Example 8.13 The supplier of electrical motors to Washtex Company—a washing machine manufacturing company—claims that the mean time between failures of the motors is at most 60 days. The quality manager of the Washtex Company wants to test the claim of the vendor. Hence, he has taken a sample of 16 motors whose mean time between failures and its variance are found to be 63 days and 25 days, respectively. Verify the claim of the vendor at a significance level of 0.01.

Solution. We have

Sample size, n	$= 16$
Population mean, μ	$= 60$ days
Sample mean, $\bar{X}$	$= 63$ days
Variance of the sample, S^2	$= 25$ days
Standard deviation of the sample, S	$= 5$ days
Significance level, α	$= 0.01$

We also have $X \sim N(\mu, \sigma^2)$. Since the sample size n is less than 30, $\bar{X} \sim t$ distribution with $(n-1)$ degrees of freedom whose statistic is given below:

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \text{ with 15 degrees of freedom}$$

where null and alternate hypotheses are:

$$H_0: \mu \leq 60$$
$$H_1: \mu > 60$$

Then

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{63 - 60}{5/\sqrt{16}} = 2.4$$

The table t value with the degrees of freedom 15 at $\alpha = 0.01$ placed at right tail is 2.602 which is shown in Fig. 8.20 along with acceptance and rejection regions.

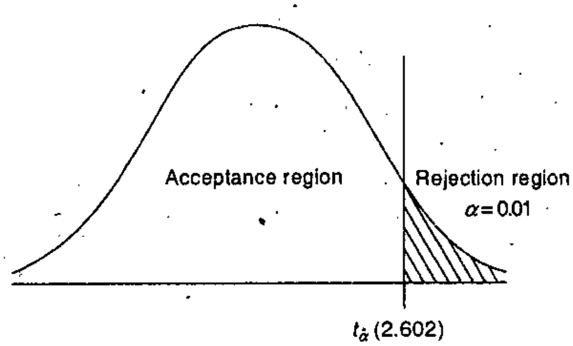

Fig. 8.20 *t* distribution curve for Example 8.13.

Since, the calculated t value (2.4) is less than table t_α value (2.602), the calculated t value falls in the acceptance region. Hence, H_0 should be accepted.

This means that the mean time between failures of the electrical motors is significantly lesser than 60 days (not greater than 60 days) which means that the claim of the vendor firm is true.

Example 8.14 The weight of costly electrodes purchased by a foundry follows normal distribution. The sales manager of the vendor firm claims that the mean weight of the electrodes is at least 100 gm. The quality manager of the foundry wants to verify this claim. So, he has taken a sample of 25 electrodes. The mean and the variance of the electrodes in the sample are found to be 97 gm and 64 gm, respectively. Verify the claim of the sales manager at a significance level of 0.05.

Solution. We have

Sample size, n	= 25
Population mean, μ	= 100 gm
Sample mean, $\bar{X}$	= 97 gm
Variance of the sample, S^2	= 64 gm
Standard deviation of the sample, S	= 8 gm
Significance level, α	= 0.05

So, $X \sim N(\mu, \sigma^2)$. Since the sample size (n) is less than 30, $\bar{X} \sim t$ distribution with $(n-1)$ degrees of freedom whose statistic is given below:

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \quad \text{with 24 degrees of freedom}$$

Here null and alternate hypotheses are:

$$H_0 : \mu \geq 100$$
$$H_1 : \mu < 100$$

Then

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{97 - 100}{8/\sqrt{25}} = -1.875$$

The table t value with the degrees of freedom 24 at $\alpha = 0.05$ placed at the left tail is -1.711 which is shown in Fig. 8.21 along with acceptance and rejection regions.

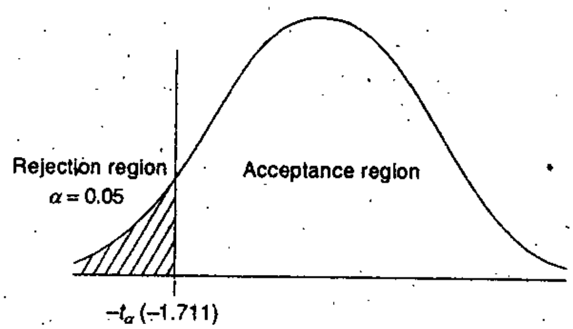

Fig. 8.21 *t* distribution curve for Example 8.14.

Since the calculated t value (-1.875) is less than $-t_\alpha$ value (-1.711), the calculated t value falls in the rejection region. Hence, H_0 should be rejected.

