Genetic Algorithms problems and solutions Final

Machine Learning (Gandhi Institute of Technology and Management (Deemed to be University))

Problems and Solutions

Q1. Suppose a Genetic Algorithm uses chromosomes of the form x=abcdefgh with a fixed length of eight genes. Each gene can be any digit between 0 and 9 . Let the fitness of individual x be calculated as:

f(x) =(a+b)-(c+d)+(e+f)- (g+h)

And let the initial population consist of four individuals x1, ..., x4 with the following chromosomes :

X1 = 6 5 4 1 3 5 3 2
X2 = 8 7 1 2 6 6 0 1
X3 = 2 3 9 2 1 2 8 5
X4= 4 1 8 5 2 0 9 4

Evaluate the fitness of each individual, showing all your workings, and arrange them in order with the fittest first and the least fit last.

For above problem perform the following crossover operations:

i) Cross the fittest two individuals using one–point crossover at the middle point.

ii) Cross the second and third fittest individuals using a two–point crossover (point's b and f).

Fitness value for Individuals

F(x1) = (6+5)-(4+1)+(3+5)-(3+2)
= 11-5+8-5
=9

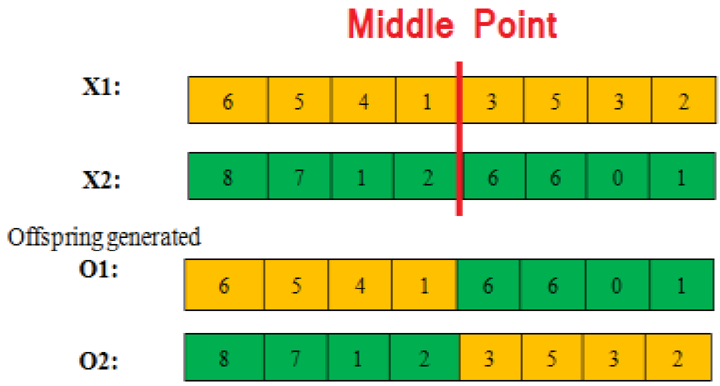
F(x2) = (8+7)-(1+2)+(6+6)-(0+1)
= 15-3+12-1
=23

F(x3) = (2+3)-(9+2)+(1+2)-(8+5)
= 5-11+3-13
=-16

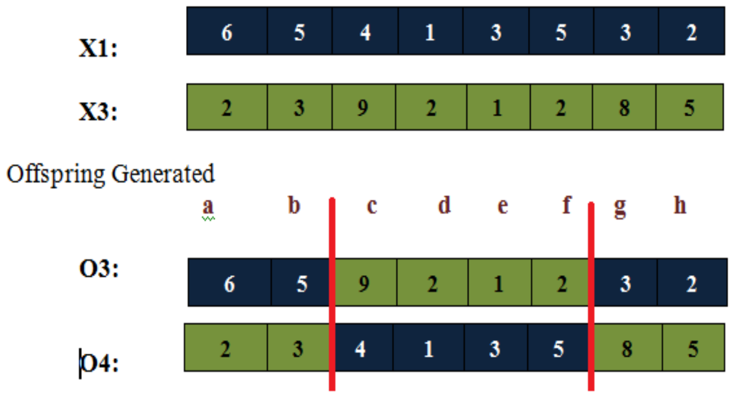
F(x4) = (4+1)-(8+5)+(2+0)-(9+4)
= 5-13+2-13
=-19

Order with the fittest first and the least fit last.
X2, X1, X3, X4

Cross the fittest two individuals using one–point crossover at the middle point.
Crossover will be performed between X2 and X1



Cross the second and third fittest individuals using a two–point crossover (points b and f).
Second and third fittest individuals are: X1 and X3



Q2. Suppose the 5 chromosomes at a given generation have fitness values listed below.

