Number System

* Binary

Radix = 2

Unique digit: 0 1

Sequence:

0
1
10
11
100
101
110
111
1000

Octal

Radix = 8

Unique digit: 0, 1, 2, 3, 4, 5, 6, 7

Sequence:

0	10 ···	70	100
1	11 ···	71	101
2	12 ···	72	102
3	13 ···	73	103
4	14 ···	74	104
5	15 ···	75	105
6	16 ···	76	106
7	17 ···	77	107

Decimal

Radix = 10

Unique digit: 0 1 2 3 4 5 6 7 8 9

Sequence:

0	10 ···	90	100
1	11	91	101
2	·	·	·
3	·	·	·
4	·	·	·
5	·	·	·
6	·	·	·
7	·	·	·
8	·	·	·
9	19	99	109

Hexadecimal

Radix = 16

Unique digit: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

Sequence:

0	10 ··	90	A0 ··	F0
1	11	·	·	·
2	12	·	·	·
3	13	·	·	·
4	14	·	·	·
5	15	·	·	·
6	16	·	·	·
7	17	·	·	·
8	18	·	·	·
9	19	·	·	·
A	1A	·	·	·
B	1B	·	·	·
C	1C	·	·	·
D	1D	·	·	·
E	1E	·	·	·
F	1F	9F	AF	FF

* Any Number System to Decimal Number System Conversion

1. $(10101.1101)_2$

Sol

$2^4 \quad 2^2 \quad 2^0 \quad 2^{-1} \quad 2^{-2} \quad 2^{-4}$

$\Rightarrow \quad 21 + \frac{1}{2^7} = (21.8\cancel{1}78)_{10}$

2. $(714.35)_8 = 7\times8^2 + 1\times8^1 + 4\times8^0 + 3\times8^{-1} + 5\times8^{-2}$

$= (460.45)_{10}$

3. $(A2.C37)_{16} = A\times16^1 + 2\times16^0 + 12\times16^{-1} + 3\times16^{-2} + 7\times16^{-3}$

$= 160 + 2 + \frac{12}{16} + \frac{3}{256} + \frac{7}{4096}$

$= (162.763)_{16}$

4. $(45.12)_6 = 4\times6 + 5 + \frac{1}{6} + \frac{2}{36}$

$= 29 + \cancel{6} = (29.12)_6$

5) $(25.31)_9$

Sol $2\times 9 + 5 + \frac{3}{9} + \frac{1}{81}$

$\Rightarrow 23 + \frac{27+1}{81}$

$\Rightarrow 23 + \frac{28}{81} = (23.34)_{10}$

* Decimal number System to any number System.

1) $(29.20)_{10} \longrightarrow (\quad)_2$

Sol

$$\begin{array}{r|l l}
2 & 29 & \\
2 & 14 & 1 \\
2 & 7 & 0 \\
2 & 3 & 1 \\
2 & 1 & 1 \\
 & 0 & 1
\end{array}$$

$(29)_{10} = (11101)_2$

0.20×2
$= 0.40 \times 2$ 0
$= 0.80 \times 2$ 0
$= 1.60$ 1
$= 0.60 \times 2$ 1
$= 1.20$
⋮

$(0.20)_2 = (0011)_2$

$(29.20)_{10} \longrightarrow (11101.00110)_2$

$0.20 \times 2 = 0.40$
$0.40 \times 2 = 0.80$
$0.80 \times 2 = 1.60$
$0.60 \times 2 = 1.20$
$0.20 \times 2 = 0.40$

② $(321.35)_{10} \longrightarrow (\quad)_8$

Sol

$$\begin{array}{c|l} 8 & 321 \\ \hline 8 & 40 \\ 8 & 5 \\ & 0 \end{array} \qquad \begin{array}{l} \longrightarrow 1 \\ \longrightarrow 0 \\ \longrightarrow 5 \end{array} \uparrow$$

$$\begin{array}{lcl} 0.35 \times 8 & = & 2.80 \\ 0.80 \times 8 & = & 6.40 \\ 0.40 \times 8 & = & 3.20 \\ 0.20 \times 8 & = & 1.60 \\ 0.60 \times 8 & = & 4.80 \\ 0.80 \times 8 & = & 6.40 \end{array}$$

$(0.35)_{10} = 263146$

$(321.35)_{10} \longleftrightarrow (501.263146)_8$

③ $(1250.25)_{10} \longleftrightarrow (\quad)_{16}$

Sol

$$\begin{array}{c|l} 16 & 1250 \\ \hline 16 & 78 \\ 16 & 4 \\ & 0 \end{array} \qquad \begin{array}{l} 2 \\ 14 \\ 4 \end{array}$$

$(2E2)_{16}$

$$\begin{array}{lcl} 0.25 \times 16 & = & 4.00 \\ 0.00 \times 16 & = & 0.00 \end{array}$$

$(.4)$

$(1250.25)_{10} \longleftrightarrow (2E2.40)$ //.

Quo. $(523.4)_6 \longleftrightarrow (\quad)_9$

$(\quad)_{10}$

Sol $(523.4)_6$

523

$5\times6^2+2\times6^1+3\times6^0+\frac{4}{6} \rightarrow (195.66)_{10}$

9	195
9	21 → 6
9	2 → 3
	0 → 2

$0.66\times9 = 5.94$

$0.94\times9 = 8.46$

$0.46\times9 = 4.14$

$0.14\times9 = 1.26$

$0.26\times9 = 2.34$

$(523.4)_6 \text{ --- } (236.5841)_9$

Que. The decimal equivalent of Binary number (10110.11) is ___

Sol $22+\frac{2+1}{4} = (22.75)_{10}$

Que. $\sqrt{(224)_r} = (13)_r \quad r=?$

$\Rightarrow \sqrt{2\times r^2+2\times r^1+4\times r^0} = 1\times r+3$

$2r^2+2r+4 = r^2+6r+9$

$r^2-4r-5=0$

$\frac{4\pm\sqrt{16+20}}{2} \rightarrow \frac{4\pm6}{2} = 5 \checkmark$ & -1

(r cannot be negative)

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Que. The number of one's present in binary Represtn of $15 \times 256 + 5 \times 16 + 3$ are 8

Sol $15 \times 16^2 + 5 \times 16^1 + 3 \times 16^0$

$(F\ 5\ 3)_{16} \longrightarrow 1111\ 101\ 011$

- Hexadecimal & Octal to Binary:

1) $(641.23)_8 \longrightarrow (\quad)_2$

6	4	1	.	2	3
110	100	001	.	010	011

$\Rightarrow (110100001.010011)_2$

II) $(C59.4)_{16} \longrightarrow (\quad)_2$

1100 0101 1001 . 0100

$\Rightarrow (110001011001.0100)_2$

Que.

* Binary to octal & Hexadecimal:

1) $(1101.10101)_2 \longrightarrow (\quad)_8$

Sol $\overleftarrow{001}\ \overleftarrow{101}\ .\ \overrightarrow{101}\ \overrightarrow{010} \rightarrow (15.52)_8$

$\overleftarrow{1101}\ .\ \overrightarrow{1010}\ \overrightarrow{1000} \longrightarrow (D.A8)_{16}$

* Octal addition

$$\begin{array}{r} \overset{1}{7}\,4\,\overset{1}{2}\,.\,\overset{1}{5}\,3 \\ 5\,4\,3\,.\,2\,7 \\ \hline (1\,5\,0\,6\,.\,0\,2)_8 \end{array}$$

OR $10-8=2$

$$\begin{array}{c|c} 8 & 10 \\ \hline 8 & 1 \\ & 0 \end{array} \quad \begin{array}{l} \rightarrow 2 \\ \rightarrow 1 \end{array}$$

Base 6 addition :

$$\begin{array}{r} \overset{1}{3}\,\overset{1}{4}\,\overset{1}{3}\,.\,\overset{1}{2}5 \\ 5\,3\,2\,.\,3\,4 \\ \hline (1\,3\,2\,0\,.\,0\,3)_6 \end{array}$$

$$\begin{array}{c|c} 6 & 9 \\ \hline & 1 \quad 3 \end{array}$$

$$\begin{array}{c|c} & 8 \\ \hline & \end{array}$$

don't divide
Subtract by radix.

Que. $(135)_x + (144)_x = (323)_x$

Sol

$$\begin{array}{r} 1\,3\,5 \\ 1\,4\,4 \\ \hline \end{array}$$

note

$9 - x = 3$

$x = 6$ //.

Que. $24 + 14 = 21$

$8 - x = 1$

$x = 7$ //.

Que. $(2.3)_4 + (1.2)_4 = (y)_4$

Sol

$$\begin{array}{r} 3 \\ 2 \\ \hline 5 - 4 = 1 \end{array}$$

$$\begin{array}{r} \overset{1}{2}\,.\,3 \\ 1\,.\,2 \\ \hline 1\,0\,.\,1 \end{array}$$

$(10.1)_4$ //.

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Que $(235)_{R_1} = (565)_{R_0} = (865)_{R_2}$

Sol $2 \times R_1^2 + 3 \times R_1 + 5 =$

$2R_1^2 + 3R_1 + 5 = 565$

$\cancel{-3 \pm \sqrt{9 - 40}}$

$\Rightarrow 2R_1^2 + 3R_1 - 560 = 0$

$\Rightarrow R_1 = \dfrac{-3 \pm \sqrt{9 + 560 \times 4 \times 2}}{4}$

$R_1 = 16$

Que. $(43)_x = (y3)_8$ no. of possible solⁿ is

Sol $4x + 3 = 8y + 3$

$4x = 8y$

$\dfrac{x}{y} = 2$

$x = 2y$ $\quad y = \cancel{1, 2}, 3$

$x = 8$

9

x	y
2	1
4	2
6	3
8	4
10	5
12	6
14	7
16	8

5 Solⁿ

Que. $(123)_5 = (x8)_y$

Sol $25 + 10 + 3 = xy + 8$

$xy = 30$

$y = \frac{30}{x}$

y	x
30	1
15	2
10	3
~~7~~	~~4~~
	~~5~~
~~5~~	~~6~~
~~6~~	~~5~~
~~1~~	~~30~~
~~2~~	~~15~~
~~3~~	~~10~~

$\therefore$ 3 possible Sol^n

$\boxed{\begin{array}{l} (1)\ (x<y) \\ (2)\ y_{min} = 9 \end{array}}$

* Representation of Signed binary Number

- Sign Magnitude form
- Complement form
  - 1's Complement
  - 2's Complement

i) $-29 \longrightarrow$ Signbit Magnitude (29)

$\underbrace{1}_{\text{signbit}}\ \underbrace{11101}_{\text{magnitude bit}}$

This is Sign Magnitude form.

ii) -29 in 1's Complement $\Rightarrow$ $\underbrace{1}_{\text{Signbit}},\underbrace{00010}_{\text{1's comp of 29}}$

iii) -29 in 2's Complement $\Rightarrow$

$$\begin{array}{r} 1\,|\,00010 \\ +1 \\ \hline 1\,|\,00011 \end{array} = 100011$$

$+29$

$\rightarrow$ SMR 011101

$\rightarrow$ 1's 00010 } X

$\rightarrow$ 2's 00011

for positive number Sign Mag. = 1's Comp = 2's Comp

		signbit
+29 =	SMR	0 11101
	1's	0.11101
	2's	011101

short cut of 2's complement
1011011 upto 1st 1 as it is
1's | 2's
0100101

Que. find 2's Complement of 100011
signbit

Sol

1's comp.	011100
2's com	011100
	+ 1
	011101 = −29 //

note 2's Comp of −ve no.
1 00011
as it is in result | 2's C. of only this

Note:- 2's Comp. of 2's Comp of a number is number itself
1's Comp. of 1's Comp. of a no. is no. itself.

Que. 2's Comp. of Some numbers are given. Find out the original no.

1) 11010

Sol 101010 $\Rightarrow$ −6 //

II)

a)	000	$\longrightarrow$	111	X number are positive
b)	001	$\longrightarrow$	111	
c)	010	$\longrightarrow$	110	
d)	011	$\longrightarrow$	101	
e)	100	$\longrightarrow$	111 $\longrightarrow$ ~~−3~~	Special Case
f)	101	$\longrightarrow$ sign bit as it is	1011 $\longrightarrow$ −3	✓
g)	110	$\longrightarrow$	1010 $\longrightarrow$ −2	
h	111	$\longrightarrow$	1001 $\longrightarrow$ −1	

000	+0
001	+1
010	+2
011	+3

✓

Special Case of 2's Complement

	no. of zeros	original no
1 0 0	2	$-2^2 = -4$
1 0 0 0 0	4	$-2^4 = -16$
1 0 0 0 0 0	5	$-2^5 = -32$

no special case in 1's complement form.

* The Range of decimal number that can be represented using n bit 2's Complement Representation is

$$-(2^{n-1}) \text{ to } (2^{n-1}-1)$$

Que. 1's Complement representation of some no are given find out the original number.

Sol

		$\xrightarrow{1's}$	
a)	0 0 0	0 0 0	+0
b)	0 0 1	0 0 1	+1
c)	0 1 0	0 1 0	+2
d)	0 1 1	0 1 1	+3
e)	1 0 0	1 1 1	−3
f)	1 0 1	1 1 0	−2
g)	1 1 0	1 0 1	−1
h)	1 1 1	1 0 0	−0

note:- In 1's Complement Representation we have two separate representation for +0 & −0 whereas in 2's Complement we have single representation for zero.

The Range of decimal numbers that can be Represented using n bit 1's comp. form is $-(2^{n-1}-1)$ to $+(2^{n-1}-1)$

Que. 11001, 1001 & 111001 Correspond to 2's Comp Representation of which of the following Set of no.

a) 25, 9, 57 b) −6, −6 & −6 c) −7 7 & −7

d) −25, −9, −57

Sol 11001 ⟶ 10111 = −7

1001 ⟶ 1 111 ⟶ −7

111001 ⟶ 100111 → −7

Que. +6 ⟶ 5 bit 2's Comp

00110 //

eg. +6 in 8 bit 2's Comp

min = 0110

00000110

In 7 bit 2's Comp 0000110

eg. −7 in 8 bit 2's

min. +7 0111

−7 1001 note

In 8 bit 11111001

Que. A no. in 4bit 2's Complement form is $x_3 x_2 x_1 x_0$. This no. when stored using 8 bit will be

Sol $x_3\, x_3\, x_3\, x_3\, x_3 x_2 x_1 x_0$

note

Que. let A = 1111 1010
B = 0000 1010 be two 8 bit 2's Comp. represent^n. Their product in 2's Comp. is

Sol. 1111 1010 ⟶ 10000110 = −6

0000 1010 ⟶ 1010 = 10

= −60

note There is no need of finding 2's c of the number

→ 1~~00~~111100

~~10111100~~ $\xrightarrow{2's}$ 11000100 //.

{ 32 — 1
16 — 1
48 — 1
8
56 — 1
4 — 0

Que. Assuming all no. in 2's Complement which of the following no. is divisible by 11111011

Sol 11111011 $\xrightarrow{2's}$ 10000101
(−5)

~~64+34+16+8+2+1~~

= ~~125~~

A) 11100111 $\xrightarrow{2's}$ −25

B) 11100100

C) 1011101

D) 1111011

* Complements Arithmetic

1) 1's ~~Twos~~ Complement

note
Answer is in 1's Comp. form. So to get original no. 1's Comp. the answer

eg. $-26 + 9$

let $-26 + 9 = x$

(1's comp of -26) + (1's comp. 9) = 1's comp. of x

$-26 \xrightarrow{1's}$ 1 1 1 0 1 0
1 0 0 1 0 1

$9 \xrightarrow{1's}$ 0 1 0 0 1
0 0 1 1 0

} donot take compl. of +ve no.

now
1 0 0 1 0 1
0 0 1 0 0 0
1 0 1 1 0 1 = 1 0 1 1 1 0 1

1's comp of 1 0 1 0 1 1 ⟶ 1 0 1 1 0 0
−17 //

eg. $+26 - 9$

Sol 26 ⟶ 0 1 1 0 1 0
−9 ⟶ 1 1 0 1 1 0

zero is used for +ve

note
To adjust equal bits in both number
for −ve number 1 is used

0 1 1 0 1 0
1 1 0 1 1 0
1 0 1 0 0 0 0
1
0 1 0 0 0 1 $\xrightarrow{1's}$ +17

note
Positive no. no need to complement

Difference betn 1's & 2's comp arithmetic

- In 1's Complement arithmetic we add the Carry (If generated) to the Result where as in 2's Complement arithmetic we Ignore the Carry (If generated)

Q1.

Concept of Overflow

Q. Using 2's Comp. arithmetic, Solve

(1) $-2+3$ (2) $-3+2$ (3) $3+2$ (4) $-3-2$

Sol 1 $-2 \xrightarrow{2's} 110$

$3 \longrightarrow 011$

$+$ ______

$1)\ 001$

Let,

$-2+3 = x$

$2'c(-2) + 2'c(3) = 2'c(x)$

Carry Ignore $= 001 \xrightarrow{2's} 001$ (+ve no)

$-3 \xrightarrow{2's} 101$

$2 \xrightarrow{2} 010$

(II) $-3+2 =$

101
010
______
$111 \xrightarrow{2's} 101 = -1$ //

(III) $3+2 =$

+ve 011
+ve 010
______
-ve $101 \xrightarrow{2's} 111$

not possible

$= -3$ ✗

(IV) $-3-2$

-ve 101
-ve 110
______
$1)\ 011 \longrightarrow 011\ +3$ ✗

+ve

NP

Note:

1) Overflow can never occur on addition of two numbers with opposite sign. Overflow may occur on addition of two number with same sign.

11) Addition of two positive number should never give a negative no. If such condⁿ occur. It indicates overflow

* 111) In MSB If $C_{in} \oplus C_{out} = 1$ Indicates overflow

$$
\begin{array}{rrrr}
C_{in} & 0 & & \\
 & 1 & 0 & 1 \\
 & 0 & 1 & 0 \\
\hline
C_{ou}\ 0 & 1 & 1 & 1
\end{array}
$$

$C_{in} \oplus C_{ou} = 0$ no overflo

$$
\begin{array}{rrrr}
C_{in} & 0 & & \\
 & 1 & 0 & 1 \\
 & 1 & 1 & 0 \\
\hline
C_{ou}\ 1 & 0 & 1 & 1
\end{array}
$$

$C_{in} \oplus C_{out} = 1$ overflow

Que.

