Optimal Sum-Rate Maximization in a NOMA System with Channel Estimation Error

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***Abstract*—Non-Orthogonal Multiple Access (NOMA) has recently been conceived as a new technology for 5G cellular communication which has the potential to provide higher throughput than conventional orthogonal multiple access. In this paper, we deal with sum-rate maximization of a downlink NOMA based system in the presence of channel estimation error which is a more practical scenario, with user's QoS and total BS power constraints. By analyzing KKT conditions, a closed form solution for optimal power allocation aiming to maximize sum-rate is derived. Simulation results depict that in terms of sum-rate, NOMA system outperforms conventional orthogonal multiple access (OMA) systems and the gain of NOMA increases with users.**

***Index Terms*—Non-Orthogonal Multiple Access (NOMA), Channel Estimation Error, Sum-Rate Maximization, KKT Conditions.**

I. Introduction

Given the prediction of dramatically increasing data traffic for 5G wireless communications, Non-Orthogonal Multiple Access (NOMA) as a candidate air interface technology to meet the overwhelming requirement of this data traffic has recently attracted much research attention [1]–[3]. NOMA technique, by using superposition coding, multiplexes users in power domain at the transmitter and successive interference cancellation (SIC) at the receiver, and allows users to share the same time/frequency resources, which can provide superior spectral efficiency compared to conventional orthogonal multiple access such as TDMA [4], [5].

In [6], the authors show that NOMA can deliver superior sum-rate than OMA systems with fixed power allocation. [7] proposed a NOMA scheme for up-link and drove a novel subcarrier and power allocation algorithm for the new NOMA scheme that maximizes the users' sum-rate. In [8], power allocation for sum rate maximization with full knowledge of channel estimation has been investigated. [9] derived a suboptimal power allocation to maximize overall sum-rate in a sub-carrier based NOMA system based on perfect CSI. Z. Ding et. al. in [10] proposed an algorithm for resource allocation in multi-carrier NOMA (MC-NOMA) systems aiming for maximizing weighted system throughput with full CSI.

Most of the aforementioned works on NOMA, consider full channel state information at the transmitter (CSIT), which is not a reasonable assumption for the transmitter to have perfect channel state information due to channel quantization and time-varying nature of the channel. Some preliminary works on NOMA with imperfect CSI are available. In [6] ergodic sum-rate with statistical CSI for SC-NOMA in downlink scenario with fixed power allocation has been considered. Average sum-rate under partial CSI has been analyzed in [11]. Imperfect CSIT due to partial and limited feedback has been investigated in [12]–[14].

In this article, we focus on optimal power allocation for sum-rate maximization in a NOMA system with impaired CSI at the transmitter. Our problem formulation contains users'QoS as user minimum rate requirement and total BS power constraint that is shown is a convex optimization problem, which ensures KKT conditions are necessary and sufficient for the optimum solution [15]. We employ the KKT conditions and drive a closed-form solution for the optimization problem.

The remainder of the paper is organized as follows. Section II describes the system and signal model, which is followed by formulation of the optimization problem In Section II-B. Simulation results are presented in Section III and Section IV concludes the paper.

II. System Model and Problem Formulation

A. System Model

We consider a single-cell system with a Base Station (BS) serving M users. All transceivers are equipped with a single antenna. BS transmits its signal according to principle of NOMA to the M users via power domain multiplexing through the same time-frequency resources (Figure 1).

The transmitted signal at BS is expressed as:

$$x = \sum_{k=1}^{M} \sqrt{p_k}\ s_k. \qquad E\left\{|s_k|^2\right\} = 1. \tag{1}$$

where p_k is power allocated to user k and s_k is the transmitted signal of k^{th} user.

We model the channel gain between transmitter and k^{th} user, $1 \leq k \leq M$, as $h_k = g_k d_k^{\frac{-\alpha}{2}}$, in which $g_k \sim \mathcal{CN}(0,1)$ is a fading coefficient with Rayleigh distribution. d_k is the distance between k^{th} user and the BS and α denotes path loss exponent. In this work we consider imperfect CSI, by assumption of MMSE channel estimator, an estimation of

