



GRADUATE SCHOOL  
OF ARTS AND SCIENCES  
COLUMBIA UNIVERSITY

# PORTFOLIO CONSTRUCTIONS AND RISK MANAGEMENT WITHIN TEN EQUITIES & ONE T-BILL

*Advisor: Prof. Hammou Elbarmi*

*Mengqi Chen            mc4396*

*Xiaochen Fan            xf2170*

*Jiaqian Yu                jy2880*

*Michael Utomo            mu2251*

## Part I. Summary

In this project, ten assets from different industries and countries are selected to ensure the diversity in portfolios. Therefore, combination of these assets could achieve higher return with lower volatility. Three categories of portfolios are constructed – minimum variance portfolio, tangency portfolio and PCA portfolio. The compare shows MVP achieves the portfolio with the minimum variance; tangency portfolios has the largest Sharpe ratio; PCA captures the market. The limitation of short sells could make portfolios more stable than ones without -- portfolios constructed with allowance of short sells provide higher return and higher risk at the same time. The risk-free asset is different from that is assumed in the lectures with zero volatility; in reality, the risk-free asset, which in this project referring to Treasury bills, also contains a little volatility. The difference should also be taken into consideration.

## Part II. Descriptive Statistics

The assets include **Apple, Delta, Exxon, GE, Honda, US Steel, Verizon, Walmart, China Eastern Airlines** and **Infosys**; the risk-free rate is 4-month T-bills, and consider SP500ETF returns as market portfolio. All are monthly data from Jan, 1, 2013 to Dec, 1, 2017. The following will descript several assets in details due to the limited pages.

	Apple	Delta	Exxon	GE	Honda	USSteel	Verizon	Walmart	CEA	Infosys
Mean	2.17%	2.46%	0.15%	-3.30%	0.07%	0.85%	0.71%	0.80%	0.86%	1.35%
SD	6.42%	7.87%	4.17%	22.09%	5.46%	19.67%	4.90%	4.77%	12.14%	14.84%
Skewness	-0.5526	-0.4313	0.0481	0.1893	-0.1431	0.3417	0.0136	0.0538	-0.5332	0.1421
Kurtosis	-0.2838	-0.3024	-0.1634	2.8439	0.2315	1.231	-0.7199	0.8537	2.2732	12.9042
Beta	1.4629	1.2867	0.6247	0.8393	0.9701	2.5408	0.5809	0.3447	1.2797	-1
Sharpe Ratio	0.3347	0.3108	0.0312	-0.1501	0.0094	0.0425	0.1416	0.1631	0.069	0.0894

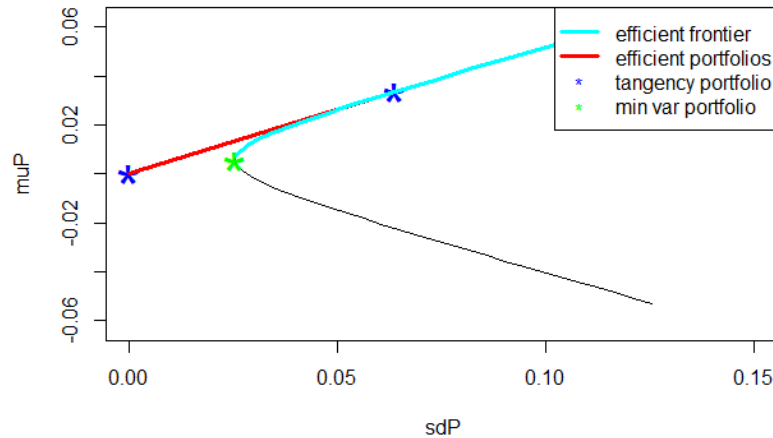
**Apple** is an American multinational technology company that designs, develops, and sells consumer electronics, computer software, and online services. The equity price had an increase tendency and the average of return rate 0.022 was larger than zero, which followed left skewed log normal distribution. It had the second largest Beta 1.46 and the largest Sharpe Ratio 0.335.

