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FX Derivatives Trader School

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For my wife and daughters: Laura, Rosie, and Emily.

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PREFACE

In 2004 I started on an FX derivatives trading desk as a graduate. I wrote down everything I learned: how markets worked, how FX derivatives contracts were risk-managed, how to quote prices, how Greek exposures evolve over time, how different pricing models work, and so on. This book is a summary of that knowledge, filtered through a decade of trading experience across the full range of FX derivatives products.

In 2011 I started sending out monthly "Trader School" e-mails to traders on the desk, covering a wide range of topics. The e-mails were particularly popular with new joiners and support functions because they gave an accessible view of derivatives trading that did not exist elsewhere. This book collects together and expands upon those e-mails.

Part I covers the basics of FX derivatives trading. This is material I wish I'd had access to when originally applying for jobs on derivatives trading desks. Part II investigates the volatility surface and the instruments that are used to define it. Part III covers vanilla FX derivatives trading and shows how the FX derivatives market can be analyzed. Part IV covers exotic FX derivatives trading, starting with the most basic products and slowly increasing the complexity up to advanced volatility and multi-asset products. This material will mostly be useful to junior traders or traders looking to build or refresh their knowledge in a particular area.

Fundamentally, the aim of the book is to explain derivatives trading from first principles in order to develop intuition about derivative risk rather than attempting to be state of the art. Within the text, experienced quant traders will find many statements that are not entirely true, but are true the vast majority of the time. Endlessly caveating each statement would make the text interminable.

Traders can only be successful if they have a good understanding of the framework in which they operate. Importantly though, for derivatives traders this is not the same as fully understanding derivative mathematics. Therefore the mathematics is kept to an accessible "advanced high school" level throughout. Some mathematical rigor is lost as a result of this, but for traders that is a price worth paying.

Also in the interests of clarity, some other important considerations are largely ignored within the analysis, most notably, credit risk (i.e., the risk of a counterparty defaulting on money owed) and interest rates (i.e., how interest rate markets work in practice). Derivative product analysis is the primary concern here and this is cleaner if those issues are ignored or simplified.

Regulations and technology are causing significant changes within the FX derivatives market structure. The most important changes are increasing electronic execution, increasing electronic market data, more visibility on transactions occurring in the market, and less clear distinctions between banks and their clients. These changes will have profound and lasting effects on the market. However, the ideas and techniques explored within the book hold true no matter how the market structure changes.

Finally, and most importantly, if you are a student or new joiner on a derivatives trading desk: *Do the practicals*. I can guarantee that if you complete the practicals, you will hit the ground running when you join a derivatives trading desk. *Do them*. Do them *all*. Do them all *in order*. Do not download the spreadsheets from the companion website unless you are completely stuck. When you're trying to learn something, taking the easy option is never the right thing to do. The practicals require the ability to set up Excel VBA (Visual Basic for Applications) functions and subroutines. If you aren't familiar with this, there is plenty of material online that covers this in detail.

The very best of luck with your studies and careers,

Giles Jewitt, London, 2015.

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This publication is intended to be educational in nature and should be used for information purposes only.

Any opinions expressed herein are given in good faith, but are subject to change without notice.

Any strategies discussed are strictly for illustrative and educational purposes only and are not to be construed as an endorsement, recommendation, or solicitation to buy or sell any financial securities.

All rates and figures used are for illustrative purposes only and do not reflect current market rates.

FX Derivatives Trader School

PART I

THE BASICS

Part I lays the foundations for understanding FX derivatives trading. Trading within a financial market, market structure, and the Black-Scholes framework are all covered from first principles. FX derivatives trading risk is then introduced with an initial focus on vanilla options since they are by far the most commonly traded contract.

Introduction to Foreign Exchange

The *foreign exchange (FX) market* is an international marketplace for trading **currencies**. In FX transactions, one currency (sometimes shortened to CCY) is *exchanged* for another. Currencies are denoted with a three-letter code and **currency pairs** are written CCY1/CCY2 where the **exchange rate** for the currency pair is the number of CCY2 it costs to buy one CCY1. Therefore, trading EUR/USD FX involves exchanging amounts of EUR and USD. If the FX rate goes higher, CCY1 is getting relatively stronger against CCY2 since it will cost more CCY2 to buy one CCY1. If the FX rate goes lower, CCY1 is getting relatively weaker against CCY2 because one CCY1 will buy fewer CCY2.

If a currency pair has both elements from the list in Exhibit 1.1, it is described as a *G10 currency pair*.

The most commonly quoted FX rate is the **spot rate**, often just called **spot**. For example, if the EUR/USD spot rate is 1.3105, EUR 1,000,000 would be exchanged for USD 1,310,500. Within a spot transaction the two cash flows actually hit the bank account (*settle*) on the **spot date**, which is usually two business days after the transaction is agreed (called T+2 settlement). However, in some currency pairs, for example, USD/CAD and USD/TRY (Turkish lira), the spot date is only one day after the transaction date (called T+1 settlement).

Another set of commonly traded FX contracts are **forwards**, sometimes called **forward outrights**. Within a forward transaction the cash flows settle on some future date other than the spot date. When rates are quoted on forwards, the **tenor** or **maturity** of the contract must also be specified. For example, if the EUR/USD 1yr (one-year) forward FX rate is 1.3245, by transacting this contract in EUR10m

EXHIBIT 1.1 **G10 Currencies**

CCY Code	Full Name	CCY Code	Full Name
AUD	Australian dollar	JPY	Japanese yen
CAD	Canadian dollar	NOK	Norwegian krone
CHF	Swiss franc	NZD	New Zealand dollar
EUR	Euro	SEK	Swedish krona
GBP	Great British pound	USD	United States dollar

(ten million euros) **notional**, each EUR will be exchanged for 1.3245 USD (i.e., EUR10m will be exchanged for USD13.245m in one year's time). In a given currency pair, the spot rate and forward rates are linked by the respective **interest rates** in each currency. By a no-arbitrage argument, delivery to the forward maturity must be equivalent to trading spot and putting the cash balances in each currency into "risk-free" investments until the maturity of the forward. This is explained in more detail in Chapter 5.

Differences between the spot rate and a forward rate are called **swap points** or **forward points**. For example, if EUR/USD spot is 1.3105 and the EUR/USD 1yr forward is 1.3245, the EUR/USD 1yr swap points are 0.0140. In the market, swap points are quoted as a number of **pips**. Pips are the smallest increment in the FX rate usually quoted for a particular currency pair. In EUR/USD, where FX rates are usually quoted to four decimal places, a pip is 0.0001. In USD/JPY, where FX rates are usually only quoted to two decimal places, a pip is 0.01. In the above example, an FX swaps trader would say that EUR/USD 1yr swap points are at 140 ("one-forty").

Pips (sometimes called "points") are also used to describe the magnitude of FX moves (e.g., "EUR/USD has jumped forty pips higher" if the EUR/USD spot rate moves from 1.3105 to 1.3145). Another term used to describe spot moves is **figure**, meaning one hundred pips (e.g., "USD/JPY has dropped a figure" if the USD/JPY spot rate moves from 101.20 to 100.20).

FX swap contracts contain two FX deals in opposite directions (one a buy, the other a sell). Most often one deal is a spot trade and the other deal is a forward trade to a specific maturity. The two trades are called the **legs** of the transaction and the notionals on the two legs of the FX swap are often equal in CCY1 terms (e.g., buy *EUR10m* EUR/USD spot against sell *EUR10m* EUR/USD 1yr forward). FX swaps are quoted in swap point terms (the difference in FX rate) between the two legs. In general, swap points change far less frequency than spot rates in a given currency pair.

A trader takes up a new FX position by buying USD10m USD/CAD spot at a rate of 0.9780. This means buying USD10m and simultaneously selling CAD9.78m. This position is described as "long ten dollar-cad," meaning USD10m has been bought and an equivalent amount of CAD has been sold. If USD10m USD/CAD had been sold at 0.9780 instead, the position is described as "short ten dollar-cad." Note that the long/short refers to the *CCY1* position. The concept of selling something you don't initially own is a strange one in the real world but it quickly becomes normal in financial markets where trading positions can flip often between long (a net bought position) and short (a net sold position).

USD/CAD spot jumps up to 0.9900 after it was bought at 0.9780: The trader is a hero! Time to sell USD/CAD spot and lock in the profit. Selling USD10m USD/CAD spot at 0.9900 results in selling USD10m against buying CAD9.9m. The initial bought USD10m and new sold USD10m cancel out, leaving no net USD position, but the initial sold CAD9.78m and new bought CAD9.9m leave CAD120k profit. This is important: FX transactions and positions are usually quoted in CCY1 terms (e.g., *USD*10m USD/CAD) while the profit and loss (P&L) from the trade is naturally generated in CCY2 terms (e.g., *CAD*120k).

A **long** position in a financial instrument *makes money* if the price of the instrument *rises* and *loses money* if the price of the instrument *falls*. Mathematically, the intraday P&L from a long spot position is:

$$P\&L_{CCY2} = Notional_{CCY1}.(S_T - S_0)$$

where S_0 is the initial spot rate and $S_T$is the new spot rate.

Exhibit 1.2 shows the P&L from a long spot position. As expected, P&L expressed in CCY2 terms is linear in spot.

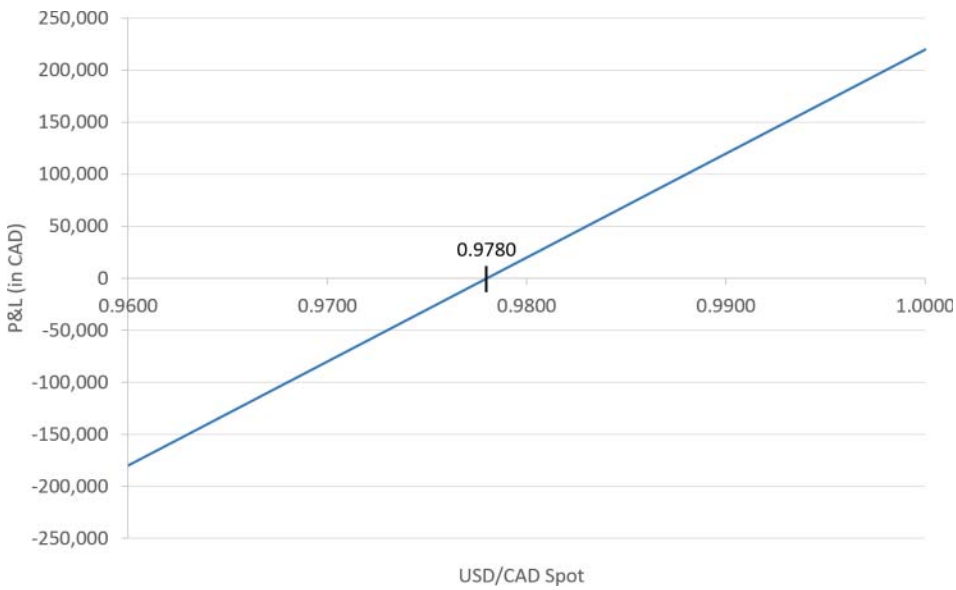


EXHIBIT 1.2 P&L from long USD10m USD/CAD spot at 0.9780

A **short** position in a financial instrument *makes money* if the price of the instrument *falls* and *loses money* if the price of the instrument *rises*. The intraday P&L from a short spot position is also:

$$P\&L_{CCY2} = Notional_{CCY1}.(S_T - S_0)$$

However, the notional will be negative to denote a short position.

Exhibit 1.3 shows the P&L from a short spot position. Again, P&L expressed in CCY2 terms is linear in spot.

If the P&L from these spot deals is brought back into CCY1 terms, the conversion between CCY2 and CCY1 takes place at the prevailing spot rate. Therefore, the CCY1 P&L from a spot position is:

$$P\&L_{CCY1} = Notional_{CCY1}.\frac{(S_T - S_0)}{S_T}$$

At lower spot levels, an amount of CCY2 will be worth relatively more CCY1 (spot lower means CCY2 stronger and CCY1 weaker). At higher spot levels, an amount of CCY2 will be worth relatively fewer CCY1 (spot higher means CCY1 stronger and CCY2 weaker). This effect introduces curvature into the P&L profile as shown in Exhibit 1.4.

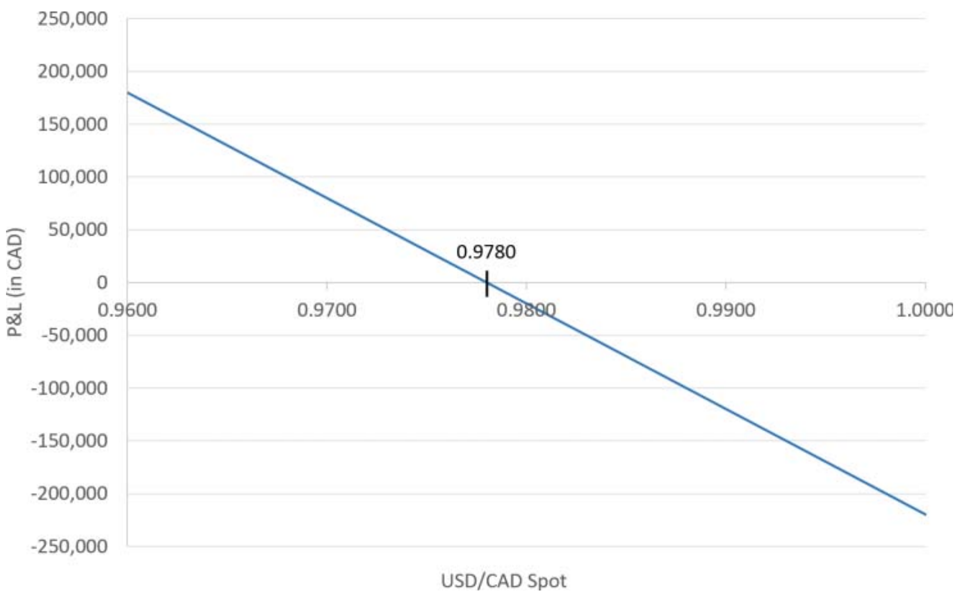


EXHIBIT 1.3 P&L from short USD10m USD/CAD spot at 0.9780

EXHIBIT 1.4 P&L from long USD100m USD/JPY spot at 101.00

Practical Aspects of the FX Market

The international foreign exchange market is enormous, with trillions of dollars' worth of deals transacted each day. The most important international center for FX is London, followed by New York. In Asia, Tokyo, Hong Kong, and Singapore are roughly equally important.

The USD is by far the most frequently traded currency with the majority of FX trades featuring USD as either CCY1 or CCY2. EUR/USD is the most traded currency pair, followed by USD/JPY and then GBP/USD.

FX traders draw a distinction between **major currency pairs**: the most commonly traded currency pairs, usually against the USD, and **cross currency pairs**. For example, EUR/USD and AUD/USD are majors while EUR/AUD is a cross. FX rates in cross pairs are primarily determined by the trading activity in the majors. The FX market is highly efficient so if EUR/USD spot is trading at 1.2000 and AUD/USD spot is trading at 0.8000, EUR/AUD spot will certainly be trading at 1.5000 (1.2/0.8).

Exhibit 1.5 is a mocked-up screen-grab of a market-data tool showing live spot rates in major G10 currency pairs. In practice these rates change (*tick*) many times a second.

Pair	Bid	Offer	Day Low	Day High
EUR/USD	1.3651	1.3652	1.3647	1.3688
GBP/USD	1.6898	1.6899	1.6856	1.6913
USD/CHF	0.8933	0.8934	0.8923	0.8950
AUD/USD	0.9237	0.9238	0.9221	0.9274
NZD/USD	0.8567	0.8568	0.8556	0.8587

EXHIBIT 1.5 Sample G10 spot rates

G10 currency pairs are (mostly) freely floating with no restrictions on their trading. The G10 FX markets are tradable 24 hours a day between Wellington Open (9 A.M. Wellington, New Zealand time) on Monday through to New York Close (5 P.M. New York time) on Friday.

In G10 pairs, the market convention for quoting a currency pair can be deduced from this ordering: EUR > GBP > AUD > NZD > USD > CAD > CHF > NOK > SEK > JPY. For example, the CAD against GBP FX rate is quoted in the market as GBP/CAD. Unfortunately, with market convention rules there are often exceptions. For example, the majority of the market quotes EUR against GBP as EUR/GBP but some U.K. corporates trade in GBP/EUR terms since GBP is their natural notional currency.

Emerging market (EM) countries often have mechanisms in place to control currency flows. For example, some EM currencies have limited spot open hours and some *peg* their currency at a fixed level or maintain it within a trading band by buying and selling spot or by restricting transactions. When trading in an emerging market currency it is vital to learn exactly how the FX market functions in that country. EM majors are quoted as the number of EM currency to buy one USD (i.e., USD/CCY).

In currency pairs with restrictions on spot transactions, **Non-Deliverable Forward** (**NDF**) contracts are often traded. NDFs settle into a single cash payment (usually in USD) at maturity rather than the two cash flows in a regular FX settlement. The **fix**, a reference FX rate published at a certain time every business day in the appropriate country, is used to determine the settlement payment.

Up-to-date FX rates can be found on the Internet using, for example, Yahoo finance (http://finance.yahoo.com/) or XE.com (http://www.xe.com/).

What Do FX Traders Call Different Currency Pairs?

Nobody on the trading floor calls USD/JPY "*you-ess-dee-jay-pee-why*." Major currency pairs have names that are well established and widely used. Standardized

EXHIBIT 1.6 **Selected G10 Currency Pair Names**

Currency Pair	Common Name
USD/CAD	"dollar-cad"
USD/JPY	"dollar-yen"
GBP/USD	"cable" (FX prices between London and New York used to be transmitted over a cable on the Atlantic ocean floor.)
EUR/USD	"euro-dollar"
AUD/USD	"aussi-dollar"
NZD/USD	"kiwi-dollar"
EUR/CHF	"euro-swiss" or "the cross"
EUR/NOK	"euro-nock" or "euro-nockie"
EUR/SEK	"euro-stock" or "euro-stockie"

EXHIBIT 1.7 **Selected EM Currency Pair Names**

Currency Pair	Common Name
USD/HKD	"dollar-honkie"
USD/CNY	"dollar-china"
USD/SGD	"dollar-sing"
USD/MXN	"dollar-mex"
USD/TRY	"dollar-try" or "dollar-turkey"
USD/ZAR	"dollar-rand"
USD/BRL	"dollar-brazil"

language is common in financial markets. It enables quick and accurate communication but it exposes those who are not experienced market participants. For this reason, using the correct market terms is important. See Exhibits 1.6 and 1.7 for common G10 and EM currency pair names.

Introduction to FX Derivatives

The FX market can be split into three main product areas with increasing complexity:

1. Spot: guaranteed currency exchange occurring on the spot date.
2. Swaps / Forwards: guaranteed currency exchange(s) occurring on a specified date(s) in the future.
3. **Derivatives**: contracts whose value is *derived* in some way from a reference FX rate (most often spot). This can be done in many different ways, but the most common FX derivative contracts are **vanilla call options** and **vanilla put options**, which are a *conditional* currency exchange occurring on a specified date in the future.

Vanilla Call and Put Options

Vanilla FX call option contracts give the *right-to-buy* spot on a specific date in the future while vanilla FX put option contracts give the *right-to-sell* spot on a specific date in the future. The term *vanilla* is used because calls and puts are the standard contract in FX derivatives. The vast majority (90%+) of derivative transactions executed by an FX derivatives trading desk are vanilla contracts as opposed to **exotic contracts**. Exotic FX derivatives (covered in Part IV) have additional features (e.g., more complex payoffs, barriers, averages).

To understand how call and put options work, forget FX for the moment and think about buying and selling apples (not Apple Inc. stock, but literally the green round things you eat). Apples currently cost 10p each. I know that I will need to buy

100 apples in one month's time. If I simply wait one month and then buy the apples, perhaps the prevailing price will be 5p and hence I can buy the apples cheaper than they currently are *or* perhaps the price will be 15p and hence more expensive *or* perhaps they will cost 10p, 1p, or 999p. The point is that there is *uncertainty* about how much the apples will cost and this uncertainty makes planning for the future of my fledgling apple juice company more difficult. Call and put options allow this uncertainty to be controlled.

One possible contract that could be purchased to control the risk is a one-month (1mth) call option with a **strike** of 10p and a **notional** of 100 apples. Note the different elements within the contract: the date in the future at which I want to complete the transaction (maturity: one month), the direction (I want to buy apples; therefore, I purchase a call option), the level at which I want to transact (strike: 10p) and the amount I want to transact (notional: 100 apples). After buying this call option, one month hence, at the maturity of the contract, if the price of apples is above the strike (e.g., at 15p) I will **exercise** the call option I bought and buy 100 apples at 10p from the seller (also known as the **writer**) of the option contract. Alternatively, if the price of apples is below the strike (e.g., at 5p), I don't want or need to use my right to buy them at 10p; hence the call option contract **expires**. Instead I will buy 100 apples directly in the market at the lower rate.

Therefore, by buying the call option, the **worst-case purchasing rate** is known; under no circumstances will I need to buy 100 apples in one month at a rate higher than 10p (the strike). This reduction in uncertainty comes at a cost: the **premium** paid upfront to purchase the call option. It is not hard to imagine that the premium of the call option will depend on the details of the contract: How long it lasts, how many apples it covers, the transaction level, plus crucially the **volatility** of the price of apples will be a key factor. The more volatile the price of apples, the more the call option will cost.

Exhibit 2.1 shows the P&L profile from this call option at maturity, presented in familiar hockey-stick diagram terms but without the initial premium included.

At the option maturity, if the price of apples is below the strike (10p), the call option has no value because the underlying can be bought cheaper in the market. If the price of apples is above the strike at maturity, the call option value rises linearly with the value of the underlying.

Mathematically, the P&L at maturity from this call option is:

$$P\&L = Notional\ .\ max(S_T - K,\ 0)$$

where *Notional* is expressed in terms of number of apples, S_T is the price of apples at the option maturity, and K is the strike. Often $max(S_T - K,\ 0)$ is written $(S_T - K)^+$.

It is worth noting that the P&L at maturity from the contract depends only on the price of apples at the moment the option contract matures; the path taken to get there is irrelevant.

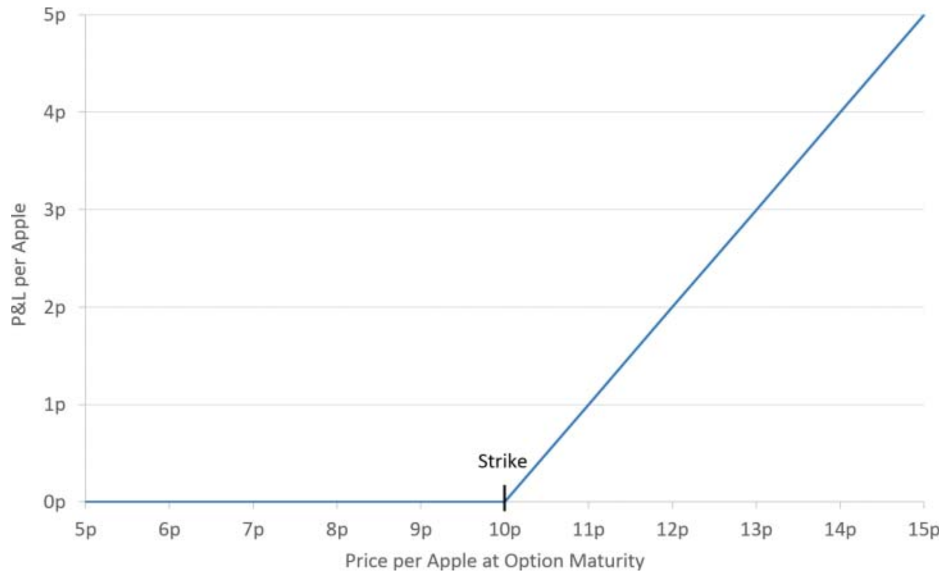


EXHIBIT 2.1 P&L per apple at maturity from call option with 10p strike

Put options are the right-to-sell the underlying. This can be conceptually tricky to grasp at first—*buying* the right to *sell*. Imagine you own a forest of apple trees. You know that by the end of August you will harvest at least 1,000 apples, which you will then want to sell. Again, uncertainty arises from the fact that the future price of apples is unknown. To control this uncertainty, a put option maturing on August 31 could be bought with a notional of 1,000 apples and a strike of 10p.

This time, at the option maturity, if the price of apples is below the strike (e.g., at 5p), the put option will be **exercised** and 1,000 apples will be sold at 10p to the option seller. Alternatively, if the price of apples is above the strike (e.g., at 15p), the put option will **expire** and 1,000 apples can instead be sold in the market at a higher rate.

By buying the put option, the **worst-case selling rate** is known; under no circumstances will I need to sell 1,000 apples at a rate lower than 10p (the strike) at the end of August. The cost of buying this derivative contract will depend on the exact contract details, plus, again, the more volatile the price of apples, the more the option will cost. Exhibit 2.2 shows the P&L profile from this put option at maturity.

Mathematically, the P&L at maturity from this put option is:

$$P\&L = Notional\ .\ max(K - S_T,\ 0)$$

Bringing these concepts into FX world, the underlying changes from the price of apples to an FX spot rate. At the option maturity, the prevailing FX spot rate will

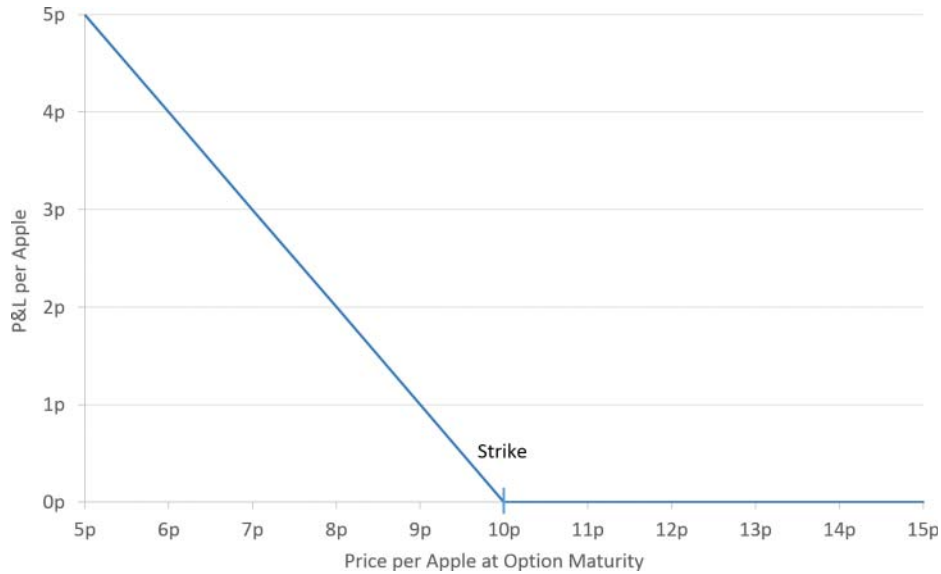


EXHIBIT 2.2 P&L per apple at maturity from put option with 10p strike

be compared to the strike to determine whether a vanilla option will be exercised or expired.

There are actually two main kinds of vanilla option:

1. European vanilla options can be exercised only *at* the option maturity.
2. American vanilla options can be exercised *at any time* before the option maturity.

European vanilla options are the standard product in the FX derivatives market because they are easier to risk manage and mathematically simpler to value. Henceforth, any mention of a *vanilla* option means a European-style contract. American vanilla options are covered in Chapter 27.

The following details are required to describe a vanilla FX option contract:

Currency pair: The spot FX rate in this currency pair is the reference rate against which the value of the vanilla option will be calculated at maturity.

Call or put (call = right-to-buy/put = right-to-sell): FX transactions exchange two currencies: one that is bought and one that is sold. Therefore, a vanilla option in a particular currency pair is simultaneously a right-to-buy one currency and a right-to-sell the other. Therefore, vanilla options are simultaneously a call on CCY1 and a put on CCY2 or vice-versa. Most often only the CCY1 direction is specified when describing the contract, so, for example, a EUR/USD call option is actually a EUR call and a USD put.

Maturity/expiry: The date on which the owner of the option decides whether to exercise their option or let it expire. There is actually a third option to *partially exercise* the option, which is explored in Chapter 9.

Cut: The exact time on the expiry date at which the option matures.

The two most common cuts in G10 currency pairs are:

- New York (NY): 10 A.M. New York time, which is usually 3 P.M. London time.
- Tokyo (TOK): 3 P.M. Tokyo time, which is 6 A.M. or 7 A.M. London time, depending on the time of year.

Strike: The rate at which the owner of the option has the right to exchange CCY1 and CCY2 at maturity.

Notional: The amount of cash (usually expressed in CCY1 terms) that can be exchanged at maturity. Vanilla option notionals can be converted between CCY1 and CCY2 terms using the strike as shown in Exhibit 2.3 since the strike is the level at which CCY1 and CCY2 are potentially exchanged at maturity.

Mathematically, the P&L at maturity from a long (bought) CCY1 call option is:

$$P\&L_{CCY2} = Notional_{CCY1} \,.\, max(S_T - K,\ 0)$$

where S_T is the spot FX rate at the option maturity and K is the strike. The CCY1 call P&L at maturity is the same as a long FX position (to the maturity date, i.e. a forward) above the strike. Exhibit 2.4 shows the P&L at maturity from a long USD/CAD call option (USD call/CAD put).

Likewise, the P&L at maturity from a long (bought) CCY1 put option is:

$$P\&L_{CCY2} = Notional_{CCY1} \,.\, max(K - S_T,\ 0)$$

The CCY1 put P&L at maturity is the same as a short FX position below the strike. Exhibit 2.5 shows the P&L at maturity from a long USD/CAD put option (USD put/CAD call).

Exhibit 2.6 shows a USD/JPY vanilla contract in an FX derivatives pricing tool. Traders use systems like this to price vanilla option contracts.

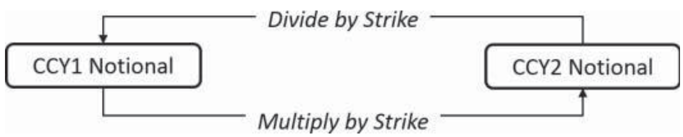


EXHIBIT 2.3 Converting between CCY1 and CCY2 notionals

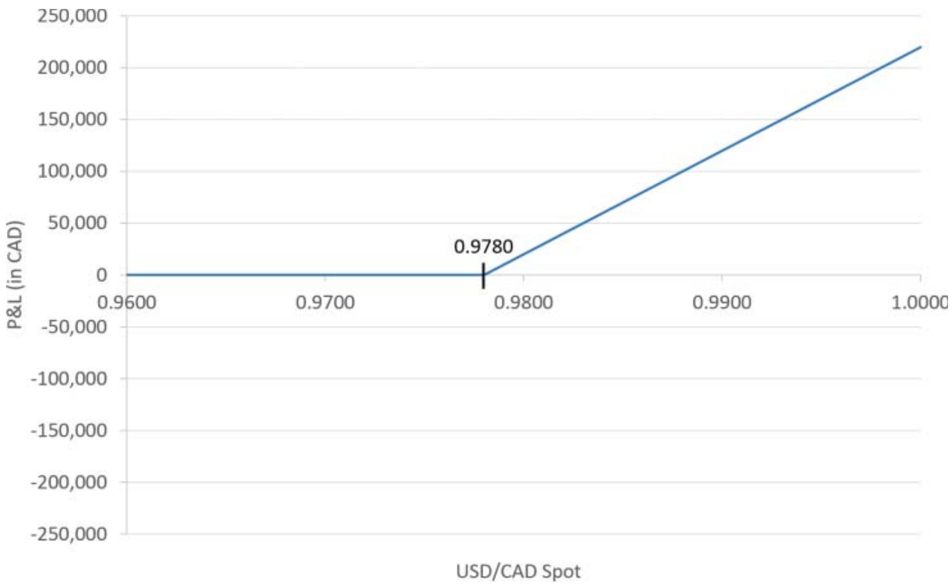


EXHIBIT 2.4 P&L at maturity from long USD10m USD/CAD call option with 0.9780 strike

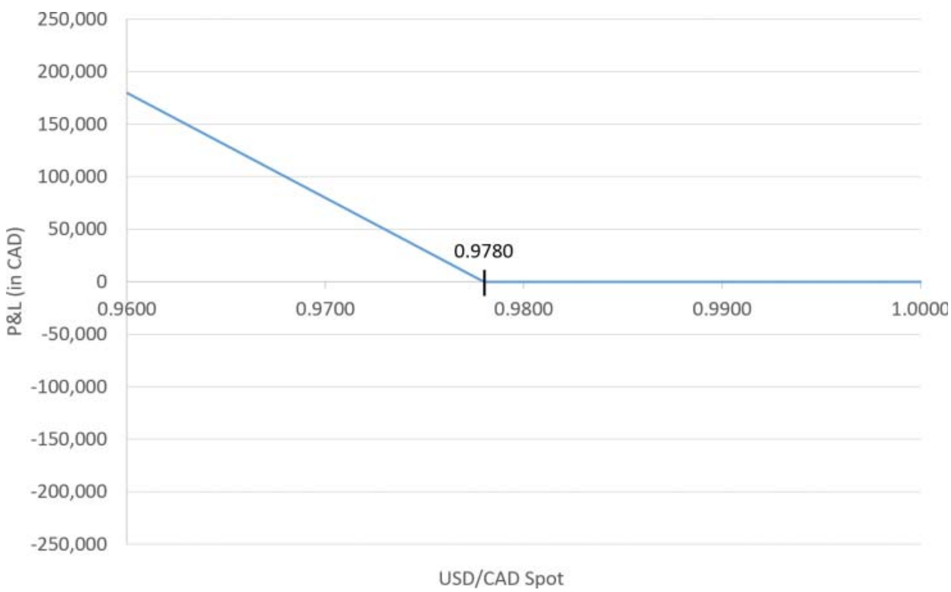


EXHIBIT 2.5 P&L at maturity from long USD10m USD/CAD put option with 0.9780 strike

Contract Details	Leg 1
Currency Pair	USD/JPY
Horizon	Fri 18-Oct-13
Spot Date	Tue 22-Oct-13
Strategy	Vanilla
Call / Put	USD Call / JPY Put
Maturity	1M
Expiry Date	Wed 20-Nov-13
Delivery Date	Fri 22-Nov-13
Cut	NY
Strike	80.00
Notional	USD5m
Market Data	
Spot	79.50
Swap Points	-2.00
Forward	79.48
Deposit (USD)	0.35%
Deposit (JPY)	0.07%
ATM Volatility	7.00%
Pricing Volatility	**7.25 / 7.65%**
Outputs	
Output Currency	USD
Premium	**0.44 / 0.48%**
Price Spread	0.04%

EXHIBIT 2.6 FX derivatives pricing tool showing a USD/JPY vanilla contract

Within the pricing tool, the **horizon** is the current date (i.e., today). On the **expiry date** (Nov. 20, 2013) at 10 A.M. NY time (since the option is priced to NY cut), the owner (buyer) of this European-style vanilla option will contact the writer (seller) of the option to inform them if they want to exercise the option.

If the spot rate at maturity is above the strike (80.00), the option is said to be **in-the-money (ITM)**. In this case the option will be exercised because the option gives its owner the right to transact at a better rate than the spot level. If the option is exercised, on the **delivery date** (Nov. 22, 2013; the delivery date is calculated from the expiry date in the same way that the spot date is calculated from the horizon—see Chapter 10 for more information on tenor calculations), the option owner will get longer USD5m versus shorter JPY400m while the option writer will get the opposite position.

If the spot rate at maturity is below the strike (80.00), the option is said to be **out-of-the-money (OTM)**. The option owner should let the option expire because USD/JPY spot can be bought more cheaply in the market.

Note that for a put option with all other contract details the same, the ITM and OTM sides flip: the ITM side is below the strike and the OTM side is above the strike.

Within the pricing tool, both volatility and premium prices are shown for the contract. Some market participants want prices quoted in volatility terms while others want prices quoted in premium terms. The **Black-Scholes formula** provides the link between volatility and premium. The formula takes as inputs the vanilla option contract details: maturity, option type (call or put), strike, plus market data: current spot, current forward/interest rates to the option maturity. The final input is *volatility* and the Black-Scholes formula can then be used to calculate the *option premium*. In Exhibit 2.6, a *two-way volatility* (explained in Chapter 3) is given and the Black-Scholes formula is used to calculate an equivalent *two-way premium*. It may seem strange that a price would be quoted in volatility terms but this is exactly how the FX derivatives market works. The essence of an FX derivative trader's job is to buy and sell exposure to FX volatility.

Practical Aspects of the FX Derivatives Market

FX derivatives trading volumes are roughly 5% of total foreign exchange trading volumes, equating to hundreds of billions of U.S. dollars' worth of transactions every day. Currency pairs that have higher trading volumes in their spot, forward, and FX swap markets tend to also have higher trading volumes in their FX derivatives markets.

The majority of FX derivatives trading occurs in contracts with maturities of one year and under. However, in some currency pairs long-dated contracts are traded—sometimes out to ten years or even longer.

Vanilla options in G10 currency pairs are usually **physically delivered**, meaning that at maturity, if the option is exercised, an exchange of cash flows (i.e., an FX spot trade) occurs.

In some emerging market currency pairs, vanilla options are **cash settled**, meaning that at maturity a fix is used to determine a settlement amount that is paid as a single (usually USD) cash flow. In G10 currency pairs it is also possible to use a fix to settle derivative contracts but this is less common.

Introduction to Trading

Fundamentally, markets are a mechanism to match buyers and sellers in order to determine the prices of goods and services. Traders interact directly with financial markets, buying and selling in order to manage their deal inventory and change their trading positions.

The process of operating within a financial market is easier to explain using a simple market. Therefore, this chapter uses a market on a single asset, like spot FX or a single equity contract, as the reference. However, the same ideas also apply to more complex markets, including FX derivatives.

Bids and Offers

The building blocks of financial markets are two types of order:

1. *Bid*: a rate at which a price maker is willing to buy
2. *Offer* (also called *Ask*): a rate at which a price maker is willing to sell

Bids and offers need to have a size associated with them. Saying, "I will buy apples for 10p each," is interesting to another market participant but not enough information; will you buy ten apples or a million apples?

There is an important distinction between price makers (also called market makers) and price takers within financial markets. Price makers leave orders in the market. Price takers come into the market and trade on existing orders. If a price taker wants to buy, the contract must be bought at a price maker's offer, and if a price taker wants to sell, the contract must be sold at a price maker's bid. Put another way, the price maker buys at their bid and sells at their offer, while the price taker sells at a price maker's bid and buys at a price maker's offer.

Within the market for a particular financial contract, if a trader wants to buy, there are essentially two ways of doing it:

1. Pay an offer (i.e., buy from someone who is showing an offer).
2. Leave a bid and hope that someone sells to you.

To sell, again, there are two possible methods:

1. Give a bid (i.e., sell to someone who is showing a bid).
2. Leave an offer and hope that someone buys from you.

There are two major differences between these approaches. The first is that trading on existing bids and offers can be executed instantly while leaving orders can take longer and may not happen at all since it requires someone else in the market to trade on your order. Generally, this requires the market to move toward the order level. The second difference is that leaving orders usually results in transacting at a better rate.

A reduced view of the current market is often given, showing only the single best bid and single best offer in a given contract. The best bid is the highest of all current bids (i.e., the most that anyone in the market is willing to pay to buy the contract). The best offer is the lowest of all current offers (i.e., the least that anyone in the market is willing to receive to sell the contract). The best bid and best offer combine to form the tightest **two-way** price in the market (i.e., the price with the tightest bid–offer spread, or put another way, the smallest difference between bid and offer).

Exhibit 3.1 shows a snapshot of an imaginary international apple market, showing bids on the left and offers on the right with shaded backgrounds. This is called the order book, which shows the **depth** in the market (i.e., all current bids and offers in the market) and a size associated with each order. Prior to transacting, this market is anonymous (i.e., it isn't known which market participant has left any particular order) and it often also isn't clear whether an order at a particular level is one order or a collection of multiple orders aggregated together.

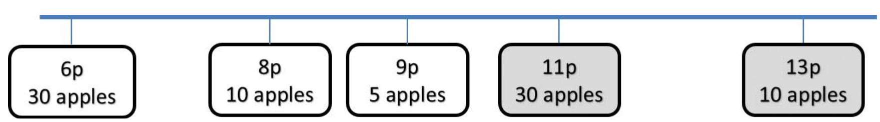


EXHIBIT 3.1 Apple market order book

In this example, the best bid is 9p and the best offer is 11p. Most of the time, the best bid is simply referred to as "the bid" and the best offer is "the offer." Lower bids and higher offers are referred to as being "behind."

For a given contract, offers are (almost) always above bids. However, in some situations, a market will be **choice**, meaning the bid and offer are at the same level. Even rarer, the offer can be below the bid—an **inverted** market. Markets rarely stay genuinely inverted for long, since the market participants showing the inverted bid and offer should be happy to trade with each other and hence restore the normal bid–offer direction. An inverted market often occurs when the relevant market participants *can't* trade with each other for some reason.

Trader X wants to buy 100 apples at 8p and trader Y wants to sell 100 apples at 8p. If trader X and trader Y each know what the other wants to do, they should be happy to trade with each other at 8p. It is therefore vital that both traders know of each other's intentions. For a market to function efficiently, *transparency* and the *flow of information* are key considerations.

Back to our apple order book: As mentioned, selling 10 apples in this market can essentially be done in two ways:

1. *Give the bids*. Five apples can be sold at 9p and five apples can be sold at 8p (so five of the 8p bid would remain). This averages at a rate of 8.5p to sell the apples but the transaction will certainly be completed.
2. *Leave an offer*. The rate of 11p already has an offer there. By "joining" (i.e., showing the same offer) the 11p offer in 10 apples, the size of that offer will go up to 40 apples.

   Note that *if* the 11p offer starts being paid, the original 11p offers will be transacted first. The offer could also be left at a higher level, at 12p or 13p, which would lead to potentially transacting at a better rate (selling higher), but the higher the offer, the lower the chance of transacting and the longer it will take.

In practice, particularly in faster markets, traders work buy or sell orders using a combination of trading on orders and placing orders. If the order was in larger size, the trader might dynamically leave (*place*) and remove (*pull*) orders, depending on how the market is reacting in order to get the best possible transaction level (*fill*).

In the apples example, if the 11p offer were to further increase in size, it is possible that bids would be pulled, or would be moved lower, since traders will see the large size on the offer and conclude that there are a large number of sellers in the market who will push the price lower. For this reason it is sometimes appropriate to transact an order in smaller chunks over time to reduce market impact.

Exchanges offer different order types that give additional control around how a bid or offer is processed. For example, *limit orders* allow buyers to define their maximum purchase price and sellers to define their minimum sale price while *fill-or-kill orders* are either executed immediately in their entirety, or else the order is canceled.

Decisions on how to transact within a certain market are based on an understanding of the relationships between transaction size, probability of transacting, transaction speed, and transaction rate. These factors are often combined into one word: **liquidity**.

Leaving Orders

Bids and offers aren't always left close to the current market. A bid could be left to, for example, buy 100 apples at 5p, with the market currently trading at 10p. In general,

- When an order sells above or buys below the current market level, this is called a **take profit** order. The term "take profit" implies there is an existing position that will make money if the market moves to the order level and the order then closes out the position, hence taking profit, although the original position may not actually exist.
- When an order buys above or sells below the current market level, this is called a **stop loss** order. Again, the term "stop loss" implies there is an existing position that will lose money if the market gets to the order level and the order then closes out the position, hence stopping the loss, although the original position may not actually exist.

Bid–Offer Spread

When a trader makes a two-way price on a contract for a client, the difference between the bid and the offer is called the **bid–offer spread**. Conceptually this spread exists to cover the market maker for the potential risk of holding the position over time if the client trades on either the bid or offer. Bid–offer spread is therefore a function of (amongst other things):

- *Contract volatility*: the more volatile a contract, the wider the bid–offer spread.
- *Average holding period*: the length of time before an offsetting (or approximately offsetting) trade can be found in the market. The longer until an offsetting trade can be found, the wider the bid–offer spread.

Traders also adjust their bid–offer spreads based on risk/reward preference:

- Showing a tighter bid–offer spread (hence a higher bid and a lower offer) increases the chance of the client trading but gives a smaller spread to protect from future price changes.

- Showing a wider bid–offer spread (hence a lower bid and a higher offer) decreases the chance of the client trading but gives a larger spread to protect from future price changes.

Bid and Offer Language

When traders pay offers they say "mine!" (i.e., they're buying it). This is sometimes accompanied with a raised index finger. When traders give bids they say "yours!" (i.e., they're selling it), sometimes accompanied with an index finger pointing down.

If bids in the market are getting "given" or "hit," this is a sign the market is moving lower. If offers in the market are getting "paid" or "lifted," this is a sign that the market is moving higher.

These terms can be confusing until they are used day-to-day, at which point they quickly become second nature.

Market Making

What follows is a simplified example of some market-making activity in our imaginary international apple market. Within this market, traders request prices directly from each other and 1,000 apples have just traded in the market at 10p. Trader B comes to trader A requesting a price in 200 apples. Trader B does not disclose a buying or selling preference so trader A makes the two-way price shown in Exhibit 3.2.

The bid is 9p and the offer is 11p. Therefore, trader A has shown a bid–offer spread of 2p. Trader A is assuming that the midmarket price of apples is still 10p. Hence if trader B transacts on either side of the price, the trade will contain some spread from the midmarket price.

Trader B now has three options:

1. *Buy* 200 apples (or fewer) at 11p (i.e., pay trader A's offer). If trader B buys, trader A sells; hence trader A would have "sold at their offer."
2. *Sell* 200 apples (or fewer) at 9p (i.e., give trader A's bid). If trader B sells, trader A buys; hence trader A would have "bought at their bid."
3. *Pass*. Trader B can decide not to buy at 11p or sell at 9p and therefore can walk away from the transaction. This could happen for many reasons; perhaps better

EXHIBIT 3.2 Trader A two-way price

prices are being shown by other traders in the market (lower offers or higher bids depending on whether trader B is a buyer or a seller) or perhaps trader B has had a change of mind and no longer wants to transact. Trader B does not have to explain to trader A why the price is being passed but it can be useful to know why (called *feedback*): If another trader is, for example, showing better bids, trader A can use that information in their future price making.

Trader B *buys* 200 apples at 11p. Therefore, Trader A has sold 200 apples and Trader A's apples position has gone *shorter* by 200. Although the actual exchange of apples for money may not occur until later, trader A's *exposure* to the price of apples changes as soon as the trade is agreed.

Trader A broadly now has two options:

1. *Warehouse* the risk and "run the position" (i.e., keep the changed exposure to the price of apples).
2. *Close out* the risk by going back into the market and trading to offset the exposure.

Trader A decides to warehouse the risk. Trader C now enters the market and requests a new two-way price in 200 apples from trader A. Here are three different possible scenarios for what happens next:

Scenario 1

Trader A quotes the same two-way price shown in Exhibit 3.3.

This time, trader C *sells* 200 apples at 9p. Hence trader A buys 200 apples, exactly offsetting the first transaction. By showing two-way prices to counterparties with opposite "interests" (buy/sell directions), trader A has managed to *buy low* (at the bid) and *sell high* (at the offer). Overall, trader A's position is back where it started, having earned 200 (contract size) × 2p (spread) = 400p for these two transactions. By market making, the trader has balanced their position (hence reducing risk) while locking in a profit. This scenario illustrates the advantages of being a market maker when counterparties have offsetting interests, called *two-way flow*.

Scenario 2

Due to the initial transaction occurring, trader A believes that the price of apples is rising in the market. Plus trader A is short from the initial transaction and doesn't want to get any shorter. Therefore, trader A makes the price shown in Exhibit 3.4,

EXHIBIT 3.3 Trader A two-way price in scenario 1

10p	12p

EXHIBIT 3.4 Trader A two-way price in scenario 2

with a relatively better (higher) bid to make it more likely that trader C will sell, and a relatively worse (higher) offer to make it less likely that trader C will buy.

Again, trader C has three options:

1. *Buy* 200 apples at 12p.
2. *Sell* 200 apples at 10p.
3. *Pass*.

If trader C is a seller, trader A is hoping that trader C decides 10p is a good price at which to sell. Raising the bid has *increased* the probability of trader C selling but it has *reduced* the amount of spread trader A will capture if trader C does sell.

Trader C was a buyer, but another trader in the market showed a lower offer, so trader C passes trader A's price. Trader A can use this information in future price making. This scenario illustrates how market makers use their *trading position* to influence their price making.

Note how important the flow of information within the market is. Depending on the market structure, the previous trade at 11p may be known by everyone or it may only be known by the traders involved in the transaction. The longer the time between transaction and reporting, the more power market makers have because they are personally involved in more trades and hence have access to better information.

Scenario 3

Once again, trader A does not want an increased short apple exposure so quotes a higher bid and a higher offer as shown in Exhibit 3.5.

Trader C pays the offer: Trader A showed a higher price but again the offer was paid. Trader A goes to the market to get a price in 400 apples to hedge ("get out of" or "offset") the position acquired from the previous two transactions. The rate received back from the market is shown in Exhibit 3.6.

10p	12p

EXHIBIT 3.5 Trader A two-way price in scenario 3

12p	14p

EXHIBIT 3.6 Market two-way price in scenario 3

The price of apples is rising and trader A is stuck with a short position. Trader A certainly does not want to sell even more apples at 12p, and if the position is bought back at 14p, an average loss of 2.5p will be locked in. A substantial part of the market has the same position. As traders go into the market to hedge their positions, the market moves higher, which causes the trader to lose money from the short apple position. This scenario illustrates the *risks* of being a market maker arising from not being in complete control of the trading position.

Price Making and Risk Management Overview

Success in price making depends on assimilating information from the market. The more information a trader has about market activity, the more likely it is that they know the current midmarket levels, which in turn increases the chances of successfully servicing clients and capturing spread. When many banks quote on the same contract in competition, to win the trade, the trader needs to show the best price of all traders quoting on the contract, but at the same time they attempt to maximize the spread earned from the midmarket level.

The key to successful risk management is to take positions (long or short) in financial contracts that make money as time passes and the market moves. Traders talk about the market "moving against" them (when losing money) or "moving for" them (when making money). For buy-side market participants (e.g., hedge funds) the risk management process is straightforward: They go into the market and transact only deals that they think will make money based on their analysis. As seen earlier, however, for market makers it is often not their decision to take a position. Rather, positions are generated as a consequence of market-making activities when the trading desk takes on the opposite position to the client when the client transacts.

Key risk management decisions for traders therefore involve **inventory management**, or in other words, deciding when to warehouse risk and when to close it out. Traders make these decisions based on their current reading of the market: direction, liquidity, sentiment, and so on plus trading decisions are also made with reference to risk limits. P&L targets and risk limits should be in line: Greater risk gives the opportunity for greater reward but it does not *guarantee* greater reward, only greater P&L volatility. Risk limits therefore keep P&L volatility to acceptable levels.

Building a Trading Simulator in Excel

This practical demonstrates how simple financial markets work and illustrates the differences between price-taking and price-making roles. Task A sets up a ticking (moving) midmarket price. Task B then introduces a two-way price (bid and offer) around the midmarket and price-taker controls whereby the trader can pay or give the market. Finally, Task C adds the ability for the trader to act as both price taker and price maker. This practical links closely to the material discussed in Chapter 3.

Task A: Set Up a Ticking Market Price

The trading simulator has one main VBA subroutine that updates the market price. The Application.OnTime command is used to pause between market ticks.

Step 1: Set Up a Ticking Midmarket Spot

Setting up the framework mainly requires VBA development. User inputs on the sheet are initial spot, time between ticks, and how much spot increments up or down at each tick. Outputs are the current time step and current spot. Control buttons for Go/Pause and Stop are also required:

Trading Simulator

Initial Spot	**1.3000**	←*Named:* ***SpotInitial***
Time Between Ticks (sec)	**1**	←*Named:* ***TickTime***
Spot Increment	**0.0010**	←*Named:* ***SpotIncrement***
Go/Pause	Stop & Reset	
Step		←*Named:* ***Step***
Midmarket Spot		←*Named:* ***SpotMidMarket***

The input cells should be named as per the screenshot. Naming cells makes development far more flexible than referencing (e.g., cell "A5" from the VBA).

The VBA module should start like this:

```
Option Explicit
Public MarketOn As Boolean
```

The first line forces all variables within the VBA to be declared using Dim statements. This makes the VBA coding more similar to languages like C++ and encourages better programming. The second line defines a global Boolean variable called MarketOn that defines whether the market is currently ticking (MarketOn = True) or not (MarketOn = False).

The GoButton subroutine should run when the Go/Pause button is pressed, flipping the MarketOn variable and initializing the sheet if required:

```
Sub GoButton1()

    'If market is ticking, stop it. If market is not ticking, start it.
    MarketOn = Not MarketOn

    'If market was previously stopped then initialize it
    If Range("Step") = "" Then
        Range("Step") = 0
        Range("SpotMidMarket") = Range("SpotInitial")
    End If

    'Run a market tick
    MarketTick1

End Sub
```

The StopButton subroutine clears the outputs and stops the market if the Stop button is pressed:

```
Sub StopButton1()

    MarketOn = False
    Range("Step").ClearContents
    Range("SpotMidMarket").ClearContents

End Sub
```

The MarketTick subroutine updates the market by moving spot up or down by the "SpotIncrement" amount at random on each market tick. Then a future market tick is scheduled. Note how time is converted from seconds terms into day terms for the OnTime function, plus "_" is used when a code statement goes over more than one line:

```
Sub MarketTick1()

    If (MarketOn) Then
        'Move spot up or down at random
        If (Rnd() > 0.5) Then
            Range("SpotMidMarket") = Range("SpotMidMarket") + _
             Range("SpotIncrement")
        Else
            Range("SpotMidMarket") = Range("SpotMidMarket") - _
             Range("SpotIncrement")
        End If

        'Increment step
        Range("Step") = Range("Step") + 1

        'Schedule a market tick in the future
        Application.OnTime TimeValue(Now() + Range("TickTime") / 24 / _
         60 / 60), "MarketTick1"
    End If

End Sub
```

When the Go/Pause button is first pressed, the market should start ticking with the frequency specified in the cell named "TickTime." Test different inputs to make sure everything is wired up properly. In particular, check that the Go/Pause button works correctly: Pressing the button while spot is ticking should pause it; then pressing again should restart the ticks.

Step 2: Record and Chart Spot

A time series of spot can now be stored on the sheet. Again, this is primarily achieved with VBA development. Columns can be set up to store the step and spot rate, with the upper-left cell of the output named: "DataOutput."

Within the VBA, the MarketTick code needs to be extended to push the data onto the sheet. The .Offset command is used to access the appropriate cell within the sheet:

```
…
If (MarketOn) Then
  'Store data
  Range("DataOutput").Offset(Range("Step"), 0) = Range("Step")
  Range("DataOutput").Offset(Range("Step"), 1) = _
   Range("SpotMidMarket")
…
```

And the StopButton code needs to be extended to clear the stored data:

```
…
'Loop around and clear the spot ticks
Count = 0
While Range("DataOutput").Offset(Count, 0) <> ""
  Range("DataOutput").Offset(Count, 0).ClearContents
  Range("DataOutput").Offset(Count, 1).ClearContents
  Count = Count + 1
Wend
…
```

When the simulator is run, spot ticks should be recorded in the table:

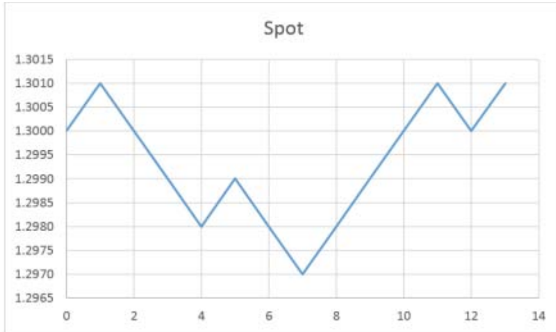
Step	Spot
0	1.3000
1	1.3010
2	1.3000
3	1.2990
4	1.3000
5	1.2990
6	1.2980
7	1.2990
8	1.3000
9	1.2990

Having the ticks stored enables them to be charted. Run the simulator for a few ticks and pause it. Then select cells, starting at the title and running a large number (500ish?) of (currently mostly blank) rows down, including both step and spot columns:

Step	Spot
0	1.3000
1	1.3010
2	1.3000
3	1.2990
4	1.2980
5	1.2990
6	1.2980
7	1.2970
8	1.2980
9	1.2990
10	1.3000
11	1.3010
12	1.3000
13	1.3010

Insert an X-Y Scatter chart with straight lines between points. When the simulator is un-paused, the data should plot with the chart automatically resizing as new data is stored (up to the number of rows originally selected):

Step	Spot
0	1.3000
1	1.3010
2	1.3000
3	1.2990
4	1.2980
5	1.2990
6	1.2980
7	1.2970
8	1.2980
9	1.2990
10	1.3000
11	1.3010
12	1.3000
13	1.3010



Task B: Set Up a Two-Way Price and Price-Taking Functionality

If price takers want to buy in the market, they must pay the offer. If price takers want to sell in the market, they must give the bid. Within this task, bid and offer prices are set up and the ability to give or pay the market is introduced.

Step 1: Set Up a Two-Way Price

Within the sheet a new bid–offer spread input is required, plus bid and offer rates must be output:

Initial Spot	**1.3000**	
Time Between Ticks (sec)	**1**	
Spot Increment	**0.0010**	
Bid-Offer Spread	**0.0030**	←*Named:* ***BidOfferSpread***
Go/Pause	Spot & Reset	
Step		
Midmarket Spot		

Bid	**Offer**
↑*Named:* ***Bid***	↑*Named:* ***Offer***

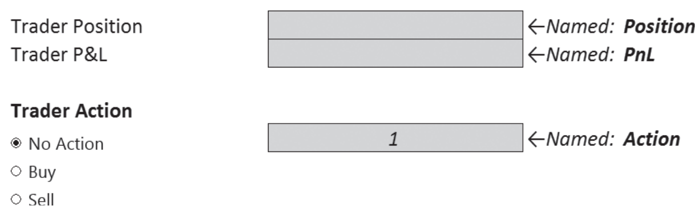
These new cells are referenced within the MarketTick VBA subroutine:

```
…
'Calculate bid and offer
Range("Bid") = Range("SpotMidMarket") - Range("BidOfferSpread") / 2
Range("Offer") = Range("SpotMidMarket") + Range("BidOfferSpread") / 2
…
```

New code also needs to be added to the GoButton and StopButton subroutines. Within GoButton, the initial bid and offer need to be set up, and within StopButton, the bid and offer output cells need to be cleared.

Step 2: Set Up Price-Taking Functionality

In order to risk manage, a trader needs to know their *position* and their *P&L*. Within the simulator both position and P&L can be added as outputs and kept updated using VBA code. In addition, at each spot tick, the trader can do one of three actions: nothing, buy (at the market offer), or sell (at the market bid), hence crossing a spread to transact. Controlling these choices could be done in many different ways in Excel but the method implemented here uses Option Buttons from the Form Control menu. These buttons should be grouped so only one of the choices can be selected at a time and the selection is then linked to an output cell that can be referenced within the VBA:



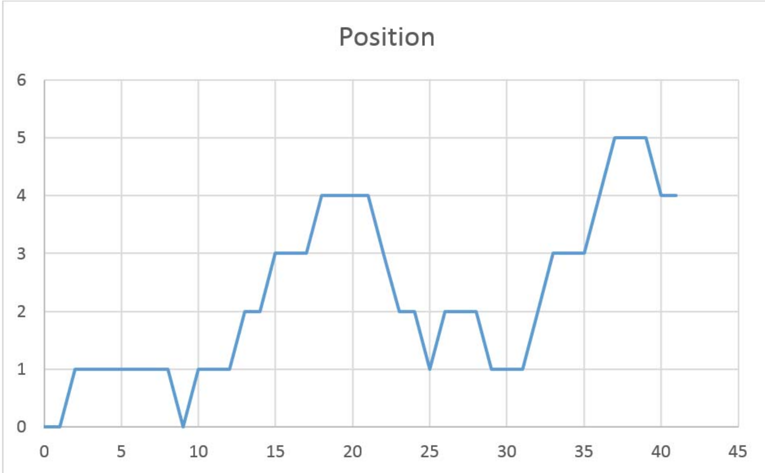
New VBA code needs to update the P&L based on the trader position and the spot move. The trader action then needs to be processed and the position updated if appropriate (and the selection reset back to "Do Nothing"):

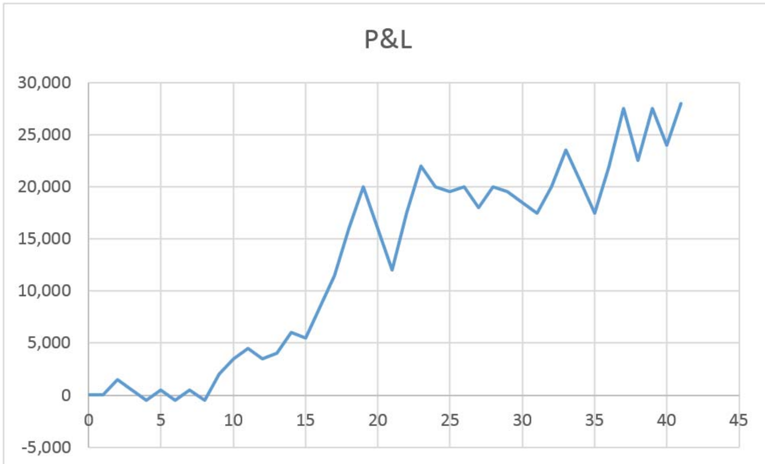
```
Sub MarketTick4()

    Dim SpotIncrement As Double

    If (MarketOn) Then
        'Store data
        Range("DataOutput").Offset(Range("Step"), 0) = Range("Step")
        Range("DataOutput").Offset(Range("Step"), 1) = _
         Range("SpotMidMarket")
        Range("DataOutput").Offset(Range("Step"), 2) = Range("Position")
```

```
        Range("DataOutput").Offset(Range("Step"), 3) = Range("PnL")

        'Calculate spot increment
        If (Rnd() > 0.5) Then
            SpotIncrement = Range("SpotIncrement")
        Else
            SpotIncrement = -Range("SpotIncrement")
        End If

        'Update P&L
        Range("Pnl") = Range("Pnl") + Range("Position") * SpotIncrement

        'Update spot and step
        Range("SpotMidMarket") = Range("SpotMidMarket") + SpotIncrement
        Range("Step") = Range("Step") + 1

        'Calculate bid and offer
        Range("Bid") = Range("SpotMidMarket") - _
         Range("BidOfferSpread") / 2
        Range("Offer") = Range("SpotMidMarket") + _
         Range("BidOfferSpread") / 2

        'Process trader action: Buy
        If Range("Action") = 2 Then
            Range("Position") = Range("Position") + 1
            Range("Pnl") = Range("Pnl") - Range("BidOfferSpread") / 2
        End If

        'Process trader action: Sell
        If Range("Action") = 3 Then
            Range("Position") = Range("Position") - 1
            Range("Pnl") = Range("Pnl") - Range("BidOfferSpread") / 2
        End If

        'Reset the trader action
        Range("Action") = 1

        'Schedule a market tick in the future
        Application.OnTime TimeValue(Now() + _
         Range("TickTime") / 24 / 60 / 60), "MarketTick4"
    End If

End Sub
```

Again, new code must also be added to the GoButton and StopButton subroutines to set up and clear the data on the sheet as appropriate.

The P&L and position are now also stored on the sheet and can be displayed in automatically updating charts using the same method as the spot chart:





The simulator is now ready to be tested. Check that spot still ticks correctly and the buy/sell controls work as expected. Each time a trade is executed there should be an initial negative P&L impact from spread cross and the trader position should correctly increment up or down. Also, the P&L must update based on market moves.

If everything is happening too quickly, slow it down; five seconds between ticks is fine to start with while the interactions between market, position, and P&L become familiar.

It should become obvious quite quickly that crossing the bid–offer spread to transact makes it difficult to make money within this framework; all transactions result in a negative P&L change so every trade reduces expected P&L. This is an important real-world trading lesson: *Don't over-trade when there is spread cross involved*.

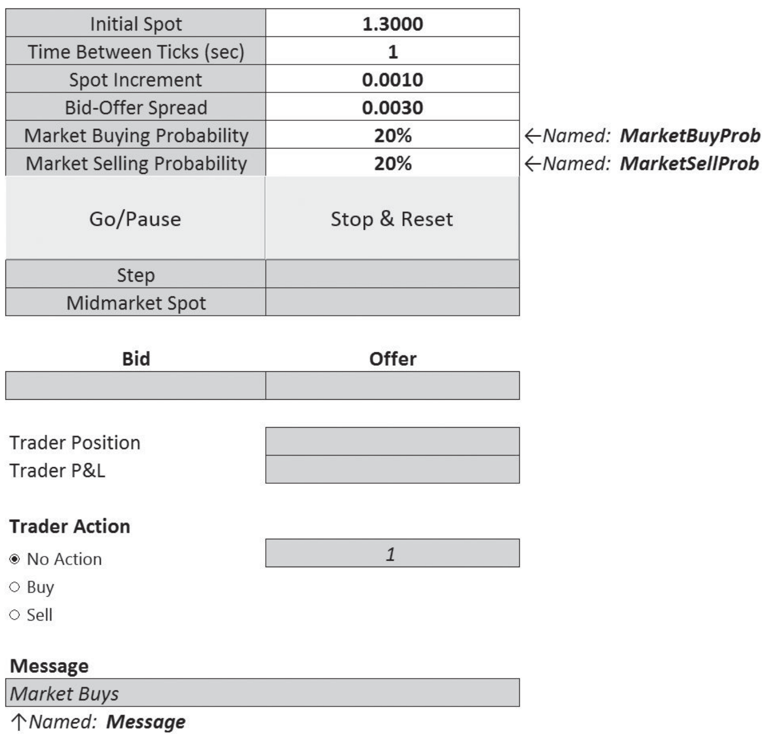
Test different combinations of bid–offer spread and spot increment to observe how their relative size impacts trading behavior and performance.

Task C: Introduce Price-Making Functionality

In practice, traders are sometimes price takers and sometimes price makers. This dynamic is achieved within the simulator by adding price-taking "market participants" to the VBA code. These price takers cause the trader position to change when they trade. Within this simplified framework, when the market participants transact they do so at the market bid and offer rather than at a price made by the trader.

This framework seeks to show how a price-making trader must deal with unpredictable flows. Should the trader wait to see if offsetting deals come in to hedge the existing position? Or should risk be immediately offset? Can the trader pre-position for the flows?

The following new inputs should be added to the sheet:



A random number within the VBA is used to determine whether the other market participants buy or sell. If there is a trade, the position and P&L must be updated accordingly.

```
…
'Process market action
MarketSignal = Rnd()
If MarketSignal < Range("MarketBuyProb") Then
    'Market Buys
    Range("Position") = Range("Position") - 1
    Range("Pnl") = Range("Pnl") + Range("BidOfferSpread") / 2
    Range("Message") = "Market Buys"
ElseIf MarketSignal >= Range("MarketBuyProb") And _
 MarketSignal < Range("MarketBuyProb") + _
 Range("MarketSellProb") Then
    'Market Sells
    Range("Position") = Range("Position") + 1
    Range("Pnl") = Range("Pnl") + Range("BidOfferSpread") / 2
    Range("Message") = "Market Sells"
Else
    'Market No Action
    Range("Message").ClearContents
End If
…
```


When running the simulator with this new functionality, the role of the trader changes. If the probability of the other market participants buying or selling is roughly equal, theoretically the trader should sit and wait for offsetting deals, reducing the position only when it gets too large and the P&L swings are too big. Skewed buy-and-sell preferences and different passive or active trading should be tested.

Extensions

The basic framework can be extended in numerous different ways to make it more realistic. Here are some suggestions:

- Add risk limits and P&L targets. Start with the limits and targets in line, then push them out of line to observe how having misaligned risk limits and P&L targets impacts trading performance.
- Evolve the spot rate over time using a volatility-based approach rather than a fixed increment, see Practical H for details.
- Introduce more interesting rules around market participant behavior so, for example, perhaps market participants are more likely to buy if spot goes lower

and sell if spot goes higher or vice versa. If the trader knows the rules, managing the flows becomes easier.

- Introduce different sized notionals. In practice, trading in larger size often means trading further away from the current midmarket.
- Most realistic (and most complicated) would be to have the trader manually making prices with the dealt side depending on the relationship between the trader price and the current market bid–offer. For example, if the current two-way market price is 1.3000/1.3030 and the trader shows a 1.3015 offer, if the price taker is a buyer, they should trade with high probability. If the trader shows a 1.3035 offer and the price taker is a buyer, they should only trade with low probability.

FX Derivatives Market Structure

Market structure is a topic that is often skipped over. In practice though, it is vitally important because it defines how clients interact with the trading desk and how the trading desk accesses liquidity to hedge their risk.

In some financial markets all participants access a centralized market or exchange anonymously on the same terms. The FX derivatives market, however, is an over-the-counter (OTC) market, meaning that there is no centralized exchange and a clear distinction exists between banks and their clients. Note that "banks" here refers to large international banks with FX derivatives trading desks.

Fundamentally, bank FX derivatives trading desks transact with clients, aggregate and offset the risk where possible, and close out unwanted residual risk. More specifically:

- Clients come to bank trading desks for prices, often via a sales desk within the bank. Usually the client simultaneously submits the same price request to multiple banks and deals on the best price as per Exhibit 4.1. Traders usually make two-way prices for clients because they do not know for certain whether a client is a buyer or a seller of a particular contract.
- Bank trading desks transact with each other either via the **interbank broker market** or the **direct market** (a price request directly between a trader at one bank and the corresponding trader at another bank). The majority of bank-to-bank transactions occur in the interbank broker market. This structure is shown in Exhibit 4.2.

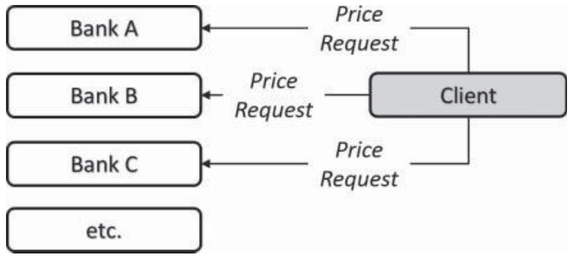


EXHIBIT 4.1 Client requesting prices from banks

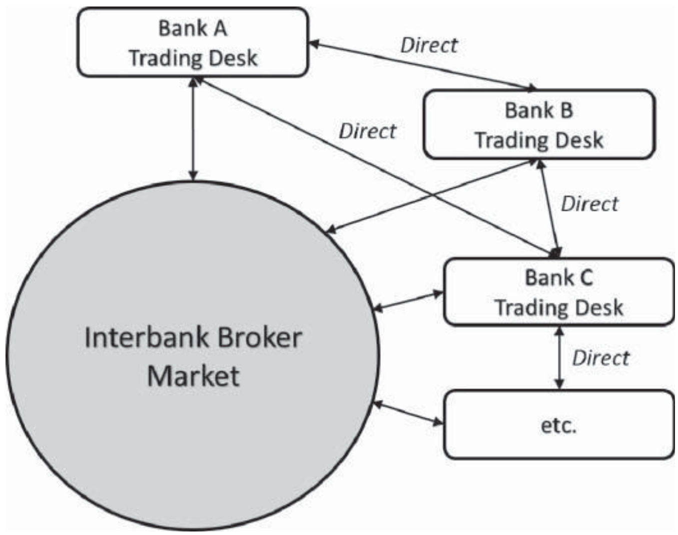


EXHIBIT 4.2 Interbank broker market interactions

Client Types

Many different types of client use FX derivatives for reasons that can generally be classified as hedging, investment, or speculation.

Corporates:

- International companies primarily concerned with managing their FX exposures and funding. For example, consider a car manufacturer with production in Europe but sales in America. This company is exposed to the EUR/USD exchange rate. If unhedged, when EUR/USD goes higher the company will be relatively less successful because the USD received from sales are worth fewer of the EUR needed to pay their workers and build new factories. Likewise, when EUR/USD goes lower the company will be relatively more successful. Fundamentally, the success of the company should depend on their ability to design, manufacture, and sell cars rather than exchange rate fluctuations. Therefore, expected future foreign exchange exposures are hedged—potentially using FX derivatives.

Institutional:

- *Real money*: Professional money managers who use foreign exchange as an asset class. Simply put, they seek to take positions that will generate positive P&L as markets move.
- *Hedge funds*: Professional money managers but typically trade to shorter time horizons than real money.
- *Sovereigns*: Central banks/NGOs (e.g., IMF/World Bank), interested in FX volatility plus potentially manage currency reserves.

Regional Banks:

- Have smaller FX derivatives trading desks or perhaps hold no risk at all and transfer (i.e., "back-to-back") all exposures to larger banks. Regional banks usually trade with international banks as clients in a *nonreciprocal* trading relationship, meaning that the international bank cannot request prices from the regional bank.

Retail:

- Individuals who trade simple FX-linked investment products, for example, *dual currency deposits* (DCDs). Within a DCD the client deposits money in one currency for a fixed term. At the end of the term, the bank has the option to return the money either in the deposited currency or in a second currency. The client has effectively sold a call option on the deposit currency, and the premium earned from selling the option gives the client an enhanced coupon on the deposit.

Bank FX Derivatives Trading Desk Structure

FX derivatives trading desks usually deliver follow-the-sun coverage to clients from three global centers, normally London, New York, and one in Asia-Pacific. Traders within the three centers are in constant communication and they aim to provide the best possible client service in terms of pricing consistency between centers, speed, and bid–offer spread.

On the trading desk in each center there are various roles. In practice these roles often overlap and what is presented in Exhibit 4.3 is enormously simplified.

Traders are responsible for keeping desk pricing in line with the market. They make prices for clients and risk manage desk trading positions. In addition they

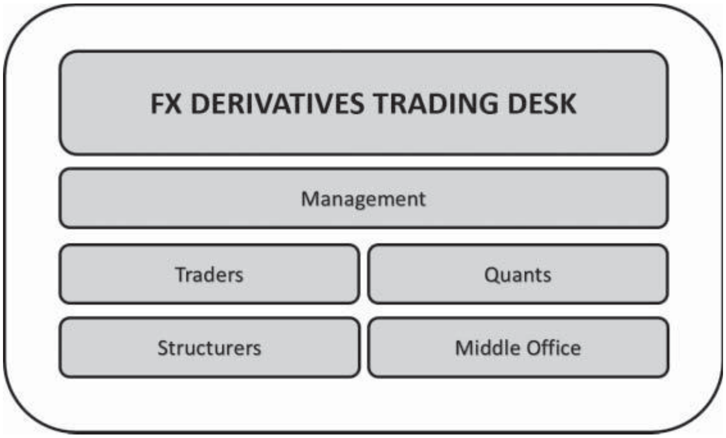


EXHIBIT 4.3 Roles on an FX derivatives trading desk

perform various other tasks that assist with their pricing or risk management, for example, market analysis and trade idea generation.

Traders are usually responsible for risk managing G10 or emerging market (EM) **currency blocks**. Related currency pairs are put together into a currency block, so, for example, the CAD block might actually contain AUD/CAD, CAD/CHF, CAD/NOK, CAD/SEK, EUR/CAD, GBP/CAD, NZD/CAD, and USD/CAD, although the majority of the risk will likely be in USD/CAD and possibly EUR/CAD. The main book-runner for a particular currency block usually sits in the most appropriate center; for example, the AUD block will normally be run out of Asia-Pacific while the Latam (Latin America) block will normally be run out of New York.

Structurers work with sales and relationship managers to understand clients' FX hedging and investment requirements. They design and construct solutions and work with traders to price more complex products. Structurers also educate sales on new derivative products offered by the trading desk.

Quants (quantitative analysts) are usually PhD-level mathematicians who develop and implement the pricing and analysis models and tools used by the trading desk.

Middle office ensure trading positions are correct and that new deals hit the trading positions quickly and accurately. In essence they are responsible for keeping desk risk management running smoothly.

Interacting with Sales Desks

FX Derivatives trading desks are, in a sense, product manufacturers. They create products for clients but it is the sales desks and relationship managers within the bank who are primarily responsible for the client relationships. The interactions between the trading and sales desks are not dwelt upon but collaboration is crucial for the overall success of the business.

Sales desks build relationships with clients by seeking to understand the clients' business and specifically their FX requirements. They provide clients with good information about what is happening in market with the aim that the trading desk is given the chance to quote on any FX contracts that the client wants to transact.

The trading desk assists the sales desk by providing them with good information about the market, coming up with relevant trade ideas, and offering quick, competitive prices in order to help build the client relationship.

Interacting with Support Functions

Trading desks do not operate in isolation; they require support from many other departments within the bank. For example, there are separate teams that do all of the following:

- Ensure the trading desk complies with their regulatory requirements.
- Monitor bank credit exposures to different counterparties across different asset classes.
- Produce official trading desk P&Ls.
- Ensure the trading desk complies with its international tax obligations.
- Deal with recruitment, contracts, and training.
- Build and maintain the desk technology infrastructure.
- Monitor trading risk to ensure risk limits are not broken.
- Ensure the trading desk has booked deals correctly and confirms deals with counterparties.
- Monitor desk pricing versus independent market sources.
- Ensure the trading desk is offering a valid range of products that can be properly priced and risk managed.
- Validate the pricing models (see Chapter 19) used by the trading desk.

Trading Internally

FX derivatives trading desks generally transact internally (i.e., within the bank) with the trading desks of other asset classes in order to hedge non-FX derivatives risk. FX spots, forwards, swaps, NDFs, interest rate products, and cash borrowing and lending will all be traded regularly by an FX derivatives trading desk.

Other departments within the bank come to the FX derivatives trading desk for advice, pricing, and execution on FX derivatives transactions. Sometimes these are for speculation. For example, a trader on the FX spot desk may wish to buy a short-dated vanilla FX option. More often, they are linked to an underlying client transaction. For example, an M&A transaction or trade finance deal may have a structured FX component that the FX derivatives trading desk will price and ultimately risk manage.

Tips for a Trading Internship

When you first start on the trading desk, don't race through any learning material you're given: It isn't a race. Developing a good understanding of the material is far more important than showing off that you've read a 40-page document in an afternoon. On the other hand, don't just sit there waiting to be told what to do; ask junior traders for assistance in getting training material if it isn't given to you.

If you're given rubbish jobs to do, just get on and do them; put the team first. Everyone went through the same thing; getting coffees and lunches for traders gives you exposure to them. By doing this they will start to get to know you and will be more likely to help you learn.

Get the right balance between project work and learning about trading. Sitting there the whole time just doing your project is folly (and one to which I fell prey during an internship in 2001). You are there to learn what the trading job involves and whether it is a career you would like to pursue. Speak to people and make connections. At the end of the internship a range of people on the desk will be asked what they thought of you; no impression is almost as bad as a negative impression.

Go and sit with the different parts of the trading desk (structuring, middle office, quants), other teams within the same asset class (sales desks and trading desks), and other asset classes (interest rates, equities, credit). The more you understand about different roles on the trading floor, the better.

Always be on the desk over economic releases and major option expiries. This is when market activity is most likely to occur and it is important to see how traders react to this.

Tips for a Junior Trader

Don't be sloppy. This is the worst possible trait for a junior trader. Be precise when describing your position, book trades properly the first time, and be able to explain your P&L and position accurately at all times.

Learn to be aware of multiple things at once: Brokers are shouting, spot traders across the room are shouting, spot is moving, and your boss is asking you a question. This is a difficult skill but it is one that must be learned. Traders become experts at flipping from chatting about sports or the weather to quickly reacting to something that has occurred in the market. If you're sitting and talking with traders, always keep half an eye on the market because you can be sure they are.

Don't get gripped watching spot moving up and down at the expense of all other work. Learn to be aware of spot without staring at it. In practice, traders view their trading positions and pick approximate spot levels where they will hedge ahead of time.

Don't panic under pressure; stay calm and keep your thoughts clear. If you lose your nerve, you might as well not be there.

Don't be lazy: If something looks wrong in the position, investigate and find what is wrong; don't assume problems will fix themselves.

Never exaggerate or bluff. Experienced traders will pick up on bluffs in picoseconds and will delight in taking you apart for it. Saying, "I don't know, let me find out," is usually acceptable. Lying is never acceptable. Also, describe market moves in terms of what has actually happened and try to avoid hyperbole (e.g., "it's going insane" or "it's getting completely destroyed" when implied volatility moves 0.2%).

Don't be afraid to admit you've made a mistake. Everyone makes mistakes. Fix it, learn from it, and make sure it doesn't happen again. Obviously, however, repeated similar mistakes can be harmful to career progression.

Learn how to round exposures and P&Ls when describing them. Traders usually only care about their deltas to the nearest one million or five million, depending on how much risk is being run, so, if asked for a delta exposure, "long ten million, three hundred and twenty-one thousand, five hundred and seven U.S. dollars" is too much information when "long ten bucks" gets the required information over.

Judging market liquidity is a skill that is acquired over time. Knowing where to price a large client trade by estimating how the market will absorb the risk if it is recycled must be experienced to be learned.

Follow as many different financial markets as you can, not just your own. Know where ten-year USD rates, the Nikkei, the Vix, and so forth are all trading. The best traders have a view on the whole market and see the connections between the different parts.

Understand what kind of trading desk you are on and how it makes its money. What were the annual P&Ls of the desk for the last five years and what is the split of that P&L between client trades and position taking? How much of a presence does the trading desk have in the interbank broker market and the direct market? Who are the main client groups (corporate, institutional, etc.) of the desk and what kind of trades do these clients like to transact?

Learn the official desk P&L currency and how the P&L conversion from the natural P&L currency per currency pair into the desk P&L currency is handled. Does this effect create a trading exposure which needs to be managed?

Always have an opinion about the market and a plan for your trading position. Always know and be able to justify your current position. The justification can be made in many different ways, but if you can't justify why you have a position, you

A> USDJPY PLS
B> HIHIHIHI
A> USDJPY 25TH AUG 103 NY IN 75M PLS
B> 4.9 / 5.5%
A> BUY PLS
B> SURE 102.50 SPOT OKAY?
A> AGREED -1 SWAP 0.15% USD DEPO 0.185 USD% 40 DELTA OKAY?
B> AGREED THANKS FOR THE DEAL BIBIBI
A> THX BIBI

EXHIBIT 4.4 Interbank direct call from July 23, 2014

shouldn't have it. Trading positions could be taken as a result of, for example, market analysis (see Chapter 17), client flows, or market positioning.

FX Derivatives Interbank Direct Market

FX derivatives traders at different banks can contact each other directly to request prices on simple vanilla contracts. The process is straightforward: Trader A calls bank B via a recorded messaging system and requests a price on a vanilla contract. Trader B picks up the request (because they are currently trading that currency pair) and makes a price on the contract in implied volatility terms. Trader A then has a short amount of time (approximately up to 20 seconds) to deal (hence *crossing* trader B's spread) or pass on the price. After an implied volatility price is dealt on, the traders agree the market data (spot, forward, deposit rate) and option premium between themselves and the deal must then be booked into both risk management systems.