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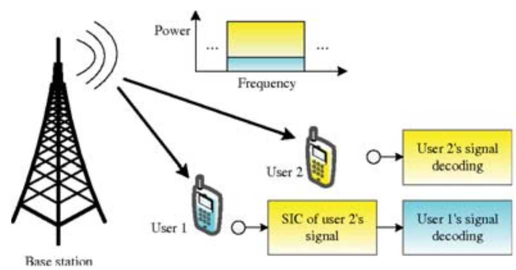


Fig. 1. System model.

h_k, is shown as $\hat{h}_k$ [16], [17]. Thus we have $h_k = \hat{h}_k + \epsilon$ where $\epsilon \sim \mathcal{CN}(0, \sigma_\epsilon^2)$ is channel estimation error. So $\hat{h}_k$ has complex Gaussian distribution with zero mean and variance $\sigma^2_{\hat{h}_k} = d_k^{-\alpha} - \sigma_\epsilon^2$, (i.e. $\hat{h}_k \sim \mathcal{CN}(0, \sigma^2_{\hat{h}_k})$)

We assume the estimated channel, $\hat{h}_k$, is uncorrelated with ϵ [17].

Without loss of generality we assume the estimated channels are sorted as

$$\left|\hat{h}_1\right|^2 > \left|\hat{h}_2\right|^2 > ... > \left|\hat{h}_M\right|^2. \tag{2}$$

Adopting the NOMA principle, more power is allocated to user with poorer channel gain, it means that power allocated to the users is sorted as $p_1 < p_2 < ... < p_M$, and $\sum_{k=1}^{M} p_k = P_t$. Received signal at the k^{th} receiver is:

$$y_k = h_k x + z_k \tag{3a}$$

$$= \hat{h}_k \sum_{j=1}^{M} \sqrt{p_j}\ s_j + \epsilon \sum_{j=1}^{M} \sqrt{p_j}\ s_j + z_k \tag{3b}$$

$$= \hat{h}_k \sqrt{p_k} s_k + \hat{h}_k \sum_{\substack{j=1 \\ (j \neq k)}}^{M} \sqrt{p_j}\ s_j + \epsilon \sum_{j=1}^{M} \sqrt{p_j}\ s_j + z_k. \tag{3c}$$

The first term at the right-hand side of (3c) is the desired signal of k^{th} user, the second term is interference of other users, the third term denotes the interference of channel estimation error and the last term indicates complex additive white Gaussian noise$\left(z_k \sim \mathcal{CN}(0, \sigma^2)\right)$.

At the receiver, SIC is used to detect the desired signal. According to SIC k^{th} user before decoding its own signal, decodes the signal of all j^{th} users, $k+1 \leq j \leq M$, remove them from the received signal, while treating the interferences of $k-1$ first users as noise.

User k can decode message of l^{th} users, $k+1 \leq l \leq M$, if the achievable rate of l^{th} user at the receiver of user k greater than or equal to the achievable rate of l^{th} user at its own receiver. It means that:

$$\begin{aligned} SINR_k^l &= \frac{p_l \left|\hat{h}_k\right|^2}{\left|\hat{h}_k\right|^2 \sum_{j=1}^{l-1} p_j + P\sigma_\epsilon^2 + \sigma^2} \\ &\geq \frac{p_l \left|\hat{h}_l\right|^2}{\left|\hat{h}_l\right|^2 \sum_{j=1}^{l-1} p_j + P\sigma_\epsilon^2 + \sigma^2} \\ &\geq SINR_l^l. \end{aligned} \tag{4}$$

In equation (4), $SINR_k^l$ indicates SINR of l^{th} user at the receiver of user k and $SINR_l^l$ is SINR of l^{th} user at its own receiver.

After performing SIC, SINR of user k is: $SINR_k = \frac{p_k \left|\hat{h}_k\right|^2}{\left|\hat{h}_k\right|^2 \sum_{j=1}^{k-1} p_j + P\sigma_\epsilon^2 + \sigma^2}$ and for user 1 (the strongest user): $SINR_1 = \frac{p_1 \left|\hat{h}_1\right|^2}{P\sigma_\epsilon^2 + \sigma^2}$. So sum rate of the users is obtained as:

$$R_{sum} = \sum_{k=1}^{M} \log_2 \left(1 + \frac{p_k \left|\hat{h}_k\right|^2}{\left|\hat{h}_k\right|^2 \sum_{j=1}^{k-1} p_j + P\sigma_\epsilon^2 + \sigma^2}\right). \tag{5}$$

