Chapter - 01 Number System

1. Number of 0's at the end

Conceptual

Q1. Find Maximum no. of 3's present in 10!

Sol $10! = 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$9 = 3^2$, $6 = 3 \times 2$, $3 = 3$

no. of 3's = 4

Apart from Solving it Manually we Can also Solve it as.

$$3's = \frac{10}{3} + \frac{10}{3^2} + \cancel{\frac{10}{3^3}} + \cancel{\frac{10}{3^4}}$$

$N < D$

$$= 3 + 1$$

No. of 3's = 4

Condition to be followed:

→ The number from which division start must be prime. 3 in this Case.

→ Consider only the Integral part when Dividing Numerator by Dlnominator.

→ Divide Numerator by Denominator as long as $N. \geq D.$

Que2. Find Maximum no. of 2's, 3's 5's & 7's present in 25! 50! 75! 100! 150! & 200!

Sol ① 25!

a. no. of 2's $= \frac{25}{2} + \frac{25}{2^2} + \frac{25}{2^3} + \frac{25}{2^4} + \cancel{\frac{25}{2^5}}$

$= 12 + 6 + 3 + 1$

$= 22$

b. no. of 3's = $\frac{25}{3} + \frac{25}{3^2} + \cancel{\frac{25}{3^3}}$

= 8 + 2

= 10 //

c. no. of 5's = $\frac{25}{5} + \frac{25}{5^2}$

= 5 + 1

= 6 //.

d. no. of 7's = $\frac{25}{7} + \cancel{\frac{25}{7^2}}$

= 3 //

ii) 50!

no. of 2's = 25 + 12 + 6 + 3 + 1

= 47

no. of 3's = 16 + 5 + 1

= 22

~~no of 4's~~ = ~~12 + 3 = 15~~

no of 5's = 10 + 2 = 12

no. of 7 = 7 + 1 = 8

iii) 75!

no. of 2's = 37 + 18 + 9 + 4 + 2 + 1

= 76

no of 3's = 25 + 8 + 2 + 0 = 35

no. of 4's = 18 + 4 + 1
= 23

no. of 5's = 15 + 3
= 18

no. of 7's = 10 + 1
= 11

iii) 100!

no. of 2's = 50 + 25 + 12 + 6 + 3 + 1
= 97

no. of 3's = 33 + 11 + 3 + 1
= 48

no. of 4's = 25 + 8 + 2
= 35

no. of 5's = 20 + 4 + 0
= 24 (CAT Exam)

no. of 7's = 14 + 2
= 16

Conceptual Que2 Find Maximum number of 2, 4, 8, 16 present in 25! 50! & 100!

Sol
= 25!

no. of 2's = $\frac{25}{2} + \frac{25}{2^2} + \frac{25}{2^3} + \frac{25}{2^4} +$

= 12 + 6 + 3 + 1

= 22

no. of 4's $\Rightarrow$ $4 = 2^2$ $\Rightarrow$ $\frac{\text{no. of 2's}}{2} = \frac{22}{2} = 11$

no. of 8's :- $8 = 2^3$ $\quad \frac{\text{No. of 2's}}{3} = \frac{22}{3} = 7$

no. of 16 :- $16 = 2^4$ $\quad \frac{22}{4} = 5$

ii) 50!

no. of 2's = 47

no. of 4's = $\frac{47}{2} = 23$

no. of 8's = $\frac{47}{3} = 15$

no. of 16 = $\frac{47}{4} = 11$

iii) 100!

no. of 2's = 97

no. of 4's = 97/2 = 48

no. of 8's = 97/3 = 32

no. of 16's = 97/4 = 24

Que find max. no. of 2's, 3's, 5, 7's 9's, 16's, 9's & 27's in 150! & 250!

Sol i) 150!

no. of 2's = $\frac{150}{2} + \frac{150}{4} + \frac{150}{8} + \frac{150}{16} + \frac{150}{32} + \frac{150}{64} + \frac{150}{128}$ $\quad 75 + 37 + 18 + 9 + 4 + 2 + 1$

$= 146$

no of 3's = $50 + 16 + 5 + 1 = 72$

no of 5's = $30 + 6 + 1 = 37$

no. of 7's = 21 + 3 = 24

no. of 9's = $\frac{72}{2}$ + 0 = 36

no. of 16's = $\frac{146}{4}$ = 36

no. of 27's = $\frac{72}{3}$ = 24 (Vizag steel question)

no. of no. of 4's = $\frac{146}{2}$ = 73

no. of 8's = $\frac{146}{3}$ = 48

ii) 250!

no. of 2's = 125 + 62 = 187 + 31 + 15 + 7 + 3 + 1 = 244

no. of 3's = 83 + 27 + 9 + 3 + 1 = 123

no. of 5's = 50 + 10 + 2 = 62

Conceptual Que 3 find Max. no. of 6's 21's & 15's in 25! 50! & 100!

25!

Sol i) No. of 2's = 12 + 6 + 3 + 1 = 22

ii) No of 3's = 8 + 2 + 1 = 10

iii) no. of 6's ⇒ $6 = 2 \times 3$, $\min(22, 10)$ = 10 {min.}

iv) no. of 21's ⇒ min(10, 3) = 3

v) no. of 7's ⇒ 3

vi) no. of 15 ⇒ min(6, 10) = 6

vii) no. of 5 ⇒ 5 + 1 = 6

ii) 100!

a) no. of 2's = 97

b) no. of 3's = 33 + 11 + 3 + 1 = 48

c) no. of 5's = 20 + 4 + 0 = 24

d) no. of 7's = 14 + 2 = 16

e) no. of 6's = min(97, 48) = 48

f) no. of 15's = min(48, 24) = 24

g) no. 21's = 16

h) no. of 12's = min(48, 48) = 48 X

i) no. of 18's = min(48, 48) = 48 X

iii) 50!

a) no. of 2's = 47

b) no. of 3's = 22

c) 4's = 15

d) 5's = 12

e) 6's = 22

f) 7's = 8

g) 15's → min(3, 5) = 12

h) 21's = min(7, 3) = 8

i) 12's = 10

j) 18's = 1

~~100!~~ 50!

h) no. of 12's in 2^{97}

$12 \rightarrow 2^2 \times 3$

$50! = 2^{47} \times 3^{22}$

$= (2^2)^{11} \times 3^{22}$

$\min(22, 22)$

no. of 12's = 22

i) no. of 18's

$18 \rightarrow 2 \times 3^2$

$50! \rightarrow 2^{47} \times 3^{22}$

$2^{47} \times (3^2)^{11}$

$\min(47, 11) = 11$ //

100!

h) no. of 12's =

$12 \rightarrow 2^2 \times 3$

$100! = 2^{97} \times 3^{48}$

$= (2^2)^{48} \times 3^{48}$

$= 48$

i) no. of 18's

$18 \rightarrow 2 \times 3^2$

$2^{97} \times (3^2)^{24}$

$= \min(97, 24)$

$= 24$

j) no. of 21's

$21 \rightarrow 7, 3$

$3^{22} \times 7^8$

$= 8$ //

~~Test~~

Note:- In any factorial number, the number of zeros at the end is equal to the no. of 5's present.

↳ min (no. of 2's, no. of 5's)

eg. find the no. of 0's at the end of the following.

i) 80!

ii) 100!

iii) 123!

Sol 80!

no. of 5 → 16 + 3

= 19

100! → 20 + 4 = 24

123! → 24 + 4 = 28

Que. find no. of 0's at the end of following

i) 77!

Sol 77! → 15 + 3 = 18

ii) a) 27 b) 27^{27} c) $27^{27}!$

Sol:- no. of zero = 0 Sol:- 0 0's

e) $27^{27}!$

Sol no. of zeros = 0

d) 27! 5 + 1 = 6 0's

e) $27!^{27}$ no. of zero = 6^{27} X

6) $27!^{27!}$

Sol no. of zeros = $6^{27!}$ X

7) $15 \times 30 \times 45 \times 60 - - - \times 1500$ X

$\Rightarrow$ ~~15~~ $50 + 11 = 61$

5) $27!^{27}$

Sol $27! \rightarrow$ 6 no. of 0's

no. of zeros $6 \times 27 = 162$

6) $27!^{27!}$

Sol no. of zeros $= 6 \times 27!$

7) $15 \times 30 \times 45 \times 60 - - - \times 1500$

$\Rightarrow (15\times1) \times (15\times2) \times (15\times3) \times (15\times4) \cdots \times (15\times100)$

$\Rightarrow 15^{100} [\, 1 \times 2 \times 3 - 100 \,]$

$15^{100} \times 100!$

$3^{100} \times 5^{100} \times 2^{97} \times 5^{24}$

$2^{97} \times 5^{124}$ = 97 no. of zeros

* To find Unit Digit Value :-

Power Cycle :- The Highest power of any number such that its unit digit doesnot repeat.

$2^1 = 2$ | $2^5 = 32$ ✗ repeated

$2^4 = 16$ — $2^2 = 4$

$2^3 = 8$

2 < Power Cycle = 4
Unit digit value = 6

$3^1 = 3$ | $3^5 = 243$ repeated ✗

$3^4 = 81$ — $3^2 = 9$

$3^3 = 27$

3 < PC = 4
UDV = 1

$7^1 = 7$ | $7^5 = 7$

$7^4 = 1$ — $7^2 = 49$

$7^3 = 343$

7 < PC = 4
UDV = 1

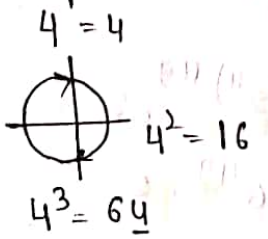
$8^1 = 8$ | $8^5 = 8$

$8^4 = 6$ — $8^2 = 64$

$8^3 = 2$

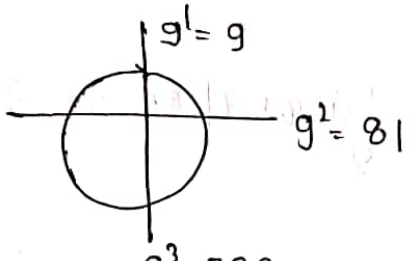
8 < PC = 4
UDV = 6

4 →



4 < PC = 2 ; UDV = 6

9 →



9 < PC = 2 ; UDV = 1

X

number	Power cycle	UDV
2	4	6
3	4	1
4	2	6
6	1	6
7	4	1
8	4	6
9	2	1

eg. 2^{23}

Sol $\frac{\text{Power}}{\text{Power cycle}} = \frac{23}{4} = 5, 3$

$2^{23} \rightarrow (2^4)^5 \cdot 2^3$

$(6)^5 \cdot 8$

$= 6 \cdot 8$

$= 8$ unit digit value.

ii) 2^{53}

Sol $(2^4)^{13} \cdot 2$

$6^{13} \cdot 2$

$\Rightarrow 6 \cdot 2$

$= 2$ //

iii) 2^{197}

Sol $(2^4)^{49} \cdot 2$

$6^{49} \cdot 2$

$6 \cdot 2$

$= 2$

iv) 2^{498}

$\Rightarrow (2^4)^n \cdot 2^0$

$= 6 \cdot 1$

$= 6$

v) $2^{98857|33}$ $\longrightarrow$ $\frac{33}{4} = 8$ 2^1

$= 2$

(No need to find n, only remainder is needed, Multiply any no with 6 give same no at U.D.)

Test

Que. find U.D. of following

i) 2^{77} $\longrightarrow$ $\frac{77}{4}$ $R = 1$ $2^1 = 2$

ii) 2^{1011} $\longrightarrow$ $1 \times 6 = 6$

iii) $2^{105|186}$ $\longrightarrow$ $6 \times 1 = 6$

$\hookrightarrow (2^4)^n \cdot 2^0 \longrightarrow$ Remainder always 0

$= 6 \cdot 1 = 6$ //

186 will only increase no. of zero at the end.

Que. find U.D. of

i) 8^{23}

Sol Same as 2^n

$\frac{23}{4}$ = Rem: 3

$8^3 \times 6 = 6 \times 2$

$= 2$

Test 02 UDV?

(I) 8^{87}

Sol $(8^4)^{21} \cdot 8^3$

$= 2$

(II) $8^{259} + 2^{359}$

Sol $\frac{59}{4}$

$= 14 \quad R:3$

$8^3 \cdot 6 + 2^3 \cdot 6$

$= 2 + 8$

$= 0$

(III) $8^{366} \times 2^{4651}$

$\frac{66}{4} \Rightarrow 8^2 \cdot 6 = 4$

$2^{4651} = 6 \qquad \frac{65}{4} = 6$

$4 \times 6 = 4$ //

Que. find UDV of -

i) 3^{23}

Sol $(3^4)^5 \cdot 3^3$ OR $\frac{23}{4}$ = Rem $3^3 \rightarrow 7$

$= 1^5 \cdot 3^3$

$= 1 \cdot 7$

$= 7$

ii) 3^{55}

Sol $\frac{55}{0} \rightarrow 13 \rightarrow 3$

3^3 = UDV = 7

iii) 3^{198}

Sol $\frac{98}{4}$ = R: 2

$3^2 = 9$ //

iv) $3^{2017!}$

Sol $(3^4)^{\alpha} \cdot 3^0$

$= 1$

v) 7^{23}

Sol $(7^4)^5 \cdot 7^3$ OR $\frac{23}{4}$ R:. 3

$1^5 \cdot 7^3$ 7^3 = 3 UDV

$= 1 \cdot 3$

$= 3$

VI) 7^{197}

Sol $\frac{197}{4}$ R 1

$7^1 = 7$

VII) $7^{2017} + 3^{2025}$

Sol $\downarrow \quad \downarrow$

$7 \; + \; 3$

$= 0$

VIII) $7^{2050} \times 2^{2051} \times 3^{2052} \times 8^{2053}$

$\downarrow \quad \downarrow^{8} \quad \downarrow \quad \downarrow$

$9 \quad 8 \quad 1 \quad 8$

$= 6$ UDV

Conceptual

Que. Find the UDV

1) 4^{23}

Sol $(4^2)^{11} \cdot 4$

$(6)^{11} \cdot 4$

$6 \cdot 4$

$= 4$

II) 4^{99}

Sol $(4^2)^{49} \cdot 4$

$6 \cdot 4$

$= 4$

iii) 4^{9999}

Sol $(4^2)^{9999}$

⇒

Note:- $(odd)^n$ = odd n = natural no

$(even)^n$ = even

$n \geq 2$ $n!$ = even

4^{odd} = 4

4^{even} = 6

$9^{odd} = 9$

$9^{even} = 1$

$4^{9999} \Rightarrow 4^{odd} = 4$

iv) $4^{99^{99}} \Rightarrow 4^{odd^n} = 4^{odd} = 4$

v) $4^{99^{99!}} \Rightarrow 4^{99^{even}} = 4^{odd} = 4$

vi) $4^{99!^{99}} \Rightarrow 4^{99!^{odd}} = 4^{even} = 6$

vii) $4^{99^{99!}} \Rightarrow 4^{odd} = 4$

viii) $4^{99!^{99!}} \Rightarrow 4^{even^{even}} = 4^{even} = 6$

HW
Que.

i) 9^{99}

ii) 9^{9999}

iii) $9^{99^{99}}$

iv) $9^{99^{99!}}$

v) $9^{99!^{99}}$

vi) $9^{99!^{99!}}$

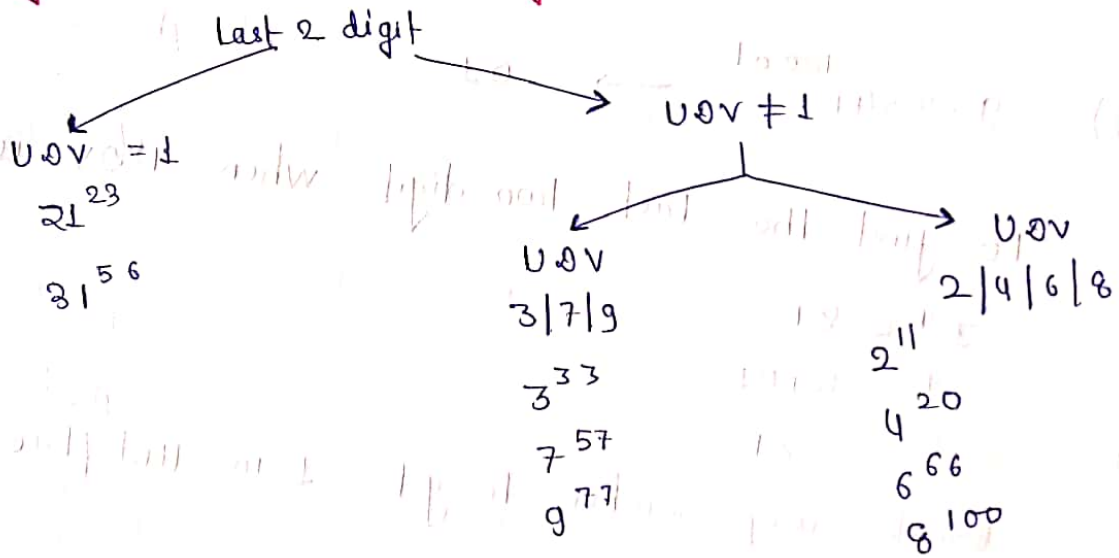
Note:- 12^{23} UDV $= 2^{23}$ UDV $= 8$

UDV of 13^{23} = UDV of 3^{23} = 7

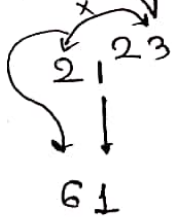
UDV of $981563^{23} = 3^{23}$

(Consider only Unit digit)

* To find the Last two Digits :



* To find last 2 digit when UDV is 1



21^{56}

$\Rightarrow$ 21^{1}

61^{99}

61^{1}

(drop unit place
& Multiply Tens place with unit place no. of power)

Que 1. i) 31^{26}

$\Rightarrow$ 81^{1}

ii) $51^{99} \longrightarrow 51$

iii) $3561^{3561!} \longrightarrow 01$

iv) $932541^{1000!} \longrightarrow 01$

* To find the last two digit when U^{OV} is 3/7/9

$3^4 = 81$

$7^4 = 2401$

$9^2 = 81$

So break every number to get 1 in Unit place

eg. $3^{23} \Rightarrow (3^4)^5 \cdot 3^3$

$(81)^5 \quad 3^3$

$01 \quad 27$

$= 27$

Test #01

i) 3^{53}

$\Rightarrow (3^4)^{13} \cdot 3$

$(81)^{13} \cdot 3$

$21 \cdot 3$

$\Rightarrow 23$

ii) 3^{199}

$\Rightarrow (3^4)^{49} \cdot 3^3$

$21 \cdot 27$

$= 67$

iii) $3^{257} + 71^{73}$

$\Rightarrow (3^4)^{64} \cdot 3^1 + 11$

$\Rightarrow (81)^{64} \cdot 9 + 11$

$\Rightarrow 21 \cdot 9 + 11$

$\Rightarrow 89 + 11$

$\Rightarrow 74$

* To find last two digit when u ⊙ v is 9

i) 9^{23}

$(9^2)^{11} \cdot 9$

$81 \cdot 9$

729

$\downarrow$

29 Ans.

{ To make u ⊙ v = 1

$9^2 = 81$

ii) 9^{99}

$\Rightarrow (9^2)^{49} \cdot 9$

$\Rightarrow (81)^{49} \cdot 9$

$\Rightarrow 21 \cdot 9$

$= 189$

$= 89$ //.

iii) $9^{29} + 3^{37} + 31^{47}$

$\Rightarrow (9^2)^{14} \cdot 9 + (3^4)^9 \cdot 3 + 31^{47}$

$\Rightarrow (81)^{14} \cdot 9 + (81)^9 \cdot 3 + 11$

$\Rightarrow 21 \cdot 9 + 21 \cdot 3 + 11$

$\Rightarrow 89 + 63 + 11$

$\Rightarrow 63$

* Last two digits when $U \otimes V = 7$

i) 7^{21}

$\Rightarrow (7^4)^5 \cdot 7$

$(01)^5 \cdot 7$

$\Rightarrow 01 \cdot 7$

$\Rightarrow 07$

ii) 7^{22}

$\Rightarrow (7^4)^5 . 7^2$

$01 \cdot 49$

$= 49$

Note only Remainder left after division by 4 will give last 2 digit

iii) 7^{2008}

$\Rightarrow (01)^{-} \cdot 7^0$

$= 01$

iv) 7^{53}

$\Rightarrow (01)^{-} \cdot 7^1$

$= 07$

v) 7^{2051}

$\Rightarrow (01)^{-} \cdot 7^3$

$= 43$

vi) $7^{2051!}$

$\Rightarrow (01)^{-} \cdot 7^0$

$= 01$

vii) $7^{89} \times 3^{89} \times 21^{89}$

$\Rightarrow (441)^{89}$

$\cdot 61$ //

* Last two digit when UOV = 2|4|6|8

$2^{10} = 1024$

$24^1 = 24$

$24^2 = 576$

$24^3 = 13824$

$24^{odd} = LTD = 24$

$24^{even} = 76$ Last Two digit

I) 2^{92}

$\Rightarrow (2^{10})^9 \cdot 2^2$

$\Rightarrow (24)^9 \cdot 4$

$\Rightarrow 24 \cdot 4$

$\Rightarrow 96$

II) 4^{63}

$\Rightarrow (2^{10})^{12} \cdot 2^6$

$\Rightarrow 76 \cdot 64$

$\Rightarrow 64$

III) 6^{121}

$\Rightarrow (2 \times 3)^{121}$

$\Rightarrow 2^{121} \times 3^{121}$

$\Rightarrow (2^{10})^{12} \cdot 2 \times (3^4)^{30} \cdot 3$

$\Rightarrow 76 \cdot 2 \times 3$

$\Rightarrow 56$

4) 8^{33}

$\Rightarrow 2^{99}$

$\Rightarrow (2^{10})^9 \cdot 2^9$

$\Rightarrow 24 \cdot 12$

$\Rightarrow 88$

Note:- 13^{23}

$(13^4)^5 \cdot 13^3$

$\Rightarrow (61)^5 \cdot 97$

$\downarrow$

$01 \cdot 97$

$= 97$

$13^4 = 13^2 \times 13^2$

$= 169 \times 169$

$= 61$

* Factorization

Any Natural number written in its prime factor form is called its factorization

eg. $12 = 2^2 \times 3$

factors $\begin{cases} 1 \times 12 \\ 2 \times 6 \\ 3 \times 4 \end{cases}$

In the above example 1, 2, 3, 4, 6, 12 are the factors of 12

and writing $12 = 2^2 \times 3$ i.e in its prime factor form is called factorization

Let N be any natural number such that

i) $N = a^p \times b^q \times c^r$

a, b, c are prime factors & p, q, r are their respective powers.