**ExxonMobil** is a US oil and gas cooperation. Its equity price followed several large ups and downs. The distribution of return rates was asymmetric and left skewed. Its SR was 0.03 and beta was 0.62. **Infosys** is an Indian information technology consulting company. Its equity price and return rate were less fluctuated but there was an exponentially increase on three and forth seasons 2015. The mean of return rate is nearly zero following a fat tail distribution, the beta was nearly -1.

**GE's** equity price had a decrease tendency and negative average return rate -0.033, following an approximately symmetric fat tail distribution. The Sharp Ratio is -0.15 and beta is 0.84.

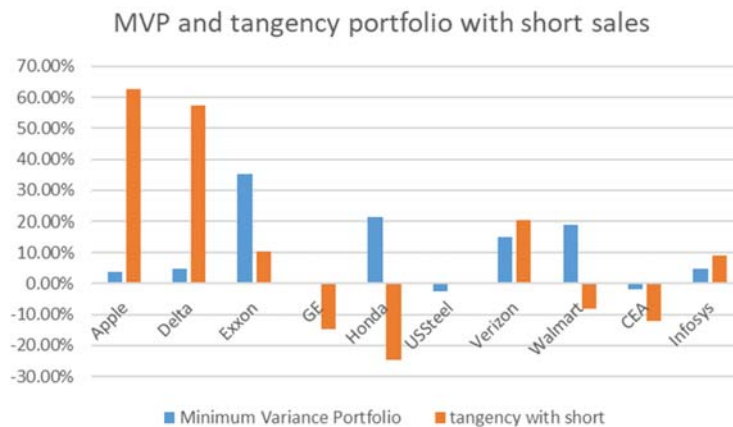
## Part III. Portfolio Theory

Markowitz approach is used to compute the efficient portfolio frontier; meanwhile, minimum variance portfolio (MVP) and tangency portfolio are also calculated. The results under the assumption that short sells are allowed will be discussed firstly.



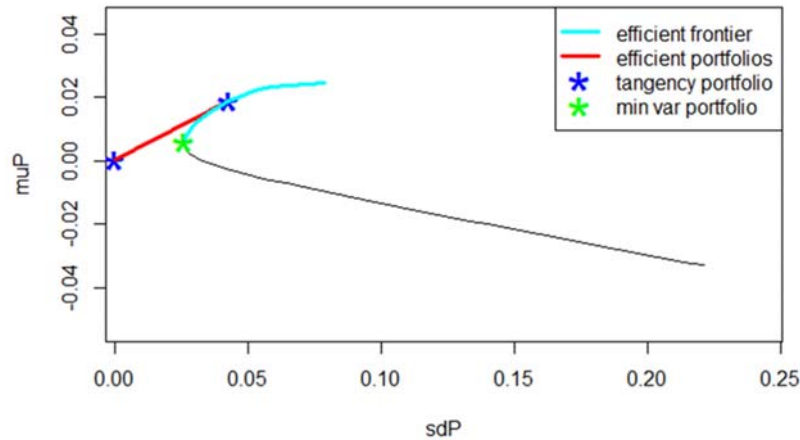
The **weights** of MVP and tangency portfolio with short sales demonstrated in the picture.

	Apple	Delta	Exxon	GE	Honda	USSteel	Verizon	Walmart	CEA	Infosys
MVP	3.68%	4.88%	35.33%	0.36%	21.38%	-2.52%	15.07%	18.86%	-1.86%	4.81%
Tangency	62.76%	57.33%	10.28%	-14.61%	-24.85%	-0.08%	20.28%	-8.28%	-12.05%	9.23%



In order to get minimum variance, larger weights will be allocated to assets with low variance such as Exxon, Honda and Walmart. Assets which have large volatility will tend to be allocated with small weights (the absolute value of the weights are small since volatility will not be influenced by the sign of the weight), such as GE, US Steel. Apple and Honda have high return rate; however, high volatility leads to small weights.

Weights of tangency portfolio are quite distinguished from that of MVP. Apple and Honda are allocated with large weights due to largest Sharpe ratio. The negative large weight of US Steel supports the idea of tangency portfolio that is to achieve highest Sharpe ratio. The results with limit of only long position is allowed will be discussed.



The **weights** of MVP and tangency portfolio without short sales are shown.