(I)

$$
\begin{array}{r}
1100 \\
1011 \\
\hline
1)\ 0111
\end{array}
$$

$0 \oplus 1 = 1$ OV

(II)

$$
\begin{array}{r}
1001 \\
1110 \\
\hline
1)\ 0111
\end{array}
$$

$1 \oplus 0 = 1$ OV

(III)

$$
\begin{array}{r}
0110 \\
1010 \\
\hline
1)\ 0000
\end{array}
$$

{ no need of checking opposite sign never gen. ov }

$C_{in} \oplus C_{ou} = 0$ no f

(IV)

$$
\begin{array}{r}
0111 \\
0110 \\
\hline
0)\ 1001
\end{array}
$$

$1 \oplus 0 =$ OV

Que. Two 2's Complement no. having Sign bit x and Y are added and the sign bit of the Result is Z which boolean function indicates the occurrence of Overflow.

x _ _   for over flow

y _ _   ~~Cout~~

z _ _

x	y	z	overflow 'f'	
0	0	0	0	
0	0	1 negat	1	overflow occu.
0	1	0	0	
0	1	1	0	
1	0	0	0	no overflo
1	0	1	0	
1	1	0	1	
1	1	1	0	

Sum of two +ve

opposite sign

x \ yz				
	0	1	0	0
	0	0	0	1

$f = \bar{x}\bar{y}z + xy\bar{z}$

$=$

Sol

	$Y+Z$	$Y+\bar{Z}$	$\bar{Y}+\bar{Z}$	$\bar{Y}+Z$
x	0	1	0	0
$\bar{x}$	0	0	0	1

$(Y+Z) * (\bar{x}+\bar{Z}) * (x+\bar{Y})$

$\Rightarrow (y+z)(\bar{x}+\bar{z})(x+\bar{y})$

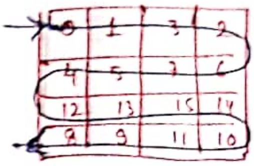
Code Converter

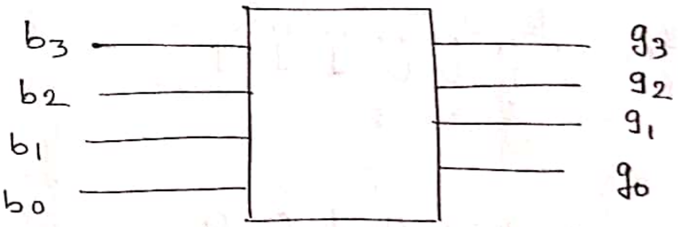
1. Binary to Gray (4bit)

	Input				Output			
	b_3	b_2	b_1	b_0	g_3	g_2	g_1	g_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
2	0	0	1	0	0	0	1	1
3	0	0	1	1	0	0	1	0
4	0	1	0	0	0	1	1	0
5	0	1	0	1	0	1	1	1
6	0	1	1	0	0	1	0	1
7	0	1	1	1	0	1	0	0
8	1	0	0	0	1	1	0	0
9	1	0	0	1	1	1	0	1
10	1	0	1	0	1	1	1	1
11	1	0	1	1	1	1	1	0
12	1	1	0	0	1	0	1	0
13	1	1	0	1	1	0	1	1
14	1	1	1	0	1	0	0	1
15	1	1	1	1	1	0	0	0

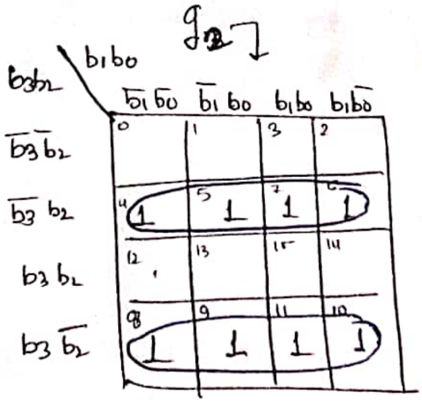
note

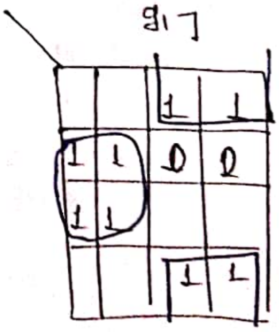
Gray Code

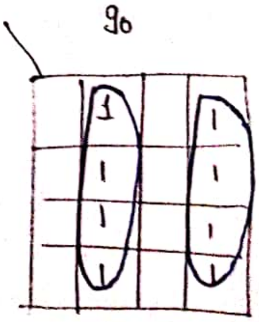




$g_3(b_3, b_2, b_1, b_0) = b_3$







$g_2 = \bar{b}_3 b_2 + b_3 \bar{b}_2$

$= b_2 \oplus b_3$

$g_1 = b_2 \oplus b_1$

$g_0 = b_1 \oplus b_0$

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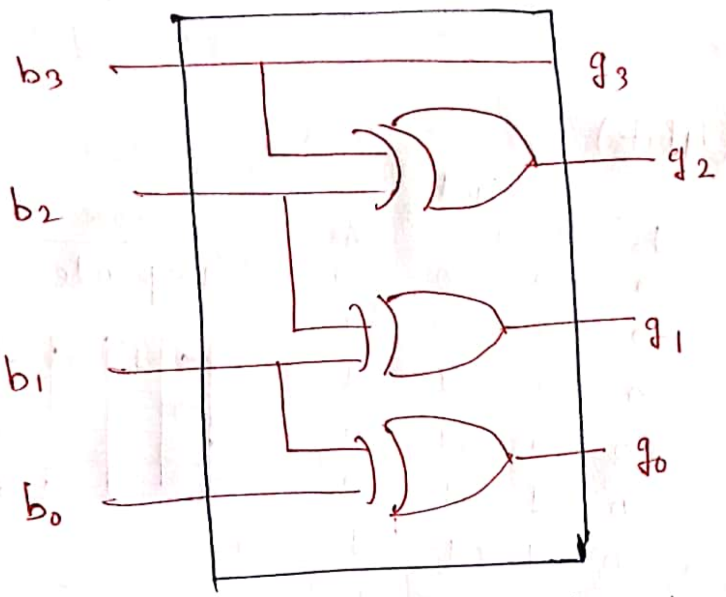


ckt. Binary to Gray Converter

eg. (0110) $\longrightarrow$ $g_3 = 0$ (0101)
$b_3 b_2 b_1 b_0$

$g_2 = 1$

$g_1 = 0$

$g_0 = 1$

eg. 0 1 1 0 → 0 1 0 1

$(0 \; 1 \; 1 \; 0)_2$

$\downarrow$

$(0 \; 1 \; 0 \; 1)$ gray

eg. 0 1 1 0 1 0 0 1 1 1

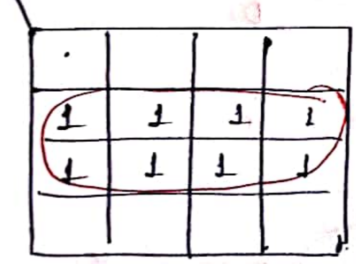
$\downarrow$

0 1 0 1 1 1 0 1 0 0

* Gray to binary (4bit) Converter:

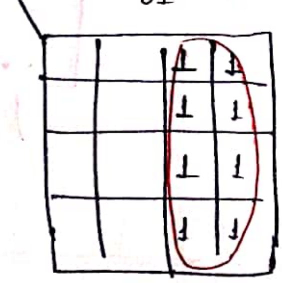
	g_3	g_2	g_1	g_0	b_3	b_2	b_1	b_0
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	1
3	0	0	1	1	0	0	1	0
2	0	0	1	0	0	0	1	1
6	0	1	1	0	0	1	0	0
7	0	1	1	1	0	1	0	1
5	0	1	0	1	0	1	1	0
4	0	1	0	0	0	1	1	1
12	1	1	0	0	1	0	0	0
13	1	1	0	1	1	0	0	1
15	1	1	1	1	1	0	1	0
14	1	1	1	0	1	0	1	1
10	1	0	1	0	1	1	0	0
11	1	0	~~0~~ 1	1	1	1	0	1
9	1	0	0	1	1	1	1	0
8	1	0	0	0	1	1	1	1

b_2

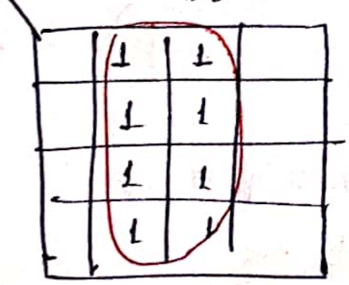


~~b_2~~ =

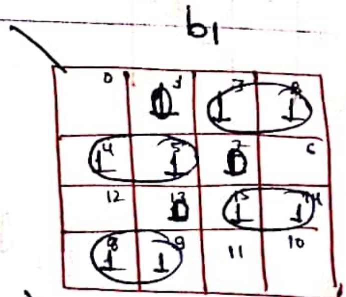
b_1



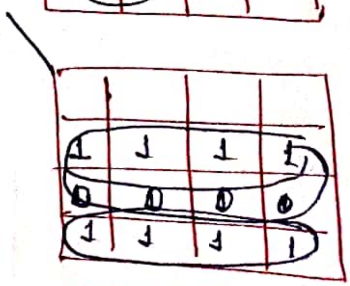
b_0



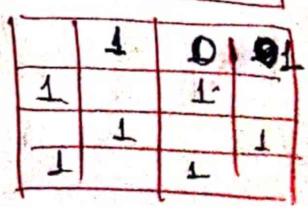
b_1



b_2



b_0



Scanned by CamScanner

$$b_3 = g_3$$

$$b_2 = g_3 \oplus g_2 = b_3 \oplus g_2$$

$$b_1 = g_3 \oplus g_2 \oplus g_1 = b_2 \oplus g_1$$

$$b_0 = \bar{g}_3 \bar{g}_2 (g_1 \oplus g_0) + \bar{g}_3 g_2 (\overline{g_1 \oplus g_0}) + g_3 g_2 (\bar{g}_1 \oplus g_0) + g_3 \bar{g}_2 (\overline{g_1 \oplus g_0})$$

$$b_0 = (g_1 \oplus g_0)(\overline{g_3 \oplus g_2}) + (\overline{g_1 \oplus g_0})(g_3 \oplus g_2)$$

$$b_0 = g_0 \oplus g_1 \oplus g_2 \oplus g_3 = b_1 \oplus g_0$$

Standard result

ab \ cd

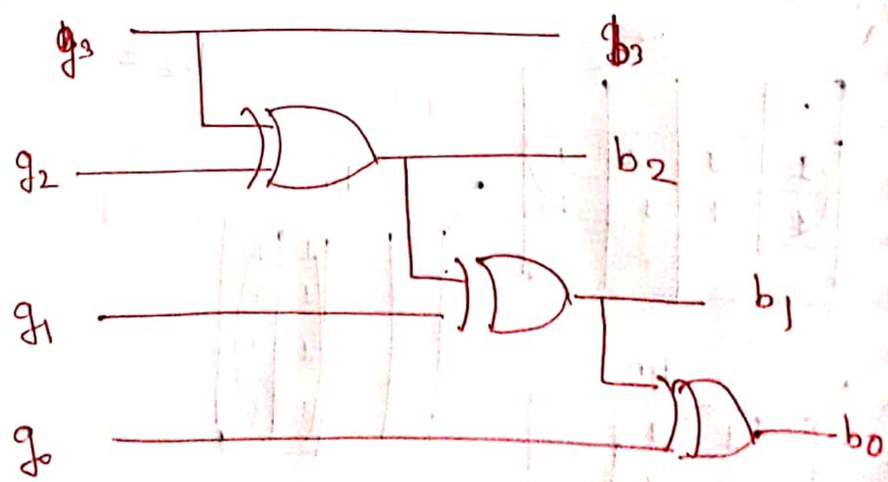
	1		1
1		1	
	1		1
1		1	

$\rightarrow a \oplus b \oplus c \oplus d$

ab \ cd

1		1	
	1		1
1		1	
	1		1

$\rightarrow a \odot b \odot c \odot d$



* delay of G to B conv. is $>$ delay of B to G conv.
  $n\Delta t$ $\qquad$ Δt sec

eg. 1 1 0 0 → 1 0 0 0

1 0 0 1 → 1 1 1 0

eg. 1 0 1 1 0 0 1 0 1 → 1 1 0 1 1 1 0 0 1

<u>Que.</u> $(24)_8 \longrightarrow$ gray

010100 ⟶ 011110

* Xs-3 Code to BCD Code

x_3	x_2	x_1	x_0	b_3	b_2	b_1	b_0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	1

eg. 1 1 0 0 → 1 0 0 0   1 0 0 1 → 1 1 1 0

eg. 1 0 1 1 0 0 1 0 1 → 1 1 0 1 1 1 0 0 1

Que. $(24)_8 \longrightarrow$ gray

010100 $\longrightarrow$ 011110

* Xs-3 Code to BCD Code

x_3	x_2	x_1	x_0	b_3	b_2	b_1	b_0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	1
0	1	0	1	0	0	1	0
0	1	1	0	0	0	1	1
0	1	1	1	0	1	0	0
1	0	0	0	0	1	0	1
1	0	0	1	0	1	1	0
1	0	1	0	0	1	1	1
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	1

B_0

$x_3x_2 \backslash x_1x_0$	$\bar{x}_1\bar{x}_0$	$\bar{x}_1x_0$	x_1x_0	$x_1\bar{x}_0$
	X	X		X
$\bar{x}_3x_2$	1		1	1
x_3x_2	1	X	X	X
$x_3\bar{x}_2$	1			1

$$B_0 = \cancel{x_3x_1x_0} + \cancel{\bar{x}_0 x_3} + \bar{x}_1\bar{x}_0$$

$$B_0 = \bar{x}_1\bar{x}_0 + x_1\bar{x}_0$$

$x_3x_2 \backslash x_1x_0$	$\bar{x}_1\bar{x}_0$	$\bar{x}_1x_0$	x_1x_0	$x_1\bar{x}_0$
$\bar{x}_3\bar{x}_2$	X	X		X
$\bar{x}_3x_2$		1		1
x_3x_2		X	X	X
$x_3\bar{x}_2$		1		1

$$B_1 = \bar{x}_1x_0 + x_1\bar{x}_0$$
$$= x_0 \oplus x_1$$

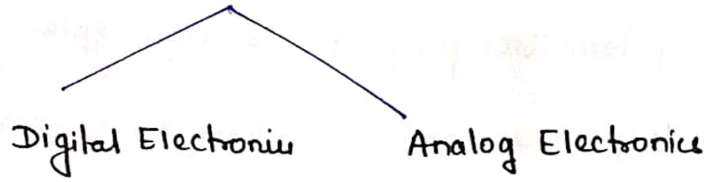
$x_3x_2 \backslash x_1x_0$	$\bar{x}_1\bar{x}_0$	$\bar{x}_1x_0$	x_1x_0	$x_1\bar{x}_0$
$\bar{x}_3\bar{x}_2$	X	X		X
$\bar{x}_3x_2$			1	
x_3x_2	1	X	X	X
$x_3\bar{x}_2$	1	1		1

$$B_2 = \bar{x}_2\bar{x}_1 + \bar{x}_0\bar{x}_2 + x_2x_1x_0$$

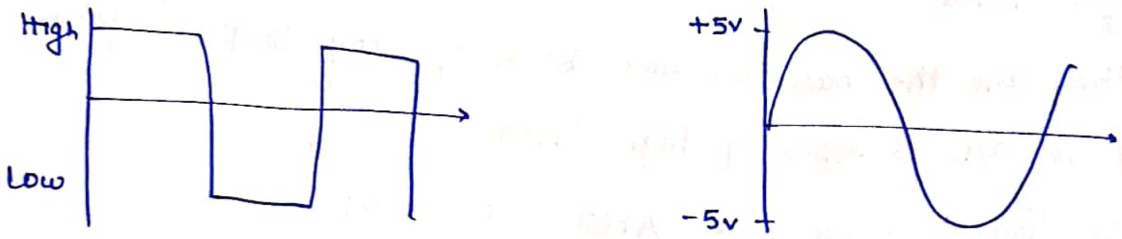
		X		
		1		

$$B_3 = \bar{x}_1x_0x_3$$

Electronics System & Circuits



- In Analog Electronics, Voltage may take any of the Infinite possible Value where as in Digital Electronics we Consider only two levels - logic high and logic low.



- Both Analog and Digital Electronics use Same Components.
- Digital Circuit is also called logic Circuit or Switching ckt.
- Transition time of digital System is assumed to be Zero.

Advantages of Digital Electronics

- Designing of Digital ckt is easy.
- Information storage is easy.
- Effect of Noise is less.
- Digital Circuits are more Versatile (Reusable)

Application of DE :-

- It is preffered in field of measurement, Control System, Comm^n, and data processing over Analog Electronics.

Disadvantage of Digital Electronics :-

- Real World is analog. Hence required ADC, DAC in the System.

* Designing of Digital System

Step 1 :- System Designing : Complete System divided into various block and their Inter Connection

Step 2 :- logic Designing : Each block Design using logic gate

Step 3 :- Circuit Designing : Technology used to design gates

* Logic Gates

→ They are the basic building block of any Digital System

There are 3 types of logic Gates

1) Basic Gates → AND OR NOT

II) Universal Gates → NAND NOR

III) Special purpose Gates → Ex-OR Ex-NOR

I. Basic Gates

1) AND Gate

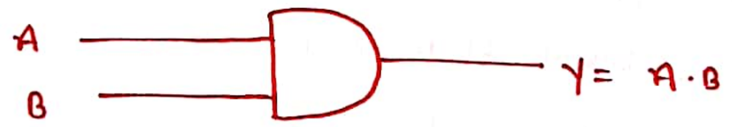


fig. Logic Symbol of 2 i/p AND gate

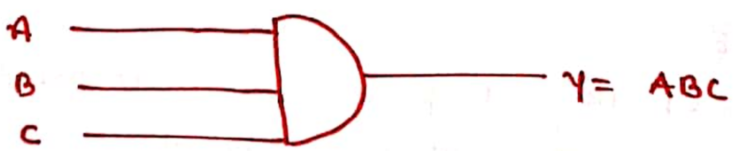


fig. Logic Symbol of 3 i/p AND gate

AND gate is a Logic gate whose o/p is high only when all the Input are high

OR AND gate is a Logic gate whose o/p is Logic Low if atleast one of the Input is at Logic Low

Truth table :-

A	B	C	Y = ABC
L	L	L	L
L	L	H	L
L	H	L	L
L	H	H	L
H	L	L	L
H	L	H	L
H	H	L	L
H	H	H	H

note :- Truth table is a table which Consist of all the possible Combination of i/p along with their Corresponding o/p.

II) OR Gate

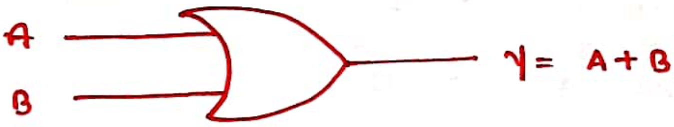


fig: Logic Symbol of 2 i/p OR gate

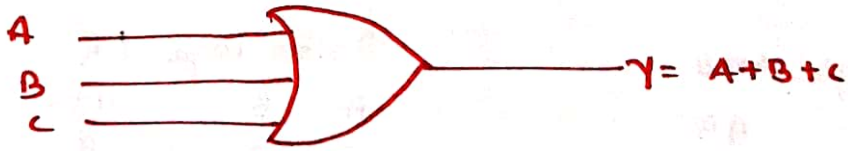


fig. 3 i/p OR gate

OR gate is a Logic gate whose o/p is Logic High when any one Input is Logic High

OR gate is a logic gate whose o/p is logic low only when all Input are Logic low.