$n = (p+1)(q+1)(r+1)$

ii) $P_n = (N)^{n/2}$

P_n = Product of factors

eg. $P_n = (12)^{6/2}$

$= 12^3$

$= 1728$

iii) $S_n = \dfrac{(a^{p+1}-1)(b^{q+1}-1)(c^{r+1}-1)}{(a-1)(b-1)(c-1)}$

eg. $S_n = \dfrac{(2^3-1)(3^3-1)}{(2-1)(3-1)}$ = Sum of all factors

$= \dfrac{7 \cdot 8}{1 \cdot 2} = 28$

Quel Find $n | P_n | S_n$

i) 24 ii) 60 iii) 90 iv) 124 v) 224

vi) 288 vii) 324

Sol: 1. $N = 24 = 2^3 \cdot 3$

$n = (3+1)(1+1)$

$= 4 \times 2 = 8$

$P_n = (24)^{8/2} = (24)^4 = 331776$

$$S_n = \frac{(2^{3+1}-1)(3^{1+1}-1)}{(2-1)(3-1)}$$

$$= \frac{15 \times 8}{2}$$

$$= 60$$

ii) 60

$N = 60$

$= 2^2 \times 3 \times 5$

$n = (2+1)(1+1)(1+1) = 12$

$P_n = (60)^{12/2} = (60)^6 =$

$$S_n = \frac{(2^{2+1}-1)(3^{1+1}-1)(5^{1+1}-1)}{(2-1)(3-1)(5-1)}$$

$$= \frac{7 \times 8 \times 24}{2 \times 4} = 168$$

iii) 288

$N = 288$

$= 2^5 \times 3^2$

$n = 6 \times 3 = 18$

$P_n = (288)^{18/2} =$

$S_n = \frac{63.26}{1.2} = 819$

2	288
2	144
2	72
2	36
3	18
3	6
2	2

note:- no. of odd factor = a^{P+1}

no. of even factor = n - n odd

i) $24 = 2^3 \times 3$

no. of odd factor = $3^{1+1} = 3^{(2)}$

$= 2$

no. of even factor = $n - 2$

$= 8 - 2 = 6$

ii) $60 = 2^2 \times 3 \times 5$

no. of odd factor = $3^{(1+1)} \times 5^{(1+1)}$

$= 2+2 = 4$

no. of even factor = $n - 4$

$= 12 - 4$

$= 8$

iii) $288 = 2^5 \times 3^2$

$n = 6 \times 3 = 18$

n. odd = [redacted] + 2 + 1 = 3

n even = $18 - 3 = 15$

note:- For obtaining no. of factor divisible by any number

12

1 × 12

2 × 6

3 × 4

÷2	04	2|4|6|12
÷3	03	3|6|12
÷4	02	4|12
÷6	02	6|12
÷12	01	12

$12 = 2^2 \times 3$

$\div 2$	$2^1 \times 3^1$	$(1+1)(1+1) = 4$
$\div 4$	3^1	$= 2$
$\div 3$	2^2	03
$\div 6$	2^1	02
$\div 12$	$2^0 \times 3^0$	01

Que. i) $24 = 2^3 \times 3$

no. of factor divisible by ⇒

$\div 2$	$2^3 \times 3$	$(3+1)(1+1) = 3\times2 = 6$
$\div 4$	$2^1 \times 3^1$	$2\times2 = 4$
$\div 3$	$2^3 \times 3^1$	$4\times1 = 4$
$\div 6$	$2^2 \times 3^0$	3
$\div 12$	$2^1 \times 3^0$	2

ii) $2^3 \times 3^4 \times 5^2$

$\div 8$	$3^4 \times 5^2$	$5\times3 = 15$
$\div 9$	$2^3 \times 3^2 \times 5^2$	$4\times3\times3 = 36$
$\div 24$	$3^3 \times 5^2$	$4\times3 = 12$
$\div 10$	$2^2 \times 3^4 \times 5^1$	$3\times5\times2 = 30$
$\div 72$	$2^0 \times 3^2 \times 5^2$	$1\times3\times3 = 9$

note :- no. of even factor = no. of factor divisible by 2

Concept Que. How many no. are there which are less than or equal to 120 and are relatively prime

Let $N = a^p \times b^q \times c^r$

→ The total no. of numbers less than or equal to N & relatively prime to N are

$$= N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$$

eg. How many natural no. less than equal to N are relatively prime to N? where N is

i) 10 iii) 90

ii) 21 iv 120

Sol i) 10 $= 2 \times 5$

$$= 10\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)$$

$$= 10 \times \frac{1}{2} \times \frac{4}{5}$$

$= 4$ no. less than equal to N. are Rel. prim

ii) $90 = 2 \times 5 \times 3 \times 3$

$= 2 \times 5 \times 3^2$

$= 90\left(1/2\right)\left(4/5\right)\left(\frac{2}{3}\right)$

$= 24$

iii) $21 = 3 \times 7$

$21\left(1-1/3\right)\left(1-1/7\right) = 21 \times \frac{2}{3} \times \frac{6}{7}$

$= 2 \times 6$

$= 12$

iv) $120 = 2 \times 3 \times 2 \times 5 \times 2$

$= 120\left(1/2\right)\left(2/3\right)\left(4/5\right) = 32$

Note:- The sum of all the natural numbers $\leq N$ and are relatively prime to N is equal to

$$\frac{N}{2}\left[N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\right]$$

I) $N = 10$

$\Rightarrow \frac{10}{2} \times 4$

$= 20$

II) 21

$$\frac{21}{2}\left[21\left(1-\frac{1}{3}\right)\left(1-\frac{1}{7}\right)\right] = \frac{21}{2} \times 21 \times \frac{2}{3} \times \frac{6}{7}$$

$\Rightarrow 63 \times 2$

$= 126$

III) 90

Sol $\frac{90}{2}[24] = 1080$

IV) 120

$\Rightarrow \frac{120}{2} \times 32 = 1920$

* Divisiblity Rule

I) If all the natural number starting from 1 are written side by side as

123456...

Then find -

a) 25^{th} b) 50^{th} c) 75^{th} d) 100^{th}

e) 150^{th} f) 200^{th} g) 500^{th} digit of the Series

1 2 3 4 5 6 7 8 9 10 11 12 ... 98 99 100 101 ... 999

9 × 1 = 9 digits

90 × 2 = 180 digits

189 digits

(i) 25^{th}

⇒ $25 - 09 = 16$

$x \times 2 = 16$

$x =$ 8 numbers after 9

10 11 12 13 14 15 16 17

↳ 25^{th} digit

(ii) 50^{th}

⇒ $50 - 09 = 41$

$x \times 2 = 41$

$x =$ 20 numbers Rem. 1

i.e 30

↳ 50^{th} digit = 3

(iii) 75^{th}

⇒ $75 - 09 = 66$

$66 = x \times 2$

$x =$ 33 numbers

i.e 32

↳ 75^{th} digit

IV 100th

Sol: $100 - 09 = 91$

$x \times 2 = 91$

$x = 45$ Rem 1

i.e 54 number 5 5

5 100th digit

V) 200th

$200 - 189 = 11$

$x \times 3 = 11$

$x = 3$ R 2

102 103

200th digit

VI) 500th

Sol $500 - 189 = 311$

$x \times 3 = 311$

$x = 103 \mid 2$

202 203

500th

Que. Find the Remainder when the above Series is divided by i) 2, 4, 8, 16

ii) 3, 9

iii) 5 & 25

* Divisiblity Rule of 2, 4, 8, 16

i) 2 :- Last digit even

eg. 32 24 100 103 (not even) ✗

ii) 4 :- Last two digit divisible by 4

eg. 924 , 4099

iii) 8 :- Last 3 digit divisible by 8

iv) 16 :- Last 4 digit divisible by 16

* Divisiblity Rule of 5, 25, 125

i) 5 :- Last digit divisible by 5

eg. 5 , 25 , 30 , 1000

ii) 25 :- Last two digit divisible by 25

eg. 625 , 125 300

iii) 125 :- Last three digit divisible by 125

eg. 93250

* Divisiblity of 3 & 9

i) 3 :- If Sum of all digit of no. is divisible by 3 then no. is divisible by 3

ii) 9 :- If Sum of all digit is divisible by 9 then no divisible by 9.

eg. $81 \rightarrow 8+1 = 9$ both 3, 9

$729 \rightarrow 7+2+9 = 18$ both 3, 9

$24 \rightarrow 2+4 = 6$ 3 but not 9

eg. i) N = 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

Check for divisiblity by 2, 4, 8, 16, 3, 9, 5, 25 & 125

Sol $\frac{7}{2} = 3$ R (1) ND $\quad \frac{7}{5} = 1$ (R) 2 ND

$\frac{17}{4} = 4$ R (1) ND $\quad \frac{17}{25} =$ (R) 17 ND

$\frac{617}{8} = 77$ R (1) ND $\quad \frac{617}{125} =$ (R) 117 ND

$\frac{1617}{16} = 101$ (R) (1) ND

for 3 :- Sum of $1+2+3+4+5+6+7+8+9$ I

$1+1+1+1+1+1+1+1+1$ II

$1+2+3+4+5+6+7$ III

Sum of n natural no. $= \frac{n(n+1)}{2}$

I $S = \frac{9(10)}{2} = 45$

II $S = 8$

III $S = \frac{7(8)}{2} = 28$

Total $S =$ 81

$\frac{81}{3} =$ (R) 0 $\underbrace{0}_{\text{divisible}}$ $\quad \frac{81}{9} =$ (R) 0 $\underbrace{0}_{\text{divisible}}$

ii) N = 1234 ..... 97 98 99 100

check divisiblity

i) 2, 4 divisible

8 not divisible

$\frac{9100}{16}$ = Not divisible

ii) 5, 25 divisible

125 not divisible

iii) Check the divisibility of 100th digit in the seq

Sol 100th digit

⇒ 100 − 9 = 91

x × 2 = 91

x = 45 (R) 1

54 5

100th no. 5

$\frac{N}{2}$ = (R) 1 ND

$\frac{N}{4}$ = $\frac{45}{4}$ (R) 1 ND

$\frac{N}{8}$ = $\frac{545}{8}$ (R) 1 ND

$\frac{N}{16}$ = $\frac{3545}{16}$ (R) ND ~~Rem 3+5+4+5 = 17~~

$\frac{N}{25}$ = (R) ND

$\frac{N}{5}$ = R = 0 D

$\frac{N}{125}$ = $\frac{545}{125}$ (R) 45

Series 1 2 3 . . . 53545

for divisible of 3 :-

$$5 \times \frac{n(n+1)}{2} = \frac{5 \times 3 \times 16}{2} = 120$$

$$100 + 25 + 15 = 220$$

note:- donot check divisible by 3 by Summing

Series:- N = 1 2 3 4 ---- 5 3 5 4 5 — 100th digit

1	10	19	28	37	46
2	11	20	29	38	47
3	12	21	30	39	48
4	13	22	31	40	49
5	14	23	32	41	50
6	15	24	33	42	51
7	16	25	34	43	52
8	17	26	35	44	53
9	18	27	36	45	54
					5

$$5 \times \frac{n(n+1)}{2} = 5 \times \frac{9(9+1)}{2} = 45 \times 5 = 225$$

$1 \times 10 = 10$
$2 \times 10 = 20$
$3 \times 10 = 30$
$4 \times 10 = 40$
$5 \times 6 = 30$
130

$$225 + 130 = 365$$

D.S = 365

$\frac{365}{3}$ = (R) NO

$\frac{365}{9}$ = (R) NO

Donot find the Sum better check Remainder by 3 & by 9

* Permutation & Combination

"+" → choice → OR → + → ∪

No choice → AND → × → ∩

Conc. Q1 How many 3 digit no. can be found using the digits –

a) 1, 2, 3, 4, 5 such that a1 Repetition allowed
a2 Repetition not allowed.

Sol a1 $\underset{1-5}{\underline{5}} \quad \underset{1-5}{\underline{5}} \quad \underset{1-5}{\underline{5}}$

$5 \times 5 \times 5 = 125$

a2 $\underset{1-5}{\underline{3}} \times \underset{1-5}{\underline{4}} \times \underset{1-5}{\underline{5}} = 60$

Que. How many 3 digit no. can be formed using the digits –

i) 1 3 5 7 8 9 < Rep. allow / Rep. not

ii) 0, 1, 2, 3, 4 < allow / not

Sol i 1.1 $\underline{6} \quad \underline{6} \quad \underline{6} = 216$

1.2 $\underline{4} \quad \underline{5} \quad \underline{6} = 120$

2.1 $\underline{4} \quad \underline{5} \quad \underline{5} = 100$

2.2 $\underline{2} \quad \underline{4} \quad \underline{5} = 40$ X

2. 3 | 2 | 4 | 5

~~5 × 4 × (3+2)~~

~~20 × 3 + 20 × 2~~

Que. 0, 1, 2, 3, 4 Repetition not allowed

Sol Method I.

4	4	3
(zero not allowed)		

= 4 × 4 × 3 = 48

Method II

	2	3	4	
	H	T	U	
Case I	2	3	4	= 24
Case II fix 0	3	4	0	= 12
Case III	3	0	4	= 12

24 + 12 + 12 = 48 (OR, OR)

Method III

Consider zero as any other No.

3 4 5 = 60

Total − not wanted

60 − 12 = 48

Que. How many 3 digit no. can be formed using 0 to 9 such that

1 → All the digit are prime & distinct

2 → All digits are odd < Rep allowed / Rep not allowed

3 → Tens place is even < Rep allowed / Rep not

Sol 1) $\underline{4}\ \underline{3}\ \underline{2} = 24$

$\underline{\cancel{4}}\ \underline{4}\ \underline{3}$

II) 2.2 $\underline{5}\ \underline{4}\ \underline{3} = 60$

2.1 $\underline{5}\ \underline{5}\ \underline{5} = 125$

III) 3.1 $\underset{H}{\underline{8}}\ \underset{T}{\underline{5}}\ \underset{O}{\underline{9}} = 450$ (9, 5, 10)

3.2 $\underline{8}\ \underline{5}\ \underline{9} =$ (move to first place after fixing cond^n)

$\underline{9}\ \underline{0}\ \underline{8} = 72$

$\underline{8}\ \underline{4}\ \underline{8} = 64 \times 4$

note: filling start from condition

$72 + 256$

$= 328$ Ans.

Test

Que. Using 0 to 9 how many 3 digit no. can be formed such that the no. are divisible by

1) 2 < Rep allowed / Rep. not allowed

II) 25

III) 4

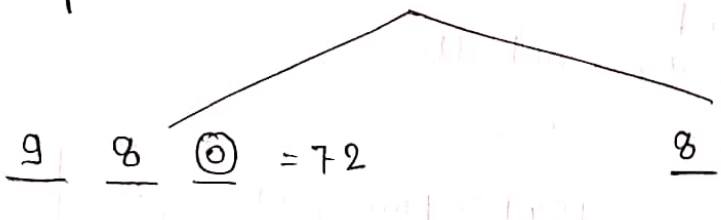
Sol

I) divisibly by 2

a) Rep allowed

9 10 5 = 450

b) Rep. not allowed



9 8 0 = 72 8 8 4 = 256

2/4/6/8

256+72 = 328

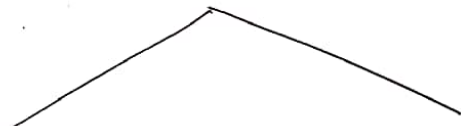
II) divisible by 5

a) Rep. allowed

9 10 2 = 180

0/5

b) Rep. not allowed



9 8 0 8 8 5

72 64

72+64 = 136

III) divisible by 25

a) Rep allowed

9 _ _ 25/50/75/00 = 9×4 = 36

b) Rep. Not allowed

_ _ _ 25/50/75

8 1 5 0 = 8

7 2 _ 5 = 14

2/7 22

iv) Divisiblity by 4

i) Rep. allowed

$9 \times 25 = 225$

00	24	48	68
04	28	52	72
08	32	56	
12	36	60	96
16	40	64	~~00~~
20	44		

ii) Rep. not allowed

00, 44, 88 not allow

Case I: $25 - 3 = 22$

22 → Z: 20 | 04 40 | 08 80 | 60 (06)

NZ: $22 - 06 = (16)$

$8 \times 06 = 48$

$7 \times$ NZ 16 $= 112$

$48 + 112 = 160$ Ans

Que. Using 0-9 How many 3 digit number can be formed such that it has exactly i) one 7.

ii) two 7.

* iii) atleast one 7

Sol i) exactly one 7

8	9	(7)	72
8	(7)	9	72
(7)	9	9	81
			225 //

ii) exactly two 7

8	(7)	(7)	= 8
(7)	9	(7)	= 9
(7)	(7)	9	= 9
			26 //

iii) At least one 7

Sol Exactly 1 → (7) + Exactly 2 → (7) + Exactly 3 → (7)

$225 + 26 + 1$

$= 252$

OR

~~At least 1 (7) = Total - no (7) case~~

$= 252 - \underline{8}\ \underline{9}\ \underline{9}$

$= 252 - 648$

$=$

Total $= \underline{9}\ \underline{10}\ \underline{10} = 900$

No 7 $= \underline{8}\ \underline{9}\ \underline{9} = 648$

252 Ans.

Que Using 0 1 2 3 4 5

i) How many 3 digit no. Can be formed Such that Tens place > Unit

Sol $\overline{H}\ \overline{T} > \overline{U}$

i) $\underset{1-5}{\underline{5}}\ \underset{2-5}{\underline{4}} > (1) = 20$

ii) $\underset{1-5}{\underline{5}}\ \underset{3-5}{\underline{3}} > (2) = 15$

iii) $\underset{1-5}{\underline{5}}\ \underset{4-5}{\underline{2}} > (3) = 10$

iv) $\underset{1-5}{\underline{5}}\ \underline{(5)} > (4) = 5$

Total = 50 //.

* Permutation & Combination

	Combination	Permutation
meaning:-	Selection	Selection followed by arrangement
formula :-	${}^nC_r = \frac{n!}{r!(n-r)!}$ $0 \le r \le n$ $= \frac{{}^nP_r}{r!}$	${}^nP_r = \frac{n!}{(n-r)!}$ $0 \le r \le n$

example :-

A|B|C

- A|B|C → ③ → ${}^3C_1 = 3$
- AB, BC, CA | ③ → ${}^3C_2 = 3$
- ABC ① → ${}^3C_3 = 1$

A|B|C → ③ ${}^3P_1 = {}^3C_1 \times 1! = 3$

AB|BA, BC|CB, CA|AC → ⑥ ${}^3P_2 = {}^3C_2 \times 2! = 6$

ABC|BAC, ACB|BCA, CAB|CBA → ⑥ ${}^3P_3 = {}^3C_0 \times 3! = 6$

note:

- ${}^nC_0 = 1$ — ${}^nP_0 = {}^nC_0 \cdot 0! = 1 \cdot 1 = 1$
- ${}^nC_1 = n$ — ${}^nP_1 = {}^nC_1 \cdot 1! = n$
- ${}^nC_n = 1$ — ${}^nP_n = {}^nC_n \cdot n! = n!$
- ${}^nC_r = {}^nC_{n-r}$ Complementary Combination

$${}^5C_2 = \frac{5 \cdot 4}{1 \cdot 2} = 10$$

$${}^{10}C_3 = \frac{10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3} = 120$$

1. DELHI

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 5! = 120$

or $5!\ \ (^5C_5 \cdot 5!)$

$= 120$ arrangement

2. AGAIN

$^5C_5 \cdot 5!$ (first select) $/\ 2!$ (A) repeated

$\Rightarrow \dfrac{5!}{2!}$

$= 60$

3. SUCCESS

$\dfrac{7!}{3!(S)\,2!(C)}$

$\Rightarrow \dfrac{7\times6\times5\times4\times3}{2}$

* Maximum Value :- $(^nC_r)_{max}$

i) when n is even

$(^nC_r)_{max} \xrightarrow{\text{when}} r = \dfrac{n}{2}$

ii) when n is odd

$(^nC_r)_{max} \xrightarrow{\text{when}} r = \dfrac{n+1}{2}$

& $r = \dfrac{n-1}{2}$

eg. Max value of $^{100}C_r$ is $^{100}C_{50}$

Max value of $^{21}C_r$ is $^{21}C_{10}$ $^{21}C_{11}$

Conc. Que 1 How many different word can be formed from Letter of word GANESHPURE Such that

i) no condition is there

Sol $10!$

$^{10}C_{10} \cdot 10! = 10!$

$= 3628800$

ii) 'G' must occupy first place

Sol (G) _ _ _ _ _ _ _ _ _

$11 \times 9!$

$= 9! = 362880$

iii) 'H' must occupy first place and 'E' must occupy last place

(H) _ _ _ _ _ _ _ _ (E)

$11 \times 8! \times 1!$ $= 8!$

$= 40320$

iv) All Vowels must put together

Sol AEUI GNSHPR

AEUI → act as one Alphabet

GNSHPR → 6 letter

$7! \times 4!$ {Vowel arrangement} $= 7! \times 4!$

v) All the vowels must occupy even places

Sol

$$\_\ \underline{2}\ \_\ \underline{4}\ \_\ \underline{6}\ \_\ \underline{8}\ \_\ \underline{10}$$

Place

$^{5}C_{4}$ · 4! = Arrangement of 4 vowel in 5 places

Vowel

$= 5!$

$^{6}C_{6} \cdot 6!$

$\therefore$ $5! \cdot 6!$ Ans.

vi) How many 5 letter can be formed such that it has 3 Cons. & 2 V

Sol

$$^{6}C_{3} \cdot {}^{4}C_{2} \cdot 5!$$

Cons. Vowel

vii) How many 5 letter can be formed by 3 Cons. & 2 vowel such that 3 Consonant occupy 3 odd places and 2 vowel occupy even places

Sol

$$^{6}C_{3} \cdot 3! \cdot {}^{4}C_{2}\, 2!$$

viii) All the vowels are never together

Sol Total arrangement − unwanted arrangement

$$10! - 7!\,4!$$

Note:-

- Pseudo Gap :- If there are two boys and two girls then total number of arrangement such that

i) no condition is there

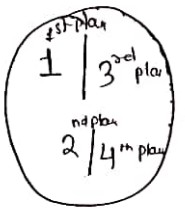
$4! = 24$

ii) Two boys must sit alternately

$B_1\ G_1\ B_2\ G_2 = 2! \cdot 2! = 4$

$G_1\ B_1\ G_2\ B_2 = 2! \cdot 2! = 4$

OR

2 Cases 

$\longrightarrow \left({}^{2}C_{1} \cdot 2!\right) 2!$

Cases, 2 boys, Girls

iii) No two boys must sit together

In addition to the above case here we can have one more case that is -

when two boys are sitting at two extreme i.e 1st & 4th place

Sol

1/3
2/4
1/4
extreme

$\left({}^{3}C_{1} \cdot 2!\right) \cdot 2!$

$= 12$

X G_1 X G_2 X

$2! \left({}^{3}C_{2} \times 2!\right) = 12$

$\left({}^{2}C_{1} + 1\right)$ → pseudo gap

2 place available for 2 boys

arrangement of two girls

arrangement of two boys

Thus we can deduce that if m boys and n girls then total number of arrangement such that no two boys must sit together

$$m\,B \mid n\,G$$

$$n! \cdot \left({}^{n+1}C_m \cdot m! \right)$$

(n+1 → pseudo gap)

e.g If we have 5G & 3B then total number of arrangement no two boys must sit together

Sol $\times G_1 \times G_2 \times G_3 \times G_4 \times G_5 \times$

$$5! \left({}^{6}C_3 \cdot 3! \right)$$

Q9. GANESH PURI

Such that no two vowel sit together

$\times G \times N \times S \times H \times P \times R \times$

$$6! \left({}^{7}C_4 \cdot 4! \right)$$

Test 01 How many seven letter words can be formed from Letters of the word ARTICLE Such that vowel must occupy even places

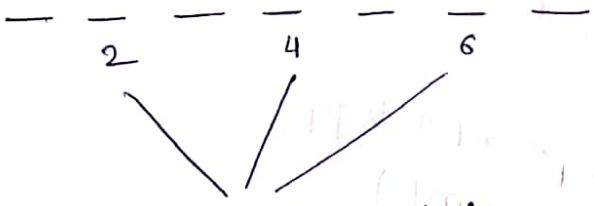
Test 02 How many six letter words can be formed from the Letters of word GARDEN. Such the vowel must come in Alphabetical order

Test 03 AAAAA BBB CCC D EE F

Total no. of ways in which all letter can be arranged

Such that no two C must come together.

Sol 1



$$({}^{3}C_{3} \cdot 3!)({}^{4}C_{4} \cdot 4!) = 3!\,4!$$
$$= 144$$

Sol 3 A A A A A B B B B C C C D E E F

(A A A A A B B B B D E E F)

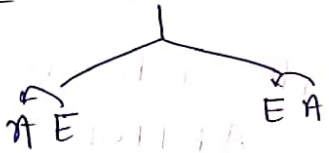
$$\frac{13!}{5!\,4!\,2!} \cdot \frac{{}^{14}C_{3} \times 3!}{3!}$$

A B E C

Sol 2 G A R D E N

Vowel must come in alphabetical order

$$6! = 720$$



$$\frac{720}{2} = 360$$

eg. 1 2 3 4

Case in which $1 > 2$ $\rightarrow \frac{4!}{2}$

$1 > 2 \& 3 \rightarrow \frac{4!}{3}$

iii) $1 > 2 > 3 = \frac{4!}{3!}$

Ja it has more than one such case

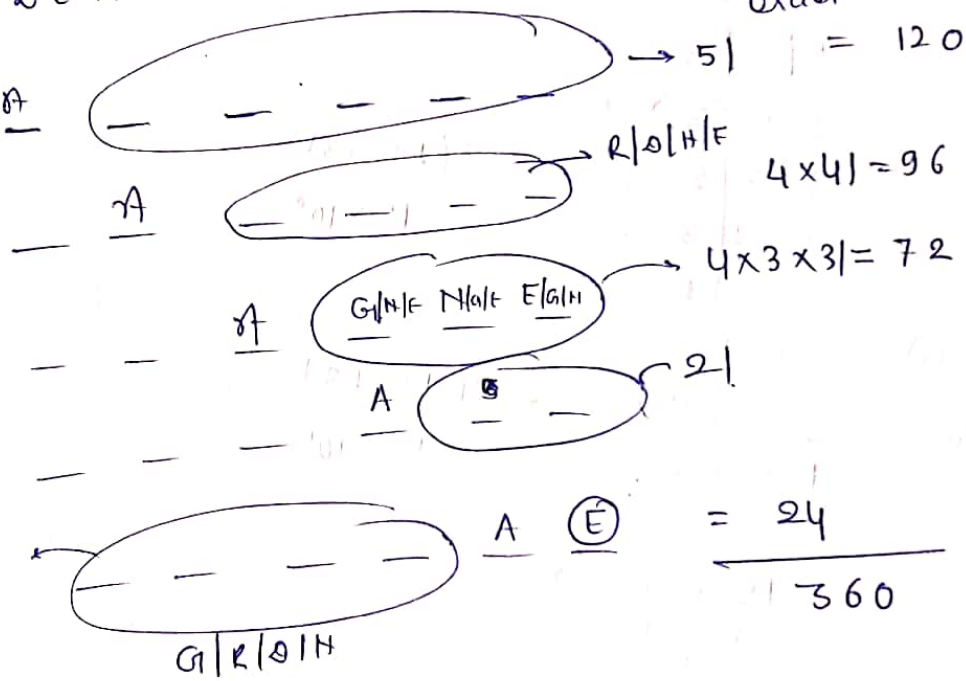
1	2	3	4
4	1	2	3

Hence use factorial

GARDEN

$A > E > D = \frac{6!}{3!} = \frac{720}{6} = 120$

Que. GARDEN vowel must come in Alphabetical order



Que. i) Given that all the Alphabet are distinct and their Value Ranges from 1 to 9

ii) If $A \times B \times C = D \times E \times F = B \times G \times E$

find G = ?

