

## HW 2

1. Do Problems 1 and 2 on pages 491 and 492.
2. Let  $f(x, y)$  be a continuous and differentiable function of  $x$  and  $y$ . the function  $f$  is said to be homogeneous of degree one if

$$f(cx, cy) = cf(x, y)$$

Euler's theorem states that if  $f$  is homogeneous of degree one, then

$$f(x, y) = x \frac{\partial}{\partial x} f(x, y) + y \frac{\partial}{\partial y} f(x, y)$$

Let  $\mu_P(x, y) = x\mu_A + y\mu_B$  and  $\sigma_P(x, y) = \sqrt{x^2\sigma_A^2 + y^2\sigma_B^2 + 2xy\sigma_{AB}}$

- (a) Show that  $\mu_P(x, y)$  and  $\sigma_P(x, y)$  are homogeneous of degree 1 (for  $\sigma_P(x, y)$  assume  $c \geq 0$ ).
- (b) In a portfolio where the portion  $x$  is invested in asset A and the portion  $y$  is invested in asset B, the partial derivatives

$$\frac{\partial}{\partial x} \sigma_P(x, y) \quad \text{and} \quad \frac{\partial}{\partial y} \sigma_P(x, y)$$

are called the marginal contributions to risk of A and B , respectively. The contributions to risk of assets A and B are given by

$$x \frac{\partial}{\partial x} \sigma_P(x, y) \quad \text{and} \quad y \frac{\partial}{\partial y} \sigma_P(x, y),$$

respectively. Find the expression of the marginal risks and the contribution to risk of assets A and B.

3. The annual estimates of the parameters for Boeing (B) and Microsoft (M) stocks are given below:

$$\mu_B = 0.1492, \mu_M = 0.3308, \sigma_B^2 = 0.0695, \sigma_M^2 = 0.1369, \rho_{BM} = -0.0083$$

- (a) Use the Lagrange multiplier method to derive the minimum variance portfolio.
- (b) Suppose you desire a portfolio with an expected return of 14%. What should be the weights of this portfolio? What is its risk equal to?
- (c) Suppose you desire a portfolio with a risk of 4%. What should be the weights of this portfolio? What is its expected return equal to?