Truth Table :-

A	B	C	$Y = A+B+C$
L	L	L	L
L	L	H	H
L	H	L	H
L	H	H	H
H	L	L	H
H	L	H	H
H	H	L	H
H	H	H	H

note:- Positive Logic Digital System

High ⟷ '1'

Low ⟷ '0'

Negative Logic Digital System

High ⟷ '0'

Low ⟷ '1'

Positive Logic AND

A	B	AB
0	0	0
0	1	0
1	0	0
1	1	1

Positive Logic OR

A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

Negative Logic AND

A	B	AB
1	1	1
1	0	1
0	1	1
0	0	0

Negative Logic OR

A	B	A+B
1	1	1
1	0	0
0	1	0
0	0	0

Keypoint :- Positive logic AND ≡ Negative logic OR
Negative logic AND ≡ Positive logic OR

III) NOT gate

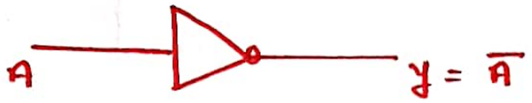


fig. logic Symbol of NOT gate

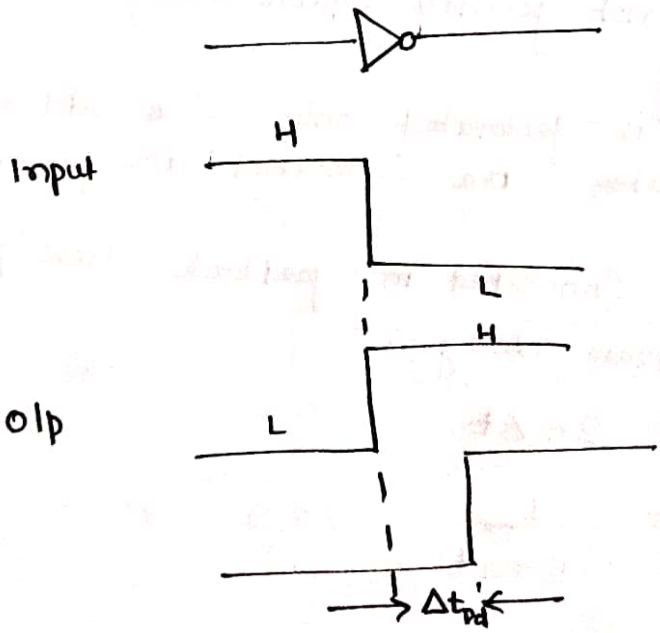
NOT gate has only one Input

Truth Table

A	$y = \overline{A}$
0	1
1	0

NOT gate is a logic gate whose o/p is Complement of its Input.

Propagation Delay :- Time taken by the gate to reflect the change in Input to the output



where, Δt_{pd} is delay time.

ex. Consider a Circuit

1)

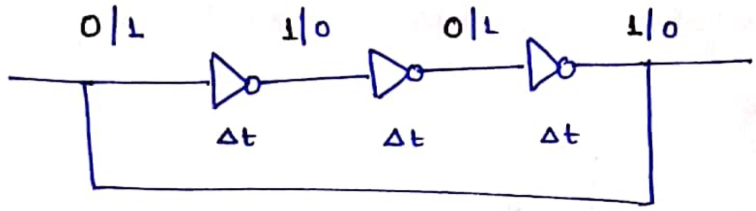
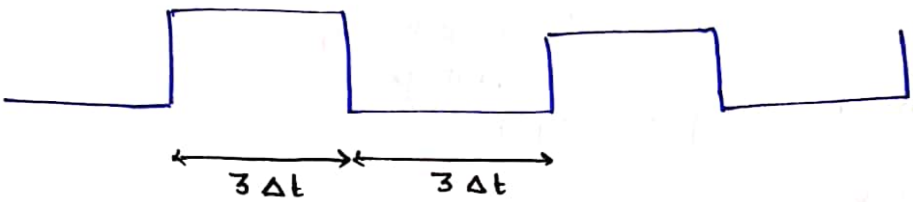


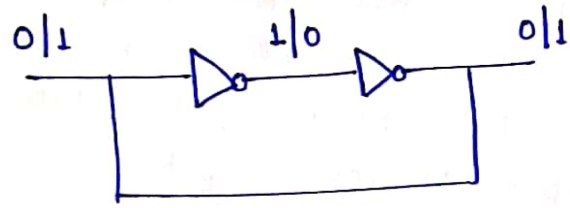
fig. Square wave Generator
OR
Clock generator
OR
Ring Oscillator
OR
Astable Multi vibrator



$T = 6\Delta t$

$f = \frac{1}{6\Delta t}$

II)



It will not generate Square waveform

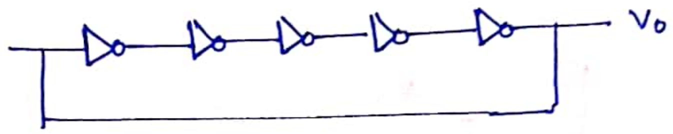
note:- Square Wave is generated only when odd no. of Cascaded Inverter are Connected in feed back.

For 'n' Inverter Connected in feed back time period of generated Square Wave form

$$T = 2n\Delta t$$

$$f = \frac{1}{2n\Delta t}$$

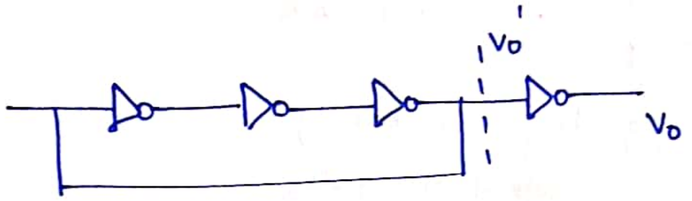
Que. For the Ring Oscillator shown in fig the propagation Delay of each Inverter is 100psec. What is fundamental frequency of the Oscillator o/p



Sol $n = 5$

$$f = \frac{1}{2 \times 5 \times 100} = 1\,GHz$$

Que.



Propagation delay of each not gate is 2ns. The Time period of generated Square wave is

Sol

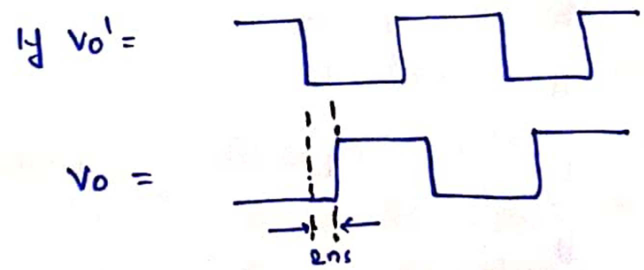
$$T = 6\Delta t + \Delta t$$

Wrong

$$= 12ns + 2ns$$

$$= 14ns$$

shifting doesnot affect time period

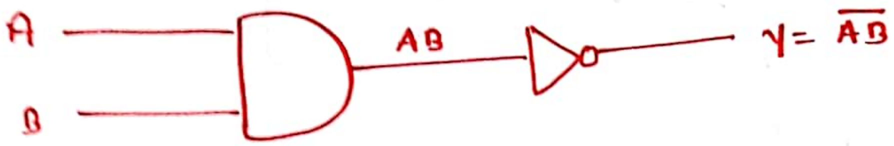


$$T = 6\Delta t$$

$$= 12ns$$ //

Universal Gate:

1) NAND gate :- It is an AND gate followed by a nor gate



OR

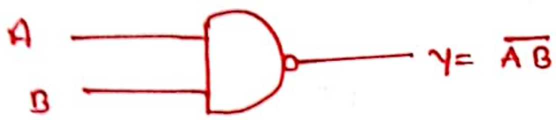


fig. Logic Symbol of 2 i/p NAND gate

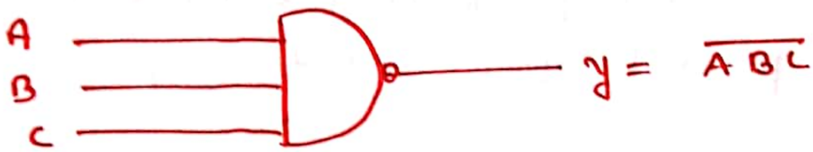


fig. Logic Symbol of 3 i/p NAND gate

Truth Table:

A	B	C	A B C	$Y = \overline{ABC}$
L	L	L	L	H
L	L	H	L	H
L	H	L	L	H
L	H	H	L	H
H	L	L	L	H
H	L	H	L	H
H	H	L	L	H
H	H	H	H	L

NAND gate is a logic gate whose output is logic low only when all the inputs are at logic high.

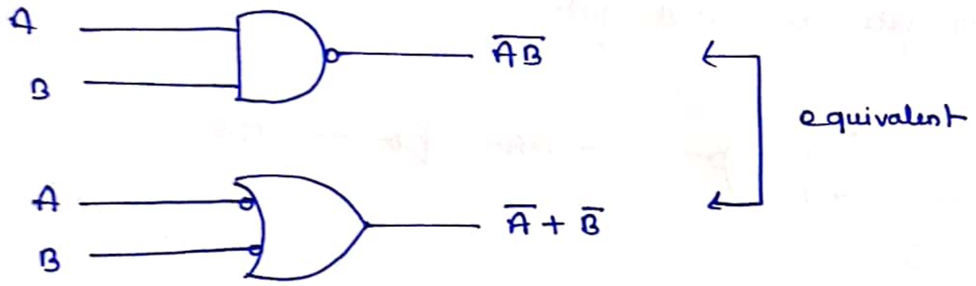
OR

NAND gate is a logic gate whose o/p is logic high when any of the inputs is at logic low.

ex.

A	B	$\overline{AB}$	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	1	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	0	0	0	0

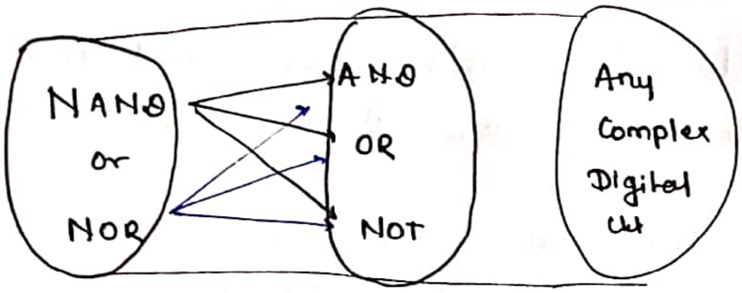
$$\overline{AB} = \overline{A} + \overline{B}$$



Keypoint: NAND gate is bubbled OR gate.
bubbled OR gate is active low OR gate or negative OR gate
Normal OR gate is active High OR gate or

Basic Gates :- AND, OR, NOT are called as basic gates as we can implement a digital ckt of any complexity using combination of these basic gates

Universal Gate :- NAND & NOR Gates are called as Universal gate as these gates (either NAND or NOR gate) are single handedly capable of designing a digital ckt of any complexity.



Que. Justify NAND gate as NOT gate

Sol

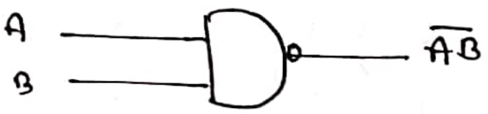


fig. NAND gate as NOT gate

NAND gate as AND gate

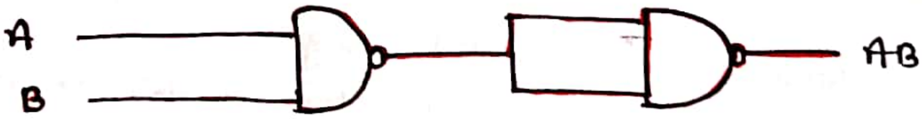


fig. NAND gate as AND gate

NAND gate as OR gate

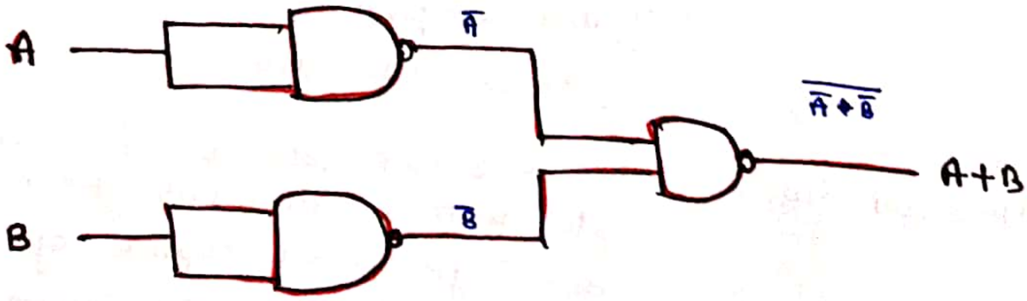


fig. NAND gate as OR gate

required function $= A+B = \overline{\overline{A+B}}$

$= \overline{\bar{A} \cdot \bar{B}}$

Que. find the Min. no. of two Input NAND gate required to design the following functions

1. $f_1 = A+B$
2. $f_2 = AB + CD$
3. $f_3 = A + BC$

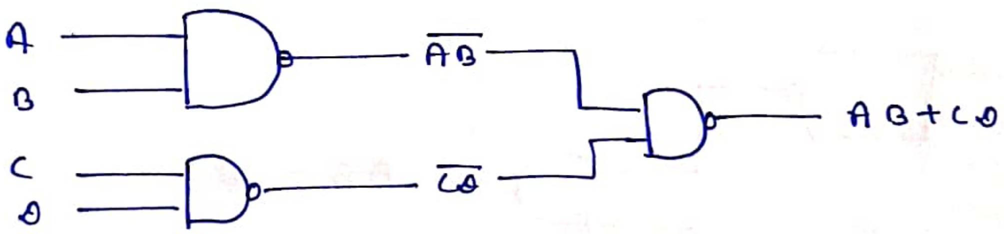
Sol 1 :- Step 1: $A+B = \overline{\overline{A+B}}$

Step 2: Demorgan Law $\overline{\overline{A+B}} = \overline{\bar{A} \cdot \bar{B}}$

$= A+B$

Sol 2. $AB+CD = \overline{\overline{AB+CD}}$

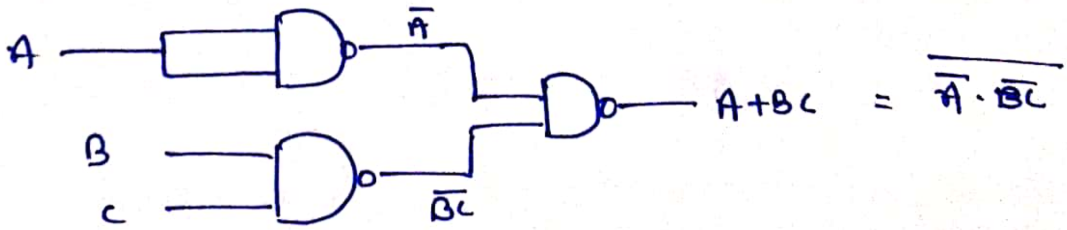
$= \overline{\overline{AB} \cdot \overline{CD}}$



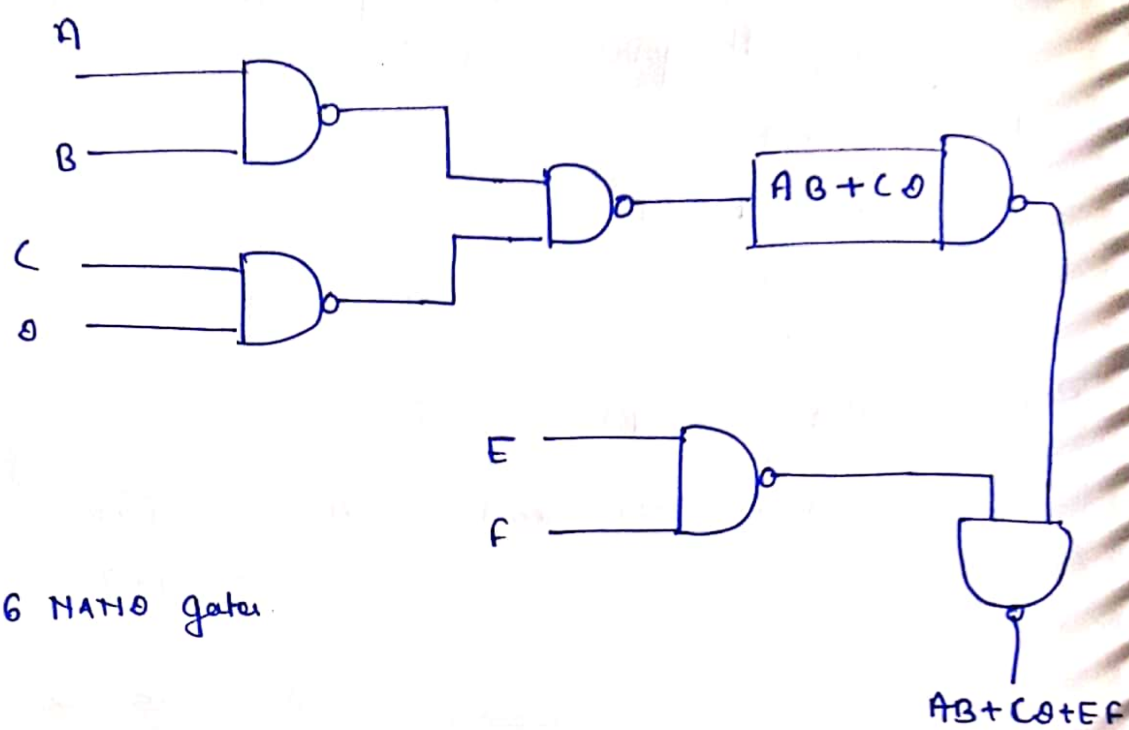
Sol 3. $f_3 = A + BC$

$= \overline{\overline{A+BC}}$

$= \overline{\bar{A} \cdot \overline{BC}}$



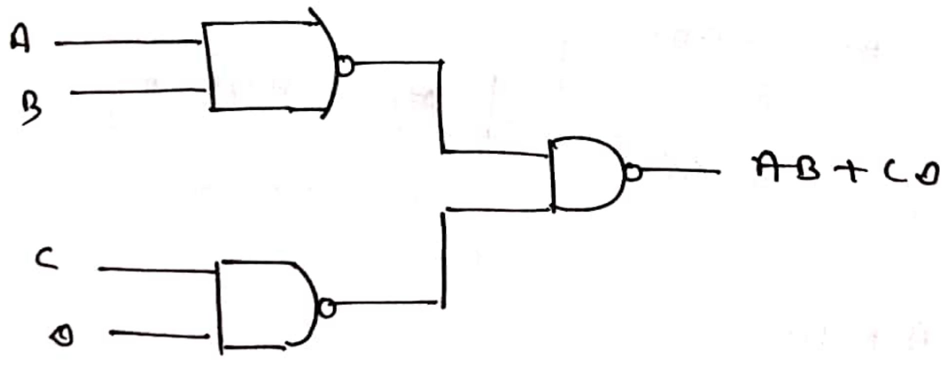
(4) $F = AB + CD + EF$



Total 6 NAND gates

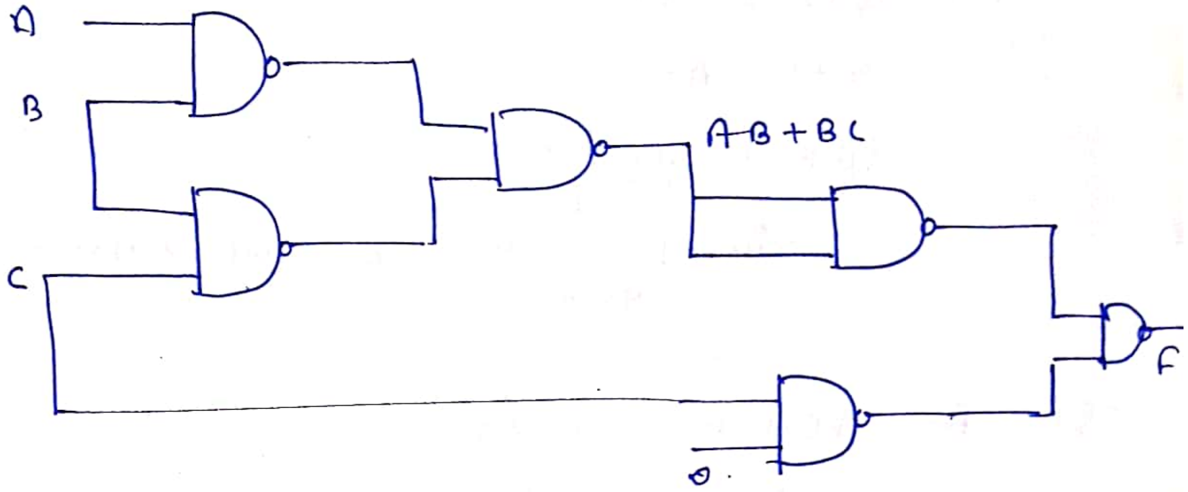
Note:

