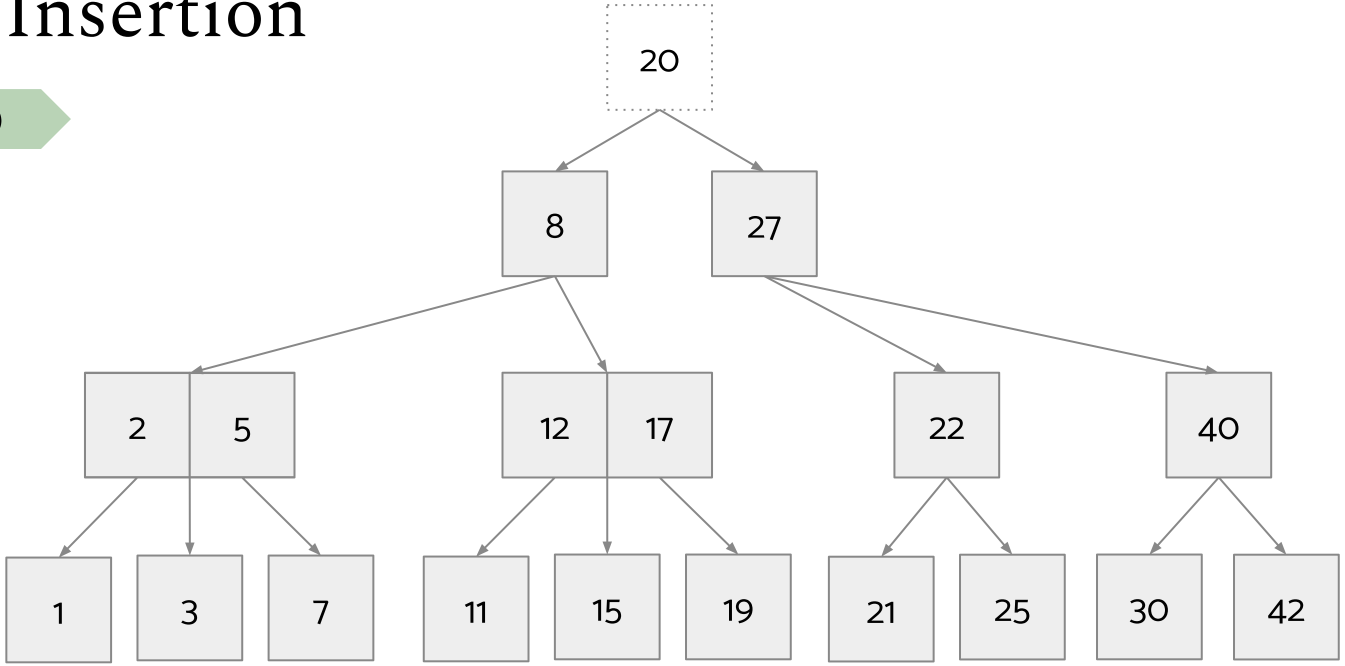
`insert(27)`



Now the root with `8`, `20`, and `27` has three elements, so we repeat the pushing process one more time.