A	D
B G	E
C	F

Que 2

```
A  B  C
   O
E  F  G
   H
   I
```

If all alphabet represent distinct value from 1 to 9
also given that $A+B+C = C+O+E = E+F+G = G+H+I = 13$
find value of all the alphabets.

*Conceptual Que. Find the sum of all the four digit numbers that can be formed using the digit 1, 2, 3, 4 exactly once.

Sol

1 | 2 | 3

1	2	3	
1	3	2	$\rightarrow 2(1+2+3) = 12\times10^0$
2	1	3	
2	3	1	
3	1	2	$2(1+2+3) = 12\times10^2$
3	2	1	$\rightarrow 2(1+2+3) = 12\times10^1$
1	3	3	2

$\Rightarrow$

$$\begin{array}{r} 1200 \\ 120 \\ 12 \\ \hline 1332 \end{array}$$

That means $1|2|3 \longrightarrow \frac{3!}{3}(1+2+3)\times(10^2*10^1*10)$

$= 1332$

for $1|2|3|4 \longrightarrow \frac{4!}{4}(1+2+3+4)(10^3*10^2+10^1+10^0)$

$= 6(10)(1111) = 66660$

$$1|2|3|\cdots n = \left[\frac{n!}{n}(1+2+3+\cdots n)(10^{n-1}+10^{n-2}+\cdots 10^3+10^2+10^1+10^0)\right]$$

↳ Sum of all n digit numbers formed by (1, 2, 3 ... n)

where,

$\frac{n!}{n}$ → Number of time each digit arriving in Vertical column

$\frac{n!}{n}(1+2+3+\cdots n)$ → denotes the digit Sum

$(10^{n-1}+10^{n-2}+\cdots 10^2+10^1+10^0)$ → Place Values of all digits

Que. find the Sum of all 5 digit numbers that can be formed using the digits — (exactly once)

I) 1, 2, 3, 4, 5

Sol $\frac{5!}{5}(5+4+3+2+1)(10^4+10^3+10^2+10^1+10^0)$

$= \frac{5!}{5}(15)(11111)$

II) 1, 1, 2, 3, 4

Sol $\frac{5!}{5\times 2!}(1+1+2+3+4)(11111)$

III) 1, 1, 2, 2, 2

Sol $\frac{5!}{5\times 3!\times 2!}(2+2+2+1+1)(11111)$

IV) 0|1|2|3|4

Sol $\frac{5!}{5}(0+1+2+3+4)(11111) - \frac{4!}{4}(1+2+3+4)\times(1111)$

↳ for Very first place zero

5) 0 | 1 | 2 | 2 | 3

Sol) $\frac{5!}{5 \times 2!}(0+1+2+2+3)(11111) - \frac{4!}{2! \times 4}(1+2+2+3)(1111)$

Challenge Que. Find the Sum of all the 5 digit numbers that using the digits

i) 0 0 1 2 3

ii) 0 0 1 1 2

Conceptual Que. If all the letters of the word AGAIN are arranged as per English dictionary then find the

1) 10^{th}

2) 25^{th}

3) 49^{th} word

4) 50^{th}

5) 54^{th}

6) 57^{th} word of the Seria

Sol

A	$\_\ \_\ \_\ \_ \rightarrow \overline{A}\,|\,\overline{G}\,|\,\overline{I}\,|\,\overline{N}$	$4! = 24$	$1^{st} - 24^{th}$
G	$\_\ \_\ \_\ \_ \rightarrow \overline{A}\,|\,\overline{A}\,|\,\overline{I}\,|\,\overline{N}$	$\frac{4!}{2!(A)} = 12$	$25^{th} - 36^{th}$
I	$\_\ \_\ \_\ \_ \rightarrow \overline{A}\,|\,\overline{A}\,|\,\overline{G}\,|\,\overline{N}$	$\frac{4!}{2!(A)} = 12$	$37^{th} - 48^{th}$
N	$\overline{A}\,|\,\overline{A}\,|\,\overline{G}\,|\,\overline{I}$	$\frac{4!}{2!(A)} = 12$	$49^{th} - 60^{th}$

57th word :

(N) | _ _ _ _ 49th - 60th
A | A | G | I

(A) | _ _ _ → 6 | A G I
A | G | I = 6

A G I → 49th → N A A G I
A I G 50th → N A A I G
G I A
G A I
I A G
I G A

(G) | _ _ _ → 3 | 2!
A | A | I

A A I
A I A
I A A

57th → N A G I A A Ans.

48th word

(I) | _ _ _ _ → 12 37th - 48th
A | A | G | N

(A) | _ _ _ → 6
A | G | N

A G N
A N G
G A N
G N A
N A G
N G A

48th = I A G N A

note:

AG A I N →

A A I G N →

(24) (12) (12) (12)

50th ⇒ 24 + 12 + 12 = 48 i.e start with N

(N) Remain A A I G
(24) (12) (12)

N A A I G

Que. OFFER

Sol 50th

EFFOR

(12) (24) (12) (1)

REE OF //

Que. MOTHER find 100th & 200th also find Rank of MOTHER

Que. ZEPHYR find Rank

Conceptual

Que. How many Committees of five members each can be formed from 8 official and 4 non official members such that—

i) No Condition

ii) 3 official & 2 Non official Members

iii) Atleast 3 official Members

iv) Atmost 3 official Member

v) A particular official Member must always be Selected.

vi) A particular official Member must always be Rejected.

vii) A particular non official Members must always be Selected.

viii) A particular non official Member must always be Rejected

Sol:- i) $^{12}C_5$

ii) $^{8}C_3 \times {}^{4}C_2$

iii)

Sol $4C_2 \cdot {}^8C_3 + {}^8C_4 \cdot {}^4C_1 + {}^8C_5 \cdot {}^4C_0$

OR Total − Unwanted

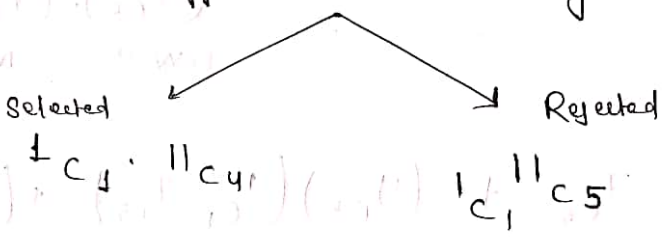
$$^{12}C_5 - \left[{}^8C_2 \cdot {}^4C_3 + {}^8C_1 \cdot {}^4C_1 + {}^8C_0 \right]$$

iv) Atmost 3 official Member

Sol $^8C_3 \cdot {}^4C_2 + {}^8C_2 \cdot {}^4C_3 + {}^8C_1 \cdot {}^4C_4 + {}^8C_0 \cdot {}^4C_5$

=

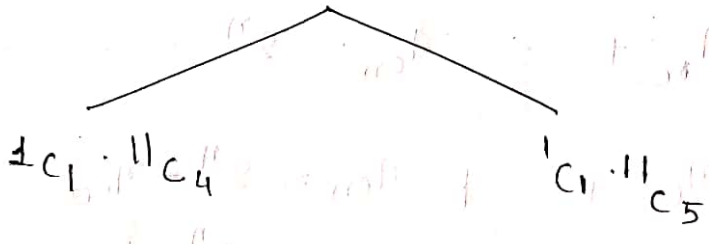
v) Particular official Member always selected



$^1C_1 \cdot {}^{11}C_4$ $\quad$ $^1C_1 {}^{11}C_5$

$= {}^{11}C_4$

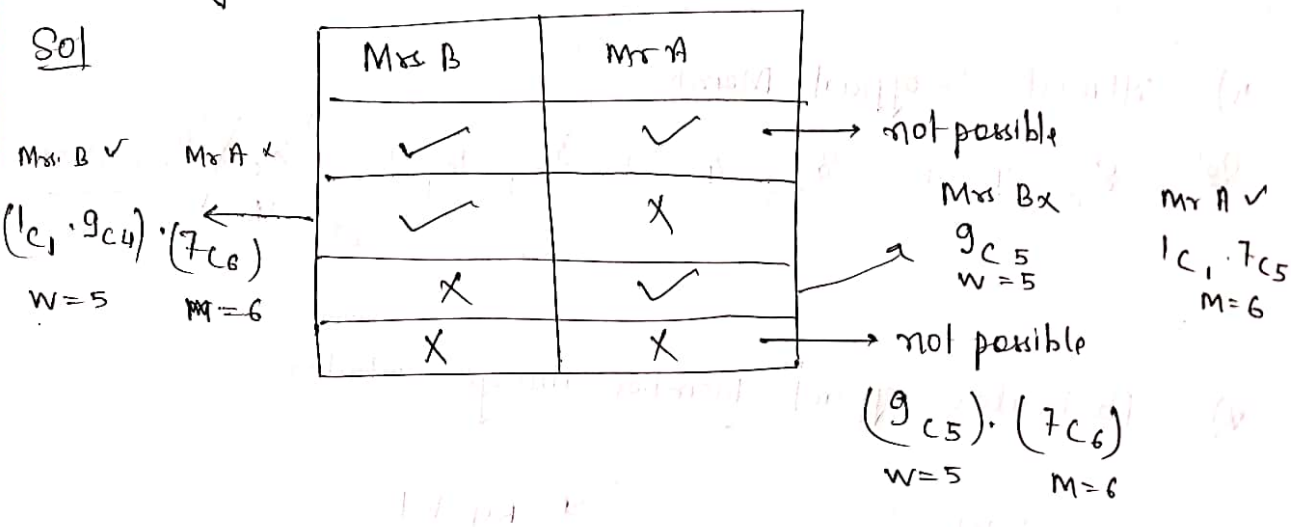
vi) Particular Non official



$^1C_1 \cdot {}^{11}C_4$ $\quad$ $^1C_1 \cdot {}^{11}C_5$

Test 01 Out of 8 Men and 10 women a Committee Consist of 6 Men and 5 Women is to be formed. How many Such Committee Can be formed when one particular Man 'A' refuses to be a Member of Committee in which his Boss B's wife is there

Sol



$$(^1C_1 \cdot {}^9C_4) \cdot {}^7C_6 + (^9C_5)(^1C_1 \cdot {}^7C_5) + (^9C_5)(^7C_6)$$

//.

Note:- Binomial theorem

$$^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n = 2^n$$

$$^nC_1 + {}^nC_2 + \cdots + {}^nC_n = 2^n - {}^nC_0$$

$$= 2^n - 1$$

eg. Question paper

1.	1
	2
	3
2	1
	2

(I) no condition $= 2^3 \cdot 2^2 = 32$

(II) $A \geq 1$ $B \times$ $= (2^3 - {}^3C_0)\, 2^2$

Total number of ways this question paper can be attempted such that -

i) no condition is there

ii) atleast one question from A and no cond^n from B.

iii) atleast two question from A and one question from B

iv) atmost two question from A and atleast one question from B.

Sol

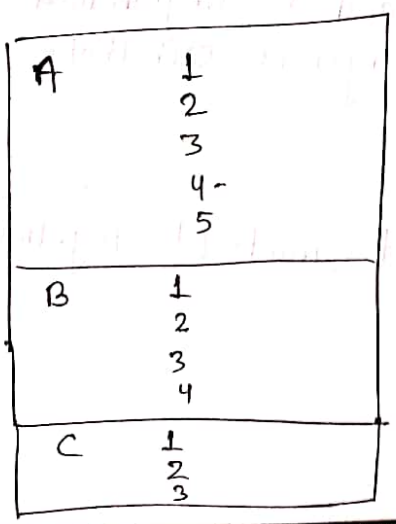
(III) $A \geq 2$, $B \geq 1$

$$\left\{ 2^3 \cdot ({}^3C_0 + {}^3C_1) \cdot \{2^2 - {}^2C_0\} \right]$$

(IV) $A \leq 2$ $B \geq 1$

$$(2^3 - {}^3C_3) \cdot (2^2 - {}^2C_0)$$

Test



1) No condition

2) $A \geq 1$, $B \times$ $C \times$

3) $A \geq 2$ $B \geq 1$ $C \geq 0$

4) $A \leq 3$, $B \geq 2$, $C \geq 1$

Sol 1 $2^5 \cdot 2^4 \cdot 2^2 = 2^{12} = 4096$

Sol 2 $(2^5 - {}^5C_0) \times 2^4 \times 2^3$

Sol 3 $2^5 - ({}^5C_0 + {}^5C_1) \times (2^4 - {}^4C_0)$

Sol 4 $\left(2^5 - ({}^5C_5 + {}^5C_4)\right] \times \left[2^4 - ({}^4C_0 + {}^4C_1)\right] \left[2^3 - ({}^3C_0)\right]$

Note:- Circular Permutation

Total number of arrangement of N different things -

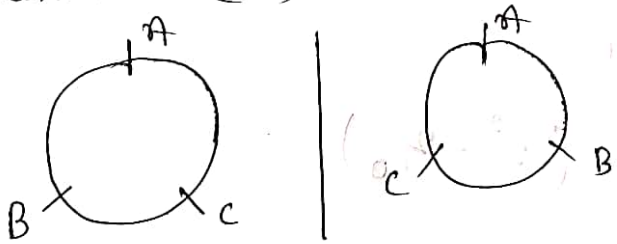
Linear arrangement = $n!$

Circular arrangement = $(n-1)!$

eg:- If there are 3 boys A, B & C then their total number of arrangement -

Linear = $3! = 6$

Circular = $(3-1)! = 2! = 2$



eg2:- If there are 5 Hindimedium and 5 English Med. student then find total number of arrangement such that -

i) No condition — Linear, Circular

ii) No two H medium student must sit together — Linear, Circular

Sol 1) ~~$^5C_5 \cdot 5! \cdot {}^5C_5 \cdot 5!$~~

Linear :- $5! \left({}^6C_5 \cdot 5! \right)$

Circular $4! \left({}^5C_5 \cdot 5! \right)$

$= 4! \, 5!$

note: In case of Garland, (Pearls) beads total no. of arrangent

$= \frac{1}{2} (n-1)!$

Note:

- Point
- $\longrightarrow$ Ray
- $\longleftrightarrow$ Line
- $\overline{AB}$ Line Segment

A $\longrightarrow$ B

${}^nC_2$

Ex. I) Geometry
II) Handshake
III) Tournament
IV) Gift exchange

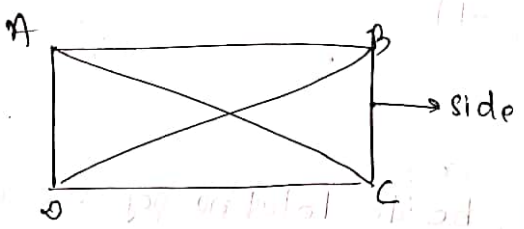
A $\longleftrightarrow$ B

$2 \times {}^nC_2$

ex. Gift exchange

Conceptual Q. How many Diagonals are there in an n side polygon

n-side diagonals



n side = n Vertices

Straight lines = Diagonals + Side

n_{C_2} = Diagonal + n

$$\text{Diagonal} = n_{C_2} - n$$

Note:- Minimum number of points Requied to draw a —

I) straight line → 2 points

II) Triangle → 3 point

III) Circle → 3 point

Eg: find the number of straight lines, Diagonals, triangles that can be formed in a Decagon.

Sol Decagon $n = 10$

Straight line $n_{C_2} = 10_{C_2} = 45$

Diagonals $n_{C_2} - n \Rightarrow {}^{10}C_2 - 10 = 35$

Triangle: $n_{C_3} = 10_{C_3} = 120$

eg 2 There are 12 points in a plane of which five points are collinear (3 or more pt.). Find the total number of -

i) straight

ii) triangle

that can be formed using these points.

Sol: straight line

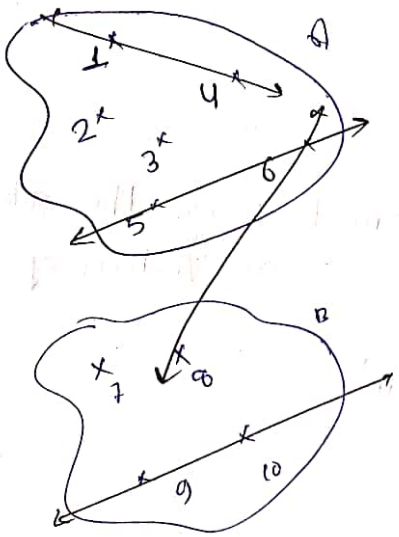
$$^{12}C_2 - {}^5C_2 + 1 = 57$$

note: Straight line = $\text{Total}_{C_2} - \text{Collinear}_{C_2} + 1$

Triangle = $\text{Total}_{C_3} - \text{Collinear}_{C_3}$

Triangle = $^{12}C_3 - {}^5C_3 = \frac{12 \times 11 \times 10}{3 \times 2} - \frac{5 \times 4 \times 3}{3 \times 2}$

$= 220 - 10$



A A + A B + B B

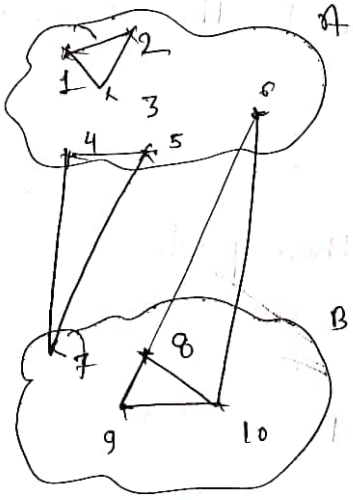
$^6C_2 + {}^6C_1 \cdot {}^4C_1 + {}^4C_2$

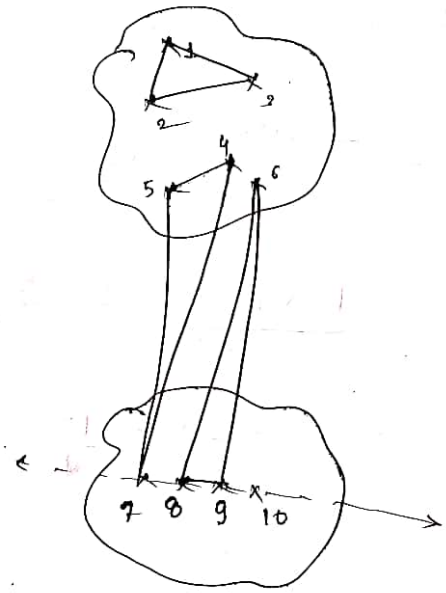
15
+24
+6
45

AAA + AAB + ABB + BBB

$^6C_3 + {}^6C_2 \cdot {}^4C_1 + {}^6C_1 \cdot {}^4C_2 + {}^4C_3$

$= 120$





AAA ⟶ $^6C_3$

\+ +

AAB ⟶ $^6C_2 \cdot {}^4C_1$

\+ +

ABB ⟶ $^6C_1 \cdot {}^4C_2$

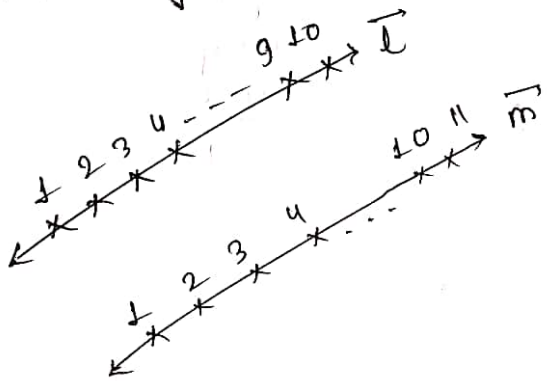
\+ +

BBB ⟶ 0

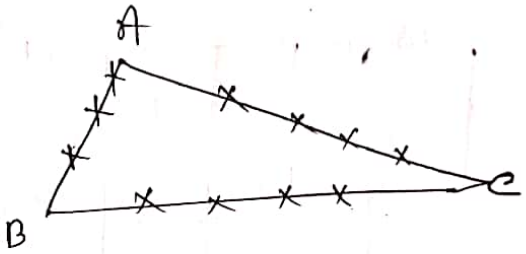
116

Test 03

① Total no. of Δ can be formed using the above point as vertices of Δ.



(2)



Sol:

1) Method 1

$\cancel{l \times l} + lm + lmm + m\cancel{\times}m$

$\cdot\ 10C_2 \cdot 11C_1 + 10C_1 \cdot 11C_2$

$= 1045$

Method 2

Total - unwanted

$21C_3 - (10C_3 + 11C_3)$

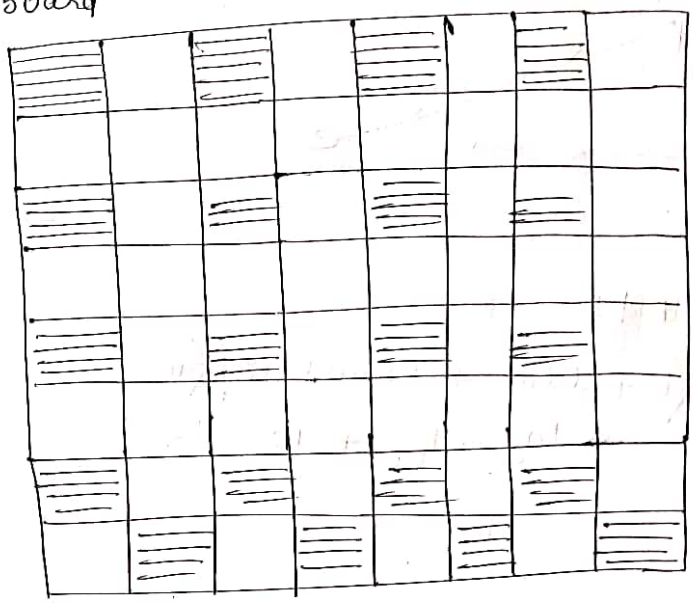
$= 1045$

Sol 2 $\quad$ Total $C_3 - (\overset{AB}{3C_3} + \overset{BC}{4C_3} + \overset{CA}{5C_3})$

OR

$3C_1\ 4C_1\ 5C_1 + 3C_2(4C_1 + 5C_1) + 4C_2(3C_1 + 5C_1) + 5C_2(5C_1 + 4C_1)$

note: Chess Board



① Squares :- $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2 + 8^2$
$(8\times8)\ (7\times7)\ (6\times6)\ (5\times5)\ (4\times4)\ (3\times3)\ (2\times2)\ (1\times1)$

$= 204$

② Rectangles :- $H_2C_2 \times V_2C_2 = {}^9C_2 \times {}^9C_2 = 1296$

= parallelogram

note: if ask Rectangles excluding square. (Rectangle is a Square small)

then $1296 - 204 = 1092$
Rect − Square

note: If ask no. of parallelogram from m parallel line intersecting n parallel line

$= {}^mC_2 \times {}^nC_2$

no. of Rectangle in Same Case = Not determined because angle is not given (90° must for Rect)

Que. Everybody in a Room shakehand with everybody else If the total number of handshake are 66 . Then total number of people in the Room.

Sol

$$n_{C_2} = 66$$

$$\frac{n!}{2\,(n-2)} = 66$$

$$\Rightarrow \quad n(n-1) = 132$$

$$= \quad n^2 - n - 132 = 0$$

$$\Rightarrow \quad n^2 - 12n - 11n - 132 = 0$$

$$\Rightarrow \quad n = 12$$

OR

Product of two Concecutive no. | even no | odd n

$$= n^2 \simeq 132$$

$$n = 12$$

Que. 930 Diwali Greeting card are exchanged among the students of a class. If every student sends a greeting card to every other student Then find total no. of student in the class

Sol:

$$2\,n_{C_2} = 930$$

$$2\,\frac{n(n-1)}{2} = 930$$

$$\Rightarrow \quad n(n-1) = 930$$

$$n^2 \simeq 930$$

$$\Rightarrow \quad n = 31$$

* Probability

B. Pascal & Kolmogorov

Experiment → Deterministic → outcome
Experiment → Random / Probabilistic → outcome range

Trial → Event → favourable / unfavourable

Sample space = fav. + unfav.

$P(E) = \frac{\text{जो चाहिए}}{\text{जो नहीं चाहिए सब कुछ}} = \frac{\text{fav}}{\text{Sample space}}$ $\quad 0 \leq P(E) \leq 1$

Conceptual Ques1. An unbias dice is thrown find the probability of getting i) an even no.

ii) a multiple of 3

iii) an even no. or multiple of 3

iv) an even no. and multiple of 3

Sol $S = 6 \quad \{1, 2, 3, 4, 5, 6\}$

i) $P(E) = \frac{3}{6} = \frac{1}{2}$

ii) $P(E) = \frac{2}{6} = \frac{1}{3}$

iii) $P(E) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$

iv) $P(E) = \frac{1}{2} \times \frac{1}{3}$

$= \frac{1}{6}$

Note:

$n(A \cup B) = n(A \text{ or } B)$

$= n(A) + n(B) - n(A \cap B)$

$A \{1, 2, 3\}$

$n(A) = 3$

$B = \{1, 1, 2\ 2, 3\ 3\ 3\}$

$n(B) = 7$ ✗

$= 3$ ✓ $\quad B = \{1, 2, 3\}$

$Z = \{1, 2, 3 \text{ --- } 100\}$

$P(Z) = 2^n = 2^{100}$

$P(A \cup B) = P(A \text{ OR } B)$

$= P(A) + P(B) - P(A \cap B)$

Sol3 $P(E \text{ or } 3n) = P(E) + P(3n) - P[E \cap 3]$

$= \frac{1}{6}[3 + 2 - 1]$

$= \frac{4}{6} = 2/3$

Sol4 $P(E \cap 3) = 1/6$

Que 2 If two dice are thrown. find the probability of getting.

i) an even no. as a Sum

ii) Multiple of 3 as a Sum

iii) Sum atleast 10

Sol $S = 36$

i) $P(E) = \frac{18}{36} = 1/2$

~~ii) P(E) = 9/36~~

$E+E = E$ ✓ $\frac{3}{2|4|6} \times \frac{3}{2|4|6} = 9$

$E+O = O$

$O+O = E$ ✓ $\frac{9}{1|3|5} \times \frac{3}{1|3|5} = 9$

3 — 1|2 2|1

6 — 3|3 1|5 5|1 4|2 2|4

9 — 6|3 3|6 ~~5|4~~ ~~4|5~~ 4|5 5|4

12

18

$P(E) = \frac{18}{36} = 1/2$ //

ii) $P(3n) = 3|6|9|12$

3 → (1,2)

6 → (1,5) (2,4) (3,3)

9 → (3,6) (4,5)

12 → (6,6)

$= \frac{12}{36}$

$= 1/3$

iii) $P(\text{sum} \geq 10) = P(\text{sum}=10) + P(\text{sum}>10)$

$(4,6) \rightarrow 2!$

$(5,5)$

11 → (5,6) → 2!