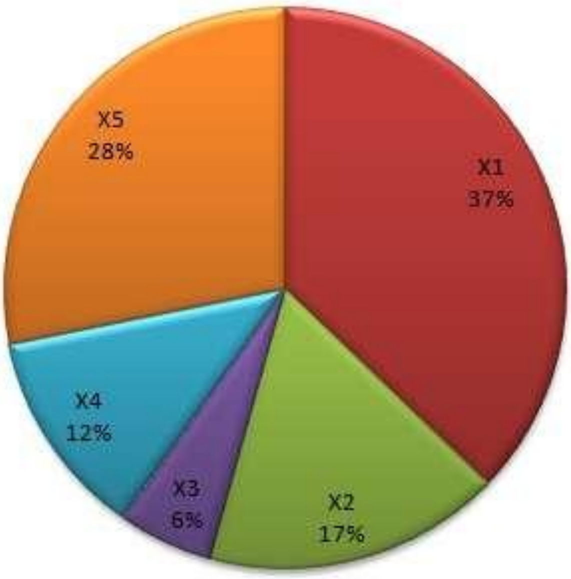
$f(x_1) = 55$
$f(x_2) = 24$
$f(x_3) = 8$
$f(x_4) = 19$
$f(x_5) = 42$

Construct the "roulette wheel" for selection of parents for crossover.

Individual	**f(X)**	$pi = \frac{fi}{\sum fi}$
X1	55	0.37
X2	24	0.17
X3	8	0.054
X4	19	0.12
X5	42	0.28
Sum of f_i	148	

$$pi = \frac{\text{fi}}{\sum \text{fi}}$$

Roulette- wheel marked for five individual according to fitness



Q-3 Giver a scenario suppose that the size of the chromosome population N is 6, and the fitness function is $15x-x^2$, and the binary code for the strings is 1100, 0100, 0001, 1110,0111,1001 here x in the fitness function is the decimal value of the given string. Perform the following:

a) Find the fitness values of the strings and draw the Roulette wheel.

b) After performing (a) do the first iteration in which crossover is done between 6th string and 2nd string at 3rd position. Onwards

c) In second iteration perform mutation in 1st and 5th string at 2nd and 3rd position.

d) Now draw the roulette wheel and tell the improvement fitness percentage.

Given that Population size N= 6 and

Sl. No.	**Individual**	**Decimal values of individual**
x1	1100	12
x2	0100	4
x3	0001	1
x4	1110	14
x5	0111	7
x6	1001	9

Step 1: computation of fitness value

$F(x) = 15x-x^2$

F(x1) = 15*12 - 12*12
=180 - 144
=36

F(x2) = 15*4 - 4*4
= 60 – 16
= 44

F(x3) = 15*1 – 1 *1
= 15 -1
= 14

F(x4) = 15* 14 – 14 * 14
= 14 (15 -14)
= 14

F(x5) = 15*7 – 7*7
= 105 – 49
= 56

F(x6) = 15*9 – 9*9
= 135 – 81
= 54

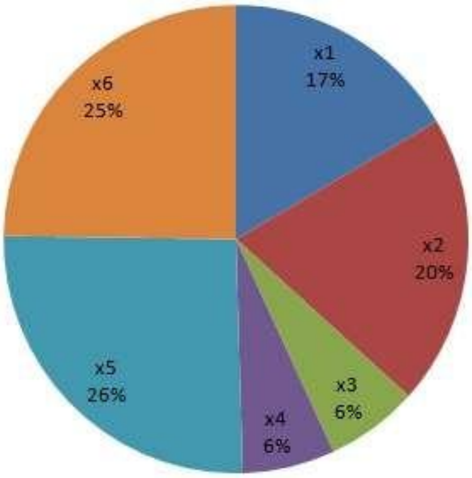
Sl. No.	**Individual**	**Decimal values of individual**	**Fitness Value**
x1	1100	12	36
x2	0100	4	44
x3	0001	1	14
x4	1110	14	14
x5	0111	7	56
x6	1001	9	54