B-Tree Insertion

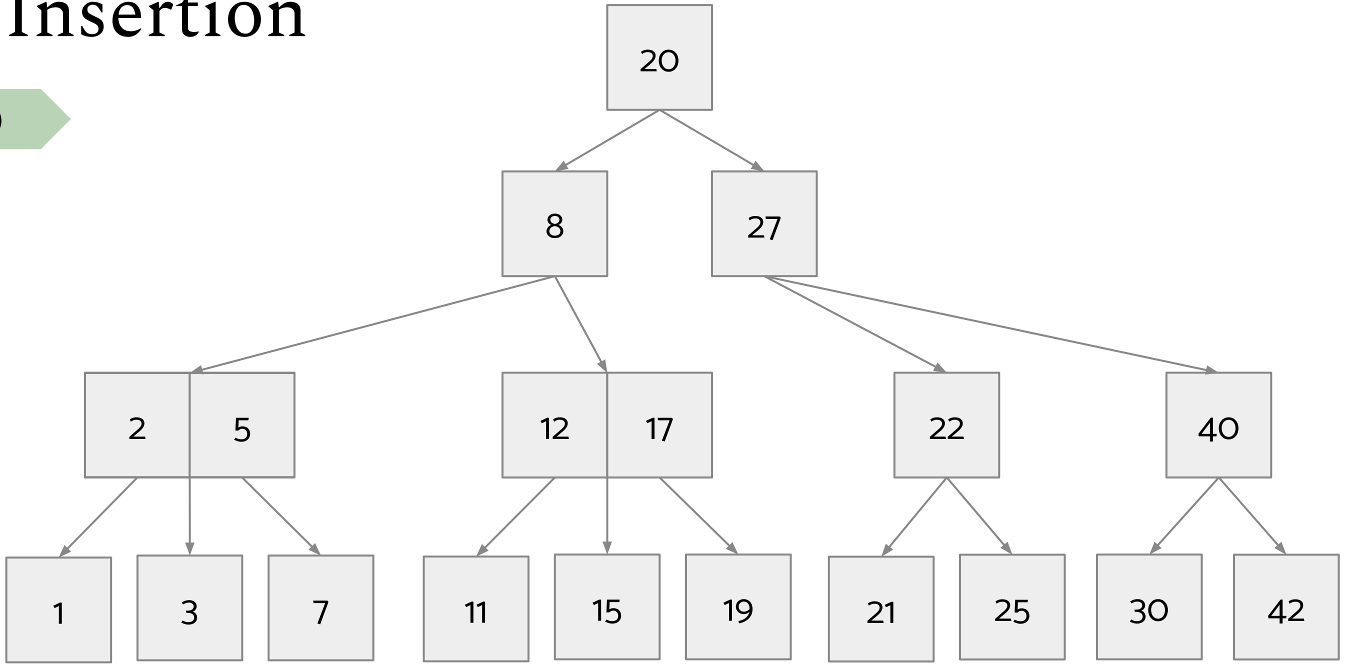
`insert(27)`



And we're done! The tree is still balanced :)

B-Tree Insertion

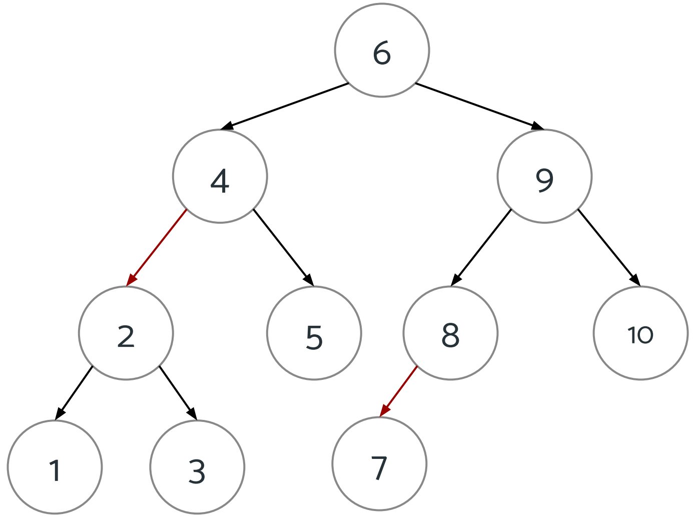
`insert(27)`



Red Black Trees

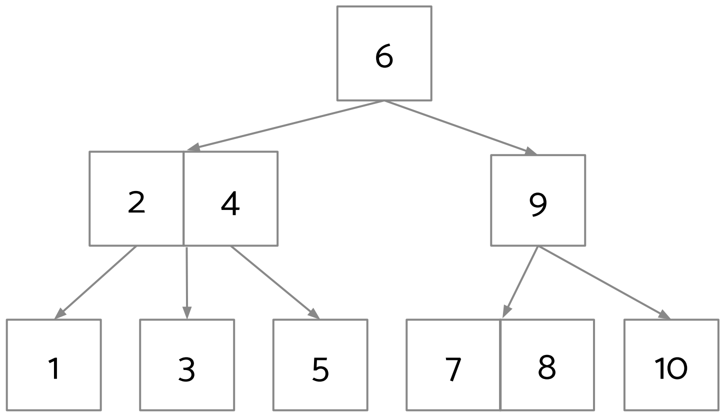
Red Black Trees

- **Red Black Trees** are BSTs that retain the properties from B-Trees
- **Black Edges** are the equivalent of a normal edge in the corresponding 2-3 tree
- **Red Edges** are the equivalent of the implicit connections between values in the same node in the corresponding 2-3 tree
- The version we use in this class is called a **Left Leaning Red Black Trees** because our red edges only point to the left.