	Apple	Delta	Exxon	GE	Honda	USSteel	Verizon	Walmart	CEA	Infosys
MVP	3.63%	5.01%	34.35%	0.00%	19.29%	0.00%	13.31%	19.45%	0.00%	4.96%
Tangency	40.36%	30.63%	0.00%	0.00%	0.00%	0.00%	19.19%	3.99%	0.00%	5.82%



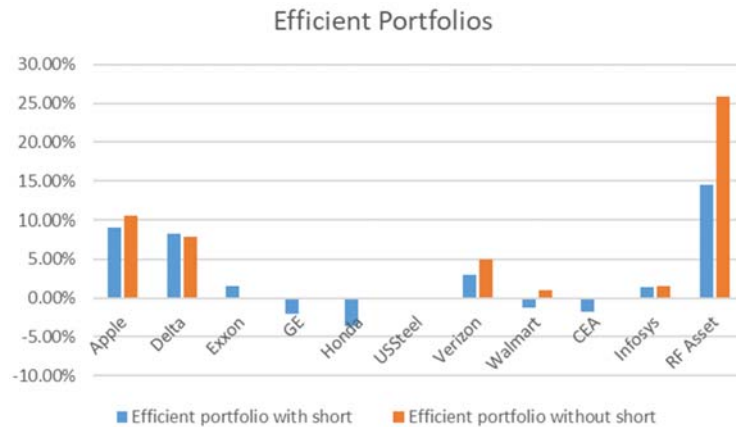
The weights are similar to the situation that short sells are allowed. The difference of the tangency portfolio weights is that the assets with small or even negative Sharpe ratio are allocated with zero weights.

The compare of risk measurement such as mean return, standard deviation, value at risk and expected shortfall and Sharpe ratio will be discussed in the Part 6 Risk Management.

## Part IV. Asset Allocation

The efficient portfolio was made of combination of tangency portfolios with the risk-free asset.

	Apple	Delta	Exxon	GE	Honda	USSteel	Verizon	Walmart	CEA	Infosys	RF Asset
With short	9.07%	8.28%	1.48%	-2.11%	-3.59%	-0.01%	2.93%	-1.20%	-1.74%	1.33%	14.45%
Without short	10.46%	7.94%	0.00%	0.00%	0.00%	0.00%	4.97%	1.03%	0.00%	1.51%	25.91%



Since the monthly mean return of tangency portfolio with and without short sales are 3.35% and 1.88% respectively, which both are much larger than expected return of 0.5% per month, both two combinations allocate large weights to the Risk-Free Asset.

VaR and ES will be discussed in details in the Part 6 .

## Part V. PCA

### Correlation

First, we compute the sample correlation matrix of the log returns for ten assets. We can see that Apple and Honda are most highly correlated with a correlation 0.3612 while Exxon and CEA are least correlated with a correlation 0.0111. Based on this sample correlation matrix, diversification can reduce risk because some of the assets have really low correlation and allocating investment on diverse assets which have low correlation could reduce risk.

#### Loadings:

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
Apple	0.408	0.167		0.299	0.235	0.536		0.581		0.172
Delta	0.351	0.135	0.375	-0.136	0.374	-0.615		0.252	0.261	-0.219
Exxon	0.268	-0.308	-0.163	-0.497	0.229	0.314	-0.566	-0.109	0.225	-0.157
GE	0.181	-0.194	0.584	-0.406		0.194	0.305	-0.196	-0.371	0.339
Honda	0.398	-0.120		0.503	0.149	0.135	0.174	-0.623	0.101	-0.315
USSteel	0.419	-0.217		-0.138	-0.680		0.175	0.253	-0.140	-0.417
Verizon	0.308	-0.102	-0.577	-0.185		-0.217	0.421	-0.120	0.240	0.477
Walmart	0.167	0.547	-0.331	-0.219	0.237			-0.141	-0.616	-0.244
CEA	0.375	0.352	0.142	0.147	-0.400	-0.160	-0.519	-0.193		0.448
Infosys		0.571	0.133	-0.323	-0.212	0.324	0.266	-0.157	0.526	-0.149