Step 2: draw the Roulette Wheel

Individuals are arranged based on their fitness value, first is the maximum fittest and last is least fit. To draw the roulette wheel first we have to compute the probability from

$$pi = \frac{\text{fi}}{\sum \text{fi}}$$

Sl. No.	Individual	Decimal values of individual	Fitness Value	Pi
x1	1100	12	36	0.165
x2	0100	4	44	0.201
x3	0001	1	14	0.064
x4	1110	14	14	0.064
x5	0111	7	56	0.256
x6	1001	9	54	0.247
			$\sum \text{fi} = 218$	

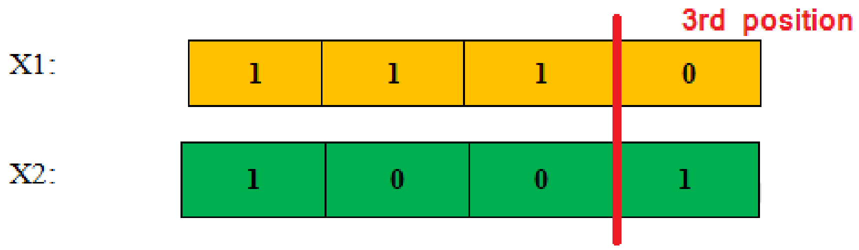


Step 3: After performing (a) do the first iteration in which crossover is done between 6^{th} string and 2^{nd} string at 3^{rd} position onwards

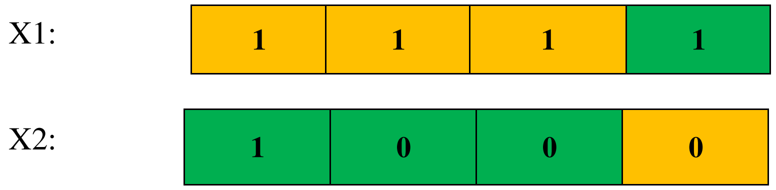
Strings according to their fitness values

No.	Sl. No.	Individual	Decimal values of individual	Fitness Value	Pi
1	x5	0111	7	56	0.256
2	x6	1001	9	54	0.247
3	x2	0100	4	44	0.201
4	x1	1100	12	36	0.165
5	x3	0001	1	14	0.064
6	x4	1110	14	14	0.064
				$\sum \text{fi} = 218$	

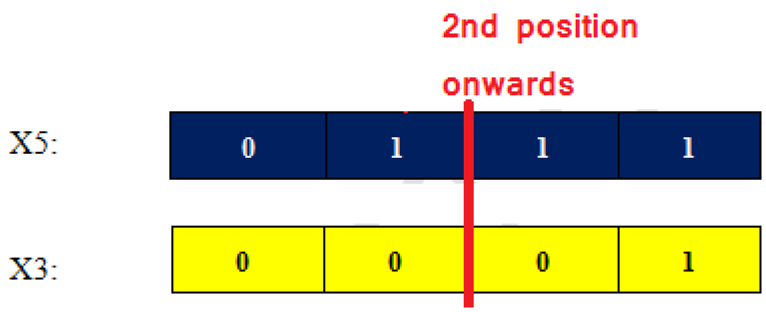
Iteration -1: Crossover between 6th string and 2nd string at 3rd position onwards
In the given problem, single point crossover will be performed between x4 and x6 i.e. 1110 and 0100