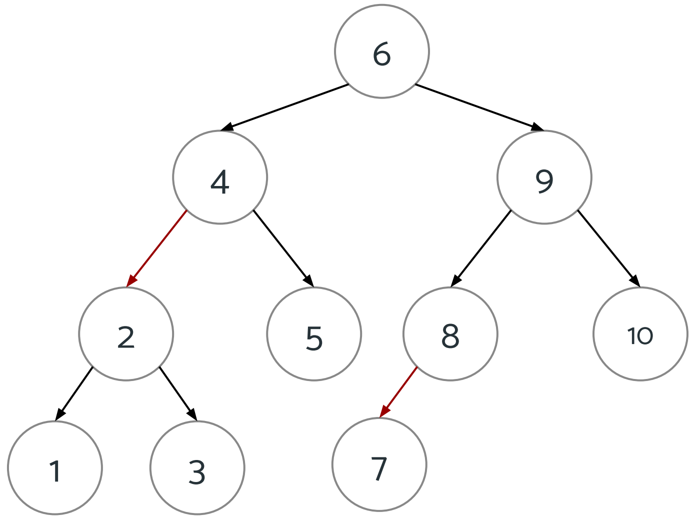


Converting Between Trees

2-3 Tree

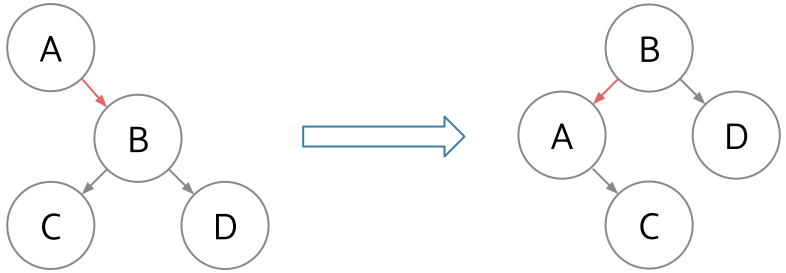


Red Black Tree

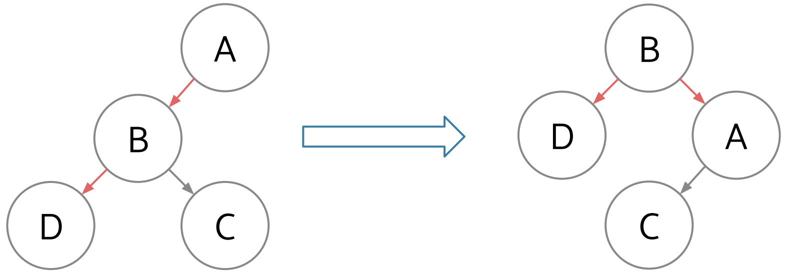


LLRB Balancing Rotations

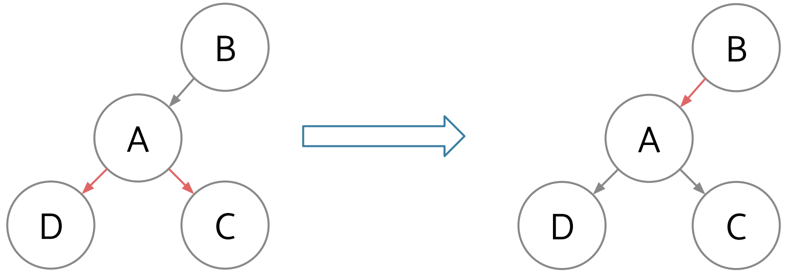
```
rotateLeft(A);
```



```
rotateRight(A);
```