12 → (6,6)

$$\frac{6!}{3^6} = \frac{1}{6}$$

Que. If three dice are thrown. find probability of getting Sum

i) At least 6

ii) Exactly 6

iii) At most

note.

$P(E) + P(\bar{E}) = 1$

occur not occur

Sol:- $n(S) = 6^3$

i) Atleast 6 i.e $P(\text{sum} < 6)$

Sol only 6 min. $= 1+1+1 = 3$

$P(S=3) + P(S=4) + P(S=5)$

$\Rightarrow \frac{1}{6^3} + \frac{3!}{6^3} + \frac{3!}{6^3}$ X

$\Rightarrow 1 - \frac{13}{6^3}$

$=$

ii) Exactly 6

$\Rightarrow P(S=6) = \frac{3!}{6^3}$ X

iii) At most 6

$\Rightarrow \left(1 - \frac{13}{6^3}\right) \frac{3!}{6^3}$ X

Sol 1 $P(6) = 1 - P(S<6)$

$P(S<6)$ —
- 3 → $(1,1,1) \rightarrow \frac{3!}{3!} = 1$
- 4 → $(1,1,2)$
- 5 → $(1,1,3)$, $(1,2,2)$ → $3!/2! = 3$

$3 \times 3 + 1$

$= 10$

$= 1 - \frac{10}{216}$

$= \frac{206}{216}$ //.

Sol 2: $P(S=6)$

$(1,1,4)$ | $(2,2,2) \rightarrow \frac{3!}{3!} = 1$

$(1,2,3)$

$\frac{3!}{2!} = 3$

$3! = 6$

$\Rightarrow 6 + 3 + 1 = 10$

$P(S=6) = \frac{10}{216} = \frac{5}{108}$

Sol At most 6

$P(\leq 6) \longrightarrow P(<6) + P(=6)$

$\frac{206}{216} + \frac{10}{216} = \frac{20}{216}$

Que. In A leap year find the probability of getting

i) 53 Sundays
ii) 53 Sun & 53 Mon
iii) 53 Sun & 53 Tue
iv) 52 Sun
v) 52 Sun & 53 Mon
vi) 52 Sun & 53 Tue
vii) 52 Sun & 52 Mon
viii) 52 Sun & 52 Tue
ix) 54 Sun

Sol leap year = 366 day

i) 52 week = 52 times all day

Sun|M ✓
M|T
T|W
W|Th
Th|Fri
Fri|Sa
Sat|Sun ✓

$P(\text{Sun}=53) = \frac{2}{7}$ < Sun|M, Sat|Sun

ii) 53 Sun & 53 Mon

$P(53S \cap 53M) = \frac{2}{7} * \frac{2}{7} \frac{1}{7}$ { Sun, Mon }

iii) $P(S \cap T) = \frac{2}{7} * \frac{2}{7}$ $0/7 = 0$

IV) Sun = 52

Sol $P(S52) = \frac{52}{52} = 1$

$P(Sun \geq 52)$

$P(Sun = 52) + P(Sun > 52)$

$5/7 + 2/7 = 7/7$

$= 1$

V) S = 52 & Mon = 53

Sol $1 \times \frac{2}{7}$ ✗

Bcz in question

$P(Sun = 52)$ { ✗ Sat | Sun, ✗ Sat | Sun

$= 1 - \frac{2}{7}$

$= 5/7$ Ans.

VI) $1 \times \frac{2}{7}$ ✗

VII) 1 ✗

VIII) 1 ✗

IX) 0 $\frac{0}{52} = 0$ //

Sol VI) S = 52 & M = 53

only M | T

$= 1/7$

Sol 8: not Com

Sun | M, M | T

S | S, T | W

$1 - 4/7$

$= 3/7$ Ans.

Sol VII) MT, TW

$2/7$

Sol VIII) only T | W, W | T, Th | F, F | S

$4/7$

Sol IX: $P(S \text{ in a}) = 0$

Que. If all the letters of word SOCIETY the what is probability of -

i) getting S must ocupy first place

ii) E must ocupy last place

iii) All the Vowels must sit together

iv) All the Vowels are never togethe

v) no two Vowels are to gether

Sol $n(S) = 7!$

i) (S) _ _ _ _ _ _

$\frac{6!}{7!} = 1/7$ Ans.

ii) $\frac{6!}{7!} = 1/7$ Ans.

iii)  _ _ _ _

O E C

$\frac{3! \times 5!}{7!} = \frac{3 \times 2}{7 \times 6 \times 1} = \frac{1}{7}$ Ans.

iv) $1 - \frac{3! \times 5!}{7!} = 6/7$

v) $\frac{3}{\cdot} - \frac{2}{\cdot} - \frac{1}{\cdot} - - \Rightarrow \frac{4! \times {}^5C_3 \, 3!}{7!}$

$\frac{3! \times 4!}{7!} \times$ $= 2/7$

Que. Two Integers are Selected at Random from 1 to 11. If the Sum is even then find the probability that —

i) Both Integers are odd

ii) Both Integers are even

E = 2 | 4 | 6 | 8 | 0

O = 1 | 3 | 5 | 7 | 9 | 11

Sol :- Sample = 2

E + E = E

O + O = E

Sample space = $^6C_2 + {}^5C_2$

$$P(\text{Even}) = \frac{{}^5C_2}{{}^6C_2 + {}^5C_2} = \frac{\frac{5\times4}{2}}{\frac{6\times5\times4}{2} + \frac{5\times4}{2}} = \frac{1}{6+1}$$

$$= \frac{10}{15+10} = 2/5 \text{ //}$$

$$P(\text{odd}) = \frac{{}^6C_2}{{}^6C_2 + {}^5C_2}$$

$$= \frac{\frac{6\times5}{2}}{\frac{6\times5}{2} + \frac{5\times4}{2}} = \frac{15}{25} = 3/5 \text{ //}$$

Que. for the Range of 1 to 100 both inclusive ~~find~~ If A number is Selected then find the probability of getting

i) a prime no.

ii) Composite no.

iii) A perfect sq.

iv) Perfect Integer

v) $\div 2$

vi) $\div 3$

vii) $\div 2 \cup \div 3$

viii) $\div 2 \cap \div 3$

ix) $\div 2 \cdot \overline{3}$

x) $\div$ by 3 but not 2

xi) no. of factors $= 6$

both 1 to 100 $1 < N < 100$

from 1 to 100 $1 \leq N \leq 100$

no. of prime

1 – 100 = 25

1 – 75 = 21

1 – 50 = 15

1 – 25 = 09

1 – 200 < 1-100 = 25, 101-200 = 21

46

Sol $S = 100$

i) $P(E) = \frac{25}{100} = \frac{1}{4} \Rightarrow \frac{25C_1}{100C_1} = 1/4$ //

iii) $P(E) = \frac{10}{100} = 1/10$ //

iv) $P(E) = \frac{100}{100} = 1$ X

Perfect Integers other than no. itself Sum of other no. give Same no.

v) $P(\div 2) = \frac{50}{100} = 1/2$ X

vi) $P(\div 3) = \frac{33}{100} = 33/10$ X

VII) $\frac{50}{100} + \frac{30}{100}$

>

VIII) $\frac{16}{100}$

IX) $P(2.\overline{3}) = \frac{50}{100} \times \frac{25}{100}$

X) $P(\pm 3.\overline{2}) = \frac{30}{100} \times \frac{25}{100}$

XI) $n = 6$ factors

II) Composite no. 74/100

IV) Perfect Integer

6

1 x 6

2 x 3

2+3+1=6

28

1 x 28

2 x 14

4 x 7

28

2/100 = 1/50 //.

3 digit	4 digit
4 9 6	8 1 2 8

Sol 11

To get $n = 6$

$$N = a^p \times b^q \times c^r$$

$$n = (p+1)(q+1)(r+1)$$

To obtain 6 $\begin{cases} 1 \times 6 & p = 5 \\ 2 \times 3 & p = 2,\ q = 1 \end{cases}$

check $p = 5 \Rightarrow a^5 \begin{cases} 2^5 = 32 \checkmark \\ 3^5 = 243 \times \text{ not in Rang.} \end{cases}$

$p = 2, q = 1 \Rightarrow a^2 \times b^1$

$2^2 \times \underline{3} = 12$ $\quad \frac{100}{4} = 25$

$= 4 \times 5 = 20$

$\vdots \qquad \vdots$

$25 \qquad 92$

X
2
3
5
7

9
11
13
17
19

Total = 8 ✓

2 won't come in place of b

$2^2 \times \underline{2} = 2^3$ i.e $p = 4$ but we need 6

$p = 2 \times q = 1$

$3^2 \times \underline{\quad}$ $\quad \frac{100}{9} = 11$

$\frac{2}{3}\times$ $\quad 1/9 = 0$

$\underline{\quad}$ 5 7 11

4 ✓

$P=2$ & $q=1$

$5^2 \times \frac{2}{25 \times 3}$

$\frac{100}{25} \rightarrow 4$

$1/2 \rightarrow 0$

$P=2$ & $q=1$

$7^2 \times 2$

factors

So, $1+8+4+2+1$

$= \frac{16}{\text{Sample space} \rightarrow 100} = 4/25$ Ans.

note:- Set theory

Q. for the Range of 1 to 100 (both Incl.) How many number are there which are i) divisible by 2

ii) divisible by 3

iii) div. by 2 or 3

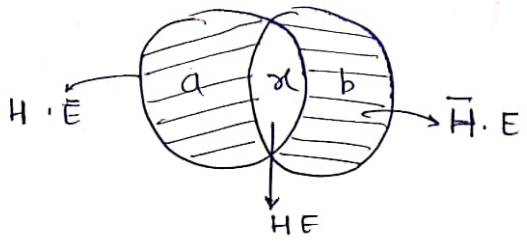
iv) ÷ 2 and 3

v) ÷ $2.\overline{3}$

vi) ÷ $\overline{2} \cdot 3$

vii) ÷ $\overline{2}$ or $\overline{3}$

viii) ÷ $2 \cdot \overline{3}$



$\eta = \bar{H} \cdot \bar{E}$

If all the student of a class appear in two test namely Hindi and English

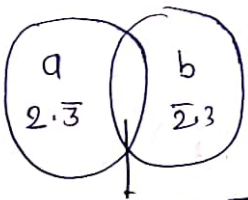
I) Total (T) = $\underset{H\bar{E}}{(a+b)} + \underset{H \cdot E}{x} + \underset{\bar{H} \cdot \bar{E}}{\eta}$

II) $n(H) = a + x$

III) $n(E) = b + x$

$a + b + 2x$

Q. $\div 2 \quad \div 3$



$x = 2 \cdot 3$ $\quad \eta = \bar{2} \cdot \bar{3}$

I) $n(2) = a + x$

$\frac{100}{2} = 50 = a + x$

$a + 16 = 50$

$a = 34$

II) $n(3) = b + x$

$\frac{100}{3} = 33$

$b + 16 = 33$

$b = 17$

(3) $x = 2,3$

$LCM(2,3) = 6$

$\frac{100}{6} = 16$

We know

$100 = (a+b) + x + n$

$n = 100 - 34 - 17 - 16$

$= 100 - 34 - 33$

$= 100 - 67$

$n = 33$

$\div 2 \Rightarrow \quad a + x = 50$

$\div 3 = \quad b + x = 33$

$\div 2 \text{ or } 3 = (\div 2 \cdot \overline{3}) + (\div 3 \cdot \overline{2}) + (\div 2 \cdot 3)$

$= a + b + x$

or

$\text{Total} - n(\overline{2} \cdot \overline{3})$

$\Rightarrow 100 - 33$

$= 67$

4) $\div 2 \text{ and } 3 = (\div 2 \cdot 3) = 16$

5) $\div 2 \cdot \overline{3} = a = 34$

6) $\div \overline{2} \cdot 3 = b = 17$

7) $\div 2$ or $\overline{3}$ $= (\div 2 \cdot 3) + (\div \overline{3} \cdot 2) + (\div 2 \cdot \overline{3})$

$= a + b + \eta$

$= 34 + 17 + 33$

$= 84$

Set Theory :-

Que. For the range of 1 to 100 (both Inclusive). How many numbers are there which are divisible by

i) $\div 2$

ii) $\div 3$

iii) $\div 5$

iv) $\div 2$ or 3

v) $\div 2$ or 5

vi) $\div 3$ or 5

vii) $\div 2$ and 5

viii) $\div 2$ and 3

ix) $\div 3$ and 5

x) $\div 2$ but not by 3 $\quad \div 2 \cdot \overline{3}$

xi) $\div 2 \cdot \overline{5}$

xii) $\div 3 \cdot \overline{5}$

xiii) $\div 2$ or 3 or 5

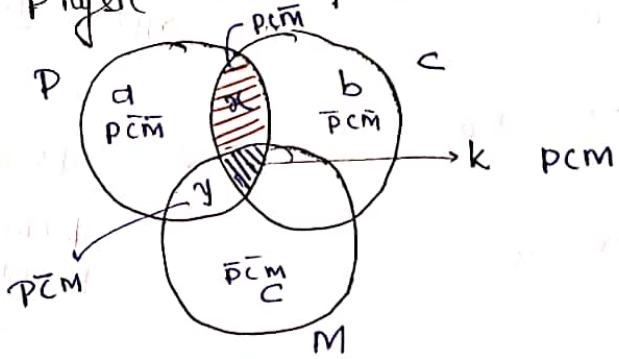
xiv) $\div 2$ and 3 and 5

xv) $\div \overline{2}$ or $\overline{3}$ or $\overline{5}$

xvi) $\div \overline{2}$ and $\overline{3}$ and $\overline{5}$

Sol:-

note:- If all the student of a class appeard in 3 test namely Physic chemistry and Maths. Such that



$\eta = \overline{P}\,\overline{C}\,\overline{m}$

Exactly

① $(a+b+c) \Rightarrow \alpha$

② $(x+y+z) \Rightarrow \beta$

③ $k \Rightarrow \gamma$

Pass

That means,

$$\text{Total} = \alpha + \beta + \gamma + \eta$$

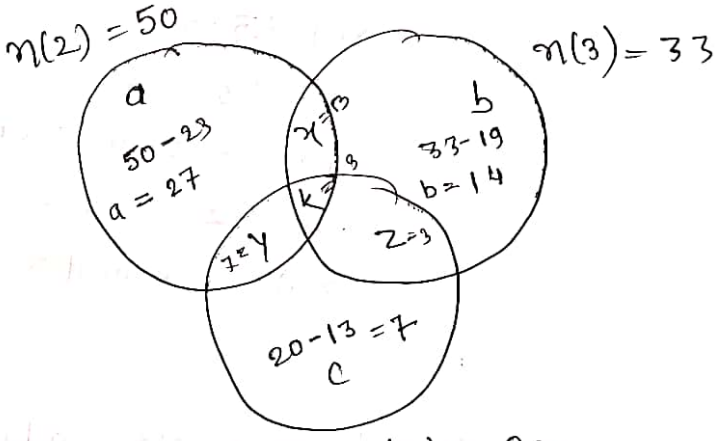
and $n(P) = a + x + y + k$

$n(C) = b + x + z + k$

$n(M) = c + y + z + k$

i.e $n(P) + n(C) + n(M) = (a+b+c) + 2(x+y+z) + 3k$

$n(P) + n(C) + n(M) = \alpha + 2\beta + 3\gamma$



i) $1 \leq N \leq 100$

$n(2) = \frac{100}{2} = 50$

ii) $n(3) = \frac{100}{3} = 33$

iii) $n(5) = \frac{100}{5} = 20$

iv) ÷ 2 and 3 and 5 = k

LCM(2,3,5)

k = 100/30

= 3

v) ÷ 2 and 3 but $\bar{5}$

So) LCM (2, 3)

= 6

but $\bar{5}$ ⇒ $\underline{LCM(2,3)} - k$

⇒ $\frac{100}{6} - \frac{3}{6}$

⇒ 16 − 3

÷ 2 and 3 but $\bar{5}$ = 13 = x

vi) ÷ 2 and 5 but $\bar{3}$

(LCM (2, 5) = 10) − k

$\frac{100}{10} - 3$

⇒ 10 − 3

= 7 = y

vii) ÷ 3 and 5 but $\bar{2}$

⇒ $\frac{100}{15} - k$

= 6 − 3

= 3 = z

i) $\div 2 = a + x + y + z$

$= 27 + 13 + 7 + 3$

$= 50$

ii) $\div 3 = x + z + k + b$

$= 33$

iii) $\div 5 = c + y + z + k = 20$

iv) $\div 2$ or $3 = (a + x + y + k) + b + z = 50 + 14 + 3 = 67$

v) $\div 2$ or $5 = a + x + y + k + c + z = 50 + 7 + 3 = 60$

vi) $\div 3$ or $5 = (x + z + k + b) + c + y = 33 + 7 + 7$

$= 47$

vii) $\div 2$ and $3 = x + k \Rightarrow 13 + 3 = 16$

viii) $\div 2$ and $5 = y + k \Rightarrow 7 + 3 = 10$

ix) $\div 3$ and $5 = z + k \Rightarrow 3 + 3 = 6$

x) $\div 2$ but $\overline{3} = (a + x + y + k) - (x + k)$

$= a + y$

$= 27 + 7$

$= 34$

xi) $\div 2$ but $\overline{5} = (a + x + y + k) - (y + k)$

$= a + x$

$= 27 + 13$

$= 40$

XII) $\div 3 \cdot \overline{5}$

Sol $x + k + z + b - z - k$

$\Rightarrow x + b$

$= 13 + 14$

$= 27$

XIII) $\div 2$ or 3 or 5

Sol $\alpha + \beta + \gamma$ OR Total - n

$\Rightarrow (a+b+c) + (x+4+z) + k$ $100 - 26$

$\Rightarrow (27+14+7) + (13+7+3) + 3$ $= 74$

$\Rightarrow 48 + 23 + 3$

$\Rightarrow 74$

14) $\div 2$ and 3 and 5

$\gamma = k$

15) $\div \overline{2} \cdot \overline{3} \cdot \overline{5}$

$n = 26$

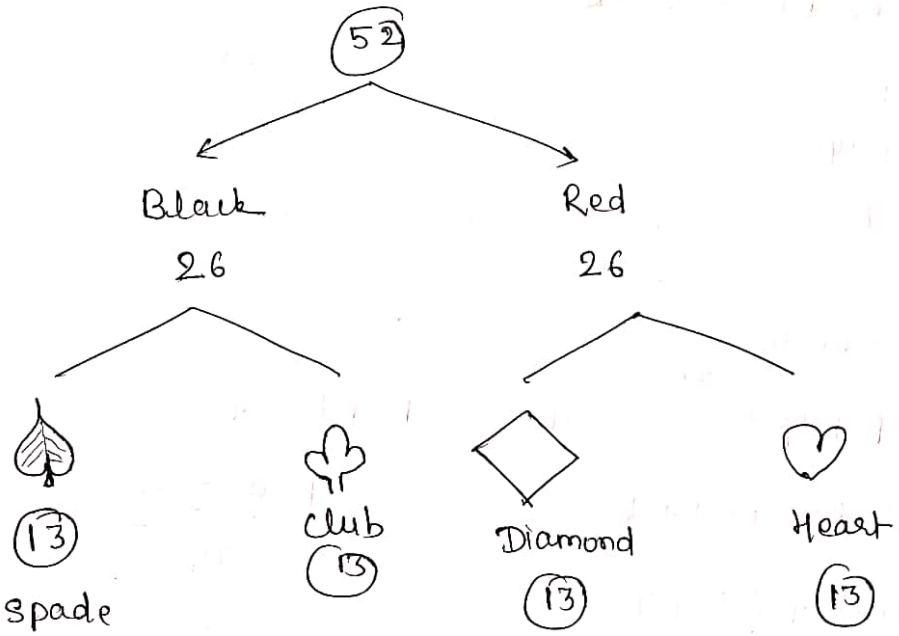
16) $\div \overline{2}$ or $\overline{3}$ or $\overline{5}$

Total - k

$\Rightarrow 100 - 3$

$= 99$

* Playing Cards



Face Card – J | Q | K

Honour Card – J | Q | K | A

note:- Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \quad ; P(B) \neq 0$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \quad P(A) \neq 0$$

Independent Event

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

Where,

$P(A) \longrightarrow$ probability of ocurance of an event A

$P(B) \longrightarrow$ probability of ocurance of event B

$P(A \cap B) \longrightarrow$ Probablity of ocurance of A and B

$P(A/B) \longrightarrow$ Probablity of ocurance of A when B already ocured

$P(B/A) \longrightarrow$ Probablity of ocurance of B when A already ocured.

Independent event

eg. 19, 20, 16, 27

Sol 19

$P(A) = 1/3$

$P(B) = 1/3$

$P(C) = 1/4$

$P(S) = P(A) + P(B) + P(C) + P(A \cap B \cap C)$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{4} + \frac{1}{36}$

$= \frac{12 + 12 + 9 + 1}{36}$

$=$

Sol P(~~Exactly 1 Solve~~)

$1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$

$\Rightarrow 1 - (1 - 1/2)(1 - 1/3)(1 - 1/4)$

$\Rightarrow 1 - 1/2 \times 2/3 \times 3/4$

$\Rightarrow 1 - 1/4$

$= 3/4$ //

Que. 16

Sol $+ - \frac{1}{7} \times \frac{1}{5}$ $\quad P(A) = 1/7$

$P(B) = 1/5$

$\Rightarrow + - \frac{1}{35}$

$\bar{H}\bar{W} = \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{5}\right) = \frac{24}{35}$

Q.20

Sol $1 - (1 - 1/3)(1 - 1/6)$

$\Rightarrow 1 - 2/3 \cdot 5/6$

$\Rightarrow 4/9$

Q27.

Sol $P(A) = 0.2$

$P(B) = 0.3$

$P(A \cup B) = 1 - P(\bar{A})\,P(\bar{B})$

$= 1 - (1-0.2)(1-0.3)$

$= 1 - 0.8 \times 0.7$

$= 0.44$

Q52.

Sol $S = 2^5 = 32$

$5C_3 \cdot 5C_2 \cdot 5!2! = \frac{5 \times 4}{2} \times \frac{5 \times 4}{2} \times 3 \times 2 \times 2 \times 1$

$\frac{}{2^5}$

$\frac{}{32}$

$= 10 \times 10$

$S = 2^5 = 32$

HHHTT $\longrightarrow \frac{5!}{3!2!} = \frac{5 \times 4}{2} = 10$

$P(S) = \frac{10}{32} = 5/16$

Q74.

Sol $S = 9C_4$

$P = \frac{4C_2 \cdot 5C_2}{9C_4}$

Q Test 1

21, 22, 23, 14

Sol 21 $\quad$ 0 2 3 5

2 2 1 $\quad$ 0|5 $\quad$ = 4

3 ~~2~~ ~~1~~ $\quad$ 0 $\quad$ = ~~18~~ ~~12~~ 6

~~20~~ 10

Total 3 3 2 1 = 18

$P = \frac{\cancel{06}}{\cancel{18}} = \cancel{16/18} = \cancel{8/}$ $\quad 10/18 = 5/9$

Q22.

Sol 1 to 100

$P = \frac{10}{100} = 1/10$

Q 23

Sol $n_1 < n_2$

$\frac{15}{36}$

Equal Case = 6

Total = 36

Half = $\frac{36-6}{2}$ = 15 Case $n_1 < n_2$

Q14.