Exhibit 4.4 shows a typical direct call between traders A and B. Note the assumptions made within the call, which enables efficient communication.

Trader A has requested a price in USD75m of 103.00 USD Call/JPY Put with August 25, 2014 NY cut expiry. Within direct calls, notional is always quoted in CCY1 terms; if a year is not specified, the next occurrence of the date is assumed and the out-of-the-money side, in this case a USD call, is always dealt (this is explained in Chapter 7).

Traders have a choice between calling direct or using the interbank broker market. Therefore, the decision to call direct is usually made because it compels the other trader to make a price with none of the safety provided by the broker market. This decision might be made because the trader wants to transact in large size and may need to trade with multiple banks at once in a way that would not be possible through the broker market, or perhaps the market is volatile and there is currently limited liquidity available in the broker market.

There is a well-established etiquette within the direct market: Traders don't call each other too often and most trading desks rarely reject a price request. Also, once a contract has been in the broker market and a price has been made, it is not usual to receive a direct call on the same contract. Most importantly, if the price received is far from what was expected, the price making trader should be given an opportunity to check their rate to avoid bigger problems later when a mistake is discovered.

Direct relationships between traders work best when they call each other with similar frequency on similar-size contracts. Mutually beneficial direct relationships exist but there is always a tension involved that arises from the price making process. When quoting a direct call, the trader thinks, "Why are they calling on this contract?" There is often a catch; otherwise, the contract could be worked in the interbank broker market and probably transacted closer to midmarket. Direct calls are dangerous because they can expose traders who aren't following the market closely enough.

FX Derivatives Interbank Broker Market

The interbank broker market is a crucial part of the FX derivatives market structure since it is where the vast majority of bank-to-bank FX derivative transactions occur. This section explains in detail how the broker market works.

In the FX derivatives market there are currently four main interbank brokerage firms. Each broker *shop* has global teams of FX derivatives brokers who between them speak to the FX derivatives traders at all the banks in the market in a structure shown in Exhibit 4.5.

Brokers are split by currency block (e.g., G10 majors/G10 crosses/Asia EM, etc.) and also by option type (e.g., short-date vanillas/long-date vanillas/exotics).

Communication between broker and trader has traditionally been done either by voice over recorded fixed telephone lines or via recorded text messaging systems. Over time, communication is moving away from free-form text messaging toward standardized electronic messaging.

Market instruments at market tenors are often quoted in the interbank broker market but *specific* contracts make up a large part of the market, too. Specifics are nonstandard vanilla contracts. For example: July 23 1.3250 NY in EUR/USD or 6mth 102.00 TOK in USD/JPY are both specific contracts.

The trader at bank A wants to transact a specific vanilla contract and requests a price from their broker at one of the interbank brokerages. This is called trader A's *interest*, which broker A is *working* as shown in Exhibit 4.6. Brokers are usually working multiple interests for different traders simultaneously. Note that the trader does not have to disclose a contract size at this stage; it may help the broker if the trader reveals that the interest is in large size, but initially a standard "market size"

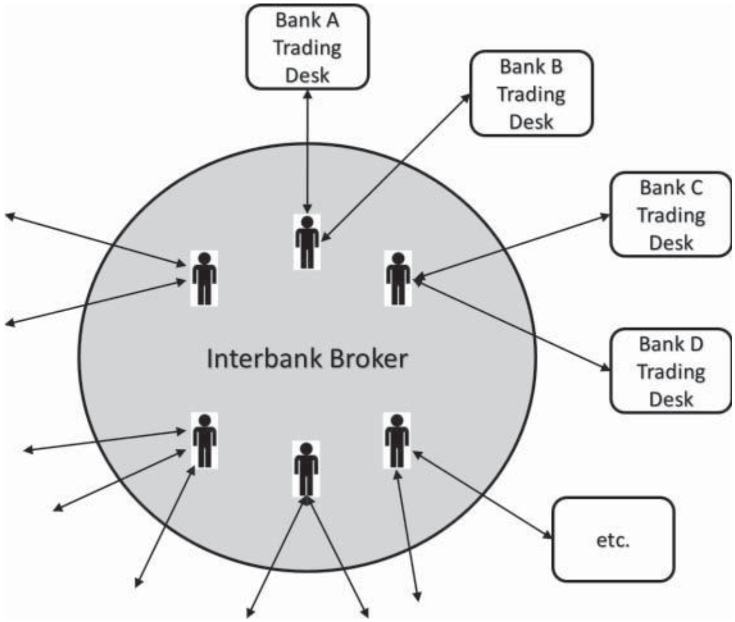


EXHIBIT 4.5 Interbank broker structure

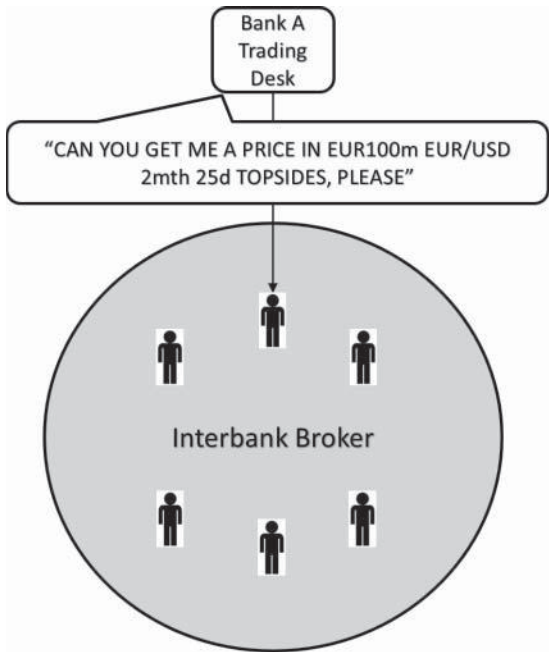


EXHIBIT 4.6 Trader A requesting a price from their broker

would be assumed. In major G10 currency pairs, market size is roughly USD30m to USD50m in normal market conditions. In cross G10 currency pairs and EM currency pairs, market size is smaller.

Broker A tells the other brokers at their shop about the interest and the brokers go to the relevant traders at all the banks in the market requesting a price as shown in Exhibit 4.7.

The traders price the contract in their pricing tools and when ready they return their prices, quoted in implied volatility terms:

- Bank B: no price
- Bank C: 6.75 / 7.25%
- Bank D: no price
- Bank E: 6.6 / 7.1%

In trader A's pricing tool, the mid-value of this vanilla contract is 7.0% implied volatility.

Note that traders are not compelled to make a price and there are many possible reasons why they might choose not to. Perhaps they are busy pricing another contract for a client, perhaps they are remarking their curves, or talking to their boss, or perhaps they are off the desk getting lunch.

The importance of the trader/broker relationship should be starting to become apparent. If a broker and trader have a good relationship, the trader is both more

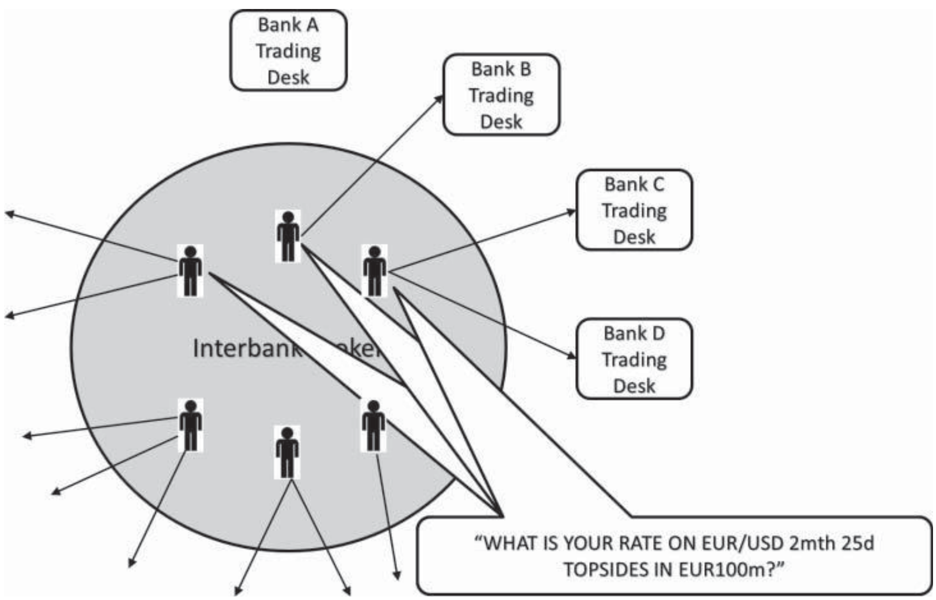


EXHIBIT 4.7 Brokers going to the market requesting a price on the contract

likely to request a price from a particular broker (remember there are four different shops to choose from), and more likely to make prices for the broker.

The relative power of the specific broker within their firm is also important. To get the most rates (prices) possible back from the market, and hence maximize the chance of getting a tight composite rate, broker A must get the other brokers to push their traders for rates. In some instances this will mean bothering a trader who does not want to make a rate. Again, relationships are vital for managing this.

The best composite rate between the two prices made is 6.75/7.1%: bid from bank C and offer from bank E. The broker goes back to trader A with this rate but does not disclose which bank is the bid or the offer. This process is shown in Exhibit 4.8.

Trader A now has five options:

1. Pay the offer.
2. Show a bid.
3. Give the bid.
4. Show an offer.
5. Pass.

If trader A is a buyer (called a *buying interest*), options 1, 2, and 5 are valid choices. If trader A is a seller (called a *selling interest*), options 3, 4, and 5 are valid choices.

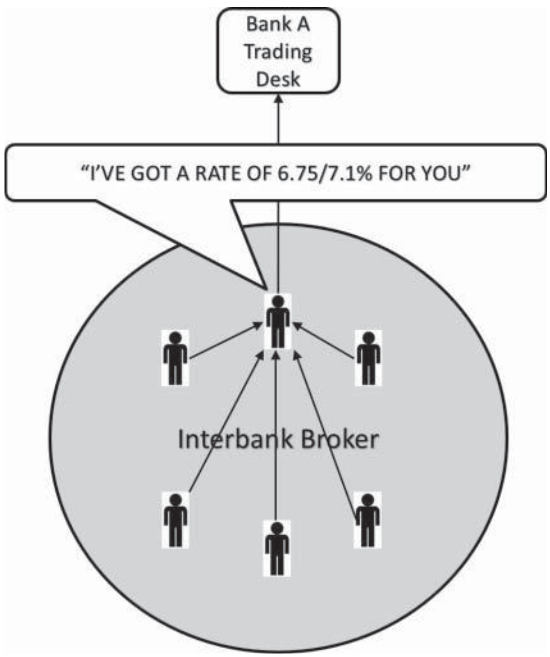


EXHIBIT 4.8 Brokers collecting prices from the market and reporting the best rate back to trader A

If trader A pays the offer or gives the bid immediately, the trade is done and the process is complete. However, in FX derivatives the interbank broker market normally works slower than that. Traders usually make a relatively wide initial rate even if they have a preference to buy or sell the contract; they wait to see whether the interest is a buyer or a seller before showing their own hand.

Therefore, in normal markets, transacting on the "opening rate" is rare. It is far more likely that trader A will show a "counter" (i.e., a bid or offer) to start a process that will result in transacting at a better level. However, in volatile or illiquid markets, where the brokers may struggle to get any rates at all, a trader may have no option but to trade on an opening rate and hence cross the full spread.

Trader A is a seller of the contract and shows a 7.0% offer. The process now becomes a negotiation between trader A and trader C only (since trader C showed the best bid) via their respective brokers as per Exhibit 4.9.

The broker goes back to trader C only and shows them the 7.0% offer (this would be described as a "seven-oh top"). Trader C then has three main options:

1. Keep the 6.75% bid at the same level ("stuck on the bid").
2. Show a higher ("better") bid.
3. Remove ("pull"/"ref"/"refer") the bid. Orders aren't usually pulled unless something has fundamentally changed in the market (i.e., spot has moved sharply) since the order was placed.

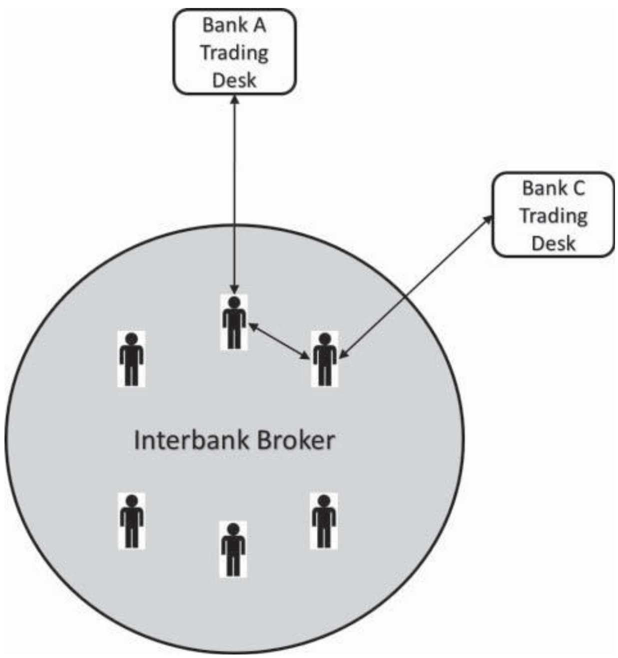


EXHIBIT 4.9 The interest negotiating with the trader who made the best price

Trader A is then told what trader C has chosen to do and similarly can either show a better offer or be stuck, assuming trader A doesn't want to walk away at this point.

This process goes back and forth between traders A and C via their brokers until either the prevailing bid is given or offer is paid and hence they transact, or both traders are "stuck" at different levels. As the price gets closer to being traded, the broker will start to discuss the potential size of the transaction to make sure that enough size exists on the bid or offer for the interest to trade in the required notional.

When both sides are stuck, the broker will ask both traders if they are happy to "show it out," which means letting all traders in the market see the two-way price with the aim of further tightening the rate, which at this point is 6.9/7.0%.

Showing the rate out is a risk for trader A since another trader may decide that 6.9% is a good bid to hit and "give it ahead of the interest." Therefore, the trader's judgment about the current state of the market is vital: Have there been more buyers or sellers of similar contracts in the market recently?

Exhibit 4.10 shows how the brokers show the rate out to the market.

Trader D, who had previously not made a rate, is alerted to this tight price and realizes that buying the contract fits their position. Trader D therefore pays trader A's 7.0% offer but only in EUR50m. Trader A has managed to sell half their full amount at their midmarket rate. The brokers inform everyone in the market about the transaction, without mentioning the names of any of the counterparties involved, known as "printing" the trade. This is shown in Exhibit 4.11.

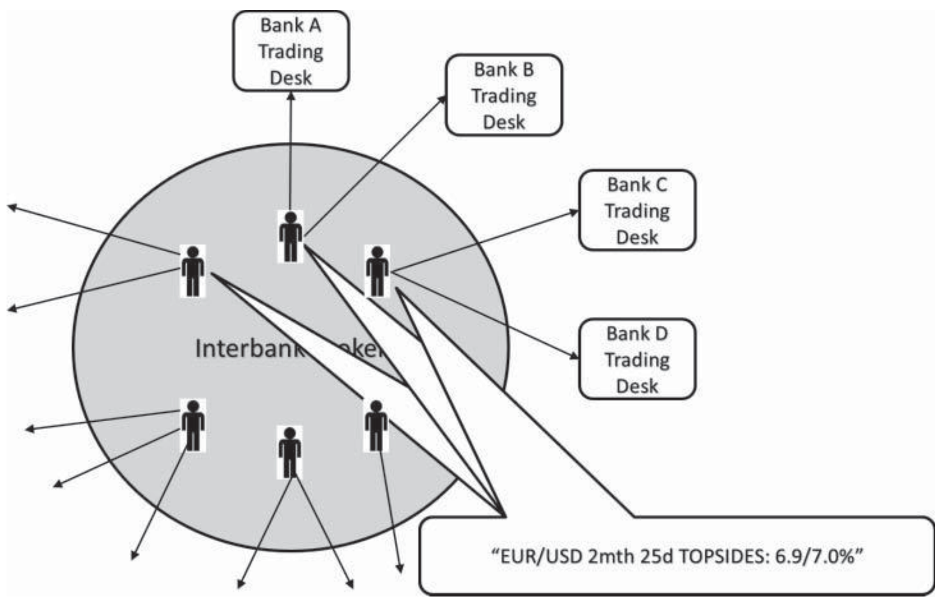


EXHIBIT 4.10 Brokers showing the rate out to the market

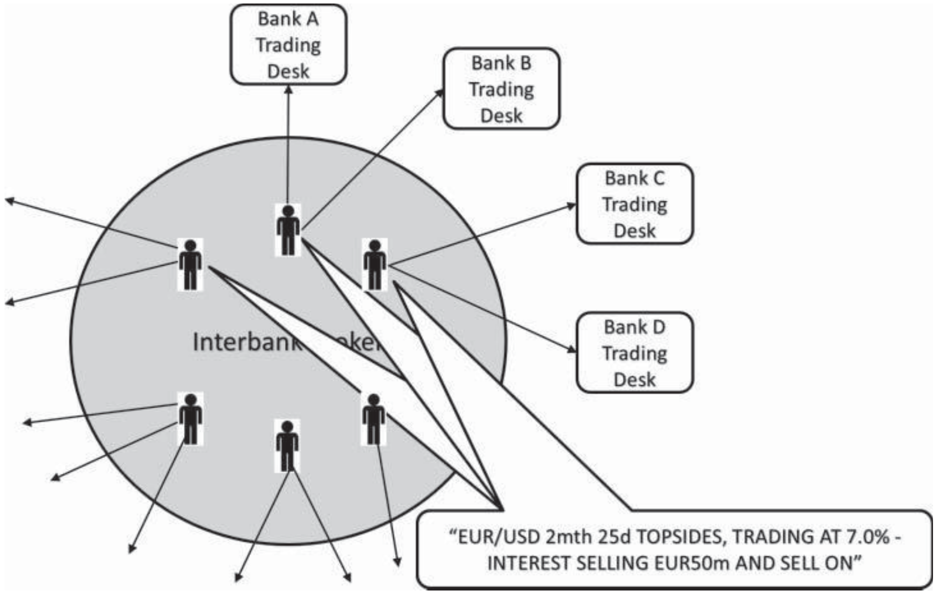


EXHIBIT 4.11 Brokers printing the trade to the market

The level at which this transaction was completed is important information for traders in the market. Even if they have been paying little attention up to this point, traders should price up the contract in their own pricing tools. If the contract is priced differently to the trading level, traders must judge whether their volatility surface needs tweaking or trading the contract represents an opportunity to buy below the midmarket level or sell above.

The information that the contract is "sell on" is also important. That there are no more buyers in the market is a sign that the price of similar contracts is falling in the market. Likewise, if traders have more of a contract to buy, with no sellers found, the brokers report the contract as "bid on" and this is a sign that the market for similar contracts is rising. If the interest managed to trade their full size, the broker would report that the interest has been "taken out."

The brokers continue to match buyers and sellers at the trading level until they are sure that everyone in the market is aware of the trade. Once there are no more interested parties, the contract dies.

The final step in the process is to agree contract details. Vanilla deals are quoted and traded in implied volatility terms but a cash premium is required when booking the deal. Therefore, the traders agree midmarket levels for spot, forward to maturity, and deposit rate (see Chapter 10) to maturity, which is then turned into a spot or forward premium using the Black-Scholes formula. Note that a forward premium (i.e., a premium paid at the delivery date rather than the spot date) will not depend on the deposit rate.

At no point during this entire process were brokers required to have opinions about the "correct" price on the interest, or any knowledge about foreign exchange, derivatives contracts, or anything except the evolution of the price of the contract. However, good brokers have all this knowledge, plus they know which traders in the market are likely to be interested in a particular contract based on recent behavior. They also have a feel for whether a rate might be paid or given ahead of the interest if shown out, and if a price made is off-market (i.e., the price is away from the correct level), they provide protection to traders.

The sidebar provides an example of a broker chat reporting market activity to a trader. Note the combination of requesting prices, reporting live prices (including the interest direction, e.g., "buyer"), and reporting trades.

Example Broker Chat from July 23, 2014

07:24:37 AUD 04 Sep 0.8875P pls
07:33:47 1yr aud trades 9.0
07:38:34 AUD/JPY 23 May 97.00C pls
07:40:29 AUD 04 Sep 0.8875 9.05/9.3 buyer
07:50:10 AUD 04 Sep 0.8875 9.05/9.2 buyer
07:55:44 CHF/JPY 16th May 116.10C 6.25/6.65 seller
07:56:00 JPY O/N 103P vs 103.25P pls
08:07:27 JPY 05 Jun 104P pls
08:20:10 CAD/JPY 12 May TK Cut 93.50 pls
08:21:55 JPY O/N 102.25C vs Tue 102.00C 50 per pls
08:43:50 tues yen 102 8.25 seller outright
08:23:25 JPY O/N 102.25C vs Tue 102.00C trades 12.0/8.3
09:33:03 JPY 1Y 115.00P pls
09:51:52 JPY Tue 103.85P 8.75/9.25 buyer
09:51:52 JPY Tue 103.85P paid 9.25
10:04:01 JPY 1Y 115.00P 9.25/9.45 seller

Brokers are paid commission on the notional (size) of trades they transact. Brokerage rates (often just called "bro" on the trading desk) are quoted in dollar-per-dollar terms, meaning the level of commission in U.S. dollar terms for each USD1m of notional transacted. It is important that traders know the brokerage rates in different broking shops for their currency pairs since that should (at least partially) determine which broker to place an interest with. In general, the more liquid the currency pair, the lower the brokerage rates.

There is an enormous amount of flexibility within this transaction structure. Brokers can work an interest in many different ways. For example, a broker can "build size" at a particular level (i.e., get USD200m on a particular bid for the interest to give all in one go) or they can work an interest more quickly or slowly to get the best fill possible. However, this flexibility inevitably means that part of the broker's skill sometimes involves manipulating the trader's impression of the market in order to get them to trade. Sales 101: Create a sense of urgency:

- "If I show this out, I'm sure it will get paid ahead of you. You should pay it yourself."
- "The guy is pretty shaky on his bid—I think he's about to ref it. You should give it."

In general, the more traders put in, the more they get out of the broker market. By making prices (i.e., being a good liquidity provider), brokers will work harder to get better fills on the trader's interests. Traders who follow the broker market closely (and therefore know the prevailing market sentiment for different types of contract) and have their own pricing up-to-date maximize their chances of transacting at good levels.

The Black-Scholes Framework

Derivatives products have been traded in one form or another for centuries, but the development of the Black-Scholes model in the 1970s enabled financial derivatives markets to flourish by enabling volatility to be consistently priced.

Financial mathematics books generally give the derivation of the Black-Scholes formula and list the reasons why the assumptions underpinning it aren't correct in practice. Traders don't need to know how to derive the Black-Scholes formula from scratch. However, it is vital that they understand the *features* of the Black-Scholes framework since it is the foundation for all derivatives valuation.

Black-Scholes Stochastic Differential Equation (SDE)

The Black-Scholes framework assumes that the price of the underlying (i.e., the FX spot rate) follows a geometric Brownian motion. The Black-Scholes stochastic differential equation (SDE) is:

$$\frac{dS_t}{S_t} = (rCCY2 - rCCY1)dt + \sigma dW_t$$

where S_t is the price of the underlying (spot) at time t, dS_t is the change in underlying at time t, $rCCY1$ and $rCCY2$ are continuously compounded (see Chapter 10) CCY1 and CCY2 interest rates respectively, σ is the volatility of the underlying's returns, generally just called "volatility," and W_t is a Brownian motion. Sometimes, $rCCY1$

is called the *foreign* interest rate and $rCCY2$ the *domestic* interest rate because, as seen in Chapters 1 and 2, P&L on standard FX contracts is naturally generated in CCY2 terms.

The left-hand side of the SDE represents *relative changes* in the underlying (often called "returns"). Relative changes are used within the model because as the underlying gets smaller (closer to zero), changes get smaller in absolute terms. Therefore, spot in the model can never hit zero, as in real life for FX (note that an equity underlying *could* go to zero).

The right-hand side of the SDE has two parts:

1. *Drift* from the interest rate differential
2. *Uncertainty* from the volatility of the underlying

Drift

Drift is a predictable, deterministic component that depends on the interest rate differential and the time passed:

$$(rCCY2 - rCCY1)dt$$

The drift gives the no-arbitrage expected future value of spot (i.e., the forward). Forward rates for different maturities in the future define the **forward path**.

If $\sigma = 0$ (i.e., no volatility), then:

$$\frac{dS_t}{S_t} = (rCCY2 - rCCY1)dt$$

which is solved by:

$$F_T = S_0 e^{(rCCY2 - rCCY1).T}$$

Plus recall from Chapter 1 that:

$$F_T = S_0 + SwapPoints_T$$

where F_T is the forward to time T and S_0 is current spot plus note the outrageous variable change from S (spot) to F (forward).

This is important: *Zero volatility does not mean that spot is static; it means that spot perfectly follows the forward path.*

Under Black-Scholes assumptions, the forward path is based on current spot and constant interest rates:

- If CCY1 and CCY2 interest rates are equal, the forward path will equal spot.
- If CCY2 interest rates are higher than CCY1 interest rates, the forward path moves higher as T increases. This is called *positive drift*.

- If CCY1 interest rates are higher than CCY2 interest rates, the forward path moves lower as T increases. This is called *negative drift*.

Within this simplified framework, at a given maturity, either the forward plus one interest rate can be used to calculate the other interest rate or two interest rates can be used to calculate the forward. All issues regarding credit risk and basis risk are ignored within this analysis.

For example: $rCCY1 = 0\%$ and $rCCY2 = 10\%$. CCY2 interest rates are higher than CCY1 interest rates and therefore there is positive drift. At shorter time-scales the forward path looks linear as shown in Exhibit 5.1.

Pushing the maturity out to ten years, the exponential nature of the function reveals itself in Exhibit 5.2.

This is important when pricing long-dated options. Exhibit 5.3 shows the USD/TRY forward path generated using constant rates to the 10yr tenor under Black-Scholes versus a market forward path generated using different interest rate instruments at different maturities.

When pricing vanilla options or any product where the payoff depends only on the spot at maturity, the forward path within the model isn't a concern so long as the forward to maturity is correct. However, the forward path is an important consideration when pricing **path-dependent options**, that is, options where the payoff depends not just on spot at expiry, but on the *path* that spot takes to get there. Many exotic options are path dependent. Consider an exotic derivative product in USD/TRY that will expire if spot ever trades above 2.5000. Using constant

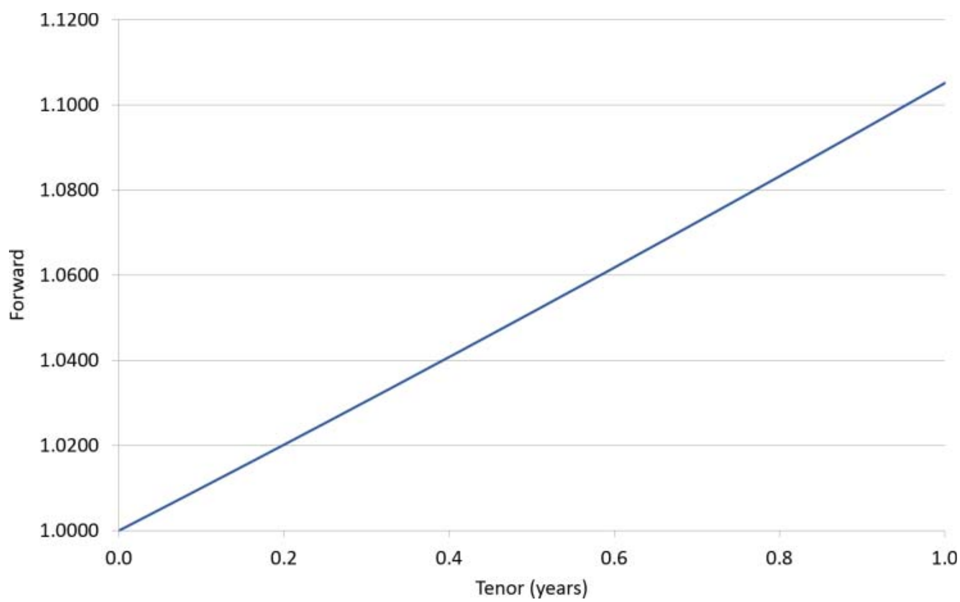


EXHIBIT 5.1 Short-term forward path

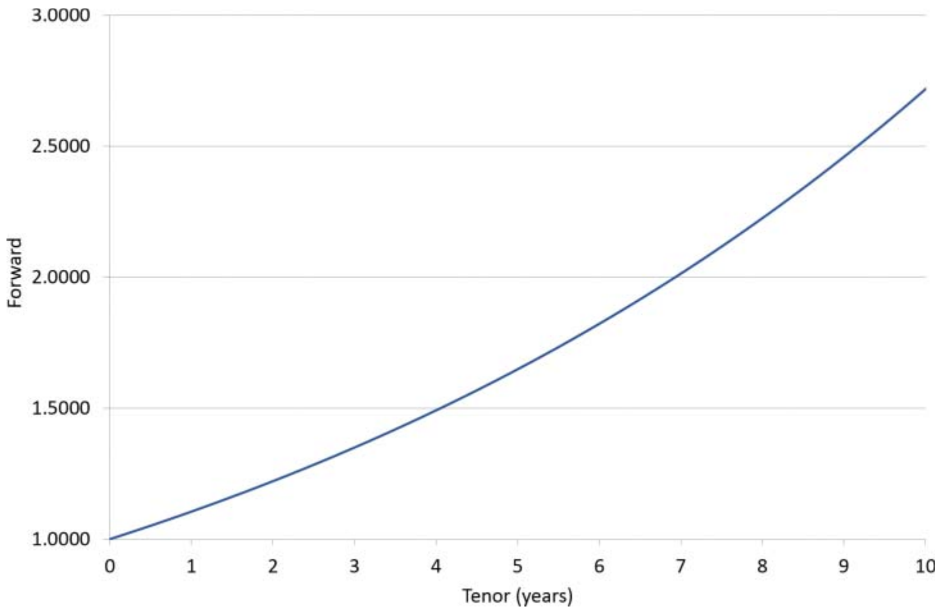


EXHIBIT 5.2 Long-term forward path

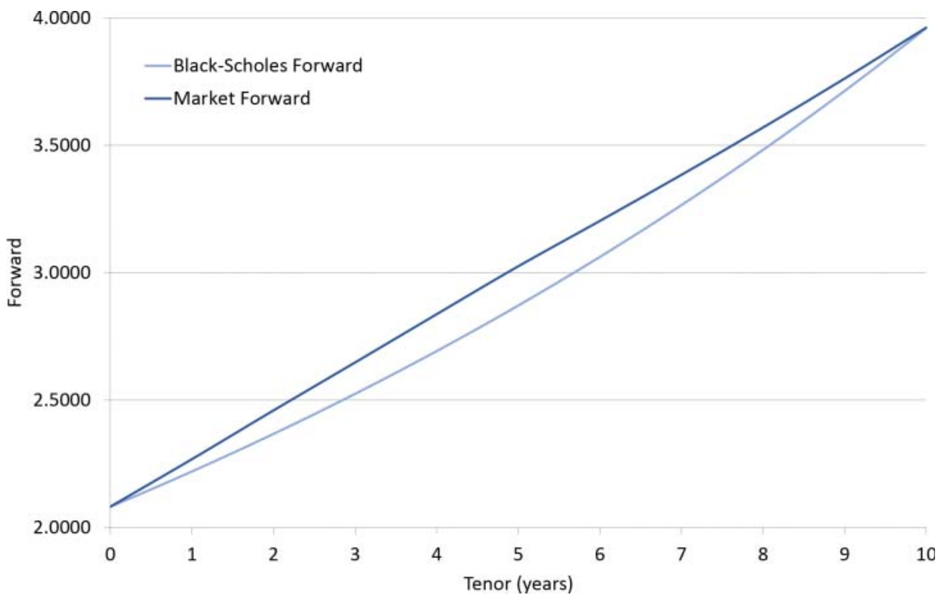


EXHIBIT 5.3 USD/TRY model versus market forward path

interest rates under Black-Scholes will generate different trading exposures than using the full market interest rate curve. Issues like this are therefore very important in practice.

Within the SDE, using full interest rate curves is equivalent to making the interest rates functions of time:

$$\frac{dS_t}{S_t} = (\boldsymbol{rCCY2}(\boldsymbol{t}) - \boldsymbol{rCCY1}(\boldsymbol{t}))dt + \sigma dW_t$$

Uncertainty

The uncertainty term in the SDE is driven by a Wiener process W_t (also called *Brownian motion*). A Wiener process is a continuous stochastic process with stationary independent increments. Translating:

- "Continuous" means "its path doesn't jump."
- "Stochastic" means "it moves."
- "Stationary" means "its probability distribution does not change over time."
- "Independent increments" means "each change does not depend on any previous changes."

Changes in W are random with this distribution:

$$(W_{t+\varepsilon} - W_t) \sim N(0, \varepsilon)$$

In words, the change in W from time *now* to time (*now* $+ \varepsilon$) is normally distributed with mean 0 and variance ε (i.e., standard deviation $\sqrt{\varepsilon}$).

The fact that the Black-Scholes SDE is driven by a normally distributed process explains why bell-curve shapes appears repeatedly within the Black-Scholes framework.

A discrete realization of W can be plotted in Excel using code shown in Exhibit 5.4 and a sample realization is plotted in Exhibit 5.5.

	A	B	C	D	E	F	G
1							
2		ε	0.003968	← *Named:* ***eta*** =1/252			
3							
4		Step	W(t)				
5		0	0				
6		1	0.04637	=C5+NORMSINV(RAND())*SQRT(eta)			
7		2	0.222032	=C6+NORMSINV(RAND())*SQRT(eta)			
8		3	0.258739	=C7+NORMSINV(RAND())*SQRT(eta)			
9		4	0.170459	etc...			
10		5	0.164376				

EXHIBIT 5.4 Excel setup for generating a realization of a Wiener process

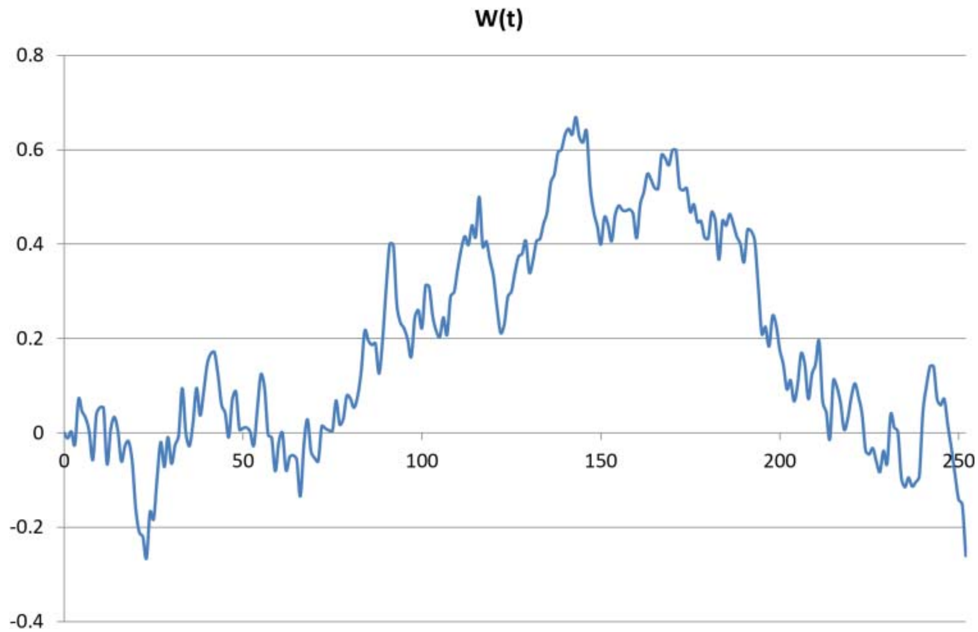


EXHIBIT 5.5 A sample realization of a Wiener process

Within the Black-Scholes SDE, the Wiener process W_t is multiplied by the volatility, meaning that, as expected, higher volatility causes spot to move more:

$$\frac{dS_t}{S_t} = (rCCY2 - rCCY1)dt + \boldsymbol{\sigma dW_t}$$

Solving the Black-Scholes SDE

The Black-Scholes SDE is solved using the magic of Itō Calculus:

$$\begin{aligned} ln\left(\frac{S_T}{S_0}\right) &= \int_0^T \left(rCCY2 - rCCY1 - \frac{\sigma^2}{2}\right) dt + \int_0^T \sigma dW_t \\ &= \left(rCCY2 - rCCY1 - \frac{\sigma^2}{2}\right) T + \int_0^T \sigma dW_t \\ &= \left(rCCY2 - rCCY1 - \frac{\sigma^2}{2}\right) T + \sigma W_T \end{aligned}$$

The key points to note are that we've moved from regular-space into log-space and the drift has been adjusted by the Itō correction term: $-\frac{\sigma^2}{2}$.

Furthermore, this term: σW_T is normally distributed with mean 0 and variance $\sigma^2 T$ (i.e., standard deviation $\sigma\sqrt{T}$). This is important because it shows how volatility and time to expiry are linked within the distribution. Ignoring the adjusted drift, multiplying time to expiry by four changes the terminal spot distribution in the same way as doubling ($\sqrt{4} = 2$) the implied volatility.

The previous formula shows that the adjusted forward drift is the central reference point of the future log-spot distribution, which at each point is normally distributed with a wider and wider distribution over time due to increasing variance. This is shown in Exhibit 5.6.

Because we're now in log-space, spot *log returns* are normally distributed. This is why log returns are always used within the realized spot volatility calculations in Chapter 17.

Understanding log-normality is important because it impacts distributions and Greek profiles particularly at higher volatility or longer maturity. For example, plotting the terminal spot distribution for 1mth EUR/USD at 8% volatility gives the standard-looking bell-shaped curve in Exhibit 5.7.

However, if volatility is raised to 30% and maturity is increased to five years, the shape of the distribution changes dramatically and the log-normality becomes apparent in Exhibit 5.8.

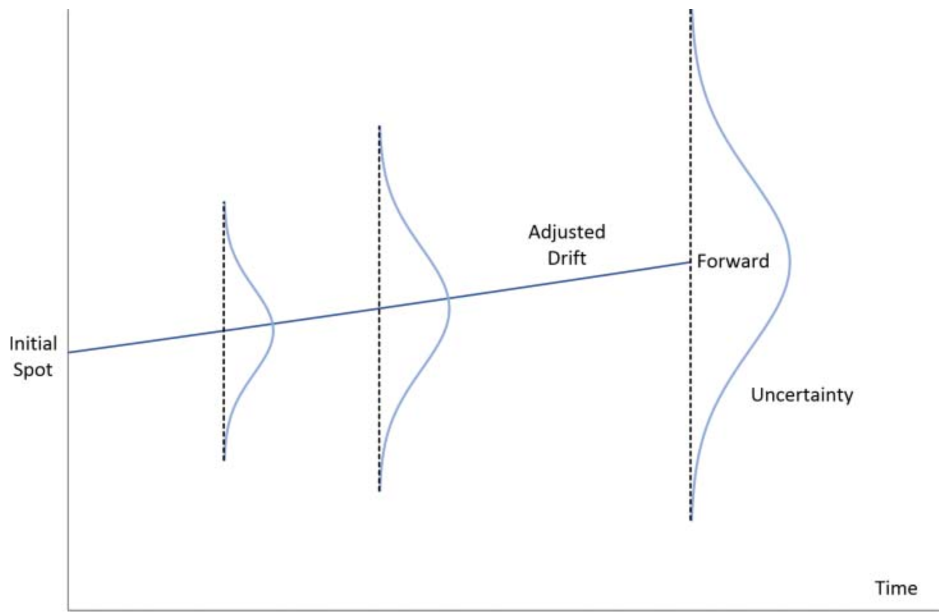


EXHIBIT 5.6 Representation of the Black-Scholes framework

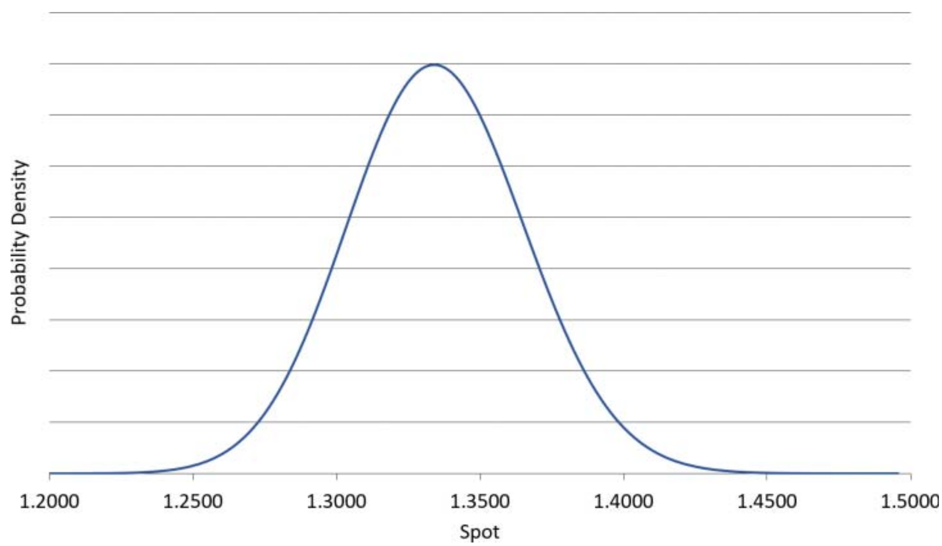


EXHIBIT 5.7 Terminal spot distribution at short tenor and low volatility

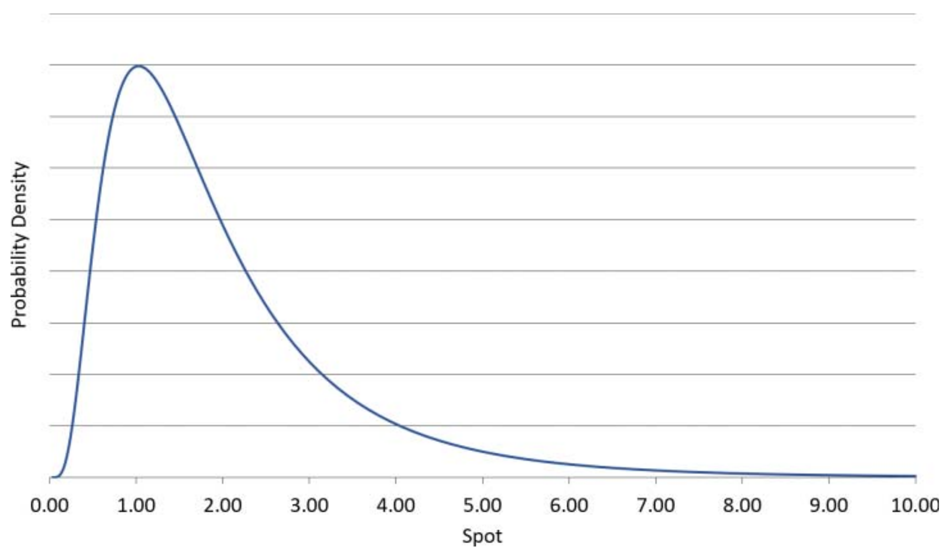


EXHIBIT 5.8 Terminal spot distribution at long tenor and high volatility

In a log-normal world, a spot move from 1.0 to 0.5 (log return = –0.693) is equal and opposite to a spot move from 1.0 to 2.0 (log return = +0.693). Hence log-normal distributions have a longer tail on the topside in regular spot space and never go below zero.

EXHIBIT 5.9 **Sample Implied Volatility Term Structure**

Tenor	Implied Volatility
1mth	5.0%
2mth	6.0%
3mth	7.0%
6mth	10.0%
1yr	15.0%

By taking exponentials, the SDE solution gives this analytic solution for S at time t:

$$S_t = S_0 e^{\left(rCCY2 - rCCY1 - \frac{\sigma^2}{2}\right)t + \sigma W_t}$$

The Black-Scholes formula uses constant volatility. This must be changed to the full ATM term structure when pricing path-dependent options. Within the SDE, this is equivalent to making volatility a function of time:

$$\frac{dS_t}{S_t} = (rCCY2 - rCCY1)dt + \sigma(t)dW_t$$

Consider the sharply upward-sloping implied volatility term structure in Exhibit 5.9.

Two realizations of S can be generated using the same W, one using flat volatility and the other using the implied volatility term structure. This is shown in Exhibit 5.10. Using the term structure of implied volatility leads to lower volatility at shorter tenors and higher volatility at longer tenors.

Within this basic Black-Scholes framework, there is only a single volatility. However, in practice, vanilla options with different maturities and strikes have different implied volatilities. In a given currency pair, the implied volatility for a given strike and expiry date is determined by the volatility surface for that pair. This idea is explored in Chapter 7.

Calculating Option Values Using Terminal Spot Distributions

Terminal spot distributions can be used to price vanilla options or any other derivative product where the payoff depends only on spot at the option maturity. The value of the option can be obtained by integrating the option payoff at maturity against the terminal spot distribution as shown in Exhibit 5.11. Intuitively, this calculation multiplies the probability of spot ending up at each point by the option payoff at that spot level. This technique is implemented in Practical B.

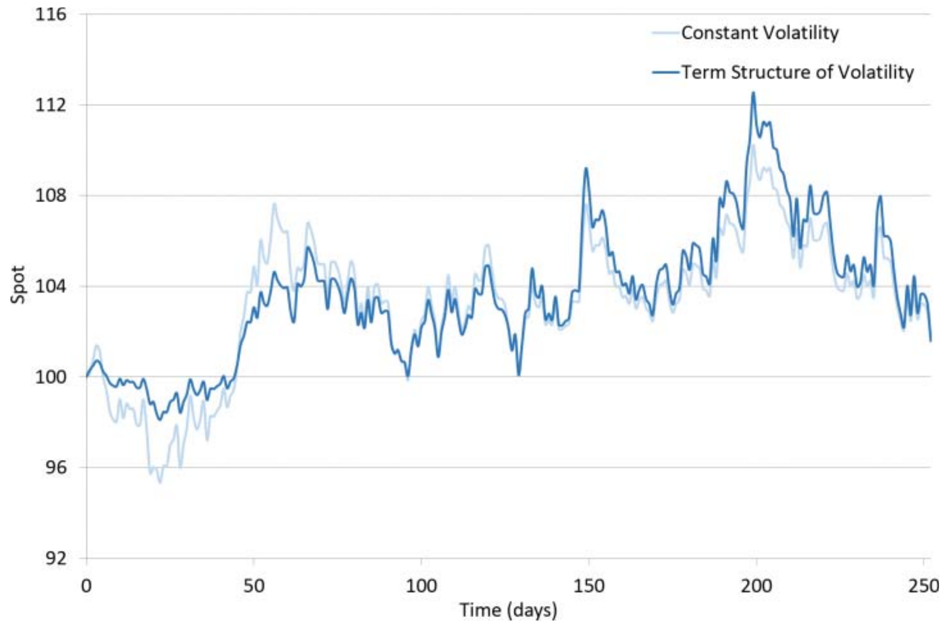


EXHIBIT 5.10 Realizations of a Wiener process using different implied volatility term structures

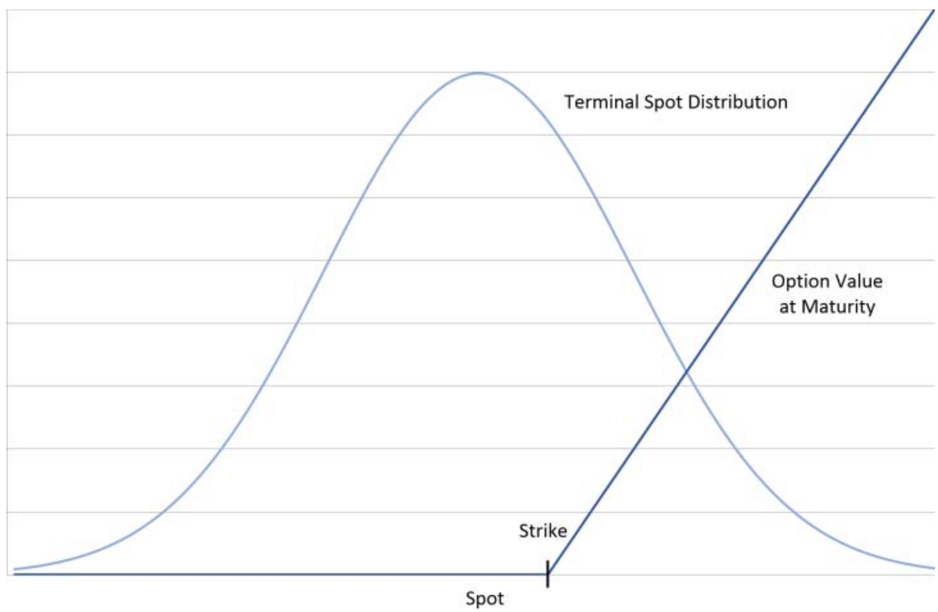


EXHIBIT 5.11 Valuing vanilla options using the terminal spot distribution

The Black-Scholes Formula

Finally, we arrive at the **Black-Scholes formula** itself, which gives prices for European vanilla calls and puts. The **Garman and Kohlhagen** (1983) formula is the FX-specific extension to the Black-Scholes formula that uses interest rates in both currencies:

$$Price_{call} = P_{call} = Se^{-rCCY1.T}N(d_1) - Ke^{-rCCY2.T}\,N(d_2)$$

$$Price_{put} = P_{put} = Ke^{-rCCY2.T}N(-d_2) - Se^{-rCCY1.T}\,N(-d_1)$$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(rCCY2 - rCCY1 + \frac{1}{2}\sigma^2\right).\,T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\ln\left(\frac{S}{K}\right) + \left(rCCY2 - rCCY1 - \frac{1}{2}\sigma^2\right).\,T}{\sigma\sqrt{T}}$$

These formulas are implemented in Practical C.

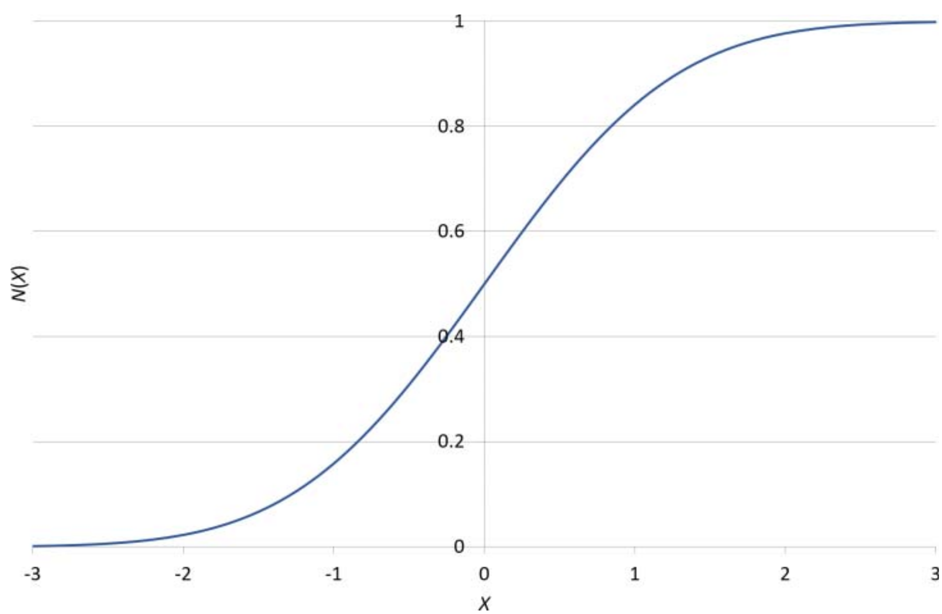


EXHIBIT 5.12 Cumulative normal distribution function

The key to the derivation of the Black-Scholes formula is the assumption that option value can be continuously delta hedged at no cost. This removes all sources of risk except for volatility. The main driver behind the formula is $N(X)$, the cumulative normal distribution function, which gives the probability that a normally distributed variable with mean 0 and standard deviation 1 will have a value less than or equal to X. Exhibit 5.12 shows a graph of the cumulative normal distribution function.

Even though the assumptions underpinning the Black-Scholes framework do not hold in practice, that isn't a day-to-day concern for traders. The main way in which the Black-Scholes formula itself is used is as a method of going between volatility pricing and premium pricing. It is instructive to note that the simplicity of the Black-Scholes framework is one of the key reasons why it is still in use decades after it was developed. Another reason is its extendibility; all the pricing models discussed in Chapter 19 extend Black-Scholes by relaxing different assumptions within the framework.

Building a Numerical Integration Option Pricer in Excel

When an option payoff depends only on the spot rate at the maturity of the contract (e.g., European vanilla options) the price of the option can be calculated using the terminal spot distribution and the option payoff.

Task A: Set Up the Terminal Spot Distribution

Step 1: Set Up the Future Spots

First, future spot levels must be generated using a log-normal distribution. The inputs to the function are:

- Spot (*S*): the current exchange rate in a given currency pair
- Interest rates (*rCCY1* and *rCCY2*): continuously compounded risk-free interest rates in CCY1 and CCY2 of the currency pair
- Time to expiry (*T*): the time between the horizon date and expiry date measured in years
- Volatility (σ): the volatility of the spot log returns

Within log-normal world:

$$Expected\ Return\ (\mu) = \left(rCCY2 - rCCY1 - \frac{\sigma^2}{2}\right).T$$

$$Standard\ Deviation = \sigma\sqrt{T}$$

For a given return of X standard deviations:

$$Return\ Level = \mu + X\ .\ \ Standard\ Deviation$$

$$Spot\ Level = S\ .\ e^{Return\ Level}$$

This framework can be set up in an Excel sheet:

Numerical Integration Option Pricer

Market Data Inputs

Spot	**1.3360**	←*Named:* ***Spot***
Time to Maturity (years)	**1.00**	←*Named:* ***T***
CCY1 Interest Rate	**0.05%**	←*Named:* ***rCCY1***
CCY2 Interest Rate	**0.02%**	←*Named:* ***rCCY2***
Volatility (σ)	**8.00%**	←*Named:* ***vol***

Calculated Values

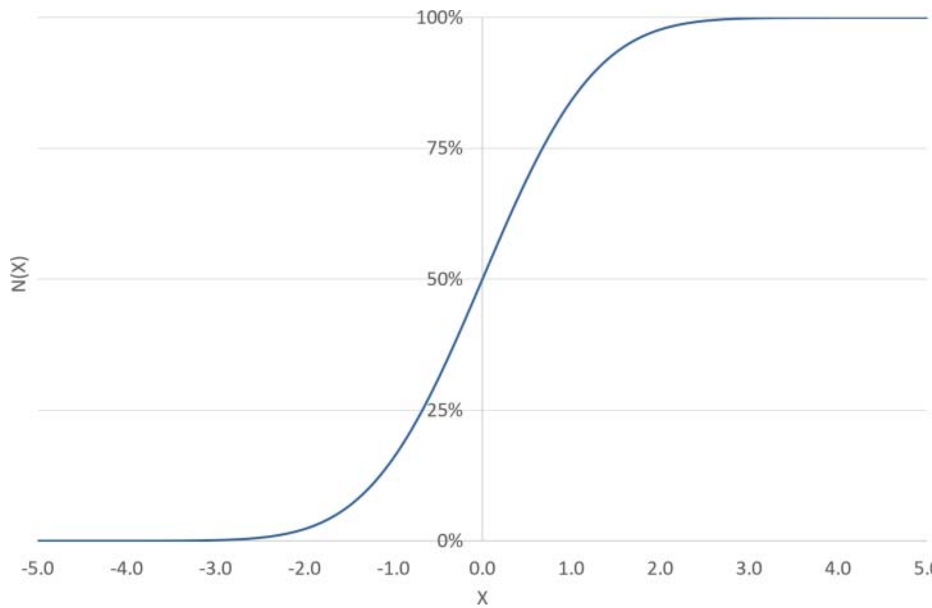
Expected Return (μ)	-0.35%	←*Named:* ***ExpectedReturn*** =(rCCY2-rCCY1-0.5*vol^2)*T
Standard Deviation ($\sigma\sqrt{T}$)	8.00%	←*Named:* ***StandardDeviation*** =vol*SQRT(T)

Under a normal distribution, a range from –5 to +5 standard deviations covers almost all possible theoretical returns. Starting with 0.1 steps, go from –5 to +5 standard deviations and calculate the return level and corresponding spot level for each standard deviation value:

	E	F	G	H	I	J	K
1							
2	**X**	**Return Level**	**Spot Level**		**Return Level**		
3	-5.0	-40.35%	0.8924		=ExpectedReturn+E3*StandardDeviation		
4	-4.9	-39.55%	0.8996		**Spot Level**		
5	-4.8	-38.75%	0.9068		=Spot*EXP(F3)		
6	-4.7	-37.95%	0.9141				
7	-4.6	-37.15%	0.9214				

Step 2: Calculate the Probability Density

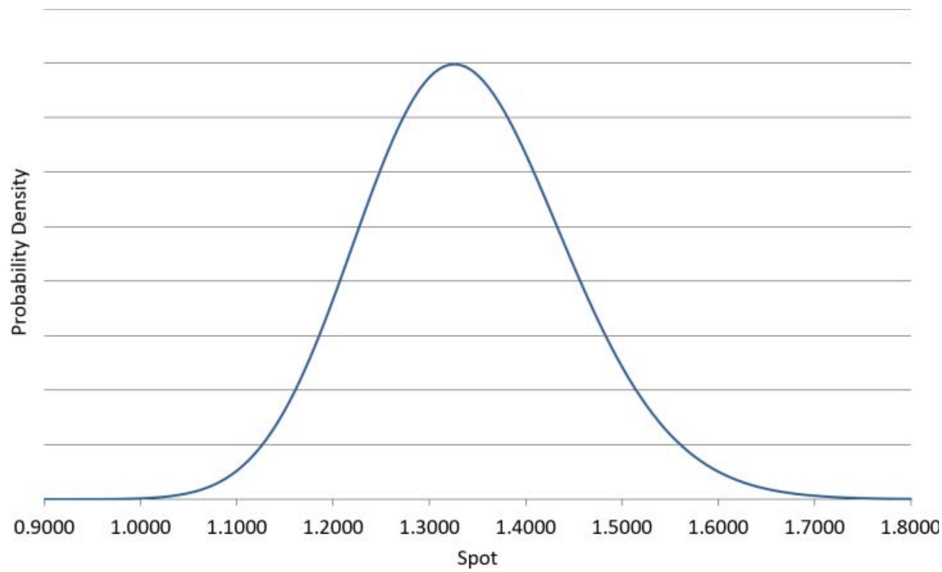
The probability density function gives the relative likelihood of a random variable falling within a particular range of values. In Excel, =NORMSDIST(X) gives the cumulative normal distribution function, which is the probability of a normally distributed random variable with mean 0 and standard deviation 1 being *at or below* the input level X. Therefore, the probability of being between two levels (i.e., the probability density) can be calculated by taking the difference between two cumulative probabilities:



	E	F	G	H	I	J	K
1							
2	X	Return Level	Spot Level	Cumulative Prob.	Prob. Density		e.g., Cumulative Probability H3
3	-5.0	-40.35%	0.8924	0.00%	0.00%		=NORMSDIST(E3)
4	-4.9	-39.55%	0.8996	0.00%	0.00%		
5	-4.8	-38.75%	0.9068	0.00%	0.00%		e.g., Probability Density I3
6	-4.7	-37.95%	0.9141	0.00%	0.00%		=(H4-H3)
7	-4.6	-37.15%	0.9214	0.00%	0.00%		
8	-4.5	-36.35%	0.9288	0.00%	0.00%		

Note how the data in the rows is lined up; the probability density in a given row gives the probability of spot ending up between that spot level and the spot level in the row below.

The probability density can be plotted against spot to visualize the terminal spot distribution:



The implementation can be tested by changing the market data and observing how the terminal spot distribution changes. As explained in Chapter 5:

- Shorter maturity or lower volatility should lead to a tighter distribution.
- Longer maturity or higher volatility should lead to a wider distribution.
- Higher CCY2 interest rates or lower CCY1 interest rates should shift the distribution higher via the forward moving higher.
- Higher CCY1 interest rates or lower CCY2 interest rates should shift the distribution lower via the forward moving lower.

Task B: Set Up the Option Payoff and Calculate the Option Price

The option payoff can now be added into the framework. This numerical integration method can be used to price any payoff that only depends on spot at maturity, no matter how complicated, but the most obvious examples are:

- Long forward: $S_T - K$
- Short forward: $K - S_T$
- Vanilla call option: $max(S_T - K, 0)$
- Vanilla put option: $max(K - S_T, 0)$

Remember that these payoffs all return values in CCY2 per CCY1 (i.e., CCY2 pips) terms.

In Excel, add a new "option payoff" column and calculate the payoff at each spot level. To price a vanilla call option, the strike must be inputted:

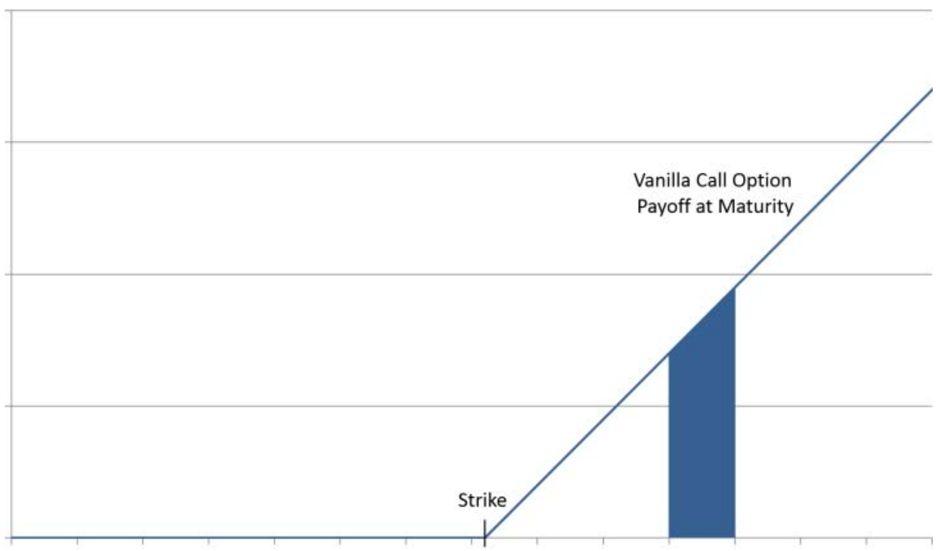
Payoff Inputs

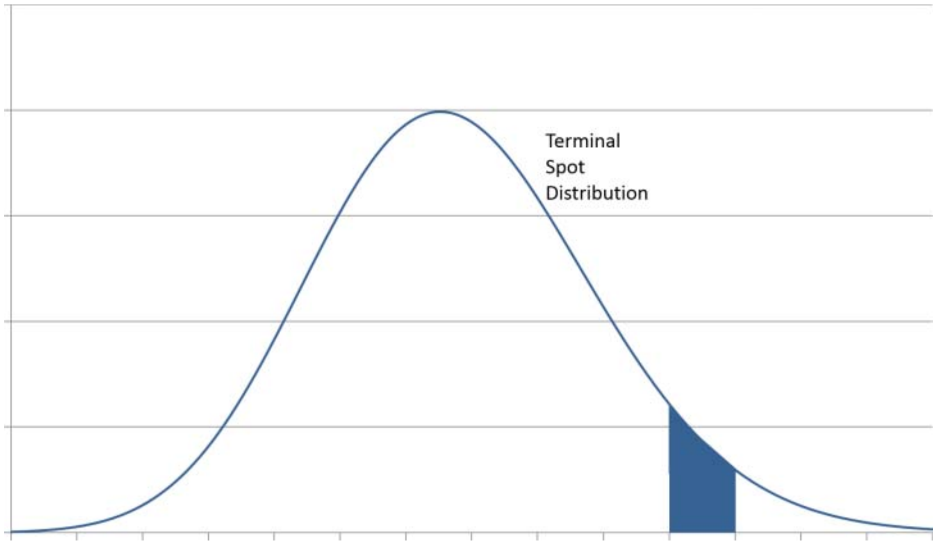
Strike	**1.3600**	←*Named:* ***Strike***

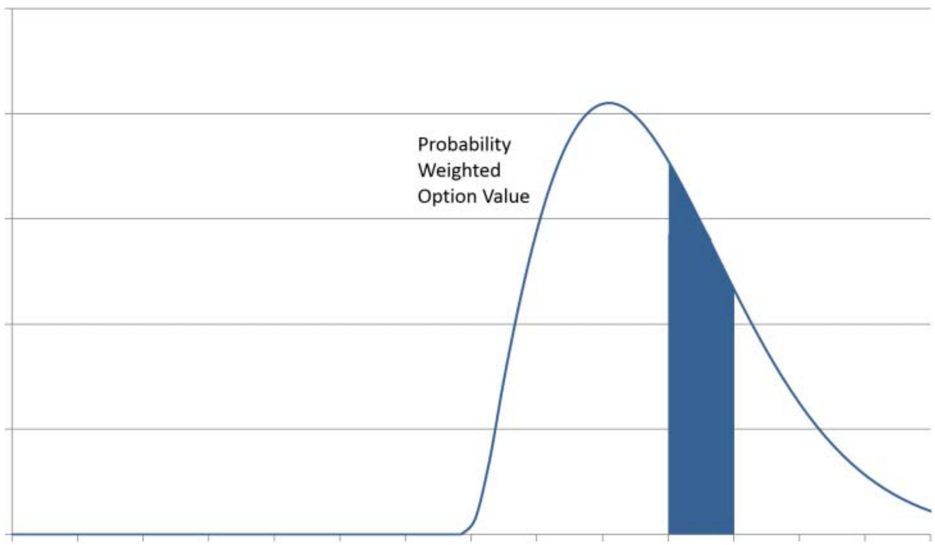
Then the option payoff at maturity can be calculated at each spot level:

	E	F	G	H	I	J	K	L
1								
2	**X**	**Return Level**	**Spot Level**	**Cumulative Prob.**	**Prob. Density**	**Option Payoff (CCY2 pips)**		
3	-5.0	-40.35%	0.8924	0.00%	0.00%	0.0000	=MAX(G3-Strike,0)	
4	-4.9	-39.55%	0.8996	0.00%	0.00%	0.0000	=MAX(G4-Strike,0)	
5	-4.8	-38.75%	0.9068	0.00%	0.00%	0.0000	...	
6	-4.7	-37.95%	0.9141	0.00%	0.00%	0.0000		

Within the numerical integration, multiply the probability of spot falling between two spot levels at maturity by the average payoff at maturity between two spot levels:







	I	J	K	L	M
1					
2	Prob. Density	Option Payoff (CCY2 pips)	Weighted Option Payoff (CCY2 pips)		
3	0.00%	0.0000	0.0000	=AVERAGE(J3,J4)*I3	
4	0.00%	0.0000	0.0000	=AVERAGE(J4,J5)*I4	
5	0.00%	0.0000	0.0000	...	
6	0.00%	0.0000	0.0000		

Probability-weighted option values are then summed to get the overall option value at maturity. The CCY2 pips option value must then be present valued (see Chapter 10) using the discount factor ($e^{-rCCY2.T}$) and converted into CCY1% by dividing by current spot:

Call Payoff Output	↓Named: ***FutValOptionValue***	
Future Valued Option Value (CCY2 pips)	0.0320	=SUM(K3:K102)
	↓Named: ***OptionValuePips***	
Present Valued Option Value (CCY2 pips)	0.0320	=EXP(-rCCY2*T)*FutValOptionValue
Present Valued Option Value (CCY1%)	**2.39%**	=OptionValuePips/Spot

Testing

Finally, the pricer can be tested:

Test 1: A forward payoff struck *at the forward* should give (approximately) zero value:

Market Data Inputs

Spot	**100.00**
Time to Maturity (years)	**1.00**
CCY1 Interest Rate	**5.00%**
CCY2 Interest Rate	**0.00%**
Volatility (σ)	**10.00%**

Derived Values

Expected Return (μ)	-5.50%
Standard Deviation ($\sigma\sqrt{T}$)	10.00%

Forward Payoff Inputs

Strike	**95.1229**	=Spot*EXP((rCCY2-rCCY1)*T)

Forward Payoff Output

Future-Valued Payoff Value (CCY2 pips)	0.0016
Payoff Value (CCY2 pips)	0.0016
Payoff Value (CCY1%)	**0.0016%**

Test 2: A vanilla CCY1 call option with $S = K = 100$, $rCCY1 = rCCY2 = 0\%$, and $T = 1.0$ should have a value very slightly under 4.00 CCY1%:

Market Data Inputs

Spot	**100.00**
Time to Maturity (years)	**1.00**
CCY1 Interest Rate	**0.00%**
CCY2 Interest Rate	**0.00%**
Volatility (σ)	**10.00%**

Derived Values

Expected Return (μ)	-0.50%
Standard Deviation ($\sigma\sqrt{T}$)	10.00%

Call Payoff Inputs

Strike	**100.0000**

Call Payoff Output

Future Valued Option Value (CCY2 pips)	3.9969
Option Value (CCY2 pips)	3.9969
Option Value (CCY1%)	**3.9969%**

Vanilla FX Derivatives Greeks

It is time to start some derivative analysis. The aim of this chapter is to introduce the basic Greek exposures on European vanilla options. This is stylized Black-Scholes analysis with zero interest rates throughout; hence the forward rate is always equal to the spot rate and discounting considerations can be ignored. The charts within this chapter can be generated in Excel after completing Practical C.

Option Value

A vanilla call option gives the right, but not the obligation, at maturity to buy spot (i.e., buy CCY1 versus sell CCY2) at the strike in the agreed notional. Exhibit 6.1 shows the value at maturity of a long (bought) vanilla call option over different spot levels. This value at maturity is often described as the option *payoff*.

For a short (sold) vanilla call option, the value at maturity is reflected in the spot-axis resulting in an increasingly negative value above the strike, as shown in Exhibit 6.2. As expected, a long position plus short position in the same contract results in zero value over all spots (i.e., no position).

The initial option premium is sometimes added into these diagrams, as per Exhibit 6.3. Adding the premium into the payoff is appropriate if the plan is to transact the option and then hold it isolated until maturity (see the section on breakevens in Chapter 17). However, in delta hedged trading portfolios, many options are risk managed together. Therefore, it is *changes* in option value caused by

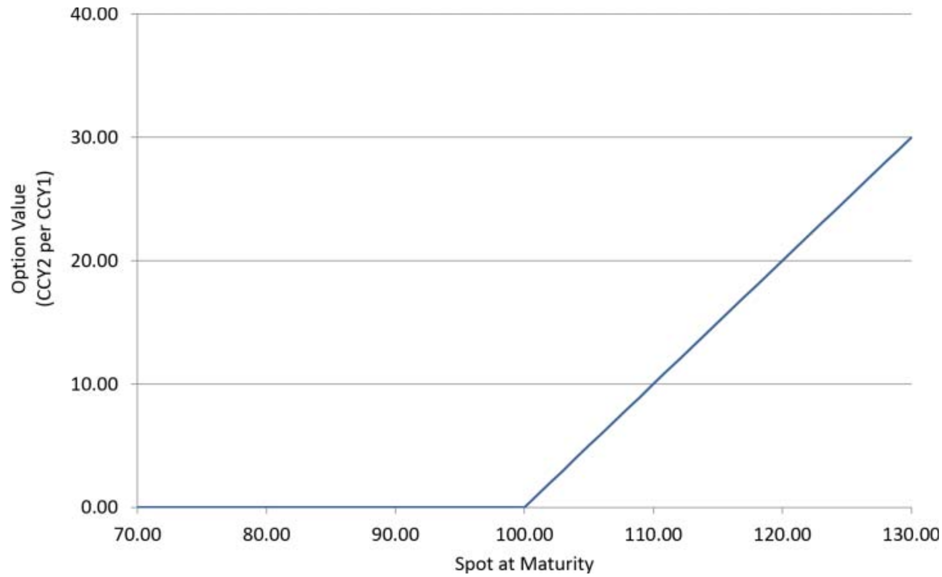


EXHIBIT 6.1 Value at maturity of long vanilla call option with 100.00 strike

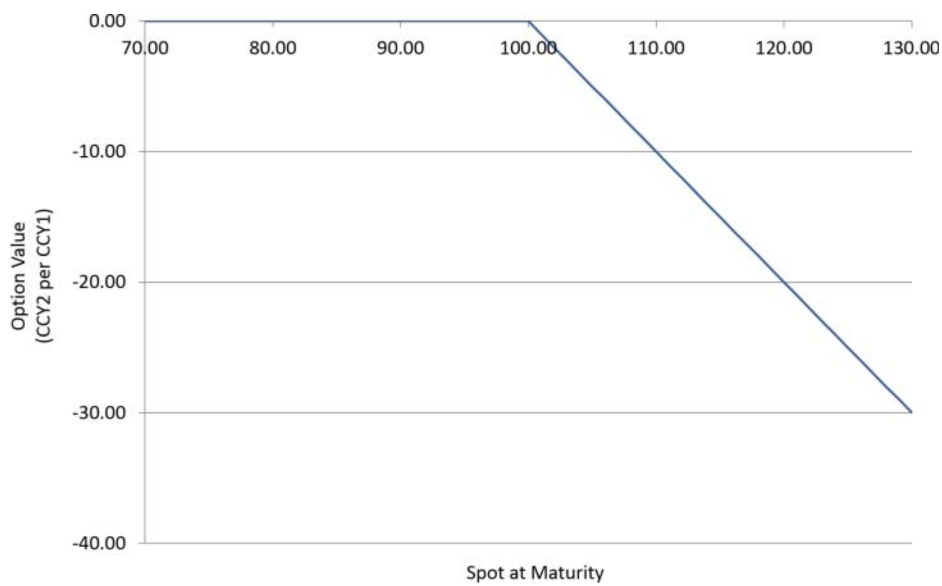


EXHIBIT 6.2 Value at maturity of short vanilla call option with 100.00 strike

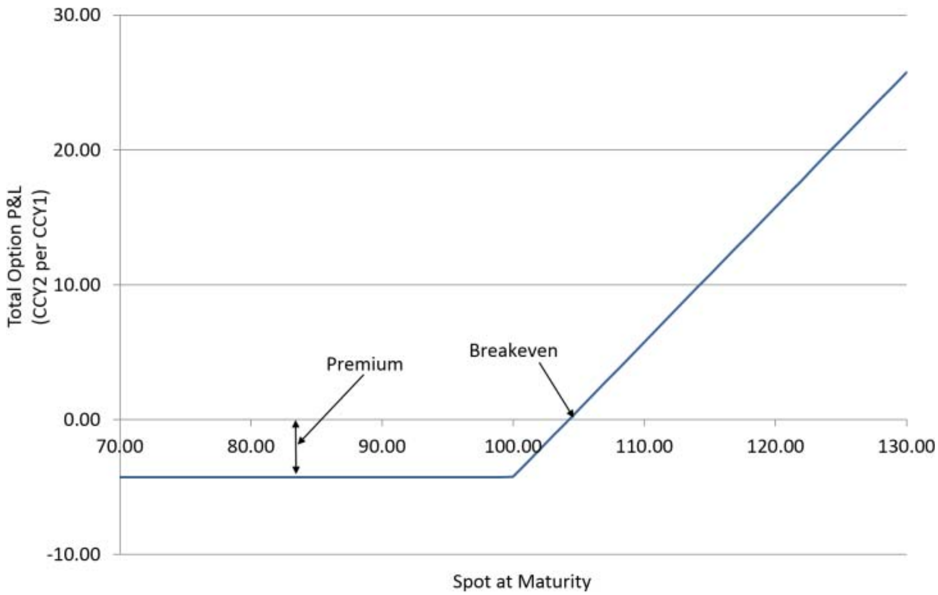


EXHIBIT 6.3 Total P&L at maturity (including initial premium) of long vanilla call option with 100.00 strike

changes in the market data that are most important, called the *exposures*. This analysis is slightly cleaner if premium is omitted.

A vanilla put option gives the right, but not the obligation, at maturity to sell spot (i.e., sell CCY1 versus buy CCY2) at the strike in the agreed notional. Exhibit 6.4 shows the value at maturity of a long vanilla put option over different spots.

Again, a short vanilla put option has a value at maturity that is reflected in the spot-axis. This results in increasingly negative value below the strike, as shown in Exhibit 6.5.

So far, so straightforward. Now see how the long vanilla call option value changes *prior to maturity* in Exhibit 6.6. Call option value prior to maturity is *convex* in spot, implying a positive second derivative (i.e., positive **gamma**, discussed later in this chapter).

The value of a vanilla option can be decomposed into **intrinsic value** plus **time value**:

- Intrinsic value is the option payoff at maturity.
- Time value is the value expected to be generated from the remaining **optionality** in the contract.

Optionality is the ability of the contract to transact spot (e.g., buy spot for a call option), *or not*, depending on the spot level at maturity. In the value at maturity charts, optionality is represented by the change in angle at the strike.

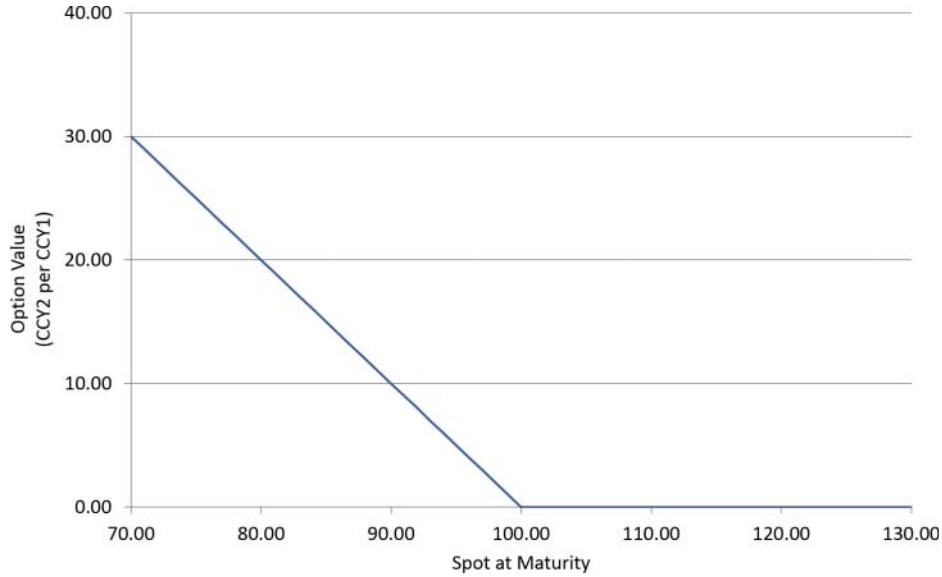


EXHIBIT 6.4 Value at maturity of long vanilla put option with 100.00 strike

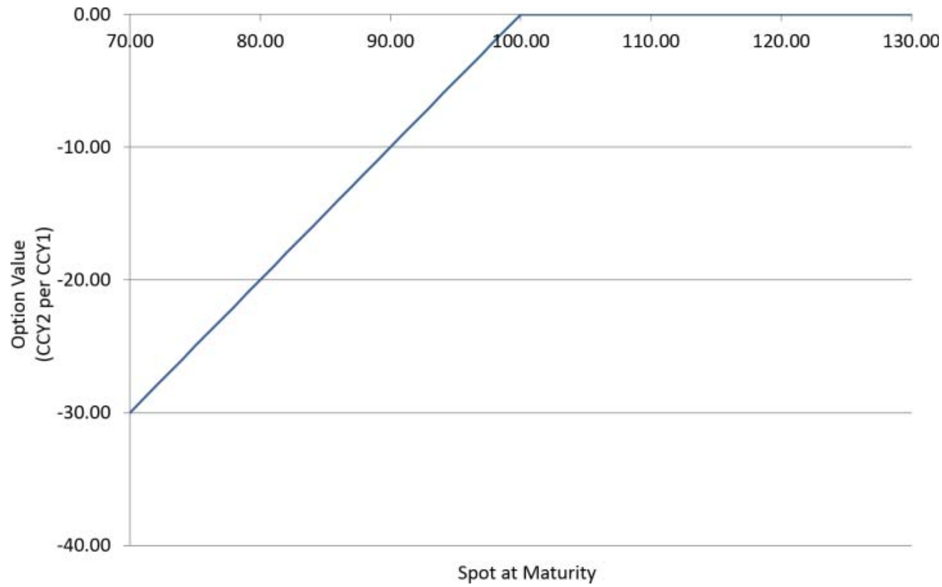


EXHIBIT 6.5 Value at maturity of short vanilla put option with 100.00 strike

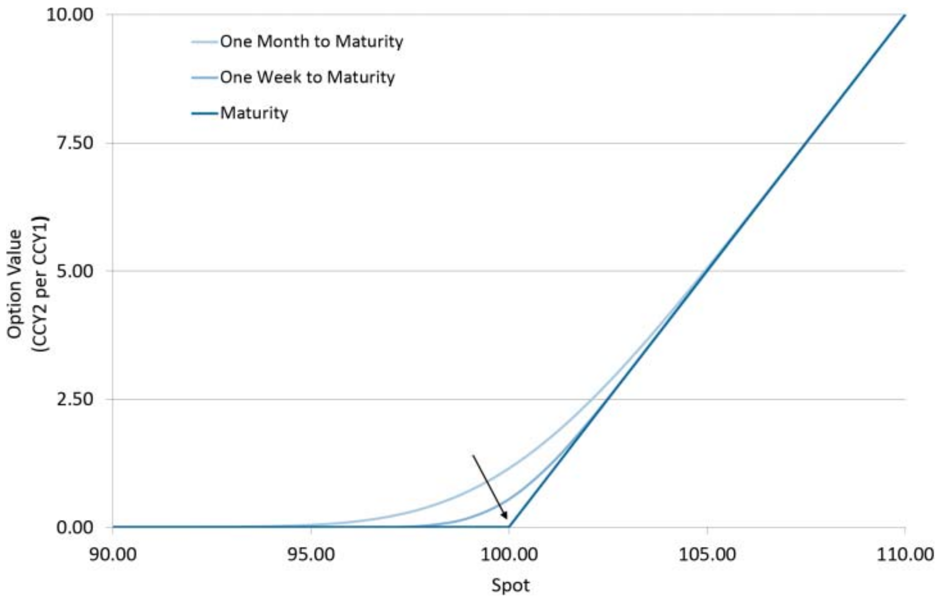


EXHIBIT 6.6 Value of long vanilla call option with 100.00 strike and 10% volatility

If the forward to maturity is far above or below the strike, the optionality has minimal value since there is little chance of spot going through the strike before maturity. Therefore, option value converges to intrinsic value away from the strike on both sides. Maximum time value occurs when the forward to maturity is equal to the strike; here the optionality is most valuable, as shown in Exhibit 6.7.

Looking at call option value at *higher volatility* in Exhibit 6.8 shows how higher volatility leads to higher time value as the distribution widens. Intuitively, if spot is more volatile, it will have more chances to go through the strike.