$F = AB + CD$ form



Try to adjust in form of $AB + CD$

Q. $F = AB + BC + CD$

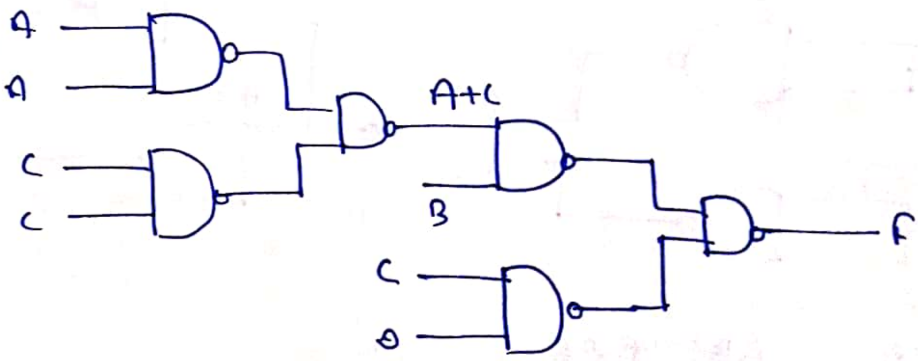


OR

$F = AB + BC + CD$

$= B(A+C) + CD$

$= B(\overline{\overline{A}\,\overline{A} + \overline{C}\,\overline{C}}) + CD$



Que. $F = AB + BC + CD + DA$

$= (A+C)B + D(C+A)$

$= \underbrace{(\overline{\overline{A}\,\overline{A} + \overline{C}\,\overline{C}})}_{X} B + D\underbrace{(\overline{\overline{C}\,\overline{C} + \overline{A}\,\overline{A}})}_{Y}$

X = 3 NAND gate

Y = 3 NAND gate

now $F = BX + DY$ $\quad$ F = 3 Nand gate $\quad$ So total 9

OR $f = AB + BC + CD + DA$

$f = B(A+C) + D(A+C)$

$= (B+D)(A+C)$

$= \underbrace{(\underbrace{B\cdot B + D\cdot D}_{3\,NAND})}_{}\underset{2\,NAND}{\cdot}(\underbrace{A\cdot A + C C}_{3\,NAND})$ = Total 8 NAND gate

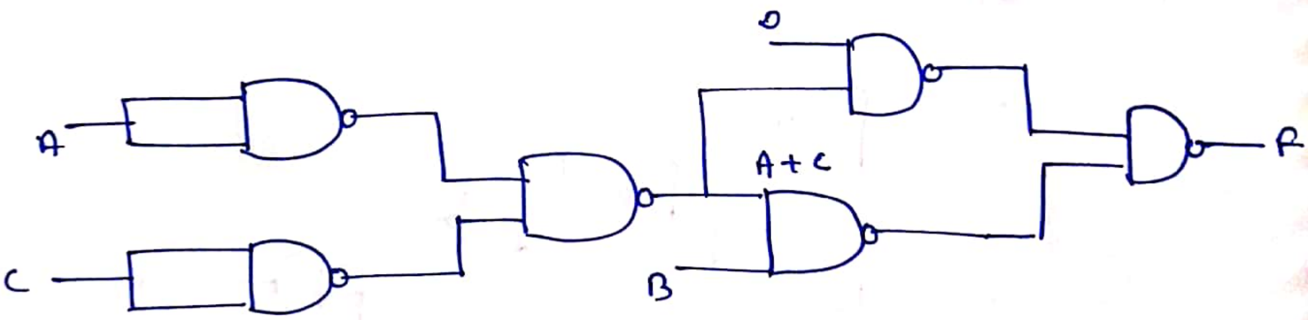
OR $f = AB + BC + CD + DA$

$= B(A+C) + D(A+C)$

$= \underbrace{BX + DX}_{3\,NAND}$

and $A + C = AA + CC = 3\,NAND$

So total 6 NAND gate //.



7. $f = ABC + DE$

$= \overline{\overline{ABC + DE}} = \overline{\overline{ABC}\cdot\overline{DE}}$

$= ABC + DE$

$= \underbrace{AX + DE}_{3\text{ gate} + 2\text{ gate }(X)}$

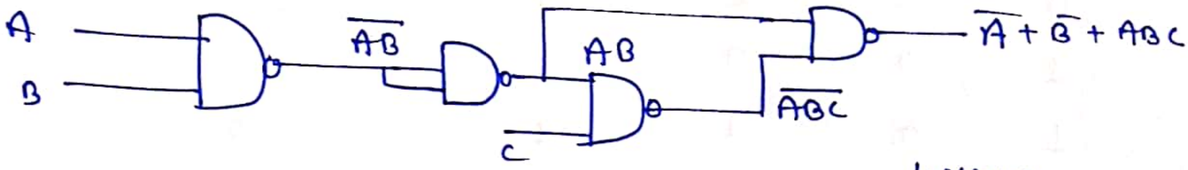
$= 5$ gate

8. $\overline{A} + \overline{B} + ABC$

Sol $\overline{A} \cdot \overline{A} + \overline{B}\,\overline{B} + A \cdot X$

$f = \overline{A} + \overline{B} + ABC$

$= \underbrace{\overline{AB}}_{1\ \text{gate}} + \underline{ABC}$



4 NAND

$f = \overline{A} + \overline{B} + ABC$

$= \overline{AB} + ABC$

$= \overline{\overline{\overline{AB} + ABC}}$

$= \overline{AB \cdot \overline{ABC}}$

II. NOR gate :- NOR gate is an OR gate followed by a NOT gate

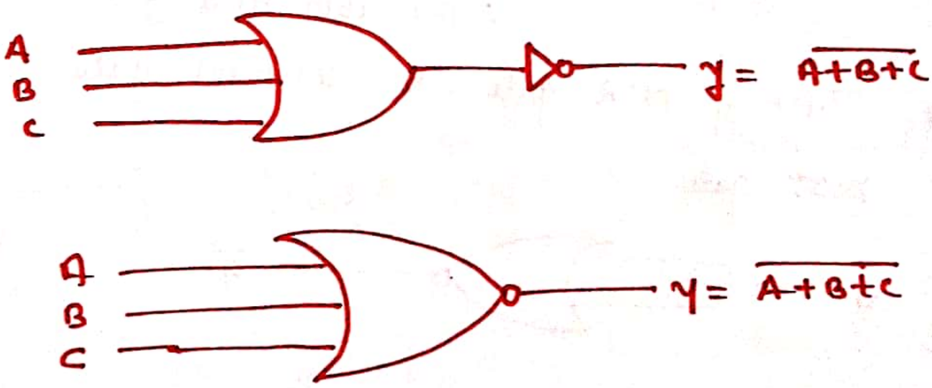


fig. logic diag. of 3 i/p NOR gate

Truth table:

A	B	C	A+B+C	$\overline{A+B+C}$
0	0	0	0	1
0	0	1	1	0
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

NOR gate is a logic gate whose o/p is logic high only when all the i/p are logic low

OR

NOR gate is a logic gate whose o/p is logic low if any of its Input is logic high.

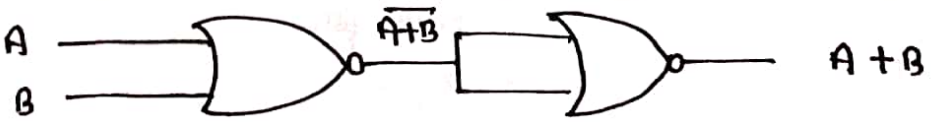
Note: NOR gate ≡ Bubbled AND gate / Negative AND Gate / Active low AND gate

Que. Justify NOR gate as universal gate

1) NOT gate

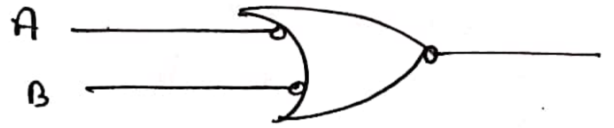
$$\left\{ \begin{array}{l} \overline{A+A} = \bar{A} \\ A+A = A \end{array} \right.$$

ii) OR gate

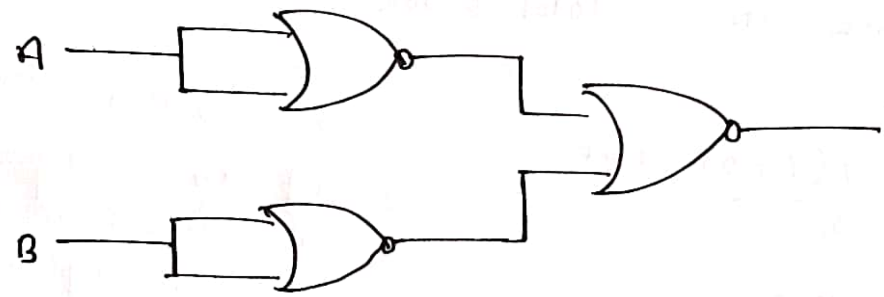


III) AND gate

$$Y = A \cdot B$$
$$= \overline{\overline{A \cdot B}}$$
$$= \overline{\bar{A} + \bar{B}}$$



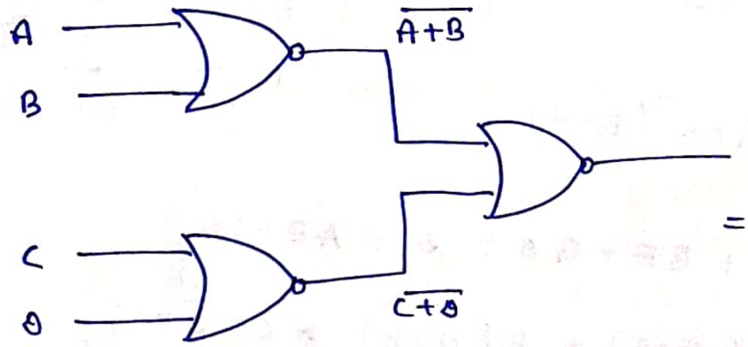
$\overline{\bar{A} + \bar{B}} = A \cdot B$



$\overline{\bar{A} + \bar{B}} = AB$

fig. AND gate using NOR

Note :

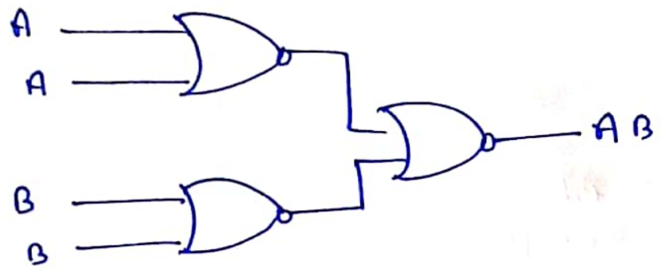


$$\overline{\overline{A+B} + \overline{C+D}}$$
$$= (A+B)(C+D)$$

Que1. Design following function using Min. no. of 2 i/p NOR gates.

1) $F = A \cdot B$

Sol $F = A \cdot B = (A+A) \cdot (B+B)$



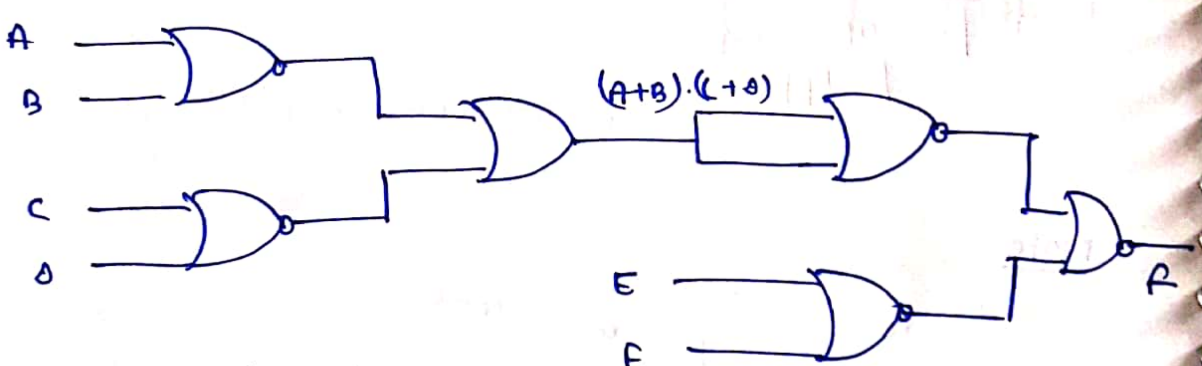
ii) $F = A(B+C)$

$= (A+A)\cdot(B+C)$

2 NOR   1 NOR   Total 3 NOR

iii) $F = \underbrace{(A+B)(C+D)}_{X}(E+F)$

Sol



$(X+X)(E+F)$

iv) $F = AD + BE + BD + CD + AE + CE$

$= D(A+B) + E(A+B) + C(D+E)$

$= (A+B)(D+E) + C(D+E)$

$= (D+E)(A+B) + (C+C)(D+E)$

$= (A+B)X + X(C+C)$

$= (A+B)(X+X) + (X+X)(C+C)$

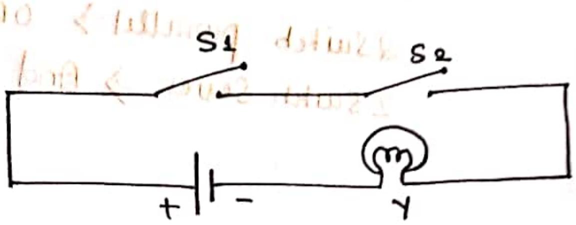
$$F = (A+B+C)(D+E)$$

$$= \underbrace{(X+C)(D+E)}_{3\text{ gate}} = 5 \text{ gate}$$

2 gate

* Switching Circuit representation of logic gates :-

1)



Switch ON ⇒ Short circuited i.e high

Switch OFF ⇒ open circuited i.e low

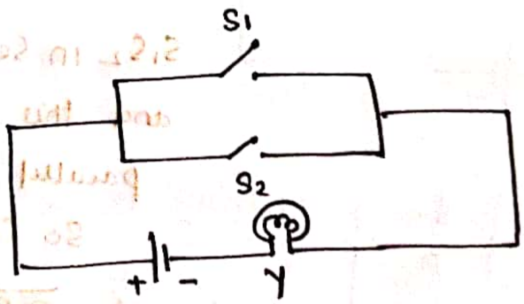
Bulb glowing ⇒ High

Bulb not glowing ⇒ low

S_1	S_2	Y
OFF 0	OFF 0	no glow 0
OFF 0	ON 1	notglow 0
ON 1	OFF 0	not glow 0
ON 1	ON 1	glow 1

Hence this Switching ckt represents AND gate.

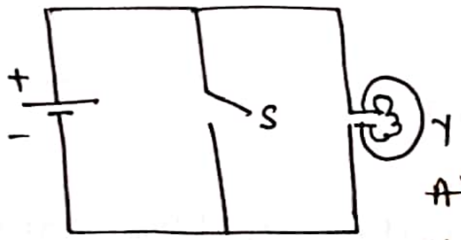
II)



S_1	S_2	Y
0	0	0
0	1	1
1	0	1
1	1	1

It Represent OR gate

iii)



~~AND → Series with bulb~~

~~NAND → parallel with bulb~~

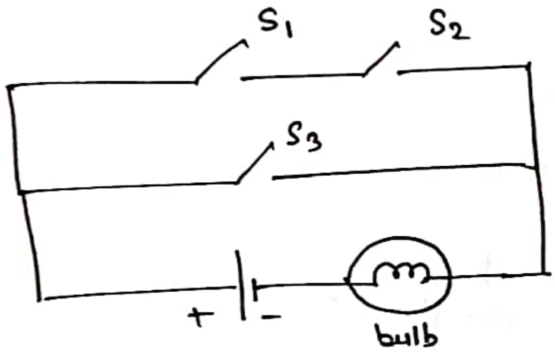
2 switch parallel ⇒ OR

2 switch series ⇒ And

S	Y
0	1
1	0

This is **NOT gate**

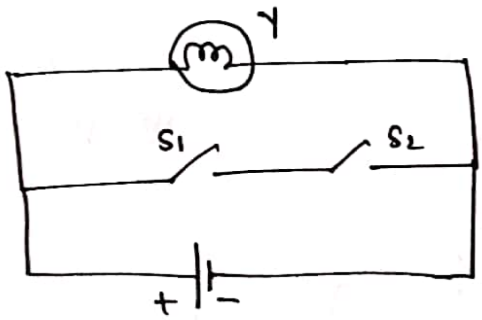
iv)



→ $S_3 + S_1 S_2$

and this arrangement series with bulb

Hence no complement applied



This is **NAND gate**

$S_1 S_2$ in series so AND

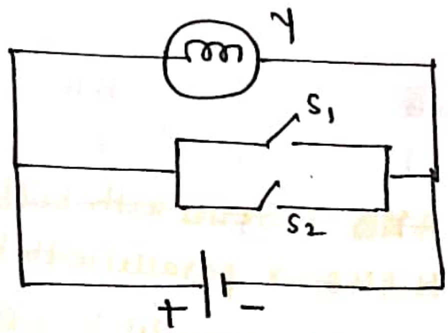
and this arrangement parallel with bulb

So complement

S_1	S_2	Y
0	0	1
0	1	1
1	0	1
1	1	0

$Y = \overline{S_1 S_2}$

v)

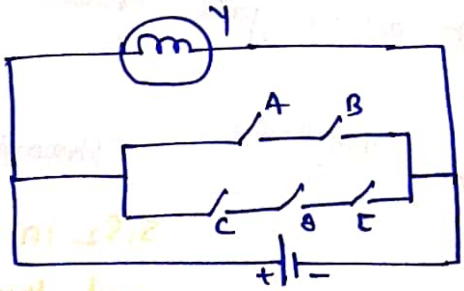


S_1	S_2	Y
0	0	1
0	1	0
1	0	0
1	1	0

This arrangement is NOR gate

Que. Draw the Truth table & Switching circuit.

$$f = \overline{AB + CDE}$$



Parallel with bulb we complement applied.

* Special Purpose gates :-

1) Exclusive OR gate (Ex-OR) :-

$A \oplus B = \bar{A}B + A\bar{B}$

fig. logic Symbol of 2 i/p ex-OR gate

Truth table

both i/p different

A	B	$\bar{A}$	$\bar{B}$	$\bar{A}B$	$A\bar{B}$	$\bar{A}B+A\bar{B}$
0	0	1	1	0	0	0
0	1	1	0	1	0	1
1	0	0	1	0	1	1
1	1	0	0	0	0	0

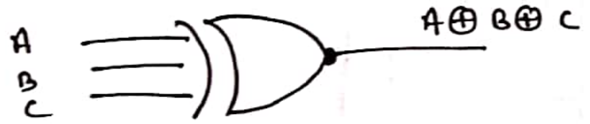
Def. :- Ex-OR gate is a logic gate whose o/p is logic high when both the Input are inequal or different

OR

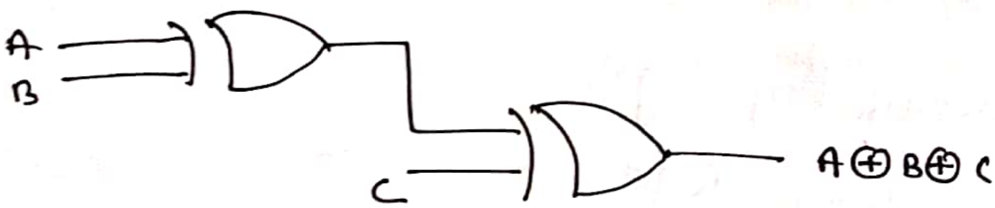
Ex-OR gate is a logic gate whose o/p is logic low when both the Input are Similar or equal.