Sol $\frac{365}{7} = 52$ $\quad$ Rem. 1

$P = 1/7$ //

Q. $a_3 > a_2 > a_1$ Total 216

~~4~~ +
~~216~~
~~54~~

6 5 4
5 4 3 ✗
4 3 2
3 2 1

Sol $a_2 > a_1$

min 2 > 1

Max 6 > 5

$\frac{5}{2-6}$ > ① $\frac{2}{5|6}$ > ④

$\frac{4}{3-6}$ > ② $\frac{1}{6}$ > ⑤

$\frac{3}{4-6}$ > ③

Total 15

$P = \frac{15}{36} = 5|12$

Sol $a_3 > a_2 > a_1$

fix. Middle no

$\frac{4}{3-6}$ > ② > $\frac{1}{}$

$\frac{3}{4-6}$ > ③ > $\frac{2}{1|2}$

$\frac{2}{5|6}$ > ④ > $\frac{3}{1-3}$ = $\frac{5}{54}$ ||

$\frac{1}{6}$ > ⑤ > $\frac{4}{1-4}$

Que1 If four Single digit whole no. from 0 to 9 are chosen at Random and Multiplied together. Then what is the probablity that the Last digit in the product is 1, 3, 7 9

a) 16/25 b) 16/625 c) 16/125 d) 16/40

e) NOT

Que2 A five digit no. is to be formed from digits 1, 2, 3, 4, 5 without Repeatation.
The probablity that the number formed is divisible by 4

a) 1/5 b) 2/4 c) 2/3 d) 4/5 e) NOT

Que3 In a plane 5 lines of Length 2, 3, 4, 5 6 cm are lying. what is the probablity that by joining three randomly chosen line and two end a triangle Cannot be formed

i) 3/10 ii) 7/10 iii) 1/2 iv) 3/20

Sol1 10 10 10 10 10^4 = 10000

Sample Space =

1 :

Sol 2 : 5 4 3 2 1 = 120 Total

— — — — —

~~00~~ 12 24 32
~~04~~ 16 ~~32~~
~~00~~ ...

~~Total 25~~

Max 4

$\frac{5}{120} = \frac{1}{}$

— —
1-5 1-4

1	2, ●	= 1
2	4	= 1
3	2	= 1
~~4~~	~~4~~	= ~~1~~
5	2	= 1

Sol 2

— — — — —
3!

12
24
32
52

$$P = \frac{4 \cdot 3!}{5!} = 1/5 \text{ //.}$$

Sol 1

unit digit

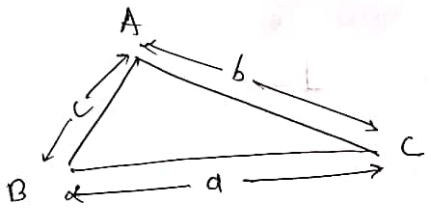
~~0~~	~~5~~
1	~~6~~
~~2~~	7
3	~~8~~
~~4~~	9

any no x 5 = 5, 0

any no. x even = even

$$P = \frac{4 \cdot 4 \cdot 4 \cdot 4}{10^4} = 4^4/10^4 = (0.4)^4$$
$$= 16/625 \text{ //.}$$

Note:



$a+b>c$ | $a \sim b < c$

$b+c>a$ → at the same time → $b \sim c < a$

$c+a>b$ | $c \sim a < b$

↓

we use $|a-b|$

Q. find the sum of all possible values of side a such that a is an integral no.

Sol $b = 12$

$c = 5$

$a = ?$

$b+c = 17 > a$ $\qquad b-c = 7 <$

from: 8 to 16

Sum of value of $a = 8+9+10+11+12+13+14+15+16$

$= \cancel{90+17} \; \cancel{+} \; \cancel{12}$

$= 90+21-3$

$= 90+18 = 108$

Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Greatest Integer function

$x = [x] + \{x\}$

Integer fractional value

2	2	0
2.1	2	0.1
2.5	2	0.5
2.9	2	0.9
-2	2	0
-2.1	-3	0.9
-2.5	-3	0.5
-2.9	-3	0.1

Sol 3 : 2 | 3 | 4 | 5 | 6 $\longrightarrow$ $^5C_3 = 10$

$a + b > c$ ✓ 2 → 3 ⊁ (5)

$a \sim b < c$ ✓ 4

(2) → 3 ⊁ (5)

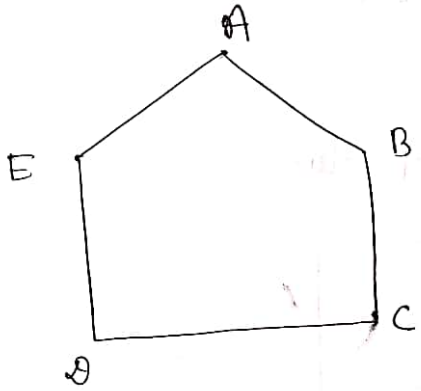
~~(3) 4 > 5~~ not valid

(2, 3, 6)

(2, 4, 6)

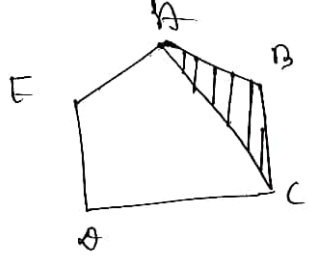
(2 3 5) $= \frac{8}{10}$ //

note

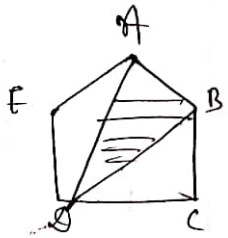


$\Delta \rightarrow \quad {}^5C_3 = {}^5C_2 = 10$

3 Consecutive point or 2 Side



A B C
B C D
C D E
D E A
E A B



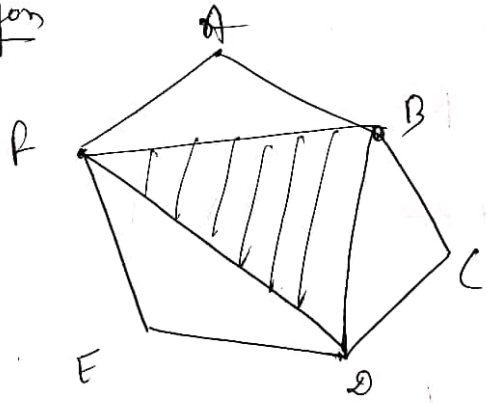
Δ using 2 Consecutive point or 1-Side

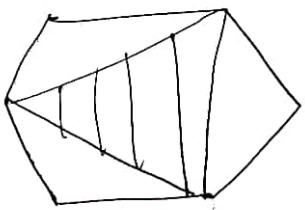
A B D
B C E
C D A
D E B
E A C

(5)

To make a Δ which doesnot ~~Vary~~ uses any side of a polygon, min' hexagon is requ.

Hexagon





no-of Δ that can be formed not using any side of polygon

$= \frac{n(n-4)(n-5)}{6}$

for hexagon $= \frac{6(2)(4)}{6}$

$= 2$ //.

Page 43

Q 5

Sol

20

✓	X
5	-
6	4
7	8
8	12
9	16

25 Q

+4 −1

55 | 63 | 68 | 70

60

✓	X
15	0
16	4
17	8

62

✓	X
16	1
17	5
18	9

68

✓	X
17	0
18	4
19	8

55

✓	X
14	1
15	5
16	9

Ans.

P-45

Q. 17-20

Tara

$\underset{4}{b} + 3x + y = 12 \Rightarrow$ 70

$3x + y = 8$ — (1)

$3 \times 3 + 2f = 12$ ✓

$\Rightarrow$ $3 \cdot 2 + 2 \rightarrow$ X

$3 \cdot 3$ X

Work and Time

P/q	%
1/1	100%
1/2	50%
1/3	33.33%
1/4	25%
1/5	20%
1/6	16.67%
1/7	14.2857 %
1/8	12.50%
1/9	11.11%
1/10	10%
1/11	9.09%
1/12	8.33%
1/13	7.7%

$\frac{1}{7}$ = 14.2857 %

$\frac{2}{7}$ = 28.5714 %

$\frac{3}{7}$ = 42.8571 %

$\frac{4}{7}$ = 57.14285 %

$\frac{5}{7}$ = 71.4285 %

$\frac{6}{7}$ = 85.7142 %

$\frac{7}{7}$ = 100%

$\frac{1}{14}$ 7.14 %.

$\frac{1}{15}$ 6.67 %.

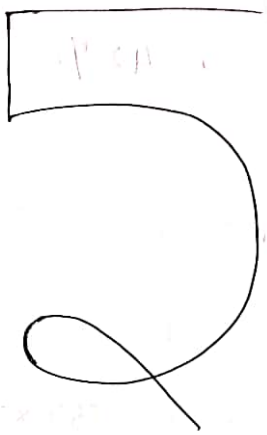
$\frac{1}{16}$ 6.25 %.

$\frac{1}{17}$ .88 %.

$\frac{1}{18}$ .55 %.

$\frac{1}{19}$ .22 %.

$\frac{1}{20}$ .00 %.



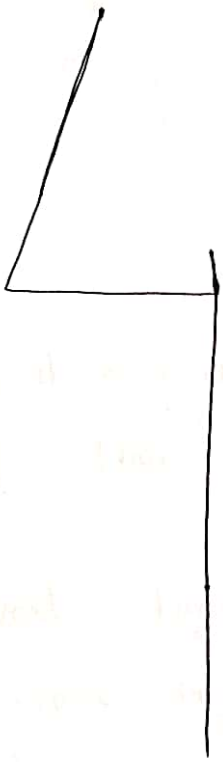
$\frac{1}{21}$ .74 %.

$\frac{1}{22}$ .54 %.

$\frac{1}{23}$ .34 %.

$\frac{1}{24}$ .14 %. $\left\{\frac{1}{24} = 4.16\%\right.$

$\frac{1}{25}$ .00 %.



$\frac{1}{26}$

$\frac{1}{27}$

$\frac{1}{28}$

$\frac{1}{29}$

$\frac{1}{30}$

3 . 84 %

-14

. 70 %

-14

. 56 %

-14

. 42 %

3.33 %

27 ⟷ 37

37 × 1 = 37
37 × 2 = 74
37 × 3 = 111
37 × 4 = 148
37 × 5 = 185
37 × 6 = 222
37 × 7 = 259
37 × 8 = 296
37 × 9 = 333
37 × 10 = 370

37 × 3 = 111
37 × 6 = 222
37 × 9 = 333
⋮
37 × 27 = 999

Concept. Que 01

If 'A' can finish a piece of work in 12 days then find the amount of work done by 'A' in 1 day, 2 day, 3days, 6days & 11 days

Sol

12 days ⟶ 1 work

1 day ⟶ $\frac{1}{12}$ work 8.33%

2 days ⟶ $\frac{1}{12} \times 2$ work 16.67%

3 days ⟶ $\frac{3}{12}$ work 25%

6 days ⟶ $\frac{1}{12} \times 6$ wo 50%

11 days ⟶ $\frac{11}{12}$ wo = 91.67%

Conceptual Que 02

If 'A' and 'B' can finish a piece of work in 12 and 15 days respectively. Then find no. of days required to finish the work —

a) When A & B work together

b) When A & B work together but alternately —

i) When A starts

ii) When B starts

Sol

	Days	one day	%
A →	12	1/12	8.33%
B →	15	1/15	6.67%
A+B →	20/3	$(\frac{1}{12}+\frac{1}{15}) = 3/20$	one day work 15%

⇒ 6.67 days

OR

1 day ⟶ 15% WD Combined

6 day ⟶ 90% ⟶ 6 day

left 10%

15% ⟶ 1

1% ⟶ $\frac{1}{15}$

10% $\frac{10}{15}$ day $= \frac{2}{3}$ ⟶ $\frac{2}{3}$ da

Combined ⇒ $6\frac{2}{3}$ days //

OR

	days
A →	X
B →	Y
A+B →	$\left(\frac{XY}{X+Y}\right)$ days

$\frac{12\times15}{12+15} = \frac{12\times15}{27_9} = \frac{20}{3}$

= 6.67 days //

b) A & B work together but —

i) when A starts

A	B
1/12	1/15

for two day A+B : $\frac{1}{12} + \frac{1}{15} = \frac{3}{20} = 15\%$

Day	1st	2nd	3rd	4th	5th
	A+B	A+B	A+B	A+B	A+B
	A	B	A	B	A	B - -

A + B

1 pattern (2 days) ⟶ 15 %

6 × 2 ⟶ 90 %

12 days ⟶ 90 %

Remain 10 %

now A will arrive A = 1/12 = 8.33 %

Day 13th 10 % − 8.33 %

= 1.67 %

Day 14th :- B arrive B = 1/15 = 6.67 %

$\frac{1.67\%}{6.67\%} \Rightarrow \frac{1/60}{1/15} = \frac{1}{4} = 0.25$

Total days $13 + 0.25$

$= 13.25$ days //.

ii) B start first

Sol B A B A

$15\% \times 6 = 90\%$

12 days = 90%

13th day B 6.67%

$10 - 6.67 = 3.33\%$

14th day A 8.33

$\frac{3.33\%}{8.33\%} \Rightarrow \frac{1/30}{1/2} = \frac{12}{30} = 2/5$

$\therefore 13\frac{2}{5}$ days //.

Concep Que 3 If A, B & C can complete a piece of work in 10, 20 and 25 days respectively then find the no. of days required for the following patterns.

	1st	2nd	3rd	4th	5th	
1	A	B	C	A	B	- -
2	A+B	A+C	A+B	A+C	A+B	- -
3	A	B+C	A	B+C	A	- - -
4	A+B	B+C	C+A	A+B	B+C	- -
5	A+B+C	A+B+C	A+B+C	-	- -	

Sol:- A = 10 days = 1/10 one day work 10%

B = 20 = 5%

C = 25 = 4%

i) A+B+C one day = 10% + 5% + 4%

= 19%

(3 days pattern)

5 × 19% ⟶ 95%

15 days ⟶ 95%

Rem. 5%

16th day $\frac{5\%}{10\%} = 1/2$

Hence 15 1/2 days. //.

ii) A+B = 15%

A+C = 14%

~~One day~~ 29% ⟶ 2 days

2 day

29% × 3 ⟶ 2 × 3 = 6 day

Rem. 13%

7th day $\frac{13}{15} = 0.866$

$6\frac{13}{15}$ //.

iii) A → 10%

B + C → 9%

Total 19%

10 days 19% × 5 = 95%

Rem. 5%

$\frac{5}{10} = 1/2$

10 1/2 days //.

iv) A + B = 15%

B + C = 9%

C + A = 14%

3 days 2 × 38% = 76%

2 day (A + B) + (B + C) = 24%

~~76~~ ~~24~~

Total = 5 days //.

v) A + B + C = 19%

5 days 19 × 5 = 95%

5%

$\frac{5}{19} = 0.26$

5 5/19 days //.

Concept

Que 4 i) If A and B can finish a piece of work in 20 & 25 days respectively. If they started the work together and worked for 5 days and then A left the work. Then find total days required to finish the work

Sol) A → 5%
B → 4%
9% one day

9 × 5

5 days = 9% × 5 → 45%

Rem. 55%

13 days 9 / 55 B → 4 × 13 days = 52%

Remaining = 3%

3% / 4% = 3/4

B% ← 4%

18 3/4 days ||.

ii) If A, B and C can finish a piece of work in 10, 12 and 15 days. resp. If they started the work together but A left the work after 2 days where as B left ~~the~~ ~~Work~~ 3 days before completion of the work. For how many days did the work last

Sol) A → 10%
B = 8.33%
C = 6.67%

2 days work = $2 \times (10 + 8.33 + 6.67)$ —— 2 day

$= 49.99$

$= 50\%$

3 day work = 25% B

3 day work C = 20% ✓ —————— 3 day

50% + 20% = 70% → (2 days A+B+C + 3 days C)

B+C = 15%

Remain ← 30%

B+C ← ~~75%~~

= ~~5~~ 2 days

2 + 3 + 2 = 7 days //.

A+B+C

25% × 2 = 50%

B+C

2 × 15% = 30%

C

6.67 × 3 = 20%

Conceptual Que 5. If A & B, B & C & C & A can complete a piece of work in 24, 30 & 40 days respectively. Then find no. of days required to finish a work — i) when A works alone

ii) when B work alone

iii) C alone iv) A+B+C

Sol $A + B = 24$ days $= 4.14\%$

$B + C = 30$ day $= 3.33\%$

$C + A = 40$ day $= 2.5\%$

1) A work along.

$$(A + B + B + C + C + A) = 9.97$$

$$2(A + B + C) = 9.97$$

$$A + B + C = 4.985\%$$

$$= 20 \text{ days}$$

A work alone.

$$A + B = 4.14\%$$

$$A + C = 2.5\%$$

$$2A + B + C = 6.64$$

$$A = \frac{6.64\% - 3.33\%}{2}$$

$$A = 1.655\%$$

$$A = 60.42 \text{ day} \quad //$$

B work alone

$$2B + A + C = 4.14 + 3.33$$

$$B = \frac{4.14 + 3.33 - 2.5}{2} = 2.485 \Rightarrow 40.24 \text{ days}$$

//

$2C + A + B = 3.33 + 2.5$

$2C = 3.33 + 2.5 - 4.14$

$C = 0.845$

$C \approx 118.3$ day ≈ 120 day

Note. Q. If A man women and child can finish in 6, 10 & 12 days a piece of work. If man start the work and work for $1/3^{rd}$ of a day, then the Women work for another $1/3^{rd}$ of the day and finally the child work for $1/3$ of day. Who was working when work got completed.

Sol $M = 6 \quad 16.67$
$W = 10 = 10$
$C = 12 = 8.33$
35% } one day work

1 day work $= \frac{35\%}{3} = 11.67\%$

8 day $= 11.67\% \times 8 = 93.31$

Remain ≈ 6.7

Men work $\frac{16.67}{3} = 5.5\%$

$6.7 - 5.5 \approx 1.12\%$ Rema.

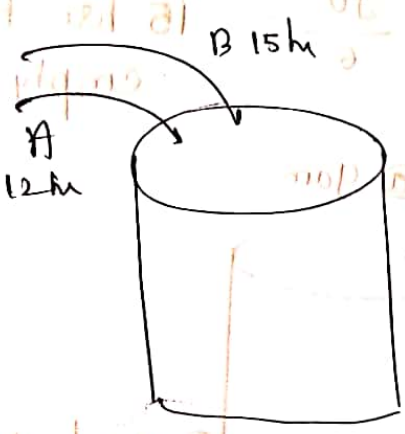
Women will complete

Pipes & Cistern

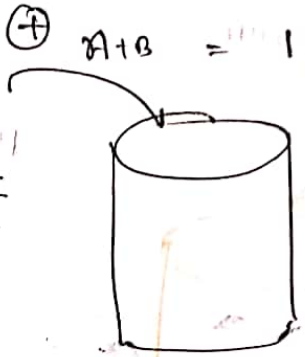
Concept Que 6

1) If two Inlet pipes A and B can fill the Cistern in 12hrs & 15hrs then find how many hrs are required to fill the Cistern.

Sol)

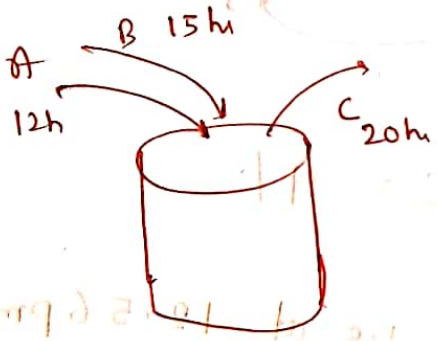


$\equiv$

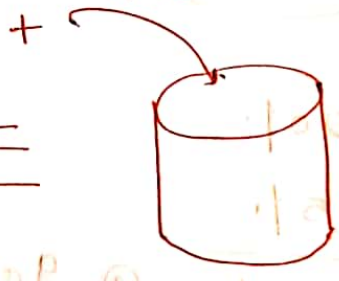


$$\frac{100\%}{15\%} = 6\frac{2}{3}\text{ hrs}$$

ii)

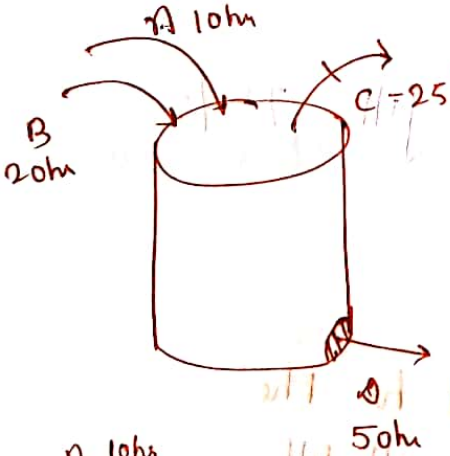


$\equiv$



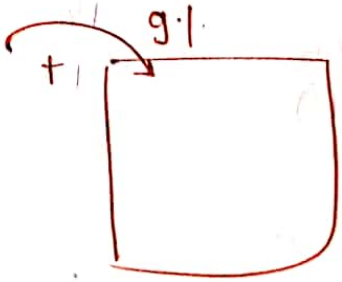
$$= \frac{100}{10} = 10\text{hr}$$

iii)



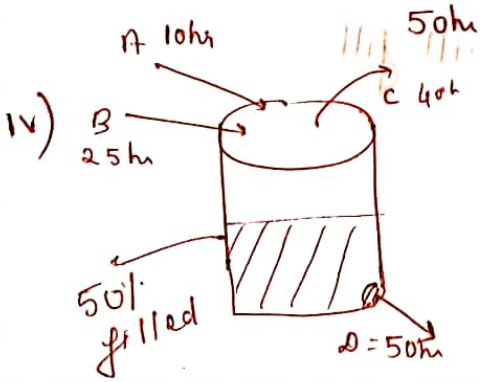
$10\% + 5\% - 4\% - 2\% = 9\%$

$\equiv$



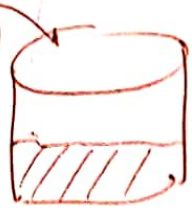
$$\frac{100}{9} = 11.11\%$$

iv)



$10\% + 2\% + 25\% - 2 = 9.5$

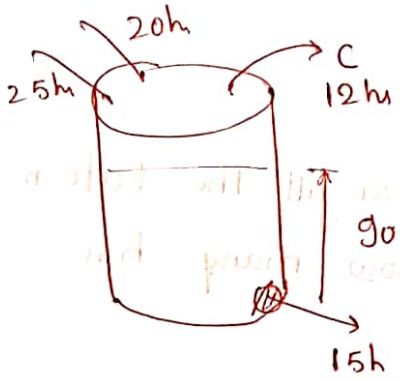
$\equiv$



$$\frac{50}{9.5} = 5.26\text{ hr}$$

Only 50% has to be filled

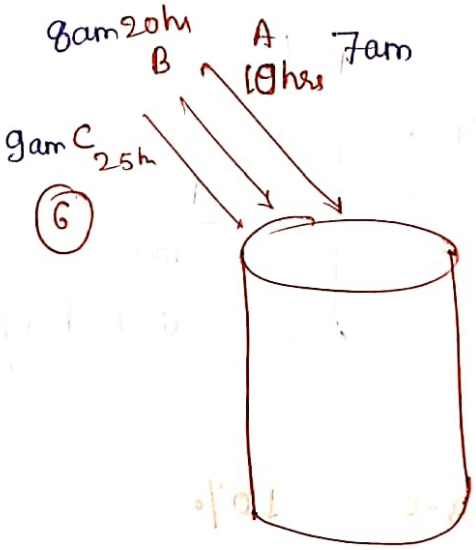
(5)

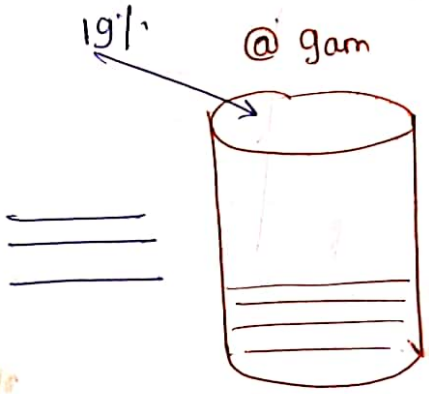


To empty 90%

$= \frac{90}{6} = 15$ hrs to empty //

(6)





A → 20%

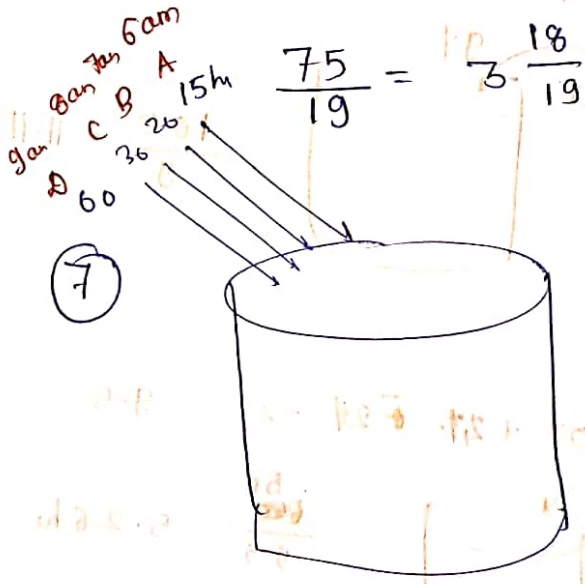
B → 5%

25% @ 9am

C = 4%

i.e at 12.56 pm

(7)

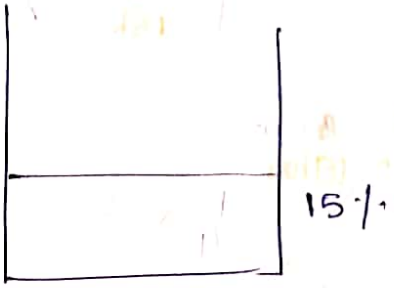


$\frac{75}{19} = 3\frac{18}{19}$ hrs to fill 75%.