Therefore, the value of a vanilla option can give information about how much optionality there is within a particular contract. For low-premium vanilla options, there must be minimal intrinsic value and minimal time value on the contract. For high-premium vanilla options, there may be high intrinsic value, high time value, or both.

Within the value charts, the strike of the option is fixed and spot is being changed. If strike were being changed instead for a fixed spot, the diagram would be approximately reflected in the strike level. For example, if the strike moves higher on a call option, the premium decreases because the payoff above the strike is being moved further away from the forward to maturity.

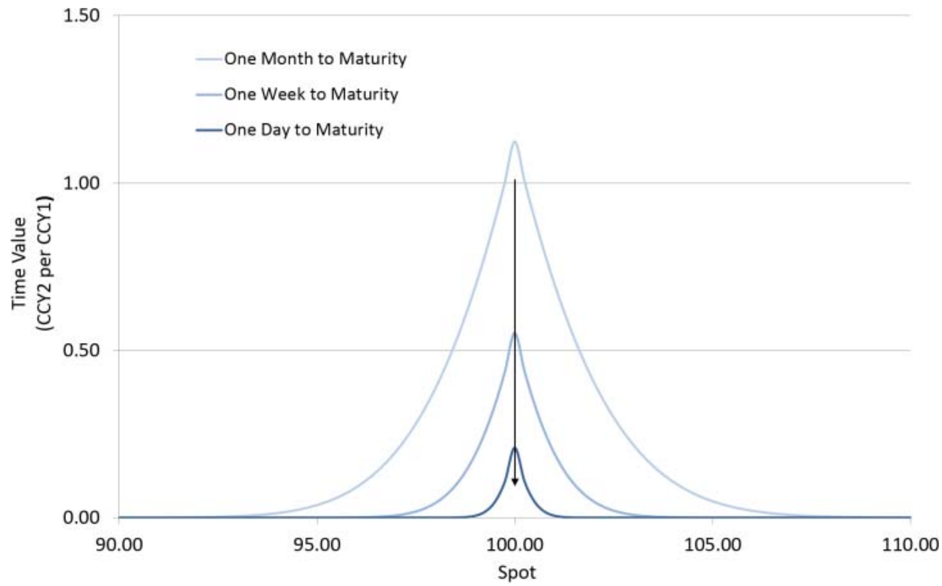


EXHIBIT 6.7 Time value of long vanilla call option with 100.00 strike and 10% volatility

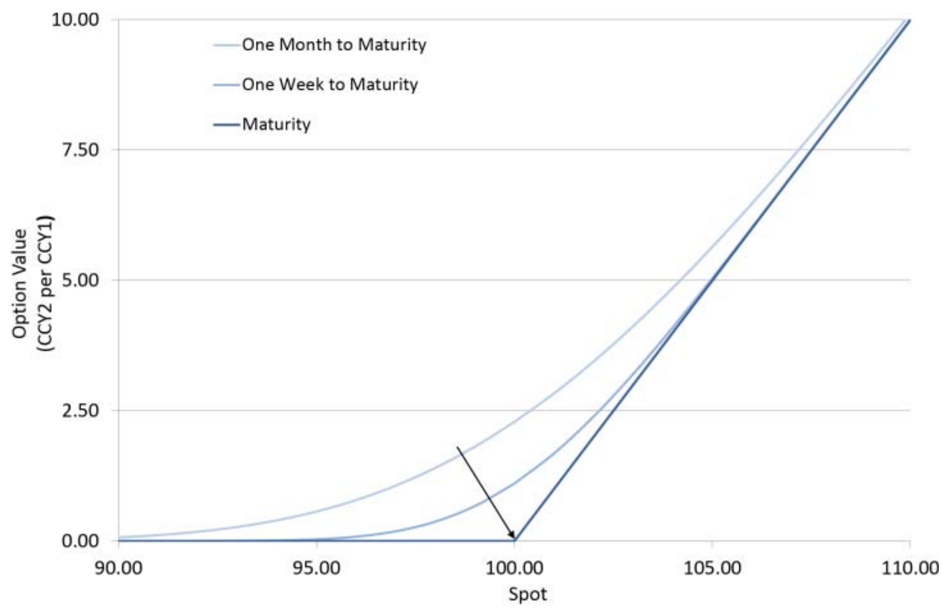


EXHIBIT 6.8 Value of long vanilla call option with 100.00 strike and 20% volatility

Delta

Taking the first derivative of option value with respect to spot gives one of the most important Greeks: **delta**, sometimes called **spot delta** for clarity. In symbols:

$$Delta\ (\Delta) = \frac{\partial P}{\partial S}$$

where P is option price and S is the spot rate. Note that traders sometimes say they are, e.g., "long spot" to mean "long exposure to spot" (i.e., long delta). In practice, delta is quoted either as a % of the notional amount, or as a cash amount in the notional currency where:

$$\Delta_{Cash} = \Delta_{\%} \times Notional$$

It is important to appreciate that forward delta can also be calculated, but this distinction will be overlooked for now:

$$Forward\ Delta\ (\Delta_F) = \frac{\partial P}{\partial F}$$

where F is the forward rate to the option maturity.

Delta is the exposure of the option value to the spot rate. The delta amount for a given option is therefore the equivalent spot notional that must be transacted (in the opposite delta direction) in order for the option plus the spot hedge to be **delta neutral** (i.e., zero delta exposure).

For a given option, delta can either be quoted in % terms or cash terms. For example, if an AUD/USD call has 25% spot delta and AUD40m of the contract is bought, AUD10m AUD/USD spot must be sold "on the hedge" in order to leave delta in the trading position unchanged. If the AUD/USD call was sold instead, spot must be bought on the hedge.

When a position is delta neutral, no P&L change results from spot moving higher or lower. However, this only works for small moves in spot if the second derivative (gamma) is non-zero.

Exhibit 6.9 shows the delta of a long vanilla call option with a 100.00 strike. This delta can be calculated by taking the *gradient* of the call option value profile chart from Exhibit 6.6.

At maturity, there is a discontinuity in delta from 0% below the strike to 100% above caused by either expiring the option below the strike (hence having no position) or exercising the option above the strike (hence buying spot in the full notional amount). Prior to maturity, the delta still goes from 0% to 100% but the change occurs over a wider spot range. Notice that the delta at all maturities is

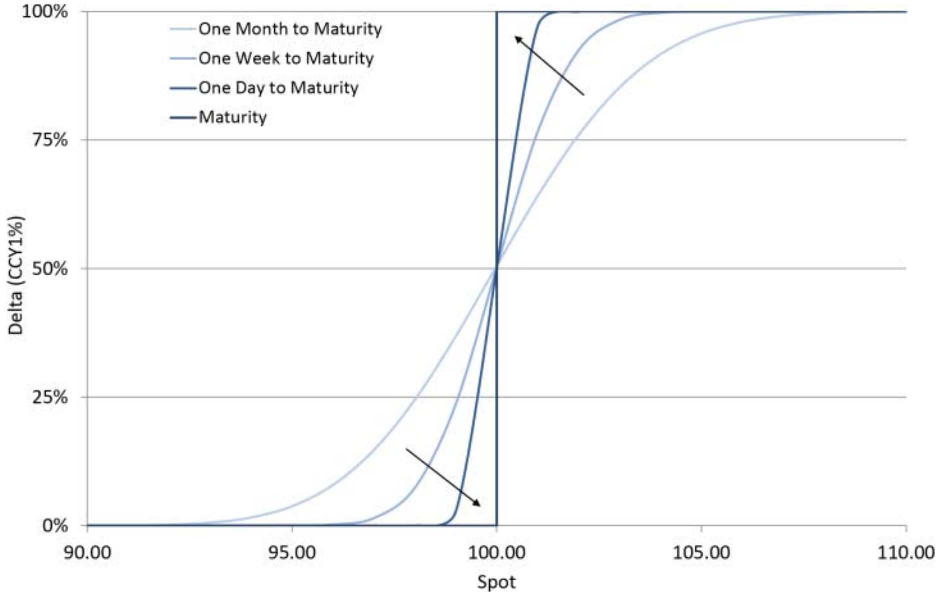


EXHIBIT 6.9 Delta of long vanilla call option with 100.00 strike

around 50% when spot (or more accurately, the forward to maturity) is equal to the strike.

Intuitively, it shouldn't be a surprise that a long call option position has a positive delta since the value of the call option increases with spot higher, the same as a long spot position.

Delta can also be (approximately) thought of as the % chance of ending up in-the-money (ITM) at maturity. Call options with strikes close to spot (the forward) have a delta of approximately 50% (i.e., a 50/50 chance of ending up ITM at maturity). As spot goes lower, the call option delta reduces as does the chance of ending up in-the-money.

Exhibit 6.10 shows the option value of a long vanilla put option with a 100.00 strike.

Again, taking the first derivative of this option value with respect to spot gives delta. The delta of the option value from Exhibit 6.10 is shown in Exhibit 6.11. Intuitively, put options have negative delta because put option value increases with spot lower, the same as a short spot position.

For example, if an AUD/USD put option has –10% spot delta and AUD80m of the contract is bought, AUD8m AUD/USD spot must be bought on the hedge in order to leave delta in the trading position unchanged. If the option was sold instead, spot must be sold on the hedge.

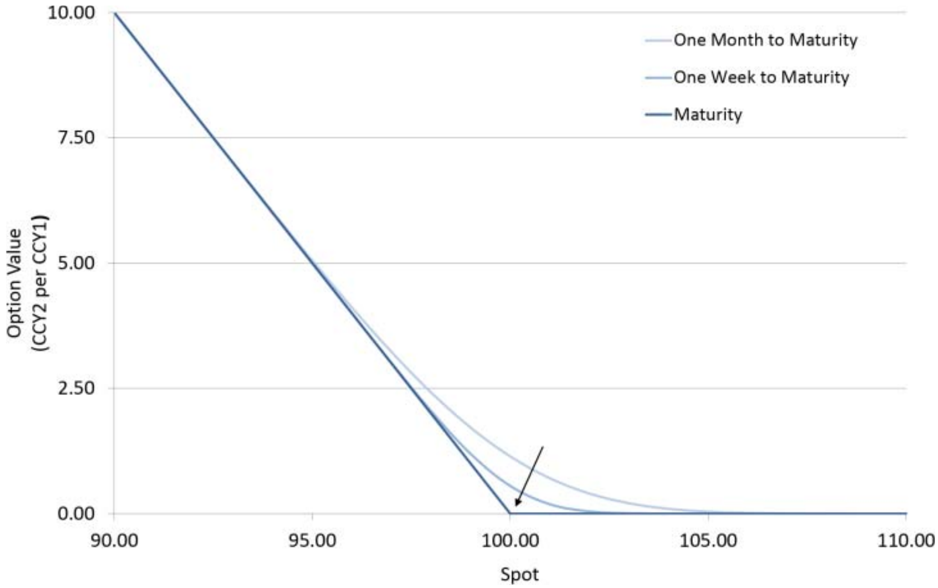


EXHIBIT 6.10 Value of long vanilla put option with 100.00 strike and 10% volatility

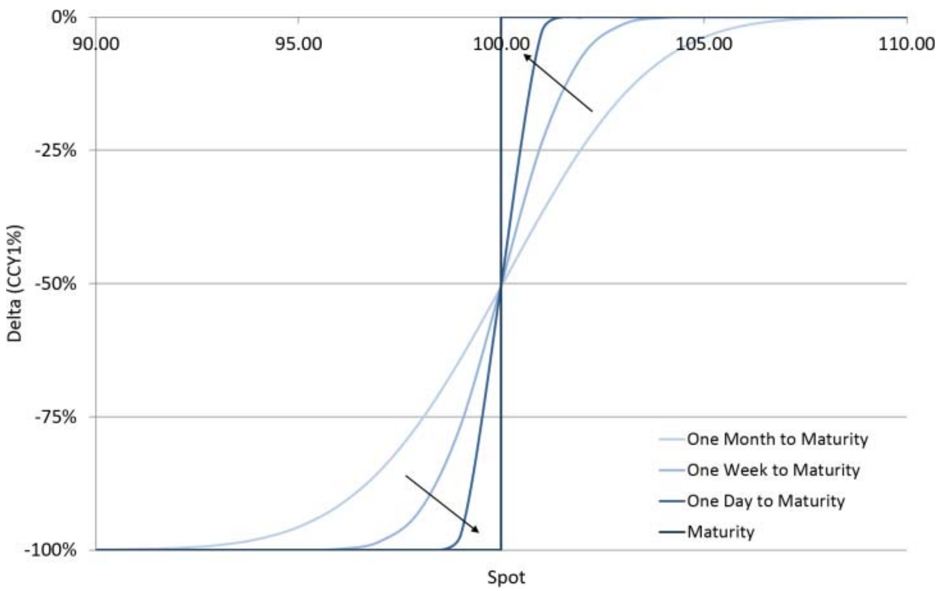


EXHIBIT 6.11 Delta of long vanilla put option with 100.00 strike

Put options with strikes close to the forward have a delta of approximately –50% (i.e., 50/50 chance of ending up ITM), and as spot goes lower, the put delta increases negatively. In practice, when traders describe deltas they often leave off the %, so a "twenty-five delta call" actually has a 25% delta. Plus, when describing put options, the negative sign is often omitted, so a "ten delta put" actually has a –10% delta.

The call delta and put delta profiles in Exhibits 6.9 and 6.11 are identical except that the put option delta is 100% lower than the call option delta over all spot values. Put another way, a call option can be converted into a put option simply by selling a notional amount of forward to maturity. In hockey-stick diagram world, a long call option (shown in Exhibit 6.12) plus a short forward with the same strike, maturity, and notional (shown in Exhibit 6.13) gives a long put option (shown in Exhibit 6.14).

This is a powerful result called **put–call parity**. In words, a vanilla option can be changed from a call into a put (or a put changed into a call) by trading the forward in the same strike, maturity, and notional:

- Long call + short forward = long put
- Long put + long forward = long call

Additionally, a forward can be constructed from a call option and a put option with all other contract details the same. This is called a **synthetic forward**:

- Long call + short put = long synthetic forward
- Short call + long put = short synthetic forward

Consider the formula for a long synthetic forward. At maturity, if spot is above the strike, the long call will be exercised (and the short put expired) and spot will be bought at the strike. If spot is below the strike, the short put will be exercised (and the long call expired) and spot will be bought at the strike. No matter where spot ends up at maturity, spot is bought at the strike—exactly the same as owning the forward contract.

One consequence of this is that since the forward has no optionality, *delta hedged calls and delta hedged puts with the same maturity and strike have the same Greek exposures*. For this reason traders don't usually think in terms of calls or puts once the option is in their trading book; they talk only in terms of *strikes* and *notionals* (e.g., "I've got a 1.3250 strike in EUR50m" rather than "I've got a 1.3250 put in EUR50m").

Another consequence of put–call parity is that *call options and put options with the same maturity and strike must always be valued at the same implied volatility*; otherwise, arbitrage would be possible via trading the forward. This also explains why approaching the trading desk and asking "who does calls and who does puts?" would be met with wide eyes.

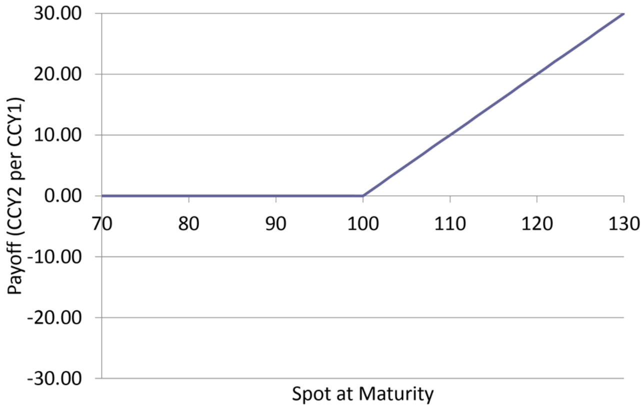


EXHIBIT 6.12 Value at maturity of long call option

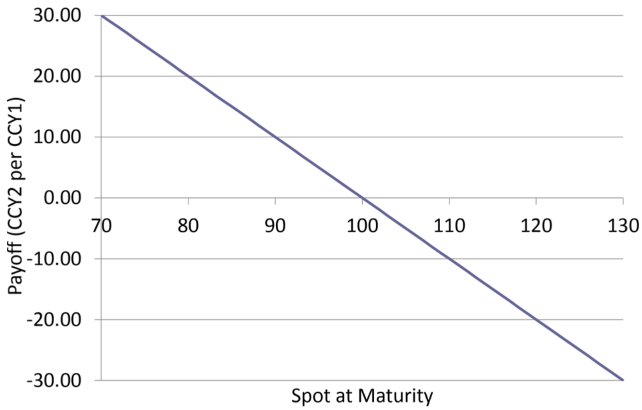


EXHIBIT 6.13 Value at maturity of short forward

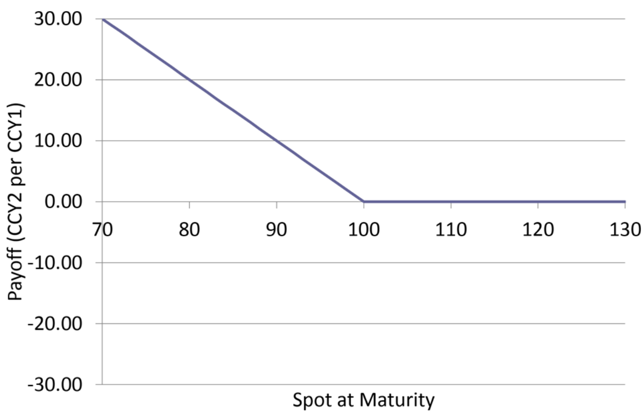


EXHIBIT 6.14 Value at maturity of long put option

Gamma

Taking the first derivative of delta (or the second derivative of option price) with respect to spot gives another important Greek: **gamma**. In symbols:

$$Gamma\ (\Gamma) = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 P}{\partial S^2}$$

In practice, gamma is quoted either as a % of the notional amount, or as a cash amount in the notional currency where:

$$\Gamma_{Cash} = \Gamma_{\%} \times Notional$$

Gamma describes how delta changes with spot and is therefore a measure of how stable delta is as spot moves. Due to put–call parity, call and put options with the same strike and maturity have the same gamma profile, shown in Exhibit 6.15. This gamma can be calculated by taking the gradient of either the call option delta profile from Exhibit 6.9 or the put option delta profile from Exhibit 6.11.

As time moves toward the option maturity, gamma increases and concentrates around the strike. Gamma *at* maturity is not shown on the graph because the discontinuity in delta cannot be neatly differentiated. As discussed previously, gamma can be seen in the *curvature* of the option value versus spot charts. At longer maturities, when there is a wider spot distribution, delta changes slowly as spot moves, hence low gamma. At shorter maturities, when there is a tighter spot distribution, delta changes quickly as spot moves, hence high gamma.

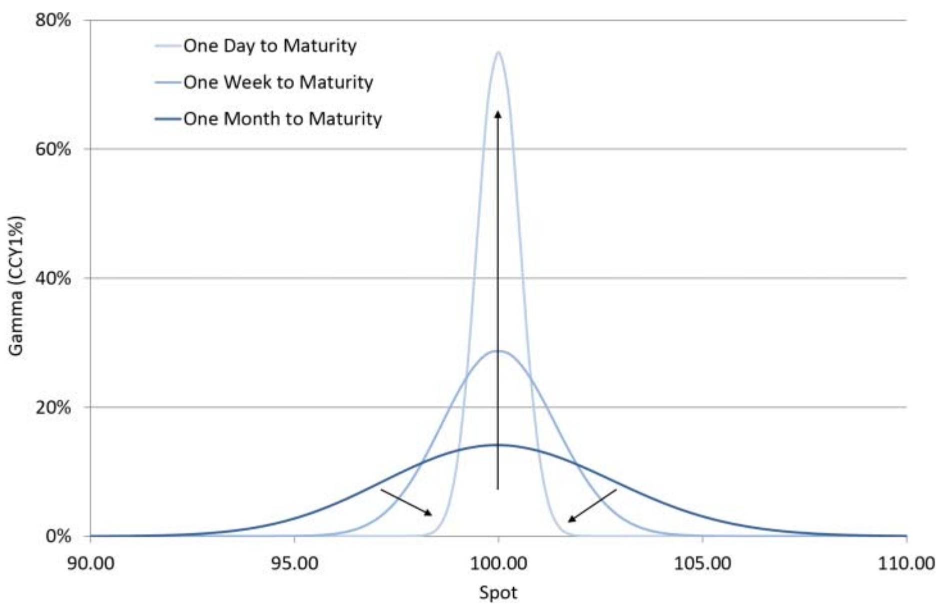


EXHIBIT 6.15 Gamma of long vanilla option with 100.00 strike

Long positions in vanilla options always have long gamma exposure because time value leads to a convex option value versus spot relationship. Likewise, short positions in vanilla options always have short gamma exposure. For a given vanilla option, peak gamma occurs at the strike because this is the point at which optionality is maximized.

Vega

Taking the first derivative of option value with respect to implied volatility gives a third important Greek: **vega**. In symbols:

$$Vega\ (\upsilon) = \frac{\partial P}{\partial \sigma}$$

In practice, vega is usually quoted either as a % of the notional amount, or as a cash amount in the notional currency where:

$$\upsilon_{Cash} = \upsilon_{\%} \times Notional$$

Note that traders describe their position as, e.g., "long vol" to mean "long exposure to implied volatility" (i.e., long vega).

Exhibit 6.16 shows how the vega profile of a long vanilla option position changes over time. Again, due to put–call parity, since forward contracts have no exposure

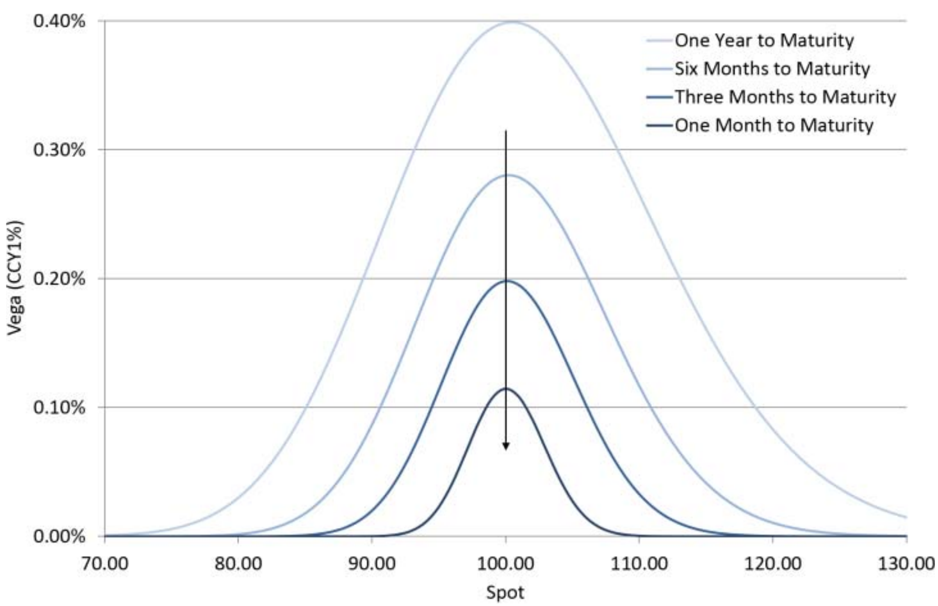


EXHIBIT 6.16 Vega of long vanilla option with 100.00 strike

to implied volatility, call and put options with the same strike and maturity have the same vega profile. The peak vega on a vanilla option reduces over time. Intuitively, vega increases at longer maturities because there is more time for a change in implied volatility to impact the payoff.

For a given vanilla option, peak vega (like gamma) occurs at the strike where optionality and time value is maximized. Far away from the optionality the option is either like a forward (if deep in-the-money) or like no position (if deep out-of-the-money). In either of these cases, changing volatility has minimal impact on option value.

Long vanilla options always have long vega exposure. Intuitively this is because higher volatility widens the distribution and therefore brings larger positive payoffs into play, hence increasing the option value. Likewise, short vanilla options always have short vega exposure.

Summary

When trading FX derivatives, the majority of trading P&L is generated from the three Greek exposures introduced in this chapter: delta, gamma, and vega.

Most often, vanilla options are booked into a trader's position *with the appropriate delta hedge* so new deals cause no net change to the delta exposure within the position, only the vega and gamma exposures are impacted. Buying delta hedged vanilla options results in longer vega and gamma exposures. Selling delta hedged vanilla options results in shorter vega and gamma exposures.

Gamma and vega both come from the optionality within the derivative contract and both are therefore maximized at the strike for vanilla options. A trading position with a long vega exposure will make money if implied volatility rises and lose money if implied volatility falls while gamma impacts how delta moves with spot. Trading long and short gamma positions is covered in Chapter 9.

Long-dated vanilla options have relatively higher vega and lower gamma exposures while short-dated vanilla options have relatively higher gamma and lower vega exposures. Therefore, the main risk on a given vanilla option changes over time from mainly vega risk to mainly gamma risk. The inflection point between the two types of risk occurs around two-month maturity.

Building a Black-Scholes Option Pricer in Excel

Building a Black-Scholes vanilla option pricing tool is one of the best ways to develop an understanding of derivatives pricing. Manipulating inputs and observing the impact on vanilla option prices is far more productive than looking at formulas in a book. This practical links closely to the material developed in Chapter 5.

Task A: Set Up a Simple Black-Scholes Options Pricer

Step 1: Set Up Spot/Rates/Time to Expiry

The first inputs to the pricer are:

- Spot (*S*): the current exchange rate in a given currency pair
- Interest rates (*rCCY1* and *rCCY2*): continuously compounded risk free interest rates in CCY1 and CCY2 of the currency pair
- Time to expiry (*T*): the time between the horizon date and expiry date measured in years

The first output is the forward to maturity T:

$$F_T = Se^{(rCCY2 - rCCY1).\ T}$$

Inputs

Spot	**1.3650**	←*Named:* ***Spot***
CCY1 Interest Rate	**1.00%**	←*Named:* ***rCCY1***
CCY2 Interest Rate	**0.40%**	←*Named:* ***rCCY2***
Time to Maturity (years)	**1.00**	←*Named:* ***T***

Outputs

Forward	1.3568	=Spot*EXP((rCCY2-rCCY1)*T)

Test to see how changing the inputs impacts the forward and note what happens when $rCCY1 = rCCY2$.

Step 2: Set Up Vanilla Option Pricing

European vanilla option payoffs are calculated using spot at maturity (S_T) and the strike (K):

- $Payoff_{call} = max(S_T - K, 0)$
- $Payoff_{put} = max(K - S_T, 0)$

As described in Chapter 5, the Garman and Kohlhagen formula calculates FX European vanilla option prices in CCY2 per CCY1 (i.e., CCY2 pips) terms:

- $Price_{call} = Se^{-rCCY1.T}N(d_1) - Ke^{-rCCY2.T}\ N(d_2)$
- $Price_{put} = Ke^{-rCCY2.T}N(-d_2) - Se^{-rCCY1.T}\ N(-d_1)$

where

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(rCCY2 - rCCY1 + \frac{\sigma^2}{2}\right).T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln\left(\frac{S}{K}\right) + \left(rCCY2 - rCCY1 - \frac{\sigma^2}{2}\right).T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

And σ is the volatility of the spot log returns.

Useful Excel functions are:

- =LN(X) for natural log
- =SQRT(X) for square root
- =EXP(X) for exponential
- =NORMSDIST(X) for the cumulative normal distribution function $N(X)$

Test that if $S = K = 1.0$, $T = 1.0$, $\sigma = 10\%$, and $rCCY1 = rCCY2 = 0\%$, the option price is very slightly under 0.04 pips (0.0399 pips):

Inputs

Spot	**1.0000**	
CCY1 Interest Rate	**0.00%**	
CCY2 Interest Rate	**0.00%**	
Time to Maturity (years)	**1.00**	
Strike	**1.0000**	←*Named:* ***K***
Implied Volatility (σ)	**10.00%**	←*Named:* ***vol***

Working

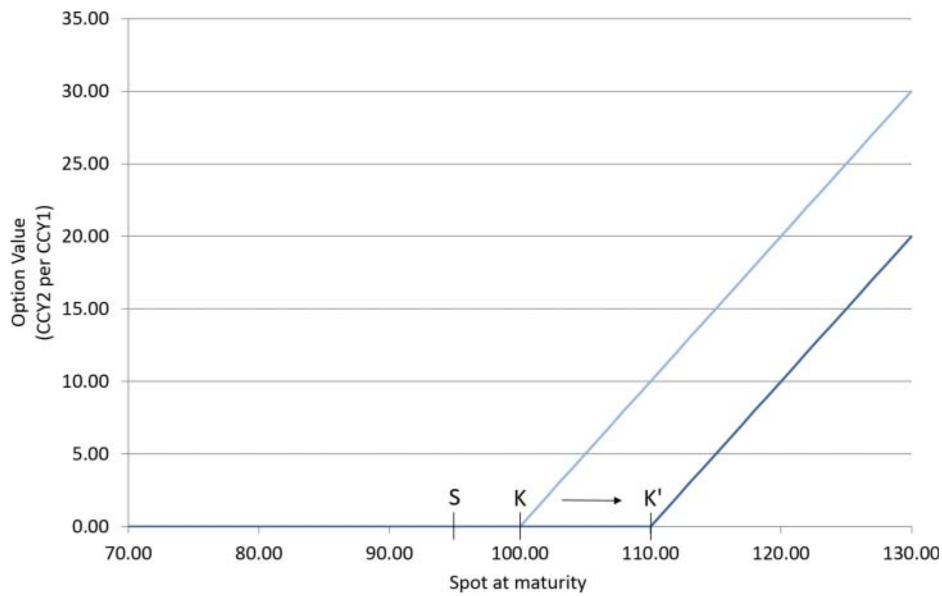
d1	0.05	=(LN(Spot/K)+(rCCY2-rCCY1+0.5*vol^2)*T)/(vol*SQRT(T))
d2	-0.05	=d_1-vol*SQRT(T)
N(d1)	0.519938806	=NORMSDIST(d_1)
N(d2)	0.480061194	=NORMSDIST(d_2)
N(-d1)	0.480061194	=NORMSDIST(-d_1)
N(-d2)	0.519938806	=NORMSDIST(-d_2)

Outputs

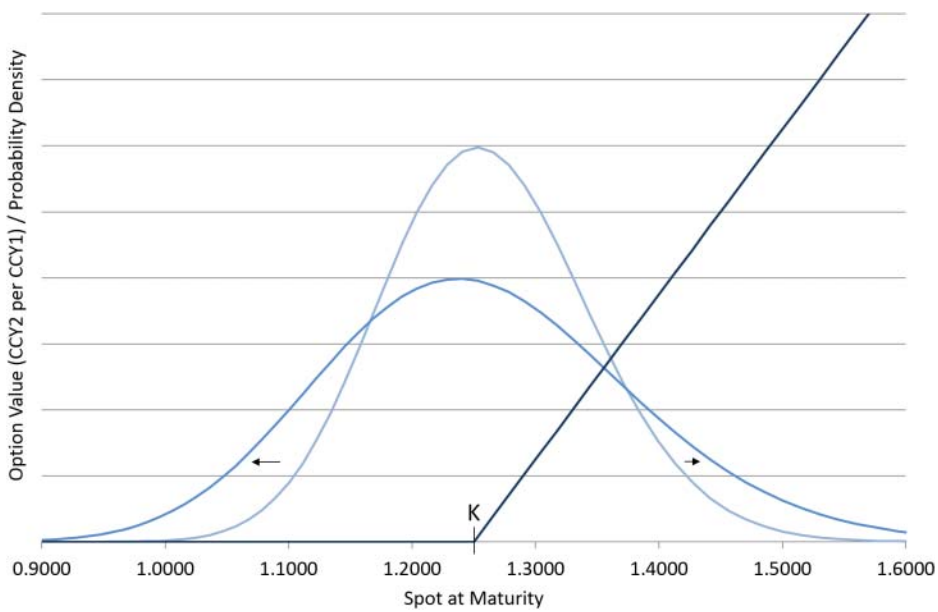
Forward	1.0000	
European Vanilla Call (CCY2 pips)	0.0399	=(Spot*EXP(-rCCY1*T)*Nd_1-K*EXP(-rCCY2*T)*Nd_2)
European Vanilla Put (CCY2 pips)	0.0399	=(K*EXP(-rCCY2*T)*Nminusd_2-Spot*EXP(-rCCY1*T)*Nminusd_1)

Once the pricing is correct, flex each parameter and work through a logical argument of how the parameter change impacts call and put vanilla option pricing. Consider the relative positioning of forward and strike, how time to maturity and implied volatility impact the terminal spot distribution, and discounting of the payoff from maturity back to the horizon (see Chapter 10).

Example 1: For a 100.00 strike call option, if the strike is moved higher, the call option price reduces because the forward is further away from the payoff. The equivalent put option will increase in value:



Example 2: For a 1.2500 strike call option, if implied volatility moves higher or time to expiry increases, both call and put option prices increase because the spot distribution moves wider, hence bringing larger payoffs into play:



Example 3: If CCY1 and CCY2 interest rates both move higher to the same level, the forward will be unchanged but both call and put option prices decrease due to increased discounting.

Step 3: Add in Option Notional and Convert to CCY1 Payoff

Option notionals are usually quoted in CCY1 terms and option prices are naturally generated in CCY2 pips (CCY2 per CCY1) terms. Given a CCY1 option notional, the **cash price** in CCY1 can therefore be calculated. This is useful because it gives the numbers a real-world feel. As in Practical B:

- To convert an option price from CCY2 pips terms into CCY2 cash terms, multiply by the CCY1 notional.

- To convert an option price from CCY2 cash terms into CCY1 cash terms, divide by current spot.

Inputs

Spot	**1.3650**	
CCY1 Interest Rate	**1.00%**	
CCY2 Interest Rate	**2.00%**	
Time to Maturity (years)	**1.00**	
Notional (CCY1)	**10,000,000**	←*Named:* ***N***
Strike	**1.3800**	
Implied Volatility (σ)	**10.00%**	

Working

d1	0.040709295
d2	-0.059290705
N(d1)	0.516236174
N(d2)	0.476360282
N(-d1)	0.483763826
N(-d2)	0.523639718

Outputs

Forward	1.3787	
European Vanilla Call (CCY2 pips)	0.0533	←*Named:* ***CallPriceCCY2Pips***
European Vanilla Put (CCY2 pips)	0.0545	←*Named:* ***PutPriceCCY2Pips***
European Vanilla Call (CCY2 cash)	532,906	=N*CallPriceCCY2Pips
European Vanilla Put (CCY2 cash)	545,468	=N*PutPriceCCY2Pips

European Vanilla Call (CCY1%)	3.90%	←*Named:* ***CallPriceCCY1Pct***	=CallPriceCCY2Pips/Spot
European Vanilla Put (CCY1%)	4.00%	←*Named:* ***PutPriceCCY1Pct***	=PutPriceCCY2Pips/Spot
European Vanilla Call (CCY1 cash)	390,407	=N*CallPriceCCY1Pct	
European Vanilla Put (CCY1 cash)	399,610	=N*PutPriceCCY1Pct	

Step 4: Investigate Put–Call Parity

In payoff terms, put–call parity is often stated as, for example, long call + short put = long forward. However, in terms of *pricing*:

$$Price_{call} - Price_{put} = F - K$$

Therefore, if the strike is set equal to the forward, the call price and the put price should be equal. This can be checked within the pricing tool:

Inputs

Spot	**1.3650**	
CCY1 Interest Rate	**1.00%**	
CCY2 Interest Rate	**2.00%**	
Time to Maturity (years)	**1.00**	
Notional (CCY1)	**10,000,000**	
Strike	**1.3787**	=Forward
Implied Volatility (σ)	**10.00%**	

Working

d1	0.05
d2	-0.05
N(d1)	0.519938806
N(d2)	0.480061194
N(-d1)	0.480061194
N(-d2)	0.519938806

Outputs

Forward	1.3787
European Vanilla Call (CCY2 pips)	0.0539
European Vanilla Put (CCY2 pips)	0.0539
European Vanilla Call (CCY2 cash)	538,913
European Vanilla Put (CCY2 cash)	538,913

European Vanilla Call (CCY1%)	3.95%
European Vanilla Put (CCY1%)	3.95%
European Vanilla Call (CCY1 cash)	394,808
European Vanilla Put (CCY1 cash)	394,808

When the strike is moved away from the forward, a subtlety reveals itself: Option prices are present valued, whereas the P&L from the forward versus strike difference is realized in the future. The $(F - K)$ value therefore needs to be present valued using the CCY2 discount factor $(e^{-rCCY2.T})$:

Put-Call Parity Check

Call less Put	-0.0012561	=CallPriceCCY2Pips-PutPriceCCY2Pips
Forward less Strike	-0.0012561	=EXP(-rCCY2*T)*(Forward-K)

Task B: Set Up a VBA Pricing Function

A lot of additional flexibility becomes possible if the pricing calculation is done within a VBA function rather than using functions constructed in the cells of the Excel sheet. The VBA pricing function should take the following inputs:

```
Public Function OptionPrice(isCall As Boolean, S As Double, K As Double, _
 T As Double, rCCY1 As Double, rCCY2 As Double, v As Double) As Double
```

Helpfully, Excel VBA uses slightly different function names:

- Log(X) is the VBA equivalent of =LN(X)
- Sqr(X) is the VBA equivalent of =SQRT(X)
- Exp(X) is the VBA equivalent of =EXP(X)
- Application.WorkbookFunction.NormSDist(X) is used to access the cumulative normal distribution function.

An explicit check for zero or negative implied volatility or time to maturity should also be included because they would cause the function to throw an error. Instead, set them to a small positive value (e.g., 10^{-10}), which will make the formula return the payoff at maturity.

The VBA option pricing function should look like this:

```
'Garman and Kohlhagen Currency Option Pricing in CCY2 Pips
Public Function OptionPrice(isCall As Boolean, S As Double, K As Double, _
 T As Double, rCCY1 As Double, rCCY2 As Double, v As Double) As Double

    Dim d1 As Double, d2 As Double

    If (T >= 0) Then T = 0.0000000001
    If (v >= 0) Then v = 0.0000000001

    d1 = (Log(S / K) + (rCCY2 - rCCY1 + v ^ 2 / 2) * T) / (v * Sqr(T))
    d2 = d1 - v * Sqr(T)

    If isCall Then
        OptionPrice = (S * Exp(-rCCY1 * T) * _
         Application.WorksheetFunction.NormSDist(d1) - K * _
         Exp(-rCCY2 * T) * Application.WorksheetFunction.NormSDist(d2))
    Else
        OptionPrice = (K * Exp(-rCCY2 * T) * _
         Application.WorksheetFunction.NormSDist(-d2) - S * _
         Exp(-rCCY1 * T) * Application.WorksheetFunction.NormSDist(-d1))
    End If

End Function
```

Call the function from the cell alongside the existing functions to test that both calculations give the same results:

VBA Working

isCall	TRUE	=IF(Payoff="Call",TRUE,FALSE)

Outputs

Forward	1.3787
European Vanilla Call (CCY2 pips)	**0.053290621**
European Vanilla Put (CCY2 pips)	0.0545
European Vanilla Call (CCY2 cash)	532,906
European Vanilla Put (CCY2 cash)	545,468

European Vanilla Call (CCY1%)	3.90%
European Vanilla Put (CCY1%)	4.00%
European Vanilla Call (CCY1 cash)	390,407
European Vanilla Put (CCY1 cash)	399,610

VBA Outputs

Option Price	**0.053290621**	=OptionPrice(isCall,Spot,Strike,T,rCCY1,rCCY2,vol)

Task C: Generate First-Order Greeks

Greek exposures are the sensitivity of an option price to changes in market parameters. As explored in Chapter 6, the most important first-order Greeks are **delta** and **vega**.

Delta (Δ) is the change in option value for a change in spot:

$$\Delta_{call} = \frac{\partial P_{call}}{\partial S} = e^{-rCCY1.T}\,N(d_1)$$

$$\Delta_{put} = \frac{\partial P_{put}}{\partial S} = e^{-rCCY1.T}[N(d_1) - 1]$$

Vega (υ) is the change in option value for a change in implied volatility:

$$\upsilon_{call} = \upsilon_{put} = \frac{\partial P_{call}}{\partial \sigma} = \frac{\partial P_{put}}{\partial \sigma} = Se^{-rCCY1.T}n(d_1)\sqrt{T}$$

where $n(X)$ is the standard normal density function. In Excel this is accessed using =NORMDIST(X, 0, 1, FALSE).

For these first-order Greeks, the Black-Scholes formula can be directly differentiated to generate the formulas. This is called a **closed-form** approach. The same exposures can also be calculated using a **finite difference** approach, which involves manually flexing a parameter (e.g., spot or volatility) a small amount up and down and taking the ratio of the change in price over the change in parameter to calculate the exposure.

In general, the closed-form approach is faster but it is not always available, particularly when pricing exotic contracts. The finite difference approach is slower but it can be applied generically to calculate any exposure for any contract.

Within the pricer both methods can be implemented for comparison. Closed-form exposures can be calculated on the sheet surface but the finite difference approach is easier in VBA. New VBA functions for calculating delta and vega must take additional *spot flex* and *vol flex* parameters respectively. The smaller these parameter flexes, the more accurate the outputs, unless the flex is smaller than the accuracy of the variables themselves within the VBA.

Greek exposures have standard market quotation conventions:

- Delta is quoted as a % of the CCY1 notional.
- Vega is often quoted in CCY1 terms (i.e., divide the function result by spot) and quoted as a % of CCY1 notional for a 1% move in implied volatility (i.e., divide the Black-Scholes vega by 100 to get it into standard market terms).

Once both functions have been implemented in the VBA, check that the values exactly match and investigate the impact of changing the flex size. Start with a 10^{-6} flex but test what happens to the outputs as flex size is increased and decreased.

The VBA code for the finite difference exposures should look like this:

```
'Finite Difference Option Delta in CCY1%
Public Function OptionDelta(isCall As Boolean, S As Double, K _
 As Double, T As Double, rCCY1 As Double, rCCY2 As Double, v As Double, _
 S_Flex As Double) As Double

    Dim OptionPriceUp As Double, OptionPriceDw As Double

    OptionPriceUp = OptionPrice(isCall, S + S_Flex, K, T, rCCY1, rCCY2, v)
    OptionPriceDw = OptionPrice(isCall, S - S_Flex, K, T, rCCY1, rCCY2, v)

    OptionDelta = (OptionPriceUp - OptionPriceDw) / (S_Flex * 2)

End Function

'Finite Difference Option Vega in CCY1%
Public Function OptionVega(isCall As Boolean, S As Double, K As Double, _
 T As Double, rCCY1 As Double, rCCY2 As Double, v As Double, _
 Vol_Flex As Double) As Double

    Dim OptionPriceUp As Double, OptionPriceDw As Double

    OptionPriceUp = OptionPrice(isCall, S, K, T, rCCY1, rCCY2, _
      v + Vol_Flex)
    OptionPriceDw = OptionPrice(isCall, S, K, T, rCCY1, rCCY2, _
      v - Vol_Flex)

    OptionVega = 0.01 * ((OptionPriceUp - OptionPriceDw) / _
      (Vol_Flex * 2)) / S

End Function
```

VBA Inputs

Option Type	**Call**	(Call or Put)
Spot Flex	**0.000001**	←*Named:* ***SpotFlex***
Vol Flex	**0.000001**	←*Named:* ***VolFlex***

VBA Working

isCall	TRUE

Sheet Outputs

Forward	1.3926
European Vanilla Call (CCY2 pips)	0.0446
European Vanilla Put (CCY2 pips)	0.0323
European Vanilla Call (CCY2 cash)	445,783
European Vanilla Put (CCY2 cash)	323,140

European Vanilla Call (CCY1%)	3.27%
European Vanilla Put (CCY1%)	2.37%
European Vanilla Call (CCY1 cash)	326,581
European Vanilla Put (CCY1 cash)	236,732

Call Delta (CCY1%)	56.217389%
Put Delta (CCY1%)	-43.283859%
Vega (CCY1%)	0.276955%

VBA Outputs

Option Price (CCY2 pips)	0.0446	=OptionPrice(isCall,Spot,Strike,T,rCCY1,rCCY2,vol)
Option Delta (CCY1%)	56.217389%	=OptionDelta(isCall,Spot,Strike,T,rCCY1,rCCY2,vol,SpotFlex)
Option Vega (CCY1%)	0.276955%	=OptionVega(isCall,Spot,Strike,T,rCCY1,rCCY2,vol,VolFlex)

Test that if $S = K = 1.0$, $T = 1.0$, $\sigma = 10\%$, and $rCCY1 = rCCY2 = 0\%$, the option delta is close to 50%, and vega is a shade under 0.40%.

Task D: Plot Exposures

The price, delta, and vega VBA functions that have been developed can be used to generate tables and charts of option prices or Greeks over different spots, interest rates, implied volatility levels, or time to expiry. These profiles are key to risk managing a portfolio of vanilla options:

C24 *fx* =OptionDelta(isCall,B24,Strike,T,rCCY1,rCCY2,vol,SpotFlex)

	A	B	C	D	E	F
20						
21		**Spot**	**Option Delta (CCY1%)**		**Spot**	**Option Vega (CCY1%)**
22		1.0000	0.00%		1.0000	0.000%
23		1.0250	0.01%		1.0250	0.000%
24		1.0500	0.02%		1.0500	0.001%
25		1.0750	0.07%		1.0750	0.002%
26		1.1000	0.19%		1.1000	0.004%
27		1.1250	0.50%		1.1250	0.010%

Interesting exposures to plot are:

- *Delta versus spot*: The gradient of this chart gives gamma, plus try extreme *rCCY1* and *rCCY2* values.
- *Vega versus spot*: Note the location of the vega peak, plus try extreme *rCCY1* and *rCCY2* values.
- *Vega versus time to expiry*: Look at the formula for vega and confirm the relationship.
- *Option value versus volatility*: Try this for strikes close to spot and strikes further away from spot.

Vanilla FX Derivatives Pricing

This chapter introduces two primary responsibilities of a vanilla FX derivatives trader: maintaining volatility surfaces and quoting vanilla price requests.

Maintaining Volatility Surfaces

In some financial markets, all relevant market prices can be observed *directly* in the market. However, in OTC (over-the-counter) derivatives markets, prices are very often requested for contracts that have *not* been directly observed in the market. This flexibility is a key advantage of the OTC market structure.

To quote consistent vanilla option prices for any expiry date and strike, traders keep a **volatility surface** updated in each currency pair. Exhibit 7.1 shows a three-dimensional representation of the volatility surface in one currency pair.

The volatility surface can be split into two components: the **ATM (at-the-money) curve** and the **volatility smile**.

ATM Curve

ATM contracts are the backbone of the volatility surface; they define the *term structure* of implied volatility. ATM contracts are vanilla contracts quoted to a specific maturity and they have a strike near (or at) the forward to the same maturity.

ATM contracts are the most important price reference points within the FX derivatives market. In the interbank broker market ATM contracts are often quoted in a **run** of prices at the **market tenors**, the most liquid expiry dates within the

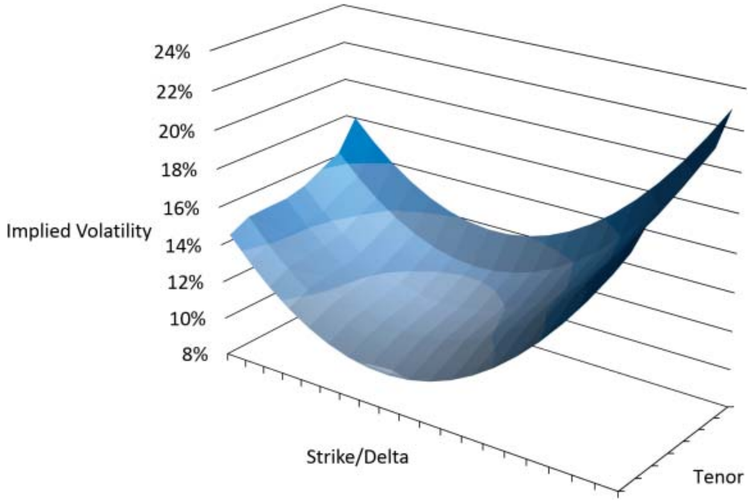


EXHIBIT 7.1 Example volatility surface

EUR/USD (NY Cut)		
Tenor	*ATM Volatility*	
O/N	4.00	5.75
1W	6.75	7.15
2W	6.50	7.20
1M	6.65	6.90
2M	6.90	7.10
3M	6.80	7.05
6M	7.40	7.60
1Y	8.10	8.25
2Y	8.60	8.90

EXHIBIT 7.2 Example ATM run

market. For example, Exhibit 7.2 gives a run of ATM prices at market tenors: O/N (overnight, i.e., tomorrow), 1wk (one week), 2wk, 1mth (one month), 2mth, 3mth, 6mth, 1yr (one year), and 2yr.

The run shows the best bid and best offer that the broker currently has for the ATM contract at each tenor. These are *tradable* rates that have been made (*contributed*) by other banks. Therefore, in Exhibit 7.2, the EUR/USD 1mth ATM NY cut two-way implied volatility price is 6.65/6.9%: The contract will cost 6.9% volatility to buy while selling it will earn 6.65% volatility.

When traders talk, ATM implied volatility is often spoken about in terms of its "vol base," meaning the approximate level of implied volatility in that currency pair. The "handle" is a similar term (e.g., "EUR/USD vol trades off a six handle" means the implied volatility in EUR/USD is 6-point-something-%). When written down, traders often use, for example, 6^1 to mean 6.1% implied volatility.

It is important to understand how this structure based around *fixed market tenors* works in practice. As the horizon date changes, the expiry date for each market tenor changes accordingly (the methodology for calculating tenor expiry dates is given in Chapter 10 and implemented in Practical D). Therefore, the contracts liquidly quoted in the market today have different expiry dates from those quoted yesterday or those quoted tomorrow. Plus as spot (and hence the forward outright) moves, the strike of the ATM contract changes too. This is the nature of an OTC market: Contracts are not standardized, although note that strikes are often rounded to the nearest five or ten pips so the ATM strike does not exactly change pip for pip as spot moves.

ATM implied volatilities at different tenors are plotted in a curve in Exhibit 7.3.

ATM curves are described as *upward sloping* if back-end (e.g., 1yr) ATM volatility is higher than front-end (e.g., 1mth) ATM volatility and as *downward sloping* if front-end ATM volatility is higher than back-end ATM volatility. In quiet markets, back-end ATM volatility tends to be higher than front-end ATM volatility. In stressed market conditions, ATM curves can become "inverted," with higher front-end volatility.

In general, ATM curves move in an orderly manner: Single tenors rarely move in isolation and changes in the ATM curve are often characterized as either parallel or weighted shifts.

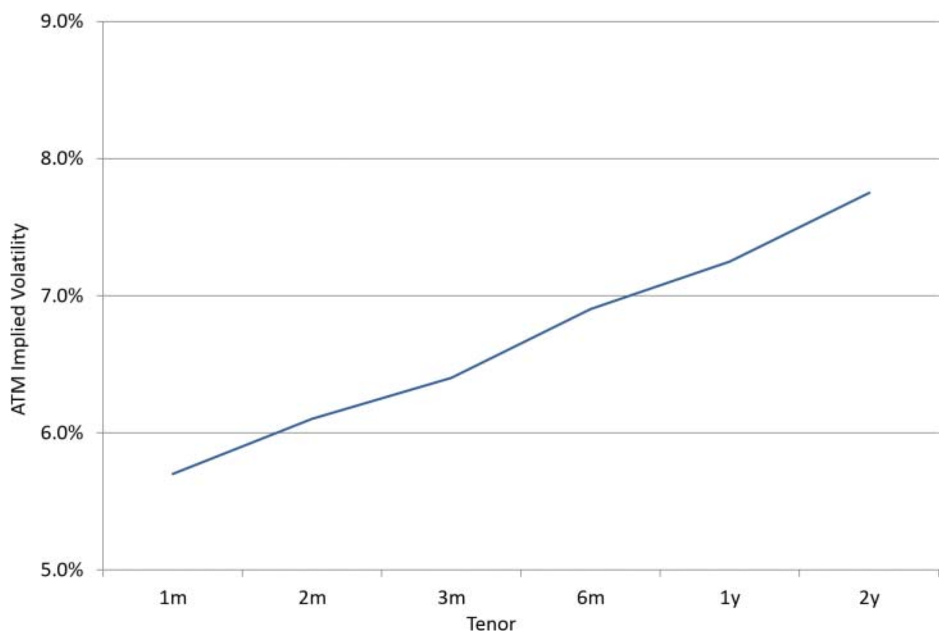


EXHIBIT 7.3 Example ATM curve

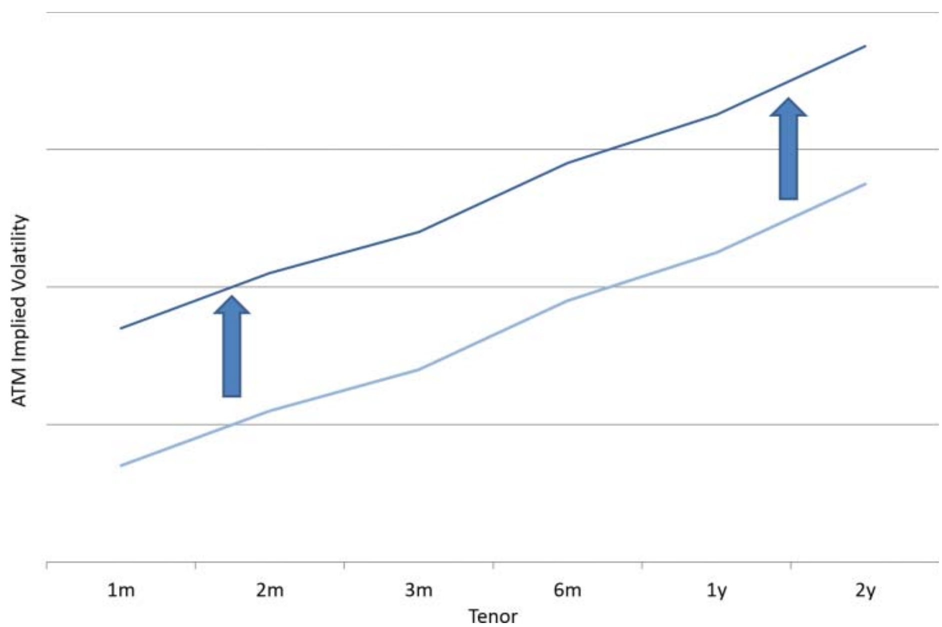


EXHIBIT 7.4 Parallel ATM curve shift

Within a **parallel** ATM shift, the ATM volatility at all maturities moves the same amount up or down as demonstrated in Exhibit 7.4.

Within a **weighted** ATM shift, ATM volatility at near maturities moves more than ATM volatility at far maturities as demonstrated in Exhibit 7.5.

Volatility Smile

The **volatility smile** defines how strikes away from the ATM strike are priced relative to the ATM implied volatility at a given tenor. Slicing through the volatility surface at a particular maturity produces a volatility smile as shown in Exhibit 7.6.

In the interbank broker market, the **butterfly** and **risk reversal** contracts are the instruments used to describe the volatility smile:

- The butterfly contract describes the **wings** of the volatility smile (i.e., how steep the sides of the volatility smile are).
- The risk reversal contract describes the **skew** of the volatility smile (i.e., how tilted the volatility smile is).

Butterfly and risk reversal contracts are quoted at market tenors like the ATM curve. In equity derivatives, lower strikes tend to have higher implied volatility than higher strikes at a given maturity because equities tend to rally slowly and

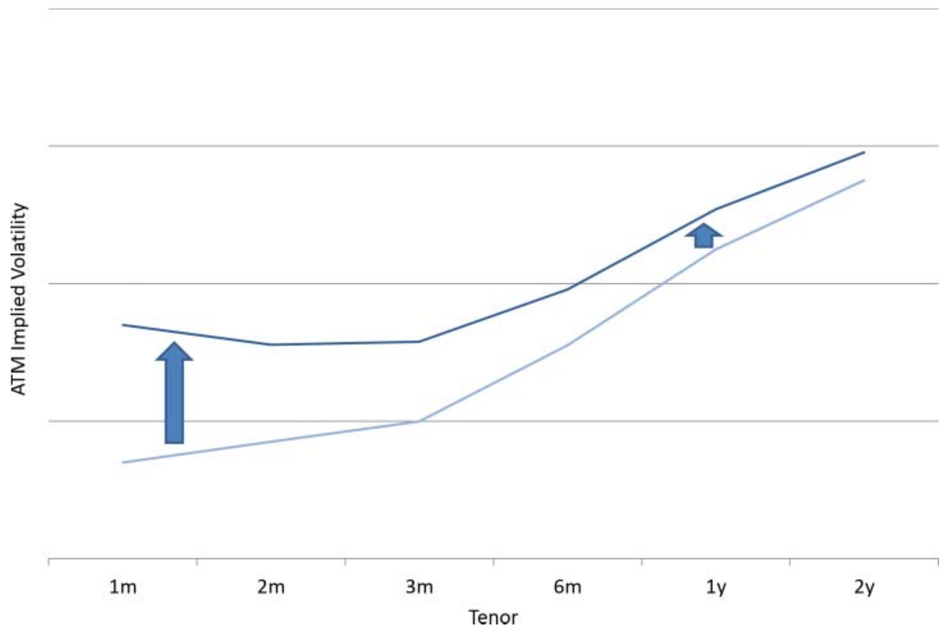


EXHIBIT 7.5 Weighted ATM curve shift

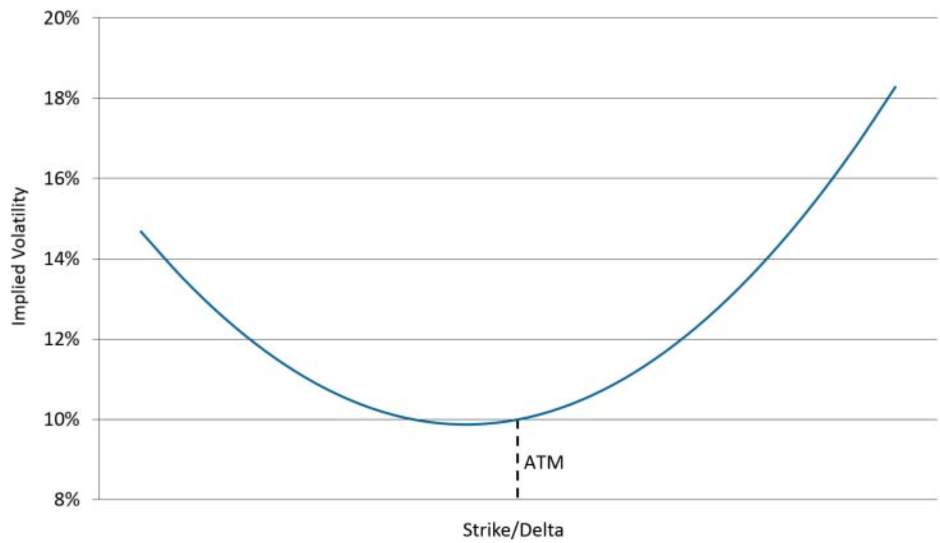


EXHIBIT 7.6 Example volatility smile

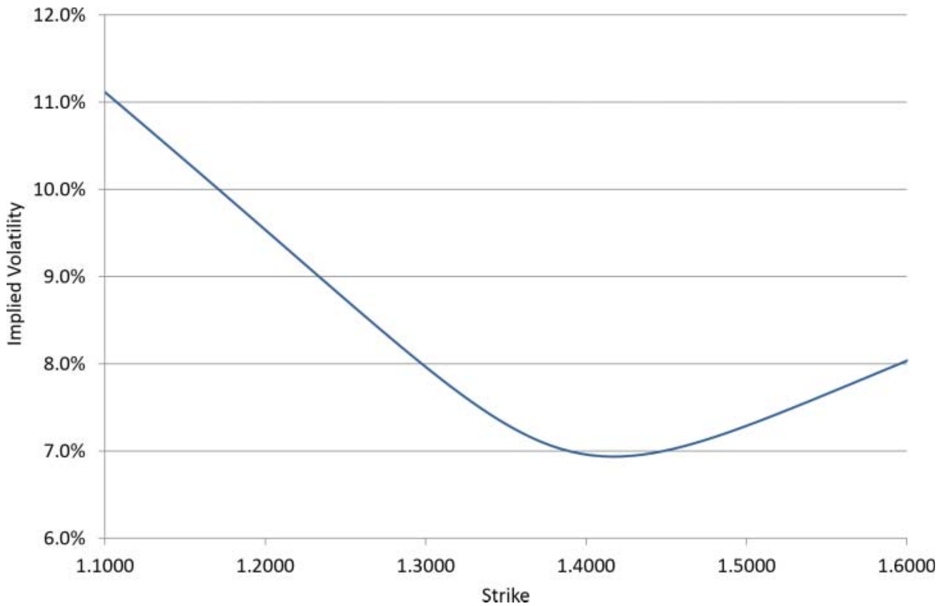


EXHIBIT 7.7 Example downward-sloping volatility smile

drop quickly, but in FX different currency pairs have differently shaped volatility smiles. Some volatility smiles are tilted such that downside strikes have higher implied volatility than topside strikes (e.g., 1yr EUR/USD in July 2014) as shown in Exhibit 7.7.

Some volatility smiles are tilted such that topside strikes have higher volatility than downside strikes (e.g., 6mth USD/ZAR [South African Rand] in July 2014) as shown in Exhibit 7.8.

Some volatility smiles are symmetric and/or have low volatility (e.g., 1mth USD/HKD in July 2014) as shown in Exhibit 7.9.

Some volatility smiles are highly skewed and/or have high volatility (e.g., 1yr AUD/JPY at the height of the 2008 financial crisis) as shown in Exhibit 7.10.

Volatility smiles have higher implied volatility on the "weaker" side of spot (i.e., the direction in which spot is more likely to jump or generally will be more volatile). For example, in major emerging market currency pairs quoted as USD/CCY, the main risk is a sharp EM currency devaluation that causes spot to jump higher. For this reason, USD versus EM currency pair volatility smiles are usually tilted so topside strikes cost more in implied volatility terms than downside strikes (see, e.g., the USD/ZAR example in Exhibit 7.8).

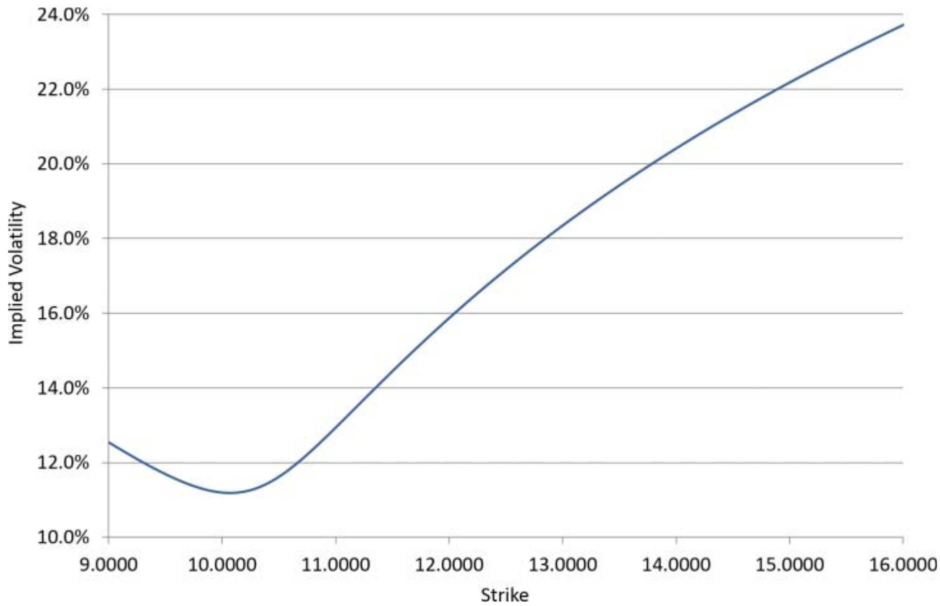


EXHIBIT 7.8 Example upward-sloping volatility smile

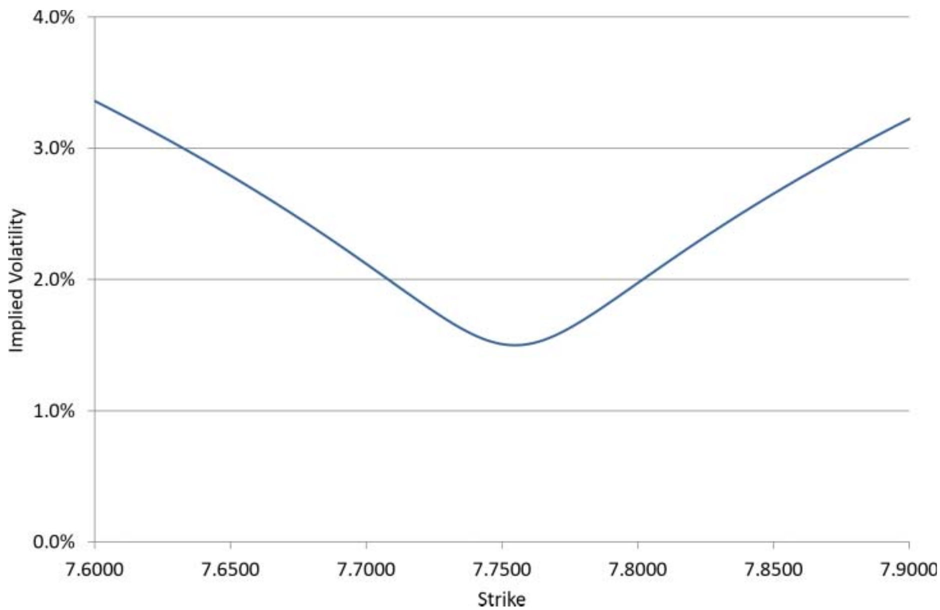


EXHIBIT 7.9 Example symmetric volatility smile

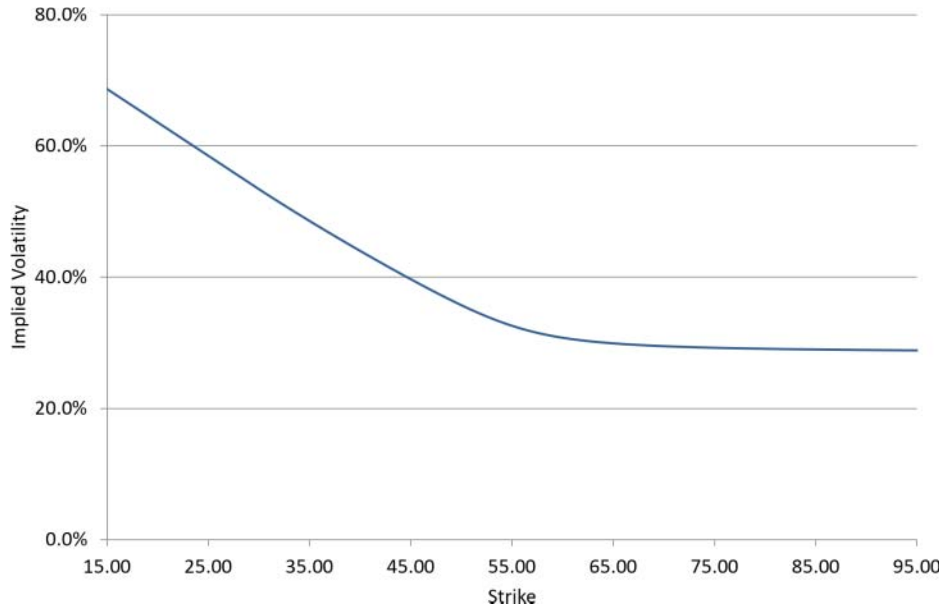


EXHIBIT 7.10 Example extreme volatility smile

Updating the Volatility Surface

Traders watch for new volatility market information in the currency pairs for which they are responsible. The primary source for this information is the interbank broker market.

When a new *price* appears or a *trade* occurs in the broker market, the trader compares that volatility level with the midmarket volatility generated by the desk volatility surface for the same contract. If differences are observed, either the trader updates the volatility surface inputs such that the midmarket volatility hits the market price or the trader may conclude that the price/trade represents a trading opportunity.

Example 1: The USD/CAD 1mth ATM contract two-way price is quoted at 4.8/5.0%. In the trader's volatility surface the current 1mth ATM is marked at 5.0%. The trader takes this as a signal that implied volatility at shorter tenors is reducing and therefore the near end of the ATM curve must be moved lower as shown in Exhibit 7.11.

Example 2: A vanilla trade occurs in the market which suggests that the market is pricing topside strikes at higher implied volatility than before. The volatility smile at the same tenor as the trade is shown in Exhibit 7.12.

The volatility smile at this tenor must be updated to hit this trading level but there are different ways in which this can be achieved. For example:

- The ATM can be moved higher, hence moving the entire volatility smile higher as shown in Exhibit 7.13.

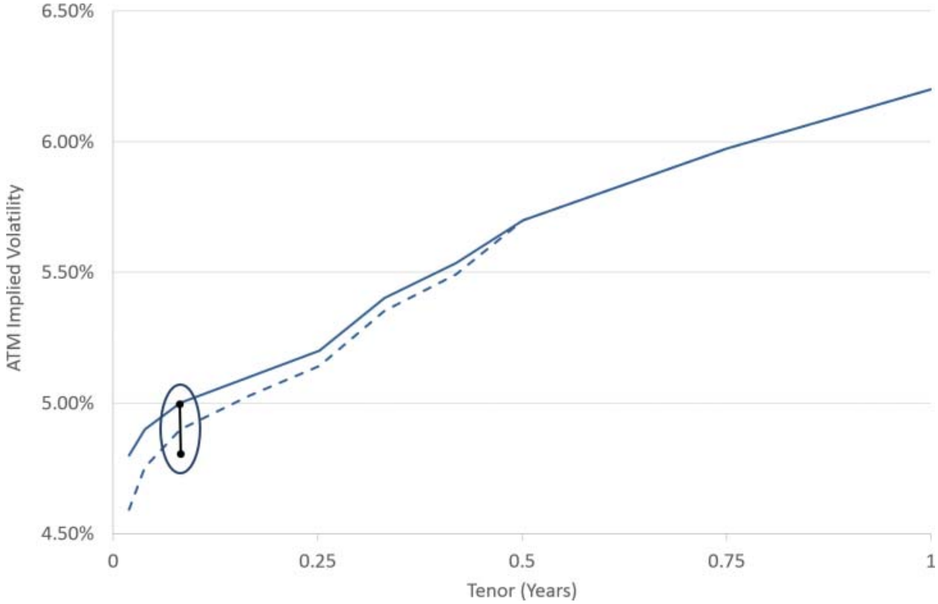


EXHIBIT 7.11 Changing the ATM curve to match new volatility market information

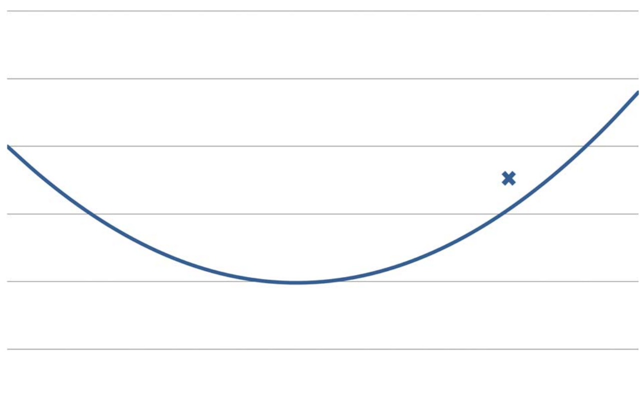


EXHIBIT 7.12 Existing volatility smile plus new volatility market information

- The wings of the volatility smile can be moved higher (i.e., strikes away from the ATM are priced relatively higher than the ATM) as shown in Exhibit 7.14.
- The skew of the volatility smile can be moved more "for the topside" (i.e., the volatility smile is tilted such that strikes above the ATM are priced higher while strikes below the ATM are priced lower). Note that the ATM stays unchanged in Exhibit 7.15.

The trader chooses between these three possibilities (or maybe a combination of them) when updating the volatility smile. As mentioned, single tenors within the

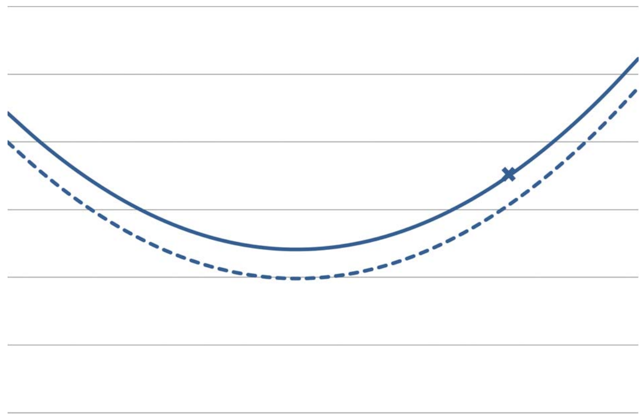


EXHIBIT 7.13 Changing the ATM to match new volatility market information

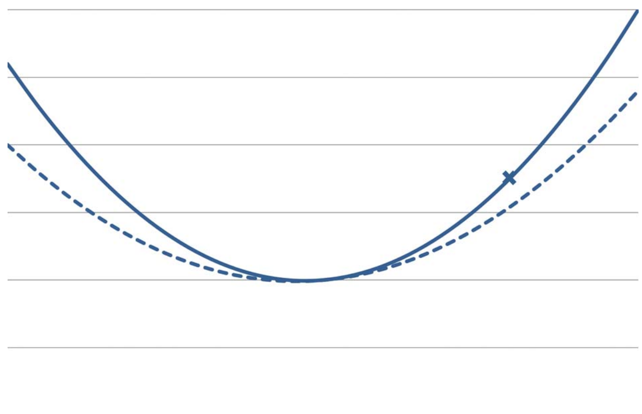


EXHIBIT 7.14 Changing the wings of the volatility smile to match new volatility market information

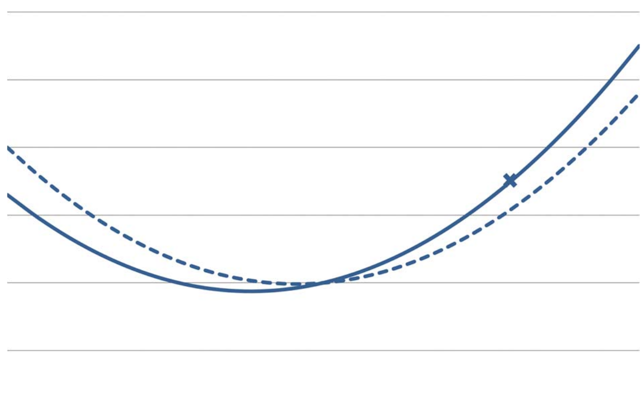


EXHIBIT 7.15 Changing the skew of the volatility smile to match new volatility market information

volatility surface rarely move in isolation so the volatility smile is usually updated over multiple tenors at once. By observing many prices and trades in the market, the trader determines how best to update the entire volatility surface.

Vanilla Bid–Offer Spreads

The volatility surface generates a *midmarket* implied volatility. A bid–offer spread is then applied around the mid-rate to give a *two-way price*. The bid–offer spread for a particular vanilla contract changes depending on its maturity and the proximity of the strike to the ATM.

In practice, standard market bid–offer spreads are observed in the interbank market. Bid–offer spreads for ATM contracts at market tenors in liquid G10 pairs under "normal" market conditions often look similar to Exhibit 7.16. Simple interpolation can be used to generate bid–offer spreads for expiry dates between tenors. In general there is also a strong relationship between spot bid-offer spread and ATM bid-offer spread because the spot bid-offer spread must be crossed when delta hedging the ATM contract to maturity.

ATM volatility spreads are wide in short-dates, reduce to a minimum from 1mth to 1yr, then go wider again at longer tenors, but why? To get a better intuition, *vega* is used to go from ATM volatility spread to ATM premium spread in Exhibit 7.17.

Recall from Chapter 6 that vega is the first derivative of option value with respect to implied volatility:

$$Vega\ (\upsilon) = \frac{\partial P}{\partial \sigma}$$

Therefore the ATM premium spread at a given tenor is calculated by multiplying the ATM volatility spread by the vega. Note that this calculation assumes that vega is unchanged as implied volatility changes.

Exhibit 7.17 shows that looking at ATM volatility spread alone is misleading when comparing bid–offer spreads at different tenors because shorter maturities have lower vega. In most currency pairs the 1mth ATM is the most liquid ATM

EXHIBIT 7.16 **Standard ATM Bid–Offer Volatility Spreads**

Tenor	ATM Volatility Spread
O/N	3.0%
1wk	1.0%
2wk	0.6%
1mth to 1yr	0.3%
2yr	0.35%
3yr	0.4%
4yr	0.45%
5yr	0.5%

EXHIBIT 7.17 **Standard ATM Bid–Offer Volatility Spreads and Premium Spreads**

Tenor	ATM Volatility Spread	Vega (CCY1% 2 d.p.)	ATM Premium Spread (CCY1% 2 d.p.)
O/N	3%	0.02%	0.06%
1wk	1%	0.06%	0.06%
2wk	0.6%	0.08%	0.05%
1mth	**0.3%**	**0.10%**	**0.03%**
2mth	0.3%	0.16%	0.05%
3mth	0.3%	0.20%	0.06%
6mth	0.3%	0.28%	0.08%
1yr	0.3%	0.40%	0.12%
2yr	0.35%	0.56%	0.20%
3yr	0.4%	0.69%	0.28%
4yr	0.45%	0.79%	0.36%
5yr	0.5%	0.87%	0.44%

contract and it therefore has the tightest bid–offer premium spread. Bid–offer spreads in less liquid currency pairs will be wider than those shown in Exhibit 7.17, and the most liquid currency pairs will be tighter, but the relative shape of the bid–offer spread curve is fairly consistent across currency pairs.

For strikes away from the ATM at a given maturity, consider two possible methods of generating the bid–offer spread:

1. Constant premium spread
2. Constant volatility spread

At a given maturity, the maximum vega exposure occurs on ATM strikes. Strikes away from the ATM have lower vega as shown in Exhibit 7.18.

A 1yr ATM option has 0.40% vega so a 0.25% ATM bid–offer volatility spread equates to a 0.10% bid–offer premium spread. If all strikes at the 1yr tenor had the same *premium bid–offer spread*, wing strikes would have a wider bid–offer volatility spread due to lower vega as shown in Exhibit 7.19.

If all strikes at the 1yr tenor had the same *volatility bid–offer spread*, wing strikes would have a tighter bid–offer premium spread due to lower vega as shown in Exhibit 7.20.

In practice, for strikes away from the ATM in liquid currency pairs, the bid–offer spread widens in volatility terms, but not as much as constant premium spread. For example, if the 1yr ATM has 0.25% volatility spread and 0.10% premium spread, a 1yr 10 delta vanilla option might have 0.35% volatility spread and hence 0.06% premium spread.

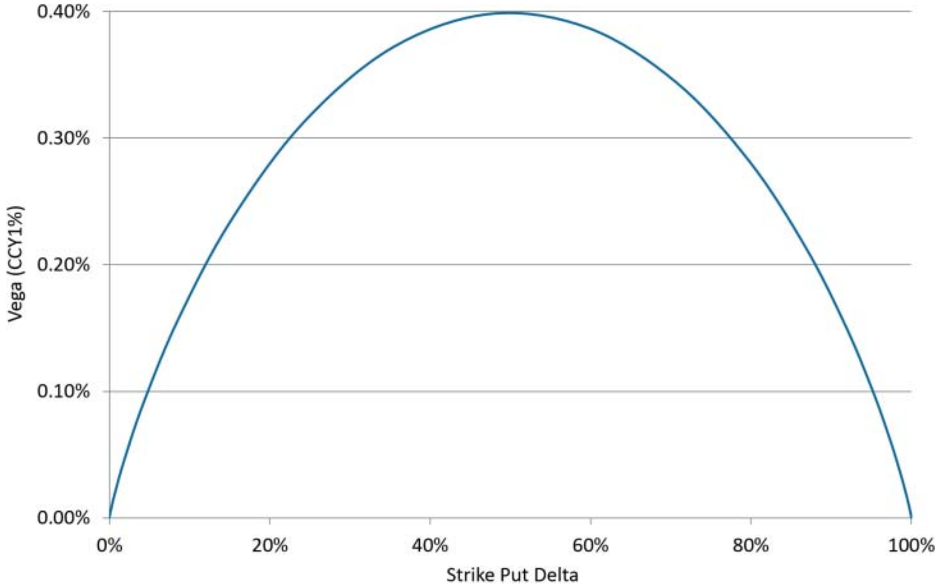


EXHIBIT 7.18 Vega versus put strike delta

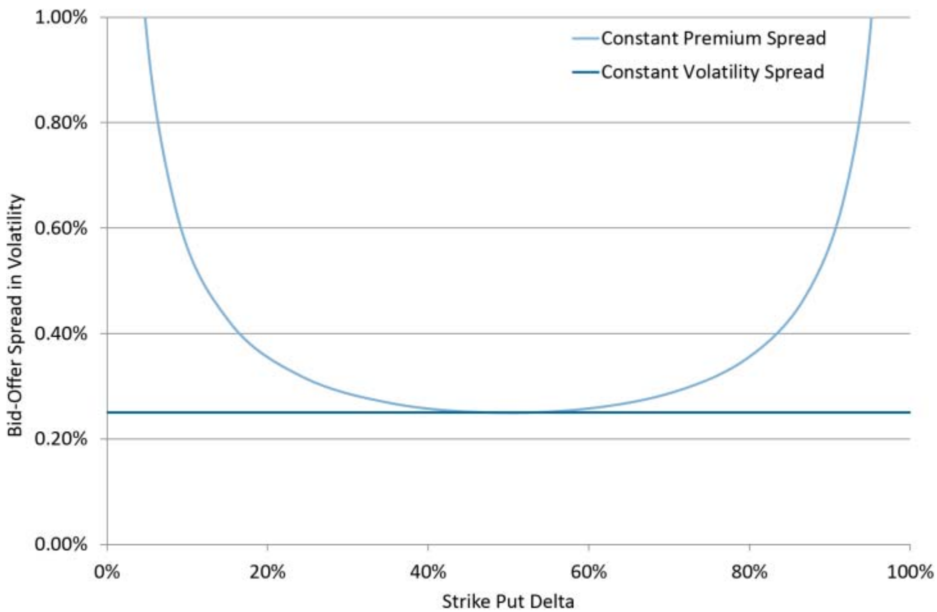


EXHIBIT 7.19 Volatility spread versus put strike delta

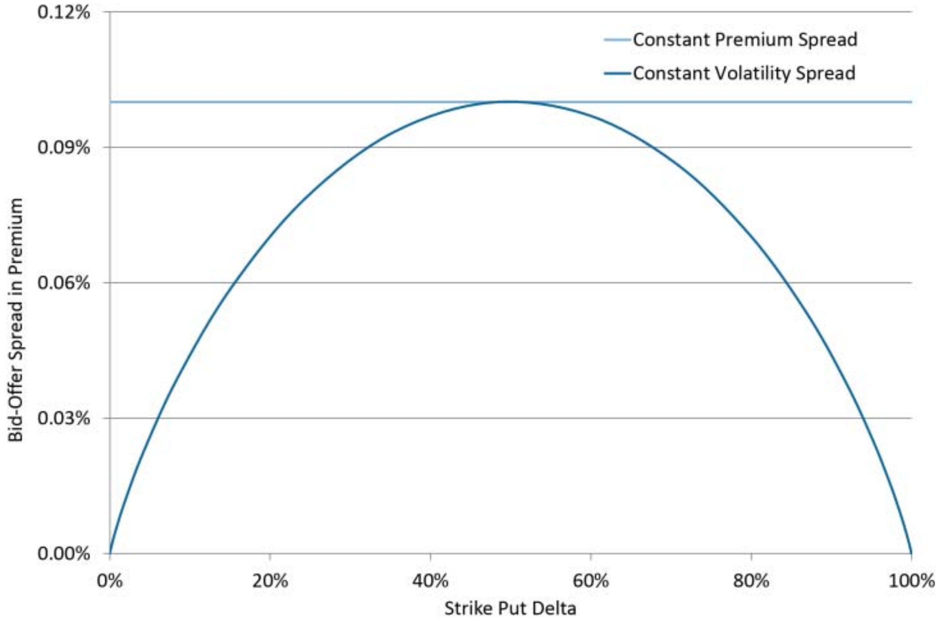


EXHIBIT 7.20 Premium spread versus put strike delta

Vanilla Price Making

When price making vanilla FX derivatives, traders adjust their prices based on market sentiment, plus contracts can either be transacted delta hedged or live.

