International Journal of Geographical Information Science

ISSN: 1365-8816 (Print) 1362-3087 (Online) Journal homepage: http://www.tandfonline.com/loi/tgis20

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To cite this article: Shihong Du, Mi Shu & Chen-Chieh Feng (2016) Representation and discovery of building patterns: a three-level relational approach, International Journal of Geographical Information Science, 30:6, 1161-1186, DOI: 10.1080/13658816.2015.1108421

To link to this article: https://doi.org/10.1080/13658816.2015.1108421

Published online: 09 Nov 2015.

INTERNATIONAL JOURNAL OF GEOGRAPHICAL INFORMATION SCIENCE, 2016
VOL. 30, NO. 6, 1161–1186
http://dx.doi.org/10.1080/13658816.2015.1108421

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Representation and discovery of building patterns: a three-level relational approach

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ABSTRACT

Building patterns exhibited collectively by a group of buildings are fundamental to understanding urban forms, classifying urban scenes, analyzing urban landscapes, and generalizing maps. The existing studies have used geometric homogeneity or regularity to represent and discover limited patterns for map generalization, or used interval and rectangle algebra to represent relations between spatial objects. These approaches, however, cannot illustrate how patterns are produced by using syntax or grammar (i.e. relations between buildings) to link words (i.e. buildings) into sentences (i.e. building patterns), making it impossible to represent and discover building patterns with diverse structures. This study presents a relation-based approach to formalize and discover arbitrary building patterns at three abstract levels. At the bottom level, a relative and local frame of reference is defined, and 169 basic relations are derived to represent relative positions between buildings. At the middle level, the 169 relations, qualitative angle description, and qualitative size are combined to formalize important semantic relations between two buildings, which include collinear, perpendicular, and parallel relations. At the top level, the relations at the bottom and middle levels are used to formalize three types of building patterns, including collinear patterns, the structured patterns with acceptable names, and other patterns of interest. Algorithms implementing the three levels of relations are presented and applied to demonstrate the effectiveness of the proposed approach in discovering building patterns from databases and querying building patterns. The results indicate that the relational approach is generic to effectively represent and discover building patterns with arbitrary structures. In addition, it complements the existing geometric methods for recognizing building patterns, and the interval and rectangle algebra for representing building relations.

ARTICLE HISTORY
Received 21 June 2015
Accepted 12 October 2015

KEYWORDS
Interval relations; building patterns; semantic qualitative relations; pattern representation and discovery

1. Introduction

Building patterns refer to visually salient structures exhibited collectively by a group of buildings. They are often described by high-level concepts (e.g. collinear, perpendicular, and parallel patterns), and have been playing important roles in map generalization (Yang 2008, Zhang *et al.* 2013, Renard and Duchêne 2014, Cetinkaya *et al.* 2015),

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semantic classification of urban buildings and scenes (Du *et al*. 2015b, Zhang *et al*. 2015), population estimation (Wu *et al*. 2005), and landscape analyses (Song *et al*. 2014). Accordingly, representing and discovering building patterns from spatial databases are fundamental to these applications. However, the most existing studies have focused on map generalization (Yang 2008, Zhang *et al*. 2013, Renard and Duchêne 2014, Cetinkaya *et al*. 2015) and used geometric methods to extract building patterns. While well-suited for finding specific patterns meaningful to map generalization, these geometric methods are unable to represent relative positions between buildings, and spatial relations that are fundamental to representing and discovering building patterns. Therefore, a more generic approach to represent and discover building patterns is needed.

Identifying building groups from individual buildings is a prerequisite for identifying building patterns. The existing geometric methods have been designed to find these groups of buildings with high geometric homogeneity or regularity within groups and high differences across groups, using similarity indices such as spacing, size, orientation, and shape (Christophe and Ruas 2002, Zhang *et al*. 2013). Detected groups of buildings are further conceptually classified into collinear, align-along-road, grid-like, and unstructured patterns (Zhang *et al*. 2013), or alignment patterns, including H-pattern, grid-pattern, Z-pattern, E-pattern, I-pattern, and stair-pattern (Yang 2008). Grouping algorithms were further presented to find building patterns fulfilling the homogeneity or regularity criteria. For example, Zhang *et al*. (2013) combined proximity graph and minimum spanning tree to find collinear and curvilinear patterns; Cetinkaya *et al*. (2015) compared four algorithms and found that density-based spatial clustering (DBSCAN) and adaptive spatial clustering based on Delaunay triangulation (ASCDT) are more efficient. Recently, some geometric methods were also presented to discover road network patterns (Yang *et al*. 2010, 2014). In summary, the existing studies relied solely on using geometric similarity (i.e. homogeneity) to group buildings, but ignored relations (including interval and rectangle relations) or syntax of building patterns. While working well for finding patterns for map generalization, they cannot illustrate how patterns are produced by using syntax or grammar (i.e. relations between buildings) to link words (i.e. buildings) into sentences (i.e. building patterns), and thereby making it impossible to compare and analyze patterns from the perspectives of both relations and syntax.

Relative positions as a type of generic spatial relation have been studied for at least three decades in artificial intelligence. In the context of this article, the most relevant works on relative positions fall under interval algebra (Allen 1983) and rectangle algebra (Guesgen 1989, Balbiani *et al*. 1999, Navarrete *et al*. 2013). Interval algebra was proposed to represent the 13 possible relations between time intervals, and was later extended to represent the relative positions between spatial objects. Since most spatial objects are two- or three-dimensional, rectangle algebra was proposed to represent knowledge of extended objects. Rectangle algebra combines two intervals at horizontal and vertical axes to approximate spatial objects and represent the possible 169 (i.e. 13×13) relative relations between them (Papadias and Theodoridis 1997). Navarrete *et al*. (2013) and Cohn *et al*. (2014) further constructed the connections between rectangle algebra and cardinal direction relations, which are important to represent spatial knowledge (Skiadopoulos *et al*. 2005), querying spatial data (Papadias and Theodoridis 1997), and modeling qualitative locations (Du *et al*. 2015a). In addition, based on rectangle algebra, cardinal direction relations were further combined with topological relations for qualitative spatial reasoning (Cohn *et al*. 2014). Despite these studies, the handling of relative locations for representing and discovering building patterns remains unsatisfactory because both interval and rectangle algebra are not invariant under rotations. In addition, the algebra used in these studies to represent cardinal direction relations are based on an absolute frame of reference, i.e. the vertical and horizontal axes always point to the geographic north and east. Accordingly, although spatial objects may be rotated, the directions of two axes never change. However, the main directions of buildings, fundamental to representing and analyzing building patterns, can be in arbitrary angles, leading to that many groups of buildings with different main directions can be described as the same patterns. As a result, a relative frame of reference, i.e. adaptively rotated with main directions of buildings, should be developed to represent and analyze building patterns. Unfortunately, such work remains lacking.

Many applications concerning building patterns often require clearly defined and formalized building patterns using relations and syntaxes. For example, sketch-based spatial query has received more attention due to its user-friendly interface in specifying query sentences (Egenhofer 1997) and retrieving spatial data from spatial databases (Nedas and Egenhofer 2008). Relations are at the core of both tasks as they are helpful to define diverse patterns and query sentences. For sub-graph-based spatial data mining (Thoma *et al*. 2010), both frequent patterns to be discovered and spatial data to be searched are represented as graphs underpinned by relations. Even for map generalization, some common building patterns cannot be captured by geometric-based methods, such as H-pattern, Z-pattern, E-pattern, I-pattern, and stair-pattern (Yang 2008). All these applications require clear specification regarding how building patterns can be formally captured by using relations, and how to handle diverse patterns. Moreover, some applications (e.g. spatial data mining) have no prior knowledge about what patterns are of interest, thus they depend on automatic tools to learn the structures and the number of building patterns. The existing studies cannot fulfill these requirements due to the following limitations. First, both interval and rectangle algebra are inappropriate to represent and analyze building patterns as they adopt absolute and global frames of reference, making those patterns variant under rotation. Second, some basic relations should be presented for defining and constructing building patterns with arbitrary structures. Third, it is unclear how to formalize different building patterns by using those basic relations and to retrieve patterns from spatial databases.

This study presents a three-level, relation-based approach to formalize and discover arbitrary building patterns. At the bottom level, with buildings being approximated by their smallest bounding rectangles (SBRs), a relative and local frame of reference is defined, and 169 SBR relations are produced to represent the relative positions between buildings. Similarly, qualitative angle descriptions and relative size relations are formalized. At the middle level, the 169 SBR relations, qualitative angle description, and qualitative size are combined to formalize some semantic relations between two buildings, such as collinear, perpendicular, and parallel relations, and the relations about complex buildings. At the top level, relations at the bottom and middle levels are combined to formalize regular and complex patterns. Algorithms are presented to handle the relations at the three levels, discover building patterns from databases, and query building patterns using the sketch-based approach. A case study using

2162 buildings is carried out to demonstrate the effectiveness of the proposed approach in formalizing and discovering building patterns with arbitrary structures.