This means that the mean weight of the electrodes is not significantly greater than 100 gm which means that the claim of the sales manager of the vendor firm is not true.

8.2.8 Two-tailed Test Concerning Single Mean (when the Variance of the Population is Unknown and the Sample Size is Small)

This is same as the one-tailed tests concerning single mean when the variance of the population is unknown and the sample size is small. But, in this test, half of the significance level ($\alpha/2$) is placed at both tails of the t distribution.

The two-tailed test is applicable to the following combination of H_0 and H_1:

$$H_0 : \mu = k$$
$$H_1 : \mu \neq k$$

where k is a constant.

The objective of this test is to check whether population is to be accepted based on the value of the sample mean by placing half of the given significance level ($\alpha/2$) at both tails of t distribution.

Example 8.15 The weight of costly catalyst electrodes purchased by a chemical company follows normal distribution. The sales manager of the vendor firm claims that the mean weight of the electrodes is 150 gm. The quality manager of the chemical company wants to verify this claim. So, he has taken a sample of 25 electrodes. The mean and the variance of the electrodes in the sample are found to be 145 gm and 100 gm, respectively. Verify the claim of the sales manager at a significance level of 0.05.

Solution. Given

Sample size, n	= 25
Population mean, μ	= 150 gm
Sample mean, $\bar{X}$	= 145 gm
Variance of the sample, S^2	= 100 gm
Standard deviation of the sample, S	= 10 gm
Significance level, α	= 0.05

So, $X \sim N(\mu, \sigma^2)$. Since the sample size, n is less than 30, $\bar{X} \sim t$ distribution with $(n-1)$ degrees of freedom whose statistic is given below:

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} \quad \text{with 24 degrees of freedom.}$$

Now, null and alternate hypotheses are:

$$H_0 : \mu = 150$$
$$H_1 : \mu \neq 150$$

Then

$$t = \frac{\bar{X} - \mu}{S/\sqrt{n}} = \frac{145-150}{10/\sqrt{25}} = -2.5$$

The table t values with the degrees of freedom of 24 at $\alpha/2 = 0.025$ placed at left and right tails are -2.064 and $+2.064$, respectively, which are shown in Fig. 8.22 along with acceptance and rejection regions.

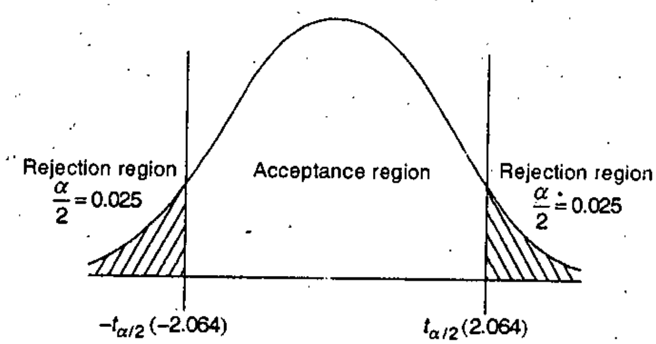

Fig. 8.22 t distribution for Example 8.15.

Since the calculated t value (-2.5) is less than $-t_{\alpha/2}$ (-2.064), the calculated t value falls in the rejection region. Hence, H_0 should be rejected.

This means that the mean weight of the catalyst electrodes is significantly different from 150 gm which means that the claim of the sales manager of the vendor firm is not true.

8.2.9 One-tailed Tests Concerning Difference between Two Means (when the Variances of the Populations are Known)

Consider a situation in which there are two random variables, X_1 and X_2 which follow two normal distributions representing two independent populations with means μ_1 and μ_2, respectively. Let the variances of these populations be σ_1^2 and σ_2^2, respectively. The objective of the one-tailed test for this situation may be to test the significance of the relative difference between the population means, as presented below:

1. *Test 1:* $H_0: \mu_1 \le \mu_2$ and $H_1: \mu_1 > \mu_2$

   or

   $H_0: \mu_1 - \mu_2 \le 0$ and $H_1: \mu_1 - \mu_2 > 0$
2. *Test 2:* $H_0: \mu_1 \ge \mu_2$ and $H_1: \mu_1 < \mu_2$

   or

   $H_0: \mu_1 - \mu_2 \ge 0$ and $H_1: \mu_1 - \mu_2 < 0$

If the distributions of two independent random variables, X_1 and X_2, have means μ_1 and μ_2, variances σ_1^2 and σ_2^2, respectively, then the distribution of their difference will have