Price Making Overview

When a new vanilla price request is received, the FX derivatives trader views the contract in their desk pricing tool. The price shown in the tool takes the midmarket volatility from the desk volatility surface and applies a default bid–offer spread.

For example, a client requests a GBP/USD 2mth 1.6800 GBP call in GBP20m. The default mid on the contract is 6.45% and the system two-way price is 6.25/6.65%. This is called a *neutral rate* because neither the bid nor the offer has been improved.

The task of the trader is then to go from the default system rate to a rate they are happy to quote. One of the main factors within the adjustment is the *position preference* of the trader. As described in Chapter 3, price making is performed with reference to the current trading position. If a trader aims to change their position (for example, by selling vega, buying gamma, or buying options with topside strikes), these preferences are reflected within price making.

The other important factor within vanilla option price making is *market sentiment*. There are two types of market sentiment: structural and temporary:

Structural market sentiment involves market-wide preferences to, for example, buy very low-premium options on the high side of the volatility smile or buy options near the ATM on the low side of the volatility smile. These effects usually apply over and above the default modeling of the volatility surface and therefore need to be adjusted for within price making.

Temporary market sentiment is the current market preference for various types of contract—that is, is the market currently neutral, bid (the market prefers to buy), or offered (the market prefers to sell) for specific event dates? short-dates? back-dates? skew? wings? For example, suppose the 1mth ATM is 7.5/7.7% but it last traded at 7.6% and was *bid on* at the time. If there is then a new client price request in something *similar* to 1mth ATM (i.e., a contract with a similar expiry date and similar strike for which the 1mth ATM would be a reasonable hedge), it is appropriate to show a better bid on the contract. If the default system two-way rate on the new contract is 8.0/8.25%, an appropriate quoted rate might be 8.1/8.25%.

Market sentiment is most easily judged by observing the interbank broker market. Traders learn how the normal transaction process for different vanilla contracts works. For example, the implied volatility rate on an ATM contract will start relatively wide—12.45/12.75%—and over time it will tighten up as better bids and offers are shown—12.55/12.7% … 12.6/12.7% … 12.6/12.65%—before eventually trading somewhere within the original rate. This process usually takes around 30 minutes but it can take anything from a few minutes to a few hours.

The trading level and the speed of transaction are linked and they give information about the current market sentiment:

- If traders can wait to transact and want to deal at the best possible level, they will work slowly toward a midmarket trading level.
- If traders need to transact quickly, they will usually need to cross a larger spread from midmarket.

At the extreme, if the opening bid is hit straightaway, that constitutes a strong sell signal, and if the opening offer is paid straightaway, that constitutes a strong buy signal.

Market sentiment should be reflected within price making because it makes risk easier to recycle and enables better (tighter) prices to be quoted for clients. Occasionally prices will be quoted with either a *midmarket bid* (i.e., the bid shown is equal to the system midmarket) or a *midmarket offer* (i.e., the offer shown is equal to the system midmarket).

Transacting Delta Hedged or Live

Vanilla FX derivative price requests are either quoted in *volatility* or *premium* terms. FX derivatives traders typically quote prices in volatility terms because this makes the price making process easier. For a vanilla contract with a fixed maturity and strike, each time the spot rate (and therefore the forward) changes, the option premium changes (although the premium is usually rounded so this reduces the impact). However, the implied volatility quoted for a given contract can stay stable for longer, perhaps up to a few minutes in normal market conditions.

Vanilla price requests that are quoted in volatility terms are traded **delta hedged.** The Black-Scholes formula is used to calculate option premium and option delta, then a spot or forward hedge is transacted at the same rate at which the deal was priced. The interbank broker market and institutional clients who trade FX volatility typically transact in this manner.

Vanilla price requests that are quoted in premium terms are traded **live** (i.e., without a delta hedge). When a client trades live, the appropriate spot or forward deal must be transacted in the market in order to make the package delta neutral for the FX derivatives trader's position. Corporate clients who want to hedge future FX flows or institutional clients who plan to hold a trade in isolation until maturity typically transact in this manner. Either a salesperson or (increasingly) a machine deals with updating the price as spot moves and then hedges the delta when the client trades.

If the client trades live, the delta must be hedged *as* the deal is transacted. In this case the two-way premium is calculated using not only a two-way implied volatility but also two-way spot and forward rates, since that spread must be crossed in the market when the trade is delta hedged.

For long-dated options, using the correct two-way forward within pricing is particularly important since a substantial amount of the total premium bid–offer spread can come from the forward spread. Exhibit 7.21 shows the same vanilla option twice in a pricing tool: Leg 1 has choiced market data; appropriate if the counterparty is going to transact delta hedged with a forward hedge. Leg 2 has spread market data; appropriate if the counterparty is going to transact live (i.e., without a delta hedge).

Market Conventions

It is important that traders know the market conventions in the currency pairs they trade. For example, different ATM contracts are traded in different currency pairs:

- In all G10 (and some EM) currency pairs, the ATM contract is traded as a straddle (i.e., a call and a put with the same maturity and strike; ATM straddles are covered in more detail in Chapter 8).

- In other EM pairs, the ATM contract is traded as a single ATMF (at-the-money-forward) vanilla option with the strike equal to the forward, plus a forward hedge.
- The final ATM contract is ATMS (at-the-money-spot), which is a single vanilla option with the strike equal to current spot.

When trading vanilla options with strikes away from the ATM, the market convention is to always trade the out-of-the-money side (i.e., trade the call or put, whichever has the *lower absolute delta*). Therefore, if a vanilla option has a strike above the ATM, it will be traded as a CCY1 call/CCY2 put. If the strike is below the ATM, it will be traded as a CCY1 put/CCY2 call. Traded contracts are shown in Exhibit 7.22.

This is important because, although the Greeks on a delta hedged call and a delta hedged put are the same (due to put–call parity), the out-of-the-money direction has a smaller premium and a smaller expected payoff at maturity and hence has less credit

Contract Details	**Leg 1**	**Leg 2**
Currency Pair	AUD/USD	AUD/USD
Horizon	Thu 25-Sep-2014	Thu 25-Sep-2014
Spot Date	Mon 29-Sep-2014	Mon 29-Sep-2014
Strategy	Vanilla	Vanilla
Call/Put	AUD Call/USD Put	AUD Call/USD Put
Maturity	5Y	5Y
Expiry Date	Thu 26-Sep-2019	Thu 26-Sep-2019
Delivery Date	Mon 30-Sep-2019	Mon 30-Sep-2019
Cut	NY	NY
Strike	0.9000	0.9000
Notional Currency	AUD	AUD
Market Data		
Spot	0.8790/0.8795	0.8795
Swap Points	-700/-680	-690
Forward	0.8090/0.8115	0.8105
Deposit (AUD)	3.60/3.70%	3.65%
Deposit (USD)	1.95%	1.95%
ATM Volatility	10.45%	10.45%
Pricing Volatility	**9.10/10.45%**	**9.10/10.45%**
Outputs		
Output Currency	**AUD**	**AUD**
Mid Price	**3.45/4.45%**	**3.50/4.40%**

EXHIBIT 7.21 Vanilla option pricing with and without delta hedge

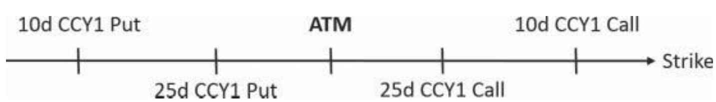


EXHIBIT 7.22 Traded vanilla option contracts

risk than the in-the-money direction. Traders should be careful when in-the-money options are requested by clients: Why wouldn't the conventional side be traded? Some banks tried to sell deep-in-the-money options as a mechanism to earn badly needed cash during the 2008 financial crisis. Although it isn't covered within this book, when trading FX derivatives contracts it is vital that the interest rates used correctly reflect the counterparty credit risk and this is particularly important for high premium in-the-money options.

When quoting in volatility terms, prices are usually rounded to the nearest 0.05% in shorter tenors and 0.025% in longer tenors, with the inflection point usually around the 2mth tenor. This rounding keeps volatility price making clean and ensures that prices quoted in volatility terms can be updated less frequently. When quoting in CCY1% premium terms, prices are usually rounded to the nearest quarter (0.0025%) or half (0.005%) a **basis point** (0.01%). A basis point is a key concept in derivatives pricing; it is always equal to a hundredth of one percent of the notional.

Vanilla FX Derivatives Structures

Vanilla options can be combined to create different payoffs. Some of these combinations are so common that the resultant structures have standardized names that are requested by clients or quoted in the interbank broker market.

For an FX derivatives trader, it is most important to understand how these combinations of long and short vanilla options impact the exposures in the trading position. Within this chapter, vega is the primary focus. If structures are traded to shorter maturities, the most important exposure will be gamma. As observed in Chapter 6; for vanilla options, gamma profiles and vega profiles have similar shapes but they evolve differently over time.

Straddle

A straddle contains two vanilla options with identical contract details (same currency pair, buy/sell direction, notional, expiry, strike, and cut) except that one is a call and the other is a put.

Exhibit 8.1 shows the value at maturity of a long USD/JPY 100.00 straddle in notional *N* per leg:

- Leg 1: Buy USD call/JPY put with strike 100.00 in notional *N*.
- Leg 2: Buy USD put/JPY call with strike 100.00 in notional *N*.

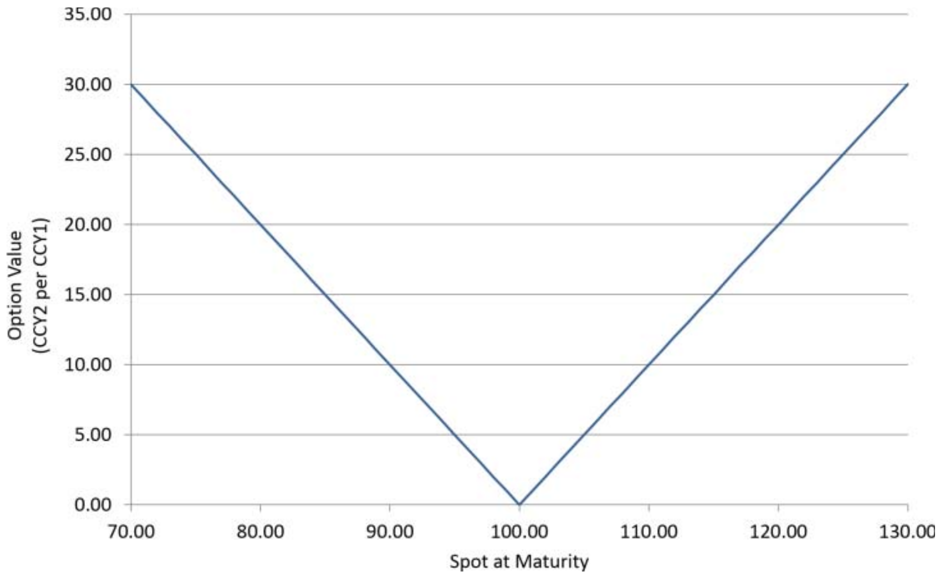


EXHIBIT 8.1 Value at maturity of long 100.00 straddle

Straddle Price Making

Thanks to put–call parity, making a volatility price for a straddle is the same as making a volatility price for a single vanilla option in the combined notional with the same expiry, strike, and cut. A single two-way volatility quote is made and, if dealt, that volatility is used to determine the premiums on both legs.

Zero-Delta ATM Straddles

By far the most commonly traded straddle contracts are **zero-delta straddles**: a straddle with its strike set such that $\Delta_{call} = -\Delta_{put}$ and therefore $\Delta_{straddle} = 0$. In all G10 and some EM currency pairs, the ATM contract is traded as a zero-delta straddle. Therefore, buying a 1mth ATM contract in EUR/USD actually means transacting a call and put, which forms a straddle.

Zero-Delta Straddle Strike Placement

The zero-delta straddle strike is positioned close to the forward for the same maturity but they are not exactly the same. Recall from Chapter 5 that the forward is derived from spot and interest rates:

$$F_T = Se^{(rCCY2 - rCCY1).T}$$

where F_T is the forward to time T (measured in years), S is the spot, and $rCCY1$ and $rCCY2$ are continuously compounded interest rates (see Chapter 10) to time T in CCY1 and CCY2 respectively.

Dipping briefly into Black-Scholes mathematics:

$$\Delta_{call} = \frac{\partial P_{call}}{\partial S} = e^{-rCCY1.T} N(d_1)$$
$$\Delta_{put} = \frac{\partial P_{put}}{\partial S} = e^{-rCCY1.T} [N(d_1) - 1]$$

where $N(x)$ is the cumulative normal distribution function and σ is volatility.

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + \left(rCCY2 - rCCY1 + \frac{1}{2}\sigma^2\right).T}{\sigma\sqrt{T}}$$

At the zero-delta straddle strike (K), $\Delta_{call} = -\Delta_{put}$. Therefore:

$$e^{-rCCY1.T} N(d_1) = -e^{-rCCY1.T}[N(d1) - 1]$$
$$N(d_1) = -[N(d1) - 1]$$
$$N(d_1) = \frac{1}{2}$$

Recalling the shape of cumulative normal distribution function from Chapter 5, if $N(d_1) = \frac{1}{2}$, then $d_1 = 0$. So

$$\frac{\ln\left(\frac{S}{K}\right) + \left(rCCY2 - rCCY1 + \frac{1}{2}\sigma^2\right).T}{\sigma\sqrt{T}} = 0$$
$$\left(rCCY2 - rCCY1 + \frac{1}{2}\sigma^2\right).T = -\ln\left(\frac{S}{K}\right)$$
$$e^{\left(rCCY2 - rCCY1 + \frac{1}{2}\sigma^2\right).T} = \frac{K}{S}$$
$$K = Se^{\left(rCCY2 - rCCY1 + \frac{1}{2}\sigma^2\right).T}$$

Therefore, the zero-delta straddle strike is *higher* than the forward due to the adjustment $+\frac{1}{2}\sigma^2 T$, which is linked to the Itō correction within the Black-Scholes framework (see Chapter 5).

Unfortunately, this is not quite the end of the story. The above is standard Black-Scholes mathematics, which assumes the premium is paid in CCY2. In, for example, EUR/USD (USD premium), this is the case; but in, for example, USD/JPY (USD premium), the market convention is to pay premium in CCY1.

In CCY1 premium pairs (see Chapter 14 for more details on premium adjusted delta):

$$\Delta_{call} = \frac{\partial P_{call}}{\partial S} = e^{-rCCY1.T}N(d_1) - \frac{P_{call}}{S}$$
$$\Delta_{put} = \frac{\partial P_{put}}{\partial S} = e^{-rCCY1.T}[N(d_1) - 1] - \frac{P_{put}}{S}$$

which works through as:

$$K = Se^{\left(rCCY2 - rCCY1 - \frac{1}{2}\sigma^2\right).T}$$

Therefore, when the premium is paid in CCY1, the zero-delta straddle strike is *lower* than the forward due to the adjustment: $-\frac{1}{2}\sigma^2 T$.

In both cases, the difference between the forward and the ATM zero-delta straddle strike increases as volatility and time to expiry get larger.

Straddle Trading Exposures

Exhibit 8.2 shows the vega profile of a straddle. It is identical to the vega profile of a single vanilla with the same (combined) notional, strike, and maturity. As expected, the vega peak occurs at the strike.

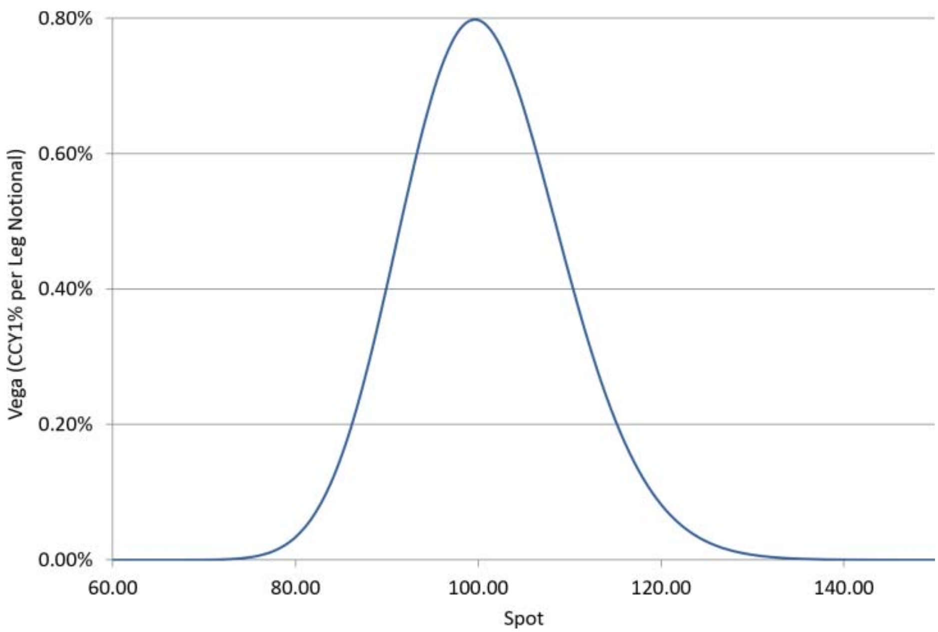


EXHIBIT 8.2 Vega profile of long 100.00 straddle

Strangle

A strangle is like a straddle except that the call and put have *different strikes*. Both strikes are placed out-of-the-money and therefore the call strike is always higher than the put strike.

Exhibit 8.3 shows the value at maturity of a long USD/JPY 90/110 strangle in notional N per leg:

- Leg 1: Buy USD call/JPY put with strike 110.00 in notional N.
- Leg 2: Buy USD put/JPY call with strike 90.00 in notional N.

Strangles are often quoted for a given delta. For example, a 25 delta strangle is constructed with strikes such that call delta = 25% and put delta = (−)25%.

Strangle Price Making

When making a price on a strangle, a *single* two-way volatility is quoted. If dealt, that volatility is used to determine the premiums on both legs. The bid–offer spread quoted on a strangle in volatility terms will usually be wider than the ATM spread to the same maturity because strikes away from the ATM have less vega (see Chapter 7).

A strangle containing strikes K_1 and K_2 has (approximate) implied volatility:

$$\sigma_{strangle} = \frac{\sigma_1 v_1 + \sigma_2 v_2}{v_1 + v_2}$$

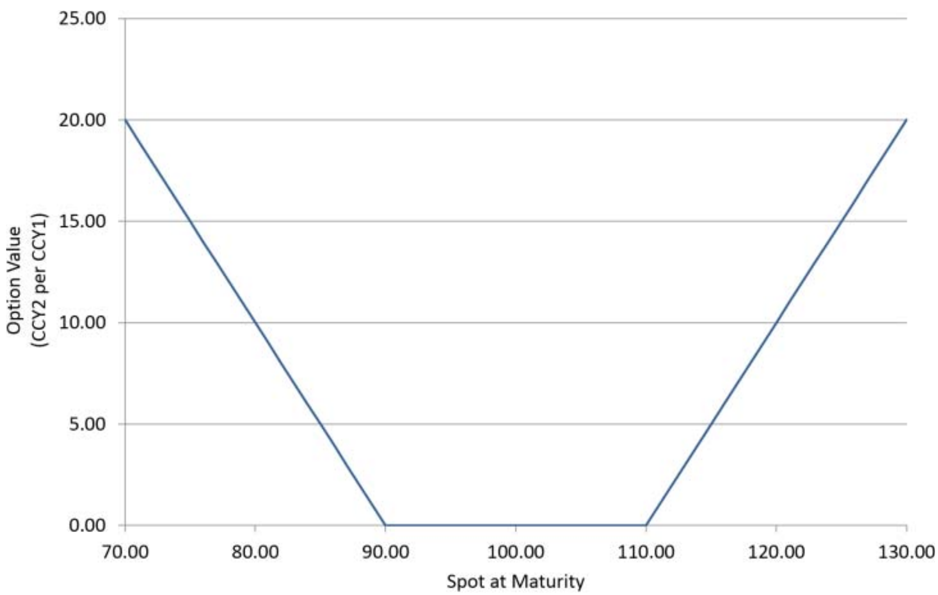


EXHIBIT 8.3 Value at maturity of long 90.00/110.00 strangle

where strike K_1 has implied volatility σ_1 and vega υ_1 while strike K_2 has implied volatility σ_2 and vega υ_2. Therefore, if the strangle contains equal delta call and put strikes with roughly equal vega, the strangle volatility will be close to the *average* of the two individual strike volatilities.

Strangle Trading Exposures

The vega profile of a strangle is similar to the vega profile of a straddle with the same maturity and notional per leg except that the strangle vega profile is wider because the strikes are spread out away from the ATM. At lower deltas the vega profile of the strangle has two distinct peaks from the two strikes as shown in Exhibit 8.4.

Butterfly (Fly)

The butterfly contract is a combination of a straddle and a strangle:

- Long butterfly = long strangle + short straddle
- Short butterfly = short strangle + long straddle

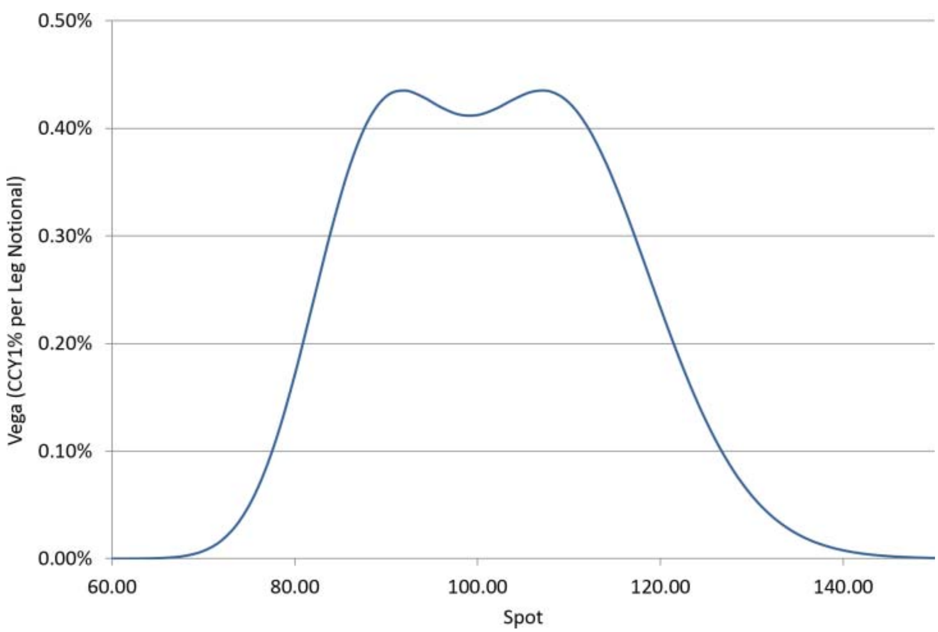


EXHIBIT 8.4 Vega profile of long 90.00/110.00 strangle

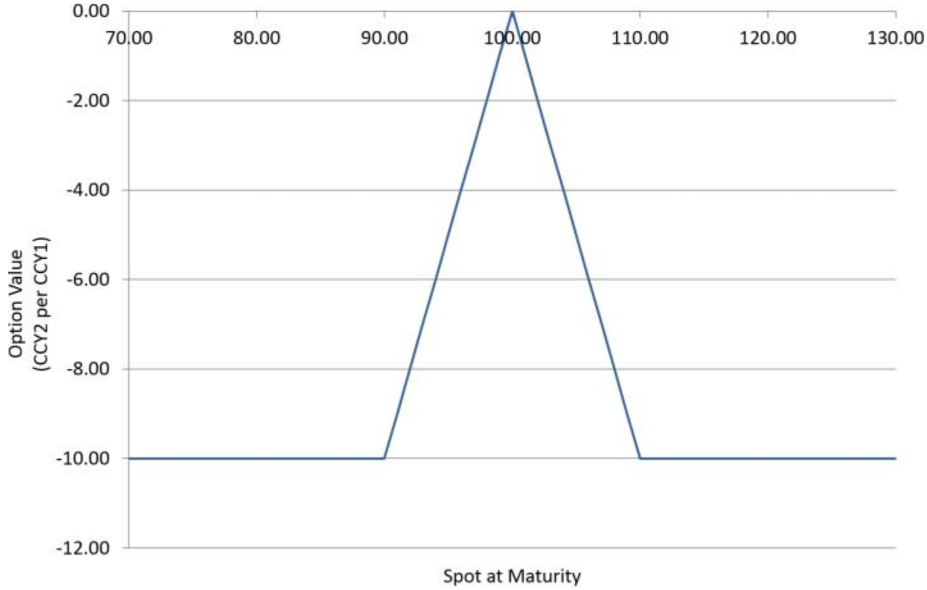


EXHIBIT 8.5 Value at maturity of long 90.00/100.00/110.00 equal notional butterfly

Buying the butterfly means buying the strangle, or put another way, buying the wings. Exhibit 8.5 shows the value at maturity of a long USD/JPY 90.00/100.00/110.00 *equal notional* butterfly:

- Leg 1: Buy USD put/JPY call with strike 90.00 in notional N.
- Leg 2: Sell USD put/JPY call with strike 100.00 in notional N.
- Leg 3: Sell USD call/JPY put with strike 100.00 in notional N.
- Leg 4: Buy USD call/JPY put with strike 110.00 in notional N.

Legs 1 and 4 form a long strangle and legs 2 and 3 form a short straddle. The contract is called a butterfly because its value at maturity looks like a butterfly if you really squint.

In the interbank broker market, the **broker fly** contract is the most commonly traded butterfly contract. A broker fly has equal notional strangle strikes and the notional on the ATM straddle is set such that the package is initially vega-neutral. Broker fly contracts are quoted for a given delta *but* the strikes for a given delta are generated using a different calculation. The broker fly product is examined in far more detail in Chapter 12.

Butterfly Price Making

In the interbank market, prices on broker fly contracts are quoted as the volatility differential between the strangle strikes and the ATM strikes. Plus the contract is quoted such that the *butterfly notional* is the *strangle notional*.

The broker fly has zero vega and minimal gamma by construction. Therefore, it is quoted significantly tighter than the strangle component in isolation. A bid–offer spread less than half the strangle bid–offer spread will often be shown.

Butterfly Trading Exposures

A long butterfly position is often flat ATM vega by construction and long vega in the wings from the long strangle strikes as shown in Exhibit 8.6.

Risk Reversal (RR)

A risk reversal contains two vanilla options with the same currency pair, notional, expiry, and cut. However, the two legs have different strikes, one is a call and the other is a put, plus one is bought while the other is sold. The contract is called a risk reversal because it transfers optionality between the two strikes.

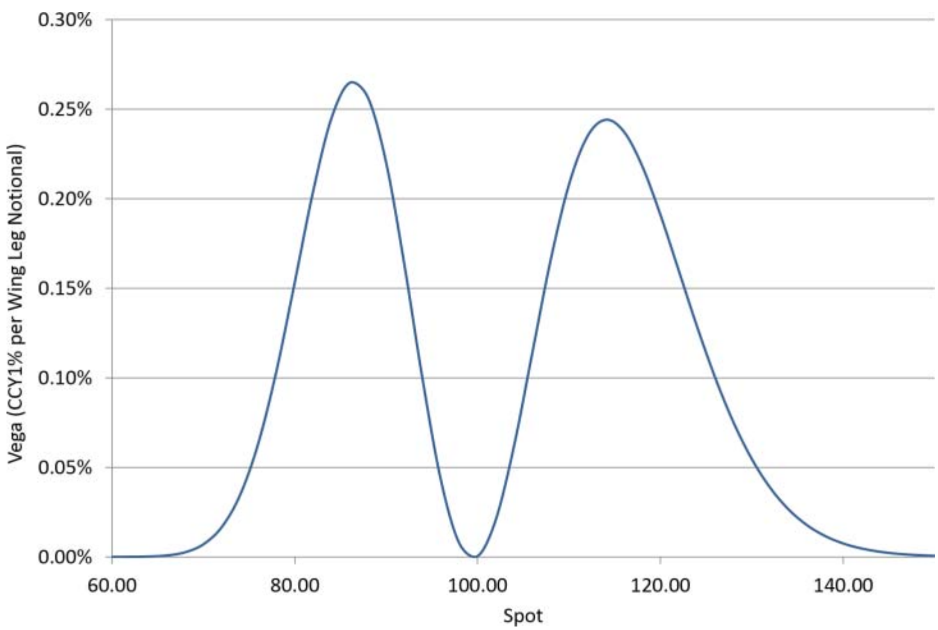


EXHIBIT 8.6 Vega profile of long 90.00/100.00/110.00 vega-neutral butterfly

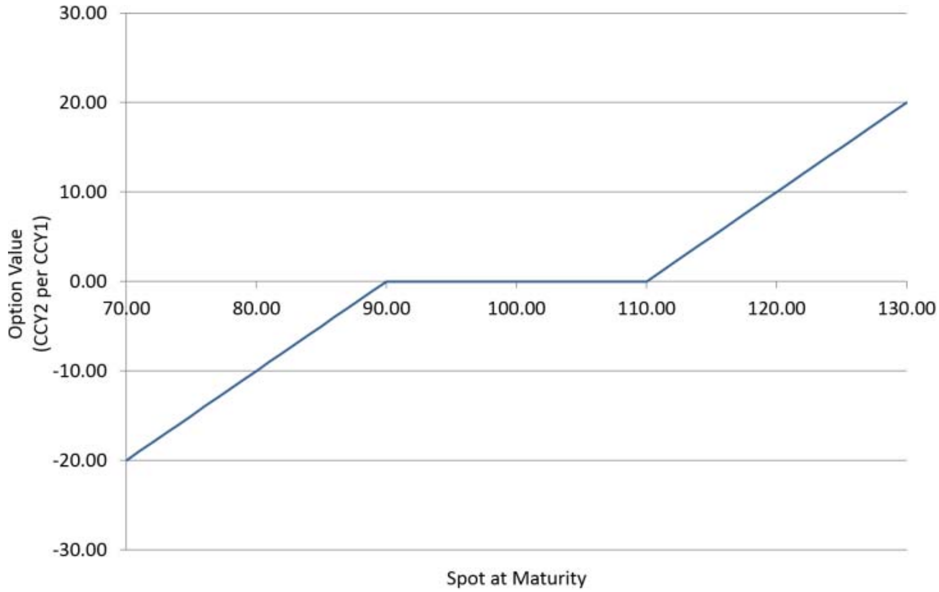


EXHIBIT 8.7 Value at maturity of 90.00/110.00 risk reversal (buying topside)

Exhibit 8.7 shows the value at maturity of a USD/JPY 90/110 risk reversal (buying topside) in notional N per leg:

- Leg 1: Buy USD call/JPY put with strike 110.00 in notional N.
- Leg 2: Sell USD put/JPY call with strike 90.00 in notional N.

Risk reversals are usually quoted to a specific delta. They are constructed using a long call and a short put (or short call and long put) and the delta exposures from the two legs therefore compound (i.e., the delta on both legs is either positive or negative). To calculate the delta hedge on the risk reversal, the delta of the two legs is summed. For example, to delta hedge a 25d risk reversal, containing a long 25d call and a short 25d put, $25\% \times 2 = 50\%$ of the single leg notional of spot must be sold.

Corporates often call risk reversals "collars" when they are used as instruments to hedge future cash flows. A popular structure is a **zero-premium collar**, which caps and floors the P&L from an FX exposure as demonstrated in Exhibit 8.8.

Risk Reversal Price Making

A risk reversal is a **spread** contract. It has two legs: one bought, one sold. There are offsetting gamma and vega exposures between the legs and therefore the product is quoted tighter than the equivalent contract where either both legs are bought or both legs are sold. When quoting a price on spread, rather than quoting tighter two-way prices on both legs, the market convention is to quote one leg with a **choice** price

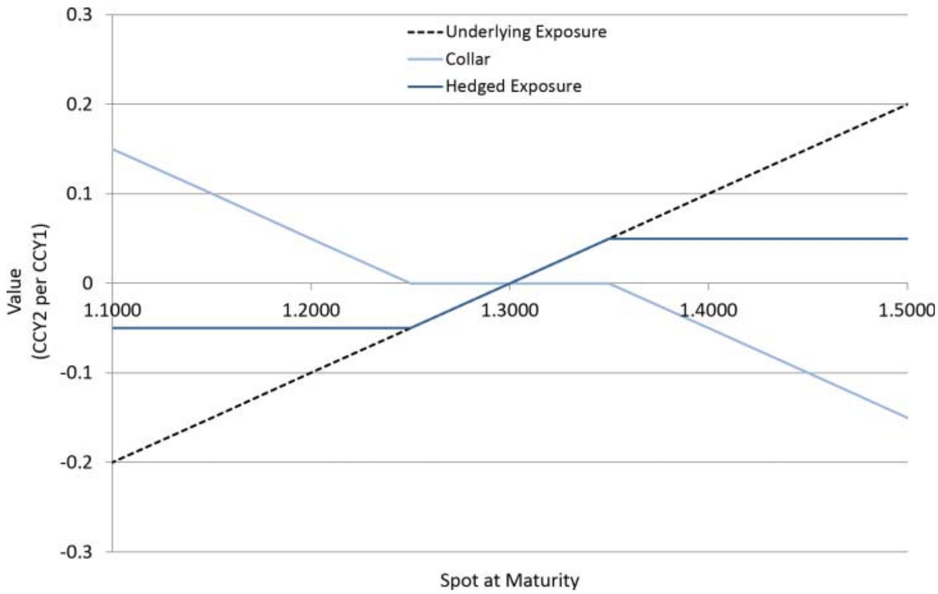


EXHIBIT 8.8 Hedging an FX exposure with a collar (risk reversal)

(i.e., a single volatility at which it is possible to buy or sell, often denoted CH) and the other leg with a **spread** price (i.e., a two-way volatility price). In other words, all the bid–offer spread is put onto one of the legs. Risk reversals are usually quoted with around half the overall bid–offer spread of the equivalent strangle.

For example, USD/BRL 1yr 25d risk reversal:

- Outright 1yr 25d call implied volatility: 14.35 / 14.85%
- Outright 1yr 25d put implied volatility: 10.75 / 11.25%
- 1yr 25d risk reversal implied volatility: 14.35 / 14.85% vs. 11.0% CH

Risk reversals for a given delta are often quoted as an *implied volatility differential* between the two strikes (e.g., 3.35/3.85% in the above USD/BRL example). This is appropriate when the risk reversals strikes have the same delta and hence there is *minimal net vega* on the structure. Once the trade is agreed in volatility differential terms, the volatility on one of the strikes is agreed and that determines the volatility of the other strike; hence both premiums can be calculated. The risk reversal product is examined in far more detail in Chapter 12.

Risk Reversal Trading Exposures

The bought leg in the risk reversal generates a long vega exposure with a peak at the long strike while the sold leg generates a short vega exposure with a (negative) peak at the short strike. This is shown in Exhibit 8.9.

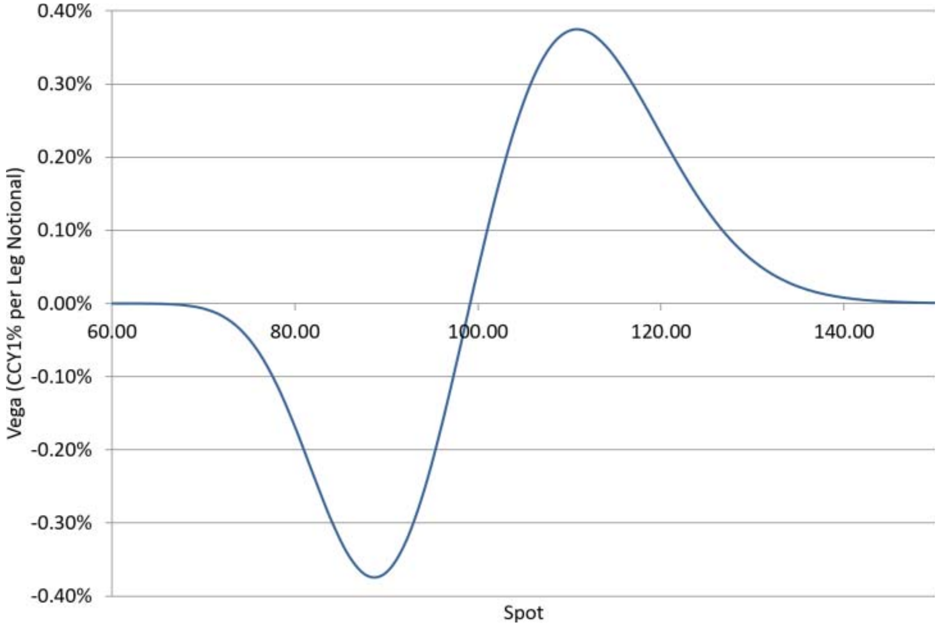


EXHIBIT 8.9 Vega profile of a 90.00/110.00 risk reversal

Leveraged Forward

Recall from Chapter 6 that a synthetic forward is constructed using a long vanilla call and short vanilla put with all other contract details the same.

A leveraged forward is an extension of a synthetic forward in which the two notional amounts are no longer equal. A 200% leveraged forward in which USD/JPY is bought at 100.00 is constructed using two legs with double the notional on the sold leg:

- Leg 1: Buy USD call/JPY put with strike 100.00 in notional *N*.
- Leg 2: Sell USD put/JPY call with strike 100.00 in notional 2 *N*.

Leveraged forwards can be decomposed into a forward in the matched notional plus a vanilla in the unmatched notional. Since forwards have no optionality, the vega trading risk is equal to a vanilla in the unmatched notional and the implied volatility price will also be equal to the volatility price of the vanilla option in the unmatched notional.

This product is popular with clients hedging FX flows because the underlying can be transacted at a better rate than the forward for zero premium. The better rate is funded by the increased notional on the sell leg.

ATM Calendar Spread

ATM calendar spreads are combinations of two ATM contracts, one bought and one sold (hence a spread), to different maturities. In old-school trading language, calendar spreads are called *horizontal spreads*.

Buying the calendar spread means *buying* the longer date (the "back date" or "far date") and *selling* the shorter date (the "near date"). If the notionals are set such that there is zero net vega on the structure, it becomes a vega-neutral ATM calendar spread.

A long 3mth/6mth equal notional ATM calendar spread is constructed using four legs, assuming the ATM is traded as a straddle:

- Leg 1: Sell 3mth ATM USD call/JPY put in notional *N*.
- Leg 2: Sell 3mth ATM USD put/JPY call in notional *N*.
- Leg 3: Buy 6mth ATM USD call/JPY put in notional *N*.
- Leg 4: Buy 6mth ATM USD put/JPY call in notional *N*.

Legs 1 and 2 form a short ATM straddle and legs 3 and 4 form a long ATM straddle.

ATM Calendar Spread Price Making

Standard vanilla spread pricing rules apply: A spread (two-way) price is shown on the ATM straddle with more vega and the other ATM straddle is shown choice. Therefore, for ATM calendar spreads the far ATM straddle is spread since vega increases at longer maturities. Calendar spreads usually have substantial offsetting vega risk and are therefore quoted tighter than two ATM contracts in the same direction: Perhaps the standard bid–offer spread will be shown on the far leg, with the near leg quoted choice.

ATM Calendar Spread Trading Exposures

ATM calendar spreads give exposures to the shape of the ATM curve since they contain offsetting vega positions in two tenors. Exhibit 8.10 shows the bucketed vega profile from the long 3mth/6mth vega-neutral ATM calendar spread. This position will make money if the 3mth ATM volatility rises more than the 6mth ATM volatility.

Call/Put Spreads

Vanilla call spreads and vanilla put spreads have two legs in the same notional and to the same maturity, either both calls or both puts. One leg is a buy and the other is a

Tenor	Vega (USDk)
O/N	-
1W	-
2W	-
1M	-
2M	-
3M	-200
6M	200
1Y	-
2Y	-
Total	-

EXHIBIT 8.10 Bucketed vega profile from long 3mth/6mth vega-neutral ATM calendar spread

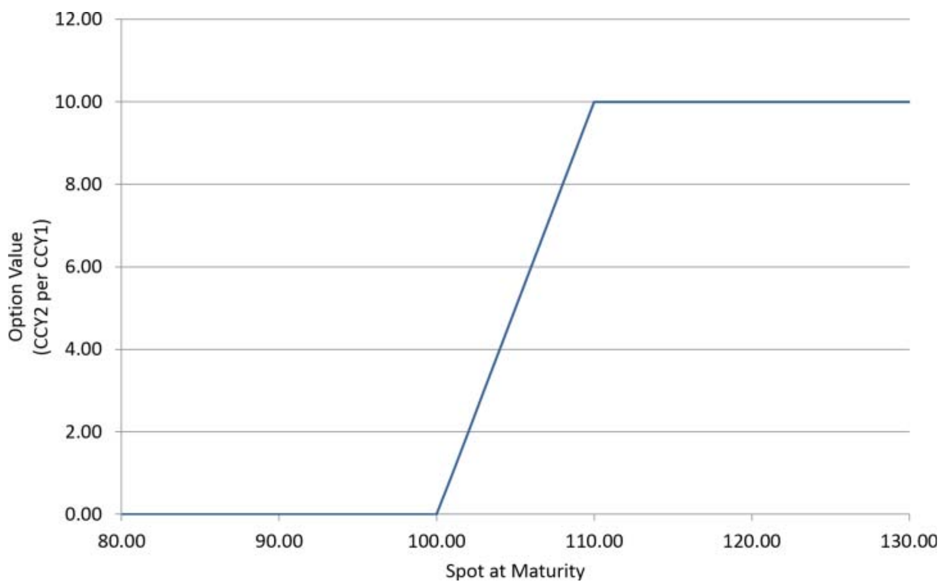


EXHIBIT 8.11 Value at maturity of long 100.00/110.00 call spread

sell; hence the structure is a spread. When buying the call/put spread, the bought leg is always more expensive, and hence further in-the-money than the sold leg.

Exhibit 8.11 shows the value at maturity of a long USD/JPY 100.00/110.00 USD call spread in notional N per leg:

- Leg 1: Buy USD call/JPY put with strike 100.00 in notional N.
- Leg 2: Sell USD call/JPY put with strike 110.00 in notional N.

Buying call/put spreads is popular with institutional clients because they enable a directional view with no possible downside other than the initial premium at a cheaper cost than the outright vanilla. In old-school trading language, call/put spreads are called *vertical spreads*.

If left unhedged until maturity:

- A long CCY1 call spread pays out if spot is above the lower strike with the payoff capped at the higher strike.
- A long CCY1 put spread pays out if spot is below the higher strike with the payoff capped at the lower strike.

Call/Put Spread Price Making

The standard vanilla spread quoting rules apply to call spreads and put spreads too: Spread the leg with more vega and choice the other leg. It is interesting to note that once call/put spreads are placed in a delta hedged trading portfolio, the vega exposures are the same as the vega exposures from a risk reversal with the same strikes; the long strike generates a long vega exposure and the short strike generates a short vega exposure.

Seagull

A long seagull contract is a long call/put spread *plus* an additional short put/call. It is most often a product sold for zero initial premium to investors who want to hedge an underlying FX exposure.

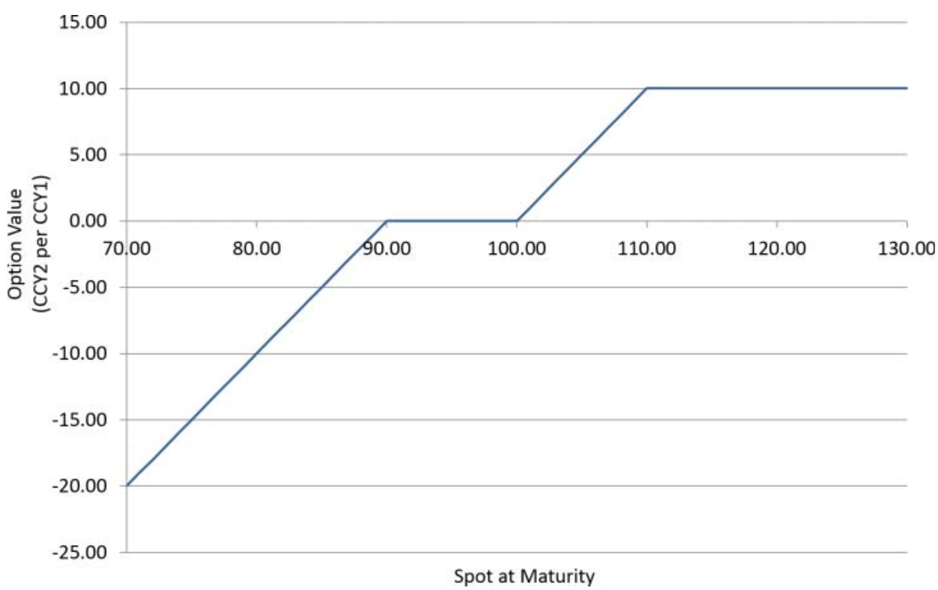


EXHIBIT 8.12 Value at maturity of long 90.00/100.00/110.00 seagull

Exhibit 8.12 shows the value at maturity of a long USD/JPY 90.00/100.00 /110.00 seagull in notional N per leg:

- Leg 1: Sell USD put/JPY call with strike 90.00 in notional N.
- Leg 2: Buy USD call/JPY put with strike 100.00 in notional N.
- Leg 3: Sell USD call/JPY put with strike 110.00 in notional N.

Legs 2 and 3 form a long call spread.

Seagull Price Making

On vanilla spreads with more than two legs there are few concrete rules on how to quote prices but generally the price is easier to understand if as many legs as possible are quoted choice. Plus the legs with the most vega should be spread because their price quoted in volatility terms will be the tightest.

CHAPTER 9

Vanilla FX Derivatives Risk Management

FX derivatives trading portfolios contain different types of deal: vanilla options, exotic options, spots, forwards, and so on. To risk manage derivatives positions traders use Greeks; the exposures of the position to market changes. Greeks are calculated on each deal in the portfolio and then aggregated together. For a given market move, some deals in the portfolio will make money or, for example, get longer vega, and others will lose money or get shorter vega: Traders only care about the *net impact* from all deals. For this reason traders primarily describe their positions in terms of long or short positions in aggregated Greek exposures. For example, an FX derivatives trader may describe their position in a given currency pair as "flat delta, short topside gamma, and long vega."

As markets move, positive or negative P&L is generated from different aggregated exposures within the position. The most important exposures are to spot (delta exposure) and ATM volatility (vega exposure). There are also exposures to CCY1 and CCY2 interest rates (rho exposures), exposures to curve moves in these instruments, plus exposures to the shape of the volatility surface. Finally, exposures are not static: Recall the gamma and vega profiles for vanilla options from Chapter 6; exposures change as the market moves or time passes.

For these reasons, trading an FX derivatives position is not straightforward. To simplify analysis, traders often consider similar types of Greeks together:

- *Short-date risk*: delta/gamma/theta
- *ATM risk*: vega/weighted vega/bucketed vega
- *Smile risk*: exposures to the shape of the volatility surface
- *Interest rate risk*: rho/swap points
- *Cross-exposure risk* (e.g., the change in interest rate position when ATM implied volatility changes)

To understand the trading risk in a vanilla derivatives trading position, a trader must investigate all these different types of risk. Fundamentally, though, focusing on higher-order risks while neglecting first-order exposures is obvious folly. The majority of FX derivative trading P&L usually comes from exposures to spot and implied volatility:

- The *short-date position* consists of vanilla options that cause delta to change significantly as spot moves. The resultant delta exposure then causes P&L to change. These exposures mainly come from short-dated vanilla options.
- The *ATM position* consists of vanilla options that generate exposures to the ATM curve. These exposures mainly come from long-dated vanilla options.

Trading Gamma

Long vanilla option positions always give long gamma exposures while short vanilla option positions always give short gamma exposures. Exhibit 9.1 recalls how the gamma exposure from a vanilla option increases and concentrates at the strike into maturity. Therefore, the *short-date position* mainly consists of vanilla options expiring within the next month or so with strikes fairly close to current spot. These options generate significant gamma exposures that cause delta and hence P&L to change as spot moves. Traders control these P&L changes by managing their delta and gamma exposures.

One common method of viewing the short-date position is a **spot ladder** that shows P&L, delta, and gamma over a range of spot values. This allows traders to anticipate how their P&L and position will change as spot moves. In particular, traders use spot ladders to determine when to hedge their delta exposures.

The gaps between spot levels in the ladder should be aligned with the spot volatility in the currency pair. The higher the spot volatility, the wider the appropriate spot spacings. Plus, if a pegged or managed currency pair has significant jump risk,

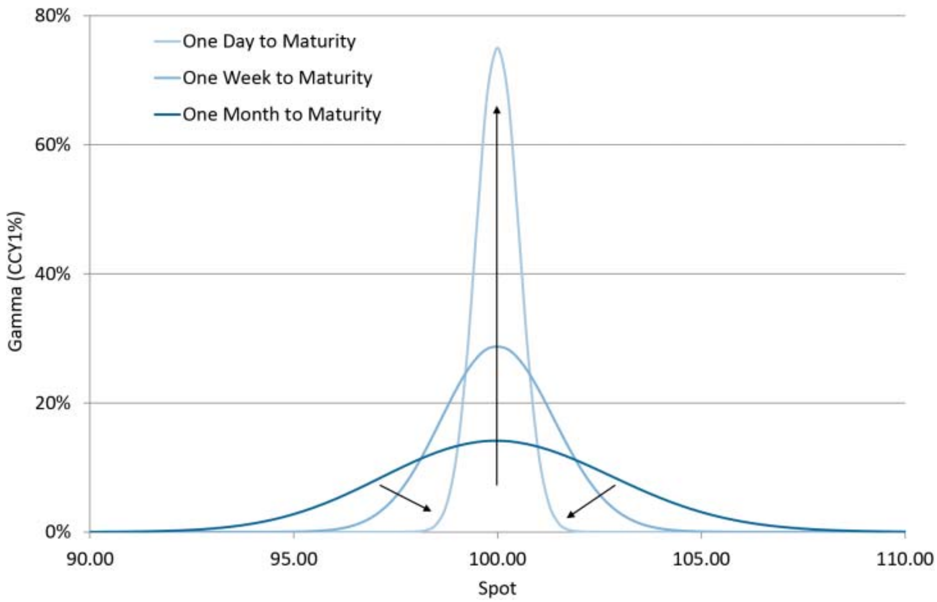


EXHIBIT 9.1 Gamma profile of long vanilla option with 100.00 strike

two ladders—one with tight spacings and one with wide spacings—might be appropriate.

Trading Long Gamma

Exhibit 9.2 shows a EUR/USD spot ladder from a *long* EUR20m 1wk ATM position, with current spot highlighted in the middle of the ladder. Values in this spot ladder are generated assuming market data (apart from spot) remains unchanged and no additional trades are executed as spot moves from its current level. This is a long

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)
1.3328	188,157	9,282,211	1,133,443
1.3262	128,980	8,502,448	2,033,607
1.3196	76,918	7,199,716	3,205,066
1.3130	35,687	5,285,401	4,436,512
1.3065	9,097	2,811,473	5,393,734
1.3000	-	**0.00**	**5,629,367**
1.2935	9,425	-2,827,304	5,400,400
1.2870	36,117	-5,309,527	4,441,749
1.2806	76,873	-7,225,145	3,204,447
1.2742	127,669	-8,523,634	2,027,795
1.2678	184,712	-9,296,817	1,125,729

Daily P&L (USD)	-

EXHIBIT 9.2 Initial EUR/USD spot ladder with long gamma exposure

vanilla option position and is therefore long gamma. Peak gamma is at current spot because it is an ATM contract. The contract is delta neutral by construction so initial delta is zero.

As spot moves, P&L change occurs faster the further spot moves in either direction. This is characteristic of being long the second derivative (i.e., long gamma). In trading positions, gamma is quoted per 1% move in spot, so, for example, on a 1% spot move higher (from 1.3000 to 1.3130), delta increases by approximately +EUR5.6m (current gamma).

Assume EUR/USD spot rises from 1.3000 to 1.3065, as shown in Exhibit 9.3:

- Spot higher and long gamma $\left(\frac{\partial delta}{\partial spot}\right)$ leads to an increasing long delta position as spot rises.
- Spot higher and long delta $\left(\frac{\partial price}{\partial spot}\right)$ produces *positive P&L change* (i.e., making money, a good thing) as seen in the spot ladder.

The trader now has a decision to make:

1. *Sell spot* in the market to reduce the long EUR2.8m delta exposure, known as "taking profit." Selling EUR2.8m EUR/USD spot will hedge (offset) the delta position back to approximately flat and the delta hedging process can start again. After selling EUR2.8m EUR/USD spot at 1.3065, the position is shown in Exhibit 9.4. Note that the P&L change is now approximately equal for a same-sized up or down move in spot; the position is "balanced."
2. *Do not hedge the delta* (i.e., let the position run). If spot continues higher, the position will make even more money, but if spot retraces back lower, the positive P&L will be lost. If delta is not hedged, the position is not balanced. By allowing a delta position to accumulate the trader has an increased exposure to the future direction of spot.

EUR/USD	*P&L Change (USD)*	*Delta (EUR)*	*Gamma (EUR)*
1.3395	242,426	9,692,767	554,939
1.3328	179,059	9,282,211	1,133,443
1.3262	119,883	8,502,448	2,033,607
1.3196	67,820	7,199,716	3,205,066
1.3130	26,590	5,285,401	4,436,512
1.3065	-	**2,811,473**	**5,393,734**
1.3000	-9,097	-14,332	5,656,990
1.2935	420	-2,840,787	5,396,886
1.2870	27,191	-5,320,602	4,435,966
1.2806	68,007	-7,233,124	3,198,182
1.2742	118,844	-8,528,676	2,022,508

Daily P&L (USD)	**9,097.47**

EXHIBIT 9.3 EUR/USD spot ladder with spot higher

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)
1.3395	150,052	6,892,767	554,939
1.3328	105,345	6,482,211	1,133,443
1.3262	64,735	5,702,448	2,033,607
1.3196	31,147	4,399,716	3,205,066
1.3130	8,299	2,485,401	4,436,512
1.3065	**-**	**11,473**	**5,393,734**
1.3000	9,194	-2,814,332	5,656,990
1.2935	36,911	-5,640,787	5,396,886
1.2870	81,790	-8,120,602	4,435,966
1.2806	140,624	-10,033,124	3,198,182
1.2742	209,389	-11,328,676	2,022,508

Daily P&L (USD)	**9,097.47**

EXHIBIT 9.4 EUR/USD spot ladder after rebalancing delta

Now, from the same starting point, assume EUR/USD spot falls from 1.3000 to 1.2935, as shown in Exhibit 9.5:

- Spot lower and long gamma $\left(\frac{\partial delta}{\partial spot}\right)$ leads to an increasing short delta position as spot falls.
- Spot lower and short delta $\left(\frac{\partial price}{\partial spot}\right)$ produces *positive P&L change* again, as seen in the spot ladder.

Once again, spot has moved, a delta position results from the long gamma, and the trader has two choices:

1. *Buy spot* in the market to reduce the delta exposure. Again, this is taking profit but the trader does not have to completely hedge the delta exposure. For example, after buying back just EUR2m EUR/USD spot at 1.2935, the position is shown

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)
1.3262	119,273	8,497,338	2,038,909
1.3196	67,255	7,191,671	3,211,333
1.3130	26,089	5,274,280	4,442,294
1.3065	-419	2,797,970	5,397,263
1.3000	-9,424	-14,332	5,656,990
1.2935	**-**	**-2,827,304**	**5,400,400**
1.2870	26,691	-5,309,527	4,441,749
1.2806	67,448	-7,225,145	3,204,447
1.2742	118,244	-8,523,634	2,027,795
1.2678	175,287	-9,296,817	1,125,729
1.2615	235,673	-9,701,277	548,266

Daily P&L (USD)	**9,425.39**

EXHIBIT 9.5 EUR/USD spot ladder with spot lower

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)
1.3262	184,598	10,497,338	2,038,909
1.3196	119,385	9,191,671	3,211,333
1.3130	65,089	7,274,280	4,442,294
1.3065	25,516	4,797,970	5,397,263
1.3000	3,511	1,985,668	5,656,990
1.2935	**-**	**-827,304**	**5,400,400**
1.2870	13,756	-3,309,527	4,441,749
1.2806	41,642	-5,225,145	3,204,447
1.2742	79,633	-6,523,634	2,027,795
1.2678	123,934	-7,296,817	1,125,729
1.2615	171,642	-7,701,277	548,266

Daily P&L (USD)	**9,425.39**

EXHIBIT 9.6 EUR/USD spot ladder after partially rebalancing delta

in Exhibit 9.6. By not buying back the full delta amount, the position is not completely balanced, but it is more balanced than it was.

2. *Do not hedge the delta* (i.e., let the position run). If spot continues lower, the position will make even more money, but if spot retraces back higher, the positive P&L will be lost.

If the delta is initially balanced, long gamma causes a positive P&L change if spot moves either up *or* down, just so long as it moves somewhere. Hedging the delta resulting from a long gamma exposure naturally leads to buying spot when it goes lower and selling spot when it goes higher. *Buying low* and *selling high* locks in P&L over the course of the day in a process known as *trading the gamma*. When leaving orders in the market to trade a long gamma position, *looped take-profit* orders are often used (e.g., sell EUR5m at 1.3130, if done, buy EUR5m at 1.3000, if done, sell EUR5m at 1.3130).

Positions with larger gamma exposures are delta hedged more frequently. In this single option example, if spot stays close to the strike as days pass, the gamma from the vanilla option increases, which will lead to more frequent delta hedging. Then, on the expiry date of the long ATM contract, Exhibit 9.7 shows the trading position with the delta balanced around the strike.

Trading risk is viewed as of a specific horizon date (usually today) and the risk on options that expire on the horizon is viewed *at expiry time*. Therefore, the option is viewed as a *strike*; an *instantaneous delta jump* equal to the option notional rather than gamma exposure. With spot at 1.3001 (above the strike), delta exposure is +EUR10m. With spot at 1.2999 (below the strike), delta exposure is –EUR10m: a EUR20m delta change, the *notional* of the option. Trading this position is similar to trading gamma, except that *all* the delta change occurs at the strike.

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)
1.3339	328,519	10,000,000	-
1.3272	262,158	10,000,000	-
1.3206	196,127	10,000,000	-
1.3140	130,425	10,000,000	-
1.3075	65,050	10,000,000	-
1.3010	**-**	**10,000,000**	**-**
1.2945	48,600	-10,000,000	-
1.2880	113,325	-10,000,000	-
1.2816	177,726	-10,000,000	-
1.2752	241,805	-10,000,000	-
1.2688	305,564	-10,000,000	-

Daily P&L (USD)	-

EXHIBIT 9.7 EUR/USD spot ladder at maturity

For a **long strike** position:

- If spot goes above the strike, the trader can sell spot in the market to hedge the long delta exposure.
- If spot goes below the strike, the trader can buy spot in the market to hedge the short delta exposure.

Once the delta position is fully hedged back to zero, it is not possible to hedge again until spot goes back through the strike.

In general, P&L volatility from a single vanilla option increases as time to expiry gets shorter if spot remains near the strike. Plus, for a given ATM vanilla option, P&L volatility from trading the strike at expiry tends to be larger than P&L volatility from trading the gamma prior to expiry.

Trading a long gamma position looks easy—buy vanilla options to get long gamma, and if spot moves higher or lower, you make money. Long gamma naturally means spot can be bought low and sold high, hence generating positive P&L. Of course, there is a cost for this and that cost is **theta** $\left(\frac{\partial price}{\partial time}\right)$, also known as **time decay** or just **decay**. For a long gamma position, theta is the *cost* of holding the trading position from one trading day to the next.

If realized volatility is larger than implied volatility there will be more opportunities to delta hedge, which usually results in larger P&L from trading a long gamma exposure than is paid in theta, hence generating positive P&L overall.

If realized volatility is smaller than implied volatility there will be fewer opportunities to delta hedge, which usually results in smaller P&L from trading a long gamma exposure than is paid in theta, hence generating negative P&L overall.

Delta Hedging in Practice

If no delta hedging is performed throughout the trading day (and assuming no P&L from any other source), the daily P&L will be a function of the difference between yesterday's end-of-day spot and today's end-of-day spot (end-of-day market data is the official reference used for P&L generation, limit checking, etc.). Alternatively, if delta hedging were performed continuously (not possible in practice), P&L would be a function of implied volatility and realized volatility (recall from Chapter 5 that this is the essence of the Black-Scholes formula derivation). An FX derivatives trader's main focus, skill, and exposures are to FX volatility. Therefore, option positions are delta hedged frequently throughout the day.

If spot followed a mathematical random walk, the primary factor in determining the optimal delta hedge frequency of a *long gamma* position would be the spot bid–offer spread size. The Euan Sinclair book "*Volatility Trading*" has a good section on this (see Further Reading). In practice, though, decisions on when and how to trade the delta exposure are based on different factors:

- *Spot bid–offer spreads*. The wider the spreads, the less often traders like to delta hedge because they must cross a spread each time they do so. If spot bid–offer spreads are wide, it often makes more sense to leave orders in the spot market rather than transacting directly. Plus, outside the main book-runner's time zone, leaving orders ensures that opportunities to delta hedge are not missed.
- *Expected spot volatility and jumps*. During quieter times, traders may hedge smaller spot moves. But if spot jumps when important economic data is released, traders often let spot run further to take advantage of their positive second derivative.
- *Time of day*. At the start of the trading day, traders tend to let deltas run further in an attempt to generate larger P&Ls, but toward the option expiry they "scalp" (trade the delta) more aggressively.
- *Key levels in the spot market (i.e., rounded spot levels where barriers will be positioned or recent low and high spot levels)*. If spot breaks through key levels, this often causes spot to run further and traders may let larger deltas accumulate before hedging.
- *How spot is moving*. If spot is whipping up and down, traders will delta hedge more aggressively, whereas if spot is trending strongly, they may let the delta run further before hedging.

Many derivatives traders love trading spot to manage their delta exposures throughout the day and they believe that they add value by doing this. A friend at another bank reported their boss proclaiming (with a straight face) "trading spot is like playing Mozart" and selling EUR/CHF spot shortly before the Swiss central bank unexpectedly intervened and sent spot over 8% higher. In stable market conditions it is debatable how much value a trader can add, but when unexpected events occur, the ability of a human trader to rapidly process new information in context can be an advantage.

Trading Short Gamma

Exhibit 9.8 shows a EUR/USD spot ladder from a *short* EUR20m 1wk ATM position.

P&L change, delta, and gamma are all equal and opposite (negative) to the long ATM position:

- Spot higher and short gamma leads to an increasing short delta position as spot rises.
- Spot higher and short delta produces *negative P&L change*.

while:

- Spot lower and short gamma leads to an increasing long delta position as spot falls.
- Spot lower and long delta again produces *negative P&L change*.

If spot moves in either direction, the delta change from a short gamma position produces a negative P&L change. The decision for the trader therefore becomes whether to *stop-loss* the delta (i.e., buy high/sell low) or let it run and potentially lose even more at an increasing rate due to the short second derivative. In practice, trading short gamma is often made easier by accepting that at least half the theta earned will be lost through stopping out. Trading with close stop-loss orders helps avoid large negative P&Ls.

A short gamma position *earns* theta but loses money throughout the trading day as spot moves. Again, if the position is delta hedged frequently, the overall P&L generated is primarily a function of implied volatility and realized volatility, but this

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)
1.3328	-188,157	-9,282,211	-1,133,443
1.3262	-128,980	-8,502,448	-2,033,607
1.3196	-76,918	-7,199,716	-3,205,066
1.3130	-35,687	-5,285,401	-4,436,512
1.3065	-9,097	-2,811,473	-5,393,734
1.3000	-	-	**-5,629,367**
1.2935	-9,425	2,827,304	-5,400,400
1.2870	-36,117	5,309,527	-4,441,749
1.2806	-76,873	7,225,145	-3,204,447
1.2742	-127,669	8,523,634	-2,027,795
1.2678	-184,712	9,296,817	-1,125,729

Daily P&L (USD)	-

EXHIBIT 9.8 Initial EUR/USD spot ladder with short gamma

time the position will generate higher positive P&L if realized volatility is below implied volatility.

Trading the Short-Date Position

Traders investigate their short-date position in order to identify the main trading risks. Exhibit 9.9 shows a spot ladder from an AUD/USD FX derivatives trading position.

The position is long gamma at current spot and to the downside, but short gamma to the topside. This gamma profile implies that at shorter maturities the position is net long downside vanillas and short topside vanillas.

The first thing to check is that the delta and gamma positions tie up:

- Long gamma and spot higher → delta gets longer
- Long gamma and spot lower → delta gets shorter
- Short gamma and spot lower → delta gets longer
- Short gamma and spot higher → delta gets shorter

The position is short gamma to the topside, yet the delta jumps longer as spot goes higher from 0.9177 to 0.9205 as highlighted in Exhibit 9.10. Gamma profiles from vanilla options are smooth. Therefore, there must be a "strike" (i.e., a vanilla option expiring today) in the position causing a delta jump through it. Moreover, it must be a long strike since the delta jumps longer with spot higher.

Details of specific options in the position can be checked using a **trade query** or **strike topography**. A trade query is used to return details of every option in the portfolio as shown in Exhibit 9.11.

Strike topographies display a grid of expiry dates and strikes in one currency pair, making it easier to visualize the positioning of strikes. This is shown in Exhibit 9.12.

AUD/USD	P&L Change (USD)	Delta (AUD)	Gamma (AUD)
0.9288	48,882	1,531,811	-9,589,063
0.9260	40,469	4,415,610	-9,459,303
0.9233	24,357	7,045,015	-7,887,526
0.9205	1,951	8,954,786	-4,722,999
0.9177	-1,510	-308,939	-224,975
0.9150	0	**-1,014,115**	**4,736,927**
0.9123	5,509	-3,137,788	9,034,647
0.9095	18,416	-6,306,965	11,653,858
0.9068	40,711	-9,921,215	12,039,969
0.9041	72,630	-13,344,891	10,541,826
0.9014	112,906	-16,121,272	7,929,408

EXHIBIT 9.9 AUD/USD spot ladder

AUD/USD	P&L Change (USD)	Delta (AUD)	Gamma (AUD)
0.9288	48,882	1,531,811	-9,589,063
0.9260	40,469	4,415,610	-9,459,303
0.9233	24,357	7,045,015	-7,887,526
0.9205	1,951	8,954,786	-4,722,999
0.9177	-1,510	-308,939	-224,975
0.9150	0	**-1,014,115**	**4,736,927**
0.9123	5,509	-3,137,788	9,034,647
0.9095	18,416	-6,306,965	11,653,858
0.9068	40,711	-9,921,215	12,039,969
0.9041	72,630	-13,344,891	10,541,826
0.9014	112,906	-16,121,272	7,929,408

EXHIBIT 9.10 AUD/USD spot ladder with delta jump highlighted

Trade ID	Currency Pair	Portfolio	Expiry	Direction	Type	Strike	Cut	C/P	CCY1 Notional
41725659	AUD/USD	214	16-Jan-13	B	Vanilla	0.9200	NY	C	10,000,000
42204333	AUD/USD	214	18-Jan-13	B	Vanilla	0.9100	NY	P	50,000,000
42204787	AUD/USD	214	23-Jan-13	S	Vanilla	0.9200	NY	C	50,000,000
42204333	AUD/USD	214	18-Jan-13	B	Vanilla	0.9100	NY	P	50,000,000

EXHIBIT 9.11 AUD/USD trade query

AUD/USD	0.9014	0.9041	0.9068	0.9095	0.9123	**0.9150**	0.9177	0.9205	0.9233	0.9260	0.9288
Wed 16-Jan-13								**0.9200 AUD10m**			
Thu 17-Jan-13											
Fri 18-Jan-13				**0.9100 AUD50m**							
Sat 19-Jan-13											
Sun 20-Jan-13											
Mon 21-Jan-13											
Tue 22-Jan-13											
Wed 23-Jan-13								**0.9200 -AUD50m**			

EXHIBIT 9.12 AUD/USD strike topography

There may be just one option or multiple long/short options at a particular expiry date and strike in the strike topography; only the net notional is displayed. This information plus other missing details (e.g., cut or counterparty) can be obtained by drilling down into a given expiry and strike level.

These views confirm that the position is long AUD10m of 0.9200 strike expiring at NY cut today. This is important information because the delta jump at 0.9200 requires particular attention from the trader. Strikes are closely risk managed due to the delta jumps they generate. The larger the notional, the more attention a strike requires. This long strike will generate negative (paying) theta, but it gives the opportunity to make money back from it by trading delta over the course of the day (before it expires).

In general, the key question for a trader is whether they *like* the short-date position—that is, *will it generate a profit?* If there are aspects of the position that the trader doesn't like, trades should be executed to change the position. However, the *cost* of achieving a preferable position must be taken into account. Very often traders put up with a position they don't particularly like because the liquidity isn't available to get a position they do like at reasonable cost.

Individual traders will assess a short-date position differently but there are several common areas to consider.

P&L Balance

P&L balance involves assessing whether the position generates similar P&L changes for similar-sized up and down spot moves. This, of course, assumes the trader has no strong *opinion* on future spot moves. P&L balance can be checked in the spot ladder.

The P&L and delta positions should tie up:

- Long delta and spot higher → P&L increases
- Long delta and spot lower → P&L decreases
- Short delta and spot lower → P&L increases
- Short delta and spot higher → P&L decreases

Any discrepancies should be investigated; unexpected P&L jumps are most likely caused by exotic risk in the position.

Assuming the trader has no opinion on spot direction, it can be checked that the P&L change on equal-sized up and down spot moves is roughly the same. Traders usually look at a one-day spot move of approximately one-and-a-half standard deviations up and down. In currency pairs with implied volatility somewhere close to 10%, spot moves around 0.75% to 1% are often considered within P&L balance. In practice, though, traders look over a wider spot range, and if there are extreme negative P&Ls at spot levels which might conceivably be reached, it is more important to concentrate on controlling the P&L in those areas rather than on pure spot up/spot down P&L balance.

In the example AUD/USD position, the P&L is roughly balanced, but a bit of additional spot could be bought to balance it better as highlighted in Exhibit 9.13.

To increase the P&L at 0.9233 by USD8k and hence reduce the P&L by USD8k at 0.9068 (the equivalent spot move lower), $8{,}000/(0.9233 - 0.9150) =$ AUD960k spot should be bought. Buying spot changes the delta exposure and hence adjusts the P&L profile over spot as shown in Exhibit 9.14.

AUD/USD	P&L Change (USD)	Delta (AUD)	Gamma (AUD)
0.9288	48,882	1,531,811	-9,589,063
0.9260	40,469	4,415,610	-9,459,303
0.9233	24,357	7,045,015	-7,887,526
0.9205	1,951	8,954,786	-4,722,999
0.9177	-1,510	-308,939	-224,975
0.9150	**0**	**-1,014,115**	**4,736,927**
0.9123	5,509	-3,137,788	9,034,647
0.9095	18,416	-6,306,965	11,653,858
0.9068	40,711	-9,921,215	12,039,969
0.9041	72,630	-13,344,891	10,541,826
0.9014	112,906	-16,121,272	7,929,408

EXHIBIT 9.13 AUD/USD spot ladder with P&L balance highlighted

AUD/USD	P&L Change (USD)	Delta (AUD)	Gamma (AUD)
0.9288	62,138	2,491,811	-9,589,063
0.9260	51,057	5,375,610	-9,459,303
0.9233	32,287	8,005,015	-7,887,526
0.9205	7,230	9,914,786	-4,722,999
0.9177	1,125	651,061	-224,975
0.9150	**0**	**-54,115**	**4,736,927**
0.9123	2,873	-2,177,788	9,034,647
0.9095	13,154	-5,346,965	11,653,858
0.9068	32,829	-8,961,215	12,039,969
0.9041	62,137	-12,384,891	10,541,826
0.9014	99,808	-15,161,272	7,929,408

EXHIBIT 9.14 AUD/USD spot ladder with better P&L balance highlighted

In practice, the delta and hence the P&L profile over different spot levels can be adjusted by buying or selling spot but full P&L balance is a multi-dimensional problem. The gamma profile must be considered as well as impacts from ATM volatility and the volatility smile.

Theta

In trading positions, theta is quoted as the cumulative change in value from one trading day to the next for all deals in the position.

If a trading position is mainly long strikes and long gamma, the long option values reduce over time and theta will be negative. The negative theta is roughly the maximum that can be *lost* from the short-date position if the delta is initially balanced since delta hedging the long gamma position will make money back.

If a position is mainly short strikes and short gamma, the short option values reduce over time and theta will be positive. The positive theta is roughly the maximum that can be *made* from the short-date position if the delta is initially balanced since delta hedging the short gamma position will cost money. A trader friend at another bank was once told, "You can have any gamma position you like, so long as you don't pay any theta!"

In Black-Scholes world, gamma and theta are proportional: Higher theta implies higher gamma exposure and hence higher P&L volatility. Put another way, with more positive or negative gamma the trader is more exposed to how spot moves. If a trader decides that theta and hence P&L volatility is too large, short-dated vanilla options should be transacted to reduce the existing gamma position.

It is worth noting here that factors other than gamma can also generate theta, for example, smile decay, roll down the ATM curve, funding cash balances, and so forth. These factors are explored in Chapter 14.

Gamma

The spot ladder shows how the gamma exposure changes at different spot levels. Recall that:

- Long gamma → want spot to move more than implied volatility suggests it will, the more the better.
- Short gamma → want spot to move less than implied volatility suggests it will, the less the better.

Fundamentally, the trader has a decision to make: If the position is long gamma, looking at the spot ladder and the theta paid, will spot move enough to make back the theta from delta hedging? If the position is short gamma, looking at the spot ladder and the theta earned, will spot cause a loss from delta hedging larger than the theta?

Strikes

For options that settle against a cut rather than a fix, at the expiry time of each option in the position, the owner of each option contacts the writer to tell them if they want to exercise the option. The most common cut time in G10 pairs is NY cut and the 20-minute period before NY cut is simply called *expiries* on most trading desks in London and New York.

In practice, from the start of the trading day, bank trading desks contact each other to exercise strikes that are very far from the current spot level. Over the course of the day, the strikes being dealt with get closer and closer to current spot and at some point responsibility passes from middle office to traders.

If spot is very close to a particular strike, communication between the option owner and the option writer will be established shortly beforehand, with the option owner asking to "hold" on the strike. Then as the cut time arrives, the decision to exercise or expire the contract is made based on the prevailing spot level.

This process is usually smooth but it can get dangerous when a trading position has multiple offsetting strikes at the same level. For example, a trader might be short a contract to one bank and long the same contract to a second bank. In order to keep an unchanged delta position the trader must wait for the bank that is long the contract to exercise or expire before they can pass the same decision onto the other bank. Often it is obvious what action should be performed, but if spot is exactly on the strike, this can be a difficult situation to manage. There are stories about traders hiding under the desk and refusing to come out until expiries are finished.

It is also possible for an option to be **partially exercised**. For example, if an option notional is AUD50m, the option owner could partially exercise it in only AUD30m. When this occurs, the new delta exposure within the trading position must be established as quickly as possible. Remaining calm when a partial exercise is requested on a large strike is a rare skill.

After expiries, if there were close strikes, the delta position may need to be rebalanced. For example, if a trading position is long USD100m USD/JPY 105.00 strike, delta may be positioned long USD50m above and short USD50m below the strike. After expiries, when the option has "rolled off," the delta exposure will remain with no protection from the strike and hence the delta must be hedged back to flat by trading spot.

For options that settle against a fix rather than a cut, if the option is in-the-money at expiry, rather than generating a spot FX transaction, the cash-settled option generates a single cash payment. In practice, this means that the delta position from the option disappears and must be replaced with an FX trade which is usually transacted off the same fix as the option settled, hence minimizing risk.

Gamma/Strike Profile

A strike topography can be used to judge how the gamma profile will change over time. Strikes disappear from the position as they expire and options "behind" start to produce more gamma as they move closer to the horizon. Based on this, a trader can judge how their gamma position will evolve. Also, if there are significant option expiries on (what traders call) "good" or "bad" dates in the future, a trader might want to unwind or offset these positions.

So-called "good dates" have events (data releases) on them and spot is therefore expected to move more than usual, for example, days on which employment or GDP data is released. Such events can cause spot to jump as the market adjusts to new information. Traders must therefore know the big events coming up in their currency pairs for the next month at least.

So-called "bad dates" are expected to be quiet and therefore spot is expected to move less than usual, for example, days over Easter or between Christmas and New Year's.

It is easier to risk manage a short-date position that is long good days and short bad days because the position will be long gamma when spot is moving more and short gamma when spot is moving less. However, this must be considered with reference to the price paid to achieve the position. Anyone can buy a Non-Farm Payroll day at 25% volatility or sell a holiday Monday between Christmas and New Year's for 3% volatility, but does that represent good value? Analysis that answers questions like these is introduced in Chapter 17.

Also, some traders are happy to run lots of strike risk (i.e., have lots of open strikes in the book), while others prefer a much cleaner position:

- Many strikes in position → position harder to manage since delta changes more → higher P&L volatility but less spread cross paid away cleaning the position.
- Few strikes in position → position easier to manage since delta changes less → lower P&L volatility but more spread cross paid away to obtain the clean position.

Traders must know the big vanilla options they have in their position, so if there is an opportunity to close out a contract with clients or in the interbank broker market with minimal spread cross (or even spread earned), they can take it. Traders may also show improved prices to clients (called "axes") to close out existing positions.

Trading the ATM Position

The ATM position contains exposures to the implied volatilities within the ATM curve. The standard way of assessing the ATM position uses vega and weighted vega exposures. Vega is a position's exposure to *parallel shifts* in the ATM curve. Recall from Chapter 6 that the vega profile on a vanilla option looks like a normal distribution bell-curve, with peak vega at the strike as per Exhibit 9.15.

Buying EUR20m 1yr ATM at 10% implied volatility gives the EUR/USD trading position shown in Exhibit 9.16. Since vega is now the focus of the spot ladder, wider spot spacings are appropriate. Total vega of roughly EUR80k is as expected since a 1yr ATM has 0.40% vega. It is important to remember that vega is proportional to the square root of time (so e.g., the 3mth ATM vega is half the 1yr ATM vega). Traders have these approximate ATM vega reference points in their head: O/N = 0.02%, 1mth = 0.10%, 3mth = 0.20%, 1yr = 0.40%. Therefore, an overnight ATM option costing 10% implied volatility will cost roughly $0.02\% \times 10 = 0.20\%$ in premium terms, although note that this approximation works for ATM options only.

If 1yr ATM implied volatility then rises to 10.5%, the position changes as per Exhibit 9.17.

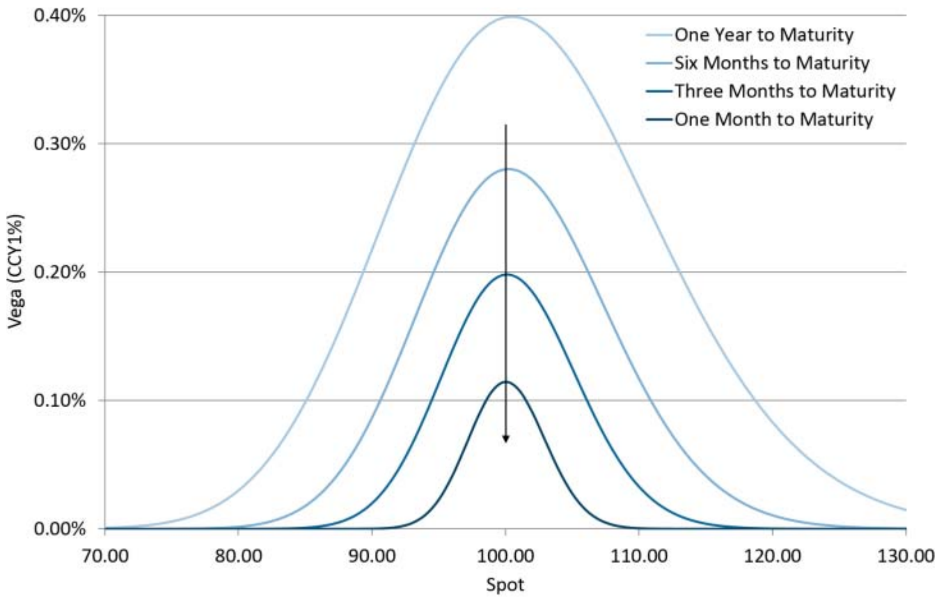


EXHIBIT 9.15 Vega profile of long vanilla option with 100.00 strike

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)	Vega (EUR)
1.5071	1,060,741	8,605,341	267,593	26,815
1.4632	702,992	7,628,744	396,520	39,711
1.4205	405,753	6,247,117	538,387	53,893
1.3792	182,915	4,455,042	669,778	67,026
1.3390	45,726	2,323,725	763,706	76,391
1.3000	**-**	**-**	**761,438**	**79,789**
1.2610	46,942	-2,394,104	761,657	76,165
1.2232	179,644	-4,576,507	662,825	66,271
1.1865	382,094	-6,392,052	525,710	52,557
1.1509	635,393	-7,769,436	379,980	37,991
1.1164	921,198	-8,722,450	250,317	25,032

Daily P&L (USD)	-

EXHIBIT 9.16 EUR/USD long vega trading position

EUR/USD	P&L Change (USD)	Delta (EUR)	Gamma (EUR)	Vega (EUR)
1.5071	1,030,074	8,421,367	280,162	29,471
1.4632	681,064	7,418,547	400,791	42,138
1.4205	392,797	6,042,695	529,649	55,664
1.3792	177,585	4,298,122	646,544	67,933
1.3390	45,076	2,253,440	729,313	76,594
1.3000	**-**	**38,635**	**758,610**	**79,788**
1.2610	43,240	-2,245,993	729,548	76,602
1.2232	168,723	-4,349,597	644,039	67,614
1.1865	362,094	-6,132,091	522,685	54,868
1.1509	606,232	-7,521,301	389,924	40,934
1.1164	884,145	-8,517,140	267,407	28,077

Daily P&L (USD)	**51,862.68**

EXHIBIT 9.17 EUR/USD trading position with higher ATM implied volatility

P&L has risen because the position is long vega and implied volatility is higher. Assuming no second-order effects:

$$P\&L_{\Delta} = \sigma_{\Delta} \times \upsilon$$

where $P\&L_{\Delta}$ is P&L change, σ_{Δ} is implied volatility change, and υ is vega. In the example, P&L change (in USD) = 0.5 (implied volatility change) × EUR79,789 (vega) × 1.3000 (conversion from EUR to USD) = USD51,863.