2. Related work

2.1. *Interval relations*

The interval algebra (IA) was introduced to model the relative relations between any two intervals in one-dimensional space (Allen 1983). The 13 basic relations determined by IA are *before* (*b*), *meets* (*m*), *overlaps* (*o*), *starts* (*s*), *during* (*d*), *finishes* (*f*), *equal* (*e*), *finishes-inverse* (*fi*), *during-inverse* (*di*), *starts-inverse* (*si*), *overlaps-inverse* (*oi*), *meets-inverse* (*mi*), and *before-inverse* (*bi*), denoted by $\mathcal{B}_{ia} = \{b, m, o, s, d, f, e, fi, di, si, oi, mi, bi\}$ (Figure 1). Among the 13 relations, 6 pairs of relations are converse to each other.

IA was further extended into rectangle algebra (RA) to model the relative positions between two extended objects in 2-D space (Papadias and Theodoridis 1997, Balbiani *et al*. 1999). In RA, the extended objects with any shapes can be approximated by rectangles, whose two sides are always parallel to the two axes of the system of coordinates. For two extended objects, their relative positions can be represented by two interval relations in two axes. Therefore, a total of 169 basic relations are produced and denoted by $\mathcal{B}_{ra}$ in this study.

2.2. *Smallest bounding rectangle (SBR)*

A building can be approximated by a minimum bounding rectangle (MBR) or smallest bounding rectangle (SBR). MBR refers to the maximum extent of a two-dimensional object (Figure 2a), and their long and short axes always point to geographic north and east. SBR refers to the smallest area rectangle containing all the points of two-dimensional objects (Figure 2b). Let SBR(a) be the SBR of object *a*. Then, μ_a and ν_a are the long and short axes of *a*, respectively (Figure 2b). Compared to MBRs, SBRs are more exact in approximating 2D objects as they have smaller areas. More importantly, SBRs provide the measurements for main directions of objects, defined as the angle of the whole building from the horizontal axis and permit the capturing of the directional difference between two buildings, represented by the angle between their long axes. Therefore, SBRs in this study are used to approximate buildings and analyze building patterns. Note that under rare circumstances,

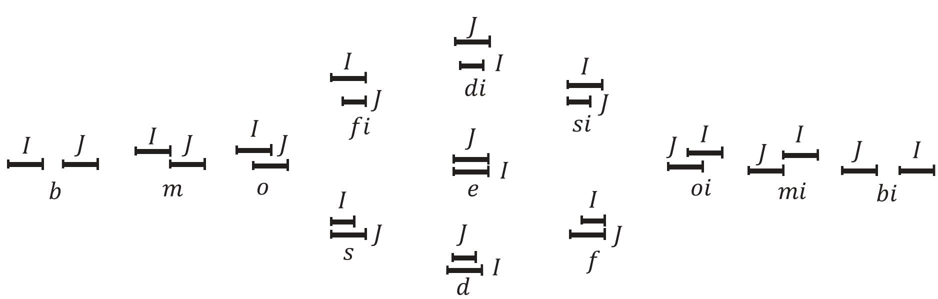

Figure 1. The 13 interval relations (Allen, 1983).

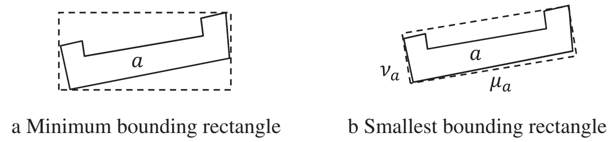

Figure 2. The approximation of a building.

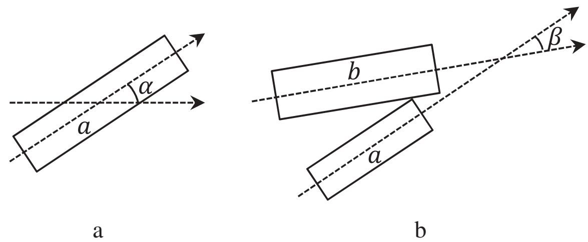

Figure 3. The main directions.

an object may have several or many different SBRs or main directions. For example, a disk has infinitely many SBRs. In this case, an arbitrary one can be chosen to find patterns.

Definition 1. The main direction of building a refers to the angle of its long axis from the horizontal axis and denoted by $\alpha(a) \in [0, \pi)$ (Figure 3a). The angle between buildings a and b is defined as the angle between their long axes, and denoted by $\beta(a, b) \in [0, \pi)$ (Figure 3b).

For example, in Figure 3a $\alpha(a) = 40$; in Figure 3b $\beta(a, b) = 30$.

3. Projection-based representation of building relations

Based on the quantitative parameters of buildings above, three types of qualitative relations can be defined to represent the relative positions between buildings, including qualitative angle descriptions, qualitative SBR relations, and relative sizes.

3.1. *Interval-based representation of SBR relations*

3.1.1. *Creating local frame of reference*

Since buildings are approximated by SBRs instead of MBRs, interval relations are no longer suitable to model the relative relations between two buildings because these relations are affected by the relative angle between the long axes of buildings, not the absolute angle from the long axis to the horizontal axis. For example, although the orientations of the building pairs are different in Figures 4a and 4b, the relations between the two buildings in both pairs are identical as their relative positions are the same.

To cope with this problem, a local frame consisting of a vertical axis y and a horizontal axis x is required. The y axis is perpendicular to the long axis of the reference

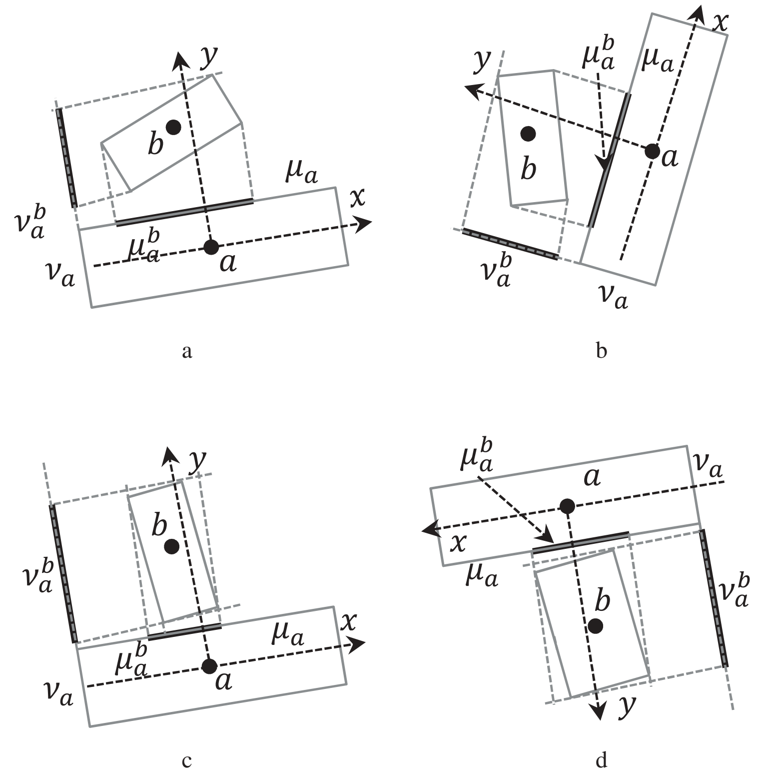

Figure 4. The SBR relations between buildings.

building (hereafter termed referent) *a* and directed from the referent to the target building (hereafter termed target) *b*, while the *x* axis is perpendicular to the *y* axis and point to the right side. Note that if the target overlaps with the *x* axis of the referent and is split into two parts, the *y* axis will point to the larger part of the target. In this way, the defined local frame with respect to a referent depends on the main direction of the referent and the relative orientation of two buildings. In Figures 4a and 4b, buildings *a* and *b* have identical relative orientations, but different main directions, thus the local frames are different. In Figure 4c, target *b* is above referent *a*, thus the *y* axis of the local frame points to the upside of the referent; while in Figure 4d, target *b* is under the referent *a*, thus the *y* axis points to the downside of the referent. Therefore, the local frames of reference based on SBRs are adaptive to the main directions of referents and relative relation from the target to the referent.

The second step is to project *b* onto the long and short axes of SBR(a) in the local frame of reference. The projections of target *b* and the axes of referent *a* are in the same 1-dimensional space. Let the long and short axes of building *a* being denoted by μ_a and ν_a, and the ones of building *b* denoted by μ_b and ν_b, then μ_a^b and ν_a^b refer to the projections of the target onto the long and short axes of SBR(a) (Figure 4), respectively.