At what hr the cistern will fill

Sol A → 6.67 %
B → 5 %
C → 3.33 %
D = 1.67 %

@ 9 am = 15 %



Remain: 75 %
16.67 %

wrong

= 4.49 hr after 9 am

Hence 9 am + 4.49 hr = 1 PM //

Sol 8 am to 9 am : A → $\frac{1}{15} \times 3 = 20\%$

7 am to 9 am : B → $\frac{1}{20} \times 2 = 10\%$

8 am to 9 am : C → $\frac{1}{30} \times 1 = 3.33$

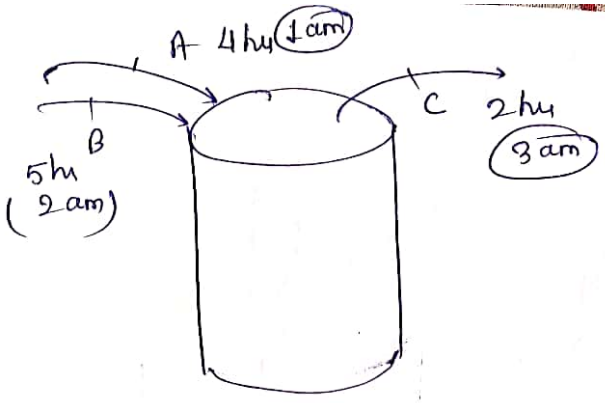
Total 33.33 %

Remain 66.67 %

$\frac{6667}{16.67} \approx 4$ hr

Hence 9 am + 4 hr = 1 pm Ans

Que.



Sol :- @ 3 am

25% A → $\frac{1}{4} \times 2 = \frac{1}{2} = 50\%$

20% B → $\frac{1}{5} \times 1 = \frac{1}{5} = 20\%$

70%

~~Remaing 30%~~

C = 50%

~~30%~~

~~(25+20-50)%~~

at 3 am

⇒ 25% + 20% - 50% = -5%

It will empty

$\frac{70}{5}$ = 14hr i.e 5 PM //

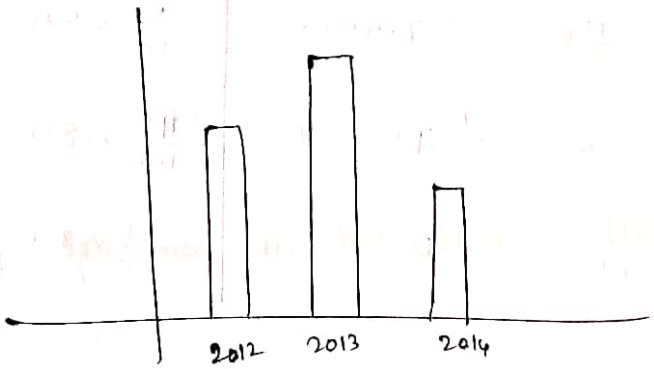
Data Interpretation

1. Table :- Data

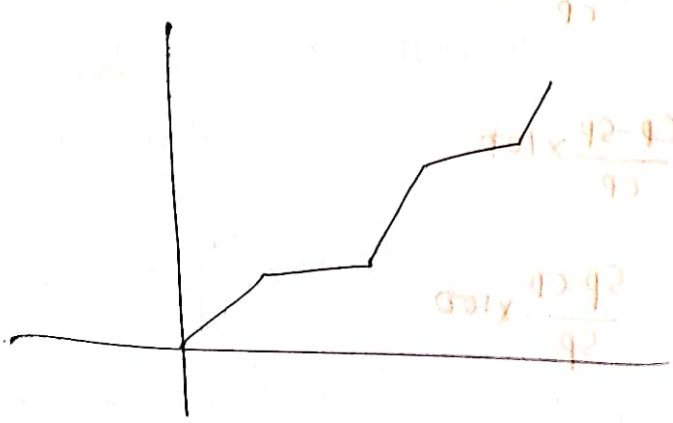
B	S	R	C

2. Pie chart → Distribution

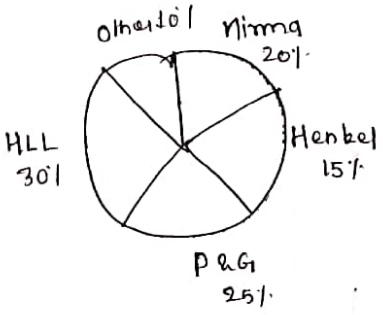
3. Bar



4. Line

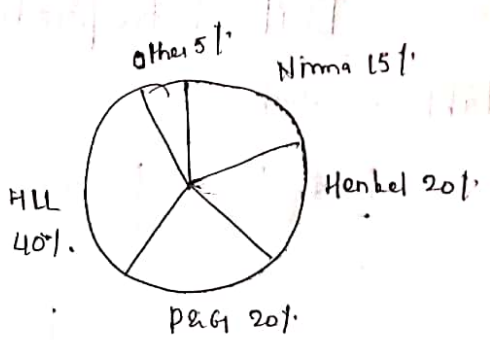


Que.
Ch-3
38



Year 2002

Rs. 150 Cr



Year 2003

Rs. 375 Cr

Given: Value wise graph

2002	2003
Nirma = $\frac{20}{100} \times 150 = 30$	$\frac{15}{100} \times 375 = 56.25$
Henkel = $\frac{15}{100} \times 150 = 22.5$	$\frac{20}{100} \times 375 = 75$
P&G = $\frac{25}{100} \times 150 = 37.50$ Cr	$\frac{20}{100} \times 375 = 75$
HLL = $\frac{30}{100} \times 150 = 45$	$\frac{40}{100} \times 375 = 150$
Other = $\frac{10}{100} \times 150 = 15$	$\frac{5}{100} \times 375 = 18.75$

(i) Gain % or profit % = $\frac{SP - CP}{CP} \times 100$

(ii) Loss % = $\frac{CP - SP}{CP} \times 100$

(iii) Margin % = $\frac{SP - CP}{SP} \times 100$

$$\% \text{ change} = \frac{\text{final} - \text{initial}}{\text{initial}} \times 100$$

Nirma = 87.5% ✓

Henkel = ~~66.67%~~ 233.33% ✓

P & G = 100% ✓

HLL = ~~233.33~~ 233.33% ✓

Other = 25% ✓

i) Overall %

$$\frac{375 - 150}{150} \times 100 = 150\%$$

ii) Other 25%

iii) 233%

iv) $\frac{75}{37.5} = 2:1$

v) Note: Increment in individual = Increment in total

$$\frac{1500}{100} = 15 \text{ Cr.}$$

vi) In 2004

Individual growth = Overall growth

Hence 10% //.

vii) $\frac{10\%}{100} \times 375 = 37.5$

In 2004 412.5

$412.5 \times \frac{1}{10} = 41.25 \Rightarrow$ In 2005 453.75 //

Q8.

2001 exp HLL = 30 Cr

2001 → 2002

$\frac{10}{100} \times 30 = 3$

in 2002 33 Cr exp.

$\text{Profit} = \frac{45-33}{33} \times 100$

$= 36.36\%$

Profit = 45 − 33 = 12

$\text{Margin} = \frac{45-33}{45} \times 100 = 26.67$ //

Opⁿ D

Q9.
Sol $(233 - 100)\% = 133\%$

Q10. $\frac{56.25}{30} = \frac{15}{8}$ //

* $\frac{DMTE}{W} = k$ (Constant)

Where

$D \rightarrow$ no. of Days
$M \rightarrow$ no. of Men/Women/& children
$T \rightarrow$ Time (Duration)
$E \rightarrow$ Efficiency
$W \rightarrow$ Work done

Conceptual Que 7.

i) A Certain no. of Men Can finish a piece of work in 60 days. If there were 8 Men more. The Work Could have been finished in 10 days less. find the number of Men present at the begining

Sol $\quad D_1 M_1 = D_2 M_2$

$60 M_1 = 50(M_1 + 8)$

$10 M_1 = 400$

$M_1 = 40$ //

ii) Three Men or 5 Women Can Complete a piece of work in 12 days. How log will it take for Six men & 5 Women to finish Same piece of work

Sol $\quad 3M = 5W \quad \text{—(1)}$

$3 \times 12 = (6M + 5W) d_2$

$36 = 9 d_2$

$d_2 = 4$ //.

iii) If 12 Men can build a wall 100m long 3m high and 0.5m thick in 25 days. In how many days will 20 men can build a wall 60m long 4m high and 0.25 m thick.

Sol

$V_1 = 150\,m^3$ Volume W_1

$V_2 = 60\,m^3$ Volume W_2

$$\frac{D_1 M_1 T_1}{W_1} = \frac{D_2 M_2 T_2}{W_2}$$

$$\frac{25 \times 12}{150} = \frac{D_2 \times 20}{60}$$

$$D_2 = 6 \text{ days} //$$

iv) If 12 pumps can raise 1218 T of water in 11 days of 9hrs each. Then how many pumps requir to raise 2030T of water in 12 days of 11hr each.

Sol

$$\frac{D_1 M_1 T_1}{W_1} = \frac{D_2 M_2 T_2}{W_2}$$

$$\frac{12 \times 9 \times 11}{1218} = \frac{12 \times 11 \times M_2}{2030}$$

$$406$$

$$M_2 = \frac{2030 \times 3}{406} = 15 \text{ pumps} //.$$

Conceptual
Que. 8

Note **Distribution of Wage**

Distribution of wage depends upon the amount of work done (and not always on the efficiency of individual)

Ex. 1 If A and B can complete a piece of work in 10 days and 15 days respectively. for which a total of Rs. 300 is to be distributed. Then find their individual share

Sol A 10 days ⟶ 10%

B 15 day ⟶ 6.67%

one 16.67% = $\frac{1}{6}$ = 6 days

A → $\frac{1}{10} \times 6 \times 300$ = 180 Rs

B → $\frac{1}{15} \times 6 \times 300$ = 120 Rs

Ex. 2 If A can do a piece of work in 15 days and B in 20 days. They finish the work with the assistance of C in 5 days and got Rs. 45 find their individual shares.

Sol A → 6.67% one day

B → 5%

$\frac{1}{15} \times 5 \times 45$

A → $\frac{1}{15}$

B → $\frac{1}{20}$

$$\left(\frac{1}{15}+\frac{1}{20}+\frac{1}{C}\right)\times 5 = 1$$

$$20C + 15C + 300 = 60 \quad \frac{300C}{5}$$

$$35C = 60C - 300$$

$$25C = 300$$

$$C = 12$$

C → $\frac{1}{12}$

C :- $\frac{1}{12}\times 5\times 45 = 18.75$ /-

B = $\frac{1}{20}\times 5\times 45 = 11.25$ /-

A = $\frac{1}{15}\times 5\times 45 = 15$ /-

A

OR

A → 6.67%

B → 5%

C → 8.33%

A+B+C → 20% one day

C = 20 - 6.67 - 5%

C = 1/12

Defined work done

$\frac{DMTC}{W} = k$; $W = DMTC$

Que. After Tsunami disaster 90 Men were required to remove the remains in 20 days but 10 men left the work after every 5th day. In how many days will the remains be cleared?

a) 25 days b) 25 1/4 days c) 26 1/4 days d) 27 days

Sol

$W = MD$

$W = 90 \times 20 = 1800$ Required work Man Day work

~~to left after every 5th~~ = ~~4 x 10 = 40~~

~~D = 1800 / 50 =~~

first 5 days = $90 \times 5 = 450$ MD work

1 - 5

6 - 10 $80 \times 5 = 400$ MD work

11 - 15 $70 \times 5 = 350$ MD

16 - 20 $60 \times 5 = 300$ MD

~~21 - 25~~

21 - 25 $50 \times 5 = 250$ MD

1750

1800 - 1750 = 50 MD Remain

26 - 30 $40 \times 5 = 200$ $\frac{50}{200} = \frac{1}{4}$

Hence 26 1/4 //.

Conceptual Que 10

Six technician working at the same rate complete the work of one server in 10hrs. If they start at 11am and one additional technician per hr is been added begining at 5 PM. At what time the work on server will be completed

Sol $W = DMT$

$= 6 \times 10 = 60$ MT work

first 6 hr $= 6 \times 6 = 36$ MD

+1M → 7th hr $= 36 + 6 = 42$

+3M → 10 hr $= 60$

5 PM

8 PM //

OR

6 T ⟶ 10 hr

1 T ⟶ $\frac{10}{6}$ hr

1 T $\xrightarrow{1\text{hr}}$ $\frac{6}{10} = 60\%$

1 T → ~~2 T~~ $\frac{1}{60} \times 7 + \frac{1}{60} \times 8 + \frac{1}{60} \times 9 = \frac{2}{5} = 40\%$

Note:

Miscellaneous:

1) $\frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + \cdots \frac{1}{20^2-1}$

Sol Note:

1) $\frac{1}{a \times b} + \frac{1}{b \times c} + \frac{1}{c \times d} = \left(\frac{1}{a} - \frac{1}{b}\right) + \left(\frac{1}{b} - \frac{1}{c}\right) + \left(\frac{1}{c} - \frac{1}{d}\right)$

$= \frac{1}{a} - \frac{1}{d}$

eg. $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4}$

$\Rightarrow \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right)$

$= 3/4$

eg. $\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10}$

Sol $\frac{1}{3}\left[\left(\frac{1}{1} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{7}\right) + \left(\frac{1}{7} - \frac{1}{10}\right)\right]$

$\rightarrow \frac{1}{3}\left[1 - \frac{1}{10}\right] = 3/10$

Sol $\frac{1}{2^2-1} + \frac{1}{4^2-1} + \frac{1}{6^2-1} + \cdots \frac{1}{20^2-1}$

Sol $\frac{1}{(2-1)(2+1)} + \frac{1}{(4-1)(4+1)} + \frac{1}{(6-1)(6+1)} \cdots \frac{1}{(20-1)(20+1)}$

$\Rightarrow \frac{1}{1\cdot3} + \frac{1}{3\cdot5} + \frac{1}{5\cdot7} + \cdots \frac{1}{19\cdot21}$

$\Rightarrow \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} \cdots \frac{1}{19} - \frac{1}{21}\right]$

$\Rightarrow \frac{1}{2}\left[1 - \frac{1}{21}\right]$

$\Rightarrow \frac{1}{21} \times \frac{20}{2}$

$= 10/21$ //

Que. $\frac{1}{1\cdot2\cdot3} + \frac{1}{2\cdot3\cdot4} + \frac{1}{3\cdot4\cdot5}$

Sol $\frac{1}{2}\left[\frac{1}{1\cdot2} - \frac{1}{2\cdot3} + \frac{1}{2\cdot3} - \frac{1}{3\cdot4} + \frac{1}{3\cdot4} - \frac{1}{4\cdot5}\right]$

$\frac{1}{2}\left[\frac{1}{2} - \frac{1}{20}\right] = 9/40$ //.

eg. $\frac{1}{1\cdot3\cdot5} + \frac{1}{3\cdot5\cdot7} + \frac{1}{5\cdot7\cdot9}$

Sol $\frac{1}{4}\left[\frac{1}{1\cdot3} - \frac{1}{3\cdot5} + \frac{1}{3\cdot5} - \frac{1}{5\cdot7} + \frac{1}{5\cdot7} - \frac{1}{7\cdot9}\right]$

$\frac{1}{4}\left[\frac{1}{3} - \frac{1}{63}\right]$

$\frac{1}{4}\left[\frac{21-1}{63}\right] = 5/63$ //.

OR

$\frac{1}{4}\left[\frac{1}{1\cdot3} - \frac{1}{7\cdot9}\right] = 5/63$ (all Middle term Cancel)

eg5 $N = 1\times1! + 2\times2! + 3\times3! + 4\times4! \dots + 10\times10!$

Q. $M = 1! + 2! + 3! + 4! + \dots + 20!$

$\frac{M}{20} = ?$ Remainder

$20 = 2^2\times5$

5! onward every no. is divisible by 20 because $2^2\times5$ appear.

$\frac{1!}{20} + \frac{2!}{20} + \frac{3!}{20} + \frac{4!}{20}$

R R R R

↓ ↓ ↓ ↓

1 + 2 + 6 + 4 = 13 //.

Que. $N = (2-1)11! + (3-1)2! + (4-1)3! + \cdots + (11+1)10!$

Sol $\frac{N+2}{11}$ $R = ?$

$N = 2! - 1! + 3! - 2! + 4! - 3! + \cdots + 11! - 10!$

$N+2 = (11! - 1!) + 2$

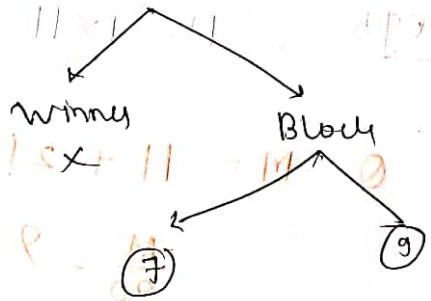
$\frac{N+2}{11} = \frac{11! + 1}{11}$ Rem = 1 //.

Puzzle

Condn
Only win or block town

1 X	2 O	3
4 X	5	6
7 O	8 O	9 X

who will win ?



last chance :

X → blocked at 9

O → would have marked at 5 and won

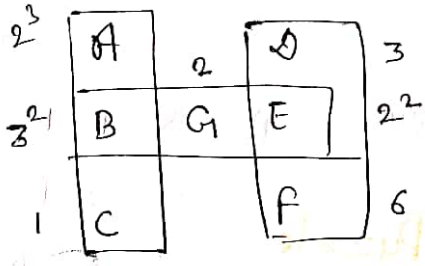
Hence last chance was X's & in next O will win.

Que. find G1 no. from 1 to 9

A X B X C = D X E X F = B X G1 X E

1 = 1x1
2 = 2x1
3 = 3x1
4 = 2^2
5 = 5x1
6 = 2x3
7 = 7x1
8 = 2^3 x1
9 = 3^3

Total 2's = $2^7 \times 3^4$



Que.

A B C

D

E F G

H

I

A+B+C =
D+D+E = } 13
E+F+G =
G+H+I =

130 = 9 + 1 + 3
~~+ 2 + 2~~
not Reapet

8 + 1 + 4
8 + 2 + 3

7 + 1 + 5
7 + 2 + 4
7 + ~~3 3~~ Repeat
~~7 + 4~~ +.

6 + 1 + 6
+ 2 + 5
+ 3 + 4

9 3
A B 1 C

0 8

E F G 2
4 7
H 6
I 5 //

Que. i) S L I D E
— D E A N
3 6 5 1

ii) N O S L E R
A S T R A L
7 2 5 6 1 3

Here all alphabet Represent distinct Value from 0 to 9

Sol i) I E
9 3
8 2
7 1

Percentage

Page 6

Q1. Q4 Q7.

Sol:- $A = 70\% \text{ of } B$

$B = 50\% \text{ of } C = \frac{50}{100} \times 100000$

$A = \frac{70}{100} \times \frac{50}{100} \times 100000$

$A = 35000$

$B = 50\% \text{ of } 100000$

$= 50000$

$A = 7 \times 5000$

$= 35000$

Sol 4 $\frac{M}{W} = \frac{2}{3}$ M $2x$ W $3x$ $5x$

$\frac{2x}{5x} \times 100 = 40\%$

late

Sol 7:- (I) $x\% \text{ of } y\% \text{ of } A = y\% \text{ of } x\% \text{ of } A$

$10\% \text{ of } 20\% \text{ of } 100 = 20\% \text{ of } 10\% \text{ of } 100$

20, 10, (2), (2)

(II) $x\% \text{ of } y\% \text{ of } A = 2x\% \text{ of } y\% \text{ of } A/2 = 2x\% \text{ of } 2y\% \text{ of } A/4$

$10\% \text{ of } 20\% \text{ of } 100 \quad 20\% \text{ of } 20\% \text{ of } 50 = 20\% \text{ of } 40\% \text{ of } 25$

20, 10, $4\frac{25}{10} \times \frac{1}{5}$, (2), (2), (2)

(III)

Sol 7 $25\% \text{ of } 250\% \times 100$

Successive percentage change

(I) a% increment ⟶ b% increment

$$a + b + \frac{ab}{100}$$

(II) a% increment ⟶ b% decrement

$$a - b - \frac{ab}{100}$$

(III) a% decrement ⟶ b% decrement

$$-a - b + \frac{ab}{100}$$

Another Method for percentage change

(I) 20% ↑ ⟶ 10% ↑

1.2 , 1.1

$1.2 \times 1.1 = 1.32$

$\frac{1.32 - 1}{1} \times 100 = 32\%$ (increment)

32% overall ↑

(II) 10% ↑ 20% ↑ 10% ↑

$1.1 \times 1.2 \times 1.1 =$

(III) 20% ↓ 10% ↓

$0.8 \times 0.9 = 0.72$

$\frac{0.72 - 1}{1} \times 100$ (for decrement)

$= -0.28 \times 100$

$= 28\%$ ↓

Example

(I) 20% of 30%

Sol $0.2 \times 0.3 = 0.06 \times 100 = 6\%$

(II) $0.2 \times 0.3 \times 0.4 = 0.024 \times 100 = 2.4\%$

(III)

Q15, 16, 19

Sol 15 :- 32%

16 :- $A = 10 \times 5$

$= 10.1 \times (5.05 + 5) =$

$= 10\% \quad 20\% \quad 5.005 m^2$

$20\% \rightarrow \frac{1}{5}$

$5\% \rightarrow \frac{1}{20}$

16 :- $A = 10 \times 5 = 50 m^2$

$A_n = (1.1 \times 10) \times (1.05 \times 5)$

$= 57.75$

Percentage change $= \frac{\text{New} - \text{old}}{\text{old}} \times 100\%$

19. radius = 20%

$r_{new} = 1.2r$

If $r \uparrow x\%$ Circumference $\uparrow x\%$

Square: Side $\uparrow x\%$ perimeter $\uparrow x\%$

side $\uparrow x\%$ Area $\uparrow$

radius 20%

$\pi r^2_{new} = (1.2)^2 = 1.44 \rightarrow 44\%$

$E = PC$

where, $E \rightarrow$ Expenditure

$P \rightarrow$ Price

$C \rightarrow$ Consumption

If Price of fuel is increased by R'/. then by what percentage the Consumption must be reduced such that the expenditure Remains Constant

$$\uparrow R'/. \longrightarrow \downarrow \left(\frac{R}{100+R} \times 100\right)'/.$$

Price — Consumption

If price of fuel is deceased by R'/. then by what percentage the Consumption must be increased such that the expenditure Remains Constant

$$\downarrow R'/. \longrightarrow \uparrow \left(\frac{R}{100-R} \times 100\right)'/.$$

price — Consumption

ex.

	Price	Cons.	
(1)	↑25'/.		$\left(\frac{25}{125} \times 100\right) = 20'/.$
(2)		↓20'/.	$\left(\frac{20}{100-20} \times 100\right) = 25'/.$
(3)	↓10'/. ($\frac{1}{10}$)	↑ $\frac{1}{9} \times 100$ ($\frac{1}{10}$ on up)	$\left(\frac{10}{100-10} \times 100\right) = 11.11'/.$
(4)	↓ $\frac{1}{21} \times 100$ ($\frac{1}{20}$ on ↓)	↑5'/. $\frac{1}{20}$	$\left(\frac{5}{105} \times 100\right) = 4.76'/.$
(5)	↑6.67'/.	↓ $\frac{1}{16} \times 100$	$\frac{6.67}{100+6.67} \times 100 = 6.25'/.$
(6)	↑37.50'/.		27.27'/.
(7)		↓62.50'/.	166.66'/.
(8)	↓73.33'/.		500.00'/.

This Method only valid for (1) $\frac{1}{n}$ form

eg. $E = PC$

$\uparrow 20\% \downarrow 10\%$

$1.2 \times 0.9 = 1.08 \longrightarrow \frac{1.08-1}{100} = 8\% \uparrow$

	xy = Constant	
expend.	$E = P\ C$	Price Consum.
Distance	$D = S\ T$	Speed time
Revenue	$R = P.S$	Price Sell
Area	$A = L\ B$	Length breadth

Q21
Sol

$A = L\uparrow B$.

$\frac{10}{100-10} \times 100 = \frac{100}{9} = 9.09\%$.

or $L\uparrow \frac{1}{10}$

$B \downarrow \frac{1}{11} = 9.09\%$.

Que 10, 11, 15, 14

Sol 15: $TS = D$

$33.33\% \uparrow \Rightarrow D \downarrow$ $\frac{33.33}{100+33.33} \times 100 = \frac{\frac{1}{3}}{100+\frac{1}{3}} \times 100$

$= 24.99\%$.

or $\frac{1}{3}\uparrow + \downarrow \frac{1}{4} = 25\%$.

12. $P - \frac{1}{4}P = \frac{3P}{4}$ $\quad 4\ M \longrightarrow 60$

$\Rightarrow \frac{5P}{100} = \frac{4}{100}$ $\quad P = 4\%$

$E = PC$

$60 \quad 1.P\downarrow$ $\quad P \quad \frac{P}{100} = 4$. $\Rightarrow \frac{4}{100} - \frac{4P}{100} = \frac{P}{100}$

$\frac{\frac{P}{100}}{100-\frac{P}{100}}$

(11) $E = P\,C$

$25\% \downarrow \quad C \uparrow \quad \frac{25}{75} \times 100 = 33.33\%$

or $\frac{1}{4} \downarrow \quad C \uparrow \quad \frac{1}{3} = 33.33\%$

(10)

Sol:

A is 37.50% more than B

what % B is less by A

$$\frac{37.50}{100 + 37.50} \times 100 = 27.27\%$$

(13) $A_{new} = 1.21A$

$A_{new} - A = 0.21A$

$r_{new}^2 = 1.21 \times r^2$

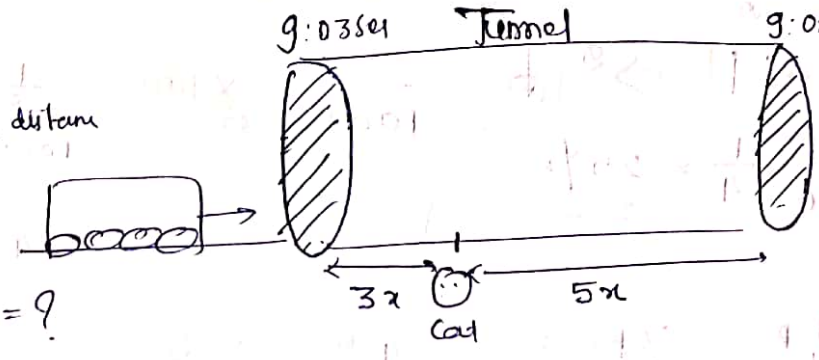
$r_{new} = 1.1r$ → $r \uparrow 10\%$ or $C \uparrow 10\%$

(12)

Today	earlier
$\frac{60}{\frac{1}{4}x}$	$\frac{60}{x} = 4$

Que.