B. Problem Formulation and Optimal Power Allocation

In this section, we address the optimization problem for maximizing sum rate of the users calculated in the previous section. The optimization problem formulated as:

P_0 :

$$\begin{aligned} &\max_{p_1, p_2, \ldots, p_M} R_{sum} = \\ &\max_{p_1, p_2, \ldots, p_M} \sum_{k=1}^{M} \log_2 \left(1 + \frac{p_k \left|\hat{h}_k\right|^2}{\left|\hat{h}_k\right|^2 \sum_{j=1}^{k-1} p_j + P\sigma_\epsilon^2 + \sigma^2}\right). \\ &\text{s.t. } C1 : \sum_{k=1}^{M} p_k \leq P. \\ &\quad C2 : p_k \geq (\omega_j - 1)\left(\sum_{j=1}^{k-1} p_j + \frac{P\sigma_\epsilon^2 + \sigma^2}{|\hat{h}_k|^2}\right), \\ &\quad \forall k = 1, 2, ..., M. \end{aligned}$$

In problem P_0, $\omega_j = 2^{R_j^{Min}}$, where R_j^{Min} is minimum required data rate for jth user. $C1$ shows total BS power constraint and in $C2$ constraints users'QoS (minimum required rate) is guaranteed.

By making Hessian Matrix for objective function of problem P_0, it is easy to show that the Hessian Matrix is negative definite, and so the objective function is a concave function and since the constraints are convex, the optimization problem P_0 is a convex optimization problem. Thus Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient for optimal solution of problem P_0, Furthermore, Lagrangian function for solving P_0 will be as follows

$$
\begin{aligned}
L(\alpha,\theta,\mu,\lambda) = {} & \sum_{k=1}^{M} \log_2 \left(1 + \frac{p_k |\hat{h}_k|^2}{|\hat{h}_k|^2 \sum_{j=1}^{k-1} p_j + P\sigma_\epsilon^2 \theta + \sigma^2}\right) \\
& + \sum_{k=1}^{M} \mu_k \left[p_k - (\omega_k - 1)\left(\sum_{j=1}^{k-1} p_j + \frac{P\sigma_\epsilon^2 + \sigma^2}{|\hat{h}_k|^2}\right)\right] \\
& + \lambda \left(P - \sum_{j=1}^{M} p_j\right).
\end{aligned} \tag{6}
$$

where λ and $\mu_i \quad i = 1, 2, ..., M$ are non-negative Lagrangian multipliers corresponding to the constraints $C1$,$C2$. Optimum power allocations are obtained using KKT conditions given in Theorem 1.

Theorem 1 : Closed form optimal power allocation for the optimization problem P_0 is obtained as:

$$
p_1 = \frac{P}{\prod_{k=2}^{M} \omega_k} - \sum_{j=2}^{M} \frac{(\omega_j - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\prod_{l=2}^{j} \omega_l |\hat{h}_j|^2}. \tag{7}
$$

and for $k = 2, 3, \ldots, M$,

$$
\begin{aligned}
p_k = {} & \frac{(\omega_k - 1)P}{\prod_{j=k}^{M} \omega_j} + \frac{(\omega_k - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\omega_k |\hat{h}_k|^2} \\
& - \sum_{j=k+1}^{M} \frac{(\omega_k - 1)(\omega_j - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\prod_{l=k}^{j} \omega_l |\hat{h}_j|^2}.
\end{aligned} \tag{8}
$$

Proof: See Appendix A.

III. Simulation results

In this section, we present numerical results to compare the performance of sum-rate for a NOMA system and OMA system. It is assumed that path loss exponent $\alpha = 3$, and small-scale fading has Rayleigh distribution, i.e., $\mathrm{g_k} \sim \mathcal{CN}(0, 1)$, which is flat through each transmission time slot and varies in next time slots, independently. Variance of noise is set to $\sigma^2 = -70$dBm. Simulation has done over 10000 random channel generations and M-user NOMA is considered, where $M = 2, 3, 4$. We use TDMA as a conventional OMA scheme for comparison.

Figure (2) demonstrates comparison of a NOMA system for $m = 2, 3, 4$ users with OMA system when full knowledge of channel state information is available and in the presence of channel estimation error. The comparison has been done for different values of total BS power. In the case of imperfect CSI, the variance of channel estimation error is set to $\sigma_\epsilon^2 = 0.01$, minimum data rate requirement set to $R^{min} = 1$ bits/sec/Hz which is assumed to be the same for all users. It can be observed that a NOMA system delivers a more superior sum-rate than OMA and NOMA gain increases with users.