3.1.2. *Modeling SBR relations*

For any two buildings, their relations can be represented as two interval relations by relating the two axes of a referent to the two projections of a target. Let a and b be two buildings, and μ_a and ν_a be the long and short axes of SBR(a), the relations between buildings a and b can be distinguished by the two interval relations $r(\mu_a, \mu_a^b) \in \mathcal{B}_{ia}$ and $r(\nu_a, \nu_a^b) \in \mathcal{B}_{ia}$, forming a rectangle relation $r(a, b) = r(\mu_a, \mu_a^b) \times r(\nu_a, \nu_a^b) \in \mathcal{B}_{ra}$. For the relation $r(a, b)$, the first parameter (i.e. a) refers to the referent and the second parameter (i.e. b) to the target. It has the following properties:

- It can be one of the 169 RA relations (Figure 4) as it can be represented as the combination of two interval relations.
- $r(a, b)$ and $r(b, a)$ are not always equal as the projections of a onto $SBR(b)$ differ from that of b onto $SBR(a)$. Accordingly, both $r(a, b)$ and $r(b, a)$ are required to represent the relations between buildings.
- Compared with RA relations, the relation $r(a, b)$ has the same domain $\mathcal{B}_{ra}$, but completely different values and operations, thus these relations are called smallest rectangle algebra (SRA).

Since the relation $r(a, b)$ is represented as the combination of two interval relations, it takes value from the 169 RA relations (Papadias and Theodoridis 1997, Balbiani *et al.* 1999) in Figure 5.

SBA relations are qualitative concepts at a lower level as they are not frequently used by people. Therefore, the 169 relations need to be either directly grouped into higher concepts or combined with other qualitative descriptions to define semantic relations at a higher level. First, the 169 basic relations can be grouped into two classes *overlapped* $\{r_{i,j}\}_{i=3,j=3}^{11,11}$ and *non-overlapped* $\mathcal{B}_{ra} - \{r_{i,j}\}_{i=3,j=3}^{11,11}$. The *non-overlapped* relations are prevalent in reality and can be further abstracted into three high-level relations parallel, perpendicular, and

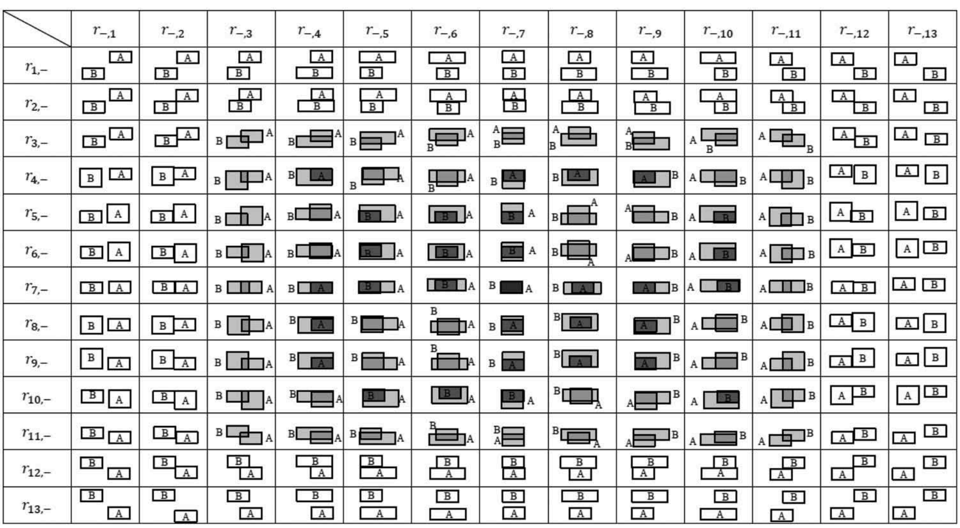

Figure 5. The 169 relations between two pairs of interval relations.

collinear (Section 3.4.2–3.4.4), while the *overlapped* relations are less common and will be represented as the combination of the basic SBR relations (Section 3.4.5).

3.1.3. *Comparing SBR with MBR relations*

Despite with the same basic relations, SRA and RA share little in common in their expressiveness, exactness, frame of reference, and related operators.

- SRA is invariant under rotation and scaling transformations, while RA is only invariant under scaling. Figure 6a and 6c are rotated into Figures 6b and 6d, respectively. For SRA, the two buildings in both Figures 6a and 6b are described as $r_{1,6}$. While for RA, the buildings in Figure 6c are described as $r_{3,6}$, but the ones in Figure 10d as $r_{3,11}$. It is clear that rotating a pair of building cannot change the description of SRA, but can change that of RA.
- SRA is more exact and powerful than RA for describing relations between buildings. Since SBRs are more exact than MBRs to approximate buildings, and more importantly SBRs consider the main directions of referents, SRA is more exact to describe relations between buildings.
- They are based on different frames of reference. SRA uses a relative and local frame instead of an absolute and global frame. Moreover, the frame of reference in SRA is adaptive to the relative orientation of two buildings and the main direction of a referent, while the ones in RA are invariant for all buildings.
- The converse operation for RA is inappropriate for SRA. For one-dimensional relations, six pairs of IA relations are converses, such as *b* and *bi*, *m* and *mi*, *o* and *oi*, *s* and *si*, *f* and *fi*, as well as *d* and *di*. However, since the local frames are rotated in SRA and the

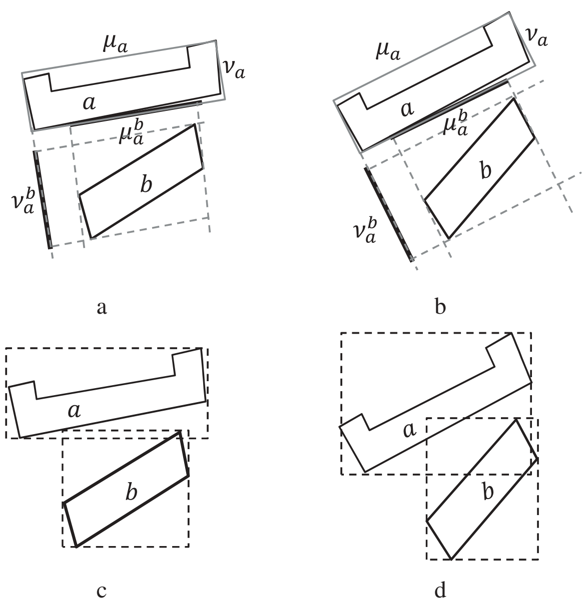

Figure 6. The comparison of SRA with RA.

projections of targets onto referents are often quite different from that of referents onto targets, the converse operator is invalid for SRA.

- The occurrence of basic relations in SRA in the real world is uneven. In reality, most buildings are *disjoint*, so are most of their SBRs. Such property leads to rare occurrence of the relations $\{r_{i,j}\}_{i=3,j=3}^{11,11}$, and frequent occurrence of the remaining SBR relations.

According to the points above, although SRA and RA have same basic relations, SRA is more suitable and powerful than RA to model the relations between buildings.

3.2. *Qualitative angle descriptions*

The angle between the main directions of two buildings $\beta(a,b)$ can be classified into four meaningful groups: parallel, perpendicular, acute, and obtuse relations. These concepts are consistent with people's cognition, thus they are helpful to describe building patterns.

Definition 2. The angle between two buildings can be qualitatively defined as the following four types at a high level:

Parallel relation: $ParAngle(a,b) =_{def} (\beta(a,b) \le \delta) \vee (180 - \beta(a,b)) \le \delta$,
Perpendicular relation: $PerAngle(a,b) =_{def} |\beta(a,b) - 90| \le \delta$,
Acute relation: $AcuAngle(a,b) =_{def} \delta < \beta(a,b) < (90 - \delta)$, and
Obtuse relation: $ObtAngle(a,b) =_{def} (\delta + 90) < \beta(a,b) < (180 - \delta)$.

Where δ is a threshold, which can be learnt from samples which include pairs of buildings and the corresponding qualitative angle descriptions. The samples are chosen from the real data by users, and the maximum likelihood or decision tree method can be used to learn the relationships between the parameter δ and the qualitative descriptions.

Figure 7 illustrates the four types of qualitative angle descriptions: parallel (Figures 7a and 7b), perpendicular (Figure 7c), acute (Figure 7d), and obtuse angles (Figure 7e). Among the four qualitative relations, the former two are symmetric, while the latter two are converse to each other. Therefore, the following holds true.

- $ParAngle(a,b) \leftrightarrow ParAngle(b,a)$,
- $PerAngle(a,b) \leftrightarrow PerAngle(b,a)$,
- $AcuAngle(a,b) \leftrightarrow ObtAngle(b,a)$, and
- $ObtAngle(a,b) \leftrightarrow AcuAngle(a,b)$.

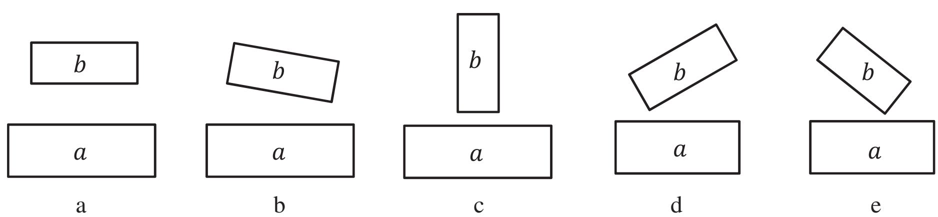

Figure 7. Classification of the angles between two buildings.