- Ex-OR gate detects inequal Inputs by producing logic high at the output So it is also Called inequality detector.

note:- 3 or more Input ex-or gate doesnot exist.



Theoritically Correct



Practically

ex.

A	B	C	$A \oplus B$	$(A \oplus B) \oplus C$	No. of 1's in A, B & C
0	0	0	0	0	even
0	0	1	0	1	→ odd
0	1	0	1	1	→ odd
0	1	1	1	0	even
1	0	0	1	1	→ odd
1	0	1	1	0	even
1	1	0	0	0	even
1	1	1	0	1	→ odd

Ex-OR gate is an odd number of 1's detector

ex. $1 \oplus 0 \oplus 1 \oplus 0 \oplus 0 \oplus 1 = 1$

Que. find

1) $A \oplus 0 = A$

2) $A \oplus 1 = \bar{A}$

3) $A \oplus A = 0$

4) $A \oplus \bar{A} = 1$

Que. If $A \oplus B = C$

$A \oplus C = ?$

Sol $\bar{A}B + A\bar{B} = C$

$A \oplus C$

$\Rightarrow A \oplus A \oplus B$

$\Rightarrow 0 \oplus B$

$= B$ //.

5) $A \oplus B = C$ Note

$A \oplus C = B$

6) If $A \oplus B = C$

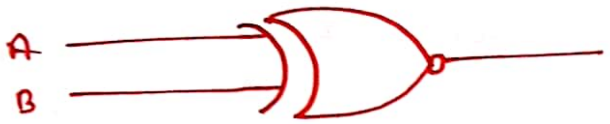
then $A \oplus B \oplus C =$

Sol

$C \oplus C = 0$

$A \oplus B \oplus C = 0$

11) Ex-NOR gate :-



$Y = A \odot B$

$= AB + \bar{A}\bar{B}$

ex-nor gate is an ex-OR gate followed by an Inverter.

	A	B	$\bar{A}$	$\bar{B}$	AB	$\bar{A}\bar{B}$	$AB+\bar{A}\bar{B}$
both i/p equal	0	0	1	1	0	1	$\boxed{1}$
	0	1	1	0	0	0	0
	1	0	0	1	0	0	0
both i/p equal	1	1	0	0	1	0	$\boxed{1}$

dy. Output is Logic high when both Input are Similar in Case of ex-nor gate.

o/p is Logic Low when both i/p are different in Case of ex-nor gate

Ex-nor gate is also Called equality detector because it give o/p logic high when both Inputs are equal.

Scanned by CamScanner

$$A \oplus B \oplus C = \overline{A \oplus B} \cdot \bar{C} + C \cdot \overline{A \oplus B}$$

$$= \overline{A \odot B} \cdot \bar{C} + C \cdot \overline{\overline{A \odot B}}$$

$$= \overline{A \odot B} \cdot \bar{C} + C \cdot A \odot B$$

$$= \bar{x}\bar{y} + xy$$

$$= x \odot y$$

$$\Rightarrow A \odot B \odot C$$

Hence $\boxed{A \oplus B \oplus C = A \odot B \odot C}$

Concept :-

$$\overline{A \oplus B} = A \odot B$$

$$A \oplus B \oplus C = A \odot B \odot C$$

$$\overline{A \oplus B \oplus C \oplus D} = A \odot B \odot C$$

XOR = $\overline{\text{XNOR}}$; for even no. of Input

XOR = XNOR ; for odd no. of I/P

for odd no. of Input

XOR = XNOR   i.e ~~both~~ XNOR is odd no. of 1's detector if I/p are odd

For even no. of Input

$\overline{\text{XOR}}$ = XNOR   i.e XNOR is even no of 1's detector if I/p are even eg1. $1 \oplus 0 \oplus 1 \oplus 0 \oplus 1 \oplus 1$ = even 1's o/p = 0

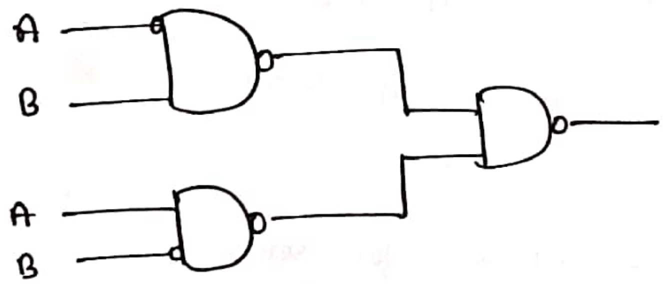
ii) $1 \oplus 1 \oplus 0 \oplus 1 \oplus 1 \oplus 1 \oplus 0$ = odd 1's o/p = 1

iii) $1 \odot 0 \odot 0 \odot 1 \odot 1 \odot 1$ = even 1's and even i/p
4 6
o/p = '1'

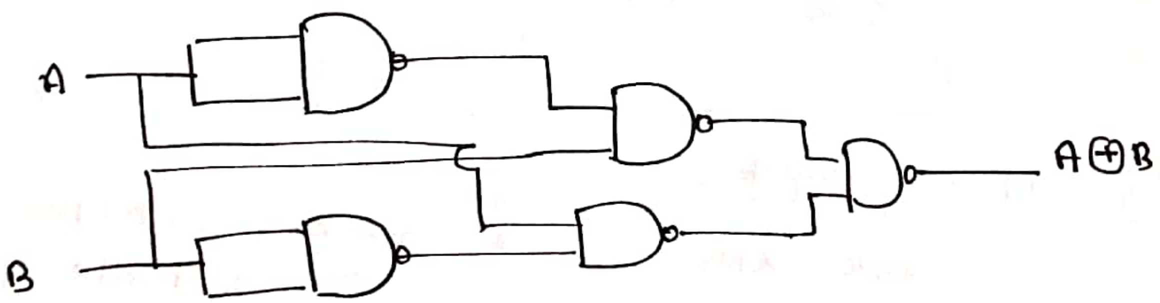
iv) $1 \odot 0 \odot 0 \odot 1 \odot 0 \odot 0 \odot 1$ = odd 1's and odd i/p
o/p = '1'

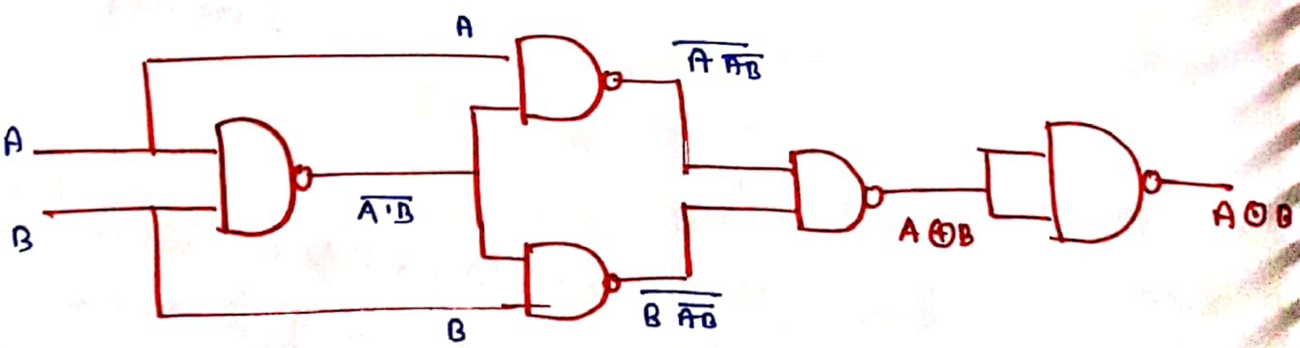
Que. find Min. two i/p NAND gate to design an XOR funⁿ.

Sol $A \oplus B$
$= \bar{A}B + A\bar{B}$



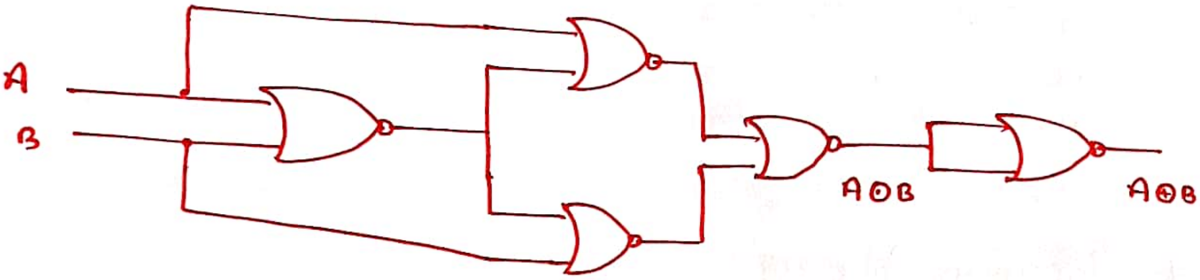
Hence total 5 NAND gate





- 4 NAND gate required to design XOR function
- 5 NAND gate required to design XNOR function

Que. Design XOR gate using NOR gate



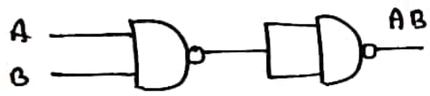
- 4 NOR gate Required to design XNOR function
- 5 NAND gate required to design XOR function

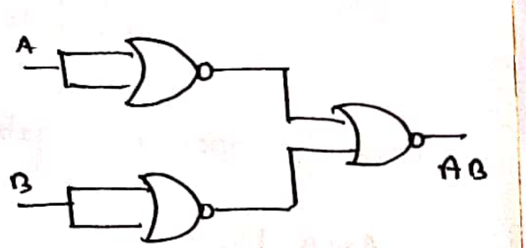
NAND Gate

NOR Gate

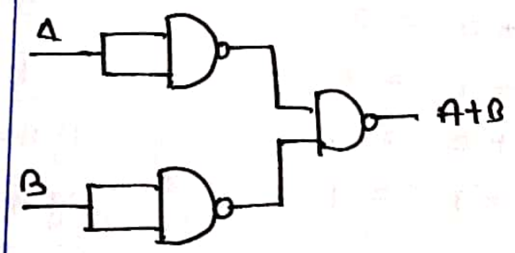
I) NOT

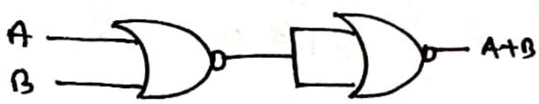
II) AND



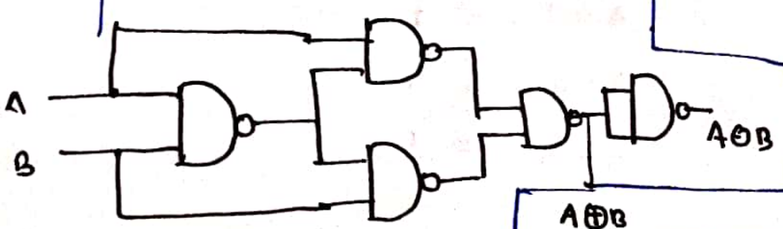


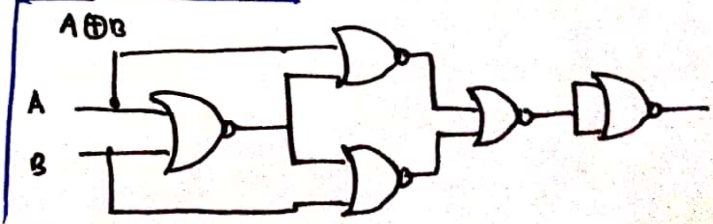
III) OR





IV) XOR





No. of Gates

	NAND	NOR
NOT	1	1
AND	2	3
OR	3	2
XOR	4	5
XNOR	5	4

* Boolean Algebra

Binary	Decimal	Boolean
$1 + 1 = 01$	$1 + 1 = 2$	$1 + 1 = 1$
Both Numerical Significance but not logical		Logical Significance

Laws of Boolean Algebra

AND laws

$0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot A = A$

$A \cdot \bar{A} = 0$

OR laws

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 1$

$A + 0 = A$

$A + 1 = 1$

$A + A = A$

$A + \bar{A} = 1$

NOT laws

$\bar{0} = 1$

$\bar{1} = 0$

If $A = 0$ $\bar{A} = 1$

If $A = 1$ $\bar{A} = 0$

$\bar{\bar{A}} = A$

Commutative Law

i) $A \cdot B = B \cdot A$

ii) $A + B = B + A$

iii) $\overline{A \cdot B} = \overline{B \cdot A}$

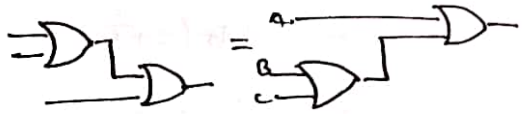
iv) $\overline{A + B} = \overline{B + A}$

v) $A \oplus B = B \oplus A$

vi) $A \odot B = B \odot A$

Associative law

i) $(A+B)+C = A+(B+C)$ OR



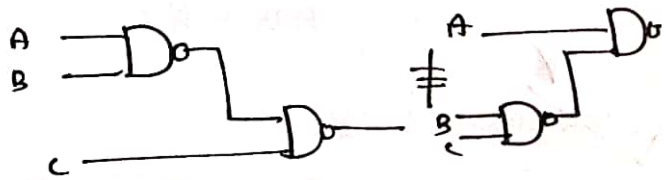
ii) $(AB)C = A(BC)$ AND

iii) $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ XOR

iv) $(A \odot B) \odot C = A \odot (B \odot C)$ XNOR

v) $\overline{\overline{(A \cdot B)} \cdot C} \neq \overline{A \cdot \overline{(B \cdot C)}}$ NAND

NAND and NOR donot follow associative Law.



Distributive Law

1) $A(B+C) = AB + AC$

** 2) $A + BC = (A+B)(A+C)$

Absortion Law

* 1) $A + AB = A$

2) $A(A+B) = A$

Que. Simplify

$F = AB + \overline{B}C + AC$ wrong

$= \cancel{AB(C+\overline{C}) + \overline{B}C(A+\overline{A}) + AC(B+\overline{B})}$

$\Rightarrow \cancel{\overline{\overline{AB + \overline{B}C + AC}}}$

$\Rightarrow \cancel{AB}$

Sol :- $AB + \overline{B}C + AC$

$AB + \overline{B}C + AC(B+\overline{B})$

$\Rightarrow AB + \overline{B}C + ACB + AC\overline{B}$

$\Rightarrow AB(1+C) + \overline{B}C(1+A)$

$\Rightarrow AB + \overline{B}C$

Consensus Law

1) $AB + BC + A\overline{C} = BC + A\overline{C}$

Hence only one Variable c present in two Variable in both normal and complemented form. Thus in output we get only those two term having c and to get o/p $AB(C+\overline{C})$ is applied

II) $\overline{A}B + BC + AC = \overline{A}B + AC$

III) $\overline{A}\overline{B} + B\overline{C} + \overline{A}\overline{C} = \overline{A}\overline{B} + B\overline{C}$

IV) $(A+B)(B+C)(\overline{A}+C) = (A+B)(\overline{A}+C)$

V) $(A+\overline{B})(B+C)(A+C) = (A+\overline{B})(B+C)$

Proof: $(A+B)(B+C)(\bar{A}+C) = (A+B)(\bar{A}+C)$

Sol LHS

$$(A+B)(B+C+0)(\bar{A}+C)$$

$$\Rightarrow (A+B)(\underbrace{B+C}_{X} + \underbrace{A\bar{A}}_{YZ})(\bar{A}+C)$$

$$\Rightarrow (A+B)(A+B+C)(\bar{A}+B+C)(\bar{A}+C)$$

$$\Rightarrow (\underbrace{A+B}_{X}+\underbrace{0}_{Y})(\underbrace{A+B}_{X}+\underbrace{C}_{Z})(\bar{A}+B+C)(\bar{A}+C+0)$$

$$\Rightarrow (A+B+0\cdot C)(\bar{A}+C+0\cdot B)$$

$$= (A+B)(\bar{A}+C) \quad //.$$

* Demorgan Theorem

i) $\overline{A+B} = \bar{A}\cdot\bar{B}$

ii) $\overline{A\cdot B} = \bar{A}+\bar{B}$

iii) $\overline{A+B+C+D+E+\cdots} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}\cdots$

iv) $\overline{A\cdot B\cdot C\cdot D\cdot E\cdots} = \bar{A}+\bar{B}+\bar{C}\cdots$

* Principle of Duality

i) '$\cdot$' $\longleftrightarrow$ '$+$'

ii) '0' $\longleftrightarrow$ '1'

iii) Donot Complement Variable

eg. i) $F = AB + C0$

$F = A \cdot B + C \cdot 0$

$F^D = (A + B) \cdot (C + 0)$

ii) $f_2 = A \cdot 1$

$f_2^D = A + 0$

Principle:- It states that "If we have a valid boolean eqⁿ then its dual eqⁿ is also valid"

eg. $A + BC = (A+B)(A+C)$

Apply dual

$A \cdot (B+C) = A \cdot B + A \cdot C$

eg. $A \cdot \bar{A} = 0$

dual ↳ $A + \bar{A} = 1$

eg. $A + AB = A$

dual $A \cdot (A+B) = A$

Que. $F = AB + BC + AC$ find dual

$F^D = (A+B)(B+C)(A+C)$

$= (AB + AC + B + BC)(A+C)$

$= AAB + AAC + AC + ABC + BAC + ACC + BC + BCC$

$= AB + AC + AC + ABC + AC + BC + BC + ABC$

$= AB + AC + BC + ABC$

$= AB + BC + AC$

$F = F^D \Rightarrow$ This type of function are called Self dual function

<u>Que.</u> How many unique function are possible using only two Variable ?

<u>Sol</u>

A	$\overline{AB}$	$A+\bar{B}$
B	AB	$\bar{A}$
$\bar{A}B$	$A+B$	$\bar{B}$
$A\bar{B}$	$\overline{A+B}$	$\bar{A}B+A\bar{B}$
0	$\bar{A}\bar{B}$	$\bar{A}\bar{B}+AB$
1	$\bar{A}+B$	$\cancel{A}+\cancel{AB}$

By o/p Combinations

A	B	Y=0	AB					$\bar{A}B+A\bar{B}$	$A+B$	$\overline{A+B}$	$\bar{A}\bar{B}+AB$	$\bar{B}$		$\bar{A}$		$\overline{A\cdot B}$	1
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	1	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
1	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
1	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1

2^{2^n} unique function Can be formed by using n Input.

- Total no. of Self dual function $2^{2^{n-1}}$

Que. $F = A \oplus B$ find Complement & dual

$\bar{F} = \overline{A \oplus B}$

$\bar{F} = \bar{A}\bar{B} + AB$

$F^{D} = \bar{A}B + A\bar{B}$

$= (\bar{A}+B)(A+\bar{B})$

$= \bar{A}A + \bar{A}\bar{B} + AB + B\bar{B}$

$= \bar{A}\bar{B} + AB$

<u>note</u> :- XOR is such function $F^{D} = \bar{F}$

Que. $x = \{(a+b\bar{c})d+e\}\bar{f}+g$

find $\bar{x}$

Sol $\overline{\{(a+b\bar{c})d+e\}\bar{f}+g}$

$\Rightarrow \overline{\{(a+b\bar{c})d+e\}\bar{f}} * \bar{g}$

$\Rightarrow \overline{(a+b\bar{c})d+e} + f * \bar{g}$

$\Rightarrow \overline{(a+b\bar{c})d} * \bar{e} + f * \bar{g}$

$\Rightarrow \{\overline{(a+b\bar{c})} + \bar{d}\}\bar{e} + f * \bar{g}$

$\Rightarrow \{\bar{a} * \overline{b\bar{c}} + \bar{d}\}\bar{e} + f * \bar{g}$

$\Rightarrow [\bar{a}\bar{e} \cdot \bar{e}(\bar{b}+c) + \bar{e}\bar{d} + f *]\bar{g}$ //.