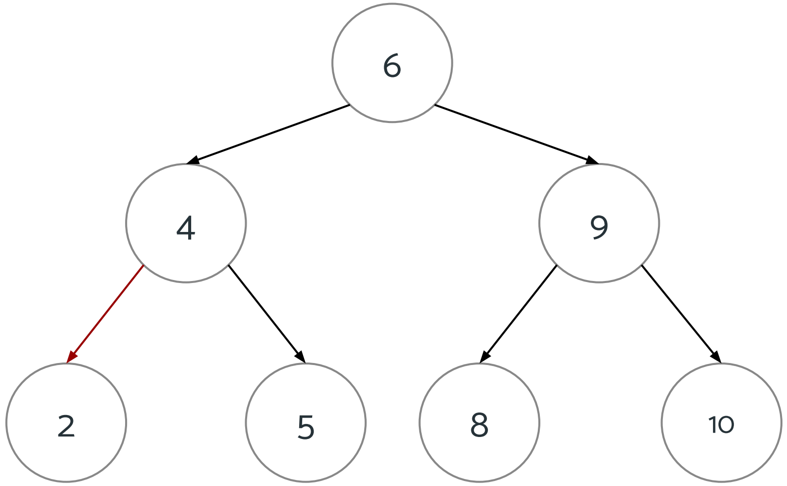


```
colorFlip(A);
```



LLRB Insertion

Nodes are always added as red connections.



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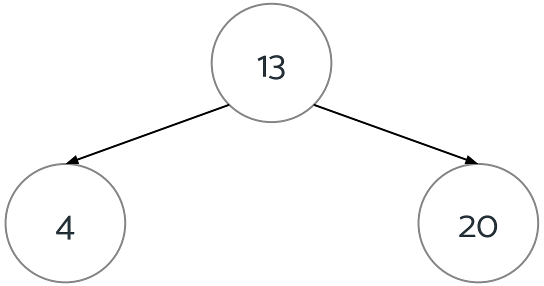

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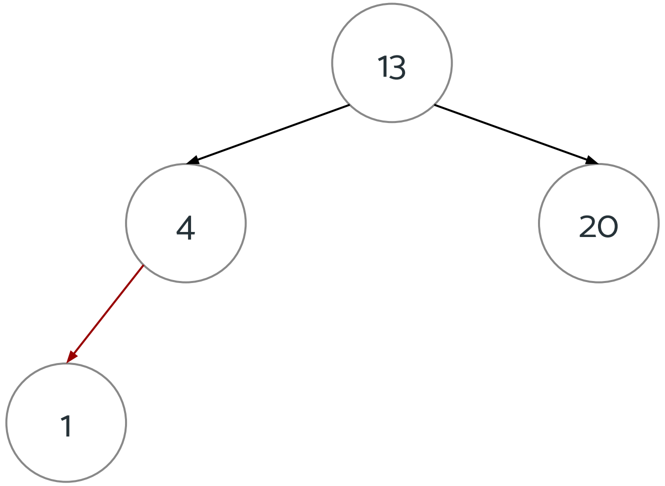
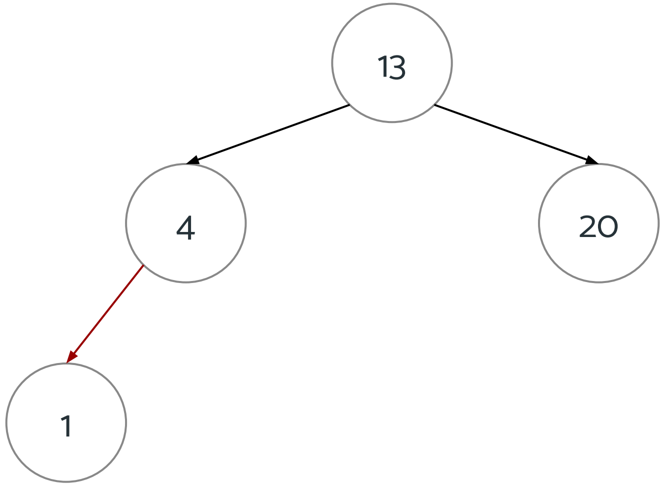


LLRB Insertion

If nodes are added to the right, the tree must be rotated until it is left leaning again.