One way of hedging the long ATM vega exposure would be to sell EUR40m of 3mth ATM, again at 10% implied volatility. This is shown in Exhibit 9.18.

The vega exposure at current spot is roughly flat but importantly the position is not flat in at least three ways:

1. If spot moves higher or lower, the tighter short vega distribution from the 3mth ATM results in a long vega position overall.
2. Since 3mth ATM options have a higher gamma exposure than 1yr ATM options, and double the notional of 3mth has been transacted, the position is overall short gamma.
3. The position is hedged if the ATM curve moves in parallel, but if the curve moves in a nonparallel manner, a P&L change will be generated.

Weighted vega is the exposure to **weighted** changes in the ATM curve. Within a weighted shift, short-dated ATM implied volatilities move more than long-dated ATM implied volatilities. See Chapter 14 for more detail on weighted vega.

Bucketed vega exposures (i.e., the vega exposure at each market tenor) are used to get a fuller view of the ATM position. For vanilla options, the price depends only on the implied volatility to the option maturity. Therefore, *all vega for a particular vanilla option is bucketed at maturity*. If the option expiry is between two market tenors, the vega will be split between them (e.g., the vega from a vanilla option with a 5mth expiry will appear as bucketed exposures in the 3mth and 6mth buckets). Exhibit 9.19 shows the bucketed vega exposures from the AUD/USD position.

The position is short both vega and weighted vega, but it is shorter vega than weighted vega, implying that the main short vega exposures are at longer maturities.

EUR/USD	*P&L Change (USD)*	*Delta (EUR)*	*Gamma (EUR)*	*Vega (EUR)*
1.5071	-2,017,397	-11,336,261	229,316	25,864
1.4632	-1,504,510	-12,025,713	207,959	35,033
1.4205	-983,502	-12,269,468	-113,324	37,756
1.3792	-496,622	-10,866,826	-911,073	27,950
1.3390	-134,578	-6,641,122	-1,926,667	9,974
1.3000	**-**	**-**	**-2,446,896**	**553**
1.2610	-137,737	6,813,856	-1,901,711	10,536
1.2232	-485,193	11,025,358	-852,758	28,944
1.1865	-920,324	12,292,873	-66,311	37,977
1.1509	-1,354,301	11,947,564	221,194	34,081
1.1164	-1,754,487	11,234,289	221,087	24,312

EXHIBIT 9.18 EUR/USD trading position with 3mth vega hedge

Tenor	Vega (AUDk)
O/N	5
1W	20
2W	-40
1M	100
2M	-165
3M	50
6M	190
1Y	-205
2Y	-50
Total Vega	**-95**
Total Weighted Vega	**-31**

EXHIBIT 9.19 AUD/USD bucketed vega profile

If a trader wanted to flatten this position, they should buy back an ATM vanilla in the 1yr tenor since it is the largest single short bucket. AUD25m (=100,000/0.40%) 1yr ATM could be bought back to hedge the vega exposure.

In general, it is not possible to trade implied volatility with the same frequency as spot; there is less liquidity and it is harder to take profit and stop-loss the position. Trading ATM and smile positions is a longer-term endeavor than trading the short-date position. However, the key question for a trader remains whether they *like* the ATM position—*will it generate a profit?* If there are aspects of the position that the trader doesn't like, trades should be executed to change the position.

FX Derivatives Trading P&L

If trades are transacted and monitored individually, the total P&L on each trade can be calculated and tracked over time. However, since FX derivatives trading positions contain many (potentially thousands of) trades that are all risk managed together, **mark-to-market** P&L is calculated. This involves periodically recalculating the total mid-value of all contracts in the position. The P&L is then the *change* in total value from some reference point to now. Typically daily (the P&L change since yesterday's end-of-day snapshot), month-to-date, and year-to-date P&Ls are monitored, with traders primarily tracking daily P&L within their risk management.

Throughout the day, P&Ls are updated as trading positions are refreshed with live market data. If new deals are entered into the position, the P&L from the new deal is calculated as the difference between the traded price and the prevailing mid-price within the risk management system.

For example, EUR/USD spot mid is 1.3450. A trader crosses a two-pip spread to sell EUR10m EUR/USD spot at 1.3448. When this trade is entered into a previously empty trading position, the P&L changes by $(1.3448 - 1.3450) \times$ EUR10m = –USD2k: the spread crossed to transact the deal. The trading position now has a short EUR10m delta. If the mid EUR/USD spot then falls to 1.3445 and the trader updates market data within the trading position, the P&L from this deal will rise by USD5k. In practice, there are many deals in FX derivative trading positions, each with their own delta exposures; the net P&L change as spot moves depends on the aggregated delta exposure from all deals.

The same methodology is applied to derivative contracts, with the *premium* additionally considered. For example, AUD/USD 1yr ATM mid is 10.2% implied volatility in the desk volatility surface. Traders say that 1yr ATM is "marked" at 10.2%. A trader buys AUD100m 1yr ATM at 10.2%, sometimes described as trading "at sheets." This deal causes *no P&L change* as it is entered into the risk management system because the deal was transacted at the current midmarket volatility: The long option position has the same mid value as the premium paid for the contract. If the contract had been bought for 10.3% instead, that would generate a negative P&L change of a tenth of the contract vega because the premium paid (calculated at 10.3% implied volatility) would be more than the value of the contract at the midmarket volatility (10.2%).

At the end of each day, a full snapshot of market data is taken: the end-of-day (EOD) data. The official daily P&L for a trading book is then calculated by taking the difference between yesterday's total EOD P&L and today's total EOD P&L where total P&L is calculated by summing the value of all contracts in the trading position using the relevant set of market data.

Finally, it is important to note that official P&Ls additionally take bid–offer spread into account. This is done because there can be differences between a "paper" mid valuation and P&L that could actually be realized. This occurs particularly in less liquid markets. For example, a USD/SGD trading position is long USD500k vega in the 5yr tenor. The 5yr USD/SGD ATM price in the market rises from 16.0/16.5% (16.25% mid) to 16.25/16.75% (16.5% mid). Once this market data is updated within the trading position, the simulation shows a profit of USD125k. However, closing out the position and realizing that profit would only be possible if better bids appear in the market.

FX Derivatives Market Language

Here is some common FX derivatives market slang and wisdom:

Phrase	What It Means
"Ones"/"Twos," etc.	1-month ATM implied volatility/2-month ATM implied volatility, etc.
"Double"	0.55% (i.e., "seven double" is 7.55%).
"Flot"	Small.

Phrase	What It Means
"Touch"	The tightest price on a contract shown by the brokers.
"Yard"	One billion.
"Quid"	One million GBP (i.e., "I'm long ten quid cable").
"Buck"	One million USD (i.e., "The notional is fifty bucks").
"Market moving left"	Generally used to describe a price moving lower, specifically used to describe FX swap points moving more negative or less positive.
"Market moving right"	Generally used to describe a price moving higher. Specifically used to describe FX swap points moving less negative or more positive.
"Buy the rumor and sell the fact."	Optionality should be bought when there is uncertainty (around a market event or a political situation) and sold when the uncertainty is removed.
"The first cut is the cheapest."	A reworking of a song lyric to reflect the fact that in the FX derivatives market, expiry cuts that occur earlier in the trading day always have a lower price.
"It's cheap because it's rubbish."	Used as a rebuke to a trading idea that involves buying something very cheap. There is a truth to this, involving the optimal balance between premium and payoff.
"The trend is your friend."	Standard market banter about following a trend.
"Long-and-wrong."	Holding a long position in a financial instrument for which the price is going lower.
"Short-and-caught."	Holding a short position in a financial instrument for which the price is going higher.
"When in trouble, double!"	Dreadful trading advice about doubling exposures in order to make back a negative P&L.
"More sellers than buyers."	A true but unhelpful explanation for the market moving lower.
"More buyers than sellers."	A true but unhelpful explanation for the market moving higher.

Vanilla FX Derivatives Miscellaneous Topics

Several FX derivatives pricing technicalities have thus far been brushed under the carpet. In this chapter, present and future valuing, tenor expiry date calculations, and premium conversions are examined.

Present Valuing and Future Valuing

It is well understood that \$10 received today is worth more than \$10 received in the future due to the time value of money. The core of the argument is that money received now has interest-earning potential if it is placed on deposit (of course, assuming interest rates are positive).

- **Present valuing** involves bringing a cash value in the future back to its equivalent value at the present day. In derivatives, option values are often calculated at maturity and must then be present valued.

- **Future valuing** is the opposite operation: taking a cash value today and pushing it to some future date.

The calculation for present and future valuing uses a **discount factors** (df) to a given maturity in the currency in which the cash value in denominated. Discount factors are generally less than 1 and:

$$Value_{present} = df \ . \ Value_{future}$$

Discount factors are calculated from interest rates but the exact calculation depends on how interest compounds on the deposited cash balance. If interest is all paid in one payment at the deposit maturity, the interest rates are called **zero interest rates** and the discount factor is calculated using:

$$df = \frac{1}{(1 + r_0 T)}$$

where T is the time to deposit maturity measured in years and r_0 is a zero interest rate.

If interest is compounded on an *annual basis*, the discount factor is calculated using:

$$df = \frac{1}{(1 + r_A)^T}$$

where r_A is an annually compounded interest rate. The market instrument called **deposits** (also called "depos") follow this convention.

In general, given a regular compounding frequency of m times a year, the discount factor is calculated using:

$$df = \frac{1}{\left(1 + \frac{r_m}{m}\right)^{Tm}}$$

where r_m is an interest rate in the given compounding rate.

As m gets larger, in the limit the rate becomes a **continuous compounded interest rate**: the rate used within the Black-Scholes mathematical framework. In this case:

$$df = e^{-rCCY. \ T}$$

where $rCCY$ is a continuously compounded interest rate.

In practice, interest rate curve building and calculations are far more complicated than this. Different interest rate curves are used and they must be bootstrapped together using a variety of quantitative techniques. These methods are important for interest rate traders but they are not day-to-day concerns for most FX derivatives traders.

Market Tenor Calculations

A well-defined logic exists for calculating expiry and delivery dates for market tenors within the FX derivatives market. Four dates are defined:

- *Horizon*: the date on which the trade originates (i.e., today)
- *Spot date*: the date on which the initial transfer of funds (the premium) often takes place and the date on which any spot hedge settles.
- *Expiry date*: the date on which the contract expires and any final transfer of funds is known.
- *Delivery date*: the date on which the final transfer of funds generated from the contract usually takes place and the date on which forward hedges usually settle.

If the final transfer of funds takes place after the natural delivery date, the option is described as **late delivery** (see Chapter 27 for examples of different late delivery vanilla options). All these dates can only ever be weekdays since the FX market is not open over the weekend.

These four dates are summarized on the timeline shown in Exhibit 10.1. This timeline may be different for overnight options since the expiry date can be before the spot date.

The term "business day" is used to describe a day that is not on a weekend and is also not a holiday in either currency within the relevant currency pair. A stylized version of these tenor calculations is implemented in Practical D.

Calculating Spot Dates

The spot date is calculated from the horizon (T). There are two possible cases:

1. If a currency pair has T+1 settlement (e.g., USD/CAD), the spot date is one day after the horizon. In this case, T+1 must be a business day and also not a U.S. holiday. If an unacceptable day is encountered, move one further day into the future and test again.
2. If a currency pair has T+2 settlement, the spot date is two days after the horizon. The calculation of T+2 must be done by considering each currency within the pair separately. For USD there must be one clear working day between the horizon and the spot date and for all non-USD currencies there must be two clear working days between the horizon and the spot date.

EXHIBIT 10.1 Timeline of the four key dates within market tenor calculations

In addition, for most currencies, no money can clear (settle) on U.S. holidays, meaning that the spot date cannot occur on a U.S. holiday even if USD is not a currency within the currency pair.

Calculating Expiry and Delivery Dates

Market tenors for FX option contracts are quoted either as "overnight" or in terms of a number of days, weeks, months, or years. In general, the expiry date can be any weekday even if it is a holiday in one or both of the currencies, except January 1. There are differing conventions for calculating expiry and delivery dates depending on the tenor.

Overnight

For overnight trades, the expiry date is the next weekday after the horizon. The delivery date is then calculated from the expiry date in the same way as the spot date is calculated from the horizon.

Days and Weeks

For a trade with a v *days* tenor, the expiry date is the day v calendar days after the horizon (unless this expiry date is a weekend or January 1, in which case the tenor is invalid) and for a trade with an x *weeks* tenor, the expiry date is $7x$ calendar days after the horizon (unless this expiry date is a weekend or January 1, in which case the tenor is invalid). The delivery date is then calculated from the expiry date in the same way as the spot date is calculated from the horizon.

Months

For a trade with a y *months* tenor, the expiry date is found by first calculating the spot date, and then moving forward y months from the spot date to the delivery date. If the delivery date is a non-business day or a U.S. holiday, move forward until an acceptable delivery date is found. Finally, the expiry date is calculated from the delivery date using an "inverse spot date" operation (i.e., find the expiry date for which the delivery date would be its spot date).

Years

For a trade with a z *years* tenor, the expiry date is found by first calculating the spot date, and then moving forward z years from the spot date to the delivery date. If the delivery date is a non-business day or a U.S. holiday, move forward until an acceptable delivery date is found. Finally, the expiry date is calculated from the delivery date using an "inverse spot date" operation (i.e., find the expiry date for which the delivery date would be its spot date).

Special Cases

There are two special cases involving trades that take place around the end of the month and have a tenor defined in month or year multiples. Defining the *target month* to lie *x* months forward from the spot date month if the tenor is *x months*, for example, if the spot date month is February and the tenor is 3M (three months), the target month is May.

1. If the spot date falls on the last business day of the month in the currency pair, then the delivery date is defined by convention to be the last business day of the target month. For example, assuming all days are business days: If the spot date is April 30, a one-month time to expiry will make the delivery date May 31. This is described as trading "end-end."
2. If the spot date falls before the end of the month but the resultant delivery date is beyond the end of the target month, then the delivery date is defined by convention to be the last business day of the target month. For example, assuming all days are business days: If the spot date is January 30, a *1 month* time to expiry implies a delivery date of February 30; however, this doesn't exist and the expiry date becomes February 28 (in a non-leap year, obviously).

Also, expiry date and delivery date calculations sometimes adjust in different time zones. For example, when trading USD/JPY for Tokyo cut in Asia time, the expiry date may be adjusted to avoid JPY holidays, but once London comes in and starts trading USD/JPY for NY cut, the expiry date will change. Therefore, expiry dates for market tenors can change not only from day to day but within the trading day.

Finally, a quick word for anyone wondering why this section looks similar to the Wikipedia entry on this subject: Documenting this process was one of my first jobs on the trading desk many years ago. Some kind soul obviously took the document and put it up on Wikipedia.

Option Premium Conversions

FX option premiums can be quoted in four ways: CCY1%, CCY2 pips (meaning a number of CCY2 for one CCY1, as spot is quoted), CCY2%, and CCY1 pips (the number of CCY1 for one CCY2).

- % prices have the same notional and premium currency. Example: Notional USD10m, premium 0.40 USD% implies a cash premium of USD40k.
- Pips prices have different notional and premium currencies. Example: Notional USD10m, premium 52 JPY pips implies a cash premium of JPY520m.

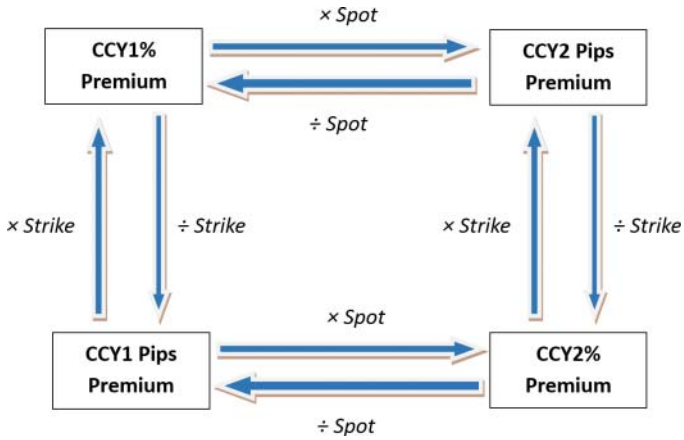


EXHIBIT 10.2 Formulas for converting options premiums

Prices on vanilla FX derivatives are usually quoted in CCY1% or CCY2 pips terms, depending on the market convention in the currency pair. For example, in EUR/JPY the notional will usually be quoted in EUR and the premium will be quoted in EUR terms (i.e., CCY1%) while in EUR/USD the notional will usually be quoted in EUR but the premium will be quoted in USD terms (i.e., CCY2 pips). Pairs where the premium is paid in CCY1 are called left-hand side (LHS) pairs, while pairs where the premium is paid in CCY2 are called right-hand side (RHS) pairs.

When quoting premiums in %, the term **basis point** is often used to mean one-hundredth of a percent (i.e., 0.01%). For example, if the price of a contract is 0.25 EUR%, it might be verbally described as "twenty-five beeps."

Exhibit 10.2 shows how to convert options premiums quoted in different terms. It is important to note that these conversions are only possible if the option contract has a strike.

Generating Tenor Dates in Excel

To build a volatility surface or quote prices based on market tenors, the expiry dates corresponding to each tenor must be calculated. In Excel, dates are internally stored as integers with 0 = Jan 1, 1900, 1 = Jan 2, 1900, and so on. Current dates are therefore over 40,000 (e.g., June 11, 2014 is 41,801). Within VBA code, dates can be represented using variables with type Long.

First, VBA functions are required to:

- Increment a date to the next business day.
- Decrement a date to the previous business day.

Note that these functions don't take holidays into account. The built-in VBA function Weekday is used to check the input day of the week:

```
Function nextBusinessDay(InputDate As Long) As Long

    If Weekday(InputDate) = 7 Then
        'Input Date = Saturday
        nextBusinessDay = InputDate + 2
    ElseIf Weekday(InputDate) = 6 Then
        'Input Date = Friday
        nextBusinessDay = InputDate + 3
    Else
        nextBusinessDay = InputDate + 1
    End If

End Function
```

```
Function previousBusinessDay(InputDate As Long) As Long

    If Weekday(InputDate) = 1 Then
        'Input Date = Sunday
        previousBusinessDay = InputDate - 2
    ElseIf Weekday(InputDate) = 2 Then
        'Input Date = Monday
        previousBusinessDay = InputDate - 3
    Else
        previousBusinessDay = InputDate - 1
    End If

End Function
```

Functions are also required to:

- Calculate the spot date from a horizon date.
- Calculate the horizon date from a spot date.

This can be achieved using VBA functions that increment and decrement a given number of business days. In this code it is assumed that the spot date is always T+2 (i.e., two business days after the horizon):

```
Function businessDayIncrement(InputDate As Long, _
 Increment As Long) As Long

    Dim Count As Long

    businessDayIncrement = InputDate
    For Count = 1 To Increment
        businessDayIncrement = nextBusinessDay(businessDayIncrement)
    Next Count

End Function

Function businessDayDecrement(InputDate As Long, _
 Decrement As Long) As Long

    Dim Count As Long

    businessDayDecrement = InputDate
    For Count = 1 To Decrement
        businessDayDecrement = previousBusinessDay(businessDayDecrement)
    Next Count

End Function

Function getSpotDateFromHorizon(InputDate As Long) As Long

    getSpotDateFromHorizon = businessDayIncrement(InputDate, 2)
```

```
End Function

Function getHorizonFromSpotDate(InputDate As Long) As Long

    getHorizonFromSpotDate = businessDayDecrement(InputDate, 2)

End Function
```

Market tenors can be specified in terms of a number of weeks (e.g., "2W"), months (e.g., "6M") or years (e.g., "5Y"), or the overnight tenor (e.g., "ON"). Therefore, the getExpiryFromTenor function must contain different logic for these different cases using the rules outlined in Chapter 10. The built-in VBA function DateAdd is used to go from spot date to delivery date, and special cases around trading "end-end," and so forth, are all ignored in this code:

```
Function getExpiryFromTenor(Horizon As Long, Tenor As String) As Long

    Dim Count As Long
    Dim SpotDate As Long, DeliveryDate As Long

    If UCase(Tenor) = "ON" Then
        getExpiryFromTenor = nextBusinessDay(Horizon)
    ElseIf Right(UCase(Tenor), 1) = "W" Then
        Count = Left(Tenor, Len(Tenor) - 1)
        getExpiryFromTenor = Horizon + Count * 7
    ElseIf Right(UCase(Tenor), 1) = "M" Then
        Count = Left(Tenor, Len(Tenor) - 1)
        SpotDate = getSpotDateFromHorizon(Horizon)
        DeliveryDate = DateAdd("M", Count, SpotDate)
        getExpiryFromTenor = getHorizonFromSpotDate(DeliveryDate)
    ElseIf Right(UCase(Tenor), 1) = "Y" Then
        Count = Left(Tenor, Len(Tenor) - 1)
        SpotDate = getSpotDateFromHorizon(Horizon)
        DeliveryDate = DateAdd("yyyy", Count, SpotDate)
        getExpiryFromTenor = getHorizonFromSpotDate(DeliveryDate)
    Else
        MsgBox "Invalid Tenor"
        getExpiryFromTenor = -1
    End If

End Function
```

The expiry dates for market tenors can now be set up in an Excel sheet. It is neater to use a subroutine that places expiry dates onto the sheet rather than using functions in the cells.

The horizon must be input and column headers for the tenors and expiry dates must be named TenorRef and ExpiryDateRef respectively. The horizon can be a user input or the Excel function =Today() can be used. It is nice to format date cells so they also show the day of the week. This is achieved by formatting cells with a custom format e.g.: “ddd dd-mmm-yy”:

Horizon	Wed 11-Jun-14	←*Named:* ***Horizon***

Populate Expiry Dates

↓*Named:* ***TenorRef***	↓*Named:* ***ExpiryDateRef***
Tenor	**Expiry Date**
ON	
1W	
2W	
1M	
2M	
3M	
6M	
1Y	
2Y	

The following subroutine can be used to populate expiry dates on the sheet:

```
Sub populateExpiryDates()

    Dim Count As Long

    Count = 1
    While Range("TenorRef").Offset(Count, 0) <> ""
        Range("ExpiryDateRef").Offset(Count, 0) = _
         getExpiryFromTenor(Range("Horizon"), _
         Range("TenorRef").Offset(Count, 0))
        Count = Count + 1
    Wend

End Sub
```

Horizon	Wed 11-Jun-14

Populate Expiry Dates

Tenor	Expiry Date
ON	Thu 12-Jun-14
1W	Wed 18-Jun-14
2W	Wed 25-Jun-14
1M	Thu 10-Jul-14
2M	Mon 11-Aug-14
3M	Thu 11-Sep-14
6M	Thu 11-Dec-14
1Y	Thu 11-Jun-15
2Y	Thu 09-Jun-16

PART II

The Volatility Surface

FX derivatives trading desks maintain volatility surfaces in all tradable currency pairs in order to determine the implied volatility for vanilla options with any expiry date and strike. It is therefore important that traders understand details about how volatility surfaces are constructed since it is a vital part of all FX derivatives valuation.

Fundamentally, a volatility surface is constructed along two axes: maturity and strike. The **ATM curve** forms the backbone of the volatility surface along different expiry dates and the **volatility smile** defines the implied volatility for strikes away from the ATM strike. Volatility surface construction is usually split into these two separate considerations.

ATM Curve Construction

ATM curves can be constructed in two steps. First, a core ATM curve is established. Then additional parameters are introduced so the correct ATM implied volatility is generated for all possible expiry dates. Note that within this chapter, some calculations are *approximate*.

Variance

The key measure for building ATM curves is

$$\boldsymbol{variance} = \sigma^2 T$$

where σ is the ATM implied volatility to time T (measured in years). For example, the variance of a 3mth ATM option with 12.0% implied volatility is $0.12^2 \times 0.25 = 0.0036$.

Variance can be thought of as a measure of *cumulative spot movement*. It has two powerful properties:

1. Variance over any time period must be nonnegative.
2. Variance is additive (i.e., variance over two days = variance on first day + variance on second day).

Variance can be used to calculate the forward ATM implied volatility (usually called **forward implied volatility** or just "forward vol" by traders) between two dates in the future. Given ATM implied volatility σ_1 to time T_1, ATM implied volatility σ_2 to time T_2, and $T_1 < T_2$:

- Variance from horizon to $T_1 = {\sigma_1}^2 T_1$
- Variance from horizon to $T_2 = {\sigma_2}^2 T_2$

Therefore, variance between T_1 and $T_2 = {\sigma_2}^2 T_2 - {\sigma_1}^2 T_1$ and forward implied volatility between T_1 and $T_2 = \sqrt{\dfrac{{\sigma_2}^2 T_2 - {\sigma_1}^2 T_1}{T_2 - T_1}}$. For example, if the 6mth ATM implied volatility is 10.5% and the 1yr ATM implied volatility is 11.7%, then the forward implied volatility from 6mth to 1yr is $\sqrt{\dfrac{11.7\%^2 \times 1.0 - 10.5\%^2 \times 0.5}{1.0 - 0.5}} =$ 12.8%.

Core ATM Curve Construction

There are two main approaches that can be used to generate core ATM curves:

1. *Input* the ATM curve at the market tenors and *interpolate* to get ATM volatility for expiry dates between the market tenors.
2. Use a *model* to generate the ATM curve and *output* the ATM volatility at the market tenors.

Recall from Part I that the standard market tenors up to two years are: O/N (overnight), 1wk, 2wk, 1mth, 2mth, 3mth, 6mth, 1yr, and 2yr.

Constructing a Core ATM Curve Using Interpolation

Core ATM curves can be constructed using interpolation between market tenors. Exhibit 11.1 shows ATM curve A—an upward-sloping ATM curve defined at market tenors.

For expiry dates between market tenors, first consider **linear volatility** interpolation as shown in Exhibit 11.2. The linear interpolation methodology can be clearly seen between market tenors.

To investigate this interpolation further, variance at each expiry date is calculated in Exhibit 11.3. The variance profile looks reasonable, rising over time as expected.

Exhibit 11.4 shows a new ATM curve B defined at market tenors. ATM implied volatility is 20% at market tenors up to 1yr, and then the next data point is 15% implied volatility at the 2yr tenor.

Interpolating ATM curve B using linear volatility and then calculating the variance at each expiry date gives the profile shown in Exhibit 11.5.

Horizon	Wed 11-Jun-14

Tenor	Expiry Date	ATM Implied Volatility
ON	Thu 12-Jun-14	10.00%
1W	Wed 18-Jun-14	11.00%
2W	Wed 25-Jun-14	12.00%
1M	Thu 10-Jul-14	13.00%
2M	Mon 11-Aug-14	14.00%
3M	Thu 11-Sep-14	15.00%
6M	Thu 11-Dec-14	16.00%
1Y	Thu 11-Jun-15	17.00%
2Y	Thu 09-Jun-16	18.00%

EXHIBIT 11.1 ATM curve A defined at market tenors

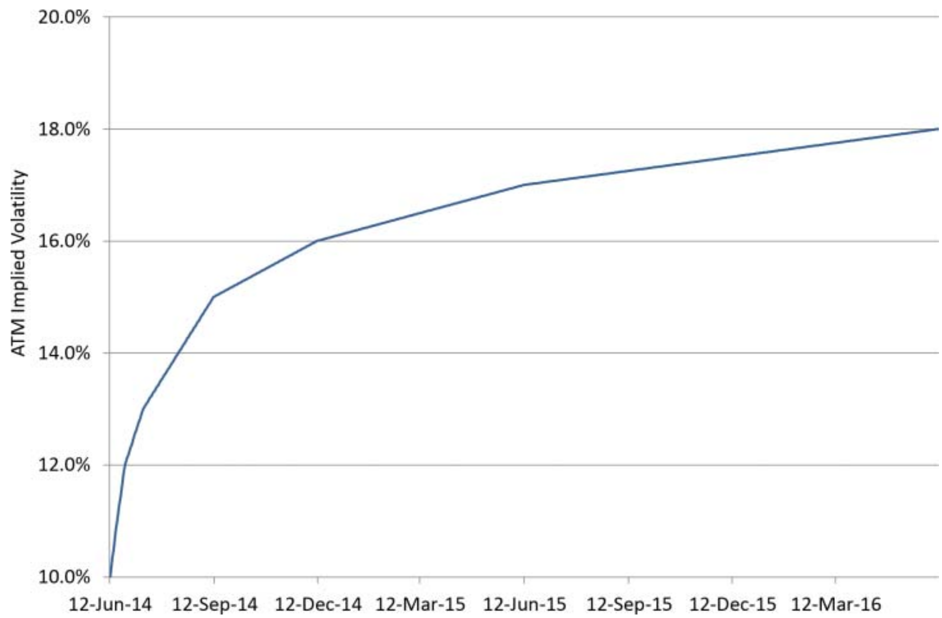


EXHIBIT 11.2 ATM curve A generated using linear volatility interpolation

Calculating *daily variance* (i.e., the change in variance for each expiry date) gives bad news in Exhibit 11.6.

Up to 1yr, implied volatility is constant so variance rises linearly with maturity, but from 1yr to 2yr variance rises and then falls. This sets alarm bells ringing: *Variance must be nonnegative*. Therefore, linear volatility interpolation has failed to build a valid ATM curve from valid inputs (variance to 2yr is larger than variance to 1yr):

- Variance between horizon and 1yr $= 20\%^2 \times 1.0 = 0.04$
- Variance between horizon and 18mth $= 17.5\%^2$ (interpolated) $\times 1.5 = 0.046$
- Variance between horizon and 2yr $= 15\%^2 \times 2.0 = 0.045$

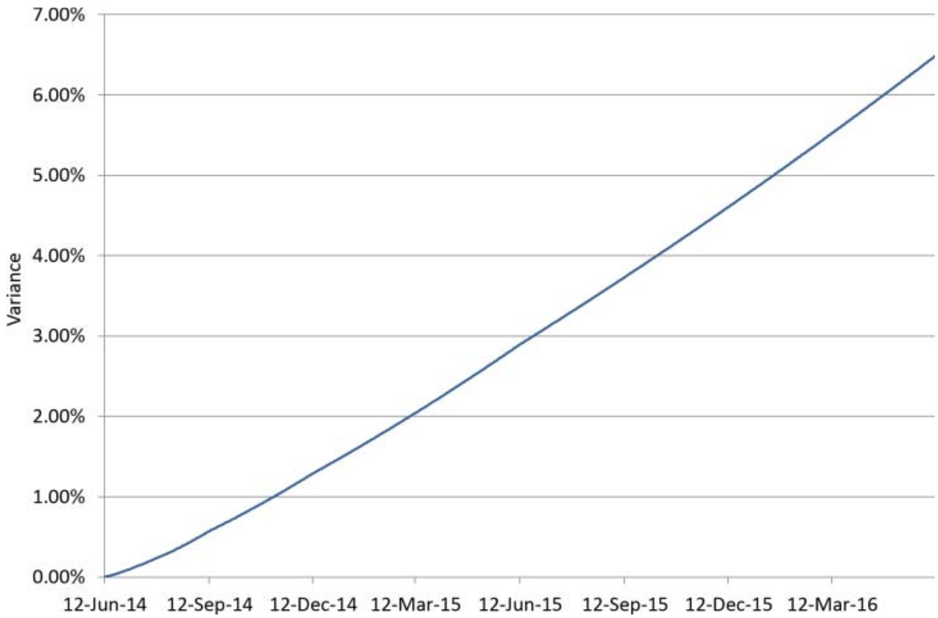


EXHIBIT 11.3 Variance profile for ATM curve A generated using linear volatility interpolation

Horizon	Wed 11-Jun-14

Tenor	Expiry Date	ATM Implied Volatility
ON	Thu 12-Jun-14	20.00%
1W	Wed 18-Jun-14	20.00%
2W	Wed 25-Jun-14	20.00%
1M	Thu 10-Jul-14	20.00%
2M	Mon 11-Aug-14	20.00%
3M	Thu 11-Sep-14	20.00%
6M	Thu 11-Dec-14	20.00%
1Y	Thu 11-Jun-15	20.00%
2Y	Thu 09-Jun-16	15.00%

EXHIBIT 11.4 ATM curve B defined at market tenors

This suggests a new interpolation methodology: **linear variance**. The variance profile resulting from a linear variance interpolation of ATM curve B is shown in Exhibit 11.7.

This variance profile has no negative daily variance and the ATM implied volatility curve shown in Exhibit 11.8 looks good, too.

Going back to ATM curve A, the profile shown in Exhibit 11.9 is generated using a linear variance methodology.

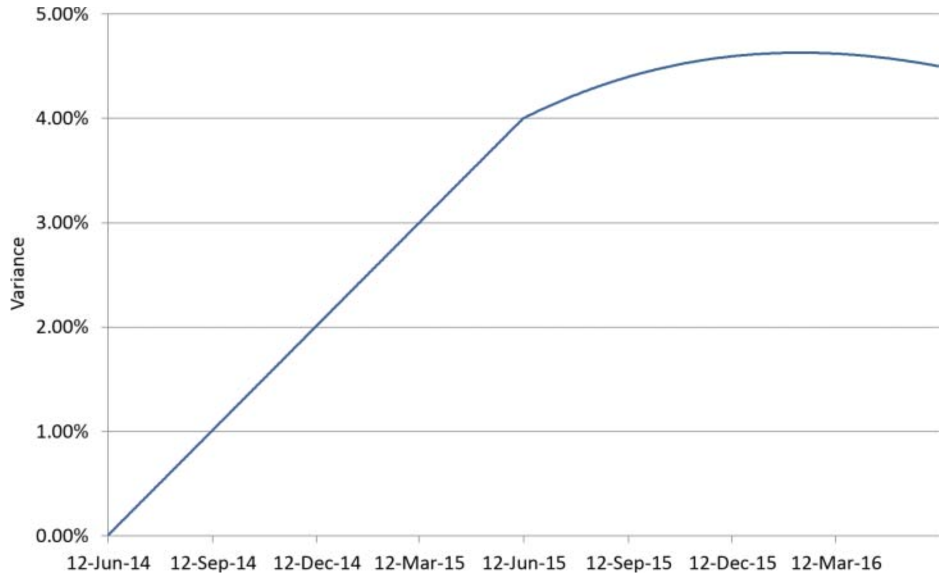


EXHIBIT 11.5 Variance profile for ATM curve B generated using linear volatility interpolation

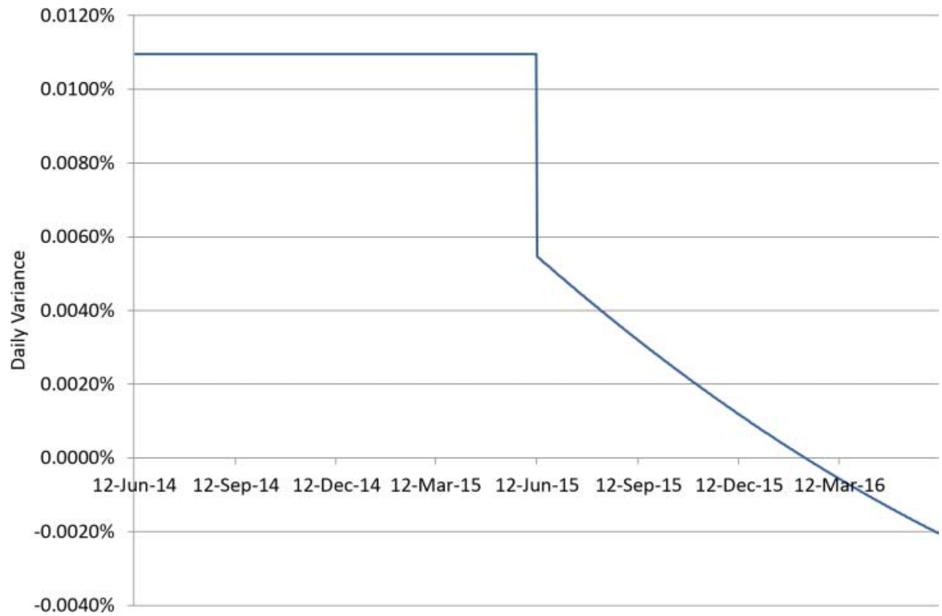


EXHIBIT 11.6 Daily variance profile for ATM curve B generated using linear volatility interpolation

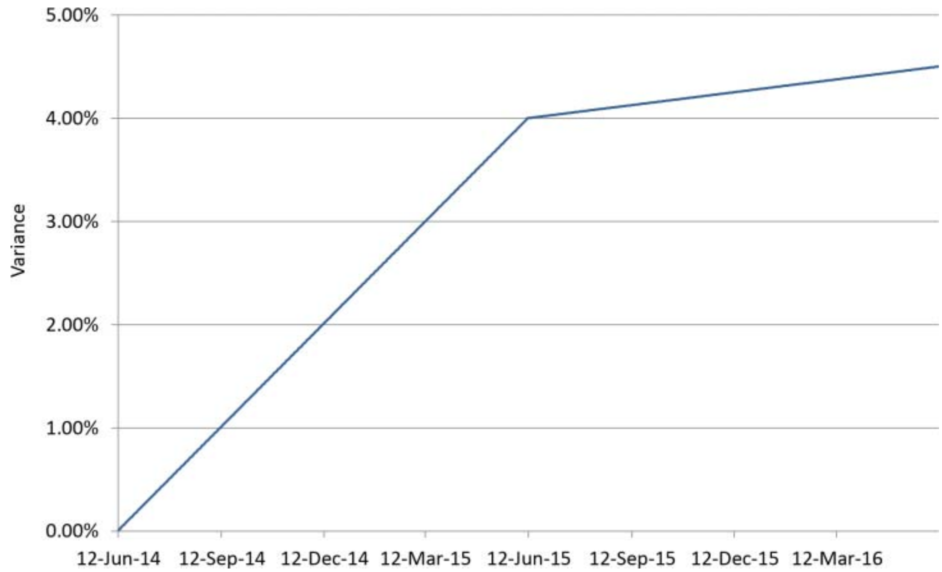


EXHIBIT 11.7 Variance profile for ATM curve B using linear variance interpolation

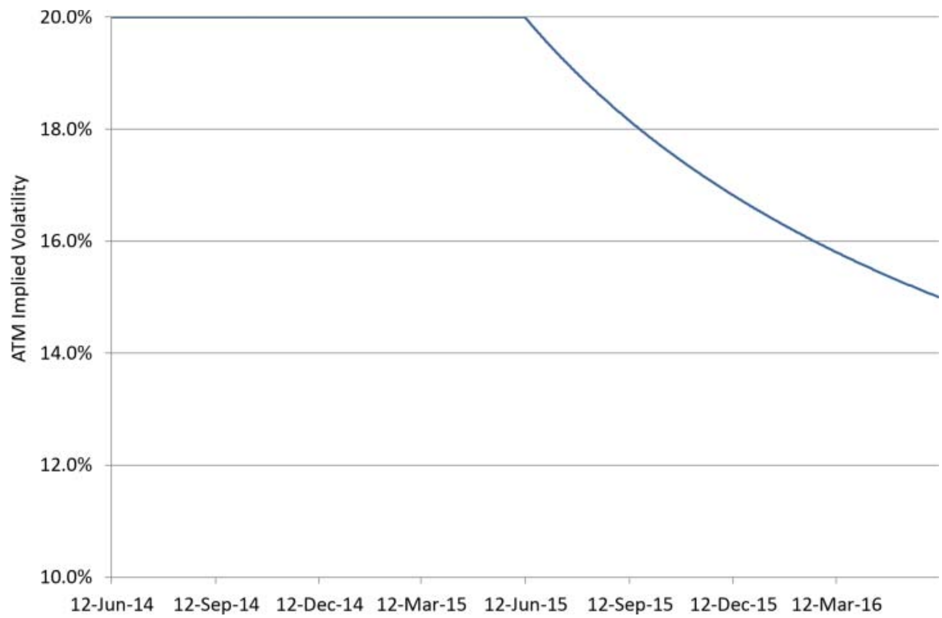


EXHIBIT 11.8 ATM curve B generated using linear variance interpolation

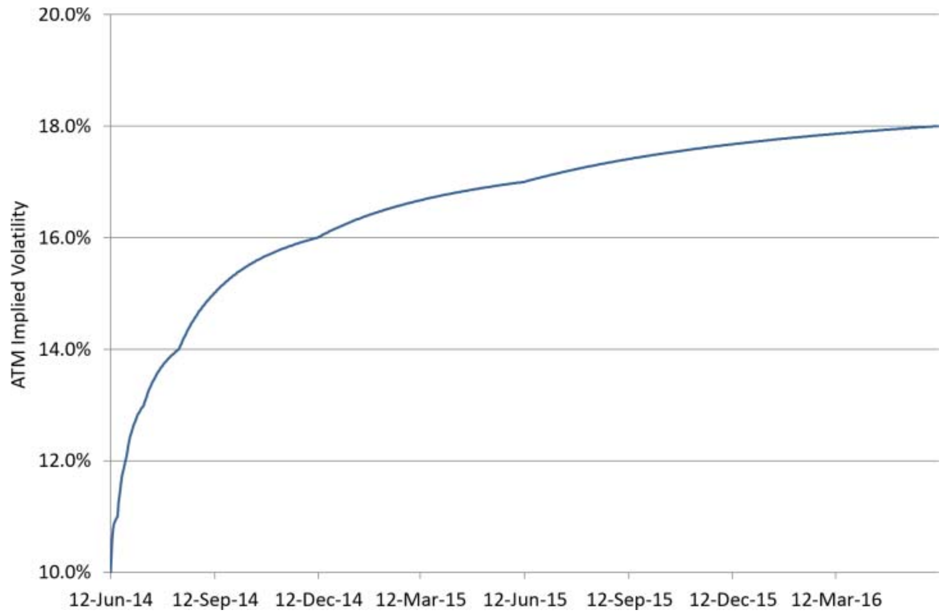


EXHIBIT 11.9 ATM curve A generated using linear variance interpolation

The ATM implied volatility between market tenors in Exhibit 11.9 looks odd. The linear variance methodology generates an ATM implied volatility profile that rises sharply initially and then flattens off between tenors for this upward-sloping ATM curve. Why is this happening? Consider the daily variance profile shown in Exhibit 11.10.

These daily variance patterns are not realistic. Intuitively it does not make sense that daily variance should jump immediately past each market tenor date. Excluding any special factors, why would daily variance one day prior to the 3mth tenor date be significantly different to daily variance one day after the 3mth tenor date? Ideally, the core daily variance function should be smooth.

Comparing these two interpolation methodologies:

1. Linear volatility interpolation often produces intuitively correct ATM curves but does not ensure positive forward variance.
2. Linear variance interpolation produces ATM curves that ensure positive forward variance (given valid inputs) but does not always create intuitively correct ATM curves.

In practice, trading desks use a combination of these approaches to produce intuitive curves with no negative forward variance. ATM curves are generally constructed in variance terms but more sophisticated schemes are used to control how daily variance evolves over time.

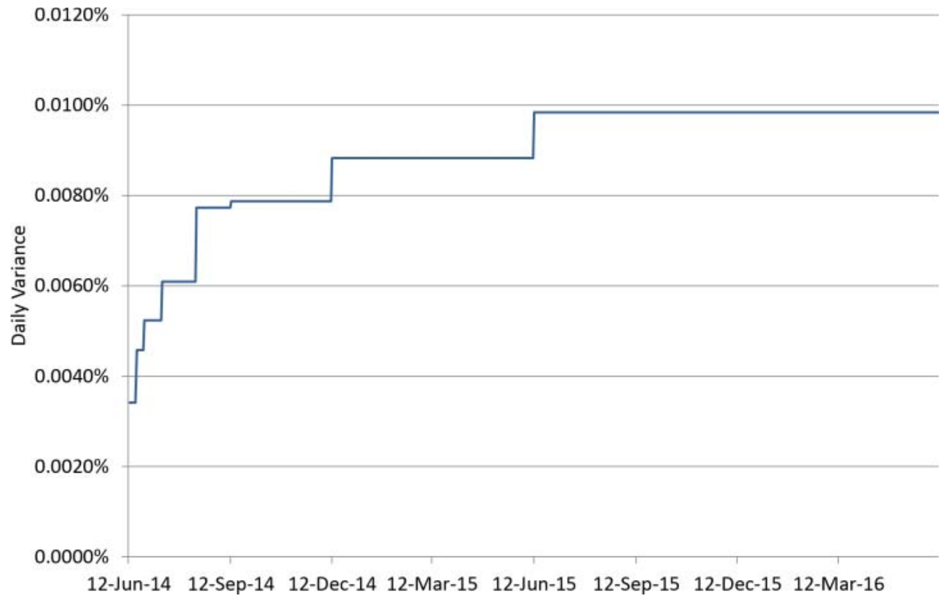


EXHIBIT 11.10 Daily variance profile for ATM curve A generated using linear variance interpolation

Constructing a Core ATM Curve Using a Model

Another possible method of constructing a core ATM curve is to use a model. Many different models are possible but fundamentally the functional form most often involves a *short-term* factor (could be volatility, variance, or daily variance), a *long-term* factor, and a *speed* of moving from short to long.

Here is one possible simple approach (that would never be used in practice because it could generate arbitragable ATM curves):

$$\sigma_t = \sigma_{short} + (\sigma_{long} - \sigma_{short}).(1 - e^{-\lambda T})$$

where σ_{short} and σ_{long} are short-term and long-term ATM volatilities respectively, λ is speed, and T is time to expiry measured in years.

The $(1 - e^{-\lambda T})$ function moves between 0 and 1 as shown in Exhibit 11.11.

Higher λ causes the function to move from 0 to 1 more quickly. This function can be fed with the T's from market tenor expiry dates to calculate ATM implied volatilities as per Exhibit 11.12.

The ATM curve is now an *output* from the model rather than being an input. This approach requires the model parameters to be calibrated to market ATM implied

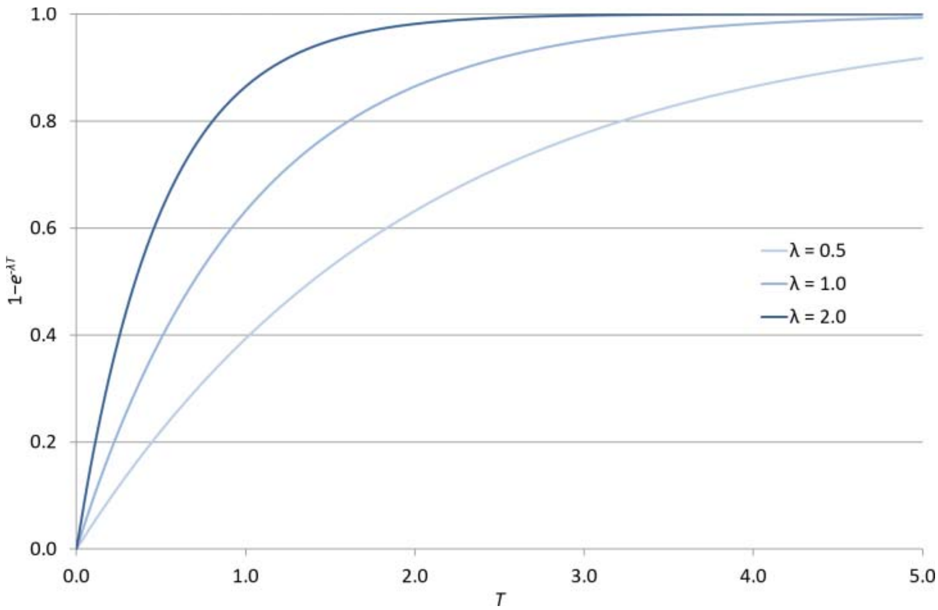


EXHIBIT 11.11 Function used within a simple ATM curve model

Horizon	Wed 11-Jun-14

Tenor	Expiry Date	T	ATM Implied Volatility
ON	Thu 12-Jun-14	0.00274	10.01%
1W	Wed 18-Jun-14	0.01918	10.09%
2W	Wed 25-Jun-14	0.03836	10.19%
1M	Thu 10-Jul-14	0.07945	10.38%
2M	Mon 11-Aug-14	0.16712	10.77%
3M	Thu 11-Sep-14	0.25205	11.11%
6M	Thu 11-Dec-14	0.50137	11.97%
1Y	Thu 11-Jun-15	1.00000	13.16%
2Y	Thu 09-Jun-16	1.99726	14.32%

EXHIBIT 11.12 ATM curve output at market tenors

volatilities. This can be time consuming initially but traders soon learn how the model parameters change as the market ATM curve moves.

Within this approach, *overrides* at market tenors are also required to ensure the system ATM implied volatility hits market mid values. For example, the ATM curve model is set up and all tenors closely match the market except for 2mth ATM, which is 0.1% lower in the market than the model suggests. The trader therefore inputs a –0.1% override at the 2mth tenor. This is useful information because it suggests that the 2mth ATM is relatively cheaper than other tenors.

ATM Curve Construction: Short-Dates

Once the core ATM curve has been constructed, additional parameters or weights are introduced in order to give traders sufficient control over the curve. This control is required because different expiry dates (or even different times within expiry dates) have different expected spot volatility and this information must be incorporated into the ATM curve.

This additional control is mainly important at shorter expiry dates. The following examples demonstrate how the same variance framework can be applied at shorter time scales, as per the diagram in Exhibit 11.13.

Example 1: 1wk (7-day) ATM implied volatility is 12.0%. A 1wk option always contains five weekdays and two weekend days where spot does not move because the market is closed (i.e., zero variance). Assume spot is equally volatile on each weekday (i.e., equal daily variance). What is the 8-day ATM implied volatility?

$$\sigma_{8-day\ ATM} = \sqrt{\frac{variance_{1wk} \cdot \frac{6}{5}}{\left(\frac{8}{365}\right)}} = 12.3\%$$

Example 2: 1wk (7-day) ATM implied volatility is 12.0%. Assume it is known that spot will be completely static during the 8th day (i.e., zero variance). What is the 8-day ATM implied volatility?

$$\sigma_{8-day\ ATM} = \sqrt{\frac{variance_{1wk}}{\left(\frac{8}{365}\right)}} = 11.25\%$$

Due to the properties of variance, this effectively forms a lower bound on the 8-day ATM implied volatility: If the 7-day ATM volatility is 12.0%, the 8-day ATM volatility must be *at least* 11.25%.

Variance and **option premium** are closely linked. If forward drift and discounting are removed from the framework, the vanilla option premium for a specific strike must rise at longer maturities since variance must rise at longer maturities. Otherwise, the ATM curve is arbitrageable.

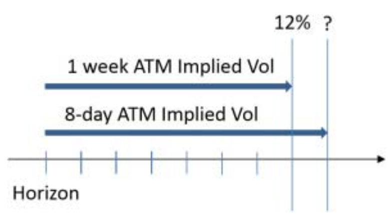


EXHIBIT 11.13 Short-date variance examples framework

In practice, however, with forward drift reintroduced, the situation becomes more complicated. Consider vanilla options on two consecutive expiry dates with the same strike. If the following trading position can be achieved for zero premium, how can a guaranteed profit be generated from these trades?

- Short 7-day call option with strike K
- Long 8-day call option with strike K

If both options are left unhedged until expiry, profitability depends on how spot moves between the two expiry dates. An overall profit will be generated if spot is more in-the-money (ITM) at the second expiry than the first. However, an overall loss will be generated if spot is more ITM at the first expiry than the second. There is no guaranteed profit locked in.

A better strategy would be to sell the second option as the first option expires. This would offer a near-certain nonnegative P&L, but Exhibit 11.14 shows how an extreme forward drift prevents a guaranteed profit. The vanilla call option value at expiry (shown in leg 1) has a higher value than the second vanilla call option (now overnight expiry) due to the large negative forward drift.

Contract Details	**Leg 1**	**Leg 2**
Currency Pair	EUR/USD	EUR/USD
Horizon	Mon 13-Oct-2014	Mon 13-Oct-2014
Spot Date	Wed 15-Oct-2014	Wed 15-Oct-2014
Strategy	Vanilla	Vanilla
Call/Put	EUR Call/USD Put	EUR Call/USD Put
Maturity	O/N	O/N
Expiry Date	Mon 13-Oct-2014	Tue 14-Oct-2014
Delivery Date	Wed 15-Oct-2014	Thu 16-Oct-2014
Cut	NY	NY
Strike	1.2000	1.2000
Notional Currency	EUR	EUR
Market Data		
Spot	1.2700	1.2700
Swap Points	0	-35
Forward	1.2700	1.2665
Deposit (EUR)	0.00%	100.00%
Deposit (USD)	0.00%	0.00%
ATM Volatility		10.00%
Pricing Volatility		10.00%
Outputs		
Output Currency	**EUR**	**EUR**
Mid Price	**5.5125%**	**5.235%**

EXHIBIT 11.14 Impact of forward drift on option prices

In practice it is hard to *guarantee* a profit on this trade, particularly once bid–offer spreads are taken into account. However, taking a step back, traders would love to transact this 7-day versus 8-day spread for zero premium. Traders often get carried away describing trades or prices as arbitrages when they really mean "fantastic trading opportunities." As a result, the "arbitrage trading books" on bank derivatives trading desks can occasionally end the year with negative P&L.

Implied Volatility Patterns over the Week

The number of market-open and market-closed days to a specific expiry date has an important impact on the ATM implied volatility. Consider (one-day) overnight ATM volatility compared to the 1wk ATM volatility. Assume each market-open day is equally volatile (i.e., equal daily variance) and no variance over the weekend:

$$variance_{1wk} = variance_{O/N} \times 5$$

$$ATM_{1wk}{}^2 \cdot \left(\frac{7}{365}\right) = ATM_{O/N}{}^2 \cdot \left(\frac{1}{365}\right) \times 5$$

$$ATM_{1wk} = ATM_{O/N} \times \sqrt{\frac{5}{7}}$$

$$ATM_{1\mathrm{wk}} < ATM_{O/N}$$

In words, the (one-day) overnight ATM implied volatility is higher than the 1wk ATM implied volatility because the 1wk expiry date contains two weekend days. This is a commonly observed feature of the FX derivatives market.

The market-open-to-total-days ratio also explains why ATM implied volatility tends to rise for future expiry dates over the working week. For a fixed horizon, a future Monday expiry will almost always have a lower implied volatility than the Friday following it because the Friday has a higher market-open-to-total-days ratio. This ATM saw-toothing, shown in Exhibit 11.15, is commonly observed within the FX derivatives market, although the effect dampens at longer maturities as the market-open-to-total-days ratio stabilizes.

FX Derivatives Market Pricing

Within the FX derivatives market, when using the Black-Scholes formula for pricing, time to expiry (T) is specified in *discrete daily steps*. This is a key feature of the FX derivatives market.

Consider a situation where the current time is 9 A.M. London time on Monday and overnight NY cut ATM implied volatility is 15.0%. The overnight expires tomorrow, so $T = \frac{1}{365}$ and:

$$variance_{O/N} = 0.15^2 \times \left(\frac{1}{365}\right) = 6.1644 \times 10^{-5}$$

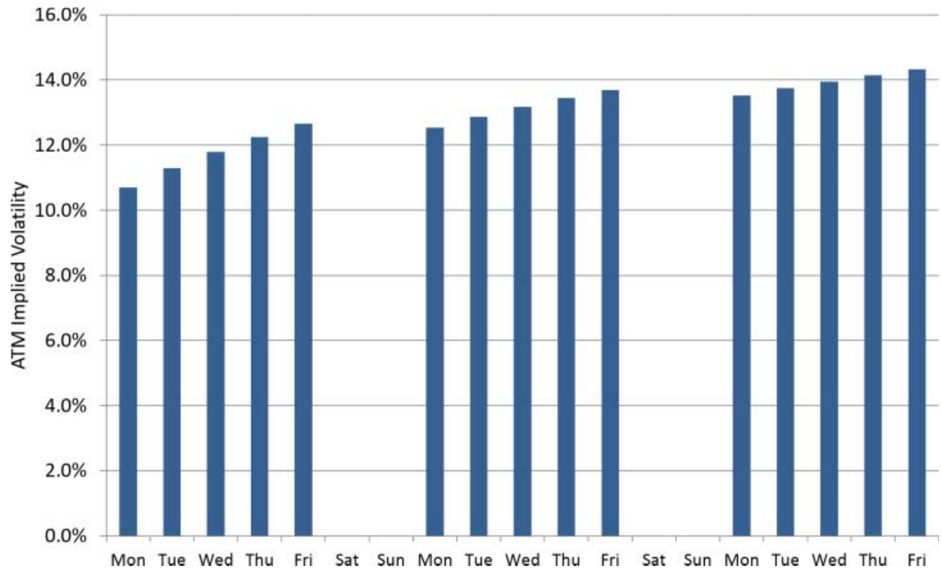


EXHIBIT 11.15 Monday to Friday ATM saw-toothing

This variance can now be split into even smaller time intervals. NY cut is at 10 A.M. New York time, which is (usually) 3 P.M. London time, so this overnight option actually expires in 30 hours. Assuming spot is equally volatile between now and NY cut tomorrow:

$$variance_{hourly} = \frac{6.1644 \times 10^{-5}}{30} = 2.0548 \times 10^{-6}$$

After one hour passes, the remaining variance on the option is:

$$29 \times variance_{hourly} = 5.9589 \times 10^{-6}$$

which implies a new overnight ATM volatility of:

$$\sigma_{O/N\ ATM} = \sqrt{\frac{5.9589 \times 10^{-6}}{\left(\frac{1}{365}\right)}} = 14.75\%$$

Note that T is unchanged within this calculation due to being specified in discrete daily steps. At the start of each trading day, when the overnight option expiry moves forward one trading day, the ATM implied volatility jumps higher due to increased variance. Then over the course of the trading day, the ATM implied volatility gradually moves lower due to reducing variance. Variance (and hence premium) to a fixed expiry date reduces as time passes but because the market uses constant daily T values within the Black-Scholes formula for pricing, implied volatility reduces instead. This effect occurs at all tenors but the impact is only visible in short-dated options, particularly the overnight.

It is interesting to consider that if the market used a more accurate T for pricing, short-dated implied volatility would be more stable throughout the day. However, since spot volatility is *not constant* throughout the day, implied volatility would not be completely static. Therefore, introducing more accuracy into T adds to the complexity of the market for only minimal benefit.

In practice, short-dated implied volatility does tend to drift lower over the course of the day, but spot behavior is also important. If there is a large spot move or spot breaks out of its recent range, implied volatility generally moves higher due to an expectation of increased future spot volatility. Traders with short gamma positions come into the market to hedge their positions and the market implied volatility increases. Alternatively, if spot is static, implied volatility often falls more quickly than its "natural rate" as traders come into the market to reduce long gamma positions where they are struggling to trade their deltas.

If traders believe short-dated implied volatility is falling slower than it should over the course of the day, gamma can be "*rented*." This involves, for example, buying the overnight ATM at the start of the trading day and then selling the same contract back at the end of the trading day. This technique is only applicable in liquid currency pairs where there is good two-way flow in short-dated vanilla options; otherwise the spread cross involved in the two transactions will kill any value in the trade.

Risk management of FX derivatives positions is also usually performed assuming discrete daily time steps. This is the reason that options expiring on the horizon date generate *delta jumps* through their strike level (as seen in Chapters 7 and 9). It is also the reason that trading positions show all options expiring on the horizon date at their own expiry times. If the trading position has options expiring at different cuts on the expiry date, this is inconsistent, but it keeps the trading risk stable. Traders adjust for this effect within their risk management.

Overnight (O/N) ATM on a Friday

On any weekday apart from Friday, the overnight option expires the following day. However, on Friday, the "overnight" option expires on a Monday; three days later rather than one. Therefore, in a market Black-Scholes pricing world, $T = \frac{3}{365}$.

Vega (υ) is a function of $\sqrt{T}$:

$$\upsilon_{call} = \upsilon_{put} = \frac{\partial P}{\partial \sigma} = Se^{-rCCY1.T} n(d_1)\sqrt{T}$$

Therefore (ignoring discounting):

$$\upsilon_{1-day\ ATM} \cdot \sqrt{3} = \upsilon_{3-day\ ATM}$$

There is no volga $\left(\frac{\partial \upsilon}{\partial \sigma}\right)$ on ATM options, so for a given tenor, roughly:

$$Price_{ATM\ call} = Price_{ATM\ put} = \upsilon \,.\, \sigma_{ATM}$$

Assuming there is no variance over the weekend and each weekday is equally volatile, the O/N ATM contract will have the same *premium* each day and therefore:

$$\sigma_{3-day\ ATM} = \frac{\sigma_{1-day\ ATM}}{\sqrt{3}}$$

In practice this means that the market overnight ATM implied volatility quoted on a Friday cannot be directly compared with the overnight ATM implied volatility quoted on other days. To get the Friday overnight ATM into the same terms it must be multiplied by $\sqrt{3}$. Furthermore, the bid–offer spread shown on a three-day overnight should be tighter in volatility terms in order to show the same premium spread. Again, the level of ATM implied volatility is being impacted by the market-open-to-total-days ratio, or put another way, the ratio of economic time (time adjusted to consider market activity only) to calendar time (see Practical E).

In practice, the market pricing of the overnight ATM contract on a Friday is closely related to the market's *weekend decay* position. The jump from Friday end-of-day to Monday morning covers three days. If this is not correctly adjusted for within risk management systems, theta from Friday to Monday will be artificially large. In a simplified world with no adjustment for this effect, a position that is long the same amount of gamma each day will, on average:

- Make money on Tuesday through to Friday as only (5 / 7) = 71% of the correct theta is paid per day.
- Lose all additional profit the following Monday as (5 / 7) × 3 = 213% of the correct theta is paid from Friday into Monday.

Amazingly, in a sophisticated financial market in the twenty-first century, this effect still produces trading opportunities as short-dated options can become too cheap on Friday as some banks oversell to reduce their weekend theta.

New York Cut versus Tokyo Cut Pricing

In G10 currency pairs the two most common expiry cuts are New York (NY) and Tokyo (TOK). The New York cut versus Tokyo cut ATM volatility differential can be analyzed using the same variance framework:

- TOK cut: 3 P.M. Tokyo time (often 6 A.M. GMT)
- NY cut: 10 A.M. New York time (often 3 P.M. GMT)

That is, NY cut options contain an extra nine hours of optionality.

Therefore, the Tokyo cut ATM implied volatility is always lower ("trades at a discount") than the New York cut ATM implied volatility because both are priced using the same discrete daily T but the Tokyo cut occurs first in the day and therefore has less variance and a lower premium.

Assuming spot is always equally volatile:

$$\frac{variance_{TOK}}{T_{TOK}} = \frac{variance_{NY}}{T_{NY}}$$

$$\frac{\sigma_{TOK}^2 . T_{market}}{T_{TOK}} = \frac{\sigma_{NY}^2 . T_{market}}{T_{NY}}$$

$$\sigma_{TOK} = \sigma_{NY} . \sqrt{\frac{T_{TOK}}{T_{NY}}}$$

where T_{market} is the time to expiry measured in years used within the market Black-Scholes pricing framework, T_{TOK} is the real time to expiry to the Tokyo cut and T_{NY} is the real time to expiry to the New York cut.

Therefore, the New York cut versus Tokyo cut volatility differential increases over the course of a given trading day.

At 9 A.M. GMT:

- O/N TOK cut = 6 A.M. GMT the next day = 21 hours
- O/N NY cut = 3 P.M. GMT the next day = 30 hours

$$\sigma_{TOK} = \sigma_{NY} . \sqrt{\frac{21}{30}} = \sigma_{NY} \times 0.84$$

At 5 P.M. GMT:

- O/N TOK cut = 6 A.M. GMT the next day = 13 hours
- O/N NY cut = 3 P.M. GMT the next day = 22 hours

$$\sigma_{TOK} = \sigma_{NY} . \sqrt{\frac{13}{22}} = \sigma_{NY} \times 0.77$$

For maturities past three months, New York and Tokyo cuts will generally be priced at the same implied volatility (assuming no events, etc., on the expiry date). For example, at the three-month tenor, approximately:

$$\sigma_{TOK} = \sigma_{NY} . \sqrt{\frac{2151\ hours}{2160\ hours}} = \sigma_{NY} \times 0.998$$

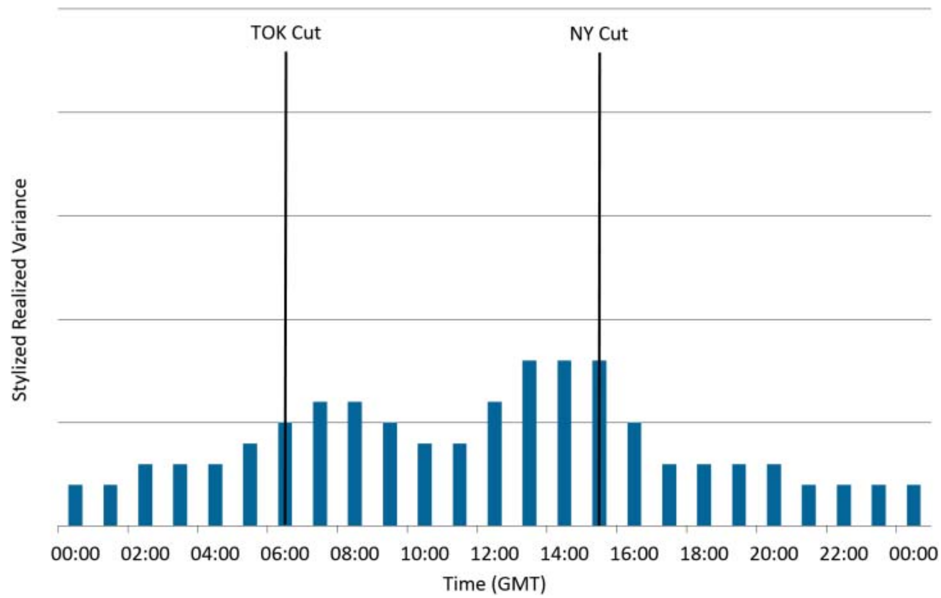


EXHIBIT 11.16 Stylized intraday hourly realized variance

Intraday Variance Patterns

The simplifying assumption that spot is equally volatile throughout the trading day is obviously not correct in practice. In liquid G10 currency pairs, realized variance follows a fairly well-established pattern shown in Exhibit 11.16 in which it:

- Starts low and builds up during Asia trading time
- Peaks around GMT 08:00 as Europe/London come in
- Dips during Europe/London lunch around GMT 11:00
- Picks up again in the afternoon with New York in and reaches day highs around GMT 14:00
- Decreases after GMT 15:00 (NY cut) to the end of the day in the New York afternoon

The intraday variance patterns are different in emerging market currency pairs where trading is concentrated in one region or the spot market opening hours are restricted. Such variance patterns should be taken into account within the option pricing framework for maximum accuracy when pricing options expire at different cut times.

Events and Holidays

Events (economic data releases, election results, etc.) cause spot to move as the market adjusts to new information. Exhibit 11.17 shows the USD/JPY spot reaction to the Non-Farm Payroll data release (an important gauge of U.S. employment usually released on the first Friday on the month) from May 2013, in which spot jumps immediately after the economic data is made public.

Event days are therefore assigned higher variance within the ATM curve, specifically in the period immediately after the data is released. This in turn increases the ATM implied volatility for options expiring on that expiry date (if the cut occurs after the event has been released) and also expiry dates following it. The exact date and time of events is known beforehand and therefore the market ATM curve incorporates this information.

On days containing important data releases, realized spot variance is usually similar or slightly lower than the spot variance on a "normal day" until the data release. Over the data release, realized spot variance increases sharply and then reverts back to the normal day as shown in Exhibit 11.18. On this expiry date, the NY cut contains the additional expected spot variance from the event but the TOK cut does not. This leads to a far larger NY cut versus TOK cut ATM implied volatility differential than usual.

The presence of an event also causes short-dated implied volatility to decay differently over the course of the day. Prior to the event, short-dated ATM implied

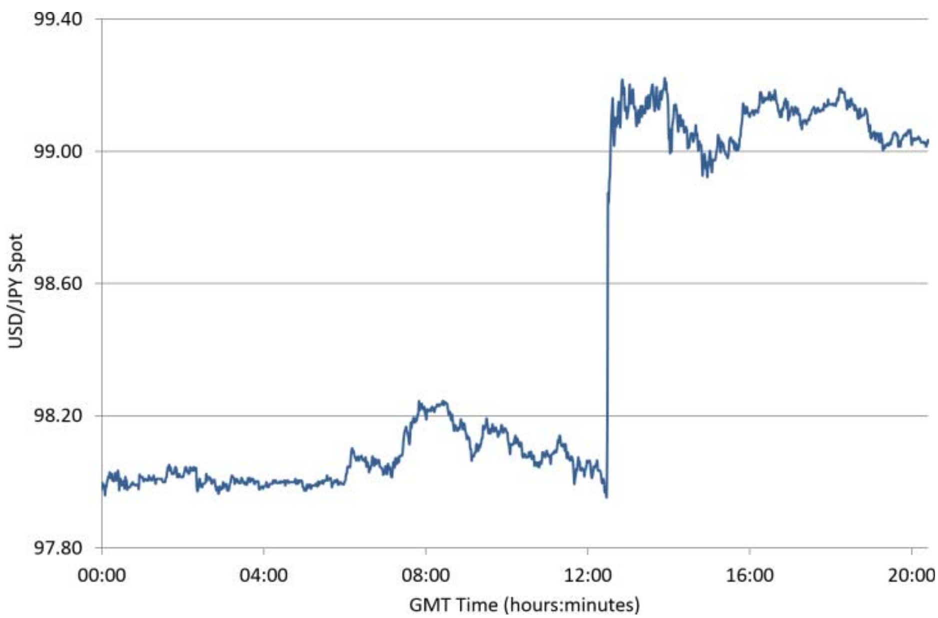


EXHIBIT 11.17 USD/JPY spot over Non-Farm Payroll data release from May 2013

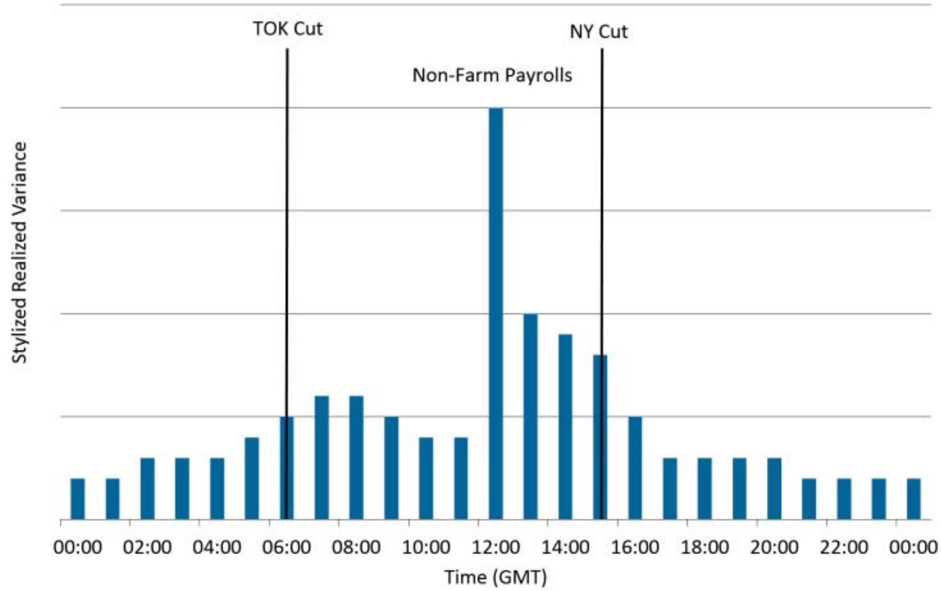


EXHIBIT 11.18 Stylized intraday hourly realized variance on Non-Farm Payroll day

volatility will move lower only slightly but after the event has occurred ATM implied volatility can drop sharply as expected future spot variance reduces.

Events usually occur in a particular currency. For example, European employment data primarily impacts spot in currency pairs that include EUR. However, for the most important events, crosses can also exhibit increased volatility if the majors move in an asynchronous manner. For example; Non-Farm Payrolls impacts USD, but if EUR/USD and AUD/USD are both more volatile but they do not move in a perfectly synchronized manner, EUR/AUD realized volatility also increases.

Public holidays also impact realized volatility and variance. There is often significantly less spot activity in pairs containing the holiday currency simply because there are fewer market participants operating that day. In addition, U.K. and U.S. public holidays are important enough to reduce spot activity across all currency pairs. Therefore, public holiday days in a particular currency have lower variance within the ATM curve.

Weekday Variance Patterns

The FX spot market often exhibits increased realized variance later in the working week, as shown in Exhibit 11.19. This effect occurs partially because there tend to be more data releases later in the week. However, even with the effect of events removed, Mondays are often less volatile than other weekdays.

Like the NY cut versus TOK cut implied volatility differential, the day of the week of a particular expiry date matters more at shorter tenors than at longer

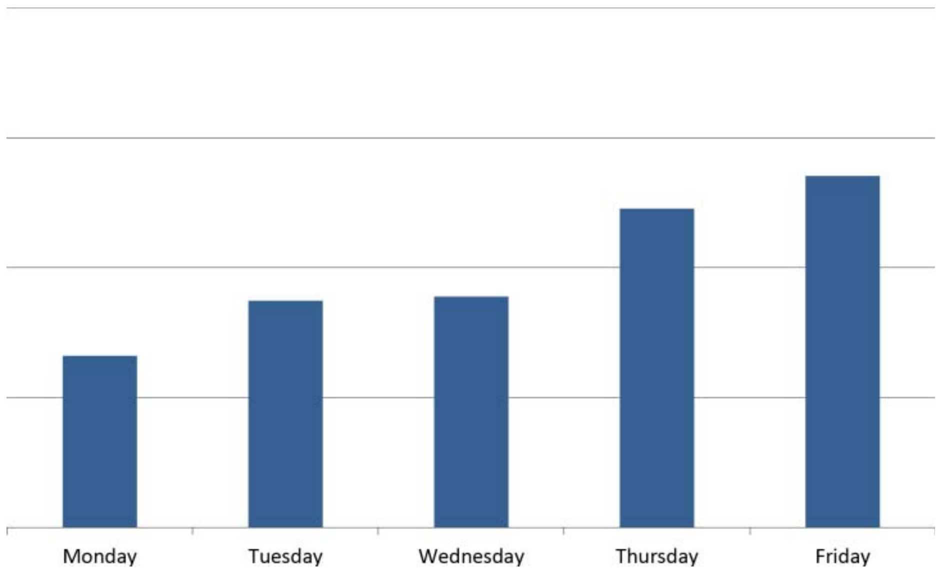


EXHIBIT 11.19 Average daily spot variance for G10 pairs in 2012

tenors. The market often has a preference to buy the next few Friday expiries and sell the next few Monday expiries but the weekday of, for example, the 6mth ATM contract is not a major concern.

Pricing Same-Day Options

Pricing options that expire later today is impossible within the standard market Black-Scholes pricing framework. The number of days is zero; hence T is zero and therefore same-day options cannot be quoted in volatility terms. Recall from Chapter 5 that in the Black-Scholes option pricing formula: $d_1 = \dfrac{\ln\left(\frac{S}{K}\right)+\left(rCCY2-rCCY1+\frac{\sigma^2}{2}\right).T}{\sigma\sqrt{T}}$ would break because the denominator is zero.

So-called **same-day options** must therefore be quoted in *premium* terms. One way to calculate the premium of a same-day option is to start with an overnight option and use the variance framework to adjust the implied volatility.

Example: At 9 A.M. GMT a client requests a price in a same-day NY cut option. This is shown in Exhibit 11.20.

The O/N NY ATM implied volatility is 12%. Therefore:

$$variance_{O/N\ NY\ Cut} = 0.12^2 \times \left(\frac{1}{365}\right) = 3.945 \times 10^{-5}$$

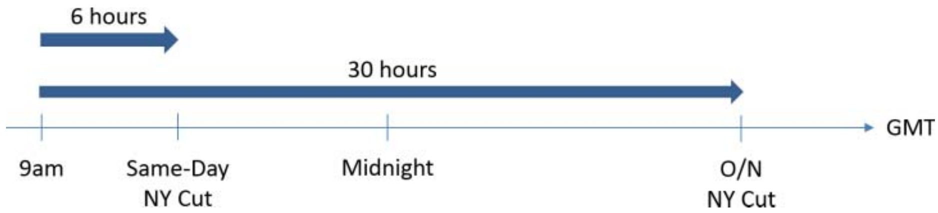


EXHIBIT 11.20 Pricing a same-day vanilla option

Assuming each hour has equal variance:

$$variance_{Same-day\ \ NY\ \ Cut} = \left(\frac{6}{30}\right) \times 3.945 \times 10^{-5} = 7.89 \times 10^{-6}$$

Therefore, the equivalent one day ATM volatility is:

$$\sigma_{1\ \ day\ \ ATM} = \sqrt{\frac{7.89 \times 10^{-6}}{\left(\frac{1}{365}\right)}} = 5.3\%$$

This implied volatility can then be used to price an overnight option which gives the same-day option premium. Note that interest rates should be set to zero within the same-day pricing since forward drift and discounting will have no impact.

In general, same-day options are nonstandard and a wider bid–offer spread should be charged. Plus it is vital to take expected intraday variance profiles and events into account. Be suspicious: Why wouldn't the counterparty be happy to wait until the standard option expiry time?

Constructing an ATM Curve in Excel

Within this practical, three methods of constructing an ATM curve are developed. First, an ATM curve is constructed using *interpolation* between market tenors. Then, an ATM curve is constructed using a parameterized *model*. Finally, *weights* are added to a simple ATM curve to demonstrate how ATM curves are maintained by traders in practice. These steps mirror the material developed in Chapter 11.