3.3. *Relative sizes*

SBR relations qualitatively describe the relative orders of two buildings in a local and relative frame (i.e. a kind of ordered relations). They thus do not consider the relations between the sizes of two buildings. To analyze building patterns, both the order and the size relations of two SBRs are required. For example, the buildings in the same pattern should be similar in both their orders and sizes.

The relative size qualitatively measures how one building is comparable with another one with respect to their sizes. For example, a building is *much larger* than another one. Such terms include *much larger, larger, roughly equal, smaller*, and *much smaller*. Since buildings can have different shapes, an effective way to define relative size of two buildings is by comparing their SBRs.

Definition 3. The relative size of two buildings can be measured by comparing their intervals on the long and short axes of SBRs, respectively. For two intervals I and J, the definitions are as follows:

$$
\begin{aligned}
&MuchLarger(I,J) =_{def} I/J > \sigma, \\
&Larger(I,J) =_{def} 1+\sigma_1 \leq I/J \leq \sigma, \\
&RoughlyEqual(I,J) =_{def} |I/J-1| < \sigma_1, \\
&Smaller(I,J) =_{def} 1/\sigma \leq I/J \leq 1-\sigma_1,\ \text{and} \\
&MuchSmaller(I,J) =_{def} I/J < 1/\sigma.
\end{aligned}
$$

Where σ is a real number larger than 1.5, and σ_1 a real number smaller than 0.5.

Similar to the four qualitative angle relations, these relative size relations have the following correspondences:

- $MuchLarge(I,J) \leftrightarrow MuchSmaller(J,I)$,
- $Larger(I,J) \leftrightarrow Smaller(J,I)$, and
- $RoughlyEqual(I,J) \leftrightarrow RoughlyEqual(J,I)$.

3.4. *Semantic relations*

The three qualitative relations – the angle relations, SBR relations and relative sizes – can characterize the relations between two buildings from the perspectives of angle, order, and size individually. However, these relations are atomic and are at a low level compared to the semantic relations people used for describing building patterns that are generally at a high level and can often be characterized by combining low level relations. Below, in Section 3.4.1, examples of building patterns characterized by certain combinations of the three low level relations are provided and their roles discussed. These examples provide intuitions for the high level semantic relations to be formalized in Section 3.4.2–3.4.5.

3.4.1. *Combination of qualitative relations*

Figures 8 and 9 show two sets of eight samples characterized mainly by two specific qualitative angle relations, i.e. perpendicular and parallel, and further characterized by SBS relations and relative sizes.

From the perspective of qualitative angle, in Figure 8 buildings *a* and *b* are mutually perpendicular if the angle between their main directions is approximately 90 degrees. From the perspective of SBR relations, the short axis of the target (*b* or *c*) is fully overlapping (Figure 8a), partly overlapping (Figure 8b) or non-overlapping (Figure 8c-d) with the long axis of the referent *a*. From the perspective of relative size, in Figures 8a-d the short axes of the targets are smaller than the long axes of the referents, while in Figures 8e-h those are nearly equal to or even larger than their counterparts. Intuitively, those relations in Figures 8a-c and 8e-g can be perceived to hold perpendicular relations, while relations in Figure 8d is better described as building *a* being perpendicular to buildings *b* or *c*; and in Figure 8h, two buildings tend to be more collinear as the size of *b*, a square, is larger than that of *a*. These facts indicate that: (1) not only qualitative angle descriptions, but SBR relations and relative size can affect the definition of perpendicularity, and (2) qualitative angle descriptions matter, SBR relations and relative size can refine the perpendicularity.

In Figures 9a-d, the long axis of building *b* or *c* is fully overlapping, partly overlapping, or non-overlapping with that of building *a*, and the short axis of *b* or *c* is equal to that of building *a*. The relations in Figures 9a-c can be conceptualized as strongly, partly and weakly parallel, respectively. The relations in Figures 9d and 9h, while could be considered being parallel, are better modeled as collinearity. For Figures 9e-h, the short axes of the targets (i.e. *b*) are larger than that of the referents (i.e. *a*). The relations in Figures 9e-f can be perceived as parallel, while the relation in Figure 9g is better described as perpendicular. Similarly, qualitative angle, SBR relation and relative size can account for conceptualizing parallel relations.

The brief analysis above gives rise to two points that require special attentions. First, square buildings need special attention as its long and short axes are identical. That is, the two directions of a square should be considered with the main direction of another building, respectively, to determine the qualitative angle descriptions, and thus the

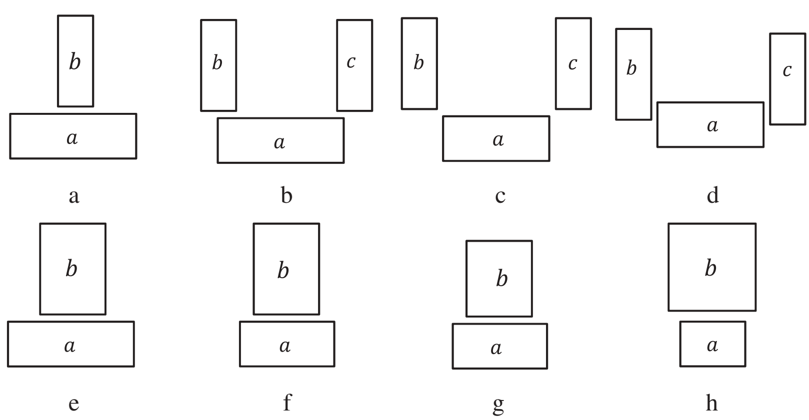

Figure 8. Combination of perpendicular relation, SBR relation, and relative size.

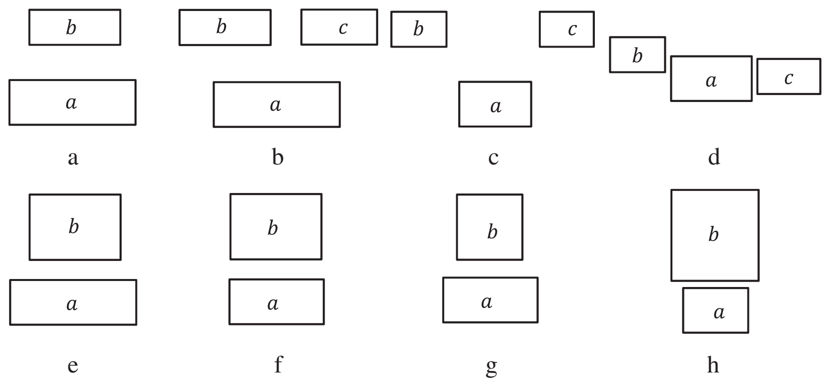

Figure 9. Combination of parallel relations, SBR relations, and relative size.

relation of a square may be modeled by multiple concepts (e.g. perpendicularity, parallelism, and collinearity). Second, for defining the three concepts, qualitative angle description matters and SBR relations refine.

3.4.2. *Parallel relations*

Two buildings *a* (referent) and *b* (target) are parallel if they are similar in sizes and main directions. Therefore, parallel relations can be modeled by the interval relations between μ_a and μ_a^b, and between v_a and v_a^b, thus $r(a,b)$ can define parallel relations. Furthermore, SBR relations and relative size can further refine parallel concepts.

Definition 4. Two buildings *a* and *b* are parallel, denoted by $BuiPal(a,b)$, if (1) their main directions are approximately parallel, i.e. $ParAnlge(a,b)$ holds, (2) $r(a,b) \in \{r_{1,-}, r_{2,-}, r_{12,-}, r_{13,-}\}$, and (3) $RoughlyEqual(\mu_a, \mu_b)$. According to SBR relations, parallel buildings can further fall into three groups: $Dis\_Pal(a,b)$, $Part\_Pal(a,b)$, and $Full\_Pal(a,b)$.

Among the three conditions, qualitative angle description is most important, followed by SBR relations and relative size. If the two buildings are very different in main directions, they cannot be parallel even if they satisfy the other two conditions. If two buildings are parallel in main directions, the short axis of the referent and the projections of the target cannot be overlapping because in this case they tend to be collinear. Among the 169 SBR relations, 117 imply that two buildings overlap on short axes, thus they cannot hold parallel relation. The remaining 52 relations are related to the concept, and fall into the following three types.

- The 16 relations, $\{r_{1,1}, r_{1,2}, r_{1,12}, r_{1,13}, r_{2,1}, r_{2,2}, r_{2,12}, r_{2,13}, r_{12,1}, r_{12,2}, r_{12,12}, r_{12,13}, r_{13,1}, r_{13,2}, r_{13,12}, r_{13,13}\}$, imply that two buildings are non-overlapping on both long and short axes, thus buildings *a* and *b* holding these relations are disjointly parallel, denoted by $Dis\_Pal(a,b)$.
- The eight relations, $\{r_{1,3}, r_{2,3}, r_{1,11}, r_{2,11}, r_{12,3}, r_{13,3}, r_{12,11}, r_{13,11}\}$, imply that two buildings do not overlap on the short axes, but partly overlap on long axes, thus

buildings a and b with these relations are partly parallel, and denoted by $Part\_Pal(a, b)$.