`insert(23)`



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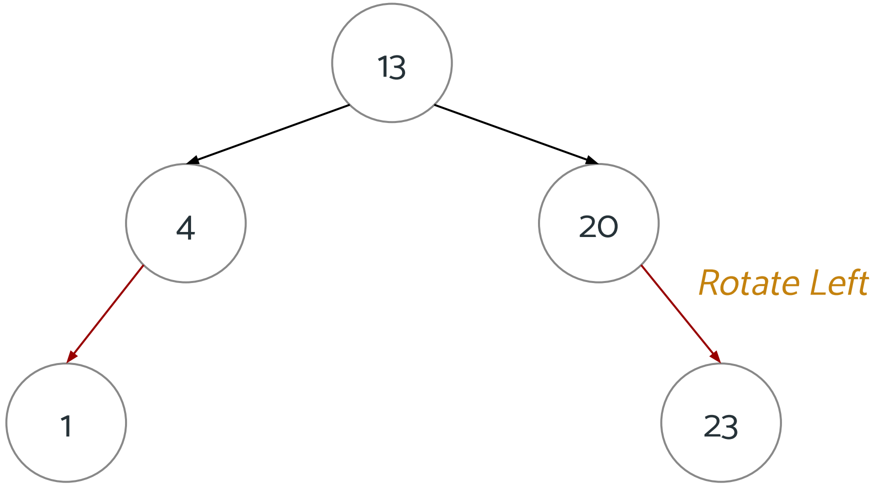

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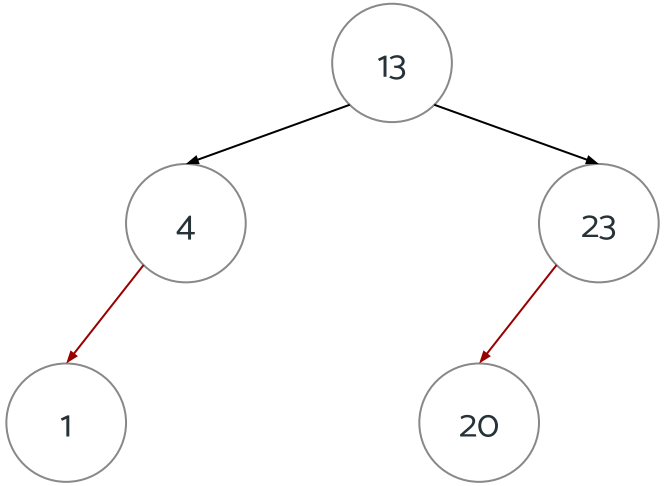
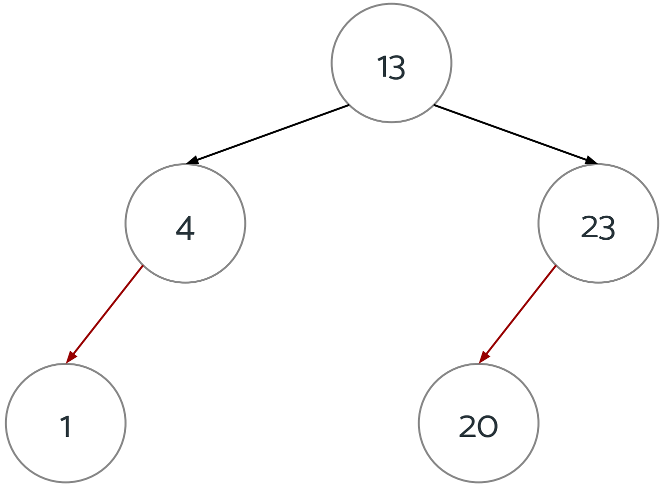


LLRB Insertion

If both the left and right branches are red, we can perform a color flip.

`insert(9)`



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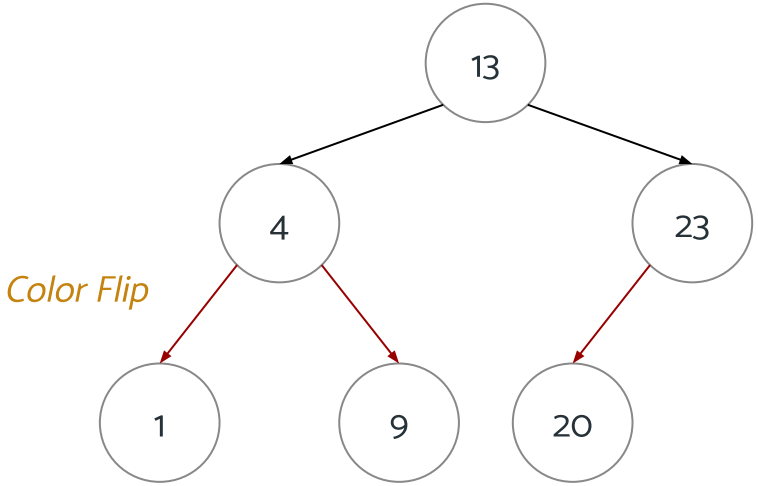

