HW7

1. Suppose the return R on a stock satisfies $R = \mu + \lambda Y$ where μ and λ are fixed and Y has a t-distribution with ν degrees of freedom.

   (a) If you hold S_0 position in this stock, show that for a one day

   $$VaR(\alpha) = -S_0(\mu + \lambda t_{\alpha,\nu})$$

   where $t_{\alpha,\nu}$ is the αth quantile of a the t-distribution with ν degrees of freedon. (Hint: $P(L > VaR(\alpha)) = \alpha$ and $L = -S_0 R$)

   (b) If $S_0 = 100000, \mu = 0.4$ and $\lambda = 0.01$, what is $VaR(0.05$ equal to if $\nu = 10$

2. Suppose that the daily returns (R_A, R_B) on Stocks A and B have a bivariate normal distributions

   $$\mu = \begin{pmatrix} 0.0002 \\ 0.0003 \end{pmatrix}$$

   and variance

   $$\Sigma = \begin{pmatrix} 0.0003 & 0.0002 \\ 0.0002 & 0.0004 \end{pmatrix}$$

   This implies in particular that $R_A \sim N(0.0002, 0.0003)$, $R_B \sim N(0.0003, 0.0004)$ and for any a and b, $aR_A + bR_B \sim N(0.0002a + 0.0003b, 0.0003a^2 + 0.0004b^2 + 0.0004ab)$

   (a) Suppose that you hold a \$1000 position in Stock A (i.e $S_0 = 1000$), compute $VaR_A(0.05)$

   (b) Suppose that you hold a \$1000 position in Stock A (i.e $S_0 = 1000$), compute $VaR_B(0.05)$

   (c) What is VaR(0.05) of a portfolio holding 500 is Stock A and 500 in Stock B?

3. Suppose the distribution of R has a pdf f. Show that

   $$ES(\alpha) = -S_0 \int_0^{q_\alpha} r f(r) dr / \alpha$$

   where q_α is the αth quantile of the distribution of R.

4. Assume $R \sim N(\mu, \sigma^2)$. Show that

   $$ES(\alpha) = -S_0 \left(\mu - \frac{\sigma}{\sqrt{2\pi}} e^{z_\alpha^2/2} / \alpha \right)$$

   where z_α is the $\alpha$the quantile of $N(0, 1)$.