- The 28 relations, $\{r_{1,j}, r_{2,j}, r_{12,j}, r_{13,j}\}_{j=4}^{10}$, imply that two buildings fully overlap on long axes only, thus they are fully parallel, and denoted by $Full\_Pal(a, b)$.

Relative size also has effect on defining parallel buildings. If the long axis of the target is much longer or shorter than the one of the referent, the two buildings are non-parallel. Only when they are approximately equal, two buildings are parallel. The similar conclusion holds for short axis.

3.4.3. *Perpendicular relations*

For two perpendicular buildings, their long axes should be mutually perpendicular. Therefore, to model the perpendicular relations between buildings, the target should be projected onto the long and short axes of the referent. The qualitative angle description is the most important relation defining perpendicularity, while SBR relations and relative size can refine this concept.

Definition 5. Building b is perpendicular to building a, denoted by $BuiPer(a, b)$, if (1) their main directions are approximately perpendicular, i.e. $PerAngle(a, b)$ holds, (2) $r(a, b) \in \{r_{1,-}, r_{2,-}, r_{12,-}, r_{13,-}\}$, and (3) $MuchSmaller(\mu_a, \mu_a^b)$ holds. According to SBR relations, perpendicular buildings can fall into three groups: $Dis\_Per(a, b)$, $Part\_Per(a, b)$, and $Full\_Per(a, b)$.

- The 16 relations $\{r_{1,1}, r_{1,2}, r_{2,1}, r_{2,2}, r_{1,12}, r_{1,13}, r_{2,12}, r_{2,13}, r_{12,1}, r_{12,2}, r_{13,1}, r_{13,2}, r_{12,12}, r_{12,13}, r_{13,12}, r_{13,13}\}$ imply that two buildings are non-overlapping on both long and short axes, thus buildings a and b with these relations are disjointly perpendicular, denoted by $Dis\_Per(a, b)$.
- The eight relations $\{r_{1,3}, r_{2,3}, r_{1,11}, r_{2,11}, r_{12,3}, r_{13,3}, r_{12,11}, r_{13,11}\}$ imply that two buildings overlap on the long axis of the referent only, thus buildings a and b with these relations are partly perpendicular, denoted by $Part\_Per(a, b)$.
- The 28 relations $\{r_{1,j}, r_{2,j}, r_{12,j}, r_{13,j}\}_{j=4}^{10}$ imply that the short axis of the target fully overlap with the long axis of the referent, but not on other axes, thus two buildings with these relations are fully perpendicular, denoted by $Full\_BuiPer(a, b)$.

Similarly, two semantic relations $Acute(a, b)$ and $Obtuse(a, b)$ can be defined and classified using Definition 5. Figure 10 illustrates parts of these relations.

3.4.4. *Collinear relations*

For two collinear buildings, their main directions must be approximately parallel, and there must be overlapping between the short axis of a referent and the projection of the short axis of a target, while not between the long axis and the projection.

Definition 6. Two buildings a and b are collinear, denoted by $BuiCol(a, b)$, if (1) their main directions are approximately parallel, (2) $r(a, b) \in \{r_{i,1}\}_{i=3}^{11} \cup \{r_{i,2}\}_{i=3}^{11} \cup \{r_{i,12}\}_{i=3}^{11} \cup \{r_{i,13}\}_{i=3}^{11}$, and (3) $RoughlyEqual(v_a, v_b)$. Furthermore, collinear buildings

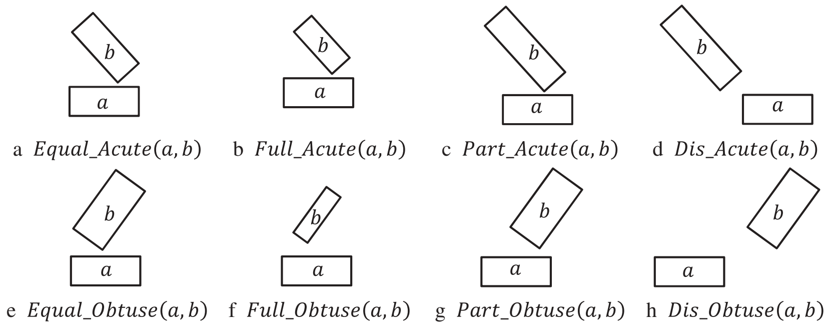

Figure 10. Semantic relations $Acute(a, b)$ and $Obtuse(a, b)$.

can fall into two types: $Strong\_BuiCol(a, b)$ and $Weak\_BuiCol(a, b)$. For the first, $r(a, b) \in \{r_{i,1}\}_{i=4}^{10} \cup \{r_{i,2}\}_{i=4}^{10} \cup \{r_{i,12}\}_{i=4}^{10} \cup \{r_{i,13}\}_{i=4}^{10}$, and for the second $r(a, b) \in \{r_{3,1}, r_{3,2}, r_{11,1}, r_{11,2}, r_{3,12}, r_{3,13}, r_{11,12}, r_{11,13}\}$ holds.

The similarity of the main directions of two buildings is still the dominant determination of collinearity. Furthermore, their short axes should be approximately equal. For the weak collinearity, the short axis and the corresponding projection on it are partly overlapping, while for the strong one, most part of them are overlapping.

Note that the two cases in Figure 11 need special attention. The first one occurs if the target is a square (Figure 11a). If one of the two directions of building b is similar to the main direction of building a, their main directions are still parallel. Therefore, to determine if they are collinear, the two axes of b must be projected onto the long or short axis of a, respectively. For the second case (Figure 11b), the target b is too small such that its long axis is approximately equal to the short one of the referent a. Their main directions are mutually perpendicular, but they still can be considered as collinearity. To accommodate this special case, the three conditions in Definition 6 can be redefined as: (1) their main directions are approximately perpendicular, (2) $r(a, b) \in \{r_{i,1}\}_{i=3}^{11} \cup \{r_{i,2}\}_{i=3}^{11} \cup \{r_{i,12}\}_{i=3}^{11} \cup \{r_{i,13}\}_{i=3}^{11}$, and (3) $Equal(v_a, \mu_b)$. Note that the collineairy of the two cases in Figure 11 is determined by cognition and needs to be proved by cognitive experiments.

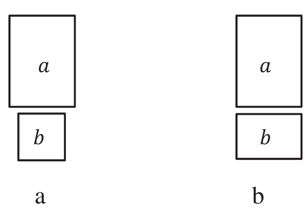

Figure 11. Two special examples.

3.4.5. *Complex relations*

In addition to the three semantic relations formalized in Sections 3.4.2–3.4.4, complex relations can occur as a result of overlapping SBRs (see Figure 12 for three such examples). Although less frequent, these cases do occur in reality and thus must be addressed. The key concern, however, is how such complex relations are described using some of the 169 relations in $\mathcal{B}_{ra}$.

To handle such complex relations, a split-and-merge approach is adapted. The intuition of the approach is based on splitting the target into parts in which none of these parts' SBRs overlap with the SBR of the referent. The semantic relation of each target part with respect to the referent is then modeled using the approaches presented in Sections 3.4.2–3.4.4. The procedure to split a target into parts is as follows. First, let x_b, x_t, y_r, and y_l be four border lines of the referent's SBR. Four half-planes denoted by s_l, s_r, s_t, and s_b are therefore formed on the left of y_l, the right of y_r, the top of x_t, and the bottom of x_b, respectively. Accordingly, the target o will be first split into four sub-parts: $o_l = s_l \cap o$, $o_r = s_r \cap o$, $o_t = s_t \cap o$ and $o_b = s_b \cap o$. The non-empty parts can then be considered separated objects. Note that the sub-part which consists of more than one component can be ignored, thus it is not necessary to model its relations.

Appling the approach to the building sample in Figure 12a, c_l, c_r, and c_t consist of one component, while c_b consists of two components, thus their semantic relation can be described as: a is full perpendicular to c_l and c_r, and parallel to c_t. For building sample in Figure 12b, a is partly perpendicular to c_l and fully to c_r and parallel to c_t. In Figure 12c, a is disjointlly perpendicular to c_l. However, a is always parallel to referent b.

4. Formalizing building patterns

With the low- and middle- level relations between two buildings formalized, it is now possible to formalize the top-level concepts of building patterns that involve a large number of buildings. This study focuses on three types of building patterns, including collinear patterns, the structured patterns, and other patterns of interest. Collinear patterns can be defined by replicating a few basic relations; the structured patterns have special

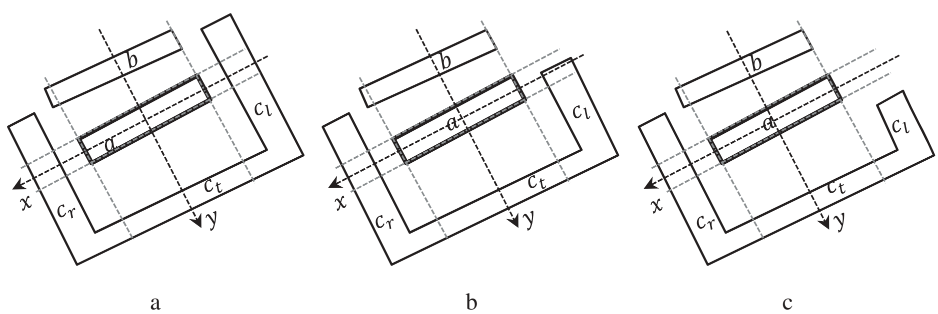

Figure 12. Complex relations between two overlapping SBRs.

configurations and acceptable names, such as Z-pattern, E-pattern, I-pattern, and L-pattern; and other patterns of interest are meaningful patterns without common names. Apparently, the latter two types of patterns have to be formalized with multiple relations, thus leading to more complex structures. This section will formalize the first two types of patterns. The third type of pattern will be handled as sketch-based approaches in Section 5.2.

