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Q.1) Cluster the following eight points (with (x,y) representing locations) into three clusters:
A1(2,10), A2(2,5), A3(8,4), A4(5,8), A5(7,5), A6(6,4), A7(1,2), A8(4,9).

Initial cluster centers are: A1(2,10), A4(5,8) and A7(1,2). ~~A4(5,8) and A7(1,2)~~.

The distance function between two points $a=(x_1,y_1)$ and $b=(x_2,y_2)$ is defined as

$$P(a,b) = |x_2 - x_1| + |y_2 - y_1|$$ Manhattan dist.

Use K-means Algorithm to find the three cluster centers after the second iteration.

Soln Let C_1, C_2, C_3 be cluster where $C_1 = A_1$, $C_2 = A_4$ & $C_3 = A_7$

Points	C_1 dist. (2,10)	C_2 dist. (5,8)	C_3 dist. (1,2)	Cluster Assignment
A1 (2,10)	$\|10-10\| + \|2-2\| = 0$	$\|5-2\| + \|8-10\| = 5$	$\|1-2\| + \|2-10\| = 9$	C_1
A2 (2,5)	$\|2-2\| + \|10-5\| = 5$	$\|5-2\| + \|8-5\| = 6$	$\|1-2\| + \|2-5\| = 4$	C_3
A3 (8,4)	$\|2-8\| + \|10-4\| = 12$	$\|5-8\| + \|8-4\| = 7$	$\|1-8\| + \|2-4\| = 9$	C_2
A4 (5,8)	$\|2-5\| + \|10-8\| = 5$	$\|5-5\| + \|8-8\| = 0$	$\|1-5\| + \|2-8\| = 10$	C_2
A5 (7,5)	$\|2-7\| + \|10-5\| = 10$	$\|5-7\| + \|8-5\| = 5$	$\|1-7\| + \|2-5\| = 9$	C_2
A6 (6,4)	$\|2-6\| + \|10-4\| = 10$	$\|5-6\| + \|8-4\| = 5$	$\|1-6\| + \|2-4\| = 7$	C_2
A7 (1,2)	$\|2-1\| + \|10-2\| = 9$	$\|5-1\| + \|8-2\| = 10$	$\|2-1\| + \|2-2\| = 0$	C_3
A8 (4,9)	$\|2-4\| + \|10-9\| = 3$	$\|5-4\| + \|8-9\| = 2$	$\|1-4\| + \|2-9\| = 10$	C_2

After 1st the iterations, Three clusters are

C_1	C_2	C_3
(2,10)	(8,4) (5,8) (7,5) (6,4) (4,9)	(2,5) (1,2)

For Iteration 2,

New mean.

- $K_1 = (2,10)$, • $K_2 = \left((8+5+7+6+4)/5 , (4+8+5+4+9)/5 \right)$

$$K_2 = (6,6)$$

- $K_3 = \left((2+1)/2 , (5+2)/2 \right)$

$K_3 = (1.5, 3.5)$

Use same formula to calculate dist. & assign cluster to minimum distance.

Point	$K_1 = (2,10)$	$K_2 = (6,6)$	K_3 (1.5, 3.5)	cluster
A_1 (2,10)	0	$\|6-2\|+\|6-10\|=8$	$\|1-5\|$ 7	C_1
A_2 (2,5)	5	5	2	C_3
A_3 (8,4)	12	4	7	C_2
A_4 (5,8)	5	3	8	C_2
A_5 (7,5)	10	2	7	C_2
A_6 (6,4)	10	2	5	C_2
A_7 (1,2)	9	9	2	C_3
A_8 (4,9	3	5	8	C_1

So, after 2nd rotation, three clusters are

C_1	C_2	C_3
A_1 (2,10)	A_3 (8,4)	A_2 (2,5)
A_8 (4,9)	A_4 (5,8)	A_7 (1,2)
	A_5 (7,5)	
	A_6 (6,4)	

And its cluster center will be

$$K_1 \equiv \left((2+4)/2, (10+9)/2\right) = (3, 9.5)$$

$$K_2 \equiv \left((8+5+7+6)/4, (4+8+5+4)/4\right) \equiv (6.5, 5.25)$$

$$K_3 \equiv \left((2+1)/2, (5+2)/2\right) \equiv (1.5, 3.5)$$

Hence
$$\boxed{\begin{array}{l} K_1\,(3, 9.5) \\ K_2\,(6.5, 5.25) \\ K_3\,(1.5, 3.5 \end{array}}$$

After 2nd iteration These are three cluster center.

