Confusion Matrix & Performance Measurement Metrics

Confusion Matrix

- A confusion matrix for a two classes (+, -) is shown below.

	C1	C2
C1	True positive	False negative
C2	False positive	True negative

	+	-
+	++	+-
-	-+	--

- There are four quadrants in the confusion matrix, which are symbolized as below.
  - True Positive (TP: f_{++}) : The number of instances that were positive (+) and correctly classified as positive (+).
  - False Negative (FN: f_{+-}): The number of instances that were positive (+) and incorrectly classified as negative (-).
  - False Positive (FP: f_{-+}): The number of instances that were negative (-) and incorrectly classified as (+).
  - True Negative (TN: f--): The number of instances that were negative (-) and correctly classified as (-).

Confusion Matrix

Note:

- N_p = TP (f_{++}) + FN (f_{+-})

  = is the total number of positive instances.

- N_n = FP(f_{-+}) + Tn(f_{--})

  = is the total number of negative instances.

- $N = N_p + N_n$

  = is the total number of instances.

- (TP + TN) denotes the number of correct classification

- (FP + FN) denotes the number of errors in classification.

- For a perfect classifier, FP = FN = 0

Confusion Matrix Example

- For example,

Class	+	-
+	52 (TP)	18 (FN)
-	21 (FP)	123 (TN)

Calculate the performance evaluation metrics

Accuracy

- It is defined as the fraction of the number of examples that are correctly classified by the classifier to the total number of instances.

$$\text{Accuracy} = \frac{TP+TN}{TP+FP+FN+TN}$$

Performance Evaluation Metrics

- We now define a number of metrics for the measurement of a classifier.
  - In our discussion, we shall make the assumptions that there are only two classes: + (positive) and – (negative)
- **True Positive Rate (TPR):** It is defined as the fraction of the positive examples predicted correctly by the classifier.

$$TPR = \frac{TP}{P} = \frac{TP}{TP+FN} = \frac{f_{++}}{f_{++}+f_{+-}}$$

  - This metrics is also known as ***Recall***, ***Sensitivity*** or ***Hit rate***.
- **False Positive Rate (FPR):** It is defined as the fraction of negative examples classified as positive class by the classifier.

$$FPR = \frac{FP}{N} = \frac{FP}{FP+TN} = \frac{f_{-+}}{f_{-+}+f_{--}}$$

Performance Evaluation Metrics

- **False Negative Rate (FNR):** It is defined as the fraction of positive examples classified as a negative class by the classifier.

$$FNR = \frac{FN}{P} = \frac{FN}{TP + FN} = \frac{f_{+-}}{f_{++} + f_{+-}}$$

- **True Negative Rate (TNR):** It is defined as the fraction of negative examples classified correctly by the classifier

$$TNR = \frac{TN}{N} = \frac{TN}{TN + FP} = \frac{f_{--}}{f_{--} + f_{-+}}$$

  - This metric is also known as ***Specificity***.

Performance Evaluation Metrics

- Both, Precision and Recall are defined by

$$Precision = \frac{TP}{TP + FP}$$

$$Recall = \frac{TP}{TP + FN}$$

Performance Evaluation Metrics

- **F_1 Score (F_1):** Recall (r) and Precision (p) are two widely used metrics employed in analysis..
  - It is defined in terms of (r or Recall) and (p or Precision) as follows.

$$F_1 Score = \frac{2 * Recall\,.\,Precision}{Recall + Precision} = \frac{2TP}{2TP + FP + FN}$$

Note

- F_1 represents the harmonic mean between recall and precision
- High value of F_1 score ensures that both Precision and Recall are reasonably high.

Analysis with Performance Measurement Metrics

- Based on the various performance metrics, we can characterize a classifier.
- We do it in terms of TPR, FPR, Precision and Recall and Accuracy
- **Case 1: Perfect Classifier**

When every instance is correctly classified, it is called the perfect classifier. In this case, $TP = P$, $TN = N$ and CM is

$$TPR = \text{TP/(TP+FN)} = \frac{P}{P} = 1$$

$$FPR = \frac{0}{N} = 0$$

$$Precision = \frac{P}{P} = 1$$

$$F_1\ Score = \frac{2\times 1}{1+1} = 1$$

$$Accuracy = \frac{P+N}{P+N} = 1$$

		Predicted Class	
		+	-
Actual class	+	P	0
	-	0	N

Analysis with Performance Measurement Metrics

Case 2: Worst Classifier

When every instance is wrongly classified, it is called the worst classifier. In this case, $TP = 0$, $TN = 0$ and the CM is

$$TPR = \frac{0}{P} = 0$$

$$FPR = \frac{N}{N} = 1$$

$$Precision = \frac{0}{N} = 0$$

$$F_1\ Score = \text{Not applicable}$$

$$\text{as } Recall + Precision = 0$$

$$\text{Accuracy} = \frac{0}{P+N} = 0$$

		Predicted Class	
		+	-
Actual class	+	0	P
	-	N	0

Analysis with Performance Measurement Metrics

Case 3: Ultra-Liberal Classifier

The classifier always predicts the + class correctly. Here, the False Negative (FN) and True Negative (TN) are zero. The CM is

$$TPR = \frac{P}{P} = 1$$

$$FPR = \frac{N}{N} = 1$$

$$Precision = \frac{P}{P+N}$$

$$F_1\ Score = \frac{2P}{2P+N}$$

$$\text{Accuracy} = \frac{P}{P+N} = 0$$

		Predicted Class	
		+	-
Actual class	+	P	0
	-	N	0

Analysis with Performance Measurement Metrics

- **Case 4: Ultra-Conservative Classifier**

This classifier always predicts the - class correctly. Here, the False Negative (FN) and True Negative (TN) are zero. The CM is

$$TPR = \frac{0}{P} = 0$$

$$FPR = \frac{0}{N} = 0$$

$$Precision = \text{Not applicable}$$

$$(\text{as } TP + FP = 0)$$

$$F_1\ Score = \text{Not applicable}$$

$$\text{Accuracy} = \frac{N}{P+N} = 0$$

		Predicted Class	
		+	-
Actual class	+	0	p
	-	0	N