Cat at $\frac{3}{8}$ distance from entrance of tunnel



$\frac{\text{Train speed}}{\text{Cat speed}} = ?$

Ans 4:1 //

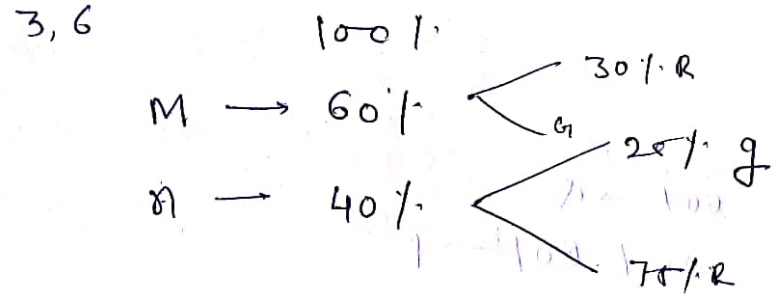
Cat speed $1\ m/s$

at 9:00

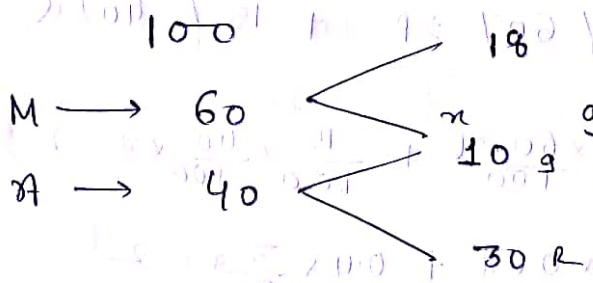
$$\text{Train speed} = \frac{D}{T} = \frac{8x}{2} = 4$$

Q2, 3, 6

Sol:



X



$\frac{42}{52} \times 100 = 80.7\%$

Q3.

Sol: Scored 50% of Max and failed by 2 mark

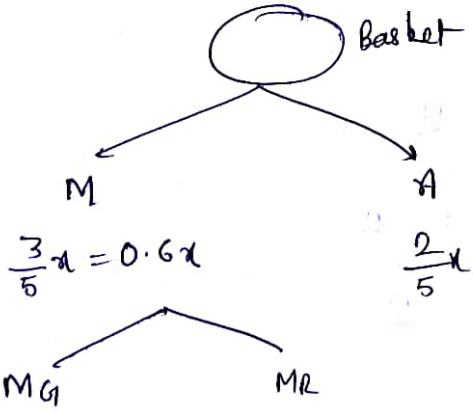
$\frac{x}{2}$ fail

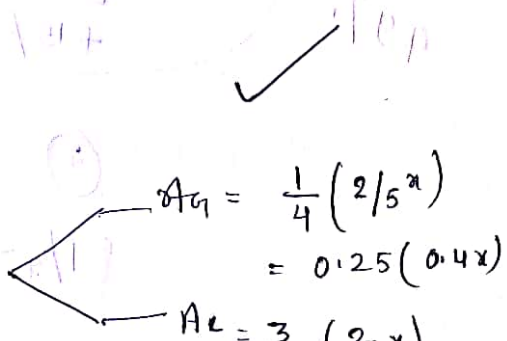
$\frac{x}{2} + 12 = 1.1 \times \frac{x}{2}$

$\frac{x}{2} + 10 \times \frac{x}{2}$

$x = 240$

Que



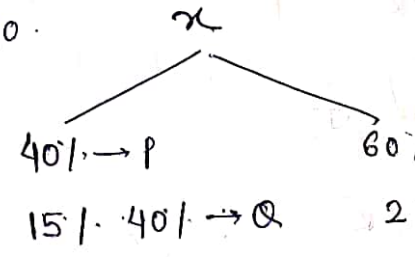


$\frac{MG}{MG + AG} \times 100$

$= \frac{0.2 \times (0.6x)}{0.2 \times 0.6x + 0.25(0.4x)} \times 100$

Page 10

Q90.



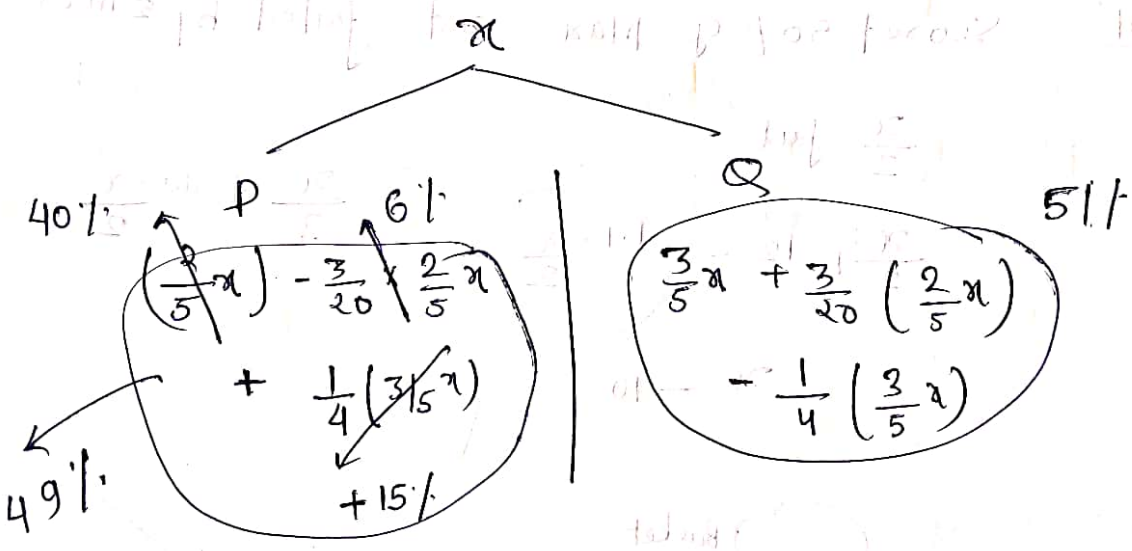
15% 40% → Q

25% 60% → P

$-25\%\ 60\% \to P + 15\%\ 40\%\ Q = 2$

$-\frac{25}{100} \times \frac{60}{100} x + \frac{15}{100} \times \frac{40}{100} \times x = 2$

$-\frac{1}{4} \times 0.6x + 0.4 \times \frac{3}{20} x = .2$



② − ① = 2

51%x − 49%x = 2

2%x = 2

Marked price MP ↓ → % discount
Selling price SP ↕ → P/L
Cost price CP

Page 09

Q65, 66, 68

Sol: CP = 10, SP = $\frac{12}{10}$ $\frac{12}{10} = 1.2$ $\frac{12}{10} = 1.2$

$\frac{10}{12} =$

12 ↑ → 10

CP 1 → $\frac{10}{12}$

SP 10 ↑ → 12

1 → 1.2

$\frac{1.20 - 0.833}{0.833} \times 100$ ~~$= \frac{0.29}{0.91} \times 100$~~

$\approx$ ~~44~~ %

Q66. 8L → 1R

CP 1L → $\frac{1}{8}$ Rs

SP 6L → 1

1L → $\frac{1}{6}$

$\frac{\frac{1}{6} - \frac{1}{8}}{\frac{1}{8}} \times 100 = \frac{8-6}{48} \times 8 \times 100 = \frac{1}{3} \times 100 = 33.33\%$

A

Q68

Sol $CP_A = 1080$

$SP_A = 0.8 \times 1080 = 864$

B

$CP_B = 864 + 56 = 900$

$SP_2 = 1.2 \times 900 = 1080$

$CP_C = 1080$ ✓

- By Selling 20 pensil A

i) Profit : A makes profit of Cost price of 5 penil, Selling price of 5 penil

ii) Loss : A makes loss of CP of 5 penil, Selling price of 5 penil.

Profit

$20SP - 20CP = 5CP$

$20SP = 25CP$ ↓

$SP = \frac{25}{20} = 5/4 = 1.25$

i.e 25% ↑

Loss

$20CP - 20SP = 5CP$

$15CP = 20SP$

$20SP = 15CP$ ↓

$SP = 15/20 = 3/4 = 0.75$

Profit

$20SP - 20CP = 5SP$

$15SP = 20CP$

$SP = \frac{20}{15} = \frac{4}{3}$

33.33 %

Loss

$20CP - 20SP = 5SP$

$25SP = 20CP$

$SP = \frac{20}{25} = \frac{4}{5} = 0.80$

20% Loss ↓

Q67. $40SP - 40CP = 10SP$

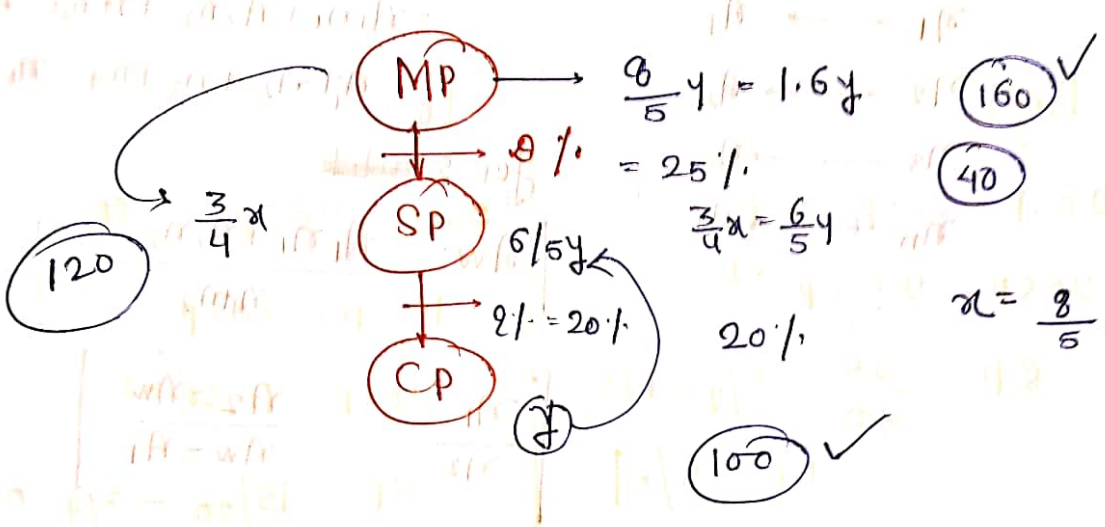
$30SP = 40CP$

$SP = 40/30 = 4/3$

$= 1.33$

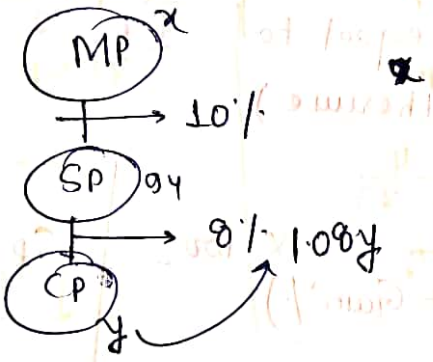
33.33

Q. Even after giving the discount of 25% a shopkeeper Manage to get profit of 20%. By what percentage has he Marked up priced of the goods



Discount = Commission

Q70.
Sol



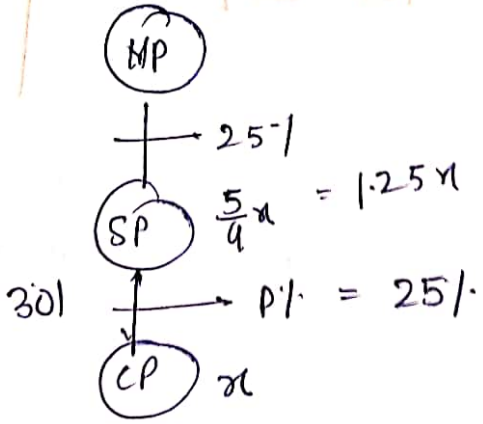
~~$\theta = \frac{SP - y}{y}$~~

~~$9y + y = SP$~~

~~$SP - 9y$~~

$x - 9y$

Q77
Sol



Mixture & Allegation

goods
$n_1 \longrightarrow A_1$
$n_2 \longrightarrow A_2$
$n_3 \longrightarrow A_3$
$n_n \longrightarrow A_4$

A_w = Average of Averg

$$= \frac{A_1 n_1 + A_2 n_2 + A_3 n_3 + \dots A_n n_n}{n_1 + n_2 + n_3 + n_4 \dots n_n}$$

for 2 ~~cubes~~

$$A_w = \frac{A_1 n_1 + A_2 n_2}{n_1 + n_2}$$

$$\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$$

Note:

i) In this topic we will Consider Costprice of Milk and coater to be equal to 1 Rs & 0 Rs resp. (Unless Mentioned otherwise)

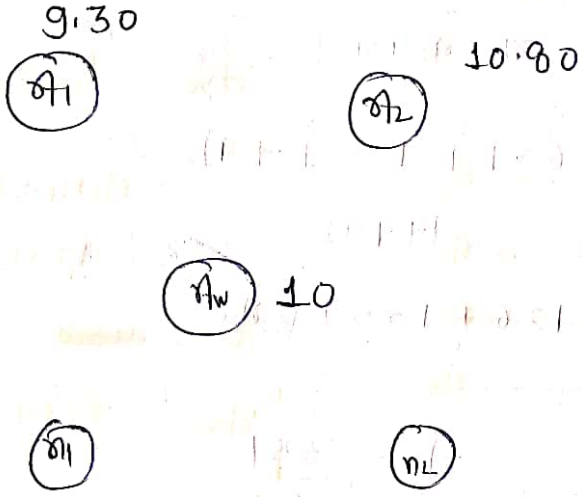
ii)

$$CP = \frac{SP}{(100 + \text{Gain}\%)} \times 100 \qquad SP = CP \times \frac{(100 + \text{Gain}\%)}{100}$$

$$= \frac{SP}{100 - \text{Loss}\%} \times 100 \qquad SP = CP \times \frac{(100 - \text{loss}\%)}{100}$$

Page 12

Q1.



$$\frac{n_1}{n_2} = \frac{\eta_2 - \eta_w}{\eta_w - \eta_1} = \frac{10.80 - 10}{10 - 9.30}$$

$$= 8:7$$

Q3

Sol Minimum wath 64.50 l/kg η_w

62 72

64.50

t_1 t_2

$$\frac{t_1}{t_2} = \frac{72 - 64.50}{64.50 - 62} = 3:1$$

Q13.

Sol $\eta_w = 153$ l / kg

$$A_w = \frac{A_1 n_1 + A_2 n_2 + A_3 n_3}{n_1 + n_2 + n_3}$$

$$153 = \frac{126 \times 1 + 135 \times 1 + A_3 \times 2}{1 + 1 + 2}$$

$$612 = 126 + 135 + 2A_3$$

$$612 - 126 - 135 = 351$$

$$A_3 = \frac{351}{2}$$

$$= 175.5 \text{ //}$$

Q5.

$$A_w = \frac{A_1 n_1 + A_2 n_2}{n_1 + n_2}$$

$$= \frac{15 \times 2 + 20 \times 3}{5}$$

$$= \frac{90}{5} = 18 \text{ //}$$

Que.2 $C_{pm} = ₹ 1$

$C_{pw} = 0$ /-

$SP_{(m+w)} = C_{pm} = ₹ 1$/-

$$C_{P(m+w)} = S_{P(m+w)} \times \frac{100}{100 + \text{gain \%}}$$

$$= 1 \times \frac{100}{100+20}$$

$$= 100/120 = 5/6$$

$$\frac{\eta_1}{\eta_2} = \frac{\eta_2 - \eta_m}{\eta_m - \eta_1}$$

$$= \frac{1 - 5/6}{5/6 - 0} = 1:5 \text{ //}$$

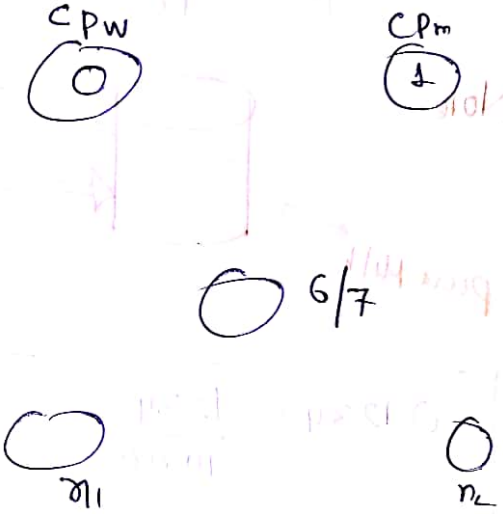
Q7. Profit $= 16\frac{2}{3}\%$

$SP = CP$

$$CP = SP \times \frac{100}{100 + \frac{50}{3}}$$

$$\rightarrow 1 \times \frac{30}{350}$$

$$\frac{30}{35} = 6/7$$

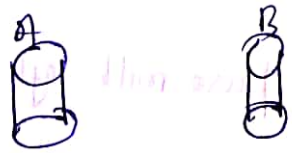


$$\frac{\eta_1}{\eta_2} = \frac{1 - \frac{6}{7}}{\frac{6}{7} - 0}$$

$$\rightarrow \frac{1}{6}$$

1 : 6 //.

Q11.



4 : 3
M W

$M_A = \frac{4}{7}$

2 : 3
M W

$M_B = \frac{2}{5}$

$$\eta_w = \frac{1}{1+1} = \frac{1}{2}$$

Ration 1:1

$$\frac{\eta_1}{\eta_2} = \frac{\frac{2}{5} - \frac{1}{2}}{\frac{1}{2} - \frac{4}{7}}$$

$$\frac{-4 + 5}{10} \times \frac{14}{1} = 1.4$$

Q.9.

Sol

$\frac{8}{13}$ $\frac{5}{7}$

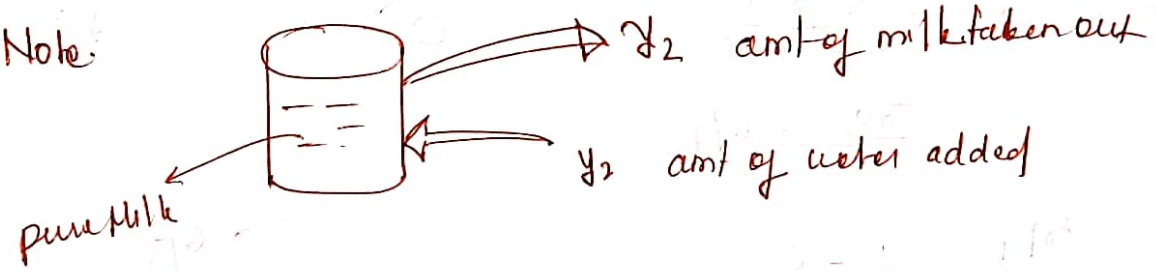
$\frac{69\times13+3}{13\times100}$ 9/15

$$-\frac{\frac{69\times13+3}{13\times100}+\frac{5}{7}}{+\frac{69\times13+3}{13\times100}}$$

x_1 x_2

$$\frac{\frac{5}{7}-\frac{9}{15}}{\frac{9}{15}-\frac{8}{13}}=\frac{2}{7}$$

Note.



$0.1234=\frac{1234}{1000}$	$0.1\overline{234}=\frac{1234-1}{9990}$
$0.\overline{1234}=\frac{1234}{9999}$	$0.12\overline{34}=\frac{1234-12}{9900}$

So after n times how much pure milk left

$y =$

After an operation the amount of pure liquid left $= x\left(1-\frac{y}{x}\right)^n$

where, x = amt of pure liquid present at the beginning or Volume of Cask

or y = amt of pure liquid (solution) replenish with water.

Q16.

$$y = 40\left(1 - \frac{4}{40}\right)^{3} \quad \text{(3) further 2 time}$$

$$= 40\left(\frac{36}{40}\right)^{3}$$

$$= \frac{40}{40^3} \times 36^3$$

$$= 29.16\ \ell \ //$$

TIME SPEED DISTANCE

Distance = Speed × Time

$$D = ST$$

Q1. A train runs at a speed of 25 km/h i) find its speed in m/s

ii) If it covers a distance of 500m in 50 sec find speed in km/hr.

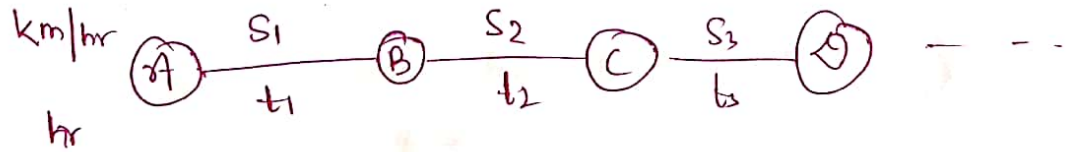
Sol i) $25 \text{ km/h} \longrightarrow \frac{25 \times 1000}{3600} = \frac{125}{18} \text{ m/s}$

ii) $\frac{500}{50} = 10 \text{ m/s} = \frac{10 \times 3600}{1000} = 36 \text{ km/hr}$

note: $1 \text{ km/hr} = \frac{5}{18} \text{ m/sec}$

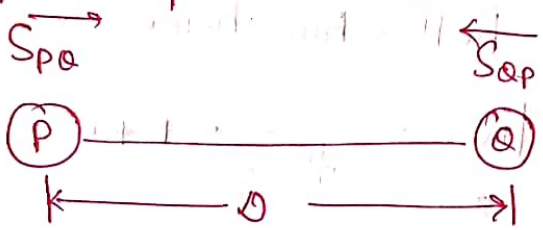
$1 \text{ m/sec} = \frac{18}{5} \text{ km/hr}$

Average speed = $\frac{\text{Total distance travelled}}{\text{Total time taken}}$



Avg. speed = $\frac{S_1t_1 + S_2t_2 + S_3t_3 \text{ --- } S_nt_n}{t_1 + t_2 + t_3 \text{ --- } t_n}$

Let us Suppose A travels P to Q at speed of S_{PQ} whereas from Q to P at speed of S_{QP}. Then the average speed of A through out the Journey is equal to —



$$\text{Avg Speed} = \frac{D_T}{T_1} = \frac{D+D}{\frac{D}{S_{PQ}} + \frac{D}{S_{QP}}} = \frac{2 S_{PQ} S_{QP}}{(S_{PQ}+S_{QP})}$$

CQ2. If 'A' travels from Delhi to Noida at a speed of 30 km/hr where as from Noida to Delhi at a speed of 70 km/hr Then find his Avg. Speed throughout the Journey.

Sol $$\text{Avg } S = \frac{D_T}{T_T} = \frac{2D(70\times30)}{(70+30)D} = \frac{2100}{100} \times 2 = 42 \text{ km/hr}$$

Q2 ii) A bird Covers a distance of 100 km at a speed of 100 km/hr. Bird Cover 2nd 100 km at speed of 200 km/hr. 3rd 100 km at speed of 300 km/hr 4th 100 km at speed of 400 km/hr. find its Avg speed during entire Journey.

$$\text{Avg speed} = \frac{400}{\frac{100}{100} + \frac{100}{200} + \frac{100}{300} + \frac{100}{400}} = 12 \times \frac{400}{12+6+4+3}$$

$$= 192 \text{ km/hr} \quad 4 \times 12 \times 4$$

Que. If Arun rides a scooter at speed of 20km/hr from his home to his office. He reaches his office 5min late where as if he drive at a speed of 30 km/hr he reaches office 5m early. find The distance betn his home & office.