Figure (3a) shows sum rate versus minimum required data rate while the total power of BS is fixed to $P = 20$dBm. It depicts that NOMA has better performance than OMA and sum-rate decreases with increasing minimum required data rate. The reason is that increasing minimum required data rate needs to allocate more power to the user with worst channel gain.

Figure (3b) plots sum-rate versus different channel estimation error variances. It is shown that sum-rate decreases with the variance of channel estimation. We also found that NOMA and OMA have the same behavior with increasing channel estimation error.

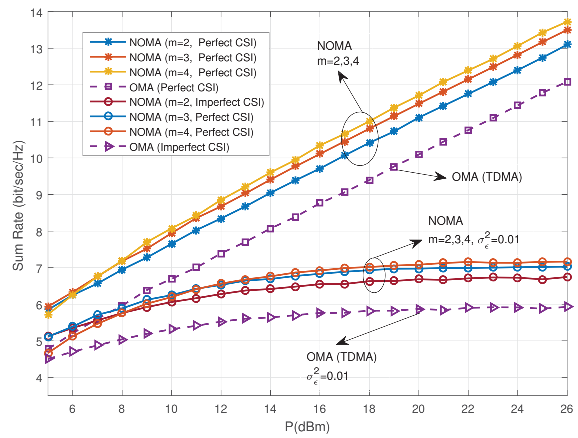


Fig. 2. Sum Rate vs. Total BS Power.

IV. Conclusion

In this paper, we have considered maximizing the sum-rate of a downlink M-user NOMA system in the presence of channel estimation error. We proved that the problem is a convex optimization problem and a closed form solution for optimum power allocation to maximize the sum rate of the system has derived. We showed that NOMA has superior sum rate compared to OMA with and without channel estimation error. And the gain of NOMA with respect to OMA increases with users.

Appendix A

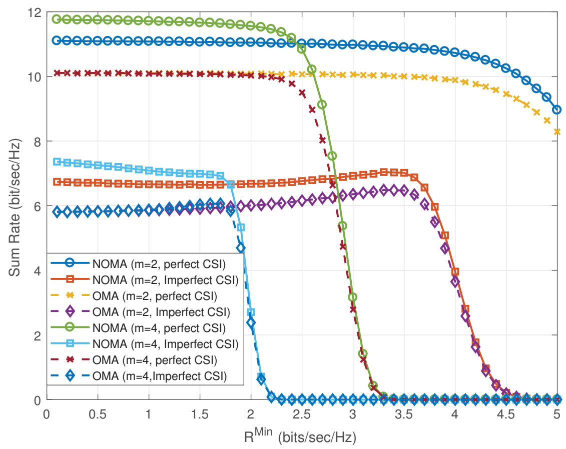
To find the solution of convex optimization problem $P0$, we reformulate the Lagrangian function as follows:

$$
L(P,\mu,\lambda) = R_{sum}(P) + \sum_{k=1}^{M} \mu_k g_k(P) + \lambda h(P). \tag{9}
$$

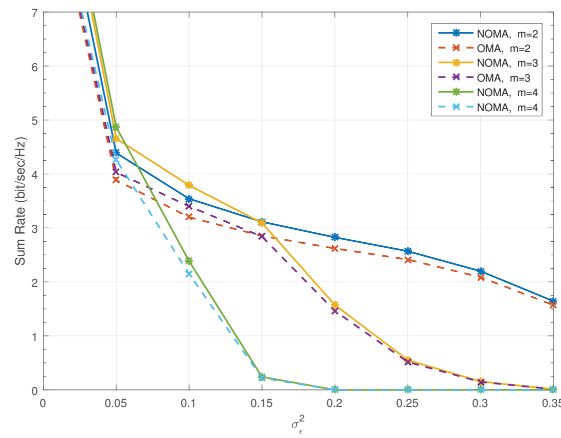
where

$$
R_{sum}(P) = \sum_{k=1}^{M} \log_2 \left(1 + \frac{p_k |\hat{h}_k|^2}{|\hat{h}_k|^2 \sum_{j=1}^{k-1} p_j + P\sigma_\epsilon^2 + \sigma^2}\right). \tag{10}
$$

$$
g_k(P) = \left[p_k - (\omega_k - 1)\left(\sum_{j=1}^{k-1} p_j + \frac{P\sigma_\epsilon^2 + \sigma^2}{|\hat{h}_k|^2}\right)\right]. \tag{11}
$$



(a) $P = 20dBm$.