$\Rightarrow (\bar{a}\bar{b}\bar{e} + \bar{a}\bar{e}c + \bar{e}\bar{d} + f)\bar{g}$ — (1)

find x^D

Sol $\{a\cdot(b+\bar{c})+d\cdot e\}+\bar{f}\cdot g$

$= [\{a\cdot(b+\bar{c})+d\}e+\bar{f}]g$

$\downarrow$ Individual Complement all Variable

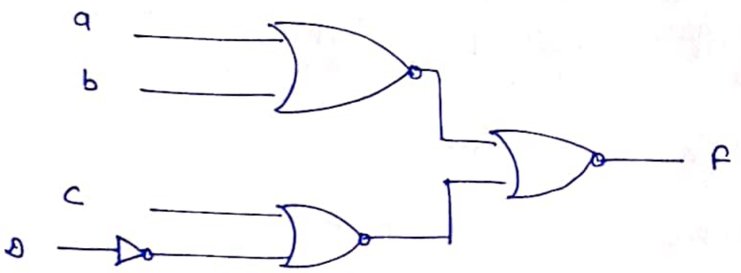
$[\{\bar{a}\cdot(\bar{b}+c)+\bar{d}\}\bar{e}+f]\bar{g}$ — (2)

eq (1) & (2) are Same Hence to find Complement

1) find Dual

11) Complement Individual

Que.



$f = ?$

a) $(a+b)(c+\bar{d})$

b) $(\bar{a}+\bar{b})(c+\bar{d})$

c) $(a+\bar{b})(c+\bar{d})$

d) $(a+b)(\bar{c}+\bar{d})$

Que. What is the Minimum no. of gate required to Implement Boolean exp. $AB + C$ if we have to use only two i/p NOR.

Sol $f = AB + C$

$= (A+C)(B+C)$

$=$ 3 NOR.

Que. The Boolean function $Y = AB + CD$ is to be implemented using only two i/p NAND. The Min. no of gate required is 3.

Que. Assume that only X & Y Logic input are available and their Complement $\bar{X}$ & $\bar{Y}$ are not available what is Min. no of two i/p NAND gate req. to Implement

$X \oplus Y$

Sol 4 NAND

Q4. The Boolean Exp. $Y = A+BC$ is to be realized using two i/p gates of only one type. What is min. no. of gate required for the Implementation

Sol

$$Y = (A+B)(A+C) \qquad 3\ \text{NOR}$$

OR

$$Y = A \cdot A + BC \qquad 3\ \text{NAND}$$

Q5. Min. no of two i/p NAND gate required to Implement the function

$$Z = A\bar{B}C$$

$$= \overline{\overline{A\bar{B}C}}$$

$$= \overline{\overline{A\bar{B}} + \bar{C}} \Rightarrow \overline{\underbrace{\overline{A\bar{B}}}_{X} + \underbrace{\overline{C \cdot C}}_{Y}}$$

$$= \bar{A} + B + \bar{C}$$

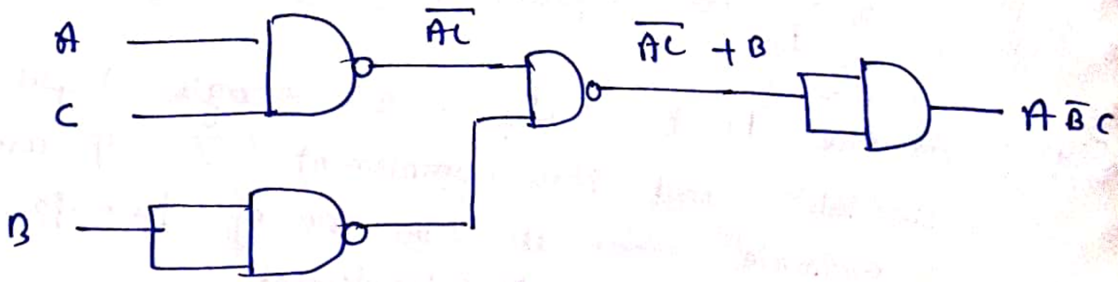
3 NAND
+ 1 $\bar{B}$
+ 1 $A\bar{B}$
+ 1 $\bar{C}$
= 6
+ 1
= 7

$$Z = A\bar{B}C$$

$$= \overline{\overline{A\bar{B}C}} \Rightarrow \overline{\overline{AC} + \bar{\bar{B}}}$$

$$= \overline{\overline{A\bar{B}} + \bar{C}} \qquad \overline{\overline{AC} + B \cdot B}$$

$$=$$



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Que. $(x+y)(x+\bar{y}) + [\overline{(x\bar{y})} + \bar{x}]$

Sol $x + x\bar{y} + xy + 0 + \overline{x\bar{y}} \cdot x$

$\Rightarrow x + xy + 0 + (\bar{x} + y)x$

$\Rightarrow x + 0 + xy$

$= x(1+y)$

$= x$ //.

$x + y\bar{y} + (\bar{x}+y)x$

$x + xy$

x //.

Que. $F(PQRS) = PQ + \bar{P}QR + \bar{P}Q\bar{R}S$

Minimal function F is

Sol $PQ + \bar{P}QR + \bar{P}Q\bar{R}S$

$\Rightarrow PQ + \bar{P}Q(R + \bar{R}S)$

$\Rightarrow Q \cdot [P + \bar{P}(R + \bar{R}S)]$

$\Rightarrow Q[P + \bar{P}(R+\bar{R})(R+S)]$

$\Rightarrow Q[P + \bar{P}(R+S)]$ $\Rightarrow$ $Q[\bar{P}$

$\Rightarrow QP + Q\bar{P}R + Q\bar{P}S$

$\Rightarrow$

Sol $Q[P + \bar{P}(R + \bar{R}S)]$

$x + yz$

$Q[(P+\bar{P})(P + R + \bar{R}S)]$

$\Rightarrow Q[P + (R+\bar{R})(R+S)]$

$\Rightarrow QP + QR + QS$ //.

Que Simplify

$$\underbrace{(P+\bar{Q}}_{A}+\underbrace{\bar{R})}_{B}\;\underbrace{(P+\bar{Q}}_{A}+\underbrace{R)}_{C}\;(P+Q+\bar{R})$$

Sol $(P+\bar{Q}+\bar{R}R)(P+Q+\bar{R})$ {Distributive law

$\Rightarrow (P+\bar{Q})(P+Q+\bar{R}) \longrightarrow P+\bar{Q}(Q+\bar{R})$

$\Rightarrow P+PQ+P\bar{R}+P\bar{Q}+\bar{Q}\bar{R}$ | $P+\bar{Q}Q+\bar{Q}\bar{R}$

$\Rightarrow P(1+\bar{R})+P(Q+\bar{Q})+\bar{Q}\bar{R}$ | $\Rightarrow P+\bar{Q}\bar{R}$

$\Rightarrow P+P+\bar{Q}\bar{R} \Rightarrow P+\bar{Q}\bar{R}$ //

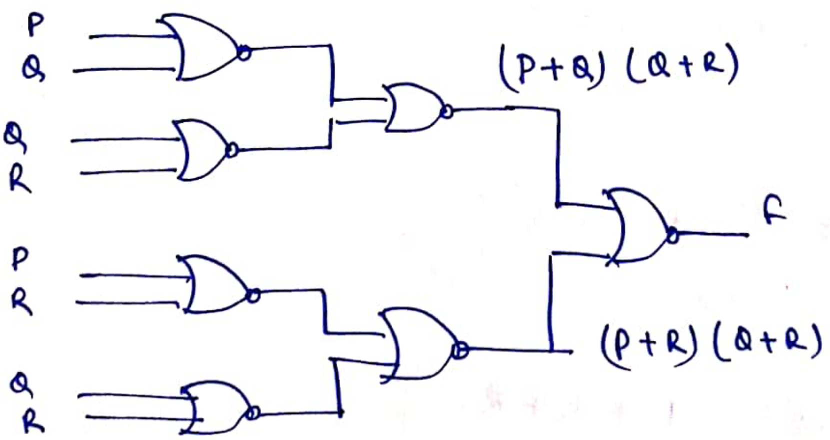
$\Rightarrow P+(Q+Q)(Q+\bar{R})$

$\Rightarrow P+Q(Q+\bar{R})$

$\Rightarrow P+Q+Q\bar{R}$ //.

$\Rightarrow P+Q$

Que. $F=?$



$$F=\overline{(P+Q)(Q+R)+(P+R)(Q+R)}$$

$$=\overline{(Q+R)(P+Q+R)}$$

{$(Q+R+0)(Q+R+P)$
$Q+R+0\cdot P$

$$=\overline{Q+R+P\cdot 0}\;// = \overline{Q+R}\;//$$

* Representation of Boolean Expressions :-

1) Sum of product form (SOP)

11) Product of Sum for (POS)

Product term	Sum terms
$f = A\bar{B}C$	$f = \bar{A}+B+C$
$f = \bar{A}B\bar{C}$	$f = A+B+\bar{C}$
$f = AC$	$f = \bar{B}+C$

Sum of product

$f = A\bar{B}C + \bar{A}B\bar{C} + AC$ B missing

Product of Sum term

$f = (\bar{A}+B+C)(A+B+\bar{C})(\bar{B}+C)$ A missing

This is not standard form of POS & SOP

In standard form there is no missing variable in all the terms.

$$f = A\bar{B}C + \bar{A}B\bar{C} + AC$$

$$= A\bar{B}C + \bar{A}B\bar{C} + AC(B+\bar{B})$$

$$= A\bar{B}C + \bar{A}B\bar{C} + ABC + A\bar{B}C$$

$$\Rightarrow A\bar{B}C + \bar{A}B\bar{C} + ABC \;//$$

now this is in standard SOP form

$$f = (\bar{A}+B+C)(A+B+\bar{C})(B+C+A\cdot\bar{A})$$

$$= (\bar{A}+B+C)(A+B+\bar{C})(A+B+C)(\bar{A}+B+C)$$

now this is std. POS

- All the product terms in standard SoP are reffered as minterm.
- All the Sum terms in standard Pos are reffered as Maxterm

Minterm table

A	B	C	minterms		Dec
0	0	0	$\bar{A}\bar{B}\bar{C}$	m_0	0
0	0	1	$\bar{A}\bar{B}C$	m_1	1
0	1	0	$\bar{A}B\bar{C}$	m_2	2
0	1	1	$\bar{A}BC$	m_3	3
1	0	0	$A\bar{B}\bar{C}$	m_4	4
1	0	1	$A\bar{B}C$	m_5	5
1	1	0	$AB\bar{C}$	m_6	6
1	1	1	ABC	m_7	7

note: Normal Variable $\longleftrightarrow$ 1 Minterm or Pos
Complemented Var. $\longleftrightarrow$ 0

Max term table

A	B	C	Max term		
0	0	0	$A+B+C$	M_0	0
0	0	1	$A+B+\bar{C}$	M_1	1
0	1	0	$A+\bar{B}+C$	M_2	2
0	1	1	$A+\bar{B}+\bar{C}$	M_3	3
1	0	0	$\bar{A}+B+C$	M_4	4
1	0	1	$\bar{A}+B+\bar{C}$	M_5	5
1	1	0	$\bar{A}+\bar{B}+C$	M_6	6
1	1	1	$\bar{A}+\bar{B}+\bar{C}$	M_7	7

Note Maxterm or SOP

'1' $\longleftrightarrow$ Complemented Var.

'0' $\longleftrightarrow$ Normal Var.

eg. $f = A\bar{B}C + \bar{A}B\bar{C} + ABC$

$\Sigma m(101 + 010 + 111)$

$= \boxed{\Sigma m(5, 2, 7)}$ Representation in std. SOP

$= m_5 + m_2 + m_7$

eg. $f = (\bar{A}+\bar{B}+C)(A+\bar{B}+C)(A+B+\bar{C})$

$= \Pi(110, 010, 001)$

$= \Pi(6, 2, 1)$

$= M_6 \cdot M_2 \cdot M_1 = \boxed{\Pi M(1, 2, 6)}$ Std POS //.

Que. $f = A + \bar{B}C$

1) SOP std:

$A(B+\bar{B})(C+\bar{C}) + \bar{B}C(A+\bar{A})$

$\Rightarrow A[BC + B\bar{C} + \bar{B}\bar{C} + \bar{B}C] + A\bar{B}C + \bar{A}\bar{B}C$

$\Rightarrow ABC + AB\bar{C} + A\bar{B}\bar{C} + \underline{A\bar{B}C + A\bar{B}C} + \bar{A}\bar{B}C$

$= ABC + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$

$= \Sigma m(7, 6, 4, 5, 1)$

II) POS std.

$f = A + \bar{B}C$

$= \cancel{A + A\bar{A} + B\bar{B}} + \cancel{\bar{B}C} + \cancel{A\bar{A}}$

$= \cancel{(A+B+C)}(\cancel{A-}$

first change in POS

$f = (A+C)(A+\bar{B})$

$$f = (A+\bar{B}+C\bar{C})(A+C+B\bar{B})$$

$$= (A+\bar{B}+C)(A+\bar{B}+\bar{C})\ (A+C+B)\ (A+C+\bar{B})$$

$$= \Pi(010,\ 011,\ 000,\ 010)$$

$$= \Pi M(0, 2, 2, 3)$$

$$= M_0 \cdot M_2 \cdot M_2 \cdot M_3$$

$$= M_0 M_2 M_3 \quad //$$

Note:- There is no need to find both Max term & Min term of any function. And only Max term or Min and find missing term to obtain Min or Max term

$$\Sigma m(1, 4, 5, 6, 7)$$

$$\Pi M(0\ 2\ 3)$$

- SOP & POS are NOT complement of Each other
SOP and POS are two different ways to represent any boolean function.

Que. Find Standard SOP form

① $f(A, B, C) = \bar{A}BC + AB\bar{C}$

MSB LSB

$$= \Sigma m(011,\ 110)$$

$$= \Sigma m(3,\ 6)$$

② $f(B, A, C) = \bar{A}BC + AB\bar{C}$

MSB LSB

$$= \Sigma m(101,\ 110)$$

$$\Sigma m(5, 6)$$

③ $f(C\ A\ B) = \bar{A}BC + AB\bar{C}$

MSB LSB

$$= \Sigma m(101,\ 011)$$

$$= \Sigma m(5, 3) = \Sigma m(3, 5)$$

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Que. $f = A + \overline{B}C$

A	B C		
x	0 1		
0	0 1	→	1
1	0 1	→	5

A	B	C		
1	x	x		
1	0	0	→	4
1	0	1	→	5
1	1	0	→	6
1	1	1	→	7

Hence $\sum m(1, 4, 5, 7)$

Que. $f(A, B, C) = \overline{A}B + BC$

A	B	C		
x	1	1		
0	1	1	→	3
1	1	1	→	7

A	B	C		
0	1	x		
0	1	0	→	2
0	1	1	→	3

$\sum m(2, 3, 7)$ //.

Que $f(A, B, C) = \overline{A} + \overline{B}C$

A	B	C		
x	0	1		
0	0	1	→	1
1	0	1	→	5

A	B	C		
0	x	x		
0	0	0	→	0
0	0	1	→	1
0	1	0	→	2
0	1	1	→	3

$\sum m(0, 1, 2, 3, 5)$ //.

Que. $f(C,A,B) = AC + \bar{B}C$

C	A	B	
1	X	X	
1	1	1	→ 7
1	1	0	→ 6

C	A	B	
1	X	0	
1	1	0	→ 6
1	0	0	→ 4

$\Sigma m(4,6,7)$ //.

Que. Identify whether SOP or POS

1) $A + B$ → POS

11) AB → SOP

111) $A \oplus B$ → Neither SOP Nor POS

IV) $A \odot B$ → Neither SOP nor POS

V) $\overline{A+B}$ → Neither SOP nor POS

VI) $\overline{A \cdot B}$ → Neither SOP nor POS

note

VII) $AB + C$ → SOP

VIII) $(A+B)(A+C)$ → POS

Que. find SOP POS

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

SOP = AB

~~POS~~ = $\cancel{AB} + \cancel{A\bar{A}} + \cancel{B\bar{B}} + AB$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

POS = $(A+B)(A+\bar{B})(\bar{A}+B)$

SOP = AB

$$Y_{POS} = (A+B)(A+\bar{B})(\bar{A}+B)$$
$$= (A + B\bar{B})(\bar{A}+B)$$
$$= A\bar{A} + AB$$
$$= AB$$
$$Y_{POS} = Y_{SOP}$$

Que. find SOP & POS

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

POS = $(A+\bar{B})(\bar{A}+B)$

SOP = $\bar{A}\bar{B} + AB$

Note:- Reason why '1' Considered for SOP & '0' Considered for POS in truth table

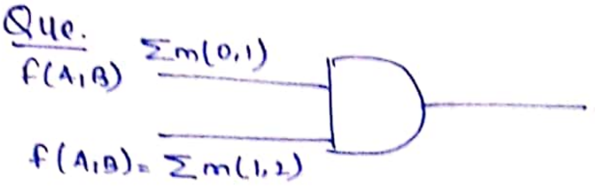
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$$SOP = \cancel{\bar{A}\bar{B}\cdot 0}^{0} + \cancel{\bar{A}B\cdot 0}^{0} + \cancel{A\bar{B}\cdot 0}^{0} + AB\cdot 1$$
$$= AB$$ i.e Correspond to 1

$$POS = (A+B+0)(A+\bar{B}+0)(\bar{A}+B+0)\cancel{(\bar{A}+\bar{B}+1)}^{1}$$

Hence 0 term for POS

Que.



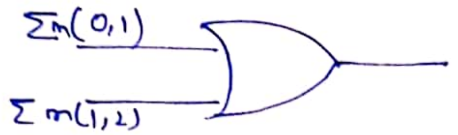
Sol :- $(\bar{A}\bar{B} + \bar{A}B)(\bar{A}B + A\bar{B})$ { Distributive Law

$\Rightarrow \bar{A}B + \bar{A}\bar{B}A\bar{B}$

$\bar{A}B$

$= \Sigma m(1)$ //.

Que.



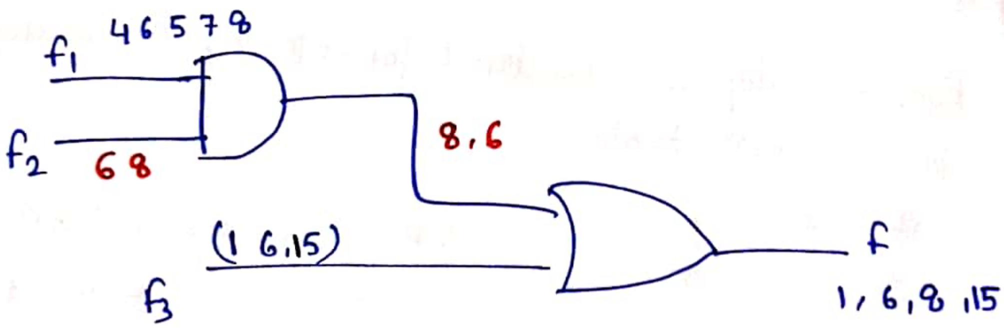
$(\bar{A}\bar{B} + \bar{A}B) + (\bar{A}B + A\bar{B})$

$\Rightarrow A\bar{B} + \bar{A}B + \bar{A}\bar{B}$

$= \Sigma m(0, 1, 2)$

Concept:- AND Gate pass term which are common in I/p
OR gate pass term which present in all I/p.