4.1. *Collinear patterns*

Collinear patterns are typically found with buildings of similar geometric properties (e.g. sizes, shapes, and orientations) and they tend to be recognized as lines. Qualitatively, collinear patterns can be formalized using the basic relations at the bottom and the middle levels. Visually, although they vary greatly in geometric representations, most collinear patterns share a common trait of buildings holding parallel (Figure 13a) or collinear (Figure 13d) relations. Therefore, they can be formalized by these two relations, respectively.

- The first kind of the collinear patterns consists of parallel relations and falls into three types, which are defined by replicating the basic relations at the bottom and middle levels.

$$Full\_Col(a_1,\ldots,a_n) =_{def} \wedge_{1\le i\ne j\le n}\left(Full\_Pal\left(a_i,a_j\right)\right), \tag{1}$$

$$Part\_Col(a_1,\ldots,a_n) =_{def} \begin{cases} \wedge_{1\le i\le n-1}\left(Part\_Pal(a_i,a_{i+1}) \wedge r_{II}(a_i,a_{i+1}) = r_{1,3}\right) \\ \wedge_{1\le i\le n-1}\left(Part\_Pal(a_i,a_{i+1}) \wedge r_{II}(a_i,a_{i+1}) = r_{1,11}\right) \\ \wedge_{1\le i\le n-1}\left(Part\_Pal(a_i,a_{i+1}) \wedge r_{II}(a_i,a_{i+1}) = r_{13,3}\right) \\ \wedge_{1\le i\le n-1}\left(Part\_Pal(a_i,a_{i+1}) \wedge r_{II}(a_i,a_{i+1}) = r_{13,11}\right) \end{cases}, \tag{2}$$

$$Dis\_Col(a_1,\ldots,a_n) =_{def} \begin{cases} \wedge_{1\le i\ne j\le n}\left(Dis\_Pal\left(a_i,a_j\right) \wedge r_{II}\left(a_i,a_j\right) = r_{1,12} \vee r_{1,13}\right) \\ \wedge_{1\le i\ne j\le n}\left(Dis\_Pal\left(a_i,a_j\right) \wedge r_{II}\left(a_i,a_j\right) = r_{1,1} \vee r_{1,2}\right) \\ \wedge_{1\le i\ne j\le n}\left(Dis\_Pal\left(a_i,a_j\right) \wedge r_{II}\left(a_i,a_j\right) = r_{13,1} \vee r_{13,2}\right) \\ \wedge_{1\le i\ne j\le n}\left(Dis\_Pal\left(a_i,a_j\right) \wedge r_{II}\left(a_i,a_j\right) = r_{13,12} \vee r_{13,13}\right) \end{cases}. \tag{3}$$

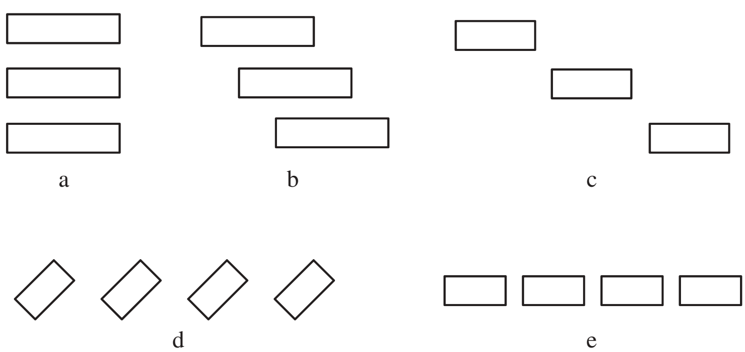

Figure 13. Collinear patterns.

- The second kind of collinear patterns is composed of collinear relations.

$$Col\_Pattern(a_1, \ldots, a_n) =_{def} \wedge_{1 \le i \ne j \le n} Full\_Col(a_i, a_j). \tag{4}$$

Figure 13 shows five collinear patterns formalized by Equations (1) and (4). All these patterns can be produced by replicating two parallel or collinear buildings. For example, for any pair of buildings a_i and a_j in Figure 13a, their relations must be $Full\_Pal(a_i, a_j)$. For the pattern in Figure 13b, each pair of buildings a_i and a_j must be $Part\_Pal(a_i, a_j)$. This pattern can be further refined into four sub-types depending on the relative locations of building a_j with respect to building a_i, i.e. top-left, upper-right, lower-left, and lower-right of building. Note that the two patterns in Figures 13c and 13d can be formalized by Equation (3) as both of them satisfy $Part\_Pal(a_i, a_j)$ and $r_{//}(a_i, a_j) = r_{1,13}$ or $r_{//}(a_i, a_j) = r_{13,1}$. Although collinear patterns have various forms with different arrangements and geometric parameters, they can be effectively formalized by the relations at the bottom and middle levels.

4.2. *Structured patterns*

Unlike collinear patterns, other structured patterns such as Z-pattern, E-pattern, I-pattern, and L-pattern cannot be represented by replicating some basic relations due to their special and diverse alignments. To formalize these structured patterns, combinations of the relations at the bottom and middle levels, thereby the combination of specific SBR relations, are more suitable. For defining the structures of these patterns, the relations between any pairs of buildings need to be specified.

$$\begin{aligned}
&E_Pattern(a, B) =_{def} B = \{b_i\}_{i=1}^{n} (n \ge 3) \wedge \exists b_i, b_j \in \\
&B\big(LSide_Per(a, b_i) \wedge RSide_Per(a, b_j)\big) \wedge \forall b_k \in \\
&B\big(i \ne j \ne k \wedge r_{13,6}(a, b_k) \wedge PerAngle(a, b_k)\big) \wedge \forall b_i, b_j \in B\big(Full_Pal(b_i, b_j)\big), \text{where} \\
&LSide_Per(a, b) =_{def} r_{13,3} \vee r_{13,5}(a, b) \wedge PerAngle(a, b) \wedge VerySmall(\mu_a, \mu_a^b) \text{ and} \\
&RSide_Per(a, b) =_{def} r_{13,10} \vee r_{13,11}(a, b) \wedge PerAngle(a, b) \wedge VerySmall(\mu_a, \mu_a^b),
\end{aligned} \tag{5}$$

$$\begin{aligned}
&Z_Pattern(a_1, a_2, b) =_{def} (Full_Pal(a_1, a_2)) \wedge In_Acute(a_1, b) \wedge In_Acute(a_2, b), \text{ where} \\
&In_Acute(a, b) =_{def} r_{13,7} \vee r_{13,6}(a, b) \wedge AcuAngle(a, b),
\end{aligned} \tag{6}$$

$$\begin{aligned}
&I_Pattern(a_1, a_2, b) =_{def} (Full_Pal(a_1, a_2) \vee Equal(a_1, a_2)) \wedge In_Per(a_1, b) \wedge \\
&In_Per(a_2, b) \text{ where } In_Per(a, b) =_{def} r_{13,5} \vee r_{13,6} \vee r_{13,10}(a, b) \wedge PerAngle(a, b),
\end{aligned} \tag{7}$$

$$\begin{aligned}
&L_Pattern(a_1, a_2, b) =_{def} Part_Pal(a_1, a_2) \wedge In_Per(a_1, b) \wedge In_Per(a_1, b) \\
&\wedge VerySmall\left(\mu_{a_1}, \mu_{a_1}^b\right) \wedge VerySmall\left(\mu_{a_2}, \mu_{a_2}^b\right).
\end{aligned} \tag{8}$$

Figure 14 illustrates some patterns with special structures that need more constraints on relations. An E-pattern (Figure 14a) can be considered as a combination of a building a and a group of collinear building pattern B, and there exist two buildings in B close to the two sides of building a can be partly or fully *perpendicular* to a while any other buildings in B must be fully perpendicular to a. The other seven patterns in Figure 14

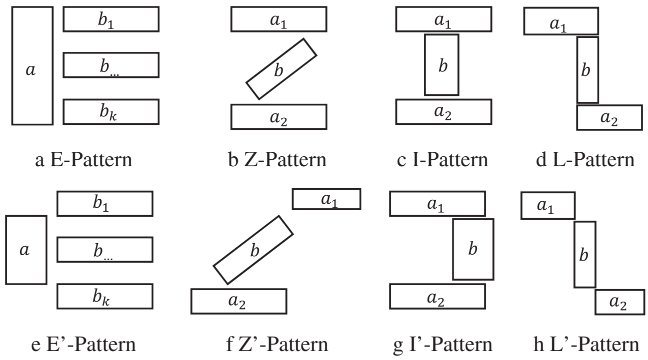

Figure 14. Structured patterns.

can be similarly defined. Note that besides those patterns (Figures 14e–14h) that can be named, many other patterns that are important and cannot be named can still be structured and formalized by the relation-based approach.