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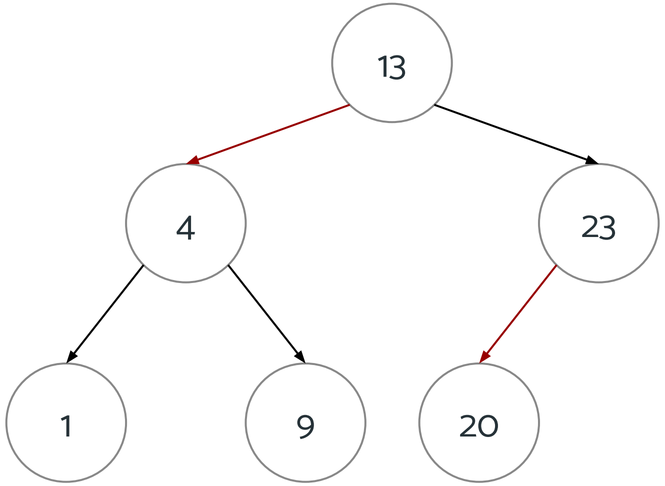
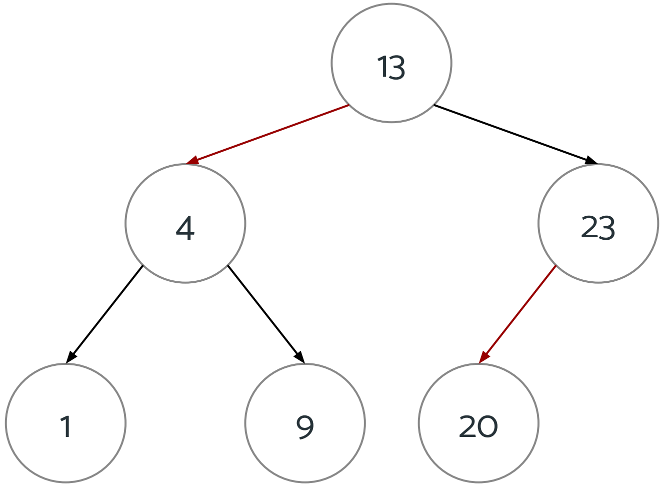


LLRB Insertion

Sometimes you will need to do multiple operations.

`insert(17)`



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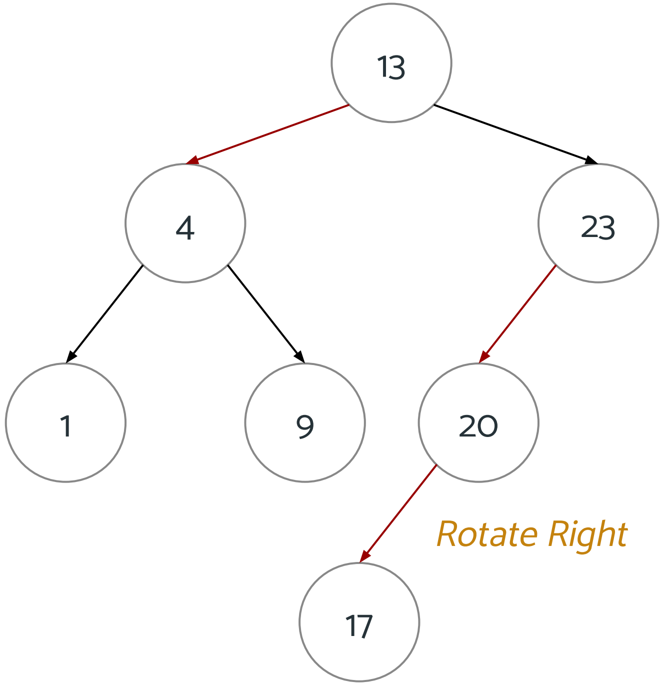