Task A: Constructing an ATM Curve Using Interpolation

When constructing an ATM curve based on market tenors, the expiry date for each market tenor must first be calculated using functions developed in Practical D. The ATM implied volatility is then manually inputted at each tenor. For the purposes of testing, a simple upward-sloping ATM curve can be used initially:

Horizon	**Wed 11-Jun-14**	←*Named:* ***Horizon***

Populate Expiry Dates

↓*Named:* ***ATMVolRef***

Tenor	Expiry Date	ATM Implied Volatility
ON	Thu 12-Jun-14	10.00%
1W	Wed 18-Jun-14	11.00%
2W	Wed 25-Jun-14	12.00%
1M	Thu 10-Jul-14	13.00%
2M	Mon 11-Aug-14	14.00%
3M	Thu 11-Sep-14	15.00%
6M	Thu 11-Dec-14	16.00%
1Y	Thu 11-Jun-15	17.00%
2Y	Thu 09-Jun-16	18.00%

Using these inputs, a VBA function can interpolate to give the ATM volatility for any date. This function references the expiry dates and ATM volatilities at market tenors using named cells, with linear interpolation used to generate ATM volatility for expiry dates between tenors:

```
Function getATMVol(QueryDate As Long) As Double

    Dim Count As Long
    Dim TimeLow As Double, TimeHigh As Double
    Dim VolLow As Double, VolHigh As Double

    'Find the relevant Expiry date row (requires the Expiry dates _
     to be ordered)
    Count = 1
    While Range("ExpiryDateRef").Offset(Count, 0) < QueryDate _
     And Range("ExpiryDateRef").Offset(Count, 0) <> ""
        Count = Count + 1
    Wend

    If Range("ExpiryDateRef").Offset(Count, 0) = "" Then
        'Query Date beyond Maximum Expiry Date
        getATMVol = -1
    ElseIf Count = 1 And Range("ExpiryDateRef").Offset(Count, 0) > _
     QueryDate Then
        'Query Date before Minimum Expiry Date
        getATMVol = -1
    ElseIf Range("ExpiryDateRef").Offset(Count, 0) = QueryDate Then
        'Exact Expiry Date Found
        getATMVol = Range("ATMVolRef").Offset(Count, 0)
    Else
        'Interpolate to get ATM Implied Volatility
        TimeLow = (Range("ExpiryDateRef").Offset(Count - 1, 0) - _
         Range("Horizon")) / 365
```

```
        TimeHigh = (Range("ExpiryDateRef").Offset(Count, 0) - _
         Range("Horizon")) / 365
        VolLow = Range("ATMVolRef").Offset(Count - 1, 0)
        VolHigh = Range("ATMVolRef").Offset(Count, 0)
        getATMVol = LinearVolatilityInterpolation(TimeLow, TimeHigh, _
         VolLow, VolHigh, (QueryDate - Range("Horizon")) / 365)
    End If

End Function

Function LinearVolatilityInterpolation (TimeLow As Double, _
 TimeHigh As Double, VolLow As Double, VolHigh As Double, _
 QueryTime As Double) As Double

    LinearVarianceInterpolation = VolLow + (VolHigh - VolLow) * _
     (QueryTime - TimeLow) / (TimeHigh - TimeLow)

End Function
```

The getATMVol function can be tested by querying for implied volatility in four different cases:

1. An expiry date before the minimum tenor expiry date
2. An expiry date after the maximum tenor expiry date
3. An expiry date at a tenor expiry date
4. An expiry date between two tenor expiry dates

Query Date	ATM Implied Volatility	
01-Jan-15	16.12%	=getATMVol(QueryDate)

↑*Named:* ***QueryDate***

The ATM volatility for daily expiry dates (starting at the overnight tenor and going for two years) can now be calculated. This subroutine (run by pressing the button) populates the ATM implied volatilities:

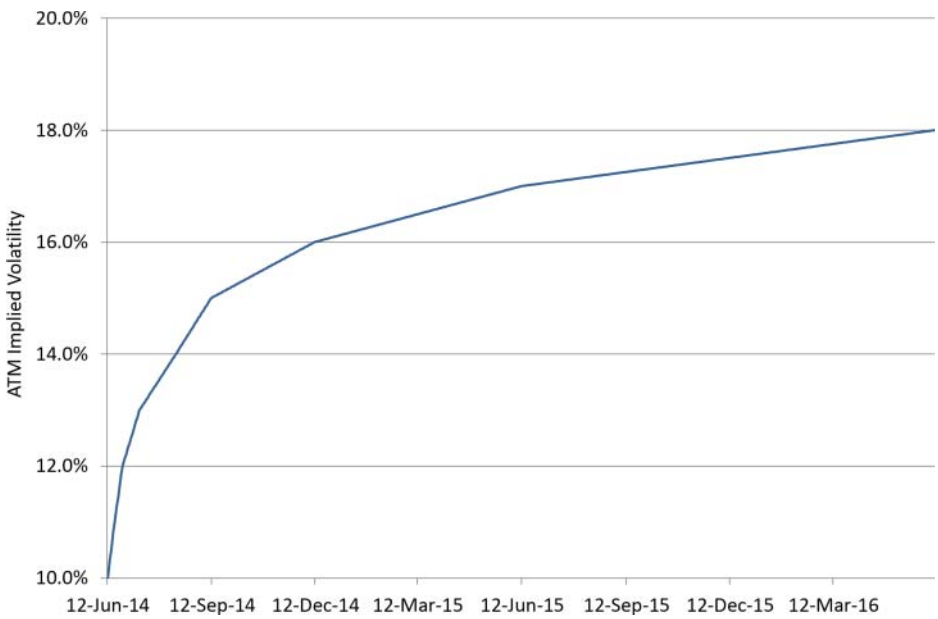
```
Sub populateATMImpliedVolatilities()

    Dim Count As Long

    Count = 1
    While Range("ChartExpiryDateRef").Offset(Count, 0) <> ""
        Range("ChartATMVolsRef").Offset(Count, 0) = _
         getATMVol(Range("ChartExpiryDateRef").Offset(Count, 0))
        Count = Count + 1
    Wend

End Sub
```

Interpolate ATM Implied Volatility

↓Named: ***ChartExpiryDateRef*** ↓Named: ***ChartATMVolsRef***

Expiry Date	ATM Implied Volatility
12-Jun-14	10.00%
13-Jun-14	10.17%
14-Jun-14	10.33%
15-Jun-14	10.50%
16-Jun-14	10.67%
17-Jun-14	10.83%
18-Jun-14	11.00%

This data can be plotted in a chart:



Variance for each expiry date can also be calculated (see Chapter 11) and pushed onto the sheet using this subroutine:

```
Sub populateVariance()

    Dim Count As Long
    Dim T As Double, vol As Double

    Count = 1
    While Range("ChartExpiryDateRef").Offset(Count, 0) <> ""
        T = (Range("ChartExpiryDateRef").Offset(Count, 0) - _
         Range("Horizon")) / 365
```

```
        vol = Range("ChartATMVolsRef").Offset(Count, 0)
        Range("ChartVarianceRef").Offset(Count, 0) = T * vol ^ 2
        Count = Count + 1
    Wend

End Sub
```

Interpolate ATM Implied Volatility | Calculate Variance

↓Named: ***ChartVarianceRef***

Expiry Date	ATM Implied Volatility	Variance
12-Jun-14	10.00%	0.00%
13-Jun-14	10.17%	0.01%
14-Jun-14	10.33%	0.01%
15-Jun-14	10.50%	0.01%
16-Jun-14	10.67%	0.02%
17-Jun-14	10.83%	0.02%

Finally, linear variance interpolation can be used instead if required:

```
Function LinearVarianceInterpolation(TimeLow As Double, TimeHigh As _
Double, VolLow As Double, VolHigh As Double, QueryTime As Double) As _
Double

    Dim VarianceLow As Double, VarianceHigh As Double, _
     QueryVariance As Double
    VarianceLow = TimeLow * VolLow ^ 2
    VarianceHigh = TimeHigh * VolHigh ^ 2

    QueryVariance = VarianceLow + (VarianceHigh - VarianceLow) * _
     (QueryTime - TimeLow) / (TimeHigh - TimeLow)
    LinearVarianceInterpolation = Sqr(QueryVariance / QueryTime)

End Function
```

Task B: Constructing an ATM Curve Using a Model

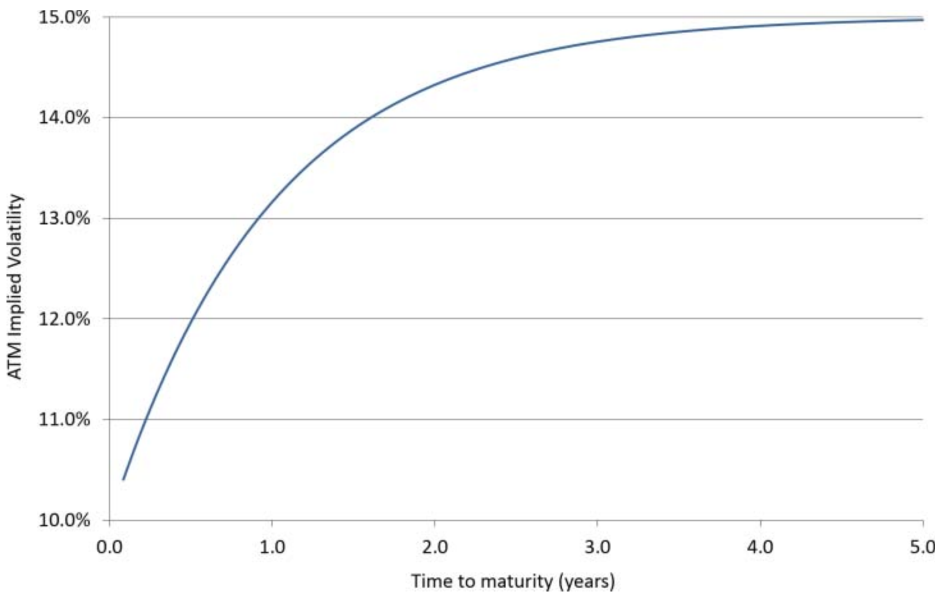
There are many possible ATM curve models. One of the simplest possible parameterizations introduced in Chapter 11 is:

$$\sigma_T = \sigma_{short} + (\sigma_{long} - \sigma_{short}).(1 - e^{-\lambda T})$$

where σ_T is the ATM implied volatility at time T (measured in years), σ_{short} and σ_{long} are the short- and long-term ATM volatilities respectively, and λ is the speed of reversion from σ_{short} to σ_{long}. This model can be implemented in an Excel sheet, with time displayed in monthly intervals (use 1/12 intervals within this stylized framework):

	A	B	C	D	E	F
3						
4		Short Vol (σ *short*)	**10.0%**	←*Named:* ***ShortVol***		
5		Long Vol (σ *long*)	**15.0%**	←*Named:* ***LongVol***		
6		Speed (λ)	**1.0**	←*Named:* ***Speed***		
7						
8		**Time**	**ATM Volatility**			
9		0.08333	10.4%	=ShortVol+(LongVol-ShortVol)*(1-EXP(-Speed*B9))		
10		0.16667	10.8%	=ShortVol+(LongVol-ShortVol)*(1-EXP(-Speed*B10))		
11		0.25000	11.1%	...		
12		0.33333	11.4%			

The output can be plotted in a chart:



The function can then be attached to the market expiry dates (and their T's) to calculate ATM implied volatility:

ATM Curve Construction

Horizon	Wed 11-Jun-14
Short Vol (σ_{short})	10.0%
Long Vol (σ_{long})	15.0%
Speed (λ)	1.0

Populate Expiry Dates

Tenor	Expiry Date	T	ATM Implied Volatility	
ON	Thu 12-Jun-14	0.00274	10.01%	=ShortVol+(LongVol-ShortVol)*(1-EXP(-Speed*D12))
1W	Wed 18-Jun-14	0.01918	10.09%	
2W	Wed 25-Jun-14	0.03836	10.19%	
1M	Thu 10-Jul-14	0.07945	10.38%	
2M	Mon 11-Aug-14	0.16712	10.77%	
3M	Thu 11-Sep-14	0.25205	11.11%	
6M	Thu 11-Dec-14	0.50137	11.97%	
1Y	Thu 11-Jun-15	1.00000	13.16%	
2Y	Thu 09-Jun-16	1.99726	14.32%	

Task C: Adding Weights to an ATM Curve

In practice, traders keep their ATM curves aligned with the market by controlling the expected variance assigned to individual dates. This control is used to, for example, assign low variance to weekends/holiday days and high variance to major event days. One common way this can be achieved is by splitting variance into discrete daily chunks based on the weight assigned to each day.

Within this model, a single flat volatility is used. By introducing a separate row for each date starting one day after the horizon (for at least a year), **calendar time** can therefore be calculated:

ATM Curve Construction

Horizon	Wed 11-Jun-14	←*Named:* ***Horizon***
Volatility	10.0%	←*Named:* ***vol***

Expiry Dates

Date	Calendar Time	
Thu 12-Jun-14	0.00274	=(B10-Horizon)/365
Fri 13-Jun-14	0.00548	=(B11-Horizon)/365
Sat 14-Jun-14	0.00822	...

Day weights can now be added. These are defined in a table:

↓*Named:* ***DayWeightRef***

Weekday	Weight
Sunday	1.0
Monday	1.0
Tuesday	1.0
Wednesday	1.0
Thursday	1.0
Friday	1.0
Saturday	1.0

A VBA subroutine can be used to push the day weights onto the expiry dates. Note how the Weekday VBA function is cunningly used to generate the offset reference to the correct cell:

```
Sub populateDayWeights()

    Dim CountExpiryDates As Long

    CountExpiryDates = 1
    While Range("DateRef").Offset(CountExpiryDates, 0) <> ""
        Range("DateRef").Offset(CountExpiryDates, 2) = _
           Range("DayWeightRef").Offset(Weekday(Range("DateRef"). _
           Offset(CountExpiryDates, 0)), 1)
        CountExpiryDates = CountExpiryDates + 1
    Wend

End Sub
```


The day weights up to a given tenor can then be summed and divided by 365 to calculate **economic time**. This calendar time versus economic time technique is used within real ATM curve models. Controlling economic time allows variance to be unevenly distributed over different days or, in more sophisticated models, over different parts of the day. For now, though, set weights of 1 on each day so calendar time and economic time are identical:

Expiry Dates

Populate Day Weights

↓*Named:* ***DateRef***

Date	Calendar Time	Day Weight	Day Weight Sum	Economic Time
Thu 12-Jun-14	0.00274	1.00	1.00	0.00274
Fri 13-Jun-14	0.00548	1.00	2.00	0.00548
Sat 14-Jun-14	0.00822	1.00	3.00	0.00822
Sun 15-Jun-14	0.01096	1.00	4.00	0.01096

When weights are added, total variance to (calendar) time t changes from:

$$var_T = \sigma^2 T$$

into

$$var_T = \sigma^2 \sum_{i=1}^{n} \omega_i dt$$

where $dt = \frac{1}{365}$.

Expiry Dates

Populate Day Weights

Date	Calendar Time	Day Weight	Day Weight Sum	Economic Time	Total Variance	
Thu 12-Jun-14	0.00274	1.00	1.00	0.00274	0.000027	=F23*vol^2
Fri 13-Jun-14	0.00548	1.00	2.00	0.00548	0.000055	=F24*vol^2
Sat 14-Jun-14	0.00822	1.00	3.00	0.00822	0.000082	...
Sun 15-Jun-14	0.01096	1.00	4.00	0.01096	0.000110	

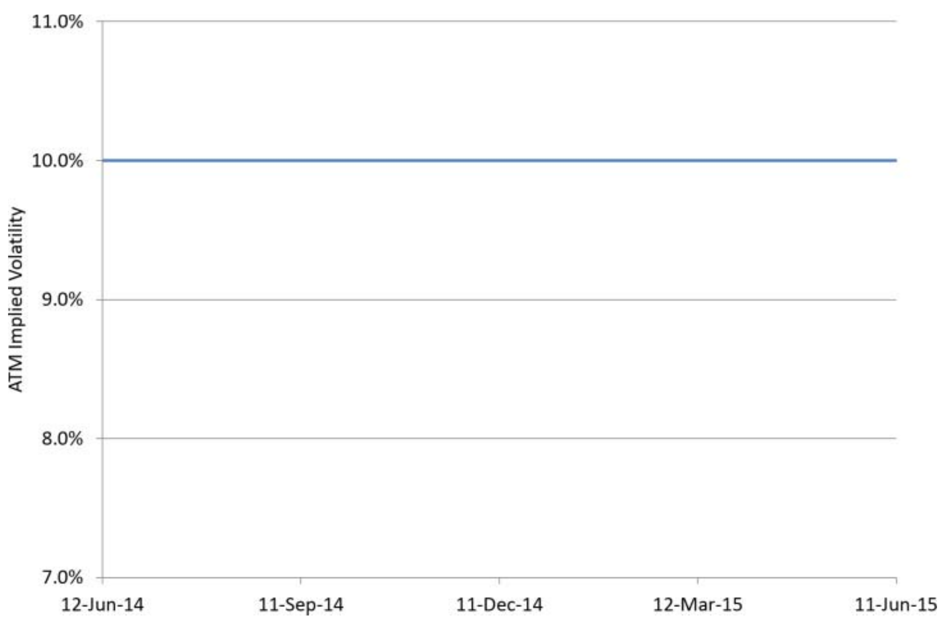
ATM volatility is then calculated from total variance using calendar time:

Expiry Dates

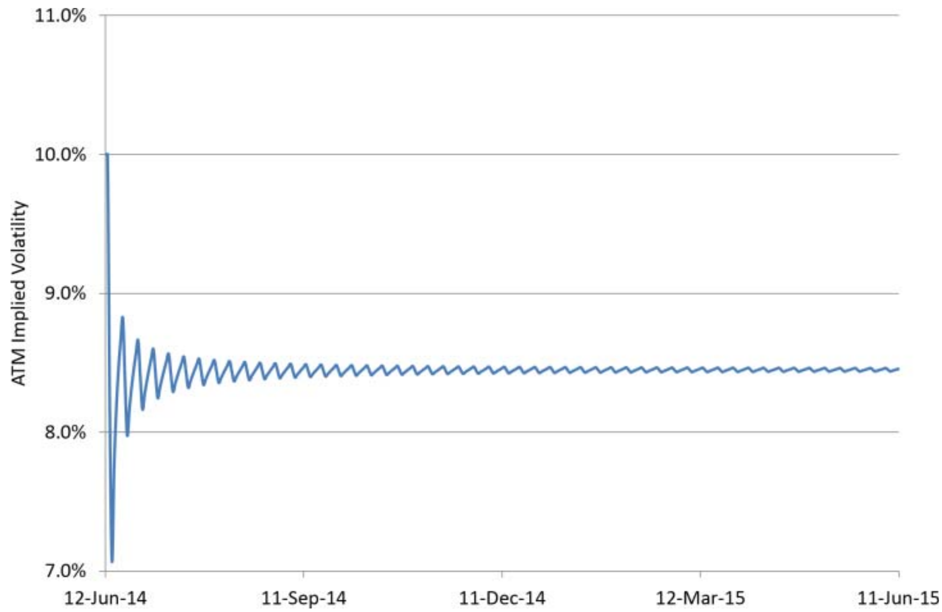
Populate Day Weights

Date	Calendar Time	Day Weight	Day Weight Sum	Economic Time	Total Variance	ATM Volatility	
Thu 12-Jun-14	0.00274	1.00	1.00	0.00274	0.00003	10.00%	=SQRT(G22/C22)
Fri 13-Jun-14	0.00548	1.00	2.00	0.00548	0.00005	10.00%	=SQRT(G22/C22)
Sat 14-Jun-14	0.00822	1.00	3.00	0.00822	0.00008	10.00%	...
Sun 15-Jun-14	0.01096	1.00	4.00	0.01096	0.00011	10.00%	

When plotted with constant day weights of 1, the ATM volatility is flat as expected:



Now comes the magic: Set the weekend day weights to zero, repopulate the expiry date day weights using the populateDayWeights subroutine, and check the graph again:



The output now contains the ATM saw-toothing observed in the real FX derivatives market. Look at the data in the sheet:

Expiry Dates

Populate Day Weights

Date	Calendar Time	Day Weight	Day Weight Sum	Economic Time	Total Variance	ATM Volatility
Thu 12-Jun-14	0.00274	1.00	1.00	0.00274	0.00003	10.00%
Fri 13-Jun-14	0.00548	1.00	2.00	0.00548	0.00005	10.00%
Sat 14-Jun-14	0.00822	0.00	2.00	0.00548	0.00005	8.16%
Sun 15-Jun-14	0.01096	0.00	2.00	0.00548	0.00005	7.07%
Mon 16-Jun-14	0.01370	1.00	3.00	0.00822	0.00008	7.75%
Tue 17-Jun-14	0.01644	1.00	4.00	0.01096	0.00011	8.16%
Wed 18-Jun-14	0.01918	1.00	5.00	0.01370	0.00014	8.45%
Thu 19-Jun-14	0.02192	1.00	6.00	0.01644	0.00016	8.66%

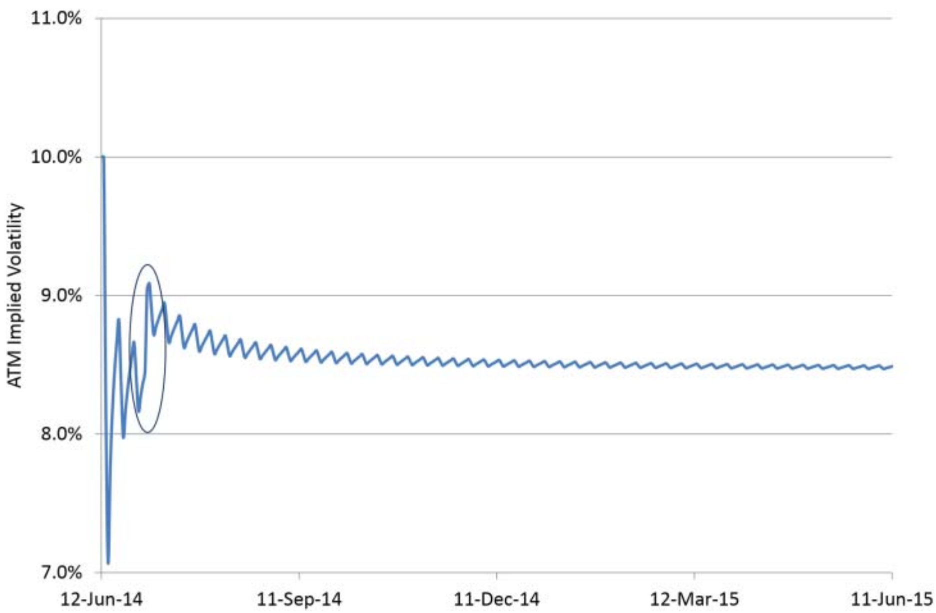
In the model, economic time stops over the weekend because the FX market isn't open on Saturday or Sunday. In practice, trading desks usually assign a small but non-zero variance to the weekend because there is a small chance that unexpected news over the weekend will cause spot to move sharply first thing on Monday morning. The reduction in the economic-time-to-calendar-time ratio causes the ATM implied volatility for the Monday expiry to be lower than the Friday preceding it.

In this case, ATM volatility tends toward a value that is lower than the flat volatility input due to the ratio of economic time to calendar time being less than 1. In practice, this effect is adjusted for when there are target volatility levels that must be hit.

Finally, consider how the model is adjusted when there is an event. On Thursday, July 3, 2014, Non-Farm Payrolls (a big USD economic indicator, which normally occurs on the first Friday of the month) is released. Therefore, expected variance on that date is higher and the ATM volatility for that date is correspondingly higher. This is achieved within the model by moving the weight for that date higher:

Expiry Dates		Populate Day Weights				
Date	**Calendar Time**	**Day Weight**	**Day Weight Sum**	**Economic Time**	**Total Variance**	**ATM Volatility**
Fri 27-Jun-14	0.04384	1.00	12.00	0.03288	0.00033	8.66%
Sat 28-Jun-14	0.04658	0.00	12.00	0.03288	0.00033	8.40%
Sun 29-Jun-14	0.04932	0.00	12.00	0.03288	0.00033	8.16%
Mon 30-Jun-14	0.05205	1.00	13.00	0.03562	0.00036	8.27%
Tue 01-Jul-14	0.05479	1.00	14.00	0.03836	0.00038	8.37%
Wed 02-Jul-14	0.05753	1.00	15.00	0.04110	0.00041	8.45%
Thu 03-Jul-14	0.06027	3.00	18.00	0.04932	0.00049	9.05%
Fri 04-Jul-14	0.06301	1.00	19.00	0.05205	0.00052	9.09%
Sat 05-Jul-14	0.06575	0.00	19.00	0.05205	0.00052	8.90%
Sun 06-Jul-14	0.06849	0.00	19.00	0.05205	0.00052	8.72%
Mon 07-Jul-14	0.07123	1.00	20.00	0.05479	0.00055	8.77%

The ATM volatility for the Non-Farm Payroll date itself moves higher plus the increased variance causes subsequent days to move higher, too. This is a real feature observed when building ATM curves: If expected variance for a given date increases, the ATM volatility for that date *and subsequent dates* rises:



Daily variance can now be calculated by taking the difference in variance between subsequent expiry dates, which in turn can be used to calculate daily ATM volatility. The daily ATM volatility is effectively the implied volatility for a strip of forward overnight ATM contracts. Traders use these forward overnight ATM volatilities to determine whether the ATM curve is overpriced or underpriced over events.

Expiry Dates

Populate Day Weights

Date	Calendar Time	Day Weight	Day Weight Sum	Economic Time	Total Variance	ATM Volatility	Daily Variance	Daily Volatility
Thu 12-Jun-14	0.00274	1.00	1.00	0.00274	0.00003	10.00%	0.00003	10.00%
Fri 13-Jun-14	0.00548	1.00	2.00	0.00548	0.00005	10.00%	0.00003	10.00%
Sat 14-Jun-14	0.00822	0.00	2.00	0.00548	0.00005	8.16%	0.00000	0.00%
Sun 15-Jun-14	0.01096	0.00	2.00	0.00548	0.00005	7.07%	0.00000	0.00%
Mon 16-Jun-14	0.01370	1.00	3.00	0.00822	0.00008	7.75%	0.00003	10.00%
Tue 17-Jun-14	0.01644	1.00	4.00	0.01096	0.00011	8.16%	0.00003	10.00%
Wed 18-Jun-14	0.01918	1.00	5.00	0.01370	0.00014	8.45%	0.00003	10.00%
Thu 19-Jun-14	0.02192	1.00	6.00	0.01644	0.00016	8.66%	0.00003	10.00%

Vanilla FX derivative traders actively update weights within their ATM curve model to match market prices observed in the interbank broker market, plus future economic release dates are assigned higher weights when release schedules are known. Holiday days in a particular currency are known far in advance, too, and are assigned lower weights to reflect the reduced expected variance.

In practice, trading desks use frameworks similar to this but more granularity is usually included within the model. Exact times of events are often specified, enabling the correct pricing of different cuts within the same day. Trading desks also require a sophisticated core ATM curve, not just flat volatility. An approach similar to those developed in Task A or Task B in this practical will be usually be taken, with weights added on top in such a way that nonnegative forward variance is guaranteed.

Volatility Smile Market Instruments and Exposures

In the interbank broker market, at each market tenor, three **market instruments** define the volatility smile:

1. *At-the-money (ATM)* contracts define the implied volatility for a specific strike close to (or exactly at, depending on the market conventions for a given currency pair) the forward for the given tenor.
2. *Butterfly (Fly)* contracts define the implied volatility differential between the wings of the volatility smile and the ATM—a measure of the height of the wings of the volatility smile.
3. *Risk reversal (RR)* contracts define the implied volatility differential between strikes above and below the ATM—a measure of how skewed or tilted the volatility smile is.

Butterfly and risk reversal contracts are most often quoted at 25 delta (25d) and 10 delta (10d) strikes. An example run of market instruments at market tenors is shown in Exhibit 12.1.

Exhibit 12.2 shows the relative positioning of different deltas within a stylized volatility smile. Recall that it is the market convention to trade the out-of-the-money side.

Tenor	Expiry Date	ATM	25d Fly	10d Fly	25d RR	10d RR
ON	26-Jun-14	5.50%	0.10%	0.25%	-0.25%	-0.40%
1W	02-Jul-14	4.10%	0.125%	0.30%	-0.25%	-0.40%
2W	09-Jul-14	5.05%	0.125%	0.325%	-0.35%	-0.55%
1M	24-Jul-14	4.625%	0.125%	0.35%	-0.40%	-0.60%
2M	25-Aug-14	4.825%	0.15%	0.45%	-0.50%	-0.85%
3M	25-Sep-14	5.15%	0.15%	0.50%	-0.60%	-0.95%
6M	23-Dec-14	5.625%	0.225%	0.75%	-0.75%	-1.35%
1Y	25-Jun-15	6.30%	0.275%	1.00%	-0.90%	-1.65%
2Y	23-Jun-16	6.90%	0.30%	1.10%	-1.00%	-1.85%

EXHIBIT 12.1 Example EUR/USD market instruments at market tenors

EXHIBIT 12.2 Deltas quoted within the volatility smile

The following *approximations* link the ATM, 25d butterfly, and 25d risk reversal instruments with the implied volatilities for the *outright* 25d call and put options at a given tenor:

- $\sigma_{Call25d} = \sigma_{ATM} + \sigma_{Fly25d} + \frac{1}{2}\sigma_{RR25d}$
- $\sigma_{Put25d} = \sigma_{ATM} + \sigma_{Fly25d} - \frac{1}{2}\sigma_{RR25d}$

Therefore:

- $\sigma_{RR25d} = \sigma_{Call25d} - \sigma_{Put25d}$
- $\sigma_{Fly25d} = \frac{(\sigma_{Call25d} + \sigma_{Put25d})}{2} - \sigma_{ATM}$

Exhibit 12.3 shows how these market instruments fit into the volatility smile.

These approximations were generalized into a single formula for any delta by Allan M. Malz in 1997:

$$\sigma_{X\ Delta\ Put} = \sigma_{ATM} + 2\ \sigma_{RR25d}.(X - 50\%) + 16\ \sigma_{Fly25d}.(X - 50\%)^2$$

In words, the ATM represents the central reference point, the butterfly lifts the wings symmetrically higher on both sides, and the risk reversal tilts the smile one way or the other. As mentioned in Chapter 7, put deltas are often quoted without the negative sign. Positive put delta values between 0% and 100% are used in the Malz formula.

If butterfly and risk reversal contracts at all deltas are zero, the volatility smile is flat as shown in Exhibit 12.4, and any strike at that tenor will be assigned the same midmarket implied volatility.

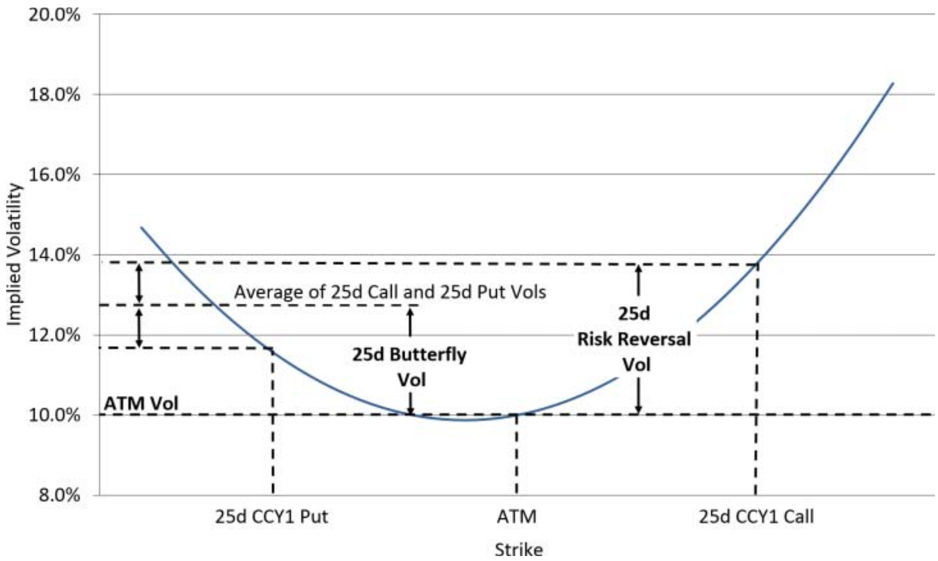


EXHIBIT 12.3 25 delta market instruments within the volatility smile

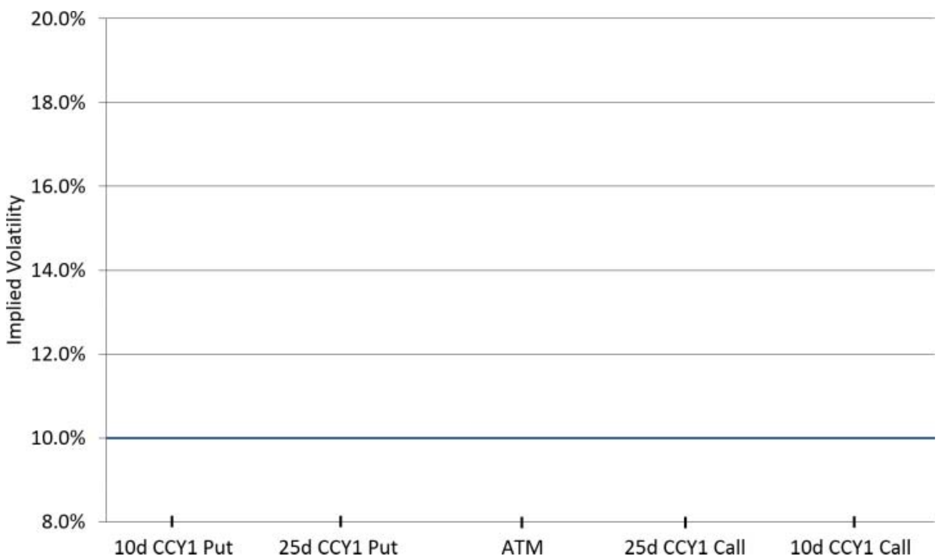


EXHIBIT 12.4 Volatility smile with zero risk reversal and zero butterfly

If the butterfly increases, the wings of the volatility smile rise symmetrically as shown in Exhibit 12.5.

With a positive risk reversal, strikes above the ATM have a higher implied volatility than the equivalent delta strikes below the ATM. This is shown in Exhibit 12.6.

With a negative risk reversal, strikes below the ATM have a higher implied volatility than the equivalent delta strikes above the ATM. This is shown in Exhibit 12.7.

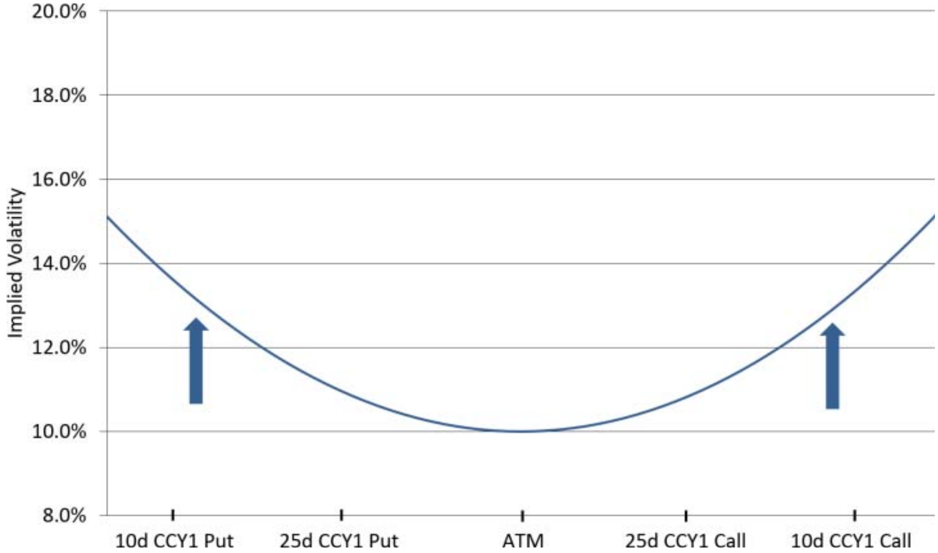


EXHIBIT 12.5 Volatility smile with zero risk reversal and positive butterfly

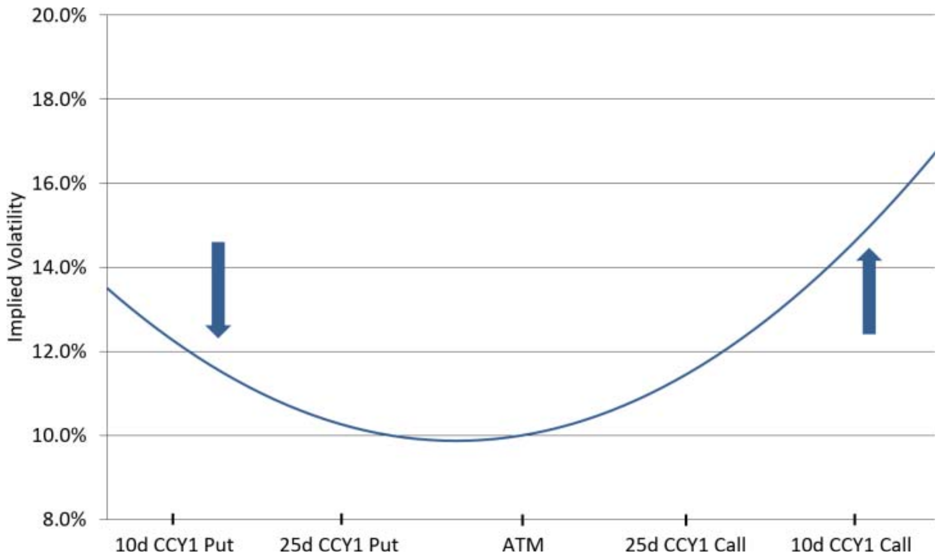


EXHIBIT 12.6 Volatility smile with positive risk reversal

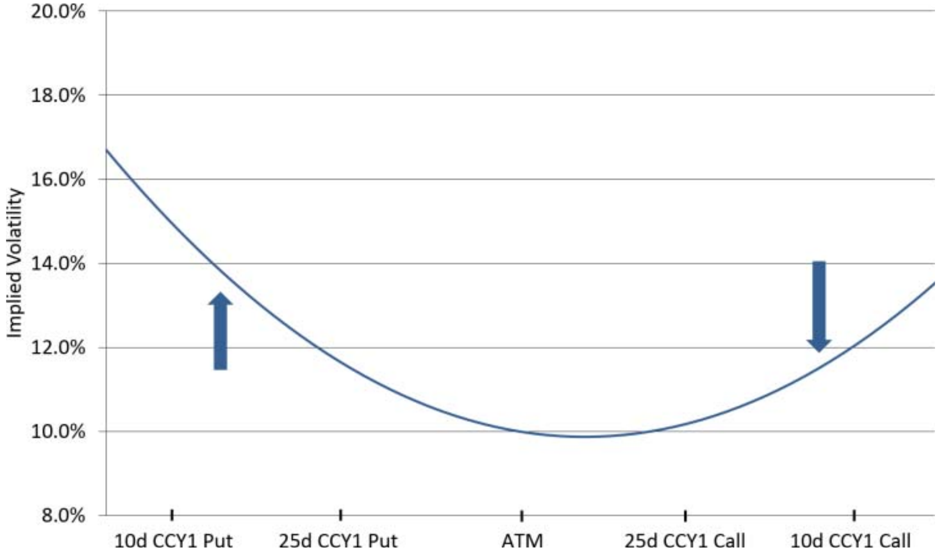


EXHIBIT 12.7 Volatility smile with negative risk reversal

Market Instrument Vega Exposures

The reason for describing the volatility smile with ATM, butterfly, and risk reversal instruments becomes clearer when the implied volatility exposures of the three market instruments are examined. The key implied volatility exposures are:

- *Vega* $\left(\frac{\partial P}{\partial \sigma}\right)$: sensitivity of price to changes in implied volatility.
- *Vanna* $\left(\frac{\partial vega}{\partial spot}\right)$: sensitivity of vega to changes in spot. Vanna can also be thought of as the sensitivity of delta to changes in implied volatility, i.e., $\left(\frac{\partial delta}{\partial \sigma}\right)$ since $\left(\frac{\partial P}{\partial \sigma}\right) / \partial spot = \left(\frac{\partial P}{\partial spot}\right) / \partial \sigma$.
- *Volga* $\left(\frac{\partial vega}{d\sigma}\right)$: sensitivity of vega to changes in implied volatility. Volga is the second derivative of price with respect to changes in implied volatility. Therefore, volga is to implied volatility as gamma is to spot and as the volatility of implied volatility rises, the expected P&L from a long volga trading position increases.

ATM Exposures

The vega profile for a long ATM vanilla option has a single peak around current spot. Exhibit 12.8 shows how, at higher volatility, the vega profile is wider since the spot distribution is wider, but vega is unchanged at the initial spot.

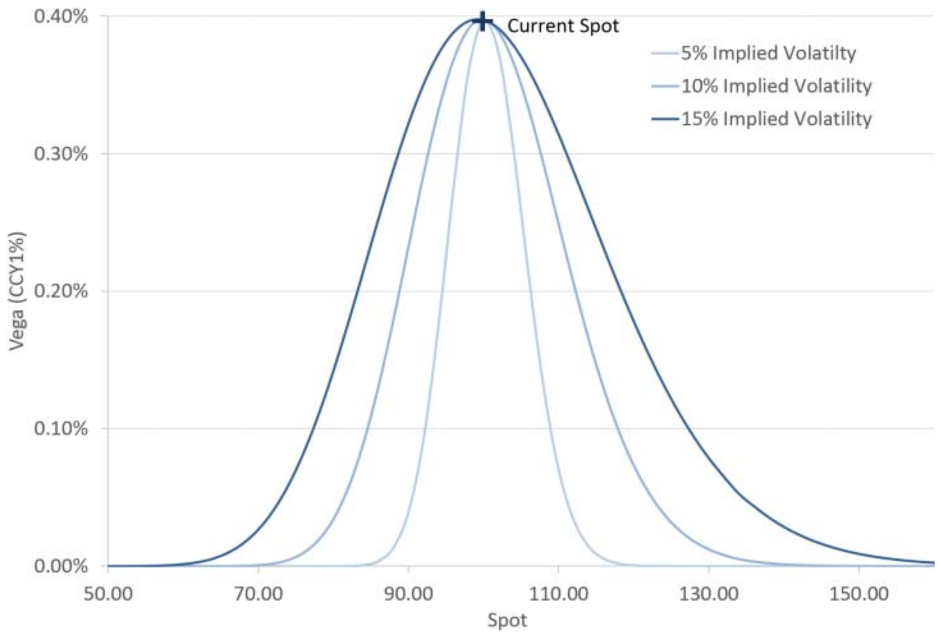


EXHIBIT 12.8 Vega profile from long ATM at different implied volatility levels

Therefore, a long ATM contract at inception has the following exposures:

- Vega: positive exposure
- Vanna (the gradient of the vega/spot chart): no exposure
- Volga (the difference between the vega profiles for different implied volatility levels at the initial spot level): no exposure

Vanna with spot above the strike is negative since vega rises into the (now) downside peak. Vanna with spot below the strike is positive since vega rises into the (now) topside peak. The vanna profile from a long ATM option is shown in Exhibit 12.9.

Recalling the dual interpretation of vanna as $\frac{\partial vega}{\partial spot}$ or $\frac{\partial delta}{\partial \sigma}$, consider an out-of-the-money topside call option (i.e., strike above spot). At current implied volatility the strike has 25% delta. If implied volatility rises, the chance of the strike ending in-the-money at maturity increases as the distribution widens and hence delta rises. Therefore, this option has a long vanna exposure.

Likewise, consider a downside (in-the-money) call strike (i.e., strike below spot). At current implied volatility the strike has 75% delta. If implied volatility rises, the chance of the strike ending in-the-money at maturity decreases as the distribution widens and hence delta falls. Therefore, this option has a short vanna exposure.

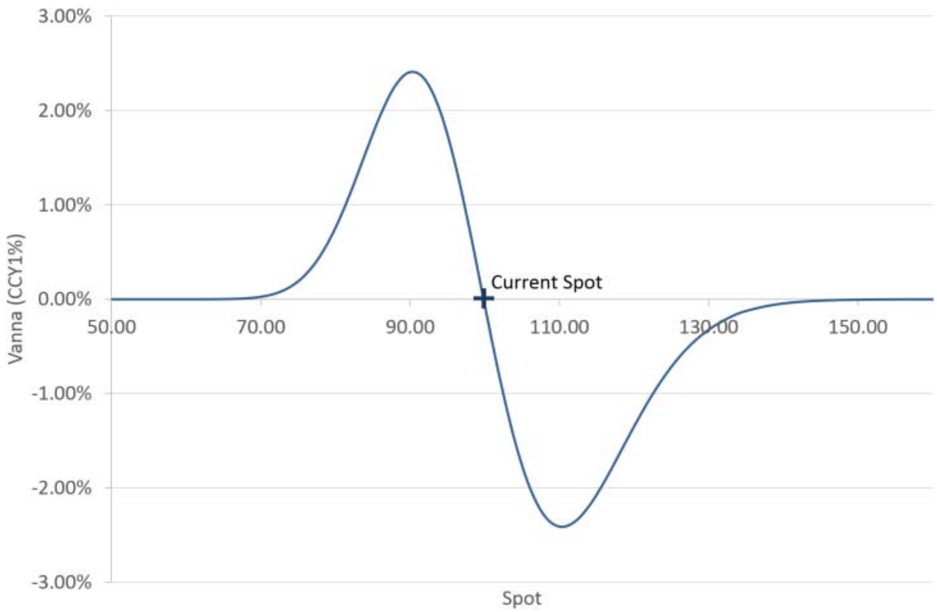


EXHIBIT 12.9 Vanna profile from long ATM

Volga with spot above or below the ATM strike is positive since long wing vanilla options generate positive volga. The volga profile from a long ATM option is shown in Exhibit 12.10.

Risk Reversal Exposures

For a risk reversal contract, again, higher implied volatility moves the vega profile wider but at initial spot the vega exposure is unchanged at zero. This is shown in Exhibit 12.11. It is important to understand that these exposure profiles are generated with fixed strikes, equivalent to calculating the exposures immediately after trading the contract.

In this instance, *buying* the risk reversal means buying the topside strike versus selling the downside strike but in different currency pairs or tenors this may be the other way around. Therefore, a *long* risk reversal position can give either a long or short vanna exposure, depending on whether topside strikes are at higher or lower implied volatility than the equivalent delta downside strikes. If the topside strikes have a higher implied volatility, traders say the risk reversal is "for topside," whereas if downside strikes have a higher implied, traders say the risk reversal is, yes, "for downside." In a currency pair where the risk reversal is for downside, a long risk reversal position initially gives a short vanna position as shown in Exhibit 12.12.

Notice that these vega profiles aren't perfectly rotationally symmetric since vega persists further to the topside. This occurs because the Black-Scholes formula is

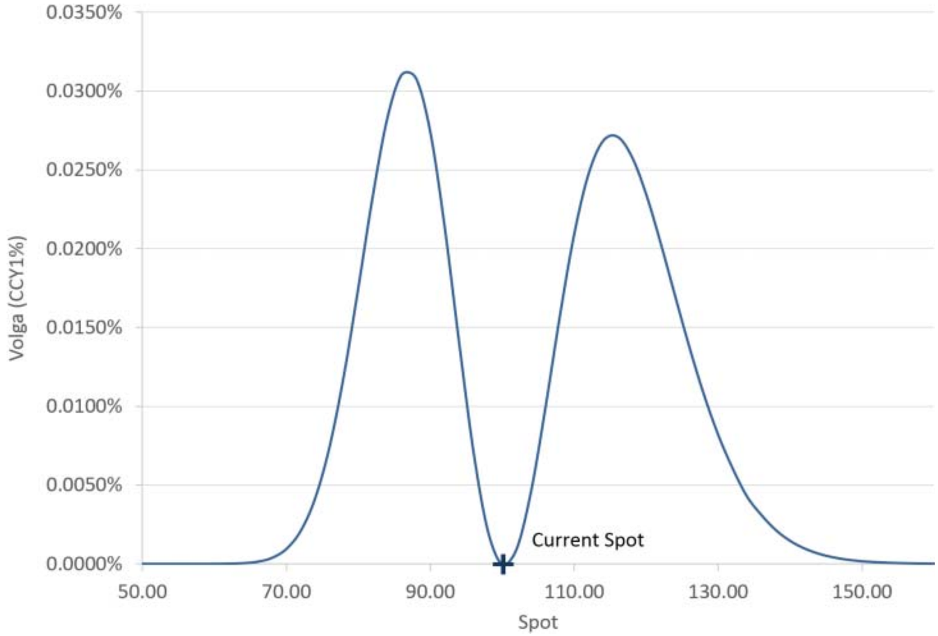


EXHIBIT 12.10 Volga profile from long ATM

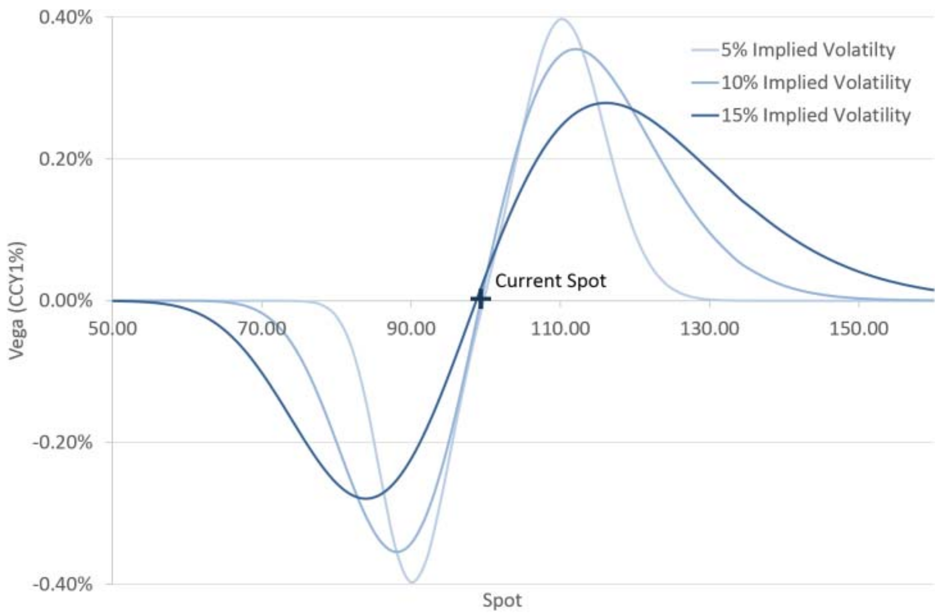


EXHIBIT 12.11 Vega profile from risk reversal (buying topside) at different implied volatility levels

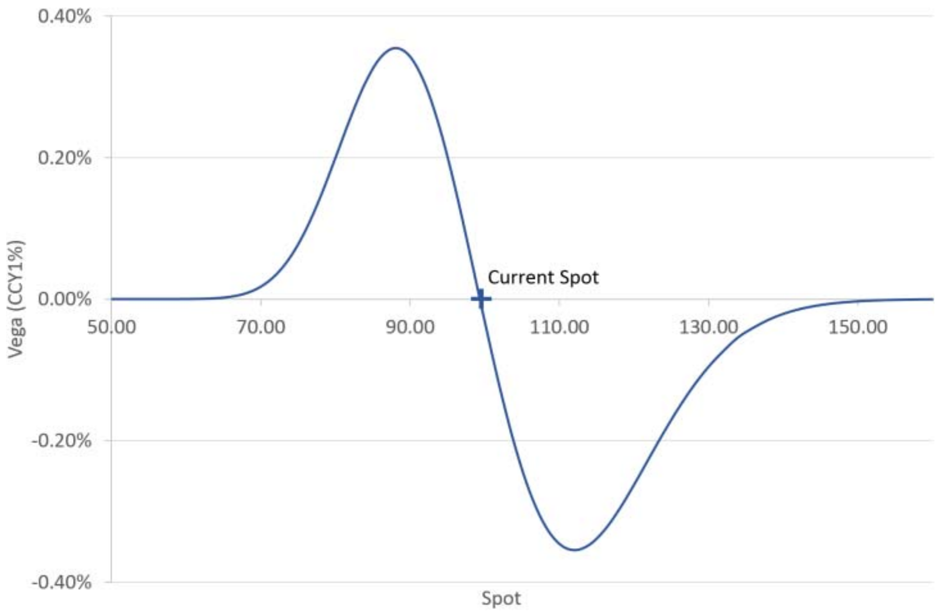


EXHIBIT 12.12 Vega versus spot profile from risk reversal (buying downside)

stated in log-return terms, which causes distances in spot space to compress toward zero. A stylized vega versus spot log-return graph for a risk reversal *is* rotationally symmetric as shown in Exhibit 12.13.

The vanna exposure on a risk reversal does not persist over all spot levels. Rather it is maximized at the initial spot as shown in Exhibit 12.14.

Therefore, a long risk reversal contract at inception has the following exposures:

- Vega: no exposure
- Vanna: positive or negative exposure depending on whether the risk reversal is "for topside" or "for downside" (i.e., whether topside or downside strikes are priced at higher implied volatility within the volatility smile)
- Volga: no exposure

Butterfly Exposures

A long butterfly contract is constructed using a long strangle (long wings) and a short straddle (ATM) with the ATM notional set such that the structure is initially vega-neutral and the call and put legs in the strangle have the same notional and delta. Exhibit 12.15 shows the vega profile from a long butterfly. Again, the butterfly strikes are fixed and hence the chart shows how the vega exposure changes at different implied volatility levels after trading.

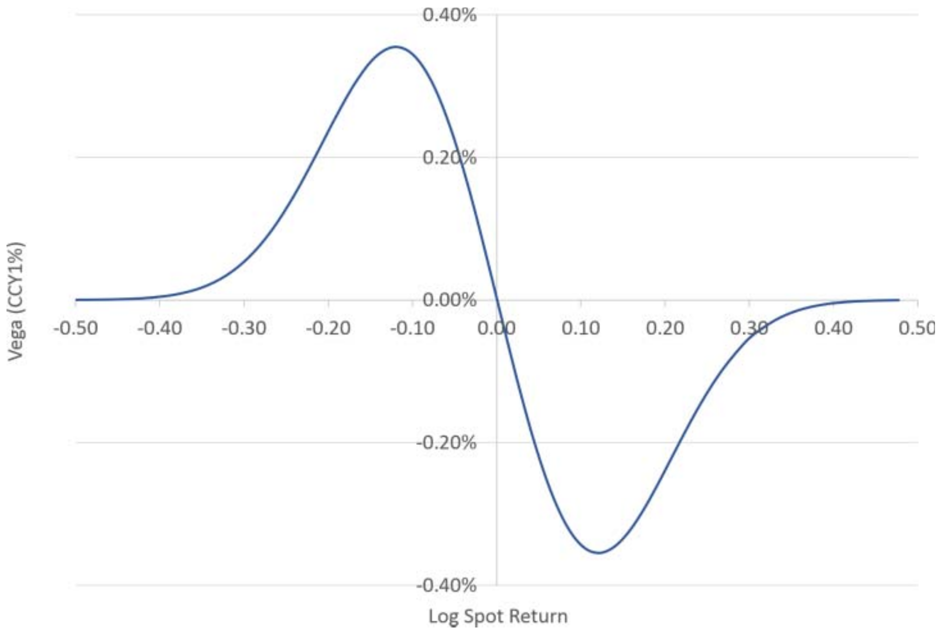


EXHIBIT 12.13 Vega versus log spot profile from risk reversal (buying downside)

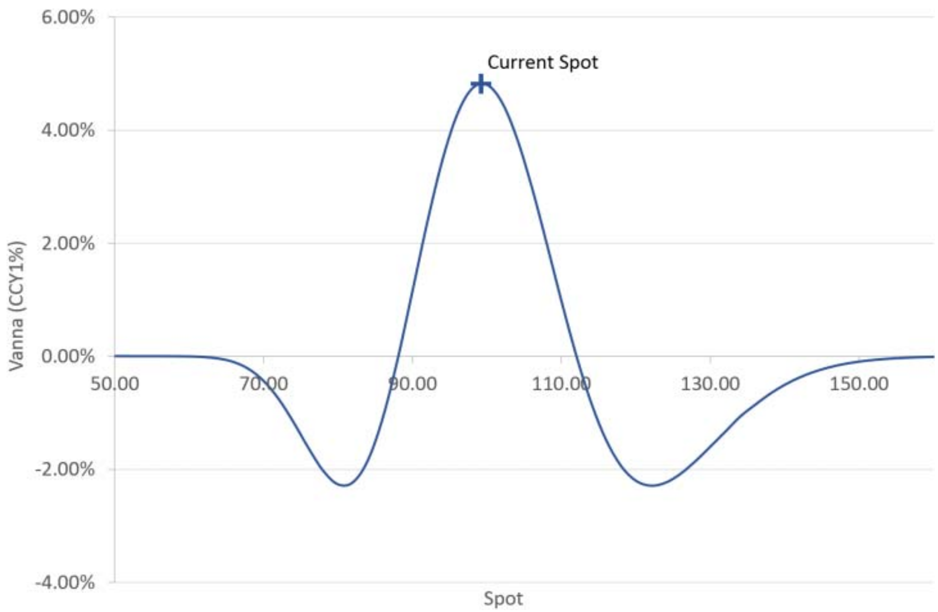


EXHIBIT 12.14 Vanna profile from risk reversal (buying topside)

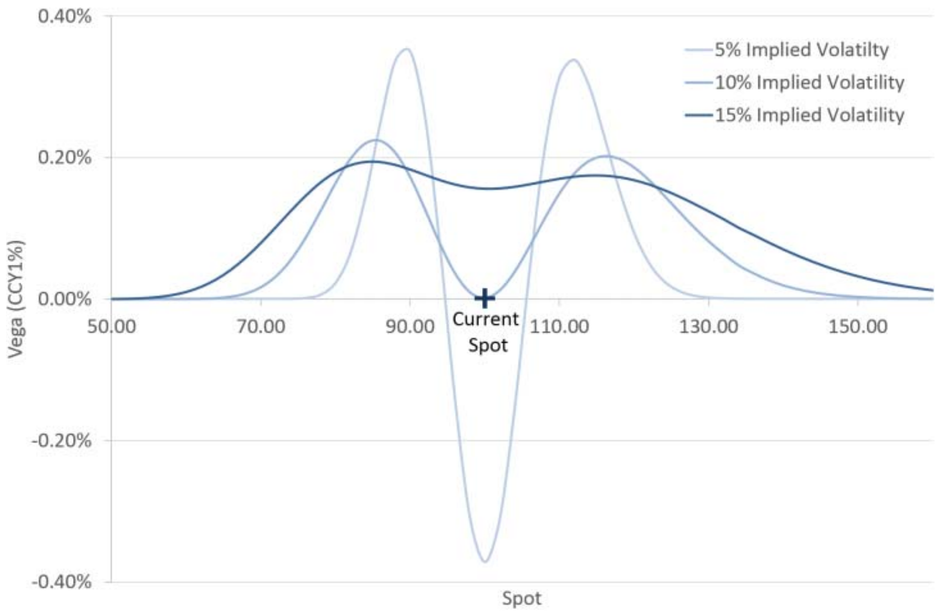


EXHIBIT 12.15 Vega profile from long butterfly at different implied volatility levels

The volga exposure on a butterfly does not persist over all spot levels. Rather it is maximized at the initial spot as shown in Exhibit 12.16.

A long butterfly contract at inception has the following exposures:

- Vega: no exposure (by construction)
- Vanna: no exposure
- Volga: positive exposure

Summary

Within this stylized analysis using a flat volatility smile and ignoring issues like adaption (explained in Chapter 14) and broker fly strike placement (explained later in this chapter), the three different market instruments give the three unique vega exposures at inception shown in Exhibit 12.17.

In practice this means that:

- ATM contracts are used to trade the *level* of implied volatility because their main exposure at inception is vega $\left(\frac{\partial P}{\partial \sigma}\right)$.
- Risk reversal contracts are used to trade the spot versus implied volatility relationship because their main exposure at inception is vanna $\left(\frac{\partial vega}{\partial spot}\right)$.

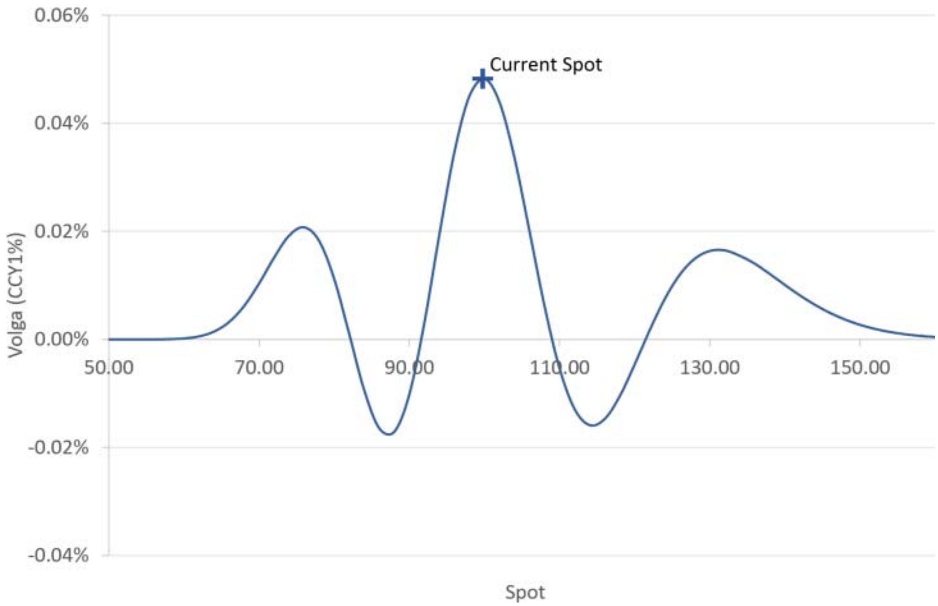


EXHIBIT 12.16 Volga profile from long butterfly

Instrument/Exposure	Vega	Vanna	Volga
ATM	✓	✗	✗
Risk Reversal	✗	✓	✗
Butterfly	✗	✗	✓

EXHIBIT 12.17 Vega exposures from market instruments

- Butterfly contracts are used to trade the volatility of implied volatility because their main exposure at inception is volga $\left(\frac{\partial vega}{\partial \sigma}\right)$.

Finally, it is mildly interesting to observe that the vanna profile of the ATM takes the same shape as the vega profile of the risk reversal while the volga of the ATM takes the same shape as the vega of the butterfly.

Risk Reversal Contract

The Black-Scholes formula assumes that the volatility of the underlying is constant. In practice, implied volatility changes depending (amongst other things) on how spot moves. Plus, in the market there is often differing supply and demand for topside or downside optionality, which leads to an asymmetric volatility smile.

The FX derivatives market expresses the amount of **skew** in the volatility smile via the risk reversal contract. Specifically, the risk reversal gives the differential between the call strike implied volatility and the put strike implied volatility for the same tenor and delta.

In the interbank broker market, risk reversals are quoted in positive terms, with the *direction* also quoted. For example, in USD/ZAR, if the 1yr 25d call is priced at 15.5% implied volatility and the 1yr 25d put is priced at 11.25% implied volatility, the 1yr 25d risk reversal would be quoted as "4.25% USD calls over," meaning that the USD call volatility is higher than the USD put volatility. In some currency pairs, it is market convention to quote the risk reversal direction in CCY2 terms. USD/JPY risk reversals are quoted as, for example, "1.4% JPY calls over" if the implied volatility for the downside strike is 1.4% higher than the implied volatility for the topside strike. As noted, *buying* the risk reversal always means *buying* the call or put strike with the higher volatility, and *selling* the other leg.

The delta used to calculate the risk reversal strikes is sometimes spot delta and sometimes forward delta, depending on market convention. Most often, short-dated G10 risk reversals are quoted using spot delta strikes while long-dated G10 and emerging market risk reversals are quoted using forward delta strikes. Whichever delta convention is used to generate the strikes will also be used to delta hedge the transaction if dealt.

When trading a risk reversal, particularly if it is long-dated, it is important to pay attention to exactly which strikes are being transacted. Strikes traded within a risk reversal are the *outright* strikes—the same strikes as if same-tenor and same-delta call or put vanillas are traded in isolation.

If a currency pair had a completely flat volatility smile, the risk reversal strikes would be positioned approximately symmetrically around the ATM strike in log-space. Therefore, the topside strike will be further away from the ATM than the downside strike in regular spot space. At short maturities this effect is small but at longer maturities the impact can be significant.

Additionally, if a currency pair has a large forward drift, at longer maturities the ATM strike will be far from current spot and it is possible that, for example, if the forward drift is large positive, the 35d put strike is positioned close to current spot.

Remembering that out-of-the-money strikes are always traded within a risk reversal contract, the volatility smile also impacts risk reversal strike placement:

- If the implied volatility for a given delta is higher on the smile, the strike moves further away from the ATM; think about the increasing chance of ending up in-the-money at higher volatility.
- If the implied volatility for a given delta is lower on the smile, the strike moves closer to the ATM; think about the decreasing chance of ending up in-the-money at lower volatility.

Finally, market conventions play an important role in risk reversal strike placement. If the premium currency is CCY1 and the ATM is a zero-delta straddle, the ATM strike is lower than the forward (see Chapter 8) and the risk reversal strikes are relatively lower also.

Exhibit 12.18 shows a typical volatility smile in AUD/JPY—a CCY1 premium pair with a large downside risk reversal.

- The AUD call strike is located on a relatively flat part of the smile, so the implied volatility from the relatively lower strike (caused by a CCY1 premium) is not too different.
- The AUD put strike is located on a steeply sloping part of the smile, so the implied volatility from the relatively lower strike (caused by a CCY1 premium) is significantly higher.

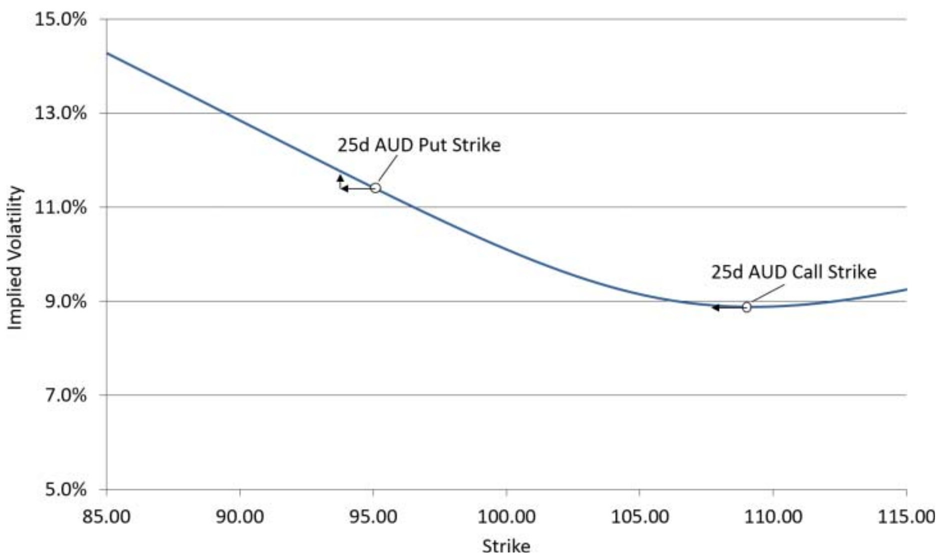


EXHIBIT 12.18 AUD/JPY volatility smile

This effect causes AUD/JPY risk reversal contracts to be valued at higher implied volatility levels and the impact gets larger for long-dated options. In practice this means that care must be taken when assessing the term-structure of long-dated risk reversals or comparing risk reversals between currency pairs with different market conventions.

What Drives the Risk Reversal in the Market?

The risk reversal contract expresses the prevailing market preference for topside versus downside optionality. This preference is a function of market positioning (see Chapter 17) but it also depends on the market's perception of expected spot moves, realized spot volatility, and implied volatility changes.

At shorter tenors the risk reversal is largely driven by expectations of spot moves and realized spot volatility. For example, if shorter tenor risk reversals go more for downside, that may imply the market expects that *if* spot goes lower, spot will be more volatile. Or it may imply that the market expects that there is an increased chance that spot will move lower.

At longer tenors the risk reversal is largely driven by expectations of spot moves and implied volatility changes. For example, if longer tenor risk reversals go more for downside, that may imply that the market expects that *if* spot goes lower, implied volatility will rise more. Or it may imply that the market expects that there is an increased chance that spot will move lower.

The risk reversal contract can be thought of as a measure of the *relative strength* of the two currencies in the currency pair. In a USD versus emerging market currency pair (e.g., USD/TRY or USD/BRL), the risk reversal will invariably be positive because there is a far higher chance of a sharp devaluation of the emerging market currency (i.e., spot jumps higher) than the USD. When spot jumps, implied volatility invariably rises and therefore a long risk reversal position with a long vega exposure to the topside will make money.

This idea of the relative strength of currencies also links to the interest rate differential (i.e., *carry*; see Chapter 17). Usually, the larger the interest rate differential in a given currency pair, the larger the risk reversal. This relationship becomes more important at longer tenors. Buying the higher-yielding currency and selling the low-yielding currency to benefit from the carry and then buying the risk reversal for protection from a blowup is a classic trading strategy in emerging market currency pairs.

Historically, interest rate differentials and risk reversals were highly correlated since low interest rate currencies (e.g., JPY or CHF) implied a more stable country with lower growth potential while high interest rate currencies (e.g., BRL or TRY) implied a country with higher growth potential but more political, social, or economic instability. However, since the 2008 financial crisis most G10

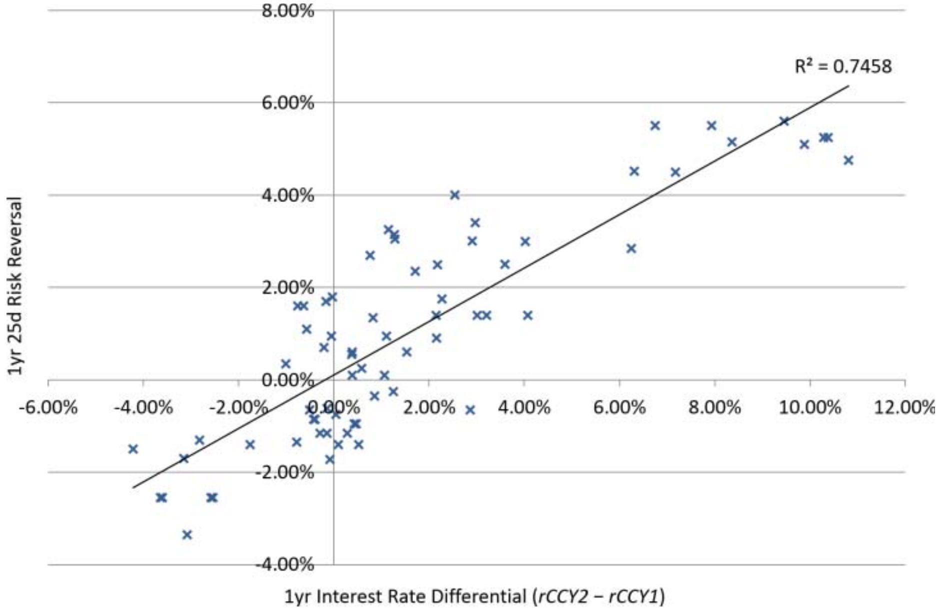


EXHIBIT 12.19 1yr interest rate differential versus 1yr 25d risk reversal scatter plot

currency pairs have low interest rates and the link between carry and risk reversals has weakened, although it remains an important factor. Exhibit 12.19 shows the relationship between 1yr interest rates and 1yr 25 delta risk reversals in 70 of the most liquid currency pairs as of October 1, 2014.

Trading the Risk Reversal

In the same way that realized volatility is often lower than implied volatility (see Chapter 17 for details), **realized skew** is often less than **implied skew** (i.e., it costs more to buy and hold the risk reversal position than can be made back from trading the spot versus implied volatility relationship). This implies that there is a risk premium associated with holding a long risk reversal position, which makes sense since the risk reversal offers protection from the most likely extreme market moves. When spot breaks out of recent trading ranges there is often a risk reversal overvaluation as the risk premium increases, particularly at longer tenors.

For risk reversals, as for all other financial instruments, traders must be careful not to fall into the trap of believing that the status quo will prevail indefinitely. Exhibit 12.20 shows a chart of USD/JPY 1yr 25 delta risk reversals over ten years with trader comments at various points.

In the interbank broker market, the risk reversal is traded in terms of the volatility differential between the two strikes. After a transaction is agreed, the actual implied volatilities must be agreed. For example, two banks could agree to transact an

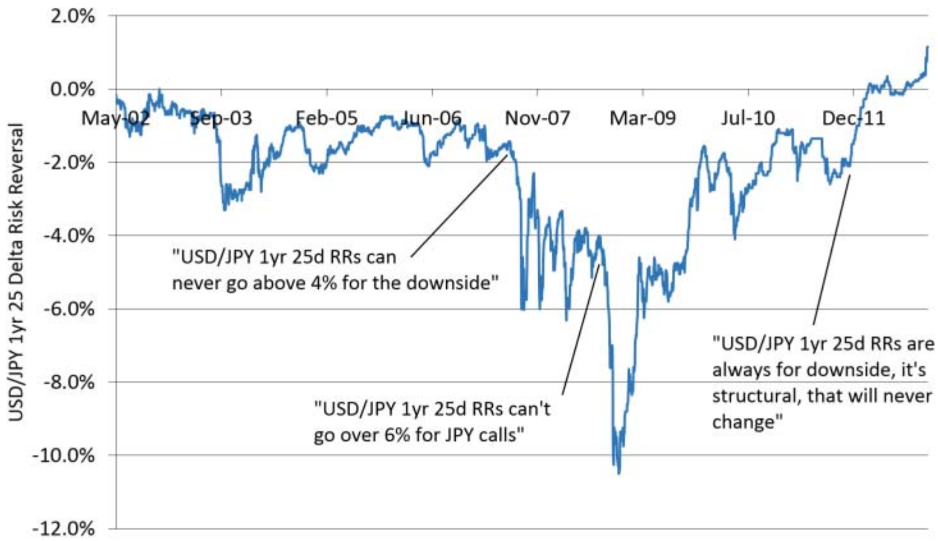


EXHIBIT 12.20 USD/JPY 1yr 25d risk reversals from May 2002 to November 2012

AUD/USD 1yr 25d RR at 2.6% AUD puts over, but then the risk reversal buying bank wants 11.6% on the AUD put (and therefore 9.0% on the AUD call) while the RR selling bank wants 11.4% on the AUD put. This disagreement occurs for two reasons:

1. *Strike placement*. The call and put strikes are backed out of an inverted Black-Scholes formula. For the risk reversal *buying* bank, the higher the agreed implied volatilities, the further away from the ATM both risk reversal strikes are positioned. The higher volatility side of the volatility smile is steeper than the lower volatility side. Therefore, by pushing the strikes further away from the ATM, the risk reversal buying bank gets a long strike which is marked even higher (i.e., better) on the volatility smile.
2. *Adapted vega* (explained in Chapter 14). Buying a risk reversal results in a short adapted vega exposure. Therefore, the risk reversal buying bank wants the highest implied volatilities possible so they get short adapted vega from the highest possible level. Likewise, the risk reversal selling bank wants the lowest implied volatilities possible so they get long adapted vega from the lowest possible level.

Traders must check the proposed market data and only agree to trade at correct implied volatility levels. There will be occasions where transacting the risk reversal is more important than these second-order effects but traders should always calculate the P&L difference from mid implied volatility levels so they know how much additional "spread" the transaction is costing.

Finally, traders use their short-dated risk reversal exposures to manage their delta positions. For example, in a currency pair with a risk reversal for topside, if a trader is long short-dated risk reversal and spot jumps higher, even if the trader has not seen any implied volatility prices in the market it is clear that implied volatility will be higher. Due to the long vanna $\left(\frac{\partial delta}{\partial \sigma}\right)$ exposure from the risk reversal the trader knows that their position will be longer delta. Assuming USD is CCY1, if vanna is long USD20m and implied volatility is approximately 1% higher, delta will be USD20m longer and additional delta can be sold at the higher spot. In effect, the long risk reversal position creates delta changes equivalent to being long gamma.

25d versus 10d Risk Reversal Contracts

Exhibit 12.21 shows vega profiles from 25d and 10d risk reversals.

The wider positioning of the strikes within the 10 delta risk reversal causes the vega peaks to be positioned further away from the ATM and the peak vega exposures to be larger since the vega offsets less when the strikes are further apart.

The risk reversal quotes at different deltas are linked. Investigating these relationships is useful for understanding the volatility smile. In many pairs, only 25d risk reversals are regularly quoted in the interbank broker market. The relationships between risk reversals at different deltas are called **risk reversal multipliers**. In almost all currency pairs, at lower delta the risk reversal quote increases, as shown in Exhibit 12.22.

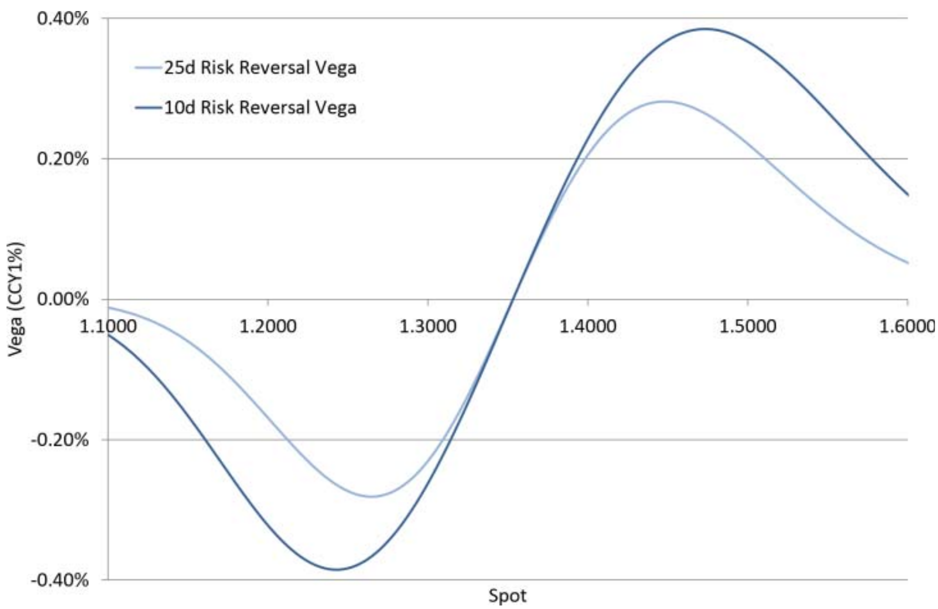


EXHIBIT 12.21 25d risk reversal vega profile versus 10d risk reversal vega profile

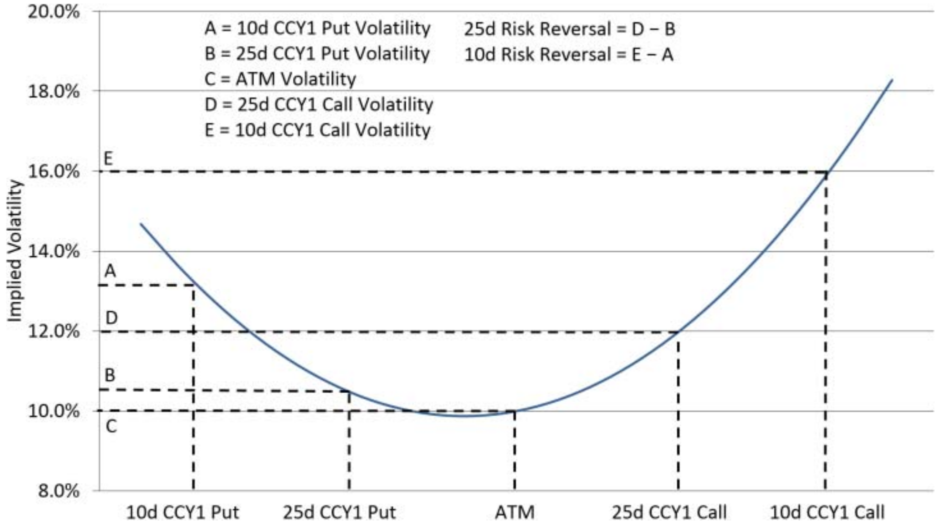


EXHIBIT 12.22 25d and 10d risk reversals on the volatility smile

Within the Malz volatility smile formula, substituting 10% put delta and 10% call delta into the formula gives:

- $\sigma_{10\%\ Delta\ Put} = \sigma_{ATM} - 0.8\ \sigma_{RR25d} + 2.56\ \sigma_{Fly25d}$
- $\sigma_{10\%\ Delta\ Call} = \sigma_{90\%\ Delta\ Put} = \sigma_{ATM} + 0.8\ \sigma_{RR25d} + 2.56\ \sigma_{Fly25d}$

Therefore:

- $\sigma_{RR10d} = \sigma_{Call10d} - \sigma_{Put10d} = 1.6\ \sigma_{RR25d}$

This 25d/10d multiplier of 1.6 is a touch lower than values typically observed in the market for liquid currency pairs where the value is usually around 1.8. Risk reversal multipliers are usually fairly stable in liquid currency pairs.

Another method for calculating risk reversal multipliers is to assume that the cost of vanna remains constant. This method is back-of-the-envelope, old-school, and circular, but it gives some intuition as to how risk reversal contracts at different deltas are linked. Exhibit 12.23 shows vega for AUD/USD 1yr outright strikes and vanna for AUD/USD 1yr long risk reversals over a range of deltas (AUD puts over hence short vanna).

The 1yr AUD/USD 25d risk reversal is –2.5%. Therefore, this risk reversal contract "costs" approximately 2.5% volatility × 0.31% vega = 0.775 AUD% more to buy in premium terms than if the risk reversal was 0%.

The 1yr AUD/USD 25d RR has –4.2% vanna while the 1yr AUD/USD 10d RR has –4.1% vanna. If the premium cost of vanna is constant, the 10d RR should cost (4.1/4.2 =) 0.975 of the 25d RR in premium terms. If the 10d RR costs (0.775% × 0.975 =) 0.755% in premium terms, that equates to a 10d risk reversal quote of

Strike	Vega (AUD%)	Vanna (AUD%)
ATM	0.40%	0.00%
35 Delta	0.36%	-2.70%
30 Delta	0.34%	-3.60%
25 Delta	0.31%	-4.20%
20 Delta	0.28%	-4.55%
15 Delta	0.23%	-4.55%
10 Delta	0.17%	-4.10%

EXHIBIT 12.23 AUD/USD 1yr outright strike vega and 1yr long risk reversal vanna

RR Strikes	RR Multiplier (Versus 25d RR)
35 Delta	0.55
30 Delta	0.78
25 Delta	1.00
20 Delta	1.20
15 Delta	1.46
10 Delta	1.78

EXHIBIT 12.24 AUD/USD 1yr risk reversal multipliers

(0.755%/0.17% vega =) –4.45%, which is (–4.45%/–2.5% =) 1.775× the 25d risk reversal.

For 1yr AUD/USD, the above methodology gives risk reversal multipliers shown in Exhibit 12.24. These multipliers are close to values often observed in the market.

Cross Risk Reversals

Given 25d risk reversals in EUR/USD and USD/JPY, how can the 25d risk reversal in EUR/JPY be calculated?

In some cases, cross risk reversals can be calculated as a fixed offset to the risk reversal in one of the major pairs. This methodology is suitable if there is clearly a dominant currency within the pair that will contribute the vast majority of the skew. For example, EUR/HKD risk reversals can be generated off EUR/USD risk reversals since USD/HKD is a managed currency pair with low implied volatility.

A **copula** approach, in its most simple form, takes probability densities (see Chapter 13 for more information on probability density functions) generated by the major volatility smiles at a given tenor and builds a cross volatility smile assuming a static correlation between the major pair spots. In some cases this works well but in others it fails to produce a smile close to the market. A more effective variation uses the copula to generate *changes* in the cross risk reversal rather than generating the absolute level.

Another possible approach is to look at a system of risk reversals in many currencies against each other, imply a relative "strength" parameter for each currency, and then use this to generate cross risk reversals (taking the level of the ATM into account each time). Alternatively, the realized historic spot versus volatility relationship can be used to imply a cross risk reversal using a regression-style calculation.

Cross risk reversals are a tricky area and this section barely scratches the surface. Different banks take different approaches but flexibility is important; finding a single method that works for all crosses all the time is very difficult.

Butterfly Contract

The Black-Scholes formula assumes constant volatility. In practice, volatility (both implied and realized) itself is volatile. This causes wing vanilla options to be often priced at higher implied volatility than the ATM due to the volga (second derivative of implied volatility) they contain.

The FX derivatives market expresses the height of the wings of the volatility smile via the butterfly contract, quoted as the average of the same-delta call strike implied volatility and put strike implied volatility less the ATM volatility at a given tenor.

Strike placement is very important within the butterfly contract. The butterfly contract that is quoted and traded in the interbank broker market is called the **broker fly**. The strikes within the 25d broker fly are *not* the outright 25d call and 25d put strikes. Therefore, the strikes within the same-tenor and same-delta risk reversal and broker fly are different. The butterfly constructed using the outright 25d call and 25d put strikes is sometimes called a **strike fly** but this instrument is rarely traded in practice.

The broker fly is a messy concept, but put as simply as possible:

> *The broker fly is the implied volatility at which the premium of the call and the put generated and priced using the ATM + broker fly volatility is equal to the premium of the same strikes on the full volatility smile.*

This statement can be broken down into two parts:

Part 1: The ATM + broker fly volatility is used to generate the call and put strikes within the broker fly. Crucially, this means the risk reversal/skew is **not** taken into account within broker fly strike placement.

Example: In EUR/USD, spot is 1.2600, the 1yr forward is 1.2660, the 1yr ATM strike is 1.2740, and the 1yr ATM implied volatility is 11.5%:

- Outright 25d call strike = 1.3690 (10.80% volatility)
- Outright 25d put strike = 1.1695 (13.15% volatility)

If the 25d broker fly volatility is +0.40%:

- Broker fly 25d call strike = 1.3810. The 25d broker fly call strike is further away from the ATM strike than the outright 25d call strike since it is generated using

11.9% volatility (11.5% ATM volatility + 0.4% broker fly volatility) rather than 10.8% volatility on the smile.

- Broker fly 25d put strike = 1.1775. The 25d broker fly put strike is closer to the ATM strike than the outright 25d put strike since it is generated using 11.9% volatility rather than 13.15% volatility on the smile.

In general, on the higher side of the volatility smile, the strike within the broker fly will be closer to the ATM than the same delta outright strike since broker fly strike volatility will be lower than the smile strike volatility. While on the lower side of the volatility smile, the strike within the broker fly will be further away from the ATM than the same delta outright strike since broker fly strike volatility will be higher than the smile strike volatility. Exhibit 12.25 gives a diagram showing broker fly strike placement.

Particularly at long-dated maturities or in high skew currency pairs the difference between outright strikes and broker fly strikes can be large.