5. Case studies

This section will present a procedure for discovering patterns from building data and demonstrate the usefulness and effectiveness of the presented relation-based approach.

5.1. *Procedure for extracting patterns from data*

The procedure for extracting buildings patterns from a large number of buildings in a real dataset consists of two steps: (1) constructing a directed graph to represent the relations between buildings at the bottom and middle levels, and (2) extracting building patterns from the graph by using sub-graph isomorphism methods.

5.1.1. *Representing building relations with a graph*

In the graph representing building relations, each node represents a building, each edge links two neighboring buildings, and the bottom and middle level relations between two nodes are considered as the properties of edges. Since the relations are antisymmetric, a directed graph is required. In addition, as only buildings within proximity can form a building group from which patterns can be extracted, two additional conditions must be satisfied before two buildings are linked in the graph: (1) the distance between them is smaller than a threshold, and (2) they cannot be separated by other buildings.

To enforce the two additional conditions, the building data are first subject to creating Delaunay triangulation. The neighboring buildings can be found in the constructed triangles if two buildings share a triangle edge (Corcoran *et al.* 2012, Zhang *et al.* 2013). Once identified, the qualitative angle descriptions, relative sizes, and the bottom and the middle level relations of two neighboring buildings are computed and stored as the properties of edges. Hereafter, the constructed graph from building data is denoted by *H*. Figure 15 illustrates a set of buildings and its graph representation.

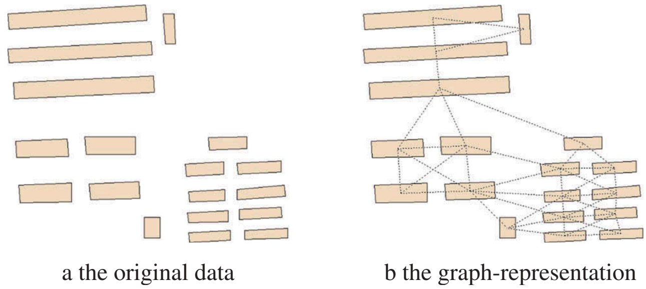

Figure 15. Graph representation of buildings.

5.1.2. *Extracting building patterns*

The extraction of building patterns of interest can be considered a graph matching process where the big graph in such process is *H*, the small graphs in such process represent the building patterns of interests, and the matching process corresponds to finding the sub-graphs from the big graph that are strictly equivalent to the small graphs.

Since collinear patterns consist of two parallel or collinear buildings, the extraction of collinear patterns aims to find *maximal cliques*, or the largest sub-graphs of *H* with each edge being marked with parallel or collinear relations. To achieve this goal, edges satisfying the conditions in Equations (1) and (4) are retrieved first from the big graph *H*, followed by examining if those edges are connected and constitute maximal cliques. However, the extraction of structured patterns is to find sub-graphs from the graph *H* that are strictly equivalent to the small graphs of the patterns. Sub-graph isomorphism (Eppstein 1999) can help find such sub-graphs from graph *H*. A simple test for sub-graph isomorphism between the small graphs of the structured patterns and the sub-graphs of *H* is sufficient to discover patterns from building data. Finally, for those patterns which are interested by users while without common names, a sketch-based approach is helpful. In this case, users first draw the sketches of interest or select them from building data, and then carry out sub-graph isomorphism test to find those patterns isomorphic to the sketches. Note that in such cases the graphs of the patterns of interest are much smaller than the graph of building data, which results in high computational complexity and robustness problems with respect to small differences in most sub-graph isomorphism methods. Fortunately, the advancements in recent years in handle sub-graph isomorphism in a big graph with billions of nodes (Sun *et al.* 2012) have remediated this issue.

5.2. *Results of discovered building patterns*

Figure 16 illustrates the parallel, collinear and perpendicular patterns extracted from 2162 buildings.

To evaluate the accuracy of the presented approach, the extracted patterns were compared with those identified manually by users. The extracted patterns are correct if they equal to the ones identified by users, and denoted by *tp*. The extracted patterns are incorrect if they are not equal to the ones identified by users, and denoted by *fp*. The identified patterns are missed by the approach are denoted by *fn*. Therefore, the

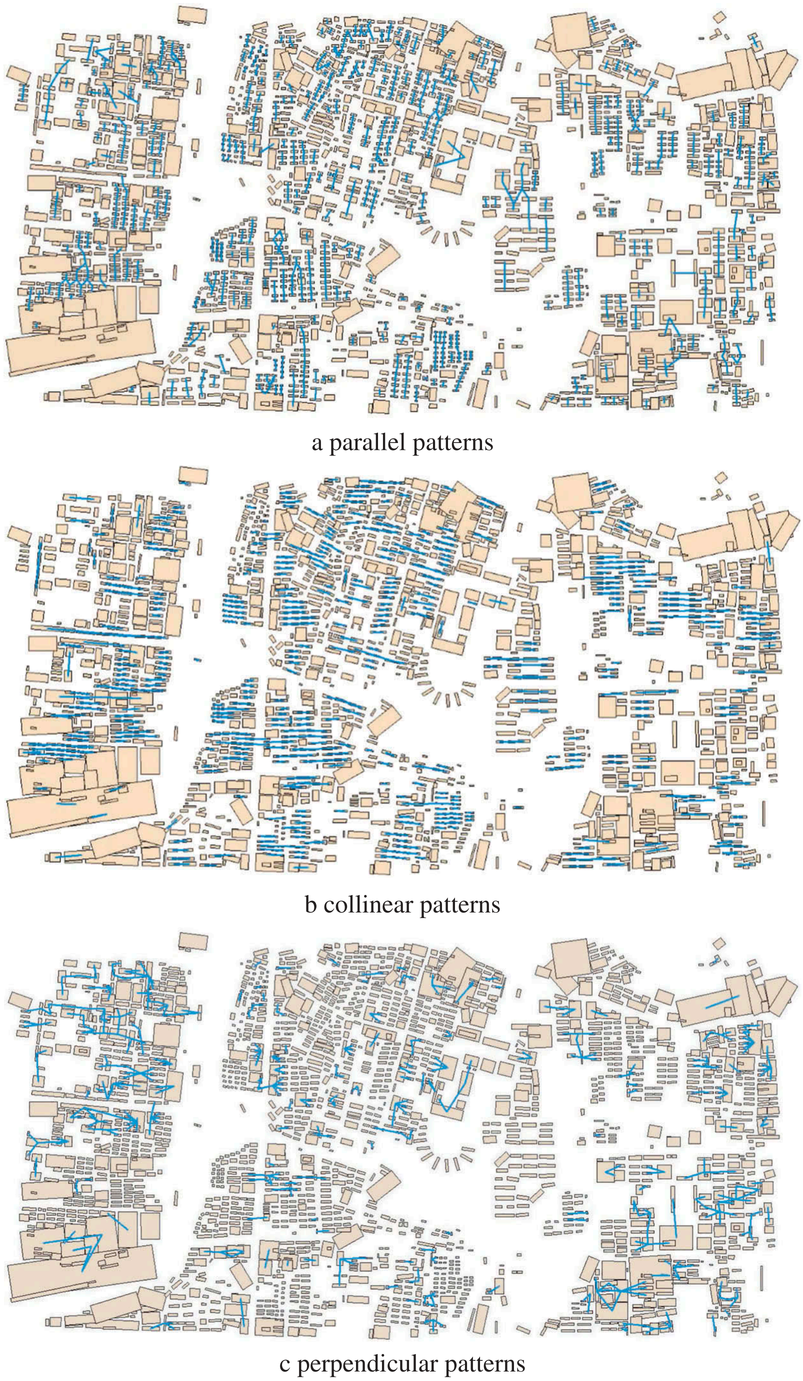

Figure 16. The extracted patterns.

correctness is defined as $tp/(tp + fp)$ and the completeness as $tp/(tp + fn)$. Both correctness and completeness can be used to evaluate the accuracy of the presented approach.

Table 1. Evaluation of extracted building patterns.

Patterns	*tp*	*fp*	*fn*	Correctness	Completeness
Parallelism	352	45	25	88.7%	93.4%
Collinearity	358	7	8	98.1%	97.8%
Perpendicularity	354	15	21	95.9%	94.4%

The results in Table 1 and Figure 16 indicate that most patterns can be successfully extracted by the proposed approach and are consistent with the ones recognized by users. The three patterns are common in the real world, and the buildings in these patterns are homogeneous in geometric properties, thus correctness and completeness are both high.