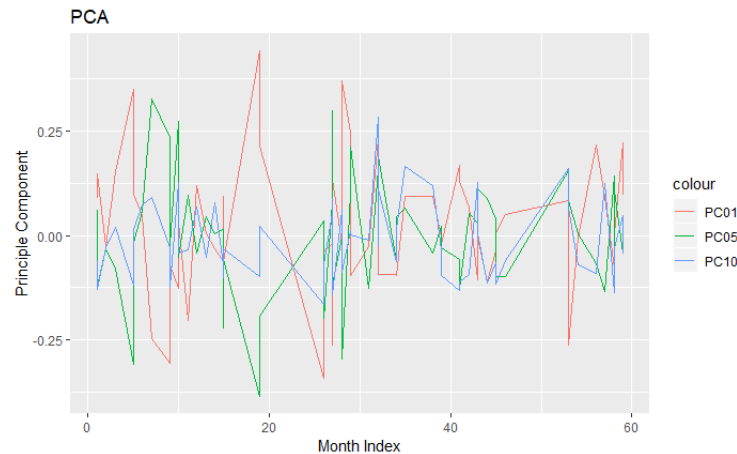
Above is the result for the PCA. From the table, we can see that the first eigenvectors have all positive values, so a change in this direction either increases or decreases the returns for all assets. The changes in the second direction will affect the convexity of the asset return curves and the changes in the third direction will affect the convexity of the asset return curves.

	Comp.1	Comp.2	Comp.3	Comp.4	Comp.5	Comp.6	Comp.7	Comp.8	Comp.9	Comp.10
Standard deviation	1.5112103	1.2873262	1.1632405	1.0558397	0.87776901	0.86459711	0.81709132	0.77062891	0.65216660	0.62150911
Proportion of Variance	0.2283757	0.1657209	0.1353129	0.1114798	0.07704784	0.07475282	0.06676382	0.05938689	0.04253213	0.03862736
Cumulative Proportion	0.2283757	0.3940965	0.5294094	0.6408891	0.71793699	0.79268980	0.85945362	0.91884051	0.96137264	1.00000000

From the table above, the standard deviation of the first principle component is 1.51 and represents 23% of the total variance. Also, the first three principal components have 53% in variation and this increases to 86% for the first seven principal components.

Plotting the picture below based on the first, the fifth and the last principle components. The first

principle component has the largest variance and the last principle component has the smallest variance.



Also, we generate a portfolio based on the first principle component via PCA. Actually, this portfolio-with the highest variance- will capture the primary market factor for this given set of assets. This portfolio could be considered as an estimation of the market portfolio. Thus, it is called “market factor portfolio”. We will analyze PCA portfolio later.

## Factor Analysis

In this part, we conduct factor analysis based on Fama 3 Factor Model. We download factors data with French’s data library. Three factors are: RMRF—weighted monthly percent log return on all the stocks minus risk free rate, SMB and HML. In order to perform a valid regression, we use percent log return minus risk-free rate as the response.

<pre>Call: lm(formula = Response[, 1] ~ RMRF + SMB + HML)  Coefficients: (Intercept)      RMRF      SMB      HML   0.3661      1.4392    -0.6322    -0.6238</pre> <pre>Call: lm(formula = Response[, 1] ~ RMRF + SMB + HML)  Residuals:     Min       1Q   Median       3Q      Max -14.3060  -2.4332   0.0747   3.3387   9.3749</pre> <pre>Coefficients:               Estimate Std. Error t value Pr(&gt; t ) (Intercept)   0.3661    0.7081    0.517  0.6072 RMRF          1.4392    0.2393    6.015 1.52e-07 *** SMB           -0.6322    0.2890   -2.188  0.0330 * HML           -0.6238    0.2854   -2.185  0.0331 * --- Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1  Residual standard error: 4.963 on 55 degrees of freedom Multiple R-squared:  0.4336,    Adjusted R-squared:  0.4027 F-statistic: 14.04 on 3 and 55 DF, p-value: 6.559e-07</pre>	<pre>Call: lm(formula = Response[, 8] ~ RMRF + SMB + HML)  Coefficients: (Intercept)      RMRF      SMB      HML   0.3072      0.3657    -0.4314    -0.2521</pre> <pre>Call: lm(formula = Response[, 8] ~ RMRF + SMB + HML)  Residuals:     Min       1Q   Median       3Q      Max -16.4755  -2.5944   0.2632   2.8244  10.8793</pre> <pre>Coefficients:               Estimate Std. Error t value Pr(&gt; t ) (Intercept)   0.3072    0.6661    0.461  0.646 RMRF          0.3657    0.2251    1.625  0.110 SMB           -0.4314    0.2718   -1.587  0.118 HML           -0.2521    0.2685   -0.939  0.352</pre> <pre>Residual standard error: 4.668 on 55 degrees of freedom Multiple R-squared:  0.09027,    Adjusted R-squared:  0.04065 F-statistic: 1.819 on 3 and 55 DF, p-value: 0.1544</pre>
--	---