Example: In AUD/JPY, spot is 80.25, the 5yr forward is 63.85, the 5yr ATM strike is 58.20, and the 5yr ATM implied volatility is 19.25%:

- Outright 25d put strike = 46.05 (22.55% volatility)
- Outright 25d call strike = 79.10 (14.15% volatility)
- Broker fly 25d put strike = 49.60 (30% forward delta on the smile)
- Broker fly 25d call strike = 82.80 (20% forward delta on the smile)

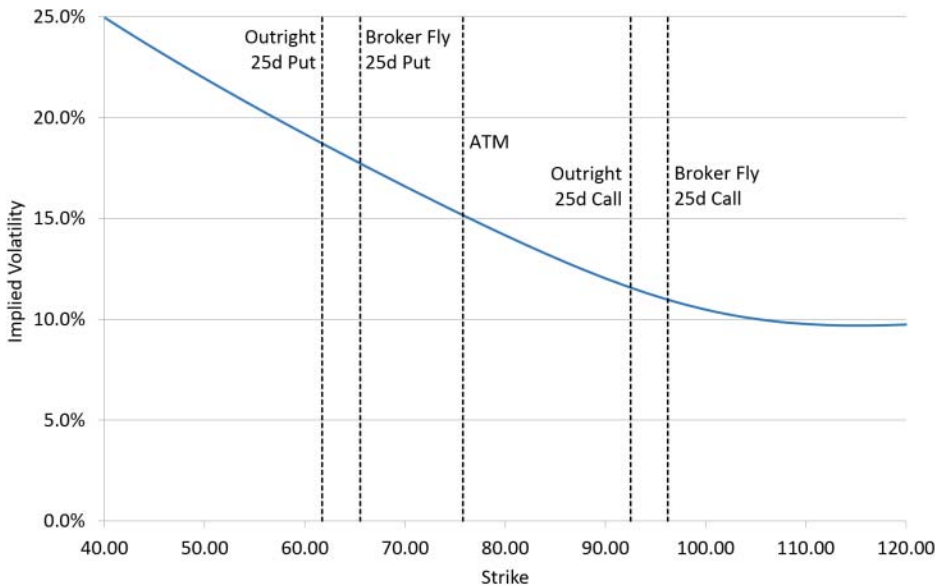


EXHIBIT 12.25 Broker fly strike placement

Part 2: The combined premium of the call and put options priced using the ATM + broker fly volatility is equal to their combined premium priced using the full volatility smile. Exhibit 12.26 shows this in a pricing tool.

The broker fly strikes are generated in leg 1 and inputted in legs 2 and 3. Look at the premiums: The cost of the strangle (the call plus the put) at ATM + broker fly volatility is the same as the cost of the call using the full smile plus the cost of put on the full smile (8.49% = 5.65% + 2.84%).

Some long-dated AUD/JPY volatility surface instruments are shown in Exhibit 12.27. The ATM and RR are both rising at longer tenors but the 25d broker flies are going more negative. This is a counterintuitive result because a higher ATM and larger skew is intuitively linked with higher wings within the volatility smile. In fact, the broker fly goes more negative because the *broker fly contract contains vanna exposure* when valued on the smile caused by the strike positioning.

In a CCY1 premium pair a long broker fly contains long vanna exposure because the CCY1 premium pulls all strikes lower. This makes the long topside strike

Contract Details	Leg 1	Leg 2	Leg 3
Currency Pair	EUR/USD	EUR/USD	EUR/USD
Horizon	Mon 03-Sep-12	Mon 03-Sep-12	Mon 03-Sep-12
Spot Date	Wed 05-Sep-12	Wed 05-Sep-12	Wed 05-Sep-12
Strategy	25d Strangle	Vanilla	Vanilla
Call/Put	N/A	EUR Put/USD Call	EUR Call/USD Put
Maturity	5Y	5Y	5Y
Expiry Date	Fri 01-Sep-17	Fri 01-Sep-17	Fri 01-Sep-17
Delivery Date	Tue 05-Sep-17	Tue 05-Sep-17	Tue 05-Sep-17
Cut	NY	NY	NY
Strikes	1.1060/1.6240	1.1060	1.6240
Notional Currency	EUR	EUR	EUR
Market Data			
Spot	1.2600	1.2600	1.2600
Swap Points	270	270	270
Forward	1.2870	1.2870	1.2870
Deposit (EUR)	0.35%	0.35%	0.35%
Deposit (USD)	0.80%	0.80%	0.80%
ATM Volatility	12.40%	12.40%	12.40%
Pricing Volatility	**12.75%**	**13.85%**	**11.55%**
Outputs			
Output Currency	EUR	EUR	EUR
Premium	**8.49%**	**5.65%**	**2.84%**

EXHIBIT 12.26 Pricing tool showing broker fly premiums

EXHIBIT 12.27 AUD/JPY Volatility Surface Instruments

Tenor	ATM	25d RR	25d Fly
1yr	15.3%	−5.7%	+0.1%
2yr	16.6%	−7.2%	**−0.5%**
3yr	17.0%	−7.8%	**−1.0%**
4yr	18.3%	−8.1%	**−1.5%**
5yr	19.25%	−8.4%	**−2.0%**

relatively closer and the long downside strike relatively further away, resulting in long vanna.

In a CCY2 premium pair, if the risk reversal is for topside, a long broker fly contains short vanna exposure, whereas if the risk reversal is for downside, a long broker fly contains long vanna exposure.

These vanna exposures significantly impact the volatility price of the broker fly contract in CCY1 premium currency pairs. If the risk reversal is for topside, the broker fly quote will be pulled higher due to the long vanna (long risk reversal exposure). If the risk reversal is for downside, the broker fly quote will be pulled lower due to the long vanna (short risk reversal exposure), as in the above AUD/JPY case.

What Drives the Butterfly in the Market?

The butterfly contract expresses the prevailing market preference for wing optionality compared with ATM optionality. This preference is a function of market positioning (see Chapter 17) but it also depends on the market's perception of expected realized and implied volatility changes.

At shorter tenors the butterfly reflects the prevailing market preference for wing gamma (i.e., gamma away from current spot) while at longer tenors the butterfly is largely driven by market expectations of how volatile implied volatility will be, plus it also reflects the prevailing market preference for wing vega.

In emerging market currency pairs, traders buy the butterfly contract as protection from sharp moves in spot and implied volatility. The cost/benefit of holding a long butterfly position is compared with the cost/benefit of holding a long risk reversal position or holding a long gamma position.

Trading the Butterfly

In the interbank market, butterfly contracts are not traded as frequently as risk reversal contracts. Butterfly contracts in a given currency pair might not trade for a few days at a time. However, traders update the wing parameters within their volatility surface more frequently in order to match implied volatility prices or trading levels on specific contracts.

25d versus 10d Butterfly Contracts

Exhibits 12.28 and 12.29 give the vega and volga profiles for 25d and 10d butterflies.

As shown in Exhibit 12.28, both 10d and 25d butterfly contracts give sharp changes in vega away from current spot. The 10d flies have over double the peak vega in the wings versus the 25d flies for *equal wing notional*, plus a wider distribution. The 25d butterflies can therefore be traded in large size without significantly impacting the trading position. Within 25d broker butterflies in CCY1 premium pairs the strikes can be positioned so closely together that they can generate large localized vanna exposures. Believe me, I found this out the hard way.

Exhibit 12.29 shows how the long volga at current spot goes flat and then short as spot moves away from the current level in the wings and how 10d flies give a wider (better) volga distribution than 25d flies. Within a butterfly contract, the maximum volga exposure occurs at current spot. Therefore, if the aim of the trader is to get longer volga at higher or lower spot levels, the butterfly (particularly the 25d butterfly) is not necessarily the best contract to trade.

Finally, it is worth noting that the broker fly strike placement prevents the existence of stable 25d/10d butterfly multiples in most currency pairs.

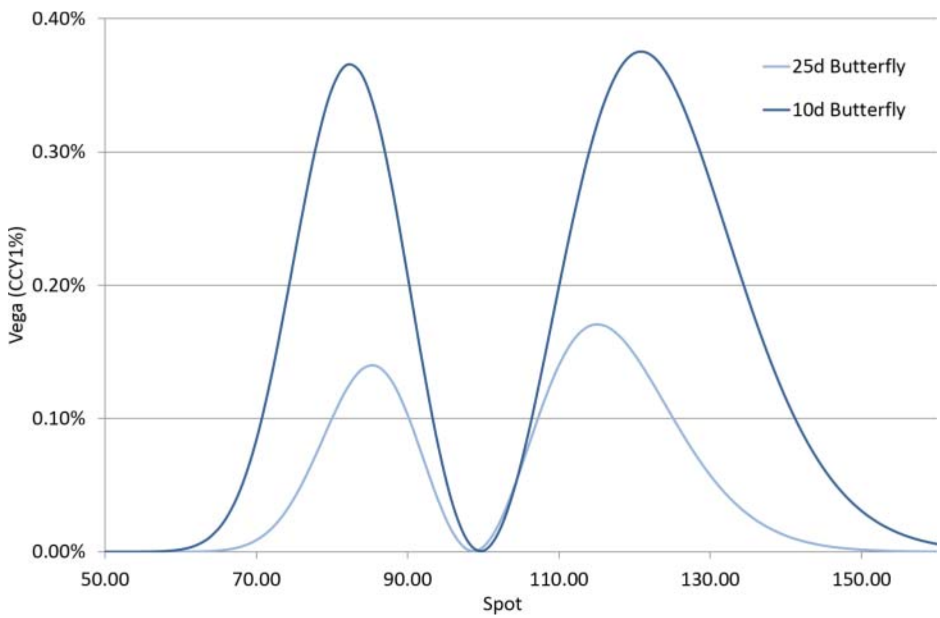


EXHIBIT 12.28 25d butterfly vega versus 10d butterfly vega profiles

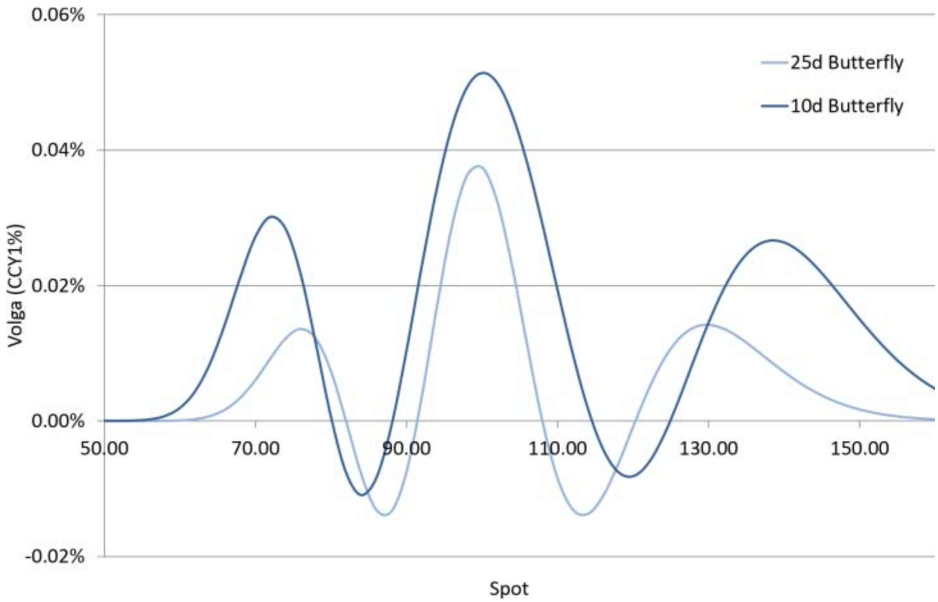


EXHIBIT 12.29 25d butterfly volga versus 10d butterfly volga profiles

Volatility Smile Risk Management

When trading an FX derivatives position it is important to understand how the vega exposure will be impacted as spot and implied volatility changes, and also the exposures to the smile instruments themselves. The smile position is therefore monitored using two sets of exposures:

1. Vanna $\left(\frac{\partial vega}{\partial spot}\right)$ and volga $\left(\frac{\partial vega}{\partial \sigma}\right)$ explain how the vega position changes as spot and implied volatility changes.
2. Rega $\left(\frac{\partial P}{\partial RR}\right)$ and sega $\left(\frac{\partial P}{\partial Fly}\right)$ explain how P&L changes as the risk reversal and butterfly prices change. Both rega and sega are quoted to whichever delta contracts are used to build the volatility surface. For example, if only 25 delta risk reversals are used to build the volatility smile, only 25 delta rega is meaningful. That rega represents the P&L generated from revaluing all contracts in the position using a volatility surface with changed 25 delta risk reversal contracts.

Vanna is usually quoted as change in vega for a change in spot, or, recalling the dual interpretation of vanna, the change in delta for a 1% change in ATM implied volatility. Therefore, if a trading position is long USD1m vanna, if spot moves 1% higher, vega will get longer by USD10k. Or, if ATM implied volatility moves 1% higher (i.e., from 8% to 9%), delta will get longer by USD1m.

Volga is usually quoted as a change in vega for a 1% move in ATM implied volatility. Therefore, if a trading position is short USD250k volga, if implied volatility rises by 0.1%, vega will get shorter by USD25k. As mentioned, volga is a second derivative like gamma. A long volga exposure therefore means that vega can be sold when implied volatility rises and bought when implied volatility falls.

Rega is usually quoted as the sensitivity to a 1% change in the volatility price of the risk reversal instrument. Therefore, if a trading position is long USD150k of 25 delta rega, if the 25d risk reversal moves from +0.8% to +1.0%, a P&L change of +USD30k will be generated.

Sega is usually quoted as the sensitivity to a 1% change in the volatility price of the butterfly instrument. Therefore, if a trading position is long USD100k of 25 delta sega, if the 25d butterfly moves lower by 0.1%, a P&L change of –USD10k will be generated.

Like other exposures, within a trading position; vanna, volga, rega, and sega will not be static; they will change as spot or ATM implied volatility moves, so all have their own higher-order derivatives (e.g., $\frac{\partial vanna}{\partial spot}$ or $\frac{\partial rega}{\partial \sigma}$, etc.). However, when analyzing derivatives trading positions it is often better to view, for example, vanna exposures within a spot ladder or implied volatility ladder rather than considering higher-order sensitivities at current spot only.

For reference, the rega on a risk reversal is approximately the average of the two absolute strike vegas while the sega on a butterfly is approximately the sum of the two wing strike vega exposures. In addition, a 25d topside strike in isolation will have a 25d rega approximately equal to *half* of its vega since, for example, a +0.1% change in the risk reversal will roughly move the implied volatility for the 25d call strike up by 0.05% and the implied volatility for the 25d put strike down by 0.05%.

The quotation conventions used for these smile exposures will differ from trading desk to trading desk. Plus, although rega and sega here are specifically the sensitivities to the market risk reversal and butterfly instruments, they can more generally be thought of as the sensitivities to the parameters that control the skew and wings within the volatility surface construction.

Risk reversal contracts have vanna and rega exposures while butterfly contracts mainly have volga and sega exposures. It is therefore natural to assume that trading positions that have vanna exposures also have corresponding rega exposures and trading positions that have long volga exposures also have long sega exposures. However, when trading a portfolio of vanilla options in high-skew pairs or when trading exotic contracts the links between vanna and rega and between volga and sega can break down. For example, a trading position might be flat volga but long sega. Therefore, traders must monitor all these exposures within their risk management.

Volatility Smile Construction Methods

The Malz volatility smile formula shows how it is possible to construct a volatility smile directly from the market instruments but in practice the process is far more complicated. As with the ATM curve, the volatility smile can be either an input or an output.

Some trading desks express the volatility smile using the market instruments directly in a process that must adjust for market conventions and strike placement issues. Within this approach, since only 25 and 10 delta market instruments are liquid, if market instruments are used to build the volatility surface, volatilities are defined only between 10 delta puts on the downside and 10 delta calls on the topside. Implied volatilities beyond the 10 delta strikes must either be controlled using extrapolation or generated automatically using a model like Stochastic Volatility Inspired (SVI)-see Gatheral's book in Further Reading for more information.

Other trading desks use models such as Hagan, Kumar, Lesniewski, and Woodward's SABR model, with traders updating the parameters of the model such that the market instruments output by the model at market tenors match the market.

In addition, the volatility smile must be interpolated between tenors. Trading desks develop their own methods for this, interpolating in e.g., delta terms, strike terms, or model parameter terms.

Other asset classes express volatility surfaces in different ways. In interest rate derivatives, the SABR model has become the market standard while in equity derivatives, implied volatility is quoted at different percentage distances from the current stock price. It is important to understand that there is nothing about the FX derivatives market that makes the ATM, risk reversal, and butterfly approach the only possible way of representing the volatility smile. It is a market convention that has developed and become the standard over time, but other parameterizations would be equally valid.

Constructing a Volatility Smile in Excel

Constructing a volatility smile using the Malz smile model builds understanding of the volatility surface market instruments. The Black-Scholes framework can then be used to calculate strikes for different deltas to show how the market instruments impact strike placement within the volatility smile.

Task A: Set Up the Malz Smile Model

Recall the Malz formula for implied volatility at a given (positive) delta put from Chapter 12:

$$\sigma_{X\ Delta\ Put} = \sigma_{ATM} + 2\ \sigma_{RR25d}.(X - 50\%) + 16\ \sigma_{Fly25d}.(X - 50\%)^2$$

This formula can be coded up in Excel:

Smile Inputs

ATM	**10.0%**	←*Named:* ***ATM***
25d RR	**2.0%**	←*Named:* ***RR***
25d Fly	**3.0%**	←*Named:* ***Fly***

Put Delta	**75%**	←*Named:* ***PutDelta***
Call Delta	25%	=1-PutDelta
Implied Vol	**14.0%**	=ATM+2*RR*(PutDelta-50%)+16*Fly*(PutDelta-50%)^2

Check that $\sigma_{50\%\ Delta\ Put} = \sigma_{ATM}$ and the 25% put delta and 25% call delta (75% put delta) implied volatility matches up with the standard approximations:

- $\sigma_{Call25d} = \sigma_{ATM} + \sigma_{Fly25d} + \frac{1}{2}\sigma_{RR25d}$
- $\sigma_{Put25d} = \sigma_{ATM} + \sigma_{Fly25d} - \frac{1}{2}\sigma_{RR25d}$

Task B: Plot Implied Volatility versus Delta and Investigate Parameters

The function output can be extended to generate a full volatility smile from 0% to 100% delta:

Smile Inputs

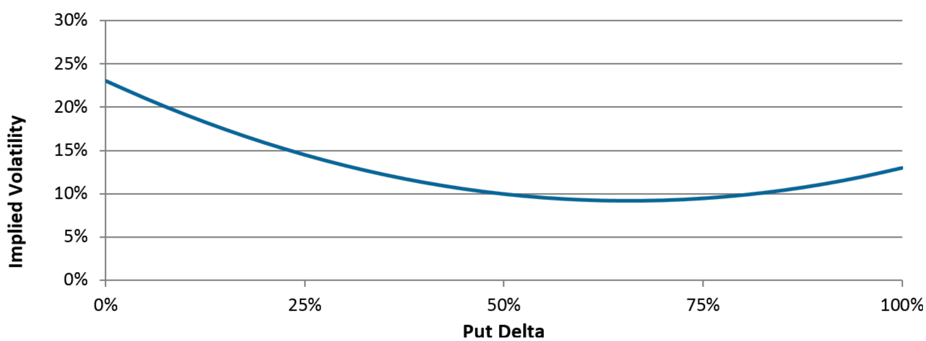
ATM	**10.0%**
25d RR	**2.0%**
25d Fly	**3.0%**

Put Delta	**0%**	**5%**	**10%**	**15%**	**20%**	**25%**	**30%**	**35%**	**40%**
Call Delta	100%	95%	90%	85%	80%	75%	70%	65%	60%
Implied Vol	20.0%	17.9%	16.1%	14.5%	13.1%	12.0%	11.1%	10.5%	10.1%

The volatility smile can then be plotted:

Smile Inputs

ATM	**10.0%**
25d RR	**-5.0%**
25d Fly	**2.0%**



Check that the volatility smile updates as expected when the ATM, risk reversal, and butterfly prices change. This becomes easier if the volatility smile chart is placed next to the inputs on the same Excel sheet, and the low and high values of the implied volatility axis in the chart are fixed rather than automatically rescaling.

Task C: Use Black-Scholes to Get Strike from Delta

The Black-Scholes framework can be used to get the equivalent strike for a given delta. Recall that:

$$\Delta_{call} = \frac{\partial P_{call}}{\partial S} = e^{-rCCY1.T} N(d_1)$$

$$\Delta_{put} = \frac{\partial P_{put}}{\partial S} = e^{-rCCY1.T} [N(d_1) - 1]$$

where S is spot, K is strike, $rCCY1$ and $rCCY2$ are continuously compounded interest rates in CCY1 and CCY2, T is time to expiry (in years), σ is implied volatility, $N(X)$ is the cumulative normal distribution function, and $d_1 = \frac{\ln\left(\frac{S}{K}\right)+\left(rCCY2-rCCY1+\frac{\sigma^2}{2}\right).T}{\sigma\sqrt{T}}$.

The formula for Δ_{put} can be inverted to get the strike from the put delta:

$$K = \frac{S}{e^{N^{-1}(e^{rCCY1.T}\Delta_{put}+1).\sigma\sqrt{T}-\left(rCCY2-rCCY1+\frac{\sigma^2}{2}\right).T}}$$

where $N^{-1}(X)$ is the inverse cumulative normal distribution function.

These functions can be implemented first on the Excel sheet using =NORMSDIST(X) for $N(X)$ and =NORMSINV(X) for $N^{-1}(X)$. A strike input is used to generate the put delta, which is itself then used to generate a strike output. If the strike input and output are equal as other inputs change, this confirms that the formulas are correctly implemented:

Market Data Inputs

Spot	100.00	←*Named:* ***Spot***
Strike Input	90.00	←*Named:* ***Strike***
CCY1 Interest Rate	0.0%	←*Named:* ***rCCY1***
CCY2 Interest Rate	0.0%	←*Named:* ***rCCY2***
Time to Maturity (years)	1.00	←*Named:* ***T***
Implied Volatility (σ)	10.0%	←*Named:* ***Vol***

Delta	-13.5%	=EXP(-rCCY1*T)*(NORMSDIST((LN(Spot/StrikeInput)+(rCCY2-rCCY1+0.5*Vol^2)*T)/Vol*SQRT(T))-1)
Strike Output	90.00	=Spot/EXP(NORMSINV(EXP(-rCCY1*T)*Delta+1)*Vol*SQRT(T)-(rCCY2-rCCY1+0.5*Vol^2)*T)

Note that the put delta used within Black-Scholes formulas is its true (negative) value rather than the positive quoted put delta.

Task D: Switch to VBA Functions and Plot Implied Volatility versus Strike

These functions are long and messy in the sheet but they are much neater as VBA functions. The Malz smile volatility function is simple:

```
Function MalzSmileVol(ATM As Double, RR25d As Double, Fly25d As _
 Double, PutDelta As Double) As Double

    MalzSmileVol = ATM + 2 * RR25d * (PutDelta - 0.5) + 16 * _
     Fly25d * (PutDelta - 0.5) ^ 2

End Function
```

The put delta from strike VBA function uses the cumulative normal distribution worksheet function:

```
Function PutDeltaFromStrike(S As Double, K As Double, rCCY1 As _
 Double, rCCY2 As Double, T As Double, v As Double) As Double

    Dim d1 As Double

    d1 = (Log(S / K) + (rCCY2 - rCCY1 + v ^ 2 / 2) * T) / (v * Sqr(T))

    PutDeltaFromStrike = Exp(-rCCY1 * T) * _
     (Application.WorksheetFunction.NormSDist(d1) - 1)

End Function
```

The strike from put delta VBA function accesses the inverse cumulative normal distribution worksheet function. The calculation is split into three parts to make it easier to follow and debug:

```
Function StrikeFromPutDelta(S As Double, PutDelta As Double, rCCY1 As _
 Double, rCCY2 As Double, T As Double, v As Double) As Double

    Dim part1 As Double, part2 As Double, part3 As Double

    part1 = Exp(rCCY1 * T) * PutDelta + 1
    part2 = v * Sqr(T)
    part3 = (rCCY2 - rCCY1 + 0.5 * v ^ 2) * T

    StrikeFromPutDelta = S / Exp(Application.WorksheetFunction _
     .NormSInv(part1) * part2 - part3)

End Function
```

Again, the VBA functions can be tested by placing them alongside the existing Excel functions:

Smile Inputs

ATM	**10.0%**
25d RR	**2.0%**
25d Fly	**3.0%**

	Excel	VBA
Put Delta	25%	
Call Delta	75%	
Implied Vol	12.00%	12.00%

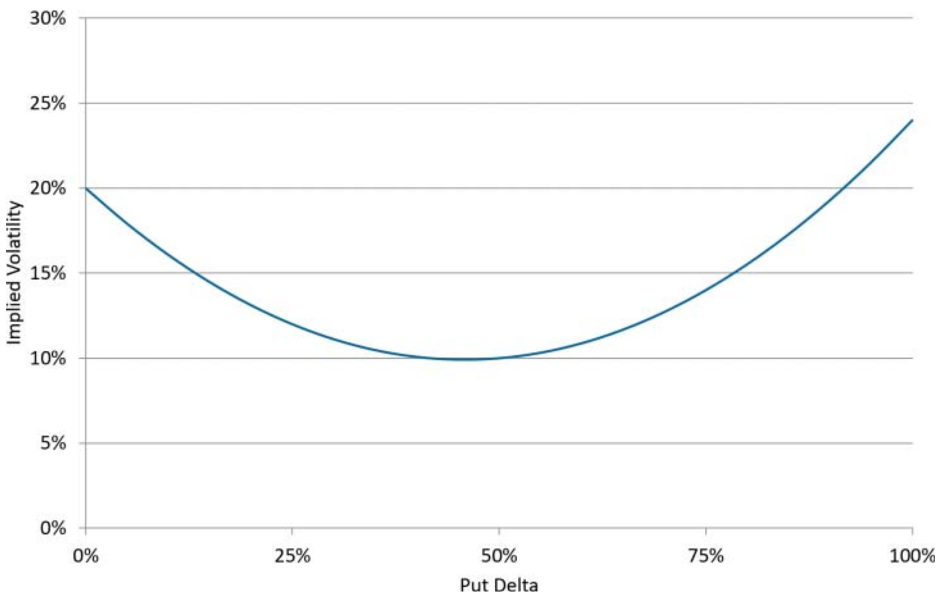
Market Data Inputs

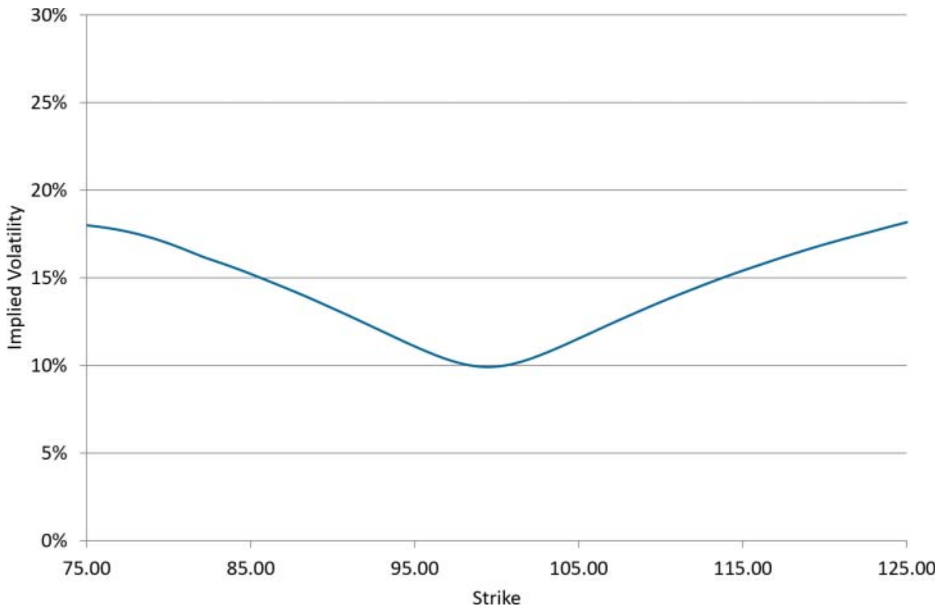
Spot	**100.00**
Strike Input	**90.00**
CCY1 Interest Rate	**0.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**1.00**
Implied Volatility (σ)	**10.0%**

	Excel	VBA
Put Delta	-13.4882%	-13.4882%
Strike Output	90.00	90.00

Once matches are confirmed, the VBA functions can be combined to plot implied volatility versus delta and implied volatility versus strike.

It is not possible to find strikes for 0 or 100 delta options, so replace 0% delta with, for example, 0.01% and replace 100% delta with, for example, 99.99%:





Task E: Investigate Volatility Smile Strike Placement

In practice, the most important strikes at a given tenor are 10 delta and 25 delta puts and calls, plus the ATM. In the Malz framework these strikes are equivalent to these put deltas: 10d, 25d, 50d, 75d, 90d. Using the functions developed, strike placement within the volatility smile can be investigated, remembering to use the negative delta to calculate the strike. With no volatility smile, the strikes for these deltas are roughly equally spaced, with relatively slightly larger differences for topside strikes due to the log-normality of the terminal spot distribution:

Market Data Inputs

Spot	**100.00**
CCY1 Interest Rate	**0.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**1.00**

Smile Inputs

ATM	**10.0%**
25d RR	**0.0%**
25d Fly	**0.0%**

Put Delta	10%	25%	50%	75%	90%
Implied Vol	10.00%	10.00%	10.00%	10.00%	10.00%
Strike	88.41	93.95	100.50	107.51	114.24
Strike Difference from ATM	-12.09	-6.56	0.00	7.01	13.74

Reducing implied volatility or time to maturity causes the terminal distribution to tighten and hence the strikes are positioned closer to the ATM:

Market Data Inputs

Spot	**100.00**
CCY1 Interest Rate	**0.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**0.25**

Smile Inputs

ATM	**10.0%**
25d RR	**0.0%**
25d Fly	**0.0%**

Put Delta	10%	25%	50%	75%	90%
Implied Vol	10.00%	10.00%	10.00%	10.00%	10.00%
Strike	93.91	96.80	100.13	103.56	106.75
Strike Difference from ATM	-6.21	-3.32	0.00	3.43	6.63

Increasing implied volatility or time to maturity causes the terminal distribution to widen and hence the strikes are positioned further from the ATM:

Market Data Inputs

Spot	**100.00**
CCY1 Interest Rate	**0.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**1.00**

Smile Inputs

ATM	**20.0%**
25d RR	**0.0%**
25d Fly	**0.0%**

Put Delta	10%	25%	50%	75%	90%
Implied Vol	20.00%	20.00%	20.00%	20.00%	20.00%
Strike	78.95	89.15	102.02	116.75	131.83
Strike Difference from ATM	-23.07	-12.87	0.00	14.73	29.81

Increasing the butterfly causes the strikes to move further from the ATM, with a larger impact at lower delta strikes due to the higher implied volatility:

Market Data Inputs

Spot	**100.00**
CCY1 Interest Rate	**0.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**1.00**

Smile Inputs

ATM	**10.0%**
25d RR	**0.0%**
25d Fly	**5.0%**

Put Delta	10%	25%	50%	75%	90%
Implied Vol	22.80%	15.00%	10.00%	15.00%	22.80%
Strike	76.63	91.40	100.50	111.90	137.46
Strike Difference from ATM	-23.87	-9.10	0.00	11.40	36.96

Changing the risk reversal moves the strikes for a given delta further away from the ATM on the high side of the volatility smile and closer to the ATM on the low side of the volatility smile:

Market Data Inputs

Spot	**100.00**
CCY1 Interest Rate	**0.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**1.00**

Smile Inputs

ATM	**10.0%**
25d RR	**5.0%**
25d Fly	**0.0%**

Put Delta	10%	25%	50%	75%	90%
Implied Vol	6.00%	7.50%	10.00%	12.50%	14.00%
Strike	92.77	95.33	100.50	109.65	120.83
Strike Difference from ATM	-7.74	-5.17	0.00	9.15	20.33

Moving CCY1 interest rates higher or CCY2 interest rates lower causes the forward to move lower and hence the whole volatility smile moves lower:

Market Data Inputs

Spot	**100.00**
CCY1 Interest Rate	**10.0%**
CCY2 Interest Rate	**0.0%**
Time to Maturity (years)	**1.00**

Smile Inputs

ATM	**10.0%**
25d RR	**0.0%**
25d Fly	**0.0%**

Put Delta	10%	25%	50%	75%	90%
Implied Vol	10.00%	10.00%	10.00%	10.00%	10.00%
Strike	80.46	85.69	92.15	100.00	117.38
Strike Difference from ATM	-11.68	-6.45	0.00	7.85	25.23

Moving CCY2 interest rates higher or CCY1 interest rates lower has the opposite effect.

Probability Density Functions

A volatility smile at a given maturity can be converted into an equivalent probability density function (pdf). The probability density function contains useful information because integrating an area under the curve gives the likelihood of spot being within the given range at maturity.

Starting with the simplest case, Exhibit 13.1 shows a 1yr volatility smile with 10% volatility for all strikes (i.e., pure Black-Scholes world).

This volatility smile generates the standard log-normal bell-shaped pdf shown in Exhibit 13.2. The method of generating pdfs from option prices is explored in Practical G.

Exhibit 13.3 shows implied volatility rising to 15% for all strikes. Increased volatility widens the distribution and the pdf extends out on both sides as shown in Exhibit 13.4.

The area under the pdf represents a probability space. Therefore the total area under the pdf is always equal to 1 and the pdf function can never go negative. If the pdf does go negative, that indicates a potentially arbitrageable volatility surface. This can manifest itself in many different ways within pricing or risk management systems, most visibly via incorrect implied volatility or unstable gamma exposures.

Exhibit 13.5 shows how the volatility smile changes when positive wings are added.

As shown in Exhibit 13.6, higher wings in the volatility smile causes the pdf to rise in the wings (often called "fat tails") and rise around the ATM but fall in between,

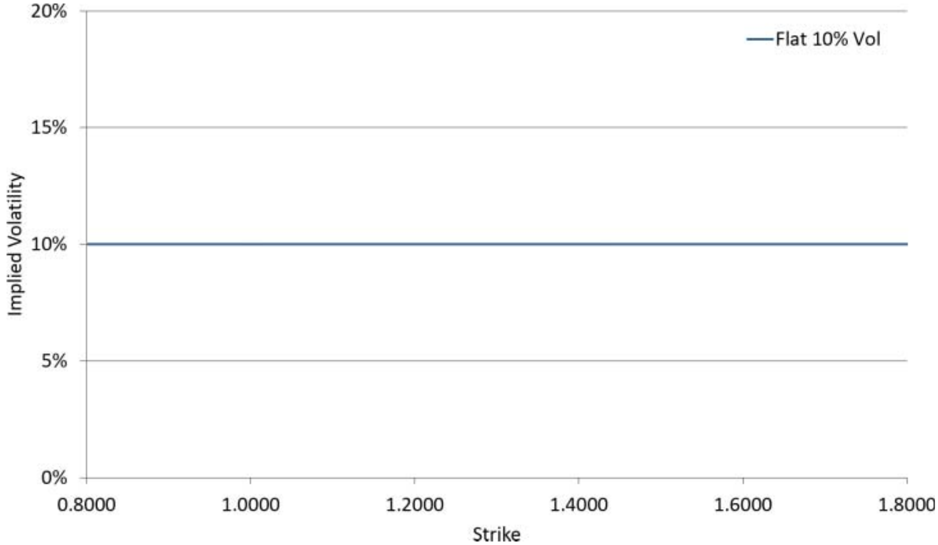


EXHIBIT 13.1 Volatility smile with flat 10% implied volatility

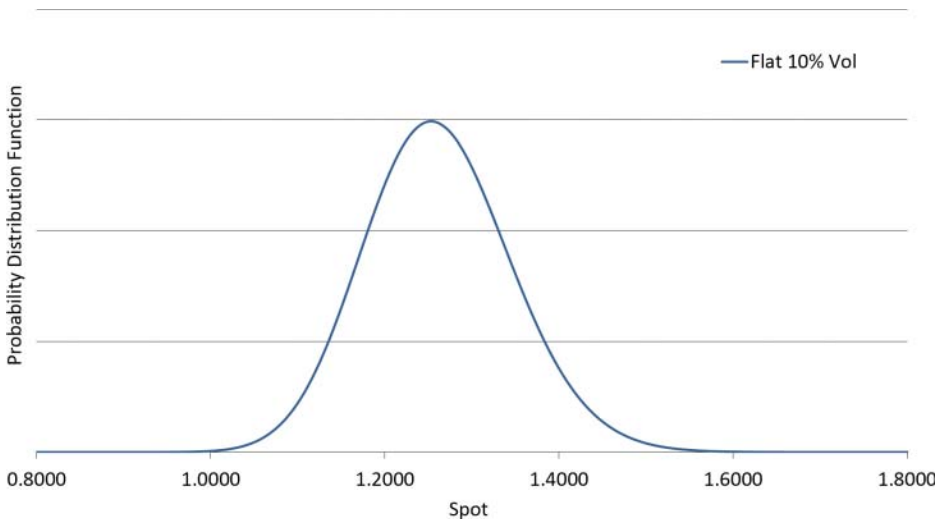


EXHIBIT 13.2 Probability density function from flat 10% volatility smile

although the total area under the pdf must remain unchanged. Distributions with this shape are known as **leptokurtotic** and they are often observed in financial markets.

Adding positive or negative skew causes the volatility smile to tilt one way or the other, as shown in Exhibit 13.7. The skew also causes the pdf to tilt. On the higher volatility side, the pdf stretches further as expected since the higher volatility causes

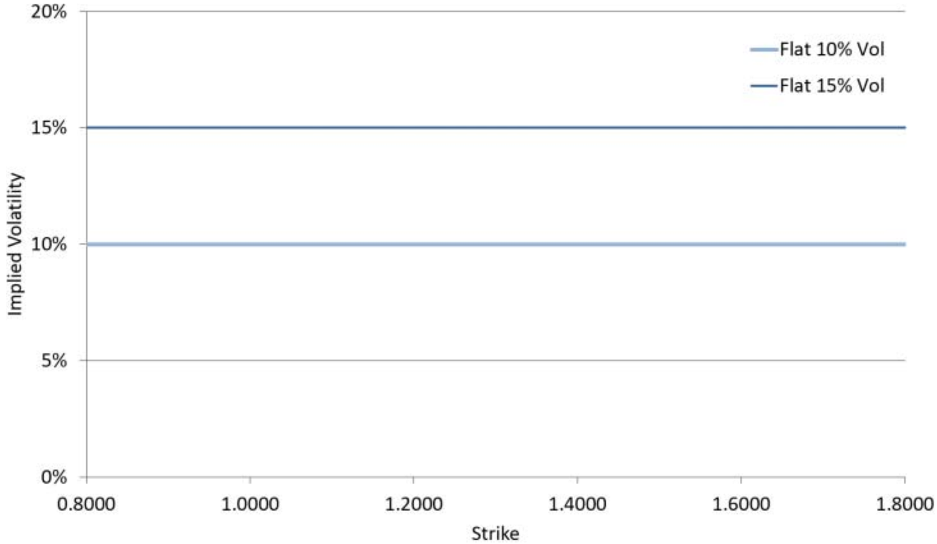


EXHIBIT 13.3 Volatility smiles with flat 10% and flat 15% implied volatility

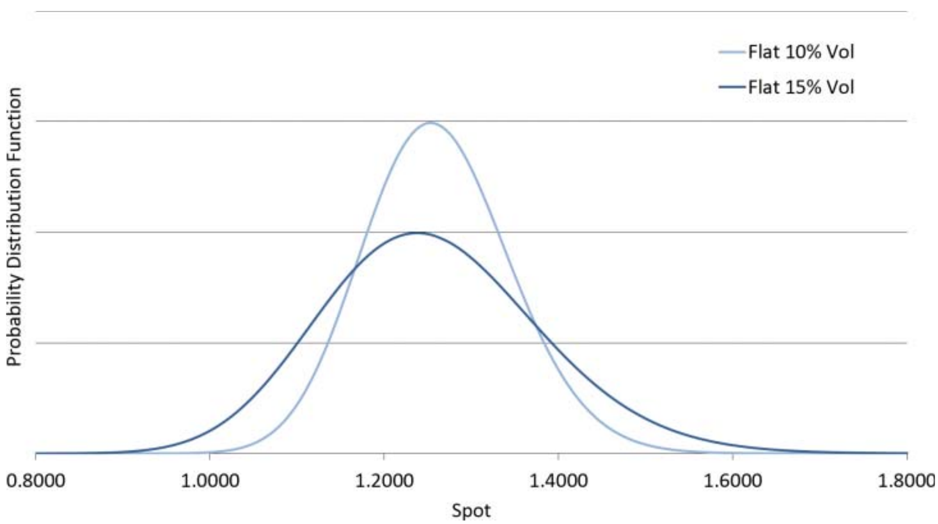


EXHIBIT 13.4 Probability density functions from flat 10% and flat 15% volatility smiles

the distribution to widen but the peak of the pdf moves the opposite direction. This occurs because the no-arbitrage condition ensures that the forward is the expected future value of spot. Therefore, if probability mass moves into the wings on one side of the smile due to higher volatility, the center of the probability mass must shift the other way. This is shown in Exhibit 13.8.

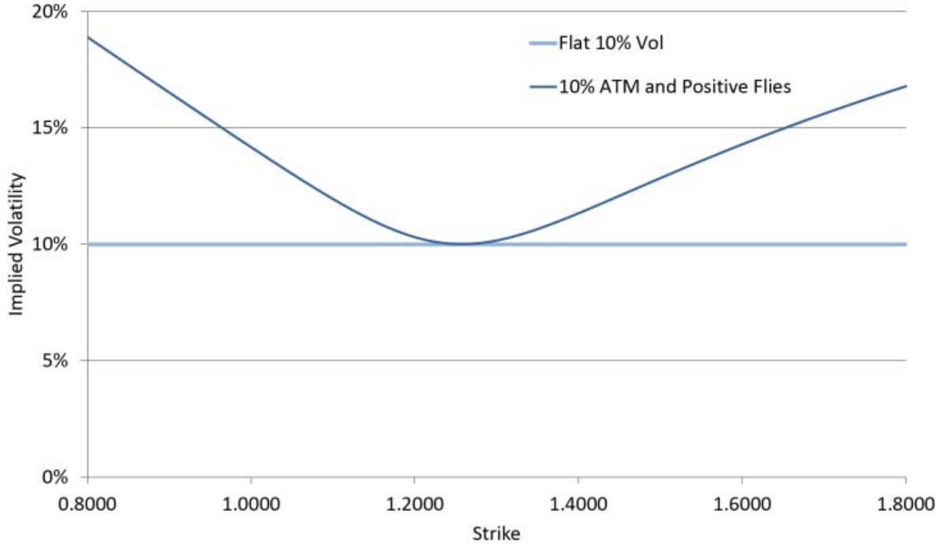


EXHIBIT 13.5 Volatility smiles with flat 10% and positive wing implied volatility

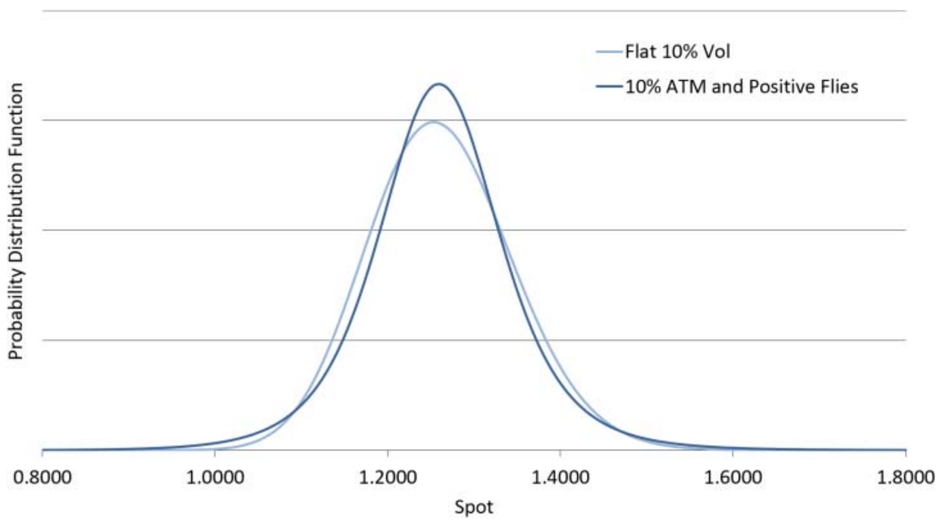


EXHIBIT 13.6 Probability density functions from flat 10% and positive wing volatility smiles

Fat-Tailed Distributions

The existence of fat-tailed distributions (i.e., excess kurtosis versus the normal distribution) in financial markets is a well-known phenomenon. Exhibit 13.9 shows the realized distribution of ten years of daily log-returns in USD/JPY spot versus the theoretical normal distribution with the same volatility.

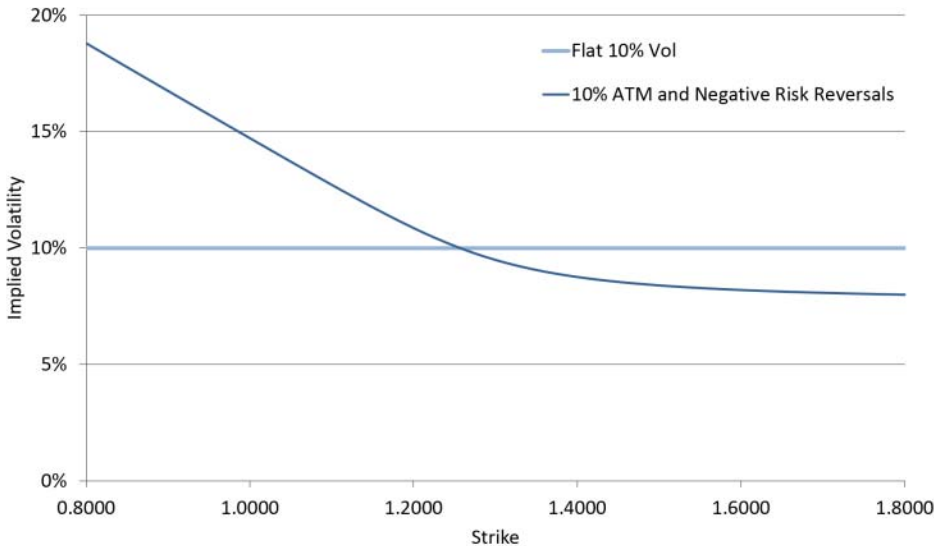


EXHIBIT 13.7 Volatility smiles with flat 10% and negative skew implied volatility

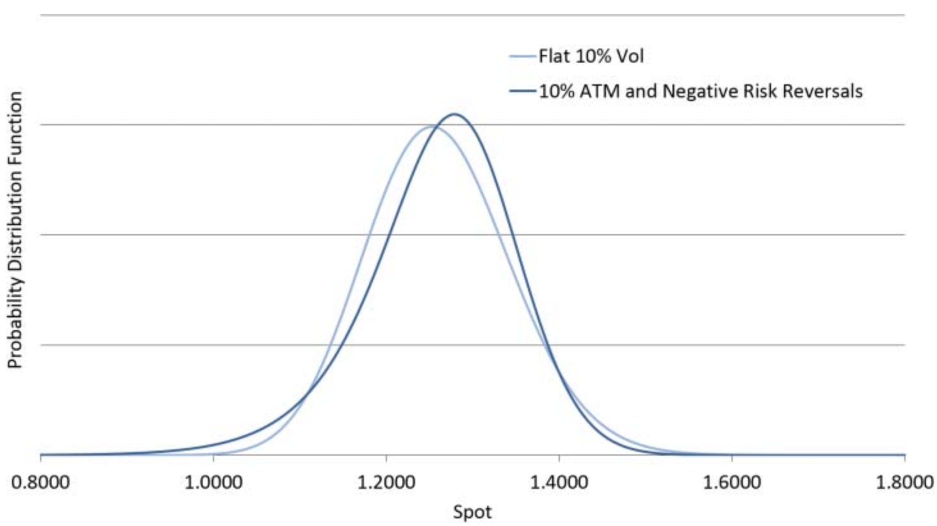


EXHIBIT 13.8 Probability density functions from flat 10% and negative skew volatility smiles

The difference around the peak is easy to see, but to see the wings more clearly a change to a log-scale for frequency is required as in Exhibit 13.10. Within this sample, the largest down moves occurred roughly ten million times more frequently than a normal distribution would suggest.

Fat tails in the spot distribution can be most easily explained by **volatility of volatility** (in freely floating currency pairs) or **spot jumps** (in emerging market

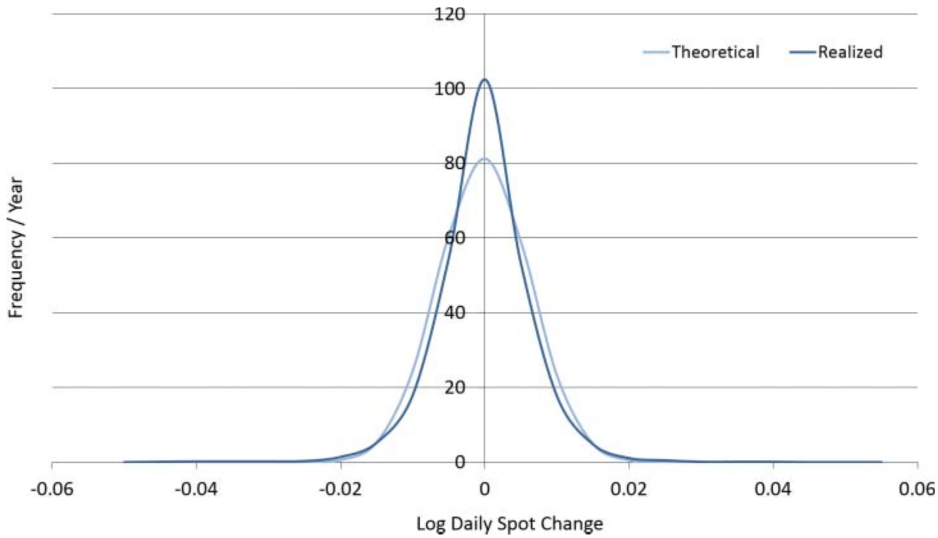


EXHIBIT 13.9 USD/JPY realized versus theoretical log daily change distribution (2003 to 2013)

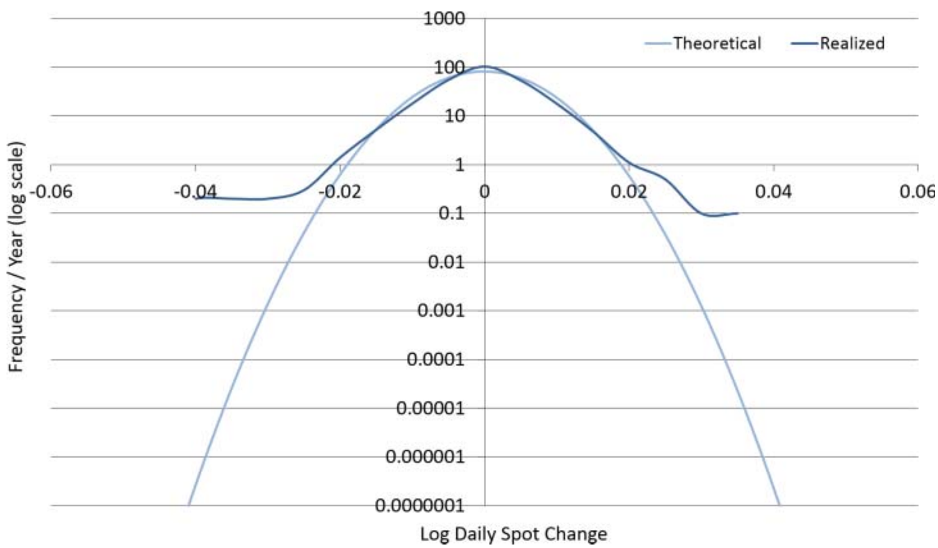


EXHIBIT 13.10 USD/JPY realized versus theoretical log daily change distribution (2003 to 2013/log scale)

or pegged/managed currency pairs). The reality is usually a combination of these two effects but for simplicity they tend to be applied in isolation within financial models (see Chapter 19).

To understand how volatility of volatility generates fatter tails within the spot distribution, consider a simple model where the spot diffusion has an equal chance of

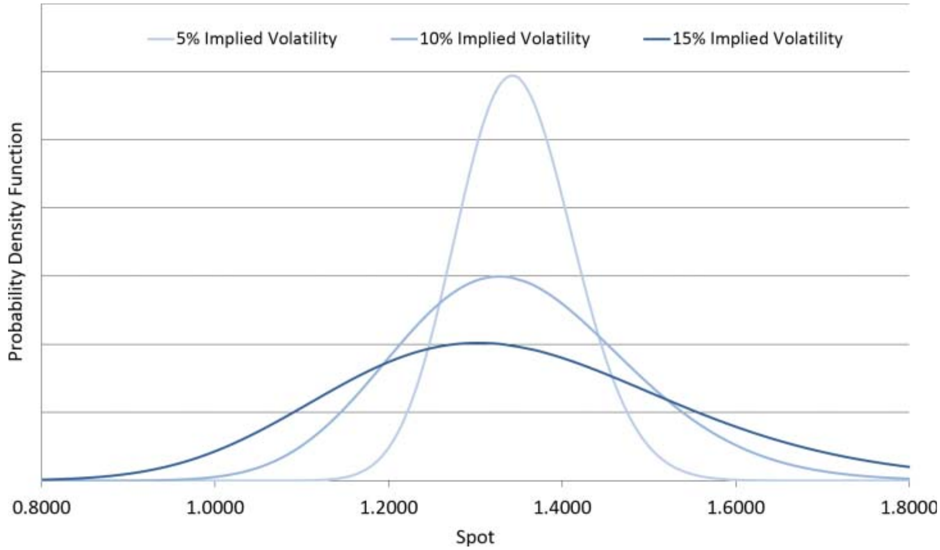


EXHIBIT 13.11 Three-state volatility model pdfs

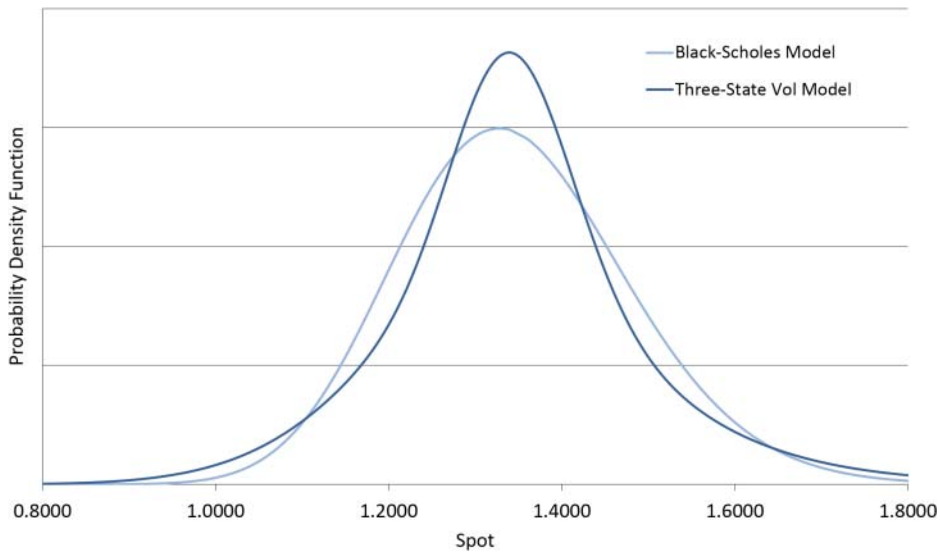


EXHIBIT 13.12 Three-state volatility model average pdf

having 5%, 10%, or 15% volatility. The three separate probability density functions are shown in Exhibit 13.11.

The probability density function of the three-state model is an equal combination of the three states. Therefore, Exhibit 13.12 shows how averaging the pdfs gives a fat-tailed pdf compared to a static 10% volatility distribution. The low-volatility state contributes the higher peak while the high-volatility state contributes the fatter tails.

The presence of spot jumps has a similar impact, pushing probability mass into the wings if a jump occurs and keeping it around current spot if a jump does not occur. As volatility of volatility increases or jumps increase in magnitude or frequency, the wings of the volatility smile move higher and the pdf gets fatter tails. See Chapter 19 for more details on FX derivatives pricing models.

Confidence Intervals

Probability density functions can also be used to generate confidence interval charts, which are an interesting way of visualizing how the volatility smile suggests spot may move in the future. For given probability intervals (e.g., 50% or 90%), the probability density function is queried for the appropriate spot levels. Exhibit 13.13 shows an example confidence interval chart in GBP/USD. The volatility surface suggests that, for example, there is a 50% chance of spot being within the 50% confidence interval bounds. Note how the bounds stretch further to the downside due to the downside skew in the GBP/USD volatility surface.

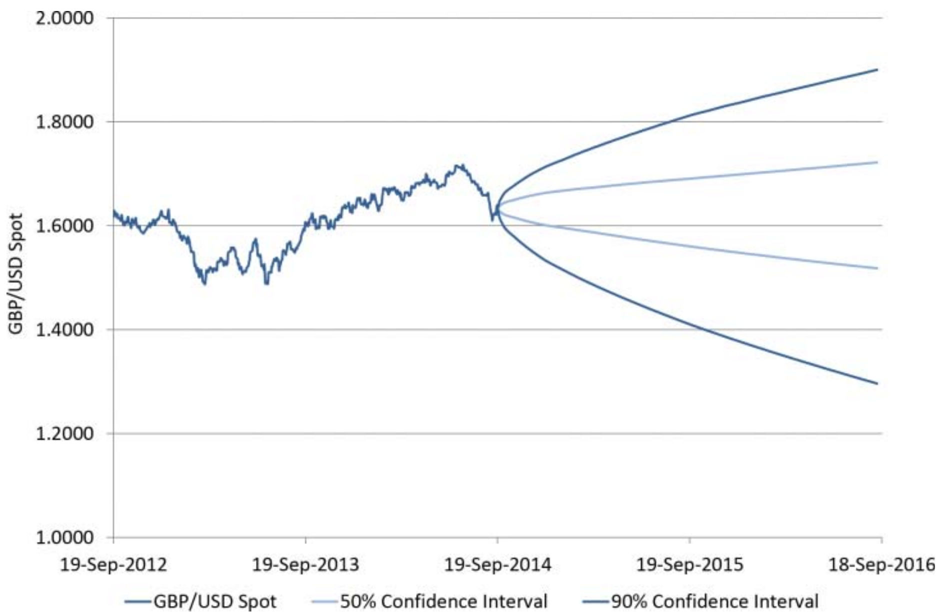


EXHIBIT 13.13 GBP/USD confidence intervals

Limitations of Volatility Smile Parameterization

Within FX derivatives trading desks, volatility smiles are expressed using a limited number of parameters. On some trading desks the five market instruments—ATM, 25d RR, 10d RR, 25d Fly, 10d Fly—are used; on others, perhaps just one ATM parameter, one wing parameter, and one skew parameter are used. In liquid currency pairs these reduced-form parameterizations work well and they allow a lot of information to be expressed in an efficient manner.

However, there are instances where these parameterizations cause problems. Exhibit 13.14 shows EUR/CHF spot from 2011 and 2012.

In mid-2011, EUR/CHF was freely floating and the EUR/CHF market instruments at June 1, 2011 were as per Exhibit 13.15.

The corresponding volatility smile and probability density function for 1yr EUR/CHF are shown in Exhibit 13.16. As expected, the higher implied volatility

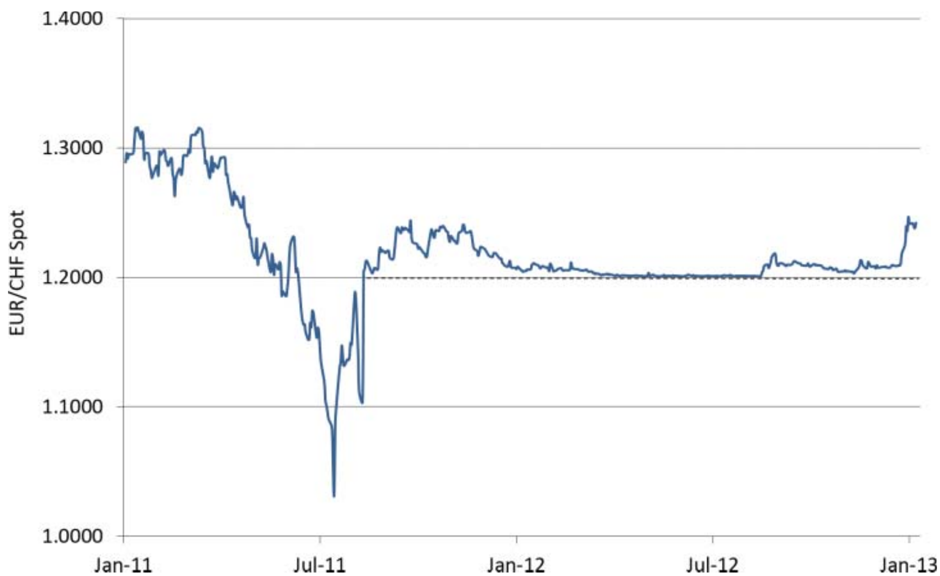


EXHIBIT 13.14 EUR/CHF spot in 2011 and 2012

EXHIBIT 13.15 **EUR/CHF 1mth and 1yr Market Instruments at June 1, 2011**

Tenor	ATM	25d RR	10d RR	25d Fly	10d Fly
1mth	11.15%	−1.75%	−2.65%	0.25%	0.70%
1yr	12.10%	−3.45%	−6.05%	0.40%	1.85%

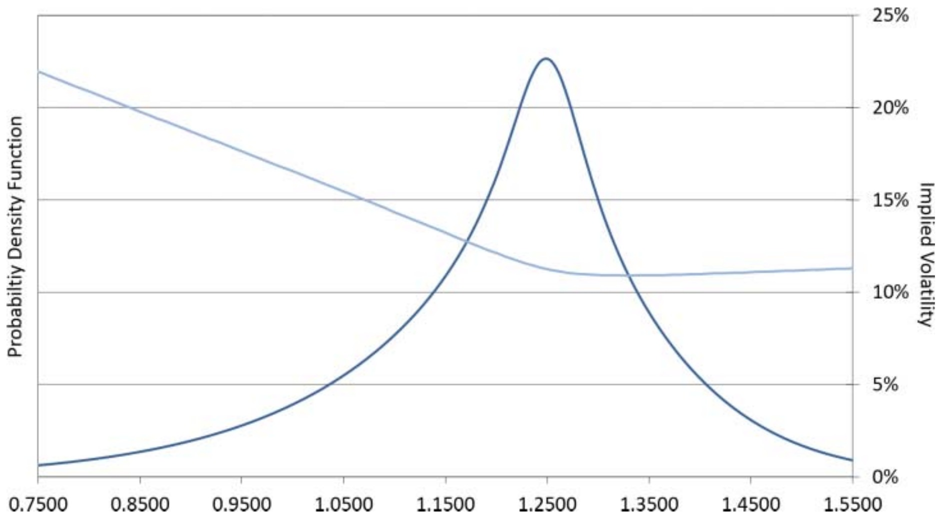


EXHIBIT 13.16 EUR/CHF 1yr implied volatility smile and pdf at June 1, 2011

for downside strikes (i.e., negative risk reversal) causes the probability density function to stretch more to the downside.

A series of interventions and announcements from late 2011 onward saw the Swiss central bank establish a floor in the spot market at 1.2000, which they actively defended (i.e., they bought EUR/CHF spot in the market in order to prevent it going below 1.2000) until early 2015.

While the floor was actively defended, what should the probability density function look like? In a vastly simplified world there are two scenarios that could potentially have played out:

1. The central bank successfully defends the 1.2000 level; spot floats freely above this level but cannot go any lower.
2. The central bank decides to change its policy, at which point spot could have a big adjustment lower.

Given these scenarios, the pdf would intuitively be expected to look like Exhibit 13.17. The most likely spot position in one year is close to, but above, the intervention level. Plus, the downside of the pdf has a longer tail, because if the intervention level was removed, there is significant risk of spot jumping lower, as happened in practice.

Exhibit 13.18 shows the market instruments one year later. The pdf derived from these quotes is overlaid onto the intuition in Exhibit 13.19.

When constructing volatility surfaces using a limited parameter set, only certain types of volatility smile can be generated. This means that only certain types of

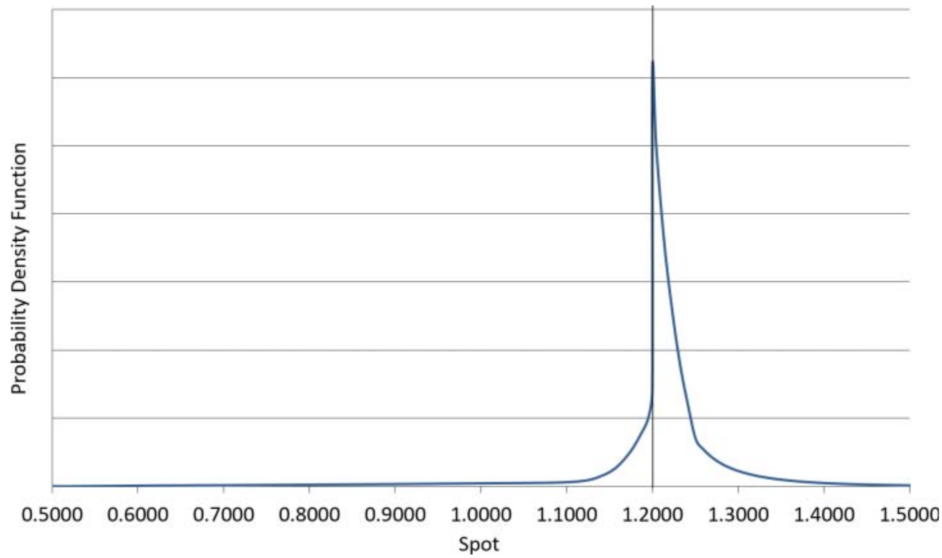


EXHIBIT 13.17 EUR/CHF 1yr intuitive pdf

EXHIBIT 13.18 **EUR/CHF 1mth and 1yr Market Instruments at June 1, 2012**

Tenor	ATM	25d RR	10d RR	25d Fly	10d Fly
1mth	2.00%	–0.70%	–1.40%	1.15%	3.75%
1yr	6.70%	–4.60%	–9.65%	1.80%	6.10%

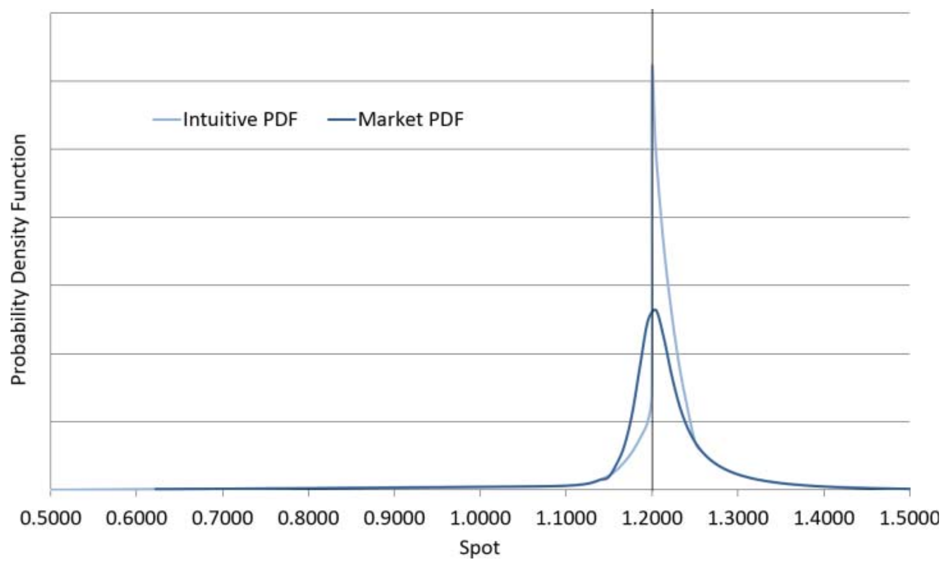


EXHIBIT 13.19 EUR/CHF 1yr actual and intuitive pdfs at June 1, 2012

unimodal (i.e., single peaked) probability density function can be generated, which presents a problem in currency pairs with more complex spot dynamics. In currency pairs where there is a peg, floor, intervention level, or similar, a more complex pdf would often better fit reality. Ideally the volatility surface construction should take into account the fact that spot ending just inside an intervention level is significantly more likely than spot ending just beyond an intervention level but there is no way to include this information within standard parameterizations.

The typical warning sign of this issue is that market instruments hit the correct market values but the implied volatility for individual strikes around key spot levels does not match the market, or vice versa.

Generating a Probability Density Function from Option Prices in Excel

This practical introduces a method of generating a probability density function from a volatility smile by numerically differentiating vanilla option prices twice with respect to the strike. The code reuses volatility smile functions developed in Practical F and vanilla options pricing functions developed in Practical C. Probability density functions are explored in detail within Chapter 13.

First, volatility smile inputs and market data must be defined within the Excel sheet. Then a range of delta values is established, from 0.1% to 99.9% in tight steps of 0.1%:

Market Data Inputs

Spot	**100.00**	←*Named:* ***Spot***
CCY1 Interest Rate	**0.0%**	←*Named:* ***rCCY1***
CCY2 Interest Rate	**0.0%**	←*Named:* ***rCCY2***
Time to Maturity (years)	**1.00**	←*Named:* ***T***

Populate Smile

Volatility Smile Inputs

ATM	**10.0%**	←*Named:* ***ATM***
25d RR	**0.0%**	←*Named:* ***RR25d***
25d Fly	**2.0%**	←*Named:* ***Fly25d***

↓ *Named:* ***VolatilitySmileRef***

Put Delta	Implied Vol	Strike
0.10%		
0.20%		
0.30%		
0.40%		
0.50%		
0.60%		
0.70%		

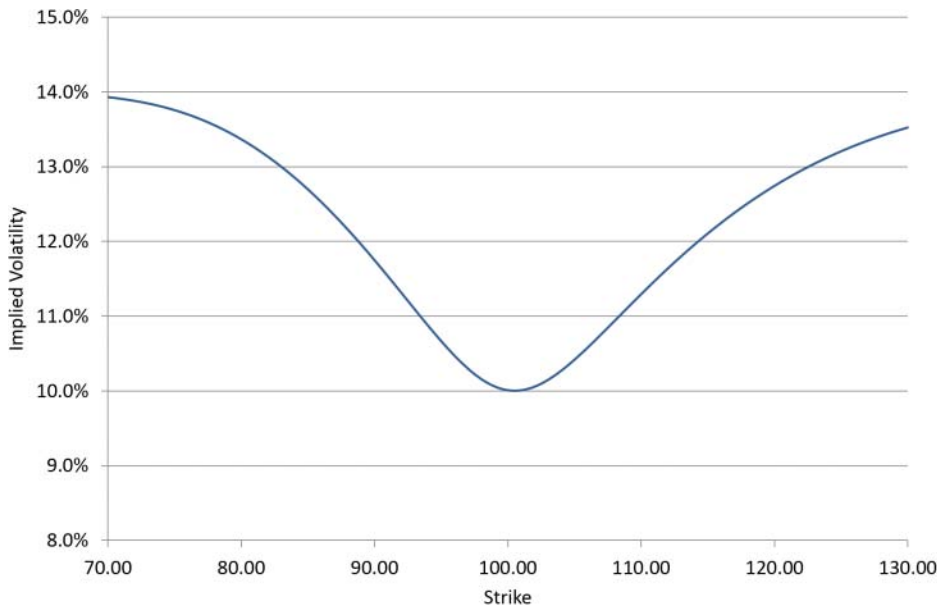
The implied volatility and strike must be calculated for each delta value. Due to the amount of data on the sheet it is better to use a VBA subroutine to calculate the values and place them on the sheet surface. The MalzSmileVol and StrikeFromPutDelta functions from Practical F can being used. Note that StrikeFromPutDelta takes a negative put delta value as input:

```
Sub populateSmileStrikesAndVols()

    Dim InputPutDelta As Double
    Dim ImpliedVol As Double
    Dim DeltaCount As Long

    DeltaCount = 1
    While Range("VolatilitySmileRef").Offset(DeltaCount, 0) <> ""
        InputPutDelta = Range("VolatilitySmileRef").Offset(DeltaCount, 0)
        ImpliedVol = MalzSmileVol(Range("ATM"), Range("RR25d"), _
         Range("Fly25d"), InputPutDelta)
        Range("VolatilitySmileRef").Offset(DeltaCount, 1) = ImpliedVol
        Range("VolatilitySmileRef").Offset(DeltaCount, 2) = _
         StrikeFromPutDelta(Range("Spot"), -InputPutDelta, _
         Range("rCCY1"), Range("rCCY2"), Range("T"), ImpliedVol)
        DeltaCount = DeltaCount + 1
    Wend

End Sub
```

Next, in a new column, define equally spaced strikes for calculating the probability density function (pdf):

↓ *Named:* ***StrikeRef***

Strike	Implied Vol	Put Price (CCY2 pips)
70.00	17.5%	0.1108
70.50	17.4%	0.1220
71.00	17.4%	0.1340
71.50	17.3%	0.1469
72.00	17.3%	0.1606
72.50	17.2%	0.1752
73.00	17.1%	0.1908



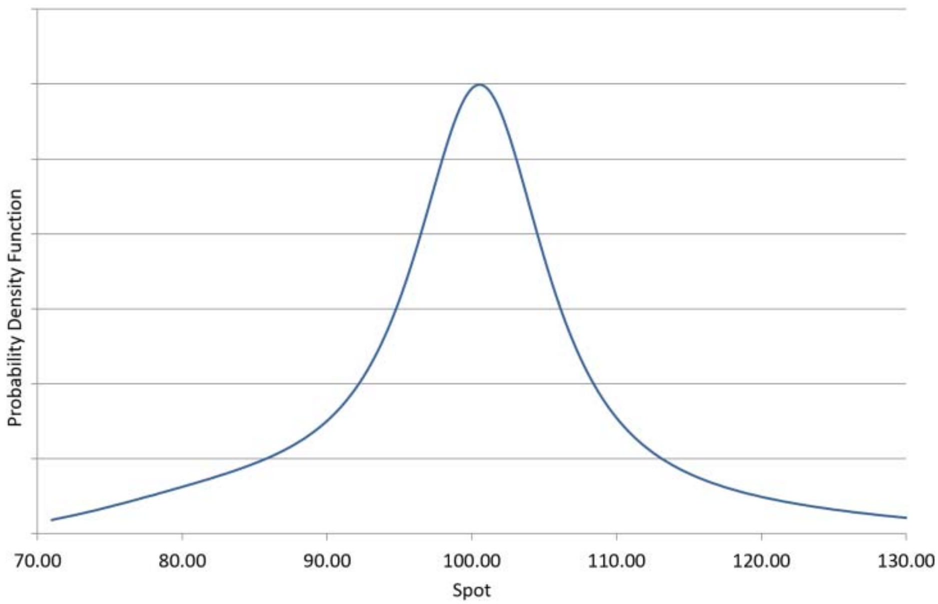
A VBA function is used to calculate the implied volatility and equivalent option price at each strike level. It is okay to use linear interpolation to generate the implied volatility since delta has small increments. The OptionPrice function from Practical C is reused:

```
Sub populatePDFImpliedVolsAndPrices()

    Dim PDFStrikeCount As Long, SmileDeltaCount As Long
```

```
    Dim LowStrike As Double, HighStrike As Double
    Dim LowVol As Double, HighVol As Double

    Dim InputStrike As Double, ImpliedVol As Double

    PDFStrikeCount = 1
    While Range("StrikeRef").Offset(PDFStrikeCount, 0) <> ""
        InputStrike = Range("StrikeRef").Offset(PDFStrikeCount, 0)
        ImpliedVol = -1
        SmileDeltaCount = 1
        While Range("VolatilitySmileRef").Offset(SmileDeltaCount + 1, 0) _
         <> ""
            LowVol = Range("VolatilitySmileRef").Offset(SmileDeltaCount, 1)
            HighVol = Range("VolatilitySmileRef").Offset(SmileDeltaCount _
             + 1, 1)
            LowStrike = Range("VolatilitySmileRef") _
             .Offset(SmileDeltaCount, 2)
            HighStrike = Range("VolatilitySmileRef") _
             .Offset(SmileDeltaCount + 1, 2)

            'Linear Interpolation to get Implied Vol
            If (InputStrike > LowStrike) And (InputStrike _
             < HighStrike) Then
                ImpliedVol = LowVol + (HighVol - LowVol) * (InputStrike - _
                 LowStrike) / (HighStrike - LowStrike)
            End If

            SmileDeltaCount = SmileDeltaCount + 1
        Wend

        Range("StrikeRef").Offset(PDFStrikeCount, 1) = ImpliedVol
        Range("StrikeRef").Offset(PDFStrikeCount, 2) = _
         OptionPrice(False, Range("Spot"), InputStrike, Range("T"), _
         Range("rCCY1"), Range("rCCY2"), ImpliedVol)

        PDFStrikeCount = PDFStrikeCount + 1
    Wend

End Sub
```

The probability density can now be calculated by finding the second derivative of price with respect to strike on the sheet:

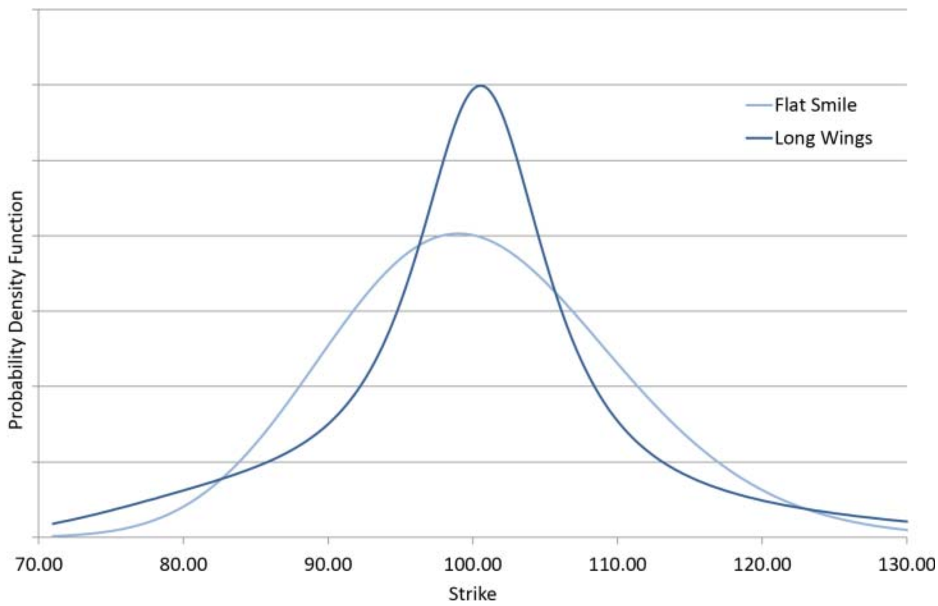
	F	G	H	I	J	K	L	M
13								↓ Probability Density
14				=F17-F16	=H17-H16	=J17/I17	=K18-K17	=L18/I18
15	**Strike**	**Implied Vol**	**Put Price (CCY2 pips)**	**dStrike**	**dPrice**	**dPrice/dStrike**	**d (dPrice/dStrike)**	**d (dPrice/dStrike) /dStrike**
16	70.00	13.9%	0.0193					
17	70.50	13.9%	0.0226	0.50	0.0033	0.0066		
18	71.00	13.9%	0.0263	0.50	0.0037	0.0075	0.0009	0.0018
19	71.50	13.9%	0.0306	0.50	0.0042	0.0085	0.0010	0.0020
20	72.00	13.9%	0.0354	0.50	0.0048	0.0096	0.0011	0.0022



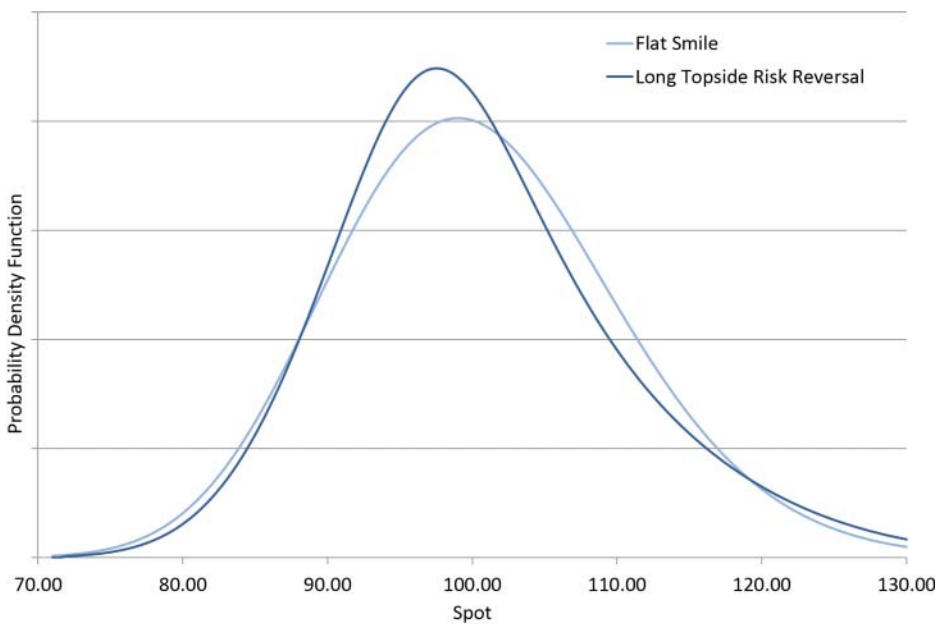
Note that the probability density (second derivative of option value with respect to strike) takes a similar shape to that of gamma (second derivative of option value with respect to spot).

Probability density functions can then be compared by copying the output values. When making changes to the volatility smile or market data inputs, remember to rerun both VBA subroutines in order to correctly set up the calculation.

Flat volatility smile versus long wings volatility smile:



Flat volatility smile versus long topside risk reversal volatility smile:



Finally, as discussed in Chapter 13, the area under the probability density function should equal one since it represents a probability mass. This can be checked by multiplying the average pdf between strikes by the change in strike at each strike level and summing the total. This indicates how accurate the output is. If the strike spacings are tight enough, the total probability value should be between 0.99 and 1.01.

PART III

Vanilla FX Derivatives Trading

The material up to this point has been developed within a stylized framework. This is important because the increased clarity makes learning easier. However, it is now time to confront some of the real-world issues faced by vanilla FX derivatives traders.

Vanilla FX Derivatives Trading Exposures

In Part I, Greek exposures were examined within a stylized framework. In practice, even the simplest Greek exposures have additional layers of complexity that must be understood by FX derivatives traders within their risk management.

Delta

Delta (Δ) is one of the most important exposures to a derivatives trader—the sensitivity of price to a change in the underlying. It is a simple concept but there are many possible variations in exactly what the delta exposure represents.

Spot Delta versus Forward Delta

Apologies if this is obvious, but the spot delta on a spot deal is 100% of the notional. Therefore, buying GBP50m GBP/USD spot results in longer GBP50m GBP/USD spot delta exposure within the trading position. For this reason, FX spot traders talk only in terms of net long or short positions, rather than their exposures. The forward delta on a forward outright contract is 100% of the notional. Selling USD100m of 1yr USD/CAD forward outright results in a shorter USD100m USD/CAD 1yr forward delta exposure within the trading position.

