STAT GU4261/GR5261 Statistical Methods in Finance

Fall 2018

Homeowork 2 Suggested Solution

If you have questions on the solution, please contact Brian at hl2902@columbia.edu

Problem 16.11.1 in textbook:

(a) $0.023w + 0.045(1 - w) = 0.03$ implies $w = 15/22$. Invest $15/22$ amount of money in A and remaining to B.

(b) $6w^2 + 11(1-w)^2 + 2w(1-w)(\sqrt{6})(\sqrt{11})(0.17) = 5.5$ gives $w = 0.94$ or $w = 0.41$. The expected return is largest when $w = 0.41$ (invest 0.41 amount of money in A and remaining to B).

Problem 16.11.2 in textbook:

$r_p = (1-w)r_f + wr_T$. Then $\sigma_p^2 = w^2\sigma_T^2$. This gives $w = \pm\frac{5}{7}$. Weights for risk-free asset, asset C and asset D are $\left(\frac{2}{7}, \frac{5}{7}\times 0.65, \frac{5}{7}\times 0.35\right) = \left(\frac{2}{7}, \frac{13}{28}, \frac{1}{4}\right)$ or $\left(\frac{12}{7}, -\frac{5}{7}\times 0.65, -\frac{5}{7}\times 0.35\right) = \left(\frac{12}{7}, -\frac{13}{28}, -\frac{1}{4}\right)$.

Problem 2

(a)

$$\mu_P(cx, cy) = cx\mu_A + cy\mu_B = c(x\mu_A + y\mu_B) = c\mu_P(x, y).$$

$$\begin{aligned}
\sigma_P(cx, cy) &= \sqrt{c^2x^2\sigma_A^2 + c^2y^2\sigma_B^2 + 2c^2xy\sigma_{AB}} \\
&= \sqrt{c^2(x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB})} \\
&= c\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}} \\
&= c\sigma_P(x, y).
\end{aligned}$$

(b) Marginal contribution to risk of asset A:

$$\begin{aligned}
\frac{\partial}{\partial x}\sigma_P(x, y) &= \frac{1}{2\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}} \cdot (2x\sigma_A^2 + 2y\sigma_{AB}) \\
&= \frac{x\sigma_A^2 + y\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.
\end{aligned}$$

Marginal contribution to risk of asset B:

$$\frac{\partial}{\partial y}\sigma_P(x, y) = \frac{y\sigma_B^2 + x\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Contribution to risk of asset A:

$$x\frac{\partial}{\partial x}\sigma_P(x, y) = \frac{x^2\sigma_A^2 + xy\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Contribution to risk of asset B:

$$y\frac{\partial}{\partial y}\sigma_P(x,y) = \frac{y^2\sigma_B^2 + xy\sigma_{AB}}{\sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}}.$$

Problem 3

(a) Let

$$L(w_1, w_2, \lambda) = w_1^2\sigma_B^2 + w_2^2\sigma_M^2 + 2w_1w_2\sigma_B\sigma_M\rho_{BM} - \lambda(w_1 + w_2 - 1).$$

Differentiate L with respect to w_1, w_2 and λ and set the derivatives to 0, we have

$$\begin{aligned}
\frac{\partial L}{\partial w_1} &= 2w_1\sigma_B^2 + 2w_2\sigma_B\sigma_B\rho_{BM} - \lambda = 0,\\
\frac{\partial L}{\partial w_2} &= 2w_2\sigma_M^2 + 2w_1\sigma_B\sigma_M\rho_{BM} - \lambda = 0,\\
\frac{\partial L}{\partial \lambda} &= w_1 + w_2 - 1 = 0.
\end{aligned}$$

Subtract the second equation from the first equation, we have

$$w_1\sigma_B^2 - w_2\sigma_M^2 + w_2\sigma_B\sigma_M\rho_{BM} - w_1\sigma_B\sigma_M\rho_{BM} = 0.$$

Substitute $w_2 = 1 - w_1$ into the above equation, we have

$$w_1\sigma_B^2 - \sigma_M^2 + w_1\sigma_M^2 + (1 - 2w_1)\sigma_B\sigma_M\rho_{BM} = 0,$$

which gives

$$w_1 = \frac{\sigma_M^2 - \sigma_B\sigma_M\rho_{BM}}{\sigma_B^2 + \sigma_M^2 - 2\sigma_B\sigma_M\rho_{BM}} = 0.662.$$

The minimum variance portfolio is to invest 0.662 amount of money to B and 0.338 to M. Remark: the λ term in L could be added or subtracted, it does not matter.

(b) Equating the expected return,

$$0.14 = 0.1492w + 0.3308(1 - w)$$

gives $w = 1.050661$. Therefore, invest 1.05 to B and -0.05 to M. The risk is

$$\sigma_p = \sqrt{w^2\sigma_B^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_B\sigma_M\rho_{BM}} = 0.278.$$

(c) We solve w from

$$0.3^2 = w^2\sigma_B^2 + (1-w)^2\sigma_M^2 + 2w(1-w)\sigma_B\sigma_M\rho_{BM}.$$

We can write this as

$$(\sigma_B^2 + \sigma_M^2 - 2\sigma_B\sigma_M\rho_{BM})w^2 + (-2\sigma_M^2 + 2\sigma_B\sigma_M\rho_{BM})w + (\sigma_M^2 - 0.09) = 0.$$

Solving this quadratic equation gives $w = 1.123$ or $w = 0.2007$. The expected value when $w = 1.123$ is 0.1268 and when $w = 0.2007$ is 0.2944.