Apple

Walmart

For Apple, the linear regression model is  $R = 0.3661 + 1.4392 \cdot RMRF - 0.6322 \cdot SMB - 0.6238 \cdot HML$  where R is the excess return. So, the sensitivity of the excess return for apple to excess market return is 1.4, to SMB and HML are -0.6, which means one unit change in excess market return will result in 1.4 unit change in excess return for Apple, and one unit change in SMB or HML will result in -0.6 unit change. Apple is sensitive to excess market return, SMB and HML. This result can also be shown by the p-value. All the p-values are less than 0.05, so we could conclude that excess market return, SMB and HML are all significant factors to excess return for Apple. This is reasonable because products from Apple are luxury goods and should be sensitive to markets. But for Walmart, the results are totally different. All the p-values for coefficients are larger than 0.05,

which means excess market return for Walmart doesn't have correlation with excess market return, SMB and HML. This is also reasonable because products from Walmart are basic necessities and should not be sensitive to markets.

## **Part VI. Risk Management**

### **Comparison of expected return, standard deviation and Sharpe ratio**

	Monthly Return	Annualized Return	Monthly SD	Annualized SD	Sharpe Ratio
MVP with short	0.54%	6.49%	2.55%	8.84%	0.2047
MVP without short	0.58%	7.00%	2.61%	9.03%	0.2166
Tangency with short	3.35%	40.22%	6.38%	22.11%	0.5222
Tangency without short	1.88%	22.52%	4.27%	14.79%	0.4352
PCA Portfolio	0.77%	9.18%	5.65%	19.55%	0.1323
Efficient portfolio with short	0.50%	6.00%	0.92%	3.20%	0.5221
Efficient portfolio without short	0.50%	6.00%	1.11%	3.83%	0.4352

Among five different portfolios constructed by individual assets, MVP have the lowest volatility while tangency portfolios have the highest return and Sharpe ratio. MVP can achieve lower volatility than any of the individual assets, meanwhile, tangency portfolio can achieve higher return and Sharpe ratio than any of the individual assets. For PCA portfolio, its Sharpe ratio is low compared with others, however, that is reasonable because it has the highest variance.

Combination with risk free asset does not change Sharpe ratio of tangency portfolios. The limitation of short sell will provide lower return, volatility and Sharpe ratio.

### **Comparison of Value at Risk and Expected Shortfall**

The second step is to compare VaR and Expected Shortfall among individual assets and all the portfolio calculated before.

	Apple	Delta	Exxon	GE	Honda	USSteel	Verizon	Walmart	CEA	Infosys
5% VaR	8053.55	9950.39	6484.44	32726.43	8524.16	27017.92	7080.22	6812.37	17396.64	20602.56
ES	10463.15	12826.98	8087.42	38460.24	10568.92	32602.29	8947.43	8638.64	21404.24	25267.02

GE has the highest VaR and expected shortfall while Exxon has the lowest.

	MVP with short	MVP without short	Tangency with short	Tangency without short	PCA portfolio	Efficient portfolio with short	Efficient portfolio without short
5% VaR	3589.15	3637.18	6898.16	5015.97	8167.35	1012.01	1311.41
ES	4607.06	4676.74	9323.43	6683.91	10288.64	1392.21	1765.83

Most of the portfolios have better performance at VaR and Expected Shortfall than individual assets except for the portfolio chosen by PCA. The possible cause is the aim of PCA method is to catch the market variance instead of reduce the variance or achieve highest Sharpe ratio.