Q.2) Suppose a genetic algorithm uses chromosomes of the form $x = abcdefgh$ with a fixed length of eight genes. Each gene can be any digit between 0 and 9. Let the fitness of individual x be calculated as:

$$f(x) = (a+b) - (c+d) + (e+f) - (g+h)$$

and let the initial population consist of four individuals with the following chromosomes:

$$x_1 = 65413532$$
$$x_2 = 87126601$$
$$x_3 = 23921285$$
$$x_4 = 41852094$$

a) Evaluate the fitness of each individual, showing all your working and arrange them in order with the fittest first and the least fit last.

Soln

Chromosomes $x = abcdefgh$	Fitness $f(x) = (a+b) - (c+d) + (e+f) - (g+h)$
$x_1 = 65413532$	$f(x_1) = (6+5) - (4+1) + (3+5) - (3+2) = 9$
$x_2 = 87126601$	$(8+7) - (1+2) + (6+6) - (0+1) = 23$
$x_3 = 23921285$	$(2+3) - (9+2) + (1+2) - (8+5) = -16$
$x_4 = 41852094$	$(4+1) - (8+5) + (\cancel{24}\,0) - (9+4) = -19$

So, the order of fitness x_2, x_1, x_3 and x_4.

b) Perform the following crossover operations

i) Cross the ~~fitness~~ fitest two individuals using one-point crossover at the middle point.

Soln: since chromosomes are of 8 digits.

so crosspoint at middle ie 4^{th}

Let pair of individual x_1 & x_2.

~~$x_1 = 8712$~~ $x_1 = 6541 | 3532$ ⇒ $O_2 = 65416601$

~~x_2~~ $x_2 = 8712 | 6601$ $O_1 = 87123532$

crosspoint

x_2 most fitest.

before cross point remain same and after it swap.

$\therefore O_1 = 8712\ 3532$

$O_2 = 6541\ 6601$

ii) Cross the second and 3rd fittest individuals using a two point crossover. (point b & f).

cross point, cross point

$x = a\ b\ c\ d\ e\ f\ g\ h$

$x_1 = 6\ 5\ 4\ 1\ 3\ 5\ 3\ 2$ ⇒ $O_3 = 65921232$

$x_3\ 2\ 3\ 9\ 2\ 1\ 2\ 8\ 5$ $O_4 = 23413585$

swap

swap[b, f]

iii) Cross the 1st and 3rd fittest individuals using a uniform crossover.

Soln Uniform means random exchange of genes

Let's say b, d and g

$x_2 = 8\ \underline{7}\ 1\ \underline{2}\ 6\ 6\ \underline{0}\ 1$ ⇒ $O_5 = 83126681$

$x_3 = 2\ \underline{3}\ 9\ \underline{2}\ 1\ 2\ \underline{8}\ 5$ $O_6 = 27921205$

c) Suppose the new population consists of the six offspring individuals received by the crossover operation operations in the above question. Evaluate the fitness of the new population, showing all your workings. Has the overall fitness improved?

Soln

Strings / new population	$f(x) = (a+b)-(c+d)+(e+f)-(g+h)$
O_1=87123532	$(8+7)-(1+2)+(3+5)-(3+2) = 15$
O_2=65416601	$(6+5)-(4+1)-(6+6)-(0+1) = 17$
O_3=65921232	$(6+5)-(9+2)+(1+2)-(3+2) = -2$
O_4=23413585	$(2+3)-(4+1)+(3+5)-(8+5) = -5$
O_5=27126201	$(2+7)-(1+2)+(6+2)-(0+1) = 13$
O_6=83921685	$(8+3)-(9+2)+(1+6)-(8+5) = -6$

$$\Downarrow$$

$$\sum f(x) = 32$$

Initial, $\sum f(x)$ was -3, and After crossover $\sum f(x) = +32$.

So, overall fitness has improved.

d) By looking at the fitness function and considering that genes can only be digits between 0 and 9. find the chromosome representing the optimal solution (i.e with the maximal fitness). Find the value of the maximum fitness.

Soln To find max value of $f(x)$ in order to get optimal solution

since $f(x) = \underset{\text{Max}}{(a+b)} - \underset{\text{Min}=0}{(c+d)} + \underset{\text{Max}}{(e+f)} - \underset{\text{Min}=0}{(g+h)}$

$\therefore x_{optimal} = 99009900$

$f(x_{optimal}) = (9+9)-(0+0)+(9+9)-(0+0) = 36.$

e) By looking at the initial population of the algorithm can you say whether it will be able to reach the optimal solution without the mutation operator?