Que.



$f_1 = \Sigma m(4, 5, 6, 7, 8)$

$f_2 = \Sigma m(1, 6, 15)$ $\quad f_2 = ?$

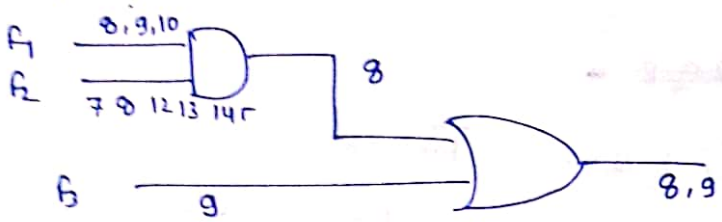
$f = \Sigma m(1, 6, 8, 15)$

11) $f_1 = \Sigma m(8, 9, 10)$

$f_2 = \Sigma m(7, 8, 12, 13, 14, 15)$

$f = \Sigma m(8, 9)$

$f_3 = ?$



$f_3 = \Sigma m(9)$

Que. $f(P, Q, R) = PQ + QR + PR$

$= PQR + PQ\bar{R} + \bar{P}QR + PQR + PQR + P\bar{Q}R$

$= PQR + PQ\bar{R} + \bar{P}QR + P\bar{Q}R$

$\quad 7 \qquad 6 \qquad 3 \qquad 5$

$= \Sigma m(3, 5, 6, 7)$

Que. $f(P\,Q\,R) = PQ + Q\bar{R} + P\bar{R}$

Sol

P Q R	P Q R	P Q R
1 1 0	0 1 0	1 0 0
1 1 1	1 1 0	1 1 0

6, 7, 2, 6, 4, 6

$\Rightarrow \Sigma m(2, 4\ 6\ 7)$

Que. The boolean expression $f(P,Q,R) = \pi(0,5)$ is to be Implemented using only 2 i/p gates which are these gate.

Sol $\Pi(0,5)$

$\Rightarrow \cancel{PQR}$

$\Rightarrow (P+Q+R)(\bar{P}+Q+\bar{R})$ { Distributive Law

$\Rightarrow \cancel{P\bar{P}} + Q + (P+R)(\bar{P}+\bar{R})$

$\Rightarrow Q + P\bar{P} + P\bar{R} + R\bar{P} + R\bar{R}$

$\Rightarrow Q + P\bar{R} + R\bar{P}$

$=$ OR & XOR //.

* Karnaugh Map (k-Map)

a \ b		$\bar{b}$ 0	b 1
$\bar{a}$	0	[0] 00	[1] 01
a	1	[2] 10	[3] 11

a	b	y
0	0	
0	1	
1	0	
1	1	

a \ bc		$\bar{b}\bar{c}$ 00	$\bar{b}c$ 01	bc 11	$b\bar{c}$ 10
$\bar{a}$	0	[0] 000	[1] 001	[3] 011	[2] 010
a	1	[4] 100	[5] 101	[7] 111	[6] 110

a	b	c	
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

$ab \backslash c$	$\bar{c}$	c
$\bar{a}\bar{b}$	0 000	1 001
$\bar{a}b$	2 010	3 011
ab	6 110	7 111
$a\bar{b}$	4 100	5 101

$ab \backslash cd$	$\bar{c}\bar{d}$ 00	$\bar{c}d$ 01	cd 11	$c\bar{d}$ 10
$\bar{a}\bar{b}$ 00	0	1	3	2
$\bar{a}b$ 01	4	5	7	6
ab 11	12	13	15	14
$a\bar{b}$ 10	8	9	11	10

$cd \backslash ab$	$\bar{a}\bar{b}$	$\bar{a}b$	ab	$a\bar{b}$
$\bar{c}\bar{d}$	0	4	12	8
$\bar{c}d$	1	5	13	9
cd	3	7	15	11
$c\bar{d}$	2	6	14	10

eg.

- Every term is 1 in Kmap o/p = 1
- Diagonal can't be group together
- group only 2^{n^2} 1's (neighbouring 1's)
- Common term in a group will be o/p. of that group

· Try to make group with max^m 1's

eg.

a \ b	$\bar{b}$	b
$\bar{a}$	1	1
a		

$y = \bar{a}$

a \ b		
$\bar{a}$	0	1
	1	0

$y = \bar{a}b + a\bar{b}$

eg

a \ bc	$\bar{b}\bar{c}$	$\bar{b}c$		$b\bar{c}$
	1	1		1
	1			1

$y = \bar{c} + \bar{a}\bar{b}$

<u>Que1.</u> Minimize

cd \ ab	$\bar{a}\bar{b}$	$\bar{a}b$	ab	$a\bar{b}$
$\bar{c}\bar{d}$	1	1		
$\bar{c}d$	1	1		
cd	1		1	1
$c\bar{d}$	1		1	

$y = \bar{a}\bar{b} + \bar{a}b\bar{c} + cda + \cancel{c\bar{d}}\ \cancel{ab}\ abc$

OR $y = \bar{a}\bar{b} + \bar{a}\bar{c} + \cancel{acd}\ \bar{b}cd + abc$

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for above question find Minimal POS form

Sol

c+d \ a+b	a+b	$a+\bar{b}$	$\bar{a}+\bar{b}$	$\bar{a}+b$
c+d	0	4	12 0	8 0
$c+\bar{d}$	1	5	13 0	9 0
$\bar{c}+\bar{d}$	3	7 0	15	11
$\bar{c}+d$	2	6 0	14	10 0

$\rightarrow (c+\bar{a})$

$\rightarrow (\bar{a}+b+d)$

$$y = (c+\bar{a}) + (a+\bar{b}+\bar{c}) + (\bar{a}+b+d)$$

$$y = (c+\bar{a})(a+\bar{b}+\bar{c})(\bar{a}+b+d)$$

note: gray code Sequence will be Similary

$$y = \Pi M(6, 7, 8, 10, 9, 10, 12, 13)$$

<u>Que.</u> Minimize in pos format

$$f(PQRS) = PR + \bar{P}QS + S$$

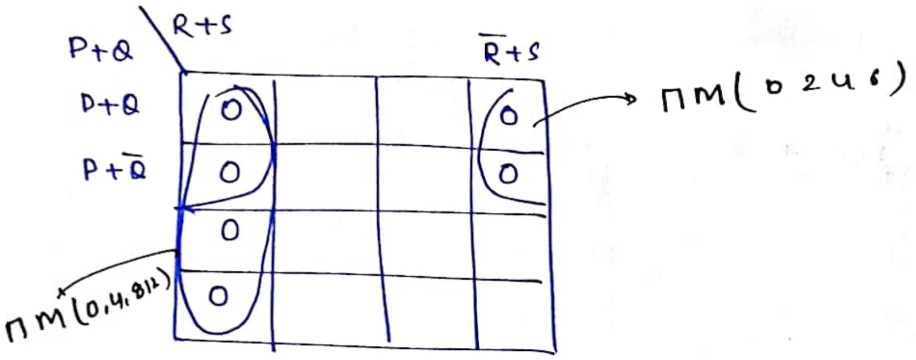
$$\Sigma m = PR(S+\bar{S})(Q+\bar{Q}) + \bar{P}QS(R+\bar{R})$$

PQRS	PQRS	PQRS
1010	0101	0001
1011	0110	0011
1110		0101
1111		0111
		1001
		1011
		1101
		1111

$$\Sigma m = \{10, 11, 14, 15, 7, 5, 1, 3, 5, 7, 9, 11, 13, 15\}$$

$$\Sigma m\{1, 3, 5, 7, 9, 10, 11, 13, 14, 15\}$$

$$\Pi M\{0\ 2\ 4\ 6\ 8\ 12\}$$

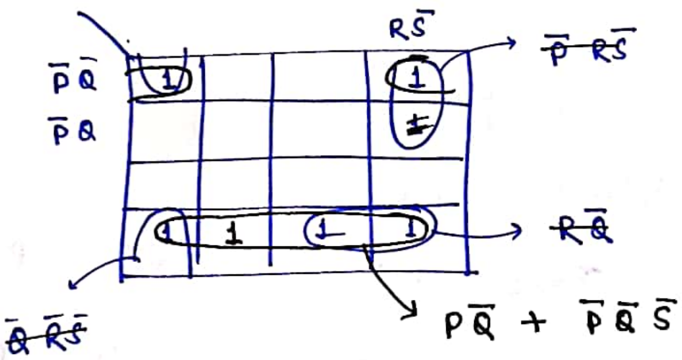


$y = (R+S)(P+S)$ //

Q1. Minimal SOP form of Boolean expression

$y = \bar{P}\bar{Q}\bar{R}\bar{S} + P\bar{Q}\bar{R}\bar{S} + P\bar{Q}\bar{R}S + P\bar{Q}RS$
$+ P\bar{Q}R\bar{S} + \bar{P}\bar{Q}R\bar{S}$ is ___

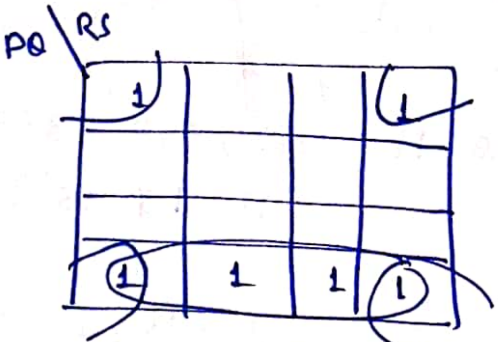
Sol $\Sigma m\{0\ 2\ 9\ 8\ 10\ 11\}$



$\cancel{y} = \cancel{\bar{P}R\bar{S}}$

$\cancel{y} = \cancel{P\bar{Q} + \bar{P}\bar{Q}\bar{S}}$

$= \cancel{\bar{Q}\{P + \bar{P}\bar{S}}$



$y = P\bar{Q} + \bar{Q}\bar{S}$

Q2.

$cd \backslash ab$				
	1	1		1
	X_1			
	X_2			
	1	1		X_1

find Minimized expression

If $X_2 = 1$ $\quad X_1 = X_2 = 0$

$\bar{a}\bar{d}$

$a\bar{b}$

$\bar{c}\bar{d}$	1	1		1
	0			
	0			
$c\bar{d}$	1	1		1

$\bar{b}\bar{d}$

$$y = \bar{b}\bar{d} + \bar{a}\bar{d}$$

If $X_1 = X_2 = 1$ $\quad X_3 = 1$

$$y = \bar{b}\bar{d} + \bar{a}\bar{d} + \bar{a}\bar{b}$$

1	1		1
1			
1			
1	1		1

Q3. $f(w,x,y,z) = \Sigma m(0,4,5,7,8,9,13,15)$

which of the following is not equivalent to f

Sol A) $\bar{x}\bar{y}\bar{z} + \bar{w}x\bar{y} + w\bar{y}z + xz$

B) $\bar{w}\bar{y}\bar{z} + w\bar{x}\bar{y} + xz$

C) $\bar{w}\bar{y}\bar{z} + w\bar{x}\bar{y} + xyz + x\bar{y}z$

D) $\bar{x}\bar{y}\bar{z} + w\bar{x}\bar{y} + \bar{w}y$

Sol Opt A

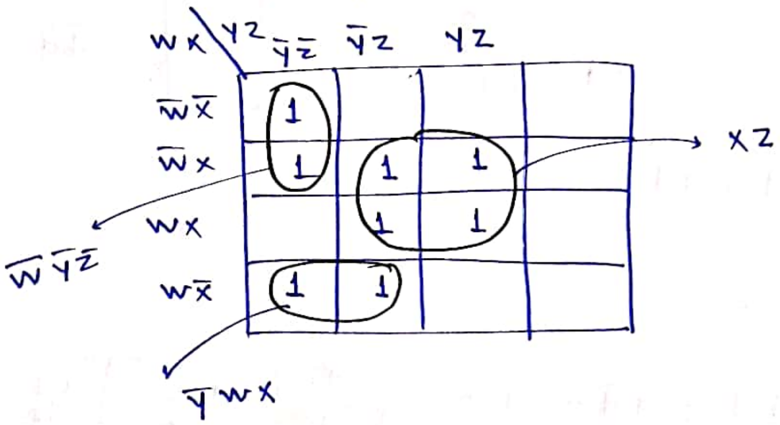
wxyz	wxyz	wxyz	wxyz
1000	0101	1001	0101
0000	0100	1101	0111
			1101
			1111

$\Sigma m(0, 4, 5, 8, 9, 7, 13, 15)$

equal to f

Opt B, C, D check in similar way

better Method: using k map



$$y = xz + \bar w \bar y \bar z + \bar y w x$$

now opt A: $\bar x \bar y \bar z + \bar w x \bar y + w \bar y z + xz$

note

If none of the sop term present as group in kmap then that option does not represent given minterms

Que.

This Quad will not be grouped

RS \ PQ	$\bar P\bar Q$	$\bar P Q$	PQ	$P\bar Q$
$\bar R\bar S$	0	1	0	0
$\bar R S$	0	1	1	1
RS	1		1	0
$R\bar S$	0	0	1	0

$\bar R\,\bar P Q + \bar R S P$
$+ QS + \bar P RS$
$+ RPQ$

<u>Wrong</u>

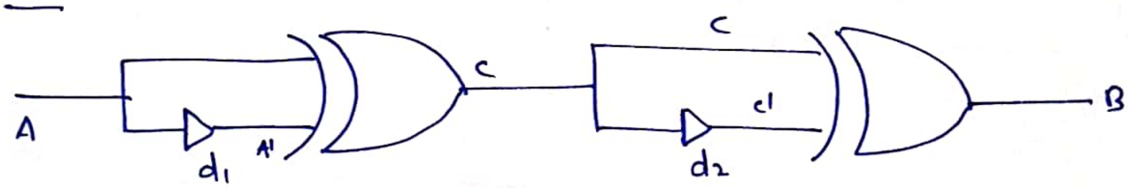
RS \ PQ				
	0	1	0	0
	0	1	1	1
	1	1	1	0
	0	0	1	0

$\bar R\bar P Q + \bar R S P + \bar P RS + RPQ$

~~Note:~~ ~~Removing~~.

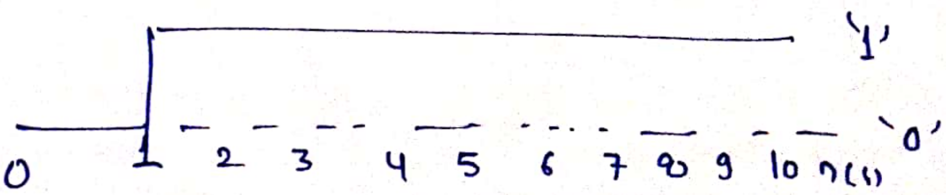
* Logic Gate Questions

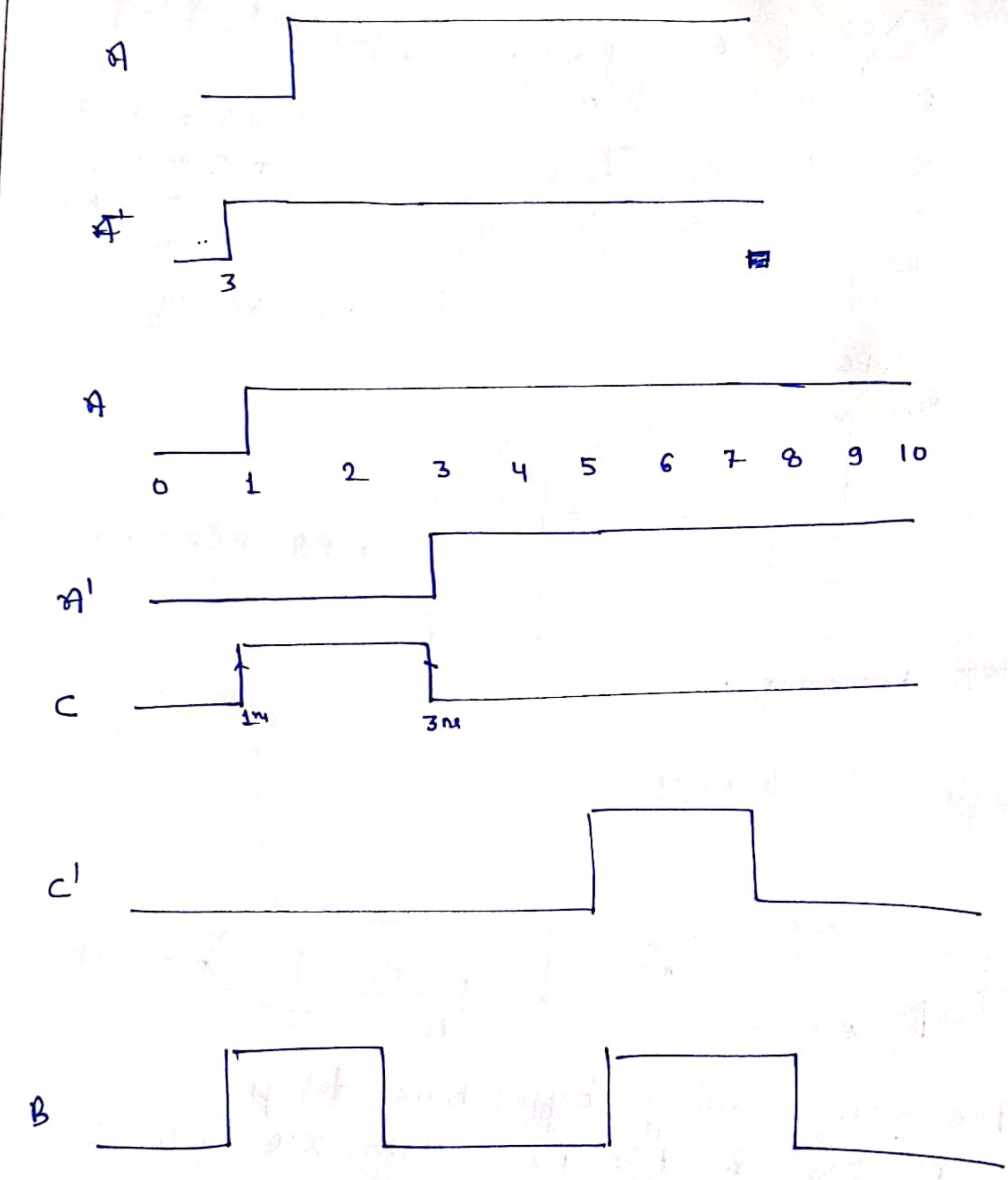
Que.



The non Inverting buffer have delays $d_1 = 2ns$ & $d_2 = 4ns$ Both XOR gate & all wire have no delay. Assume all the gate i/p and o/p are stable at time Zero. If following wave form is applied at i/p A. How many Xsition will be there at o/p B ?

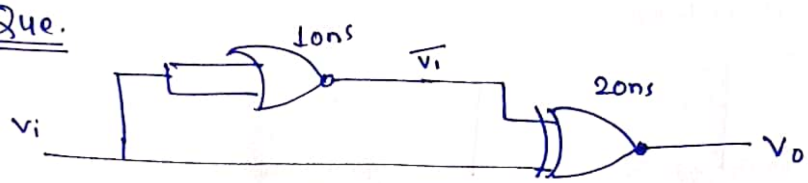
note
if will not invert i/p

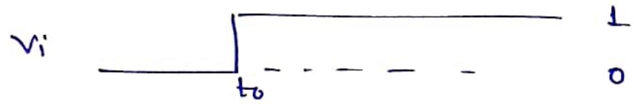




Two xsition in o/p

Que.