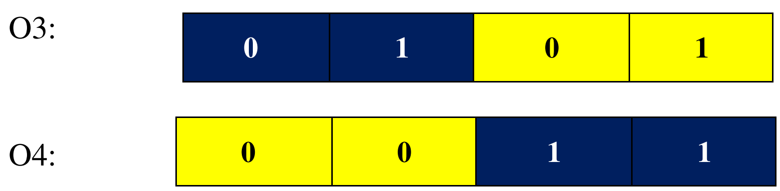


After performing single point crossover at 3rd position offspring's will be

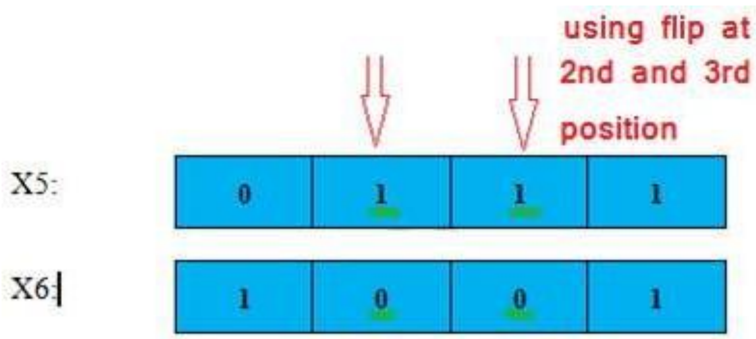


Crossover between 1st string and 5th string at 2nd position onwards
In the given problem, single point crossover will be performed between x5 and x3 i.e. 0111 and 0001





Iteration 2: Mutation in 1st and 2nd string at 2nd and 3rd position.
First string x5, i.e. 0111
Fifth string x6,i.e. 1001



After Mutation

X5’:	**0**	**0**	**0**	**1**
X6’:	**1**	**1**	**1**	**1**

After first and second iteration, we have following individuals

No.	**Sl. No.**	**Individual**	**Decimal values of individual**
1	O1	1111	15
2	O2	1000	8
3	O3	0101	5
4	O4	0011	3
5	X5’	0001	1
6	X6’	1111	15

F(O1) = 15*15 – 15*15
=225-225
=0
F(O2) = 15*8 - 8*8
= 120 – 64
= 56
F(O3) = 15*5 – 5 *5
= 75 -25
= 50
F(O4) = 15*3 – 3*3
= 45 – 9
= 36
F(X5’) = 15*1 – 1 *1
= 15 -1
= 14
F(x6’) = 15*15 – 15*15
= 225 – 225
= 0

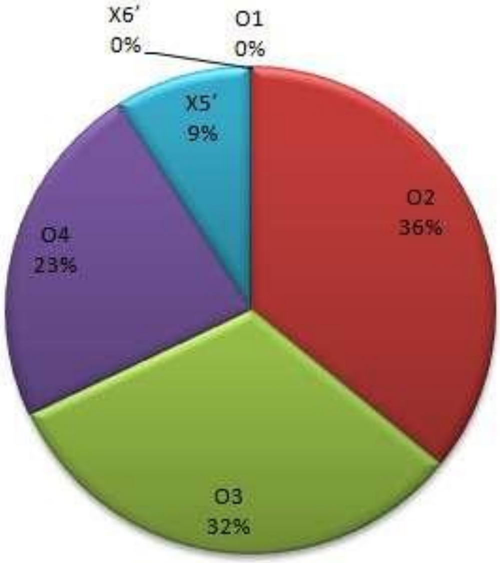
Sl. No.	**Individual**	**Decimal values of individual**	**Fitness Value**
O1	1111	15	0
O2	1000	8	56
O3	0101	5	50
O4	0011	3	36
X5’	0001	1	14
X6’	1111	15	0
			$\sum fi = 156$

Step 2: draw the Roulette Wheel

To draw the roulette wheel first we have to compute the probability from

$$pi = \frac{\text{fi}}{\sum \text{fi}}$$

Sl. No.	Individual	Decimal values of individual	Fitness Value	Pi
O1	1111	15	0	0.0
O2	1000	8	56	0.358
O3	0101	5	50	0.320
O4	0011	3	36	0.230
X5’	0001	1	14	0.09
X6’	1111	15	0	0
			$\sum \text{fi} = 156$	



In first generation the maximum fitness is 26% while in second generation it is 36%
There is a improvement of 10%

Q4: Consider the scenario suppose the size of the chromosome is 8. The fitness function is the sum of positions of 1 in the chromosome. Position of most significant bit is considered as 1 and least significant bit position is 8.
For Example in a chromosome 11000110 the fitness function will be 1+2+6+7= 16. Binary coded strings are 10100000, 10101100, 10101110, 10101010, 11101010, 11001010, 00111010 and 10111010.