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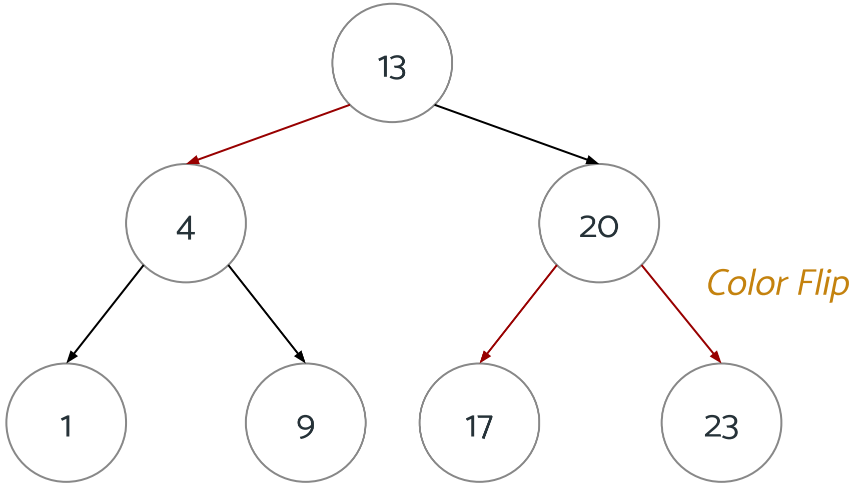

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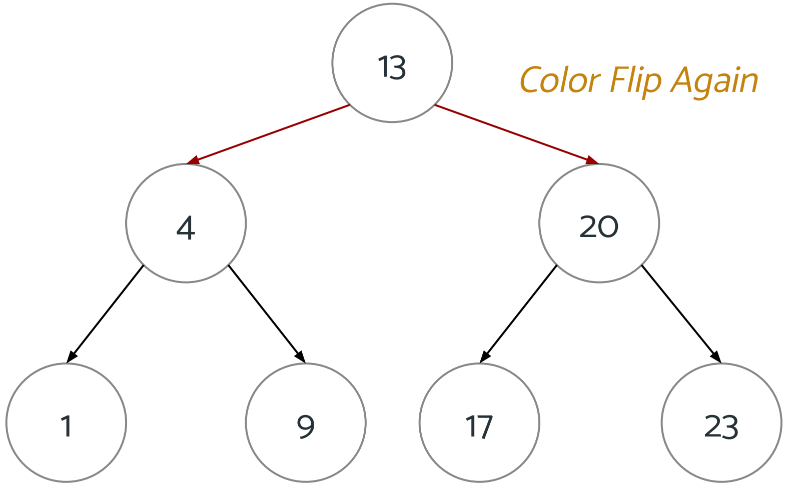
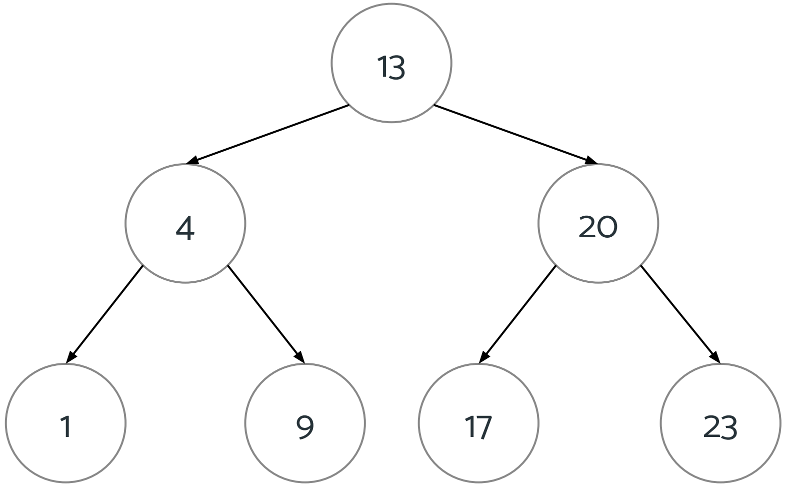


Color Flip Again

LLRB Insertion

Sometimes you will need to do multiple operations.

`insert(17)`



Summary of Red Black Trees

A balancing operation is necessary under the following conditions:

1. A right leaning branch is present
2. There are two left leaning branches in a row

How do we make sure an LLRB is valid?

1. It is still a BST - all those rules apply
2. Once you convert it to a B-Tree, it should follow all the rules laid out for B-Trees
3. Every path from the root to a null reference must have the same number of Black links