11、解：电子元件为一等品的概率为：

$$e^{-2} \approx 0.1353$$

设1000件中该电子元件的一等品数为X，则

$$X \sim b(1000, 0.1353)$$

由中心极限定理，

$$\frac{X - 1000 \times 0.1353}{\sqrt{1000 \times 0.1353 \times 0.8647}} \sim N(0,1)$$

故

$$P\{X > 150\}$$

$$\approx P\left\{\frac{X - 1000 \times 0.1353}{\sqrt{1000 \times 0.1353 \times 0.8647}} > 1.36\right\}$$

$$\approx 1 - \Phi(1.36)$$

$$= 0.0869$$

2. 解：$EZ_1 = E(X)\cdot E(\frac{1}{Y}) = 0$

$EZ_2 = E(X)\cdot E(Y) = 0$

$E(Z_1\cdot Z_2) = E(X^2) = D(X) + (EX)^2 = 3 + 0^2 = 3$

$cov(Z_1, Z_2) = E(Z_1 Z_2) - EZ_1 \cdot EZ_2 = 3$

$E(\frac{1}{Y^2}) = \int_1^{\infty} \frac{1}{x^2}\cdot 3x^{-4} dx = \int_1^{\infty} 3x^{-6} dx = \frac{3}{5}$

$E(Z_1^2) = E(X^2)\cdot E(\frac{1}{Y^2}) = 3 \times \frac{3}{5} = \frac{9}{5}$

$E(Y^2) = \int_1^{\infty} x^2 \cdot 3x^{-4} dx = \int_1^{\infty} 3x^{-2} dx = 3$

$E(Z_2^2) = E(X^2)\cdot E(Y^2) = 3 \times 3 = 9$

$D(Z_1) = E(Z_1^2) - (EZ_1)^2 = \frac{9}{5} - 0^2 = \frac{9}{5}$

$D(Z_2) = E(Z_2^2) - (EZ_2)^2 = 9$

$$\rho = \frac{cov(Z_1, Z_2)}{\sqrt{D(Z_1)\cdot D(Z_2)}} = \frac{3}{\sqrt{\frac{9}{5}\times 9}} = \frac{\sqrt{5}}{3}$$

3. 解：(1) 设 $x_1, x_2, \cdots, x_n$ 为样本观测值，则

$$L(\theta) = \prod_{i=1}^{n} f(x_i) = \prod_{i=1}^{n}\left(2x_i\theta^{-1}\exp\{-x_i^2\theta^{-1}\}\right)$$

$$= 2^n\left(\prod_{i=1}^{n}x_i\right)\theta^{-n}\exp\left\{-\theta^{-1}\sum_{i=1}^{n}x_i^2\right\}$$

$$\ln L(\theta) = \left(n\ln 2 + \sum_{i=1}^{n}\ln x_i\right) - n\ln\theta - \theta^{-1}\sum_{i=1}^{n}x_i^2$$

令 $\left(\ln L(\theta)\right)' = -\frac{n}{\theta} + \theta^{-2}\sum_{i=1}^{n}x_i^2 = 0$，解得

$$\hat{\theta} = \frac{\sum_{i=1}^{n}x_i^2}{n}$$

故 θ 的最大似然估计量为 $\hat{\theta} = \frac{\sum_{i=1}^{n}X_i^2}{n}$.

(2) $E(X^2) = \int_0^{\infty} 2x^3\cdot\theta^{-1}\exp\{-x^2\theta^{-1}\}\,dx.$

$$\xlongequal{y=x\cdot\theta^{-\frac{1}{2}}} 2\theta\int_0^{\infty} y^3\exp\{-y^2\}\,dy.$$

$$\xlongequal{t=y^2} \theta\int_0^{\infty} t e^{-t}\,dt.$$

$$= \theta$$

故 $E\hat{\theta} = \frac{\sum_{i=1}^{n}E(X_i^2)}{n} = E(X^2) = \theta.$

即 $\hat{\theta}$ 是 θ 的无偏估计量.

4. 解: $\bar{x}=19$, $S^2=\frac{40}{3}\approx 13.3333$

$S=3.65$, $n=10$, $\alpha=0.05$.