a) Find the fitness value and draw the roulette wheel.
b) Generate the mating pool of size five if random numbers to select the individual from the population are 0.42, 0.61, 0.81, 0.002 and 0.95 to

Sl. No.	**Individual**
x1	10100000
x2	10101100
x3	10101110
x4	10101010
x5	11101010
x6	11001010
x7	00111010
x8	10111010

For Individual 1

1	2	3	4	5	6	7	8
1	0	1	0	0	0	0	0

F(x1) = 1+3 = 4

For Individual 2

1	2	3	4	5	6	7	8
1	0	1	0	1	1	0	0

F(x2) = 1+3+5+6 = 15

For Individual 3

1	2	3	4	5	6	7	8
1	0	1	0	1	1	1	0

F(x3) = 1+3+5+6+7 = 22

For Individual 4

1	2	3	4	5	6	7	8
1	0	1	0	1	0	1	0

F(x4) = 1+3+5+7 = 16

For Individual 5

1	2	3	4	5	6	7	8
1	1	1	0	1	0	1	0

F(x5) = 1+2+3+5+7 = 18

For Individual 6

1	2	3	4	5	6	7	8
1	1	0	0	1	0	1	0

F(x6) = 1+2+5+7 = 15

For Individual 7

1	2	3	4	5	6	7	8
0	0	1	1	1	0	1	0

F(x7) = 3+4+5+7 = 19

For Individual 8

1	2	3	4	5	6	7	8
1	0	1	1	1	0	1	0

F(x8) = 1+3+4+5+7 = 20

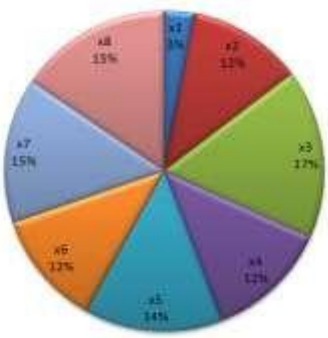
Sl. No.	**Individual**	**Fitness value**
x1	10100000	4
x2	10101100	15
x3	10101110	22
x4	10101010	16
x5	11101010	18
x6	11001010	15
x7	00111010	19
x8	10111010	20
		$\sum \mathrm{fi} = 129$

(a) Draw the roulette wheel

To draw the roulette wheel first we have to compute the probability from

$$pi = \frac{\mathrm{fi}}{\sum \mathrm{fi}}$$

Sl. No.	**Individual**	**Fitness value**	**pi**	**Pi (cumulative pi)**
x1	10100000	4	0.031	0.031
x2	10101100	15	0.116	0.147
x3	10101110	22	0.17	0.317
x4	10101010	16	0.124	0.441
x5	11101010	18	0.139	0.58
x6	11001010	15	0.116	0.696
x7	00111010	19	0.147	0.843
x8	10111010	20	0.155	0.998
		$\sum \mathrm{fi} = 129$		



Roulette Wheel marked for eight individual according to fitness

(b) The given random numbers are: 0.42, 0.61, 0.81, 0.002 and 0.95

Sl. No.	Individual	Fitness value	pi	Pi (cumulative pi)	Random number(r)
x1	10100000	4	0.031	0.031	0.42
x2	10101100	15	0.116	0.147	0.61
x3	10101110	22	0.17	0.317	0.81
x4	10101010	16	0.124	0.441	0.002
x5	11101010	18	0.139	0.58	0.95
x6	11001010	15	0.116	0.696	
x7	00111010	19	0.147	0.843	
x8	10111010	20	0.155	0.998	
		$\sum fi = 129$			

According to the roulette wheel method, individual j will be selected if

$$P_{j-1} < r < P_j$$

Sl. No.	Individual	Fitness value	pi	Pi (cumulative pi)	Random number(r)	Selected Individual
x1	10100000	4	0.031	0.031	0.42	x4
x2	10101100	15	0.116	0.147	0.61	x6
x3	10101110	22	0.17	0.317	0.81	x7
x4	10101010	16	0.124	0.441	0.002	x1
x5	11101010	18	0.139	0.58	0.95	x8
x6	11001010	15	0.116	0.696		
x7	00111010	19	0.147	0.843		
x8	10111010	20	0.155	0.998		
		$\sum fi = 129$				

Mating pool will be 10101010, 11001010, 00111010, 10100000 and 10111010.