The values of VaR and ES among seven portfolios demonstrates the different aims of different

portfolios. MVP tends to reduce the volatility, meanwhile, also reduce the risk of extreme situation. Tangency portfolio tends to achieve highest Sharpe ratio, however, increase the risk of extremely high loss at the same time.

The limitation of short sells tends to reduce VaR and Expected Shortfall.

### **Comparison of different methods calculating VaR and expected shortfall**

The third section is to compute VaR and ES using two methods which is parametric approach and nonparametric approach.

	5% VaR	5% ES	5% NPVaR	5% NPES
MVP with short	3589.149	4607.065	4243.172	5067.564
MVP without short	3637.176	4676.74	5156.763	5424.294
Tangency with short	6898.16	9323.429	6539.769	10829.93
Tangency without short	5015.972	6683.911	4933.675	7248.371
PCA Portfolio	8167.348	10288.64	9011.742	10318.28
Efficient portfolio with short	1012.006	1392.214	956.7567	1625.169
Efficient portfolio without short	1311.406	1765.834	1292.143	1917.993

Nonparametric approach is based on real data. For portfolio constructed only by individual assets, parametric approach tends to underestimate the extreme situation. This result could also support the argument that it is not reasonable to assume the normal distribution of these assets.

When portfolio consists risk free asset, the situation is a little different. This may be the result caused by overestimated volatility since the real data of risk free asset also reveals variance.

### **Bootstrap**

	sdeVar	sdeES	CIVar.95%	CIVar.5%	CIES.95%	CIES.5%
MVP with short	540.4835	613.2666	2754.211	4491.173	3655.306	5664.368
MVP without short	513.0319	587.8169	2868.917	4501.39	3784.803	5679.375
Tangency with short	1639.472	1855.285	4258.694	9595.138	6458.643	12354.88
Tangency without short	930.1192	1058.353	3626.536	6500.032	5071.782	8332.026
PCA Portfolio	987.5841	1119.337	6733.62	9969.047	8680.494	12390.42
Efficient portfolio with short	244.2408	283.9603	596.0722	1416.796	917.6533	1868.64
Efficient portfolio without short	259.534	301.2228	867.0047	1712.342	1229.401	2230.161

## **Part VII. Copula**

In this part, we try to find out the best Copula that fit our data which means it can describe the dependence in log-return of ten assets.

We try to fit six different Copulas to our data and use AIC to choose the best one. We use the pseudo maximal likelihood method calculate parameters in different copulas. We first transform data to empirical CDF so that the first step of pseudo maximum likelihood will be a nonparametric method and we don't need to specify the marginal distribution for every single asset.

Actually, Gumbel and Joe Copula requires that the data should not have negative dependence. But it is clearly shown from the previous part that our data has negative dependence, so these two Copula



may not fit well and this is the same with the AIC result.

Here we print the AIC for different models.

Normal Copula	T Copula	Gumbel Copula	Joe Copula	Clayton Copula	Frank Copula
3.823124	-2.728730	-31.334537	-22.883793	-34.579922	-28.841570

From the result, we can see that the best Copula is Clayton Copula because it has the smallest AIC value. So, Clayton Copula can best describe the dependence of returns in the ten assets. Since Clayton Copula exclude the region with small U1 and U2 values (where U1 and U2 are uniformed transformed data), we may conclude that there are really few points in the region with small U1 and U2 values. Copula is really important because it can provide us with the dependence structure so in portfolio selection, we can avoid choosing assets with high dependence in order to reduce risk.

## Part VIII. Conclusion

Given specific assets which necessary diversification are achieved, Minimum Variance Portfolio can construct the linear combination that has the minimum variance, in another word, can achieve the combination with the lowest risk.

Tangency portfolio can get the highest Sharpe ratio, which means, the largest proportional return compared to the risk it takes. In real world, tangency portfolio is often used to construct linear combination with risk free asset, which is efficient portfolio.

PCA portfolio does not perform well in reducing the volatility and achieving high Sharpe ratio, however, the portfolio computed by PCA is the linear combination of these specific assets that can reveals the comprehensive tendency of the whole market the most significantly.