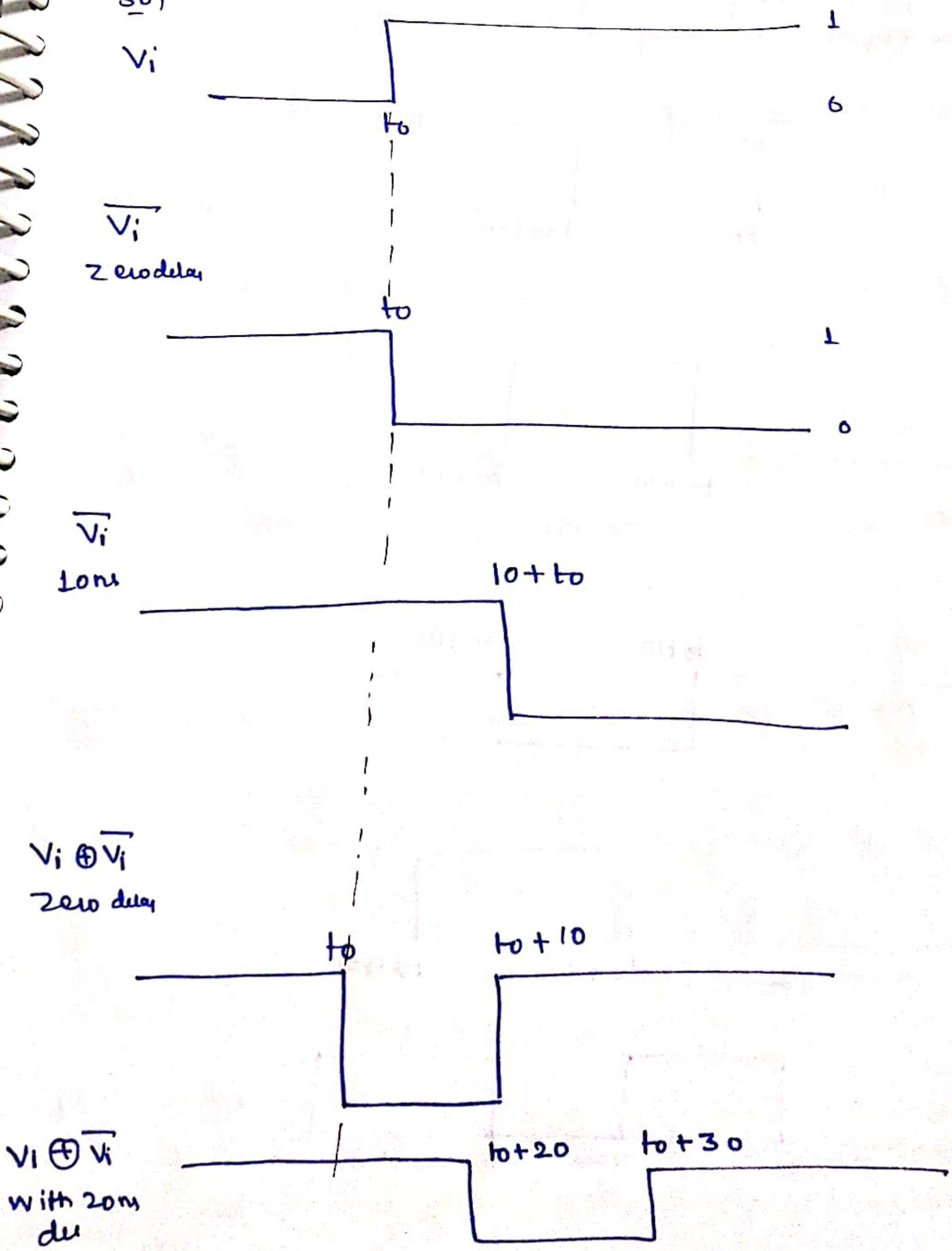




Draw waveform for V_o

note :- first Make zero delay waveform and then apply delay

Sol



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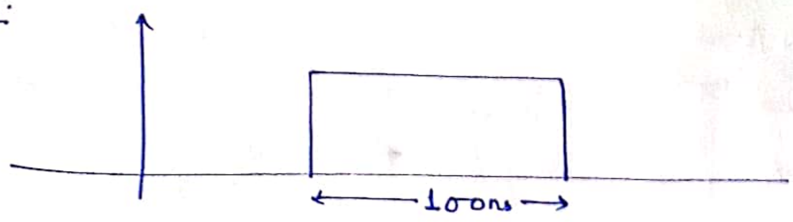
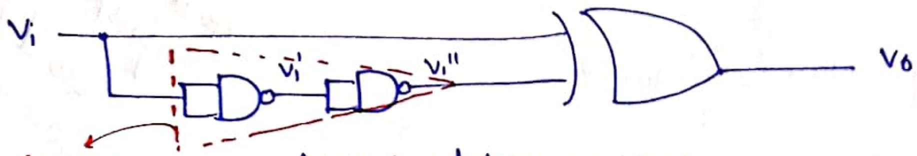
Que.

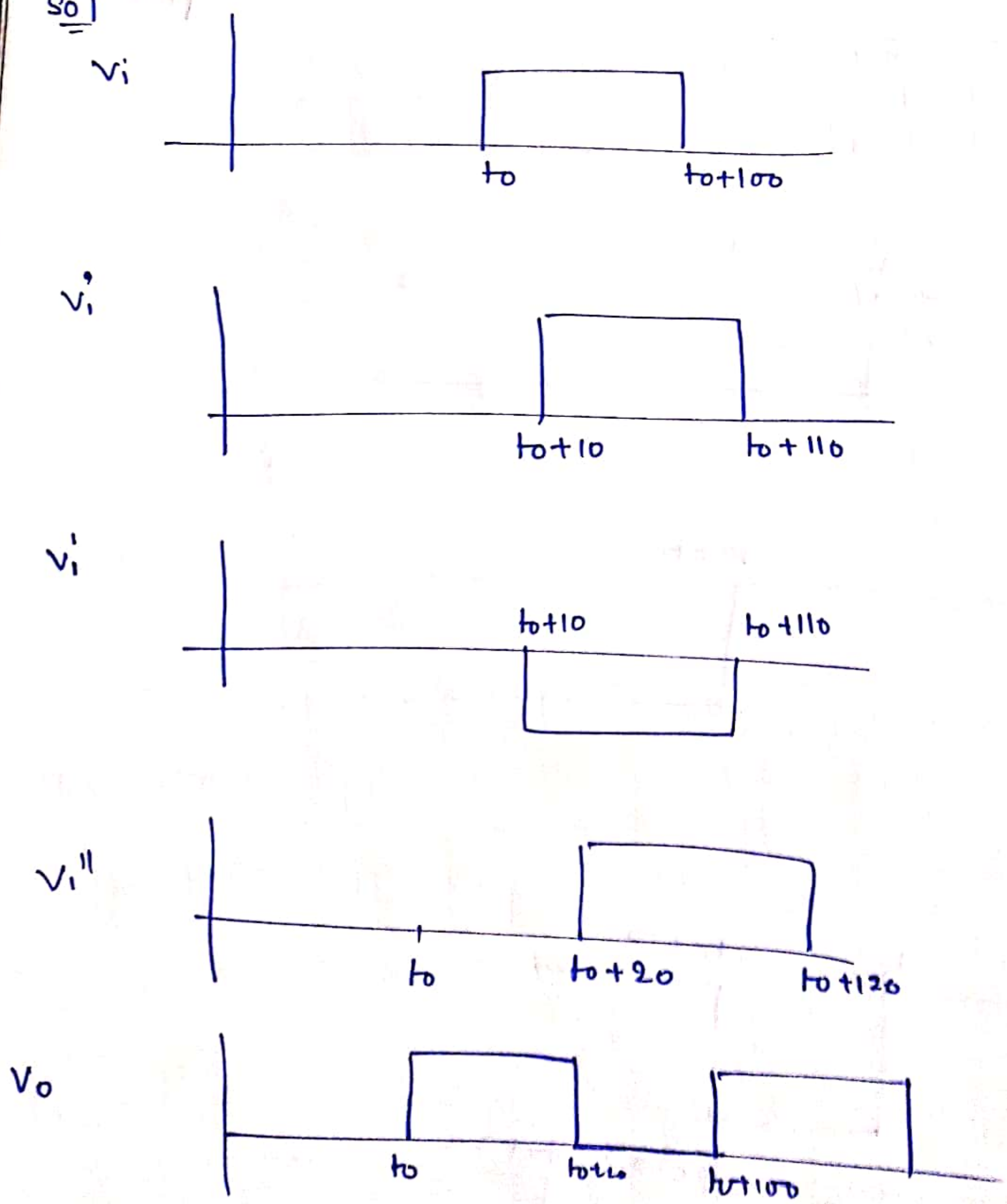




buffer ckt   All gate delay = 10ns

Same sig with 20ns delay

Sol



//.

* Implicants, Prime Implicants & Essential Implicants ÷

1) Implicants

let, $f = \sum m(0,2,3,6,8,9,12,13,15)$

1) $g_1 = \sum m(0,3,8,12,15)$

f is Covering function $g_1 \Rightarrow g_1$ is Subfunction of f i.e g_1 is an Implicant of f

ii) $g_2 = \sum m(2\ 6\ 9\ 13\ 15)$

g_2 is Subfunction of f i.e g_2 is an Implicant of f

iii) $g_3 = \sum m(0,3,6,9,10^*)$

f is not Covering function $g_3 \Rightarrow g_3$ is not Subfunction of f i.e g_3 is not an Implicant of f.

iv) $g_4 = \sum m(2,6,9,14^*)$

f is not Covering function $g_4 \Rightarrow g_4$ is not Subfunction of f ie g_4 is not an Implicant of f.

Que1. $F(A,B,C,D) = \sum m(1,5,7,8,10,12,13,15)$

$h_1 = BD$

$h_2 = \bar{A}\bar{C}D$

$h_3 = A\bar{B}\bar{D}$

$h_4 = A\bar{C}\bar{D}$

$h_5 = AB\bar{C}$

$h_6 = B\bar{C}D$

$h_7 = \bar{A}BD$

$h_8 = B\bar{C}D$

$h_9 = \bar{A}\bar{B}\bar{C}$

Sol :- $h_1 = BD$

A	B	C	D	
0	1	0	1	→ 5
0	1	1	1	→ 7
1	1	0	1	→ 13
1	1	1	1	→ 15

$h_2 = \bar{A}\bar{C}D$

A	B	C	D	
0	0	0	1	→ 1
0	1	0	1	→ 5

$h_3 = A\bar{B}\bar{D}$

1	0	0	0	→ 8
1	0	1	0	→ 10

$h_4 = A\bar{C}\bar{D}$

A	B	C	D	
1	0	0	0	→ 8
1	1	0	0	→ 12

$h_5 = AB\bar{C}\bar{D}$

1	1	0	0	→ 12
1	1	0	1	→ 13

$h_6 =$

A	B	C	D	
0	1	0	1	→ 5
1	1	0	1	→ 13

h_7 $\bar{A}BD$

A	B	C	D	
0	1	0	1	→ 5
0	1	1	1	→ 7

h_8 . $B\bar{C}\bar{D}$

A	B	C	D	
0	1	0	0	→ 4*
1	1	0	0	→ 12

It is not Implicant

h_9

A	B	C	D	
0	0	0	0	→ 0*
0	0	0	1	→ 1

h_9 is not Implicant

Hence only h_8, h_9 are not Implicant of F.

k map

Implicant :- If group possible ?

Yes → Implicant

No → not prime Implicant

Prime Implicant :- Are best group possible ?

Yes → Prime Implicant

kmap for f

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1: 1	3	2
$\bar{A}B$	4	5: 1	7: 1	6: D
AB	12: 1	13: 1	15: 1	14
$A\bar{B}$	8: 1	9	11	10: 1

Prime Implicants → (5 7 13 15)

h_2 (1, 5) PI

h_3 (8, 10) PI

h_4 (8, 12) PI

h_5 (12, 13) PI

h_6 (5, 13) Not prime Implicant, They Can Include in Quad

h_7 (5, 7) Not prime Implicant

h_1 (5 7 13 15) Prime Implicant.

If any term Cannot come under Pair Quad they they are PI

Essential prime Implicants

h_1(1, 5) EPI due to 1

h_3(8, 10) EPI due to 10 because 10 has no option of group (8, 10) is Compulsory

h_4(8, 12) Not EPI (8 - 10) (12 - 13) Can be possible

h_5(12, 13) Not EPI.

h_6 (5, 13) Not EPI

h_1 (5, 7, 13, 15) ~~Not~~ EPI Pair is not alternative of Quad Quad itself is best group

due to 7 & 15.

note: Implicant → any group

Prime Imp. → all possible group, can be other option group

EPI → no other option for group

Redundant PI → group which Can't be formed but exist

Que.

AB \ CD	00	01	11	10
00	1	0	1	1
01	0	1	0	0
11	0	1	1	0
10	1	0	0	1

no. of EPI = ?

Sol (3, 2) (12, 15) (5, 13) (0, 2, 8, 10)

4 essential prime Implicants.

Que.

CD \ AB				
	1	1	0	1
	0	0	0	1
	1	0	0	0
	1	0	0	1

no. of essential prime Imp

= 4

Que. Which are the EPI of following Boolean function

$$f(a,b,c) = a'c + ac' + b'c$$

A) $a'c$ & ac' B) $a'c$ & $b'c$ C) ac' only d) ac' & bc'

Sol $\Sigma m(3, 1, 4, 6, 5)$

a \ bc				
		1	1	D
	1	1	1	D

b

$a\bar{b}$

Wrong

a \ bc				
		1	1	
	1	1		1

$c\bar{a}$

$a\bar{c}$

Hence opt A

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Que. In SOP function

$$f(xyz) = \Sigma m(2,3,4,5)$$

the prime Implicants are

i) $\bar{x}y,\ x\bar{y}$ ii) $\bar{x}y,\ x\bar{y}\bar{z},\ x\bar{y}z$ iii) $\bar{x}y\bar{z},\ \bar{x}yz\ xy$

iv) $\bar{x}y\bar{z},\ \bar{x}yz$

$x\bar{y}\bar{z},\ x\bar{y}z$

x \ yz				
			1	1
	1	1		

$\bar{x}y$

$x\bar{y}$

Que. $f(w,x,y,z) = wy + xy + \bar{w}xyz + \bar{w}\bar{x}y + xz + \bar{x}\bar{y}\bar{z}$

which of the following is complete set of EPI?

wx \ yz	$\bar{y}\bar{z}$	$\bar{y}z$	yz	$y\bar{z}$
$\bar{w}\bar{x}$	1		1	1
$\bar{w}x$		1	1	1
wx		1	1	1
$w\bar{x}$	1		1	1

note

Directly put is in kmap from expression no need to find all minterm

xz $\bar{x}\bar{z}$, y

wy :- $wxyz$
1010 → 10
1011 → 11
1110 → 14
1111 → 15

xy :- $wxyz$
0110 → 6
0111 → 7
1110 → 14
1111 → 15

$\bar{w}xyz$ → 7

$\bar{w}\bar{x}y$

(Avoid this calⁿ)

Hence. $xz,\ \bar{x}\bar{z},\ y$ //

Que $F(x_1 x_2 x_3) = \Sigma m(1,3,6,7)$

Redundand PI is –

x_1 \ x_2x_3	$\bar{x}_2\bar{x}_3$	$\bar{x}_2x_3$	x_2x_3	$x_2\bar{x}_3$
$\bar{x}_1$		1	1	
x_1			1	1

Redundant PI

x_2x_3 i.e

Pair which have optⁿ to for alternate pair

* ## BCD Codes

Binary Coded decimal :- Every decimal digit is encoded with four binary bit

eg. $(12)_{10} \xrightarrow{\text{Binary}} (\ \ )_2$

$\frac{12}{2}$	0
$\frac{6}{2}$	0
$\frac{3}{2}$	1
$\frac{1}{2}$	1

$(12)_{10} \longrightarrow (1100)_2$ Binary

eg1 $(12)_{10} \longrightarrow (\ \ )_{BCD}$

1 2

0001 0010

$(0001\ 0010)_{BCD}$

eg 2) $(243)_{10} \longrightarrow \overset{8421}{(0010}\ \overset{8421}{0100}\ \overset{8421}{0011)}_{BCD}$ (8421)

BCD
- Weighted BCD
  - Positively weighted 8421
  - Negatively weighted 84-2-1
- Non weighted Excess-3

	8421	84-2-1	XS-3
7	0111	1001	1010
5	0101	1011	1000

$(57)_{10} = (0101\ \ 0111)_{8421} = (1011\ 1001)_{84-2-1}$

$= (1000\ \ 1010)_{XS-3}$

Decimal	8421 BCD (10 used code 6 unused)	84-2-1 BCD (10 used 6 unused)	XS-3 BCD (10 used 6 unused)	2421 (All 16 codes are used)	2421
0	0000	0000	0011	0000	0000
1	0001	0111	0100	0001	0001
2	0010	0110	0101	1000 0010	0010
3	0011	0101	0110	0011	1001
4	0100	0100	0111	0100	1010
5	0101	1011	1000	0101	0101
6	0110	1010	1001	0110	0110
7	0111	1001	1010	0111	1101
8	1000	1000	1011	1110	~~00~~ 1110
9	1001	1111	1100	1111	1111

<u>Note</u>: Sequential Code :- By adding 1 to a codeword we get next codeword

eg. BCD & XS-3

Non Sequential Code

eg. 84-2-1 BCD

Advantage of Sequential Codes

→ Arithmetic operation are Possible in Sq. code

eg. 8421 BCD arithmetic

XS-3 arithmetic

* Self Complementing Code

Number System

- (R−1)'s Complement
- R's Complement

eg.

Decimal (R = 10): 9's, 10's

Octal (R = 8): 7's, 8's

Binary (R = 2): 1's, 2's

782 →

9's	10's	7's	8's	1's	2's
999	217	777	064	111	101
782	1	713	1	010	1
217	218	064	065	101	110

$$\boxed{r\text{'s comp} = (r-1)\text{'s} + 1}$$

9's Complement pair

9 ⟷ 0

8 ⟷ 1

7 ⟷ 2

6 ⟷ 3

5 ⟷ 4

eg. ~~BCD~~ ~~8421~~, 84-2-1, XS-3 all are having 9's pair

Decimal

8421 is not Self Complementing

- Self Complementing Code is a Code in which we can obtain Codeword for 9's Complement of N by from the Code of N by Complementing each bit in Code of N

- 2421 is a Self Complementing Code. Any Code which has repeated weight (2 in this Case) are Self Complementing.

- Self Complementing property must hold to write Codewords. Hence there are many Sequence of 2421 BCD are possible

* Gray Code

Distance	Gray Code	
	0 0 0 0	
1	0 0 0 1	Mirror
1	0 0 1 1	
1	0 0 1 0	Mirror
1	0 1 1 0	
1	0 1 1 1	
1	0 1 0 1	
1	0 1 0 0	Mirror
1	1 1 0 0	
1	1 1 0 1	
1	1 1 1 1	
1	1 1 1 0	
1	1 0 1 0	
1	1 0 1 1	
1	1 0 0 1	
1	1 0 0 0	

1

- Gray code is non weighted code.
- Gray Code is also Called reflective Code
- Gray code is not a BCD code because Gray code Can be of 2 bit, 3 bit 4 bit & bit but BCD is only 4 bit.

note:- Gray Code are Unit distance Code

Distance between Code → difference in no. of bit

- Gray Code is also Called Cyclic Code