Sol

20 km/hr ——— $t+5$

30 km/hr ——— $t-5$

$$D = 20 \times \frac{(t+5)}{60} = 30 \left(\frac{t-5}{60}\right)$$

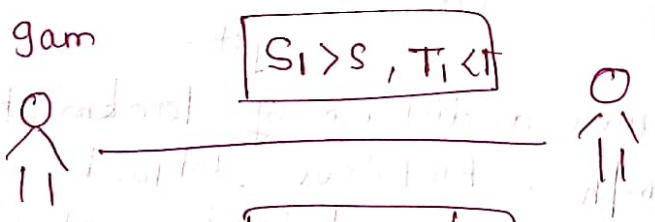
$$20t + 100 = 30t - 150$$

$$10t = 250$$

$$t = 25 \text{ min}$$

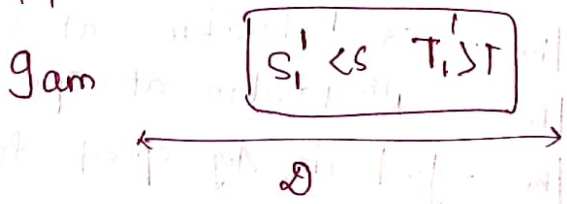
$$D = \frac{20}{\frac{km}{hr}} \times \frac{30}{min} = \frac{600}{60} = 10 \text{ km} \quad //$$

Time Difference



8:50 am ⇒ 9 − 8:50 = 10m

$$\frac{D}{S} - \frac{D}{S_1} = \frac{10}{60}$$



9:15 am ⇒ 9:15 − 9 = 15m/n

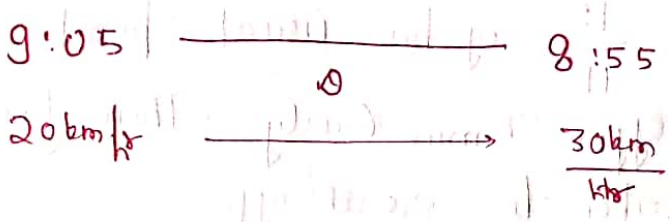
$$\frac{D}{S_1'} - \frac{D}{S} = \frac{15}{60}$$

9 am

9:30 am ⇒ 9 − 8:30 = 30m

$$= \frac{D}{S} - \frac{D}{S_2} = \frac{30}{60}$$

Ex.



$9:05 - 8:55 = 10 \text{ min}$

$$\frac{D}{20} - \frac{D}{30} = \frac{10}{60}$$

$$D = \frac{6 \times 10}{6} = 10 \text{ km} \quad //.$$

OR

Previous Que

Sol

20 km/hr	30 km/hr
5 min late	5 min early
9:05	8:55

$$T_{diff} = 10 \text{ min} = \frac{D}{20} - \frac{D}{30}$$

$$D = 10 \text{ km}$$

Que. Walking at $5/7^{th}$ of his usual speed A reaches office 16 min late. find usual time taken to reach office.

Sol

$$16 \text{ min} = 9:16 - 9:06$$

$$16 = \frac{7D}{5S} - \frac{D}{S}$$

$$16 = \frac{D}{S}\left(\frac{7-5}{5}\right)$$

$$\frac{D}{S} = \frac{16 \times 5}{2} = 40 \text{ min}.$$

ii) Walking at $\frac{12}{11}$ of his usual speed a Man reaches his office 5 min Early. Then find his usual time taken to reach office

Sol

$$5 = \frac{D}{\frac{12}{11}S} + \frac{D}{S}$$

(8:55 am, 9 am)

$$\Rightarrow \frac{D}{S}\left(1 - \frac{11}{12}\right)$$

$$60 \text{ min} = \frac{D}{S} \checkmark$$

OR

By $D = ST$

$$\text{Const} = \frac{ST}{D}$$

Inverse to Make $D = \text{Const}$

$$= \left(\frac{12}{11}S\right)\left(\frac{11}{12}T\right)$$

$$\Rightarrow \frac{11}{12}T = \frac{12-1}{12}$$

$$\frac{1}{12}T = 5 \text{ min}$$

$$T = 60 \text{ min} \checkmark$$

Que. A train Running between 2 town arrives at its destination 10 min late when travelling at a speed of 60 km/hr where as it reaches its destination 16 min late when travelling at 40 km/hr. find distance between two towns

Sol

9:am 9:10 = 10 min

~~10 min~~ = 9:10 − 9

$= \frac{D}{60} - \frac{D}{60}$

16 min − 10 m = 9:16 9:10

$\frac{6 \text{ m}}{60} = \frac{D}{40} - \frac{D}{60}$

$D = \frac{6 \times 40^{2} \times 60}{20 \times 60}$

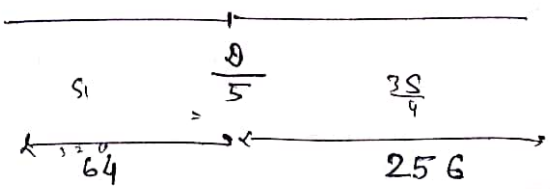
$= 12 \text{ km}$ //

~~OR~~

~~$D = ST$~~

Que. A train has 320 km to run. After going $1/5^{th}$ of distance. The engine breaks down and it can only run remaining part of Journey at $\frac{3}{4}^{th}$ of original speed. If it arrive 2 hr 40 min late then what is the original speed of train

Sol



Ans 2h 40 min 11:40 am → 9 am

$$\Rightarrow 2\frac{40}{60} = \frac{8}{3} = \left(\frac{64}{x} + \frac{256}{\frac{3}{4}x}\right) - \frac{320}{x}$$

$$\frac{8}{3} = \frac{64}{x} + \frac{256 \times 4}{3x} - \frac{320}{x}$$

$$= \frac{64 \times 3 + 256 \times 4 - 320 \times 3}{3x}$$

$$\frac{8}{3} = \frac{192 + 1024 - 960}{3x}$$

$$\Rightarrow x = \frac{1216 - 960}{8} = \frac{256}{8} = 32 \text{ km/hr}$$

Relative Speed

km/h

$x \longrightarrow$ (diff.) $(x \sim y)$ km/hr

$y \longrightarrow$

$x \longrightarrow (x+y)$ km/h

$y \longleftarrow$

CQ1 A Carriage driving in a fog passed a Man who was walking at the rate of 3km/hr in the same direction. If he could see the carriage for 4min and if it was visible to him upto a distance of 100m. Then find speed of the Carriage.

a) 3.5 km/hr b) 3.6 km/hr c) 4.6km/hr d) NOT

Sol Relative Speed $= |S_c - S_m|$ or $S_c \sim S_m$

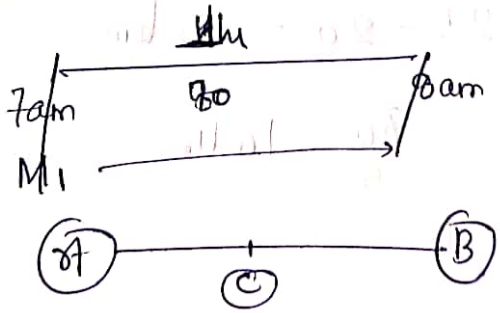
$S_{cm} = S_c - 3$

$\frac{100 \times 60}{4 \times 1000} + 3 = S_c$

$S_c = \frac{6}{4} + 3 = \frac{18}{4} = 4.5$ km/hr

ii) The distance between two Cities A & B is 80km. A Motor cyclist starts from A towards B at 7am. at a speed of 10km/hr where as another Motor Cyclist starts from B towards A, at 8am at a speed of 25 km/hr. At what time will they cross each other

Sol



80-10 = 70km

M1
D = ST
= 10·1
= 10km

at. Speed = 10 + 25

= 35 km/r

$$D = S\,T$$

$$T = \frac{D}{S}$$

$$T = \frac{70}{35} = 2\text{ hr}$$

$8\text{ am} + 2\text{ hr} = 10\text{ am}$ //

Que. Two train start at same time from two station and proceed toward each other at the rate of 20 km/hr and 25 km/hr respectively. When they meet, it was found that one train has travelled 80 km more than the other find distance bet^n two station

Sol

20 km/hr ~ 25 km/hr

= 5 min

A B

20 25

40 50

80 ~ 100

$$\Delta = 25 - 20 = 5\text{ km}$$

$$T = \frac{80}{5} = 16\text{ hr}$$

$D = ST$

$= (20+25)\,16$

$= 45 \times 16$

$D = 720$ km //.

Que. Two bodies A and B move toward each other such that A moves at a constant speed of 4km/h where as B at the speed of 2 km/h for 1st hr 2.5km/h for 2nd hr 3 km/hr for 3rd hr and so on. If the distance between two bodies at the begining was 72km when will they meet

Sol

$R_s = 2$ km/h

$D = R_s \times T$

	A	B
		?
1hr	4	
2hr	4	2.5
3hr	4	3

Distance sum

Here $(4+2) + (4+2.5) + (4+3) + \dots = 72$

$6 + 6.5 + 7 + 7.5 + \dots = 72$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$72 = \frac{t}{2}\left[2\times 6 + (t-1)\frac{1}{2}\right]$$

Ans 9

$$\Rightarrow 72 = 6t + \frac{t^2}{4} - \frac{t}{4}$$

$$= \frac{t^2}{4} - \frac{23t}{4} = 72\times 4$$

$$144 = 12t + t^2 - t$$

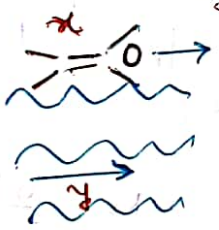
$$0 = t^2 + 11t - 144$$

$$= \frac{-11 \pm \sqrt{121 + 576}}{2}$$

$$= \frac{-11 \pm 26.4}{2}$$

Boats & Stream

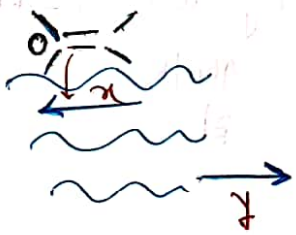
Down



$(x+y)$ km/h

$(x+y)$ km/h

Up



$(x-y)$ km/h

$(x-y)$ km/h

Down: $x+y = \frac{d}{t_1} = S_1$

Up $x-y = \frac{d}{t} = S_2$

$D = St$

$T = \frac{D}{S}$

$T = t_d + t_{up}$

$= \frac{d}{x+y} + \frac{d}{x-y}$

Ex1 A boat goes 2km against the current of the stream in 1hr whereas 1km along the current of stream in 10 min. How long will it take to cover a distance of 5km in stationary water

Sol $D = S\,T$

$2 = S_1 \times 1$

$D = S_2 \frac{10}{60}$

$S_2 = 6$

$x + y_1 = 6$

$x - y_1 = 2$

~~$2y_1$~~ $= \cdots$

$2x = 8$

$x = 4$

Stationary

$y = 0$

$D = ST$

$T = D/S = \frac{5}{4}$

$= 1.25$

$1h\ 0.25h$

$= 1h\ 0.25 \times 60min$

$= 1hr\ 15min$

Q. A boat can travel at a speed of 13 km/hr in still water. If the speed of stream is 4km/hr. Then find time taken to cover a distance of 68 km down stream.

Sol $x = 13$

$y = 4$

$d = S\,t$

$68 = (13+4)\,t$

$t = 68/17 = 4$ hrs.

Quet A boat speed is 15 km/hr in still water goes thirty 30 km down stream and comes back in a total of 4hr & 30min. find the speed of stream

Sol

$$15 + y = 30$$

$$15 - y = 30$$

$$2y = 60$$

$$y = 30$$

$$T = 4\text{hr } 30\text{min}$$

$$= 4\text{hr } \frac{30}{60}$$

$$T = 4.5$$

$$T = \frac{d}{s}$$

$$4.5 = \frac{30}{x+y} + \frac{30}{x-y}$$

$$\Rightarrow 4.5 = \frac{30}{15+y} + \frac{30}{15-y}$$

$$\Rightarrow \frac{4.5}{30} = \frac{15-y + 15+y}{15^2 - y^2}$$

$$\Rightarrow y = 5 \quad //$$

Que. The speed of boat in still water is 10 km/hr. If it can travel 26 km downstream and 14 km upstream in the same time. find speed of stream

Sol $x = 10$

$x + y = 26$

$x - y = 14$

$10 + y = 26$

$+$

$$T = \frac{d}{S}$$

$$= \frac{14}{x-y} = \frac{26}{x+y}$$

$$\Rightarrow \frac{14}{10-y} = \frac{26}{10+y}$$

$$\Rightarrow 70 + 7y = 130 - 13y$$

$$20y = 60$$

$$y = 3 \text{ km/hr}$$

Que. A man can Row to a place 48 km distance and back in 14 hr. He finds that he can Row 4km with the stream in the same time as 3 km against the stream. find speed of the stream.

$D = 48 \times 2$

$T = 14 \text{ hr}$

$D = st$

$\frac{4}{T} = \text{Sd}, \text{Sup} \frac{3}{T}$ ⇒ $T = \frac{st}{4} = \frac{5v}{5}$

$D = \frac{S \times T}{\Phi}$

$T = \frac{48}{\frac{4}{T}+y} + \frac{48}{\frac{3}{T}-y}$

$T = \frac{48\left(\frac{3}{T}-y\right) + 48\left(\frac{4}{T}+y\right)}{\left(\frac{4}{T}+y\right)\left(\frac{3}{T}-y\right)}$

⇒ $14 = \frac{48\left(\frac{3}{14}-y\right) + 48\left(\frac{4}{14}+y\right)}{\left(\frac{4}{14}+y\right)\left(\frac{3}{14}-y\right)}$

$T = \frac{48}{}$

$t_{down} = t_{up}$

$\left(\frac{D}{S}\right)_{down} = \left(\frac{D}{S}\right)_{up}$

$\frac{4}{x+y} = \frac{3}{x-y}$

$x = 7y$

$T = t_{down} + t_{up}$

$$14 = \frac{d}{x+y} + \frac{d}{x-y}$$

$$14 = \frac{48}{x+y} + \frac{48}{x-y}$$

$$14 = \frac{48}{7y+y} + \frac{48}{7y-y}$$

$$\frac{14}{48} = \frac{1}{8y} + \frac{1}{6y}$$

$$\frac{14}{48} \Rightarrow \frac{14y}{48y^2}$$

$$\boxed{y = 1}$$

Linear Race

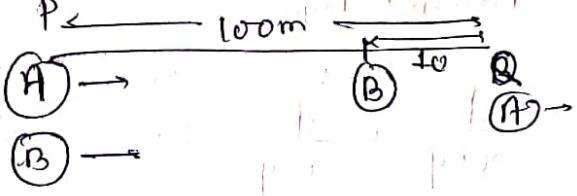
The time at which A finishes the Race. at that time location of B

$D \propto S$ Time Constant

CQ1 A and B run a 100m race where A beats B by distance of 10m. To do a favour to B, A starts 10m behind the starting line in the second 100m Race. If they both at their usual speed then who will win the Race.

Sol

Sol : $D \propto S$



$$\frac{S_A}{S_B} = \frac{D_A}{D_B}$$

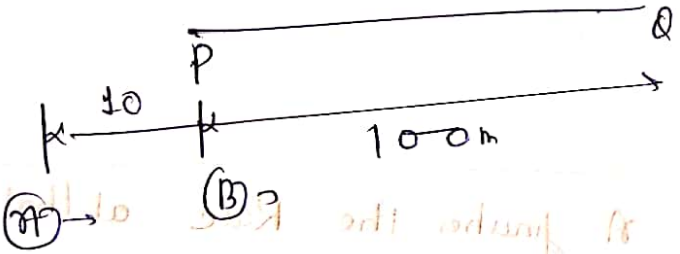
$$= \frac{100}{90}$$

$$= \frac{10}{9}$$

now 2nd Race

$$\frac{S_A}{S_B} = \frac{D_A}{D_B}$$

$$D_A = \left(\frac{S_A}{S_B}\right) D_B \qquad \left| \qquad D_B = \left(\frac{S_B}{S_A}\right) D_A \right.$$



for win A must cover 110 m

B must cover 100

checking

$$D_B = \frac{S_B}{S_A} D_A$$

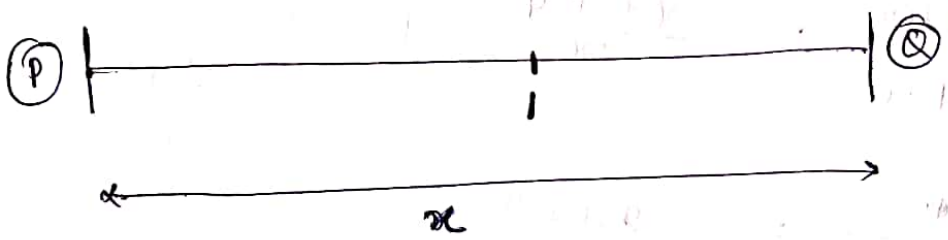
$$= \frac{9}{10} \times 110$$

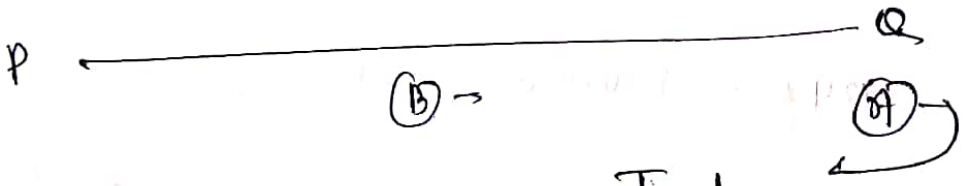
$$D_B = 99$$

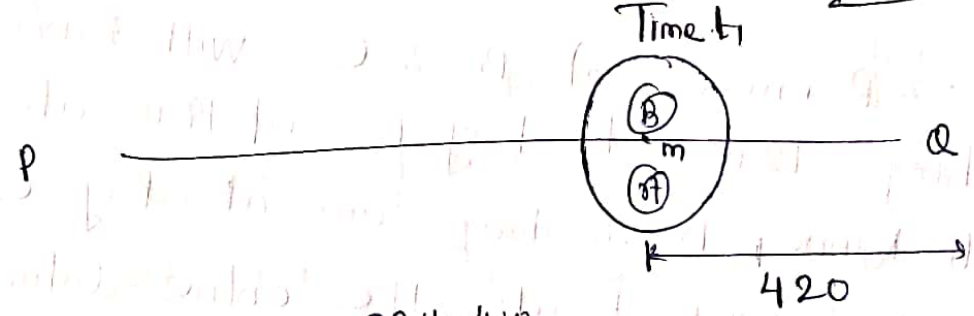
B at 99 Loser Race.

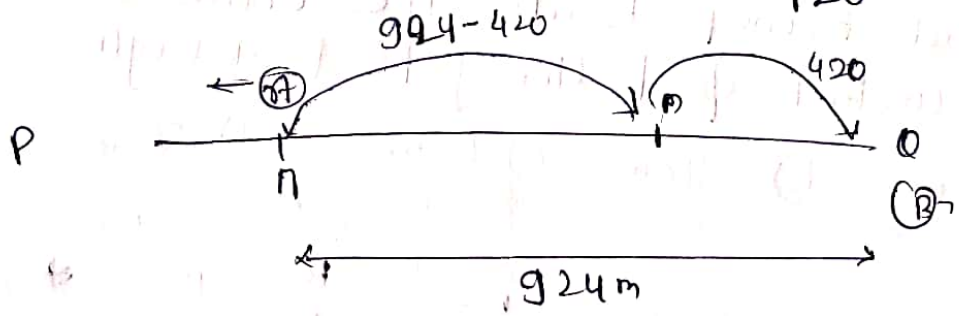
Que. There is a Race between A & B, after winning the Race A started to move backwards with the same speed and meets B at a distance of 420m from the finish line and kept Running. When B finish the Race A was at a distance of 924 m from finish line. find Ratio of Speed A to speed B

Sol









At t_1 : $\frac{D_A}{D_B} = \frac{S_A}{S_B}$

linear relation

$\frac{S_A}{S_B} = \frac{924-420}{420} = \frac{504}{420} = \frac{126}{105} \quad \frac{6}{5}$

OR

$$\frac{S_A}{S_B} = \frac{D_A}{D_B} = \frac{x+420}{x-420} \quad \text{---(1)}$$

$$\frac{S_A}{S_B} = \frac{D_A}{D_B} = \frac{x+924}{x}$$

$$\frac{x+420}{x-420} = \frac{x+924}{x}$$

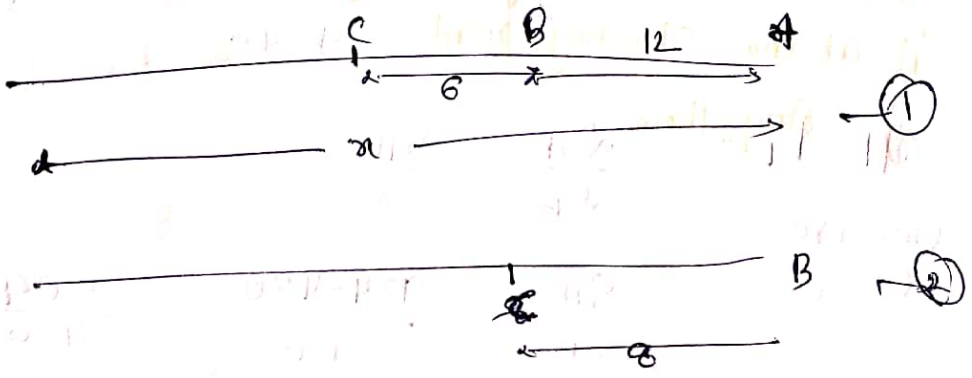
$$\Rightarrow \frac{2x}{x-420} = \frac{2x+924}{x}$$

$$\Rightarrow 924x - 924 \times 420$$

Que. 3 Runners A, B & C with Runner A finishing 12m ahead of B and 18m ahead of C. While Runner B finishing 6m ahead of C. If each Runner travels the entire distance at a constant speed then find track length.

a) 36m b) 48m c) 60m d) 72m

Sol



$$\frac{S_B}{S_C} = \frac{D_B}{D_C} = \frac{x-12}{x-18} \quad \text{---(1)}$$

$$\frac{S_B}{S_C} = \frac{D_B}{D_C} = \frac{x}{x-8} \quad \text{---(2)}$$

$$\frac{x-12}{x-18} = \frac{x}{x-8}$$

$$x^2 - 12x - 8x + 12 \times 8 = x^2 - 18x$$

$$-12x + 10x = \frac{12 \times 8}{1}$$

$$2x = 12 \times 8$$

$$x = 6 \times 8 = 48 \text{ //}$$

Circular Races

A & B run a Circular path of Circumference of 1200m. If Speed of A is 150 m/min B is 70 m/min If they start from the same point and Run

i) In the same direction

ii) Opposite Direction

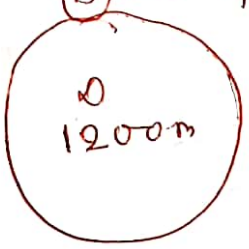
Then when will they be first together again. —

i) at the starting point

ii) Any Where

(A) → 150m/min

(B) → 70m/min



Same Direction

Lcm $(t_A\ t_B)$ = Lcm $(a_0, \frac{120}{7})$ { Lcm of fraction

= 120min At 120 min they meet at starting pt = $\frac{\text{Lcm Num}}{\text{HCF Den}}$

$t_A = D/S_A$

$= \frac{1200}{150} = 8$ min

$t_B = \frac{1200}{70} = \frac{120}{7}$

Check

120min

(A) $D_A = 150 \times 120 = 18000$

(B) $D_B = 70 \times 120$

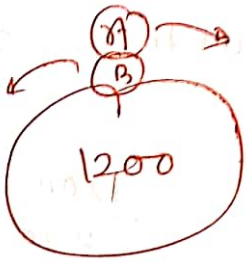
=

no. of Round A = $\frac{150 \times 120}{1200} = 15$

no. of Round B = $\frac{70 \times 120}{1200} = 7$

~~After~~ 8th 120 min. A Completed 15 Round while B Completed 7 Round.

Opposite dirn



Same Calculation

120 min.

Similarly for three body starting point they meet

= Lcm (T_1 T_2 T_3)

(Same dr or opposite dr)

valid

- Body travelling in same direction and meeting anywhere –

Relative speed = 150 − 70 = 80 m/mn

$D = 1200$

$D = ST$

$T = \frac{1200}{80} = 15$ min

Check (15m)

$D_A = 150 \times 15$

$\frac{150 \times 15}{1200} = \frac{2250}{1200}$ 45 24

$D_B = 70 \times 15$

$\frac{70 \times 15}{120} = 1050$

Meeting Anywhere when Moving opposite direction

Rel-speed = 150+70 = 220 m/m

$D = ST$

$T = D/S = \frac{1200}{220} = \frac{60}{11} = 5.45$ m

Check

D_A $150 \times 5.45 = 817.5$

D_B $70 \times 5.45 = \pm 382.5$

1200

* Three body meeting anywhere Same direction

B → 130 m/m
B → 70 m/m
A → 150 m/m

1200 m

Sol $R_{AB} = 150 - 70 = 80$ m/m $\quad t_{AB} = \frac{D}{S} = \frac{1200}{80} = 15$ m

$R_{BC} = 150 - 70 = 60$ m/m $\quad t_{BC} = \frac{1200}{60} = 20$ m

$R_{AC} = 150 - 130 = 20$ m/min $\quad t_{AC}$ 60 m

LCM (t_{AB}, t_{BC}, t_{AC}) =

(15, 20, 60) = 60

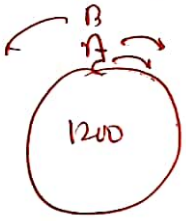
Check

$D_A = 150 \times 60 \Rightarrow \frac{150 \times 60}{1200} = 7 \mid 600$

$D_B = 70 \times 60 \Rightarrow \frac{70 \times 60}{1200} = 3 \mid 600$

$D_C = 130 \times 60 \Rightarrow \frac{130 \times 60}{1200} = 6 \mid 600$

- opposite direction Meeting any where



Rel-Speed

$A|B = 150+70 = 220 m/m \Rightarrow t_{AB} = \frac{D}{S_{AB}} = \frac{60}{11}$

$B|C = 150+70 = 220 \Rightarrow t_{BC} = \frac{D}{S_{BC}} = 6 min$

$A|C = 150 - 130 = 20 m/m \quad t_{CA} = \frac{1200}{20} = 60 mn$

$Lcm\left(\frac{60}{11}, 6, 60\right) = 60 min$