Q5: Suppose the problem is to evolve a binary string of length n which is symmetric. If the string positions are numbered from 0, then a symmetric string will have a 1 in position i if and only if there is a 1 in position (n − 1) − i. For example, 001100 is symmetric since it has a 1 at index 2 and a 1 at index (6 − 1) − 2 = 3. Similarly, 110011 is symmetric, and 011011 is not. The initial population is a randomly generated set of binary strings of length n, where n is an even number.

(i) Give a suitable fitness function for this problem.

(ii) Will the offspring of parents with a high fitness value generally also have a high fitness value, given your fitness function? Explain your answer.

Answer. A possible fitness function is n − x, where x is the number of unmatched 1s in both halves of the string.

(ii) For the function above, no (the probability that the result of crossover of two almost symmetric strings is almost symmetric is low).

For example: x1: 11111111 f(x1)= 8-0 = 8 x2: 11100111 f(x2)= 8-0 = 8

After performing single point crossover at fourth position

O1: 11110111 f(O1) = 8- 1 = 7

O2: 11101111 f(O2) = 8-1 = 7

While if we perform single point crossover at 3rd position then

O1: 11100111 f(O1) = 8- 0 = 8

O2: 11111111 f(O2) = 8-0 = 8

So, it is not necessary that offspring of parents with a high fitness value also have a high fitness value.

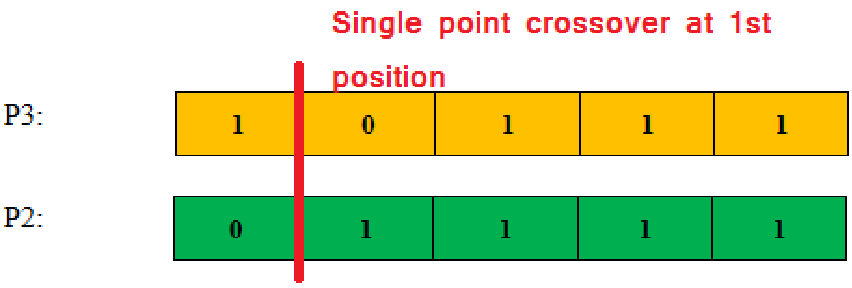
Q6: Assume we have the following function

$F(x)= x^3 – 60*x^2 +900*x+100$

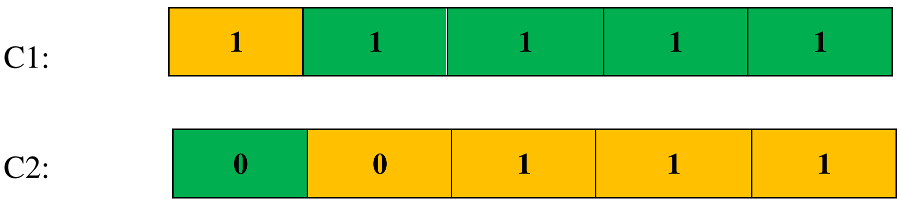
Where x is considered to 0..31. We wish to maximize f(x) (the optimal is x=10), using a binary representation we can represent x using five binary digits. Given the four chromosomes as 11100, 01111, 10111 and 00100, give the value of x and f(x)