5.3. *Results of the sketch-based pattern query*

Figure 17 illustrates four sketches used for querying patterns with special structures and no names are provided, but they are of interest to some users. In this case, these sketches can be used as constraints to extract patterns isomorphic to them. Figure 18 shows the results retrieved from 1956 buildings, and Table 2 indicates the results of accuracy evaluation. The correctness of the four queries is very high, while the completeness is low compared to correctness. This is not surprising because the exact graph-isomorphism approaches adopted in this study use a stricter relation, i.e. equivalence to extract patterns. Therefore, only those buildings which have identical relations with the relations in the sketches are considered as candidates, while those buildings with similar structures are ignored. Accordingly, some similar patterns with the sketches are lost, leading to a low completeness.

6. Discussion

6.1. *Advantages of the proposed approach*

This study presents the relation-based approach for describing the semantics of building patterns at three levels. The approach brings at least two advantages over the existing geometric-based approaches. First, the formalization of the relations at the three levels, bottom level with the interval-based relations, middle level with the semantic relations, and top level with building patterns, enables automatic identification of building patterns in a more intuitive and user-friendly way. The existing geometric-based approaches relying on geometric similarity cannot support such kind of analysis. Second, it presents a model for describing the relative positions of buildings in a local and relative frame of reference. The existing models of metric relations describe

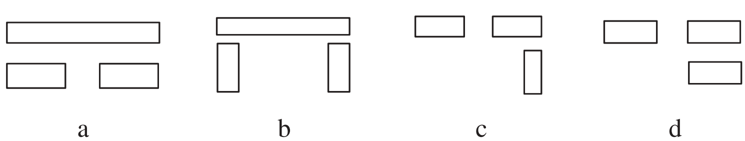

Figure 17. Four sketches.

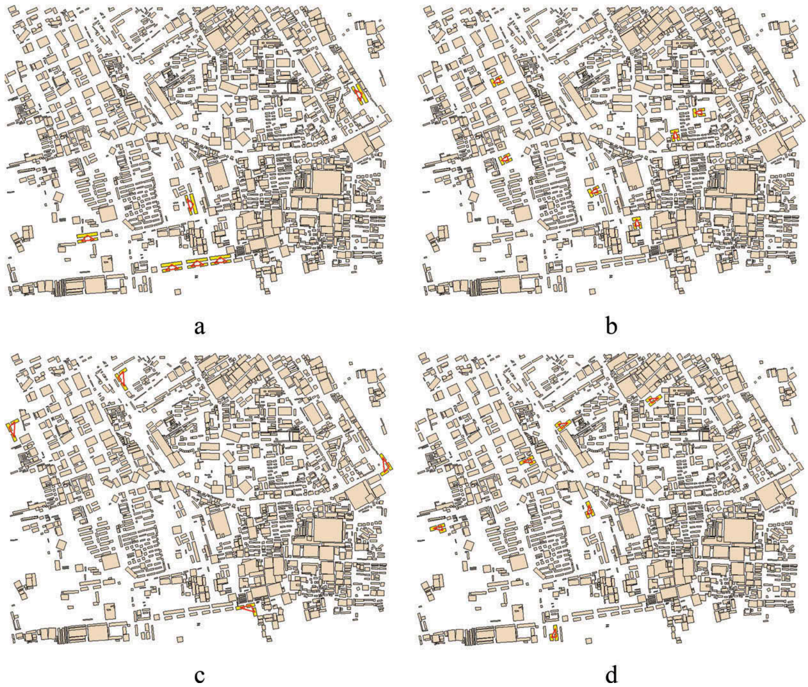

Figure 18. The results of the four sketch-based queries.

Table 2. Evaluation of sketch-based building patterns.

Patterns	*tp*	*fp*	*fn*	Correctness	Completeness
a	6	0	3	100%	66.7%
b	6	0	3	100%	66.7%
c	4	0	1	100%	80.0%
d	6	0	2	100%	75.0%

buildings in a global and absolute frame of reference, making it difficult to distinguish similar building patterns with different main directions.

Compared with geometric-based approaches, the proposed relation-based approach is more generic to represent the semantics and structures of building patterns and extract building patterns. This study first abstracts the relations between two buildings into 169 SBR relations at the bottom level. These qualitative relations are sufficient to handle all possible building relations, but they are too many to be understood or used by humans. Therefore, these 169 relations are further grouped into three simple semantic relations at the middle level – collinear, parallel, and perpendicular relations – and complex semantic relations that are easy to be understood and used by people on the daily basis. However, they can only handle two buildings. Finally, the 169 relations at the bottom level, the semantic relations at the middle level, and those qualitative angle relations, relative size relations are combined to formally define building patterns (i.e. groups of buildings). Actually, building patterns can be formally defined as a graph for describing their structures and semantics, and extracted from a large number of buildings by using graph-isomorphism. In contrast, the existing studies (Christophe and Ruas 2002, Yang 2008, Zhang *et al*. 2013, Cetinkaya *et al*. 2015) first compute geometric properties of two buildings and then extract buildings patterns using geometric similarities. That is, the existing geometric-based approaches directly build the mapping between geometric similarities and building patterns without considering the relational, semantic and structural information of buildings. As a result, they can only extract limited number of patterns because it is hard to establish the correspondence between geometric similarities and most patterns. The relation-based approach in this study is more generic and can define most patterns using the relations at the three levels. For example, the sketch-based query can be implemented by the presented approach as the sketches can be represented as graphs, while it is hard to be handled by geometric-based approaches as the mapping between geometric parameters and the sketches are unknown.

Compared to the existing studies on cardinal direction relations, the relations presented here are based on a local and relative frame of reference, which is more powerful to handle the relations between buildings. Building relations or patterns are independent on the main directions of buildings and geographic north. For any pairs of buildings with similar relative orientations, they tend to be perceived to be in identical pattern regardless of their absolute orientations to geographic north. However, the existing approaches including rectangle algebra (Papadias and Theodoridis 1997, Balbiani *et al*. 1999, Navarrete *et al*. 2013) and cardinal direction relations (Skiadopoulos *et al*. 2005, Kurata and Shi 2009, Cohn *et al*. 2014) are based on a global and absolute frame of reference. As the existing approaches always adopt geographic north to be their directional north, they cannot handle the variations of metric relation relative to buildings with different main directions, leading to limited capability in representing and extracting building patterns. On the other hand, the presented model in this study is invariant to rotation, and a local and relative frame of reference is adopted to represent building relations, thus it can help to represent and extract building patterns with different structures. The sketch-based query of building patterns further demonstrates this point.

The work in this study is also somewhat relevant to topological relations (Egenhofer and Herring 1991). Generally, topological relations can be derived from cardinal direction relations (Papadias and Theodoridis 1997, Guo and Du 2009, Li and Cohn 2012). However, because the existing studies normally adopted MBR relations to derive topological relations, the results are not precise due to the coarse approximation of MBRs to spatial objects. The SBR relations proposed in this study can improve precision of the derived results as SBRs are more exact than MBRs to approximate spatial objects.

6.2. *Limitations of the proposed approach*

This study currently focuses on qualitative aspect of building patterns. Although qualitative knowledge contributes much more to representing and understanding building patterns, and are more generic to handle various patterns, there are still some limitations. First, geometric parameters also play some roles in refining different concepts. For example, the collinear relations between two buildings can be distinguished qualitatively into three types at the bottom level. However, the distinctions between the three types are sometimes uncertain. In such a case, the geometric parameters between buildings can help to refine the distinctions among different concepts of patterns. Second, the strict graph-isomorphism approaches used to find strictly equivalent patterns with the formalization or sketches can lead to the omission of certain similar patterns. Accordingly, the geometric parameters and relation relaxation (Nedas and Egenhofer 2008) will be considered to extract similar patterns instead of equivalent ones.

7. Conclusions

This study presented a relation-based approach to represent and extract building patterns. A local and relative frame of reference is adopted to represent the relative positions between buildings. This model is invariant under rotation transformation and thus can help represent various patterns with different main directions. Therefore, this work complements current studies on rectangle algebra or cardinal direction relations, which cannot readily represent and extract building patterns. A three-level mechanism is presented to formalize the semantics and extract building patterns with various structures. The patterns at the high levels can be formalized by replicating a few relations at the bottom and middle levels, or represented as graphs. The graph approaches (i.e. maximal cliques and association graph) are used to extract building patterns from the graph representation of buildings. The experimental results indicate that the presented approach can find various patterns with very high correctness and completeness.

Future work will focus on the refinement of the presented approach. The geometric properties will be used to refine the relation representation. Special attention will be paid to refining the relations at the middle level and the patterns at the high level and extract similar patterns using geometric properties. Specifically, it is necessary to analyze which geometric properties are related to each concept and what ranges of each related property should be used for each concept. For extracting patterns from databases, similarity measurements should be developed to define similar patterns by considering both qualitative criteria and geometric properties. The presented approach can also be extended to support the detection of multi-scale building patterns.

Acknowledgements

Comments from the editor and three anonymous reviewers are greatly appreciated.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

The work of the first author was supported by the National Natural Science Foundation of China [41171297]. The work of the second author was supported by the National University of Singapore Academic Research Fund [R-109-000-112-112].

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