STAT GU4261/GR5261 Statistical Methods in Finance

Fall 2018

Homeowork 3 Suggested Solution

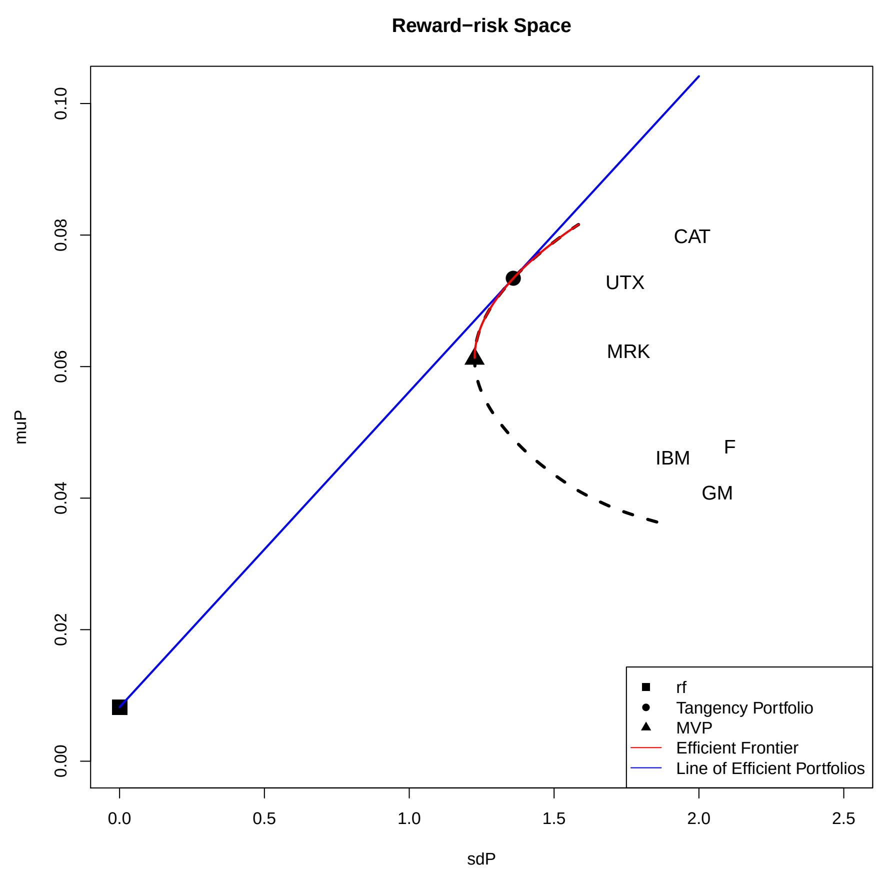
If you have questions on the solution, please contact Brian at hl2902@columbia.edu

1. 16.10.1 R lab of Chapter 16

   (Problem 1) Note: The code is attached in the R file.

   Weight of the tangency portfolio: $(-0.0920, -0.0032, 0.3363, 0.3844, 0.3196, 0.0549)$.

   Weight of the minimum variance portfolio: $(0.0839, 0.0581, 0.1275, 0.2344, 0.2959, 0.2002)$.



   Remark: because of the constraints $-0.1 \leq w_i \leq 0.5$ for each $i = 1, \ldots, 6$, there is a limitation on the minimum and maximum expected returns that could be achieved. Outside the range [min return, max return], there will be no solution when you do the optimization. Hence, one should only consider the targeted in this range. The minimum and maximum expected returns could be obtained by hand (don't need to solve a linear programming to find that) easily, see the code for details.

(Problem 2) Let $r_p = wr_T + (1-w)r_f$. Then $w = \frac{0.07-r_f}{\mu_T - r_f} = 0.9473557$. Hence, the proportion of capital to invest in the six stocks is $(-0.0871, -0.0030, 0.3186, 0.3642, 0.3028, 0.0520)$.

(Problem 3) Yes.

2. Note that

$$\begin{aligned}
0.01 &= \mathbb{P}(R_p < -0.1) \\
&= \mathbb{P}(wR_A + (1-w)0.04 < -0.1) \\
&= \mathbb{P}(\mathcal{N}(0.14w + (1-w)0.04, 0.25^2 w^2) < -0.1) \\
&= \Phi\left(\frac{-0.1 - (0.04 + 0.1w)}{0.25w}\right).
\end{aligned}$$

Hence,

$$\frac{-0.1 - (0.04 + 0.1w)}{0.25w} = \Phi^{-1}(0.01),$$

which we can find that $w = 0.2907$.

3. Risk here is defined as the standard deviation of the portfolio's return.

(a)

$$\begin{aligned}
w_{\text{MVP}} &= \frac{\Omega^{-1}1}{1^T\Omega^{-1}1} = (0.441, 0.366, 0.193)^T, \\
\mu_{\text{MVP}} &= w_{\text{MVP}}^T \mu = 0.02489, \\
\sigma^2_{\text{MVP}} &= w_{\text{MVP}}^T \Omega w_{\text{MVP}} = 0.005282.
\end{aligned}$$

(b) The required weight is given by

$$w^* = \theta w_1 + (1-\theta) w_2,$$

where

$$\begin{aligned}
w_1 &= \frac{\Omega^{-1}1}{1^T\Omega^{-1}1}, \\
w_2 &= \frac{\Omega^{-1}\mu}{1^T\Omega^{-1}\mu}, \\
\theta &= \frac{\mu_p - w_2^T\mu}{w_1^T\mu - w_2^T\mu}.
\end{aligned}$$

Hence, $w^* = (0.828, -0.091, 0.263)^T$ and $\sigma^* = \sqrt{(w^*)^T \Sigma w^*} = 0.09166$.

(c)

$$w_T = \frac{\Omega^{-1}(\mu - \mu_f 1)}{1^T\Omega^{-1}(\mu - \mu_f 1)} = (0.9093, -0.1873, 0.2780)^T.$$

(d) First, $\mu_T = w_T^T\mu = 0.046469$. Then we solve w from

$$0.0427 = w\mu_T + (1-w)\mu_f.$$

Hence, $w = 0.9188$. The risk is 0.09125034.