HW 1

1. In the R lab section on pages 13 and 14, do problems 4, 5, 6

2. Problems 1, 3, 4, 7 (Page 40)

3. Problem 13 on page 41

4. Problem15 on page 42

5. Problem 21 on page 43

6. If X is a continuous random variable with a strictly increasing distribution function F, find the distribution of of $U = F(X)$ (show all your work to get a full credit)

7. Let X have a normal distribution with mean μ and variance σ^2 and let $Y = e^X$. Y is said to have a lognormal distribution with parameters μ and σ^2 (since $X = \log(Y)$ has a normal distribution).

   (a) Find the density $f_Y(y)$. (Hint: compute $F_Y(y) = P(Y \leq y)$)

   (b) Find the mean and the variance of Y. (Hint : if $X \sim N(\mu, \sigma^2)$, then $E(e^{tX}) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$)

8. Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from a distribution F with mean μ and variance σ^2 and

$$F_n(x) = \frac{1}{n}\sum_{i=1}^{n} I(X_i \leq x) = \frac{\text{number of } X_i\text{s less or equal to x}}{n}$$

   (Here $I(X_i \leq x) = 1$ if $X_i \leq x$ and 0 otherwise). F_n is the empirical distribution function and it is used to estimate of F.

   (a) Show that $E(F_n(x)) = F(x)$ (that is $F_n(x)$ is an unbiased estimator of $F(x)$) Hint: $Z_i = I(X_i \leq x)$ is a Bernoulli random variable with parameter $p = F(x)$

   (b) Show that $\mathrm{VAR}(F_n(x)) = F(x)(1 - F(x))/n$

   (c) What is the asymptotic distribution of

$$\frac{\sqrt{n}(F_n(x) - F(x))}{\sqrt{F(x)(1 - F(x))}}?$$

   and why? (you do not need to give a proof, you only need to quote a theorem)