(b) $P = 20dBm$.

Fig. 3. Figure (3a) plots sum-rate vs. minimum required data rate of users and compares a NOMA system with OMA, and (3b) shows sum-rate vs. different values of variance of channel estimation.

$$h\left(P\right) = \left(P - \sum_{j=1}^{M} p_j\right). \tag{12}$$

Let us assume $M = 3$, which is a 3-user down link NOMA system. By using KKT analysis we have the following equations:

$$\frac{\partial R_{sum}}{\partial p_2} + \mu_2 - (\omega_3 - 1)\mu_3 - \lambda = 0. \tag{13a}$$

$$\frac{\partial R_{sum}}{\partial p_3} + \mu_3 - \lambda = 0. \tag{13b}$$

By substituting λ from (13b) into (13a) and with some algebraic operation we have:

$$\frac{\partial R_{sum}}{\partial p_2} - \frac{\partial R_{sum}}{\partial p_3} = \omega_3\mu_3 - \mu_2. \tag{14}$$

After some mathematic simplification we have:

$$\frac{\partial R_{sum}}{\partial p_2} - \frac{\partial R_{sum}}{\partial p_3} = \frac{(\sigma_\epsilon^2 P + \sigma^2)(|\hat{h}_2|^2 - |\hat{h}_3|^2))}{(A_2)(A_3)}. \tag{15a}$$

where

$$A_2 = |\hat{h}_2|^2(p_1 + p_2) + \sigma_\epsilon^2 P + \sigma^2. \tag{15b}$$

$$A_3 = |\hat{h}_3|^2(p_1 + p_2) + \sigma_\epsilon^2 P + \sigma^2. \tag{15c}$$

Owing the fact that $|\hat{h}_2|^2 > |\hat{h}_3|^2$, It is easy to show that,

$$\frac{\partial R_{sum}}{\partial p_2} > \frac{\partial R_{sum}}{\partial p_3}. \tag{16}$$

Hence, left side of the equation (14) is always positive. As a result of this, $\omega_3\mu_3 > \mu_2$, and considering the KKT conditions $\mu_3 \geq 0,\ \mu_2 \geq 0$, it is clear that μ_2 and μ_3 and also λ are positive. So the corrsponding constraints g_2, g_3 and h are active. Mainly

$$p_k - (\omega_k - 1)\left(\sum_{j=1}^{k-1} p_j + \frac{P\sigma_\epsilon^2 + \sigma^2}{|\hat{h}_k|^2}\right) = 0,\ for\ k = 2, 3. \tag{17a}$$

$$P = p_1 + p_2 + p_3. \tag{17b}$$

Now, by solving the equations (17), the power allocations can be obtained as follow:

$$\begin{aligned} p_2 &= \frac{(\omega_2 - 1)P}{\omega_2\omega_3} + \frac{(\omega_2 - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\omega_2|\hat{h}_2|^2} \\ &- \frac{(\omega_2 - 1)(\omega_3 - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\omega_2\omega_3|\hat{h}_3|^2}. \\ p_3 &= \frac{(\omega_3 - 1)P}{\omega_3} + \frac{(\omega_3 - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\omega_3|\hat{h}_3|^2}. \\ p_1 &= P - (p_2 + p_3). \end{aligned}$$

We can generalize the same approach to the other cases, $M > 3$, easily. After mathematical derivations, the optimal solution for power allocation, p_ks, is obtained as follows:

$$\begin{aligned} p_k &= \frac{(\omega_k - 1)P}{\prod_{j=k}^{M}\omega_j} + \frac{(\omega_k - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\omega_k|\hat{h}_k|^2} \\ &- \sum_{j=k+1}^{M} \frac{(\omega_k - 1)(\omega_j - 1)(\sigma_\epsilon^2 P + \sigma^2)}{\left(\prod_{l=k}^{j}\omega_l\right)|\hat{h}_j|^2}. \end{aligned}$$

Finally, $p_1 = P - \sum_{k=2}^{M} p_k$.

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