血压增高均值的95%置信区间为

$$\left(\bar{x}-\frac{S}{\sqrt{n}}t_{\alpha/2}(n-1),\ \bar{x}+\frac{S}{\sqrt{n}}t_{\alpha/2}(n-1)\right)$$

$$=\left(19-\frac{3.65}{\sqrt{10}}\times 2.2622,\ 19+\frac{3.65}{\sqrt{10}}\times 2.2622\right)$$

$$=(16.39,\ 21.61).$$

血压增高~~均值~~标准差的95%置信区间为

$$\left(\frac{\sqrt{n-1}\,S}{\sqrt{\chi^2_{\alpha/2}(n-1)}},\ \frac{\sqrt{n-1}\,S}{\sqrt{\chi^2_{1-\frac{\alpha}{2}}(n-1)}}\right)$$

$$=\left(\frac{\sqrt{9}\times 3.65}{\sqrt{19.022}},\ \frac{\sqrt{9}\times 3.65}{\sqrt{2.7}}\right)$$

$$=(2.51,\ 6.66).$$

$X \sim N(\mu_1, \sigma^2)$，$Y \sim N(\mu_2, \sigma^2)$

5. 解：检验 $H_0: \mu_1 = \mu_2$，$H_1: \mu_1 \neq \mu_2$.

检验统计量：

$$t = \frac{\bar{x} - \bar{y}}{S_w \cdot \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{，其中 } S_w = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

拒绝域：

$$t < -t_{\alpha/2}(n_1+n_2-2)\text{，或 } t > t_{\alpha/2}(n_1+n_2-2).$$

$\alpha = 0.05$，$n_1 = 6$，$n_2 = 4$，

$$t_{0.025}(8) = 2.3060$$

$S_w^2 = 3.28$，$S_w = \sqrt{3.28} = 1.81$，

$$t = 1.37 \in (-2.3060, 2.3060)$$

未落入拒绝域，故两种农药残留时间无显著差异.

6. 解：H_0：$P\{X=k\}=e^{-\lambda}\cdot\frac{\lambda^k}{k!}$，$k=0,1,2,\cdots$

$\hat{\lambda}=\bar{x}=0.6$，　$n=200$

x_i	f_i		$\hat{p}_i$	$n\hat{p}_i$	
0	116		0.5488	109.76	
1	56		0.3293	65.76	
2	22		0.0988	19.76	
3	4		0.0198	3.96	
4	2	28	0.003	0.6	24.38
≥5	0		0.0003	0.06	

合并后组数为 $k=3$，

$$\chi^2=\sum_{i=0}^{2}\frac{f_i^2}{n\hat{p}_i}-n=2.37<3.843=\chi^2_{0.05}(1)$$

无法拒绝原假设，故认为次品件数服从泊松分布.

7. 解：(1) $\bar{x}=19.5$，$\bar{y}=69.5$，$n=10$.

$$S_{xx}=\sum_{i=1}^{n}x_i^2-n\bar{x}^2=82.5$$

$$S_{xy}=\sum_{i=1}^{n}x_iy_i-n\bar{x}\bar{y}=233$$

$$\hat{b}=\frac{S_{xy}}{S_{xx}}=2.8242$$

$$\hat{a}=\bar{y}-\hat{b}\bar{x}\approx 13.53$$

$$\hat{y}=13.53+2.8242x$$

(2) $H_0: b=0$，$H_1: b\neq 0$.

检验统计量：$t=\frac{\hat{b}}{\hat{\sigma}}\sqrt{S_{xx}}$，

其中 $\hat{\sigma}=\sqrt{\frac{Q_e}{n-2}}$，$Q_e=S_{yy}-\hat{b}S_{xy}$

拒绝域 $|t|>t_{\frac{\alpha}{2}}(n-2)$.

计算得 $S_{yy}=\sum_{i=1}^{n}y_i^2-n\bar{y}^2=724.4$

$Q_e=66.3614$，$\hat{\sigma}=2.88$，

$|t|=8.907>2.3060=t_{0.025}(8)$

落入拒绝域，故回归效果显著。

8. 解: $\bar{y}_1=8$, $\bar{y}_2=6.8$, $\bar{y}_3=7.2$, $\bar{y}_4=8.5$

$\bar{y}=7.6154$.

方差分析表

方差来源	平方和	自由度	均方	F比
因素	4.8139	3	1.6	3.2
误差	4.5	9	0.5	
总和	9.3139	12		

$F=3.2<3.86=F_{0.05}(3,9)$.

未落入拒绝域，故认为无显著影响.