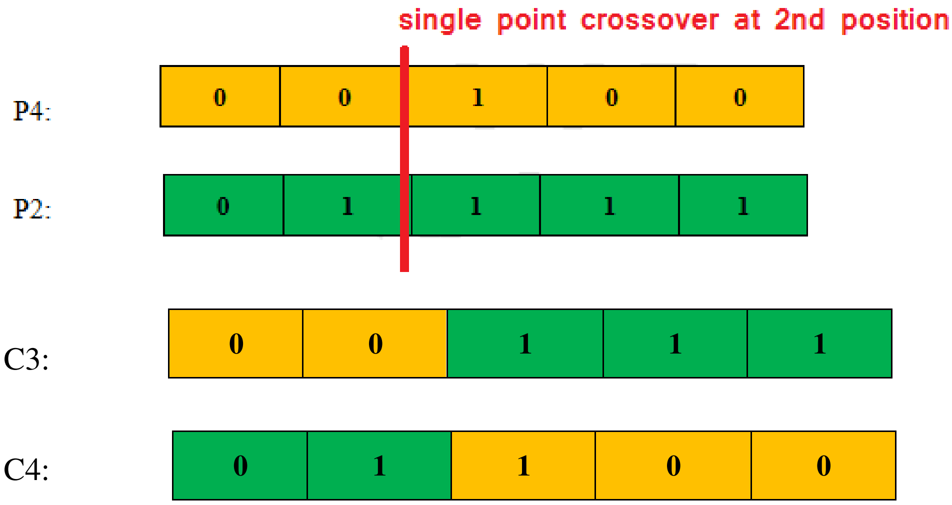
a) If p3 and p2 are chosen as parents and we apply one point crossover show the resulting children C1 and C2. Use a crossover of 1(where 0 is to the very left of the chromosome),
b) Do the same using P4 and P2 with a crossover point of 2 and create C3 and C4.
c) Calculate the value of x and f(x) for C1.. C4.
d) Assume the initial population was x= 17, 21, 4 and 28. Using one point crossover, what is the probability to finding the optimal solution? Explain your reasons.

Chromosome	String	Value (x)	Fitness value f(x)
P1	11100	28	212
P2	01111	15	3475
P3	10111	23	1227
P4	00100	4	2804



To find the 0 on the very left of the chromosome, single point crossover is performed at position 1. After crossover





Chromosome	**String**	**Value (x)**	**Fitness value f(x)**
C1	11111	31	131
C2	00111	7	3803
C3	00111	7	3803
C4	01100	12	3998

As given the optimal solution is at x=10 and four following strings

String	**Value (x)**
10001	17
10101	21
00100	4
11100	28

Binary for x=10 is 01010
To achieve the x=10 we required 1 in second and 4th position, while in the given strings no string have 1 at 4th position.
Thus, we can't find the optimal solution by applying any of the crossovers.
So, mutation is only the solution to finding the optimal solution
On performing mutation on 00100 at position 2nd, 3rd and 4th position we can get 01010.

Q7: Given the following parents, P1 and P2 and the template T

P1	A	B	C	D	E	F	G	H	I	J
J	E	F	J	H	B	C	I	A	D	G
T	1	0	1	1	0	0	0	1	0	1

Perform the following crossover
a) Uniform crossover
b) order-based crossover

Answer:

Since in the uniform crossover
When there is 1 in the template Gene, bit is copied from the first parent and
When there is 0 in the template Gene, bit is copied from the second parent

P1	A	B	C	D	E	F	G	H	I	J
P2	E	F	J	H	B	C	I	A	D	G
T	1	0	1	1	0	0	0	1	0	1
C1	A	F	C	D	B	C	I	H	D	J

P1	E	F	J	H	B	C	I	A	D	G
P2	A	B	C	D	E	F	G	H	I	J
T	1	0	1	1	0	0	0	1	0	1
C2	E	B	J	H	E	F	G	A	I	G

Order based crossover

In the order based crossover, Firstly we fill all the positions for bit 1 and bit is copied from the first parent.

For the bit value 0, Bit is copied from the second parent from the starting position of second parent and with the condition that bit value is not already present in the offspring

P1	A	B	C	D	E	F	G	H	I	J
P2	E	F	J	H	B	C	I	A	D	G
T	1	0	1	1	0	0	0	1	0	1
C1	A	E	C	D	F	B	I	H	G	J

P1	E	F	J	H	B	C	I	A	D	G
P2	A	B	C	D	E	F	G	H	I	J
T	1	0	1	1	0	0	0	1	0	1
C2	E	B	J	H	C	D	F	A	I	G