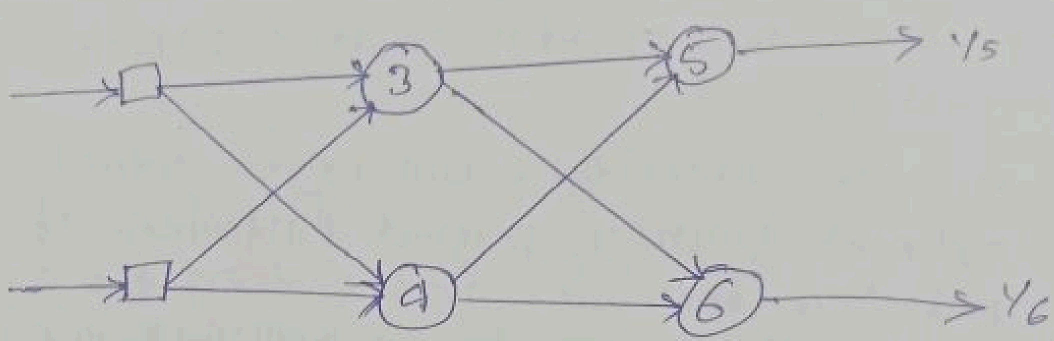
Ans: No, the algorithm will never reach the optimal solution without mutation. If the mutation does not occur, then the only way to change genes is by applying the crossover operator.

Regardless of the way crossover is performed, its only outcome is an exchange of genes of parents at certain positions in the chromosome. This means that the first gene in the chromosome of children can only be either 6, 8, 2 or 4 and none of the individuals in the initial population begins with gene 9. the crossover operator alone will never be able to produce an offspring with gene 9 in the beginning.

since optimal solution, $n = 99009900$.

Thus, without mutation, this genetic algorithm will not be able to reach the optimal solution.

Q.3) The following diagram represents a feed-forward neural network with one hidden layer.



A weight on connection between i and j is denoted by w_{ij}, such as w_{13} is the weight on the connection between nodes 1 and 3. The following tables list all the weights in the network.

$w_{13} = -2$ $w_{23} = 3$	$w_{35} = 1$ $w_{45} = -1$
$w_{14} = 4$ $w_{24} = -1$	$w_{36} = -1$ $w_{46} = 1$

Each of the nodes 3, 4, 5 and 6 uses the following activation function

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where v denotes the weighted sum of a node. Each of the input nodes (1 and 2) can only receive binary values. Calculate the output of the network for each of the input patterns.

Pattern	P_1	P_2	P_3	P_4
Node 1:	0	1	0	1
Node 2:	0	0	1	1

Soln. In order to find the output of the network it is necessary to calculate weighted sums of hidden nodes 3 and 4.

$$v_3 = w_{13} x_1 + w_{23} x_2$$
$$v_4 = w_{14} x_1 + w_{24} x_2$$

Then find the outputs from hidden nodes using activation function φ.

$$y_3 = \varphi(v_3)$$
$$y_4 = \varphi(v_4)$$

Use the outputs from hidden nodes ~~using~~ y_3 ~~activation function φ~~ & y_4 as the input values to the output layer (nodes 5 and node 6) and find weighted sums of output nodes 5 and 6.

$$v_5 = w_{35} y_3 + w_{45} y_4$$
$$v_6 = w_{36} y_3 + w_{46} y_4$$

Finally, find the outputs from nodes 5 and 6.

$$y_5 = \varphi(v_5)$$
$$y_6 = \varphi(v_6).$$

The output pattern will be (y_5, y_6). Perform these calculation for each input pattern.

- Two variable input

pattern P_1 $(0,0)$

$v_3 = w_{13} x_1 + w_{23} x_2 = -2\cdot 0 + 3\cdot 0 = 0$, $y_3 = \varphi(0) = 1$

$v_4 = 4\cdot 0 - 1\cdot 0 = 0$ $\quad y_4 = \varphi(0) = 1$

$v_5 = 1\cdot 1 - 1\cdot 1 = 0$ $\quad y_5 = \varphi(0) = 1$

$v_6 = -1\cdot 1 + 1\cdot 1 = 0$ $\quad y_6 = \varphi(0) = 1$

So, the output of network is $(y_5, y_6) = (1,1)$.

for pattern P_2. $(1,0) = (x_1, x_2)$

$$v_3 = -2 \cdot 1 + 3 \cdot 0 = -2 \qquad y_3 = \varphi(-2) = 0$$
$$v_4 = 4 \cdot 1 - 1 \cdot 0 = 4 \qquad y_4 = \varphi(4) = 1$$
$$v_5 = 1 \cdot 0 - 1 \cdot 1 = -1 \qquad y_5 = \varphi(-1) = 0$$
$$v_6 = -1 \cdot 0 + 1 \cdot 1 = 1 \qquad y_6 = \varphi(1) = 1$$

Here used activation function to find output.

