STAT GU4261/GR5261 Statistical Methods in Finance

Fall 2018

Homeowork 4 Suggested Solution

If you have questions on the solution, please contact Brian at hl2902@columbia.edu

1. (a) $\mu_j = \mu_f + \beta_j(\mu_M - \mu_f) = 0.03 + 0.75(0.1 - 0.03) = 0.0825$.

   (b) Since the expected return implied by CAPM is lower than 0.09, the stock is underpriced.

2. <u>Problem 17.10.3 in textbook:</u>

   (a) $\mu_R = \mu_f + \frac{\mu_M - \mu_f}{\sigma_M}\sigma_R = 0.05508333$.

   (b) $\beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = 0.2777778$.

   (c) Let $r_P = \frac{1}{2}r_B + \frac{1}{2}r_C$.

   Note that $\mu_B = \mu_f + \beta_B(\mu_M - \mu_f) = 0.1385$ and $\mu_C = \mu_f + \beta_C(\mu_M - \mu_f) = 0.1616$. Hence, the expected return of P is $\frac{1}{2}\mu_B + \frac{1}{2}\mu_C = 0.15005$.

   (ii)

$$
\begin{aligned}
\operatorname{Var}(r_P) &= \frac{1}{4}\operatorname{Var}(r_B) + \frac{1}{4}\operatorname{Var}(r_C) + \frac{1}{2}\operatorname{Cov}(r_B, r_C) \\
&= \frac{1}{4}(\beta_B^2\sigma_M^2 + \sigma_{\varepsilon,B}^2) + \frac{1}{4}(\beta_C^2\sigma_M^2 + \sigma_{\varepsilon,C}^2) + \frac{1}{2}\beta_B\beta_C\sigma_M^2 \\
&= 0.043304.
\end{aligned}
$$

   Therefore, $\sigma_P = 0.2080961$.

   <u>Problem 17.10.6 in textbook:</u>

   (a) $\beta_A = \frac{\sigma_{A,M}}{\sigma_M^2} = \frac{15}{11}$.

   (b) $\mu_A = r_f + \beta_A(\mu_M - r_f) = \frac{41}{275}$.

   (c) (This question is problematic, exclude this question for grading) If we assume that the characteristic line model is true, then this is not possible, as variance due to market is $\beta_A^2\sigma_M^2 = 0.0225 > 0.022$, which is the variance of r_A.

3. (a)

$$
\begin{aligned}
\boldsymbol{w}_{\min} &= \frac{\Omega^{-1}\mathbf{1}}{\mathbf{1}^T\Omega^{-1}\mathbf{1}} = (0.500, 0.375, 0.125) \\
\mu_{\min} &= \boldsymbol{w}_{\min}^T\mu = 0.075 \\
\operatorname{Var}_{\min} &= \boldsymbol{w}_{\min}^T\Omega\boldsymbol{w}_{\min} = 0.01125.
\end{aligned}
$$

   (b) Let

$$
\begin{aligned}
w_1 &= \frac{\Omega^{-1}1}{1^T\Omega^{-1}1}, \\
w_2 &= \frac{\Omega^{-1}\mu}{1^T\Omega^{-1}\mu}, \\
\theta &= \frac{\mu_p - w_2^T\mu}{w_1^T\mu - w_2^T\mu}.
\end{aligned}
$$

Then $w = \theta w_1 + (1-\theta)w_2 = (0.1666667, 0.2500000, 0.5833333)$. Risk is $\sqrt{w^T \Omega w} = 0.1607275$.

(c)

$$\boldsymbol{w}_T = \frac{\Omega^{-1}(\boldsymbol{\mu} - \mu_f \mathbf{1})}{\mathbf{1}^T \Omega^{-1}(\boldsymbol{\mu} - \mu_f \mathbf{1})} = (0.3163265, 0.3061224, 0.3775510).$$

(d) First, compute μ_T and σ_T. Then, setting $0.1 = \alpha\mu_T + (1-\alpha)0.04$ gives $\alpha = 1.230126$. Hence, $\sigma_p = \alpha\sigma_T = 0.1540255$.

4. (a) To estimate the CAPM model, we perform linear regressions of the excess returns of stocks against the excess return of market. Comment on the beta values: the aggressiveness of the stocks: MSOFT > IBM > GM > GE. (any reasonable answer is ok)

	MSOFT	GE	GM	IBM
alpha	0.0102	0.0059	-0.0023	0.0068
beta	1.4299	0.9830	1.0744	1.2683
alpha_pvalue	0.2502	0.2329	0.7518	0.3491

(b) Since the p-values for the alphas are all greater 0.05, we do not reject the null hypothesis that $\alpha = 0$ for all the stocks at 0.05 significance level.

(c) The 95% CIs are:
Microsoft: [1.06, 1.80]
GE: [0.78, 1.19]
GM: [0.77, 1.38]
IBM: [0.96, 1.57]

Although the point estimates of betas appear to be greater than 1 for MSOFT, GM and IBM, the confidence intervals show the uncertainties involved in the estimates are actually quite large. (any reasonable answer is ok)

(d) Based on (c), we reject H_0 only for Microsoft as 1 does not lie in its CI.

(e) Reject when $\hat{\beta}_{MSOFT} \geq 1 + \hat{\sigma}_{\hat{\beta}_{MSOFT}} z_{0.95} = 1.301$. Therefore, we reject H_0 for Microsoft.