Delta exposures can be present valued or future valued like cash flows. Therefore, on a specific trade:

$$\Delta = \Delta_F \,.\, df$$

where df is the discount factor in the delta currency to the forward maturity, Δ is the spot delta, and Δ_F is the forward delta. When interest rates in the delta currency are positive (as they usually are), the discount factor is below 1 and $|\Delta| < |\Delta_F|$. For example, buying AUD50m of AUD/USD 5yr forward outright has a spot delta exposure of approximately AUD42m when the 5yr AUD discount factor is around 0.84. This means that, for example, when AUD100m of spot versus AUD100m of forward is traded in an equal notional FX swap, a residual spot delta exposure remains.

Spot delta is the delta exposure most often used within FX derivatives risk management because spot is most often used to hedge delta, particularly in G10 currency pairs. When this is the case, exposures to interest rates and swap points must be additionally monitored.

Delta Quoting Conventions

Traders most often quote option delta in CCY1% or CCY1 cash terms with CCY1 determined by market convention for that currency pair. Spot and forward deals are generally traded in CCY1 terms, so when the option delta is also given in CCY1 terms the hedge amount and direction are immediately clear. For example, an option has *long* CCY1 delta exposure: *Sell* spot on the hedge. Quoting delta in CCY2 terms can cause mistakes. For example, if the delta on a GBP/USD option is quoted as *short* USD16m, the delta hedge at current spot would be to *sell* GBP10m GBP/USD.

To convert CCY1% spot delta to CCY2% spot delta the following formula is used:

$$\Delta_{CCY1\%} = -\Delta_{CCY2\%} \cdot \frac{S}{K}$$

where S is spot and K is the strike. Note that when $S = K$, the delta is unchanged between CCY1% and CCY2% terms.

To convert CCY1% forward delta to CCY2% forward delta the following formula is used:

$$\Delta_{F\ CCY1\%} = -\Delta_{F\ CCY2\%} \cdot \frac{F}{K}$$

where F is the forward.

The negative signs are present within these formulas because CCY1 is bought while CCY2 is sold, or vice versa within an FX transaction.

CCY1 versus CCY2 Premium Delta

The standard delta profile for a long vanilla call option goes from 0% (with spot far below the strike) to +100% (with spot far above the strike), hitting +50% around the ATM, as shown in Exhibit 14.1.

This analysis assumes that the option premium is paid in CCY2 (the domestic currency). If the option premium is paid in CCY1 (the foreign currency), the delta exposure changes.

In USD/JPY, consider a long USD call option with JPY as the natural P&L currency but premium paid in USD by market convention. When buying this option, an additional *short USD delta* exposure is generated since USD are paid out in the premium.

In general:

$$\Delta_{CCY1\ Premium} = \Delta_{CCY2\ Premium} - Premium$$

where delta and premium are quoted in the same terms.

The intuition that the inclusion of premium moves delta shorter is as follows:

- At lower spot, the USD premium will be relatively cheaper to pay in JPY (domestic) terms.
- At higher spot, the USD premium will be relatively more expensive to pay in JPY (domestic) terms.

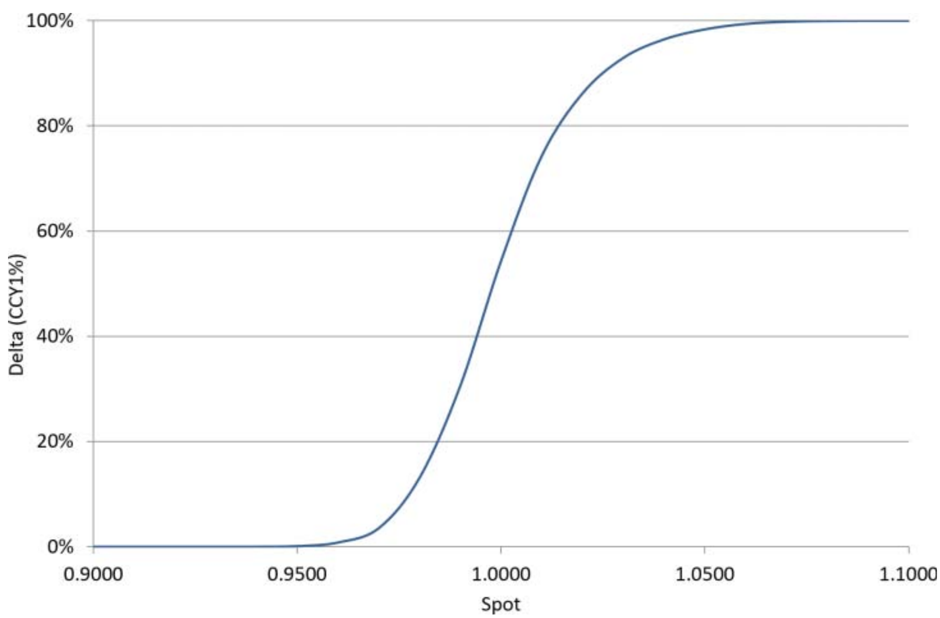


EXHIBIT 14.1 Long vanilla call delta (premium paid in CCY2) with 1.0000 strike

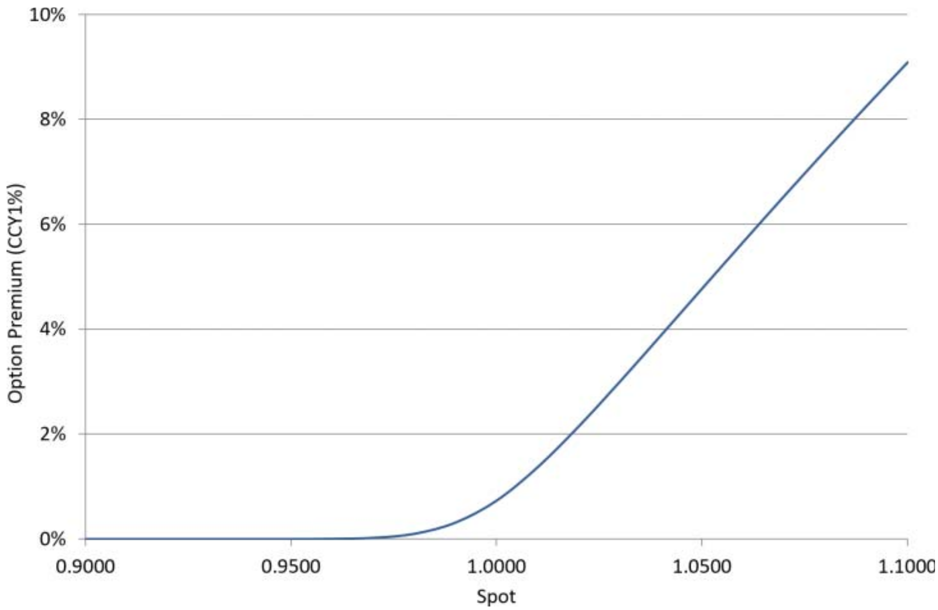


EXHIBIT 14.2 Long vanilla call option premium with 1.0000 strike

The long call CCY1% option premium profile is shown in Exhibit 14.2. The delta adjustment will be particularly large when the option is deep in-the-money and therefore has a large premium. This is shown in Exhibit 14.3. At longer tenors this effect can cause the delta of ATM options to be far away from the stylized 50%.

For a long put option, the stylized delta profile goes from –100% (with spot far below the strike) to 0% (with spot far above the strike). Again, a CCY1 premium causes an additional short delta, which this time has a larger impact with spot in-the-money to the downside as shown in Exhibit 14.4.

The premium currency in a particular currency pair is determined by market convention. If the currency pair contains USD, then the premium currency will be USD. Other G10 currencies can be approximately ordered: EUR > GBP > AUD > NZD > CAD > CHF > NOK > SEK > JPY.

In G10 versus emerging market currency pairs, the premium is usually paid in the G10 currency.

Delta Bleed

Greek exposures within derivatives trading positions do not stay constant over time. Traders therefore investigate how exposures change (bleed) over the next few trading days. Specifically, traders roll the horizon date forward near the end of the

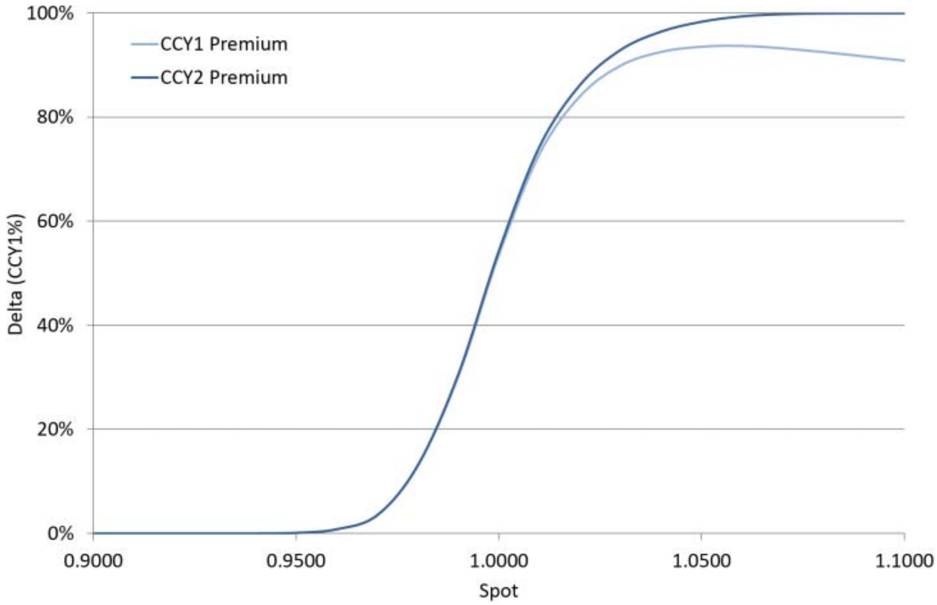


EXHIBIT 14.3 Long vanilla call option delta (premium paid in CCY1 or CCY2) with 1.0000 strike

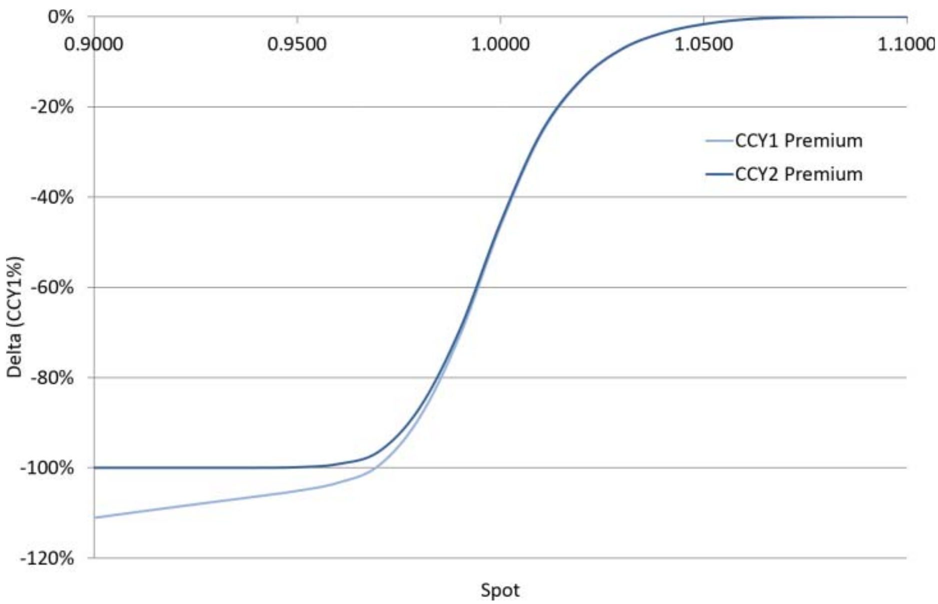


EXHIBIT 14.4 Long vanilla put option delta (premium paid in CCY1 or CCY2) with 1.0000 strike

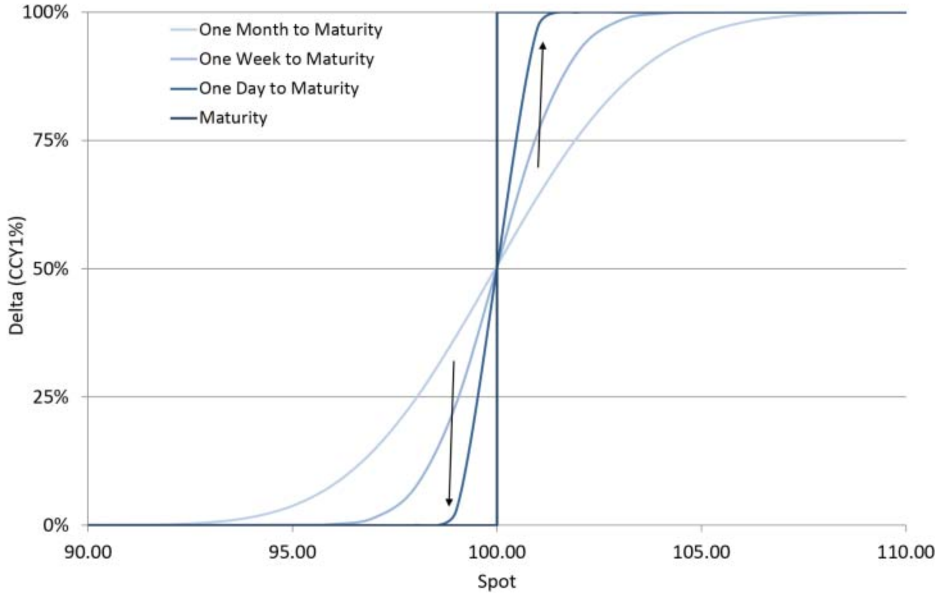


EXHIBIT 14.5 Vanilla call option delta with 100.00 strike over time

day to see how their position will look on the next trading day. This allows them to assess position bleed, particularly **delta bleed**.

Exhibit 14.5 shows how delta on a vanilla call option changes over time. Prior to the expiry date there are three possible cases for the relative positioning of spot and the strike of a long vanilla call option:

- For an out-of-the-money (OTM) spot (i.e., spot below strike), delta bleeds shorter.
- For an at-the-money (ATM) spot (i.e., spot close to the strike), delta remains roughly unchanged over time.
- For an in-the-money (ITM) spot (i.e., spot above strike), delta bleeds longer.

Example: EUR/USD spot: 1.3650.

- 7-day 1.3750 EUR call/USD put: 31% delta
- 6-day 1.3750 EUR call/USD put: 29% delta (i.e., –2% delta bleed)

Intuitively this makes sense when delta is thought of as the chance of ending up in-the-money at maturity. Lower time to maturity gives spot less time to move through the strike. Therefore:

- For an OTM option, as time to expiry shortens, the probability of ending up ITM at maturity (and hence the delta) reduces.

- For an ATM option, as time to expiry shortens, the probability of ending up ITM at maturity (and hence the delta) remains roughly unchanged at approximately 50%.
- For an ITM option, as time to expiry shortens, the probability of ending up ITM at maturity (and hence the delta) increases.

For a long put option the ITM versus OTM side is switched (e.g., spot below the strike is OTM for a call but ITM for a put) but additionally, put deltas are negative so the delta bleed on call options and put options ends up being equivalent, as expected due to put–call parity.

Example: EUR/USD spot: 1.3650.

- 7-day 1.3750 EUR put/USD call: –69% delta
- 6-day 1.3750 EUR put/USD call: –71% delta (i.e., –2% delta bleed)

The delta bleed for a particular strike increases as the expiry date approaches with the most dramatic delta bleed occurring into the expiry date itself. On vanilla options with large notionals, traders pay close attention to these delta changes.

Consider a long call option:

- On the day before maturity, if spot is slightly above the strike, delta will be approximately +50%. On the expiry date itself and with spot unchanged delta will be +100% above the strike. Therefore, delta bleed will be +50% of the option notional.
- On the day before maturity, if spot is slightly below the strike, delta will be approximately +50%. On the expiry date itself and with spot unchanged delta will be 0% below the strike. Therefore, delta bleed will be –50% of the option notional.

Generally, delta bleed *assists* traders with their risk management:

- If the trading position is mainly long downside or short topside vanilla options, delta will bleed longer.
- If the trading position is mainly long topside or short downside vanilla options, delta will bleed shorter.

In each of these scenarios, the delta bleed matches a natural risk management preference to run delta into short gamma areas (i.e., running long delta if short topside gamma) and to run delta away from long gamma areas (i.e., running long delta if long downside gamma). Delta bleed therefore often helps to produce a balanced P&L profile.

Gamma and Theta

As discussed in Chapter 9, buying vanilla options produces long gamma exposures, which costs theta over time. Selling vanilla options produces short gamma exposures, which earns theta over time.

Gamma (Γ) is the rate at which delta changes as spot moves and in risk management tools it is usually quoted for a 1% move in spot.

Theta (θ) is the rate at which price changes over time and in risk management tools it is usually quoted as the P&L from a *one-day shift* forward in the horizon date. It therefore represents the net cost (negative theta) or net benefit (positive theta) from holding the trading position for one full trading day. This seems clear enough but it is important that traders know what assumptions are used within the calculation. For example, what happens to the implied volatility and interest rate curves as the horizon date rolls forward?

In practice, cumulative position value changes with time in two different ways. First, when the horizon is rolled forward, there is a P&L jump (i.e., theta). This P&L jump will be particularly large for slightly out-of-the-money vanilla options expiring on the new horizon date. Second, throughout the day, implied volatility moving lower at shorter maturities is an additional source of value change (as per Chapter 11).

Under Black-Scholes, gamma and theta are linked by this formula:

$$\theta = -\frac{1}{2}\Gamma\sigma^2 S^2$$

where σ is volatility and S is spot. The negative sign exists because, for example, long (positive) gamma causes paying (negative) theta.

Looking at the formula, gamma and (negative) theta are proportional ($\Gamma \propto -\theta$). This makes intuitive sense because, e.g., long gamma gives the ability to make money out of spot moves and the more delta changes as spot moves, the more money can be made from delta hedging. Put another way, more theta must be paid for the privilege of more long gamma.

Since gamma and theta are proportional, the theta profile shown in Exhibit 14.6 takes an identical shape to the corresponding gamma profile; rising into the strike at maturity.

The next thing to note from the gamma-theta formula is that for the same amount of gamma, higher volatility (σ) causes theta to rise negatively. Intuitively, vanilla options have higher premium at higher implied volatility and therefore they have more value to decay away.

Additionally, for the same amount of theta, higher volatility causes lower gamma. To understand why, consider the delta profile: If volatility is low, there will be a big delta change over a given spot change, whereas if volatility is high, there will

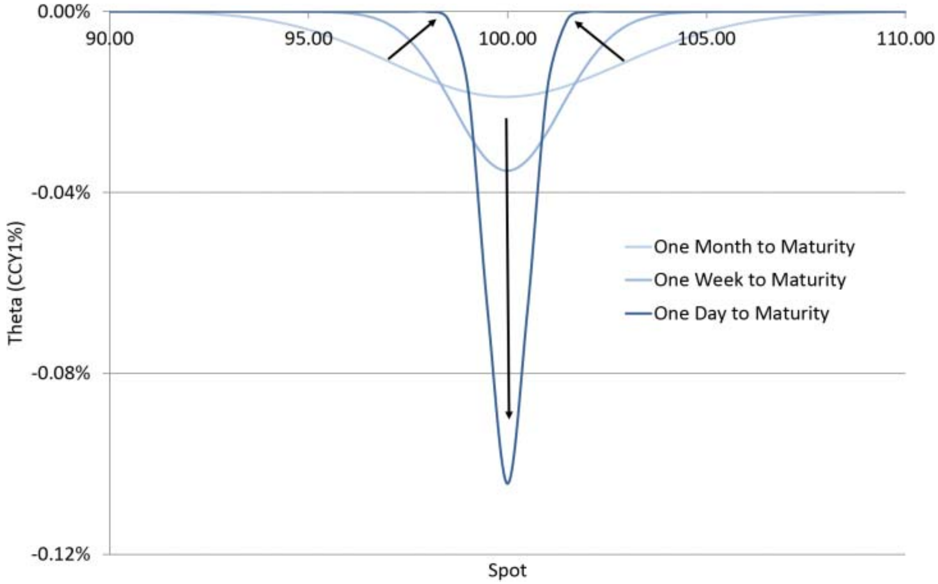


EXHIBIT 14.6 Long vanilla option with 100.00 strike theta over time

be a much smaller delta change over the same fixed spot change. During the 2008 financial crisis, buying 1mth AUD/JPY ATM at 50% volatility gave 3% gamma while buying 1mth USD/HKD ATM today at 0.25% volatility gives 490% gamma.

Traders actively track the gamma/theta ratio in their trading positions. Within the Black-Scholes framework, theta simply pays for gamma, but in practice theta comes from different aspects of the trading position.

ATM Curve and Volatility Smile Roll Theta

Over time, options roll down the ATM curve as an option maturity goes from, for example, 365 days to 364 days, and so on. If the ATM curve is upward sloping (i.e., 1yr ATM higher than 1mth ATM), this effect causes additional theta to be paid on long vanilla option positions (and additional theta to be earned on short vanilla option positions) as the ATM implied volatility drops over time as the expiry date moves closer to the horizon. If the ATM curve is downward sloping (e.g., 1yr ATM lower than 1mth ATM), the opposite effect occurs. The more steeply sloped the ATM curve, the larger the impact. Exhibit 14.7 demonstrates the ATM curve roll process.

In practice, this effect occurs within the entire volatility surface as all strikes roll to closer maturities.

Forward Roll Theta

With everything else fixed constant, but the horizon shifted forward by one day, the forward outright to a fixed expiry date drifts toward spot. For a given option this is

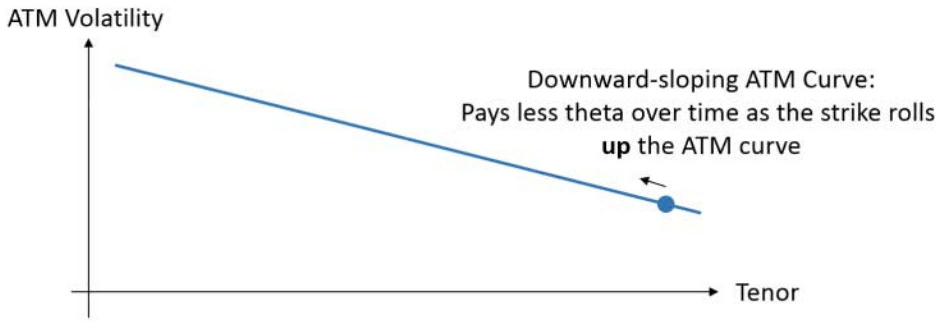


EXHIBIT 14.7 ATM curve roll

similar to a free spot move equal to the one-day swap points. The P&L impact from this is only a small part of the total option theta, but in high interest rate differential currency pairs it can be important.

Cash Balance Theta

Consider what happens when spot deals in a derivatives trading position settle at maturity. Long AUD10m 0.9500 AUD/USD spot settles into long AUD10m and short USD9.5m on the spot date. Cash balances from settled spot or forward deals, option premiums, or other cash payments accumulate in a trading position over time. These cash balances can be managed by putting long cash balances on deposit, hence earning interest on the cash and borrowing to offset short cash balances, hence paying interest on the cash. Alternatively, FX swaps can be traded, usually against the USD, to push cash balances into the future. It is important that traders know how to monitor these cash balances in their position and understand how they are managed day-to-day.

Smile Gamma

One aspect of a trading position with a significant impact on the gamma/theta ratio is the *smile position*. Owning options high on the volatility smile results in a lower gamma/theta ratio due to the higher implied volatility (recall the gamma-theta formula). This can be thought of as one of the costs of holding a long smile position. In high-skew currency pairs, long smile positions are expensive to hold over time.

Vanilla traders sometimes look at their gamma exposure calculated from vanilla options valued using the ATM curve (called *ATM gamma*) and compare it to the gamma exposure calculated from options valued using the full volatility surface. The difference is the **smile gamma effect**; the amount of gamma coming from the smile:

- If the trading position is mainly long vanilla options priced at higher implied volatility and/or short vanilla options priced at lower implied volatility on the smile, the net smile gamma effect will be negative.

- If the trading position is mainly long vanilla options priced at lower implied volatility and/or short vanilla options priced at higher implied volatility on the smile, the net smile gamma effect will be positive.

A common issue for risk managers is a low gamma/theta ratio within their trading position. To fix this issue, traders search for options in their position with large negative smile gamma effect. It is worth noting that in currency pairs where the smile is high compared to the volatility base, this approach can overlook options that have very low premiums priced using the ATM volatility (e.g., up to 0.02%) but higher premiums priced using the smile volatility (e.g., 0.10% to 0.20%). These options have almost no smile gamma effect (they contain little gamma of any kind because they are low delta), yet long positions in them can contribute significant negative theta.

P&L Distributions from Long Gamma or Short Gamma

On a given trading day in a stylized Black-Scholes world, the maximum P&L that can be *lost* from a long gamma trading position (ignoring P&L from vega, rho, new deals, etc.) is theta while more (potentially much more) can be made if spot is highly volatile. Exhibit 14.8 shows the P&L distribution from a long gamma position; it contains many small losses and few large gains, although the expectation of the distribution is zero.

Similarly, the most P&L that can be *made* from a short gamma trading position is theta while (much) more can be lost if spot is highly volatile. Exhibit 14.9 shows

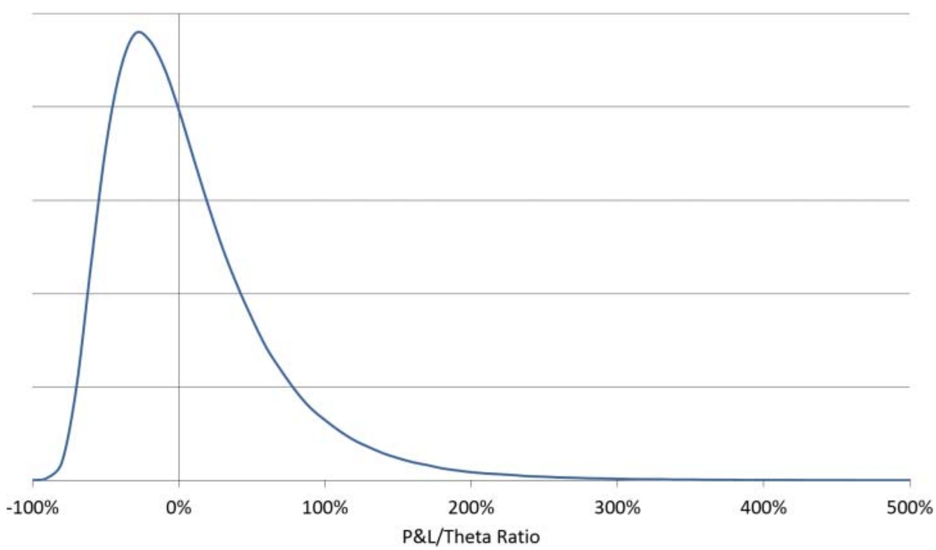


EXHIBIT 14.8 Stylized P&L/theta distribution from long gamma position

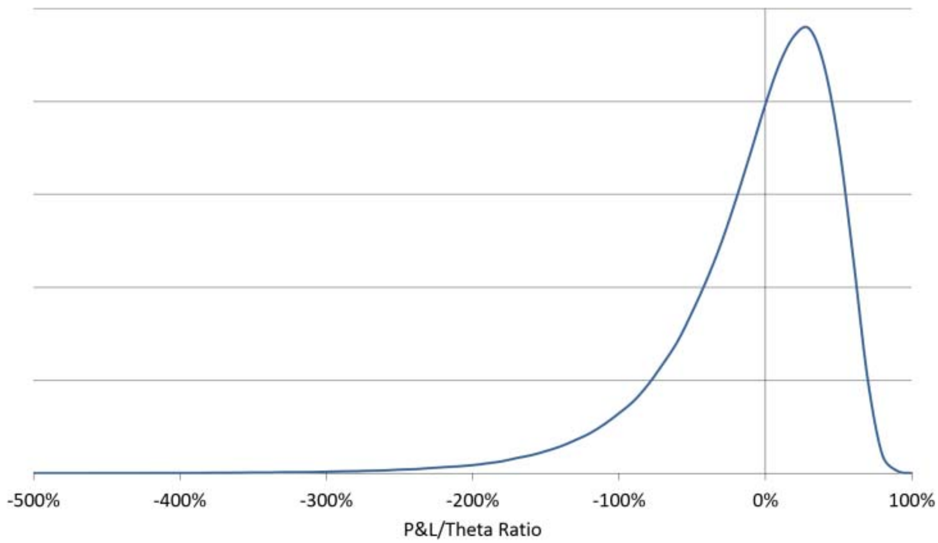


EXHIBIT 14.9 Stylized P&L/theta distribution from short gamma position

the P&L distribution from a short gamma position: It contains many small gains and few large losses, although, again, the expectation of the distribution is zero.

Preference for a long or short gamma P&L distribution impacts how risk managers position their trading books. Some traders prefer to trade long gamma positions while others prefer to trade short gamma positions. In a job interview, though, the correct answer is obviously that you are equally happy trading with whichever gamma position gives the best risk/reward at the time.

Vega and Weighted Vega

The most important exposure on most derivatives contracts is vega. Like delta, vega discounts in the currency it is expressed, so while stylized vega generally changes proportionally to the square root of time, in practice discounting can have a large effect at longer maturities. It is therefore not safe to assume that vega on ATM contracts always increases at longer maturities. For example, Exhibit 14.10 shows long-dated AUD/JPY ATM vega.

Traders most often view vega exposures in CCY1 terms, but sometimes vega will be viewed in CCY2 if that is the P&L currency. For example, in EUR/USD, a vega exposure quoted in USD may be preferred.

A standard vega calculation assumes the ATM curve moves in parallel. A **weighted vega** calculation assumes the ATM curve moves proportional to $\frac{1}{\sqrt{t}}$. If the ATM curve moves in a weighted manner, it is therefore expected that, for

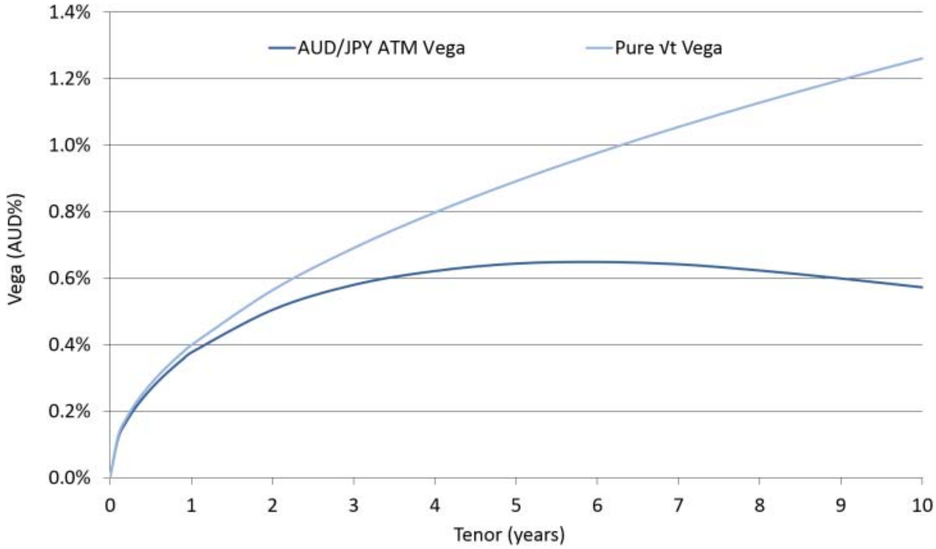


EXHIBIT 14.10 AUD/JPY ATM vega

example, the 3mth ATM moves twice as much as the 1yr ATM. This links to vega exposures where 3mth ATM vega is half that of the 1yr ATM.

The weighted vega calculation folds all exposures into a single reference pillar (usually 1mth or 3mth depending on which tenor is most liquid in a particular currency pair). For example, a 3mth weighted vega of USD400k implies that the full vega exposure in the position is equivalent to a vega exposure of USD400k *in the 3mth tenor*. It is therefore important to know which reference pillar the weighted vega calculation uses.

The following formula is used to convert vega (υ) at time t into weighted vega ($\upsilon_{weighted}$) at the weighted reference time $t_{weighted}$.

$$\upsilon_{weighted} = \sqrt{\frac{t_{weighted}}{t}} \,.\, \upsilon$$

Exhibit 14.11 gives vega multipliers for converting bucketed vega at market tenors into 1mth weighted vega. The same methodology can also be used to calculate weighted vega on each option individually.

If the ATM curve is moving in a parallel manner, *vega* will most accurately predict P&L changes caused by changes to the ATM curve, whereas if the ATM curve is moving in a weighted manner, *weighted vega* will most accurately predict P&L changes caused by changes to the ATM curve. Most often, the ATM curve moves in *neither* a parallel nor weighted manner but some combination of the two. Traders therefore view both measures within their risk management and assess how the ATM curve is currently moving when deciding which exposure to pay closest attention to.

EXHIBIT 14.11 **Weighted Vega Multipliers (1mth reference)**

Tenor	t (years)	$\sqrt{t}$	Weighted Vega Multipliers (1mth reference)
O/N	0.00274	0.052342	5.515
1wk	0.01923	0.138675	2.082
2wk	0.03846	0.196116	1.472
1mth	0.08333	0.288675	1.000
2mth	0.16667	0.408248	0.707
3mth	0.25000	0.500000	0.577
6mth	0.50000	0.707107	0.408
9mth	0.75000	0.866025	0.333
1yr	1.00000	1.000000	0.289
2yr	2.00000	1.414214	0.204
5yr	5.00000	2.236068	0.129

Adapted Greeks

Greek exposures for vanilla options can be calculated using a **sticky strike** or a **sticky delta** methodology.

Under sticky strike, the implied volatility for a given expiry date and *strike* stays constant as the forward moves. Standard Black-Scholes Greeks are calculated assuming a sticky strike methodology since only one constant volatility is used within the Black-Scholes framework.

Under sticky delta, the implied volatility for a given expiry date and *delta* stays constant as the forward moves. These so-called **adapted Greeks** are calculated assuming a sticky delta methodology.

It is important that traders understand the differences between standard exposures and adapted exposures so they can risk manage using the correct exposures given prevailing market behavior.

Throughout this analysis, *Black-Scholes delta* refers to the delta calculated under Black-Scholes using the smile volatility and it is easiest to think of the delta on a long vanilla call option when following these logical arguments.

Adapted Delta

Consider a finite difference spot delta calculation in which spot is flexed up and down a small amount and the resultant premium change is divided by the spot flex to calculate delta. In the standard Black-Scholes delta calculation, the implied volatility at which the vanilla is valued remains constant but in an adapted delta calculation the implied volatility for the fixed strike changes as spot flexes up and down: This is the **adaption**.

Adapted delta is such a well-accepted concept within the FX derivatives market that in some emerging market currency pairs, risk reversals are traded in the interbank broker market with an adapted delta forward hedge.

In symbols:

$$\Delta_{Black-Scholes} = \frac{\partial P(S)}{\partial S}$$

$$\Delta_{Adapted} = \frac{dP(S,\ \sigma(S))}{dS} = \frac{\partial P}{\partial S} + \frac{\partial P}{\partial \sigma}.\frac{d\sigma}{dS} = \Delta_{Black-Scholes} + \upsilon.\frac{d\sigma}{dS}$$

The formula for adapted delta comes from the chain rule. In words, adapted delta is equal to Black-Scholes delta plus vega (υ) multiplied by the change in implied volatility for a change in spot. To understand this $\frac{d\sigma}{dS}$ quantity, consider a finite difference delta calculation within a volatility smile with the risk reversal *for downside* shown in Exhibit 14.12.

Within the volatility smile:

- When spot is flexed *up*, the whole volatility smile moves higher with the flex and the fixed strike rolls up to a *higher* implied volatility. This makes the vanilla option premium relatively higher.
- When spot is flexed *down*, the whole volatility smile moves lower with the flex and the fixed strike rolls down to a *lower* implied volatility. This makes the vanilla option premium relatively lower.

Therefore, $\frac{d\sigma}{dS}$ is positive.

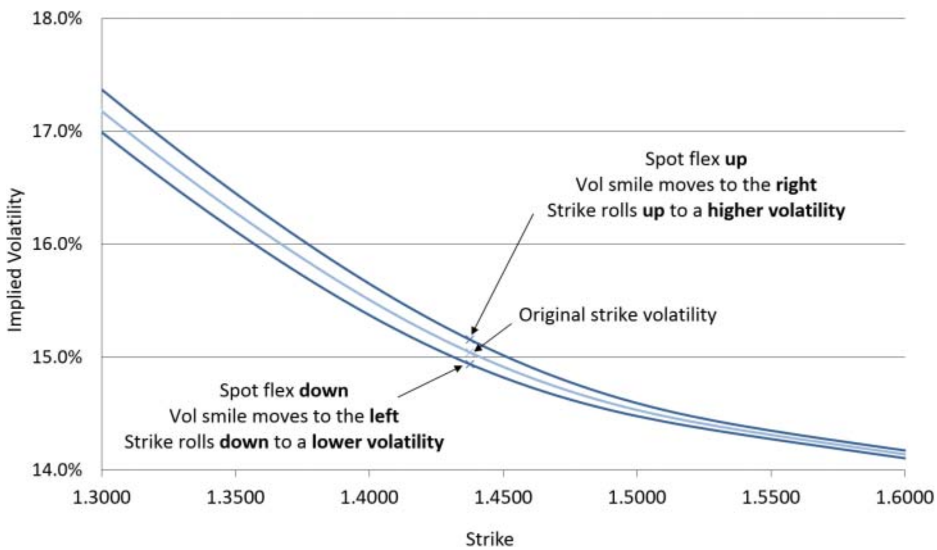


EXHIBIT 14.12 Adapted delta finite difference calculation shown on volatility smile

Looking back at the adapted delta formula, vega (υ) for a long vanilla option is always positive. Therefore, the direction of the adapted delta versus Black-Scholes delta difference comes from the $\frac{d\sigma}{dS}$ quantity. The whole volatility smile moves with spot under the adaption:

- If implied volatility increases at lower strikes, the implied volatility for a fixed strike will rise as spot rises and $\frac{d\sigma}{dS}$ will be positive (as in the previous example).
- If implied volatility increases at higher strikes, the implied volatility for a fixed strike will fall as spot rises and $\frac{d\sigma}{dS}$ will be negative.

In addition, vega decreases away from the strike so Black-Scholes delta versus adapted delta differences reduce to zero in the wings.

Therefore, in a currency pair with relatively higher skew and lower wings:

- If the risk reversal is for *topside*, adapted delta is usually *shorter* than Black-Scholes delta.
- If the risk reversal is for *downside*, adapted delta is usually *longer* than Black-Scholes delta.

In a currency pair with relatively higher wings and lower skew, adapted delta versus Black-Scholes delta differences change sharply over different spot levels around the ATM. Exhibit 14.13 shows a symmetric volatility smile with zero skew and positive wings.

When an ATM contract is traded, the implied volatility will be at the lowest point of the volatility smile, technically called "at the bottom of the bucket."

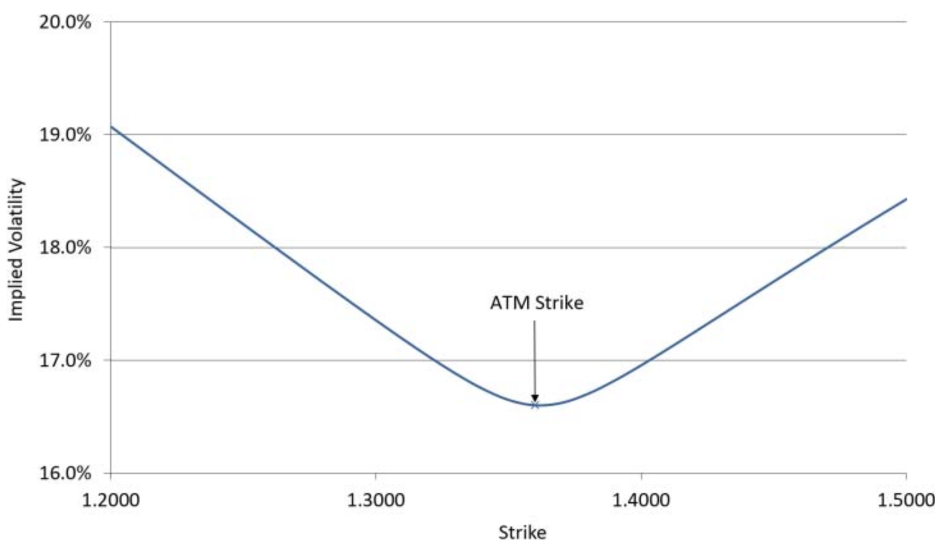


EXHIBIT 14.13 Symmetric volatility smile

If spot moves higher, the volatility smile at the fixed strike becomes locally sloped higher to the downside, hence $\frac{d\sigma}{dS}$ is positive and:

$$\Delta_{Adapted} > \Delta_{Black-Scholes}$$

If spot moves lower, the volatility smile at the fixed strike becomes locally sloping up to the topside, hence $\frac{d\sigma}{dS}$ is negative and:

$$\Delta_{Adapted} < \Delta_{Black-Scholes}$$

Adapted Gamma

Adapted gamma takes into account how a fixed strike rolls around the volatility smile in exactly the same way as adapted delta does. The important measure from a risk management perspective is the **gamma adaption effect**—the difference between adapted gamma and Black-Scholes gamma.

Consider a symmetric volatility smile with no skew but positive wings. The adapted gamma argument is visualized in Exhibit 14.14.

As described in the previous section on adapted delta, for a strike that is initially ATM:

- Spot flexed higher → fixed strike becomes downside → $\Delta_{Adapted} > \Delta_{Black-Scholes}$
- Spot flexed lower → fixed strike becomes topside → $\Delta_{Adapted} < \Delta_{Black-Scholes}$

Therefore, under adaption, delta moves more for both up and down spot flexes. Hence around the ATM point there is *positive* gamma adaption effect and the

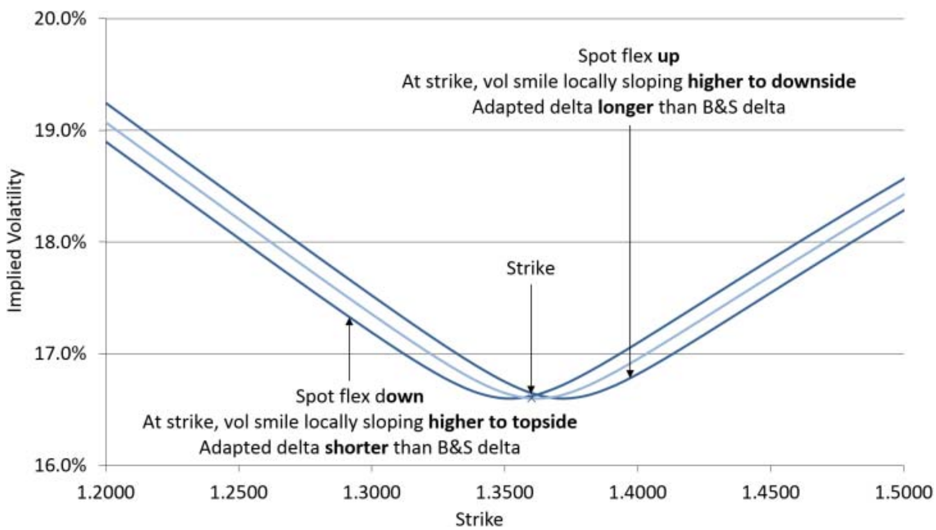


EXHIBIT 14.14 Adapted gamma finite difference calculation for a symmetric volatility smile

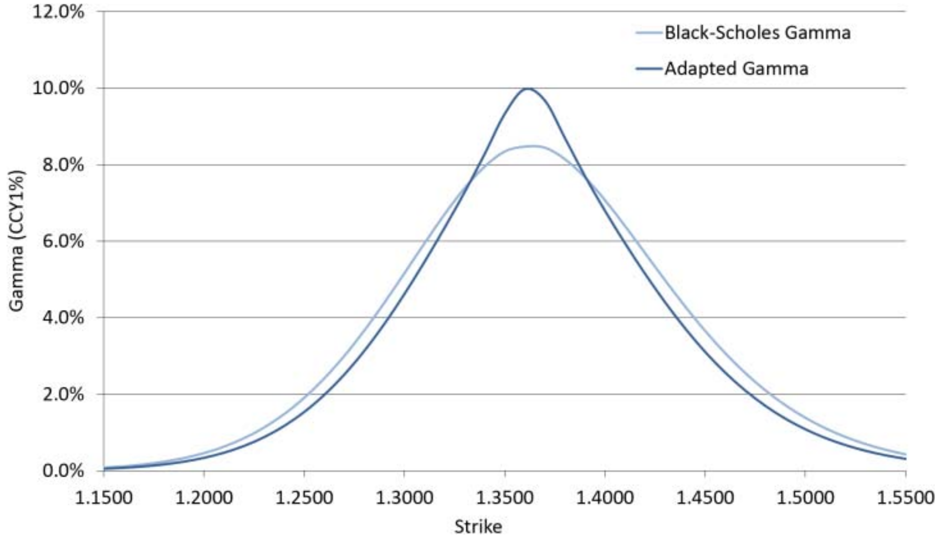


EXHIBIT 14.15 Adapted gamma versus Black-Scholes gamma in a symmetric volatility smile

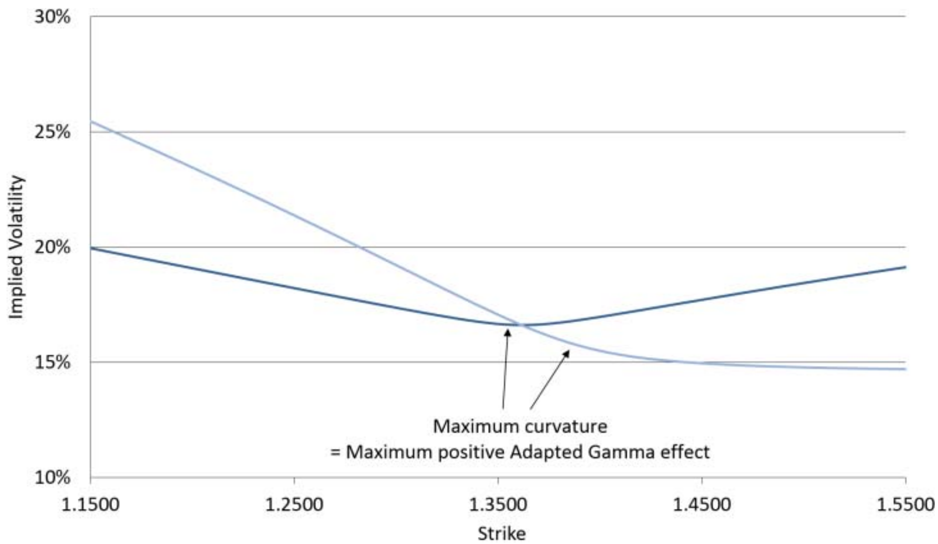


EXHIBIT 14.16 Maximum curvature points for symmetric and non-symmetric volatility smiles

effect is larger the more pronounced the wings of the smile. Exhibit 14.15 shows Black-Scholes gamma and adapted gamma for an ATM strike within a symmetric volatility smile.

The largest positive gamma adaption effect occurs at the point of maximum curvature in the volatility smile (i.e., maximum $\frac{d^2\sigma}{dS^2}$). When a skew component

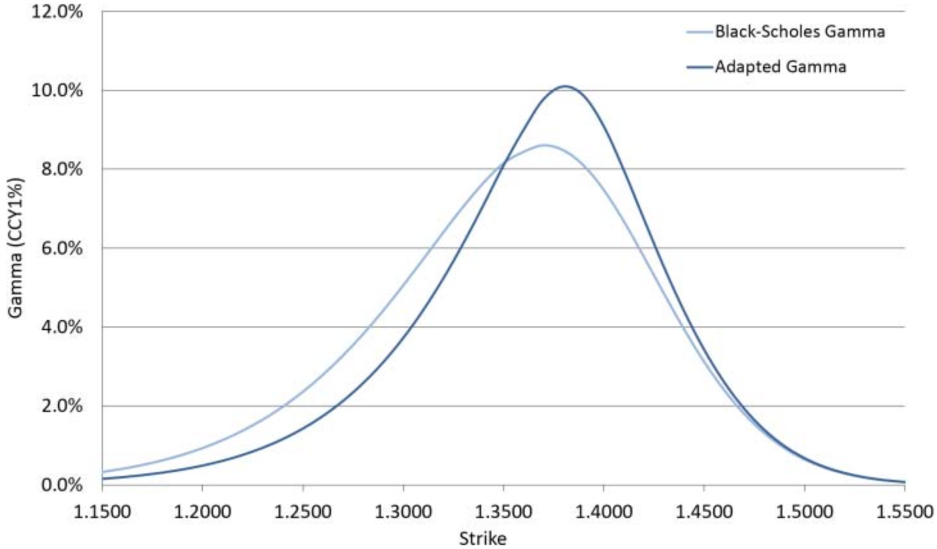


EXHIBIT 14.17 Adapted gamma versus Black-Scholes gamma profiles for a non-symmetric volatility smile

is added into the volatility smile, this point of maximum curvature moves to the lower volatility side of the volatility smile. In Exhibit 14.16, the risk reversal is for downside and the point of maximum curvature has moved to the topside.

This results in the gamma profiles shown in Exhibit 14.17, which in turn results in the gamma adaption effect profile shown in Exhibit 14.18.

In general, the gamma adaption effect follows these rules of thumb:

- Strikes on the high side of the volatility smile usually have a *negative* gamma adaption effect.
- Strikes on the low side of the volatility smile usually have a *positive* gamma adaption effect.

Therefore, buying vanillas high on the volatility smile (i.e., at an implied volatility above the ATM) results in a worse gamma/theta ratio under adaption than under Black-Scholes. *In extremis* it is possible for a trading position to be *short* adapted gamma yet *paying* theta—a difficult situation to risk manage.

Adapted Vega

Vega (υ) is the exposure to ATM volatility changes. Consider a finite difference vega calculation in which ATM implied volatility is flexed up and down a small amount and the resultant premium change is divided by the ATM flex to calculate vega. In the standard Black-Scholes vega calculation the implied volatility at which a vanilla

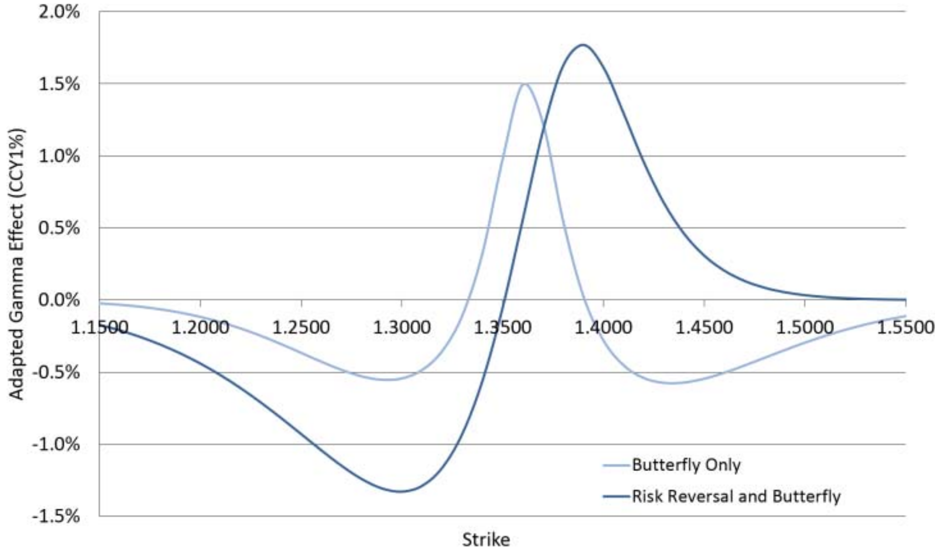


EXHIBIT 14.18 Gamma adaption effect profile for a non-symmetric volatility smile

is priced changes exactly as much as the ATM flex but in an adapted vega calculation the implied volatility changes differently because the whole volatility curve rebuilds under the adaption.

When the ATM is flexed higher, the strikes for a specific delta move away from the ATM. This pushes the whole volatility smile wider and causes the implied volatility for a specific strike to rise less on the high side of the smile. This effect is shown in Exhibit 14.19.

Therefore, strikes on the high side of the volatility smile have $\upsilon_{Adapted} < \upsilon_{Black-Scholes}$ and strikes on the low side of the volatility smile have $\upsilon_{Adapted} > \upsilon_{Black-Scholes}$. Traders observe this effect most frequently when a bought risk reversal contract gives a short adapted vega exposure and a sold risk reversal contract gives a long adapted vega exposure.

Risk Managing with Adapted Greeks

A trader has done some analysis and concludes that 1yr USD/JPY risk reversals are too high. Therefore they go into the market and sell USD100m/leg of 1yr 25d risk reversal. In USD/JPY the risk reversal is for downside so selling the risk reversal involves selling the downside strike versus buying the topside strike. The risk reversal is delta hedged by selling USD50m of USD/JPY 1yr forward. This results in the trading position shown in Exhibit 14.20. The exposures in Exhibit 14.20 are calculated using the standard Black-Scholes methodology that assumes volatility for a given strike stays unchanged as spot moves (i.e., sticky strike).

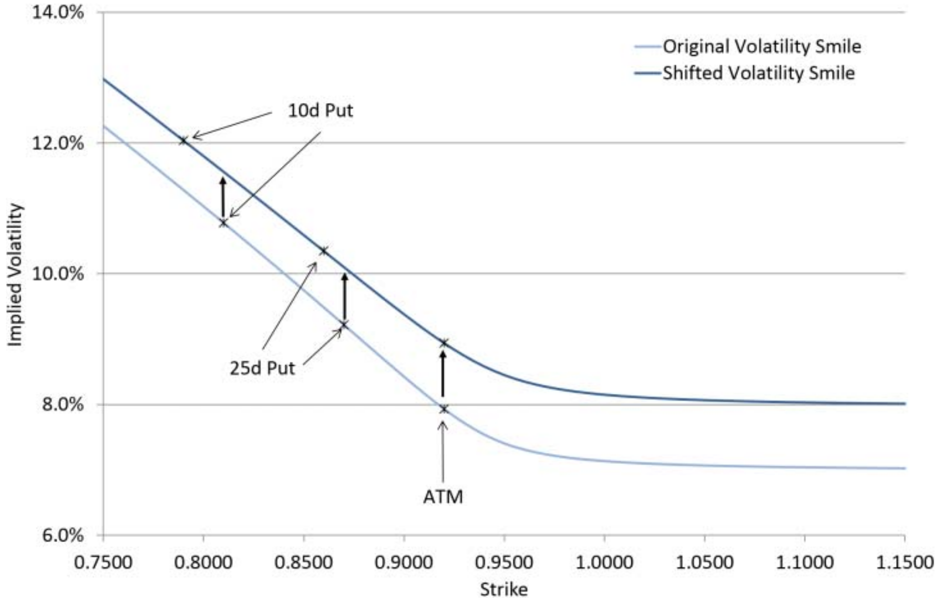


EXHIBIT 14.19 Volatility smile adjustment at higher ATM volatility

USD/JPY	P&L Change (USD)	Delta (USD)	Gamma (USD)	Vega (USD)
116.58	2,834,782	36,382,477	1,834,940	150,114
113.54	1,806,624	32,707,504	2,444,849	182,744
110.50	875,257	26,554,805	3,122,341	211,548
107.46	185,515	17,433,223	3,442,592	211,455
104.42	-119,800	7,056,247	2,728,312	148,375
101.38	**-**	**175,555**	**692,797**	**16,217**
98.33	287,975	1,722,526	-1,853,292	-136,078
95.29	290,955	11,668,370	-3,576,380	-239,513
92.25	-258,190	25,241,657	-3,708,416	-250,840
89.21	-1,388,173	36,638,252	-2,781,588	-201,084
86.17	-2,887,062	44,833,891	-1,771,521	-137,062

EXHIBIT 14.20 Spot ladder from selling USD/JPY 1yr risk reversal (Black-Scholes exposures)

However, the P&Ls within Exhibit 14.20 are generated by changing spot to each level in the ladder, rebuilding the volatility smile using the new spot level as the reference point, and revaluing all the options within the portfolio. This is equivalent to a sticky delta methodology and therefore the delta exposures and P&L change do not tie in between different spot levels within the spot ladder. For example, delta is roughly flat at 101.38 and yet there is significant positive P&L at 98.33 and significant negative P&L at 104.42.

Put another way, if the method used to calculate P&L within the spot ladder and the method used to calculate the exposures within the ladder are not aligned, delta exposures and P&L changes will not be aligned.

USD/JPY	P&L Change (USD)	Delta (USD)	Gamma (USD)	Vega (USD)
116.58	2,834,782	40,129,976	1,076,353	119,024
113.54	1,806,624	37,249,157	1,995,306	161,485
110.50	875,257	30,514,174	3,770,450	215,015
107.46	185,515	18,003,437	5,383,001	245,376
104.42	-119,800	2,973,223	5,490,513	201,361
101.38	-	**-9,381,917**	**1,998,537**	**43,285**
98.33	287,975	-6,688,560	-3,491,865	-149,308
95.29	290,955	7,354,156	-5,410,616	-265,644
92.25	-258,190	26,420,817	-5,095,665	-253,564
89.21	-1,388,173	39,386,742	-2,240,424	-177,100
86.17	-2,887,062	46,475,717	-1,168,101	-115,891

EXHIBIT 14.21 Spot ladder from selling USD/JPY 1yr risk reversal (adapted exposures)

If adapted exposures are displayed in the spot ladder instead as in Exhibit 14.21, delta at 101.38 spot is shorter and P&L changes are aligned with the delta exposures.

When a spot ladder is constructed as per Exhibit 14.21, volatility surface changes need to be additionally considered. This risk reversal trade therefore will be risk managed based on how implied volatility for the two strikes in the risk reversal is expected to change as spot moves:

- If the implied volatility surface inputs are not changing as spot moves, adapted exposures should be used. Under adaption, the trading position looks too short delta and some spot or forward could be bought back to balance the delta position better.
- If the implied volatility surface inputs are changing as spot moves such that the implied volatility for the transacted strikes remains (roughly) constant, the Black-Scholes exposures should be used and the delta position looks balanced.

Holding the short risk reversal position will cause the trading book to earn money over time if everything stays static as the short smile value decays positively. However, as spot moves, ATM implied volatility changes will probably cost money since in a currency pair with downside risk reversal:

- Spot lower → ATM implied volatility higher while the position gets shorter vega.
- Spot higher → ATM implied volatility lower while the position gets longer vega.

Traders often have a choice whether to risk manage vanilla positions using Black-Scholes exposures or adapted exposures. For risk management it is best to view both exposures and judge how the market is currently trading:

- If the volatility surface is not changing as spot moves, adapted exposures should be used.
- If the volatility surface changes and particularly the ATM changes as suggested by the volatility smile as spot moves, Black-Scholes exposures should be used.

In practice, for small spot moves the volatility surface often remains unchanged (i.e., use adapted exposures), but for larger spot moves the volatility surface often reacts such that the implied volatility for specific strikes remains constant (i.e., use Black-Scholes exposures). The speed of spot moves also matters, since implied volatility reacts more if spot moves more quickly. In some sense, the volatility surface therefore contains information about expected speed of spot moves higher or lower.

Zeta

The zeta of a vanilla option is the premium difference between the value of the option priced using the smile volatility less its value priced using the ATM volatility. Zeta therefore quantifies the value in the option which is due to the volatility smile.

The zeta for a vanilla option is approximately (ignoring second-order effects) vega multiplied by the smile volatility less ATM volatility difference. Therefore:

- $\sigma_{Smile} > \sigma_{ATM} \rightarrow$ positive zeta
- $\sigma_{Smile} < \sigma_{ATM} \rightarrow$ negative zeta

In AUD/JPY, the risk reversal is for downside as shown in Exhibit 14.22.

The equivalent zeta curve shown in Exhibit 14.23 reflects this. Of course, the zeta for an ATM strike is zero.

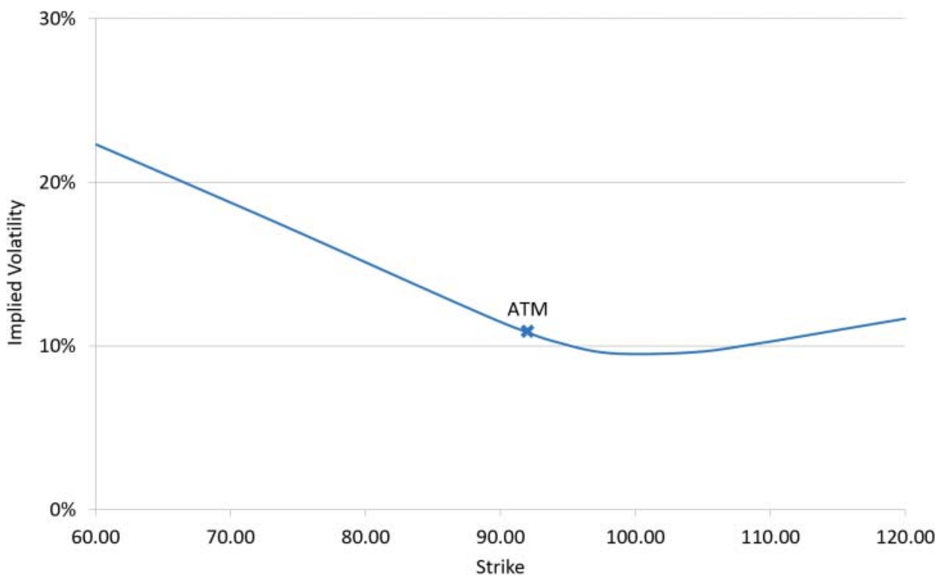


EXHIBIT 14.22 AUD/JPY 1yr volatility smile

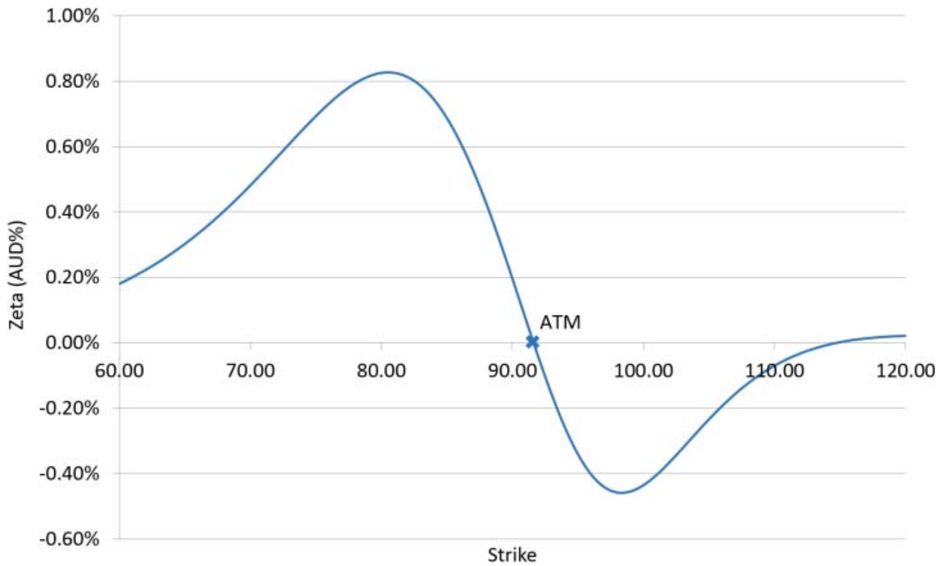


EXHIBIT 14.23 AUD/JPY 1yr zeta profile

In high-skew currency pairs:

- The high point of the zeta curve is often located around the 15d point on the high side of the smile. Being high on the zeta curve causes options to decay more quickly because of their additional value due to the smile.
- The low point of the zeta curve is often located around the 35d point on the low side of the smile.

It is important that traders know approximately where these points are in strike and delta terms because market participants often try to sell vanilla options around the most expensive point on the volatility smile and buy vanilla options around the cheapest point on the volatility smile.

Another popular strategy in high-skew currency pairs is to sell 15 delta vanilla options and buy 2 delta vanilla options, both on the high side of the smile. The idea behind this strategy is as follows:

- If spot doesn't move, the short 15d option decays (positively) more quickly than the long 2d option since the 2d option has a far lower premium.
- If spot blows up to the high side of the volatility smile, the 2d option relatively picks up in value far more than the 15d option as the ATM and risk reversals contracts increase sharply in price.

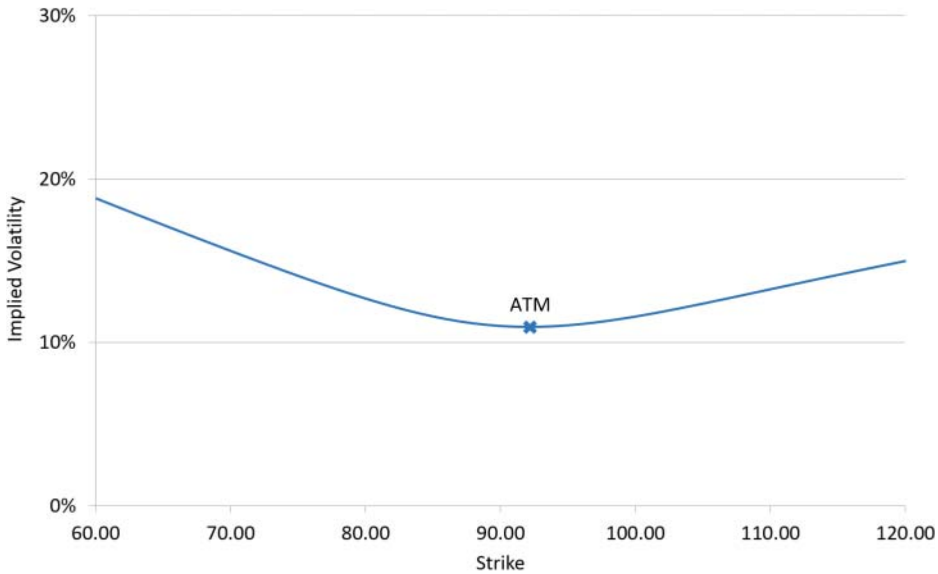


EXHIBIT 14.24 Volatility smile with positive wings and no skew

Unfortunately, the market does not make implementing this strategy easy: Offers on very low delta options on the high side of the risk reversal are often well above their theoretical midmarket (see Chapter 15).

Finally, in a currency pair with no risk reversal, the volatility smile is symmetric and the implied volatility never goes below the ATM as shown in Exhibit 14.24.

This results in the zeta profile shown in Exhibit 14.25, which is roughly symmetric and always positive.

Interest Rate Risk (Rho)

Within standard Black-Scholes mathematics, rho1 (ρ_1) and rho2 (ρ_2) are sensitivities to changes in the CCY1 and CCY2 continuously compounded risk-free interest rates respectively. In FX derivatives there are always two rho values to consider: one for each currency in the currency pair. Rho is most often quoted in per basis point (pbp) terms—the P&L change for a 0.01% change in interest rates.

The following analysis examines rho exposures for vanilla contracts using stylized Black-Scholes analysis where premiums and hence subtleties around the P&L currency are ignored.

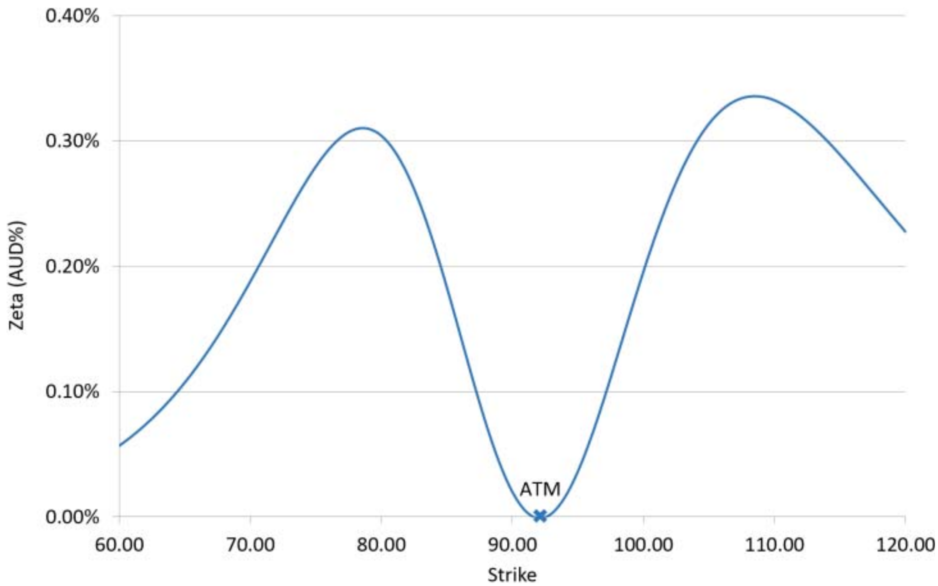


EXHIBIT 14.25 Zeta profile for volatility smile with positive wings and no skew

Future Cash and Forwards

Very generally, long-dated contracts have more interest rate risk than short-dated contracts. Future cash payments generate negative rho exposure linearly proportional to notional and tenor. In per basis point terms:

$$\rho_{Cash} \approx -0.01\% \,.Notional\, .\, T$$

where *Notional* is future cash notional and T is the time (in years) to the future cash delivery date. The negative sign in the formula indicates that, for example, USD must be sold forward to get longer USD rho. This can be confusing because when USD cash is *held*, higher return is generated with higher USD rates. The difference comes from holding the cash position versus taking delivery of a fixed cash amount in the future.

The same formula can be applied to forward contracts by considering each currency separately. For example, buying EUR100m EUR/USD 1yr forward at 1.3100 generates short EUR10k pbp EUR rho and long USD13.1k pbp USD rho.

Long Vanilla Call Option

The per basis point rho exposure on a long vanilla call option prior to maturity is shown in Exhibit 14.26:

- If the forward to expiry is far below the strike, the long CCY1 call vanilla has no payoff, no risk, and no rho exposures.

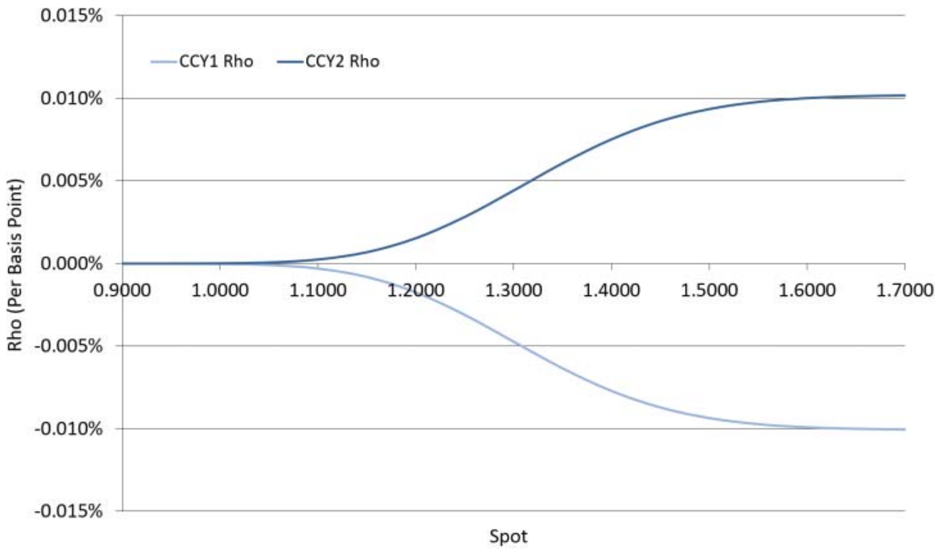


EXHIBIT 14.26 Long 1yr call option with 1.3100 strike rho

- If the forward to expiry is far above the strike, the long CCY1 call vanilla payoff behaves like a long forward and hence generates negative CCY1 rho and positive CCY2 rho exposures.

The rho exposure on a vanilla option contract is linked to its forward delta exposure. Exhibit 14.27 shows how a long 1yr CCY1 call vanilla has a long forward delta that produces short CCY1 rho and long CCY2 rho:

- *At 0% forward delta*: Per basis point rho is 0%.
- *Around the ATM*: Per basis point rho is 0.005%.
- *At 100% forward delta*: Per basis point rho is 0.01%.

Therefore:

$$\rho_{CCY1\ \ Call} \approx -0.01\%\ .\Delta_F\ .Notional\ .\ T$$

and

$$\rho_{ATM\ \ CCY1\ \ Call} \approx -0.01\%\ .\frac{1}{2}.Notional\ .T$$

where Δ_F is the forward delta expressed in notional currency %.

As noted, longer tenor options have higher rho exposure. Therefore, over time the rho exposures on a vanilla reduce as shown in Exhibit 14.28.

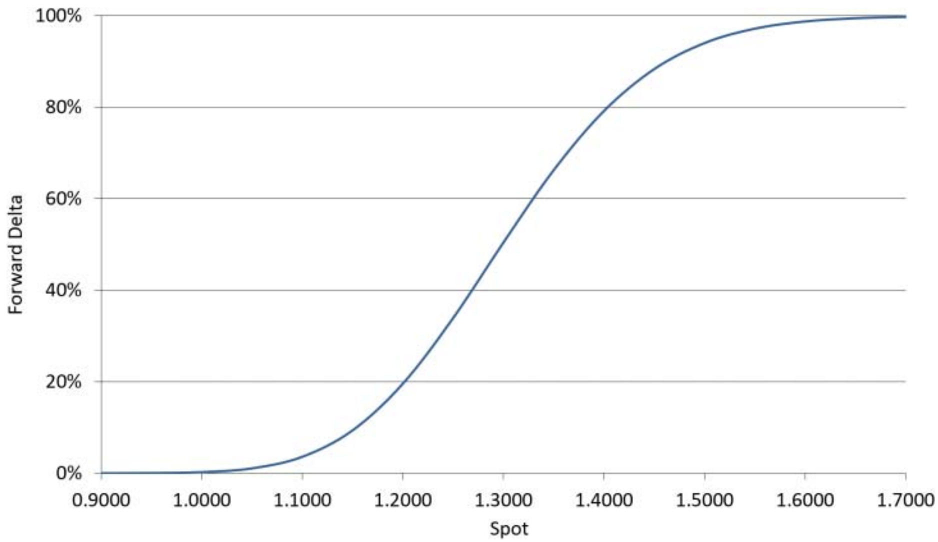


EXHIBIT 14.27 Long 1yr call option with 1.3100 strike forward delta

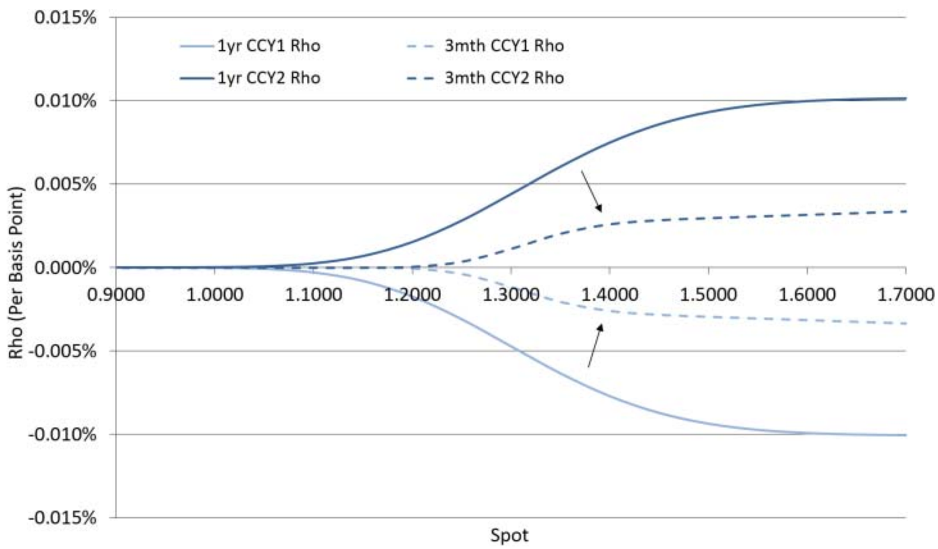


EXHIBIT 14.28 Long 1yr call option with 1.3100 strike rho over time

Long Vanilla Put Option

There are no surprises with the rho for a long vanilla put option. Prior to maturity, if the forward to expiry is far above the strike, the CCY1 put vanilla has no payoff and no risk. If the forward to expiry is far below the strike, the long CCY1 put vanilla is essentially a short forward position and hence generates long CCY1 rho and short CCY2 rho. These rho profiles are shown in Exhibit 14.29.

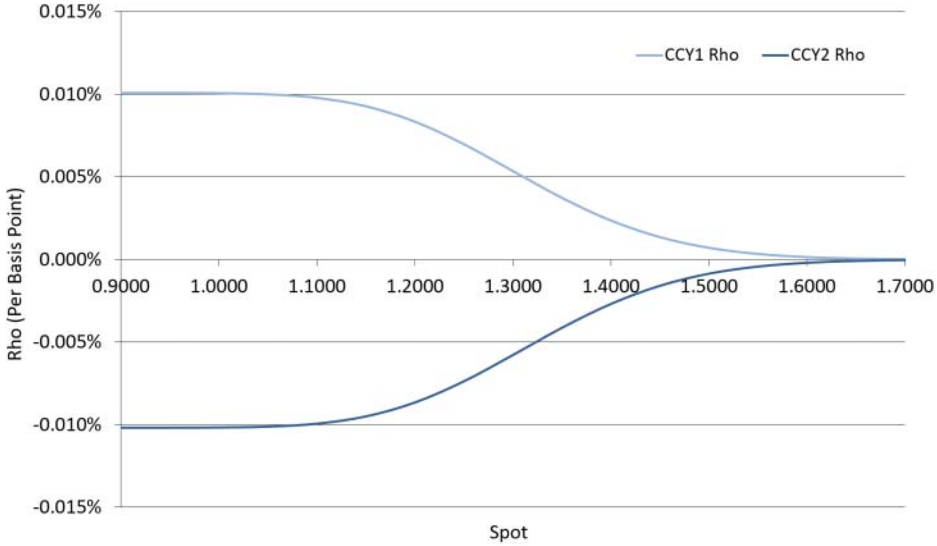


EXHIBIT 14.29 Long 1yr put option with 1.3100 strike rho

Long ATM Straddle

Combining long CCY1 call and CCY1 put vanillas with the same ATM strike and maturity gives a long ATM straddle position:

- With a lower forward to maturity, forward delta gets shorter, CCY1 rho gets longer, and CCY2 rho gets shorter.
- With a higher forward to maturity, forward delta gets longer, CCY1 rho gets shorter, and CCY2 rho gets longer.

The crossing point occurs around the ATM as shown in Exhibit 14.30. It is instructive to note that the same rho exposures as Exhibit 14.30 are generated by this straddle, a forward-hedged CCY1 call option or a forward-hedged CCY1 put vanilla option.

In general, if there is a negative correlation between spot and CCY1 interest rates or a positive correlation between spot and CCY2 interest rates, buying an ATM straddle results in a desirable trading position.

Exhibit 14.31 shows how the ATM straddle rho exposures change with implied volatility:

- At higher implied volatility, the rho exposures stretch out as the distribution widens.
- At lower implied volatility, the rho exposures compress as the distribution tightens.

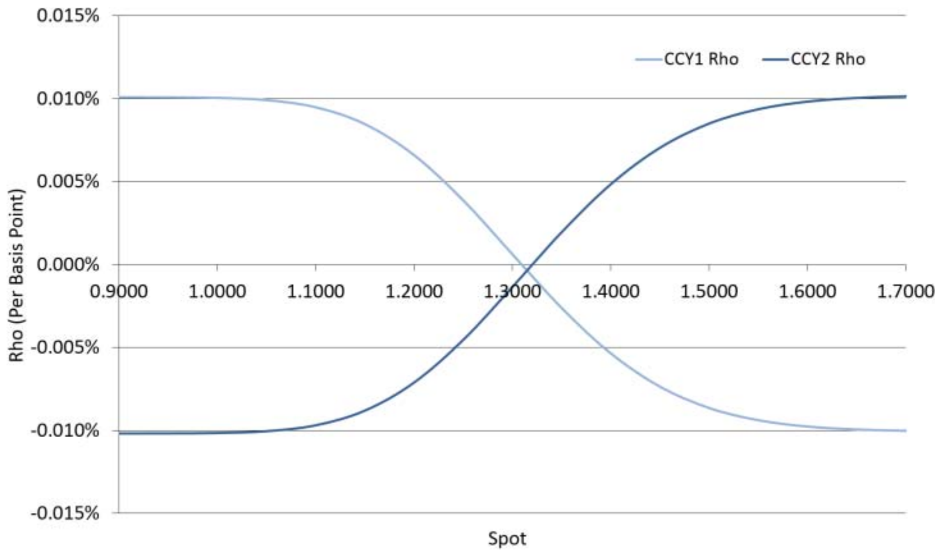


EXHIBIT 14.30 Long 1yr ATM straddle with 1.3100 strike rho

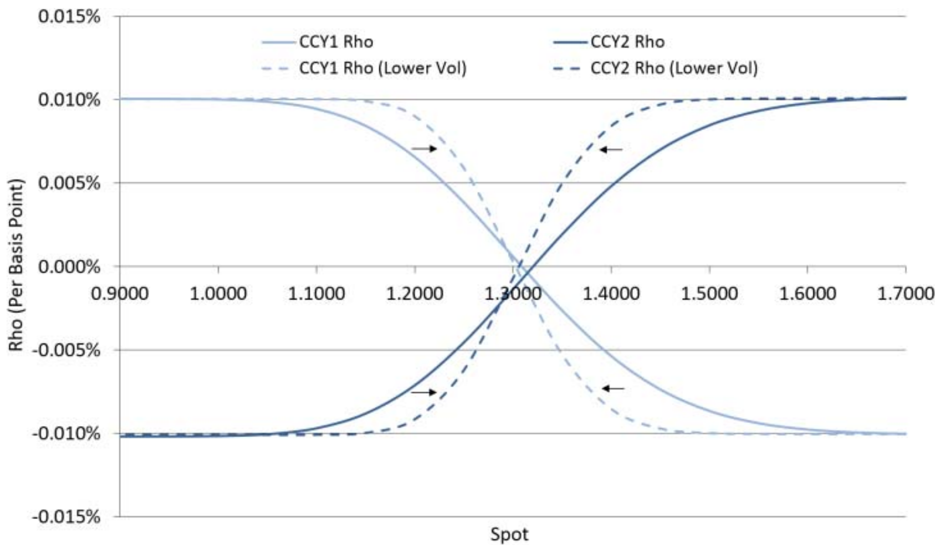


EXHIBIT 14.31 Long 1yr ATM straddle with 1.3100 strike rho at different implied volatility levels

Finally, changing interest rates with a fixed spot impacts the forward and this feeds through to the rho exposures on an ATM straddle:

- CCY1 rates higher → forward lower → longer CCY1 rho
- CCY2 rates higher → forward higher → longer CCY2 rho

Hence a long ATM straddle position contains *long interest rate gamma*, although this effect only becomes significant at longer tenors.