The output of the network is $(0,1)$

for pattern P_3 $(0,1) \equiv (x_1, x_2)$

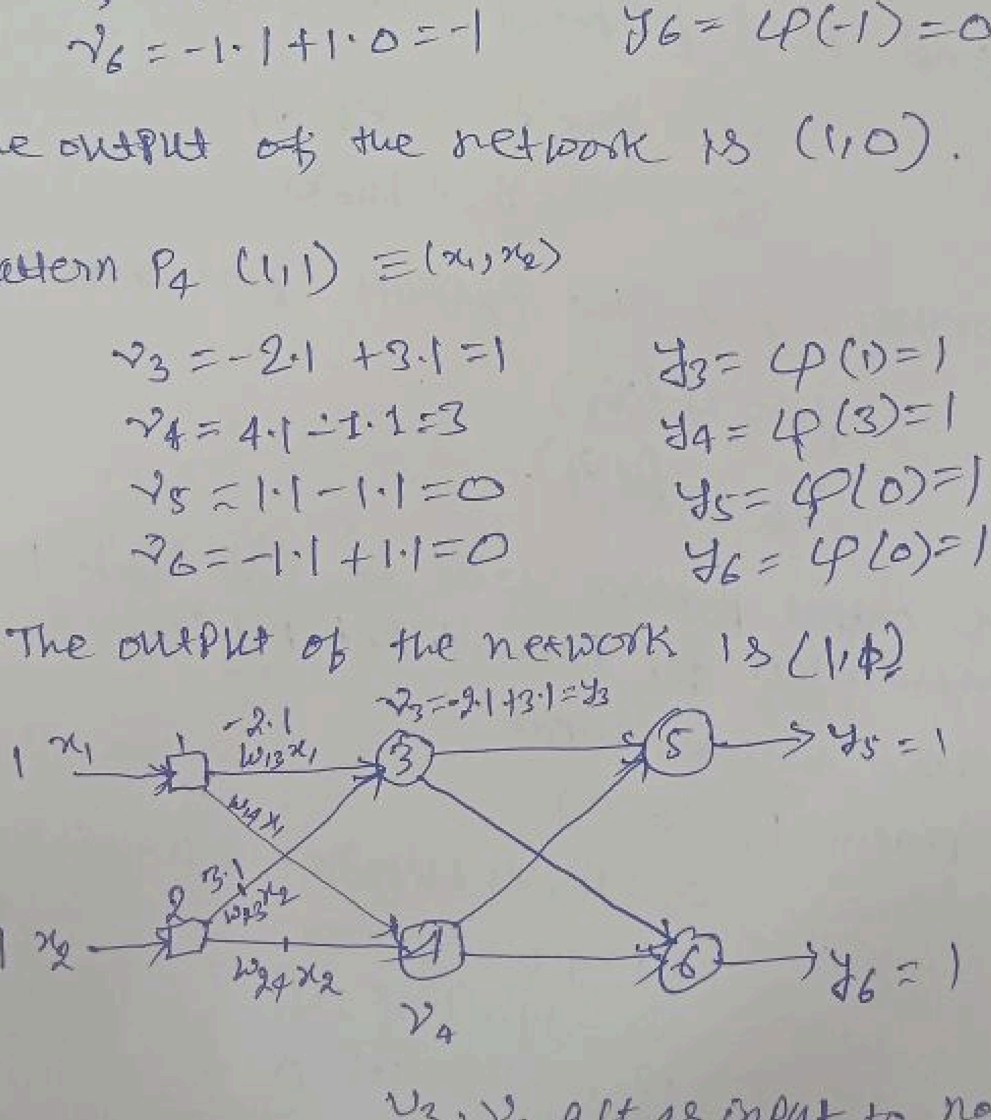
$$v_3 = -2 \cdot 0 + 3 \cdot 1 = 3 \qquad y_3 = \varphi(3) = 1$$
$$v_4 = 4 \cdot 0 - 1 \cdot 1 = -1 \qquad y_4 = \varphi(-1) = 0$$
$$v_5 = 1 \cdot 1 - 1 \cdot 0 = 1 \qquad y_5 = \varphi(1) = 1$$
$$v_6 = -1 \cdot 1 + 1 \cdot 0 = -1 \qquad y_6 = \varphi(-1) = 0$$

The output of the network is $(1,0)$.

for pattern P_4 $(1,1) \equiv (x_1, x_2)$

$$v_3 = -2 \cdot 1 + 3 \cdot 1 = 1 \qquad y_3 = \varphi(1) = 1$$
$$v_4 = 4 \cdot 1 - 1 \cdot 1 = 3 \qquad y_4 = \varphi(3) = 1$$
$$v_5 = 1 \cdot 1 - 1 \cdot 1 = 0 \qquad y_5 = \varphi(0) = 1$$
$$v_6 = -1 \cdot 1 + 1 \cdot 1 = 0 \qquad y_6 = \varphi(0) = 1$$

The output of the network is $(1,1)$



v_3, v_4 act as input to node 5 & 6.