EM600 - Engineering Economics and Cost Analysis

Lecture 03: *3 "Worths" and Rates of Return*

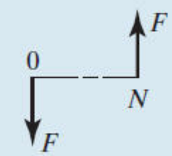
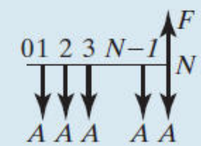
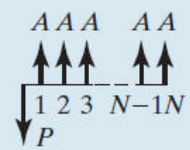
- References:

  – Park, Chan S. Contemporary Engineering Economics. New Jersey: Pearson Prentice Hall, 2006 (Chapter 4, 5, 7)
  – Ganguly, A. Engineering Economics Using Excel. New Jersey: SSE, 2008

After completing this module you should understand the following:

- Three Worths:
  - Present Worth, PW
  - Annual Equivalence, AE
  - Future Worth, FW
- Mutually Exclusive Alternatives
- IRR: Internal Rate of Return
- Incremental IRR

TABLE 3.6 Summary of Discrete Compounding Formulas with Discrete Payments

Flow Type	Factor Notation	Formula	Excel Command	Cash Flow Diagram
SINGLE	Compound amount (F/P, i, N)	$F = P(1+i)^N$	=FV(i,N,0,P)	
	Present worth (P/F, i, N)	$P = F(1+i)^{-N}$	=PV(i,N,0,F)	
EQUAL PAYMENT SERIES	Compound amount (F/A, i, N)	$F = A\left[\dfrac{(1+i)^N - 1}{i}\right]$	=FV(i,N,A)	
	Sinking fund (A/F, i, N)	$A = F\left[\dfrac{i}{(1+i)^N - 1}\right]$	=PMT(i,N,0,F)	
	Present worth (P/A, i, N)	$P = A\left[\dfrac{(1+i)^N - 1}{i(1+i)^N}\right]$	=PV(i,N,A)	
	Capital recovery (A/P, i, N)	$A = P\left[\dfrac{i(1+i)^N}{(1+i)^N - 1}\right]$	=PMT(i,N,P)	

Effective interest rate per Payment Period

- The effective interest rate can be assessed per payment period (periodic interest rate).

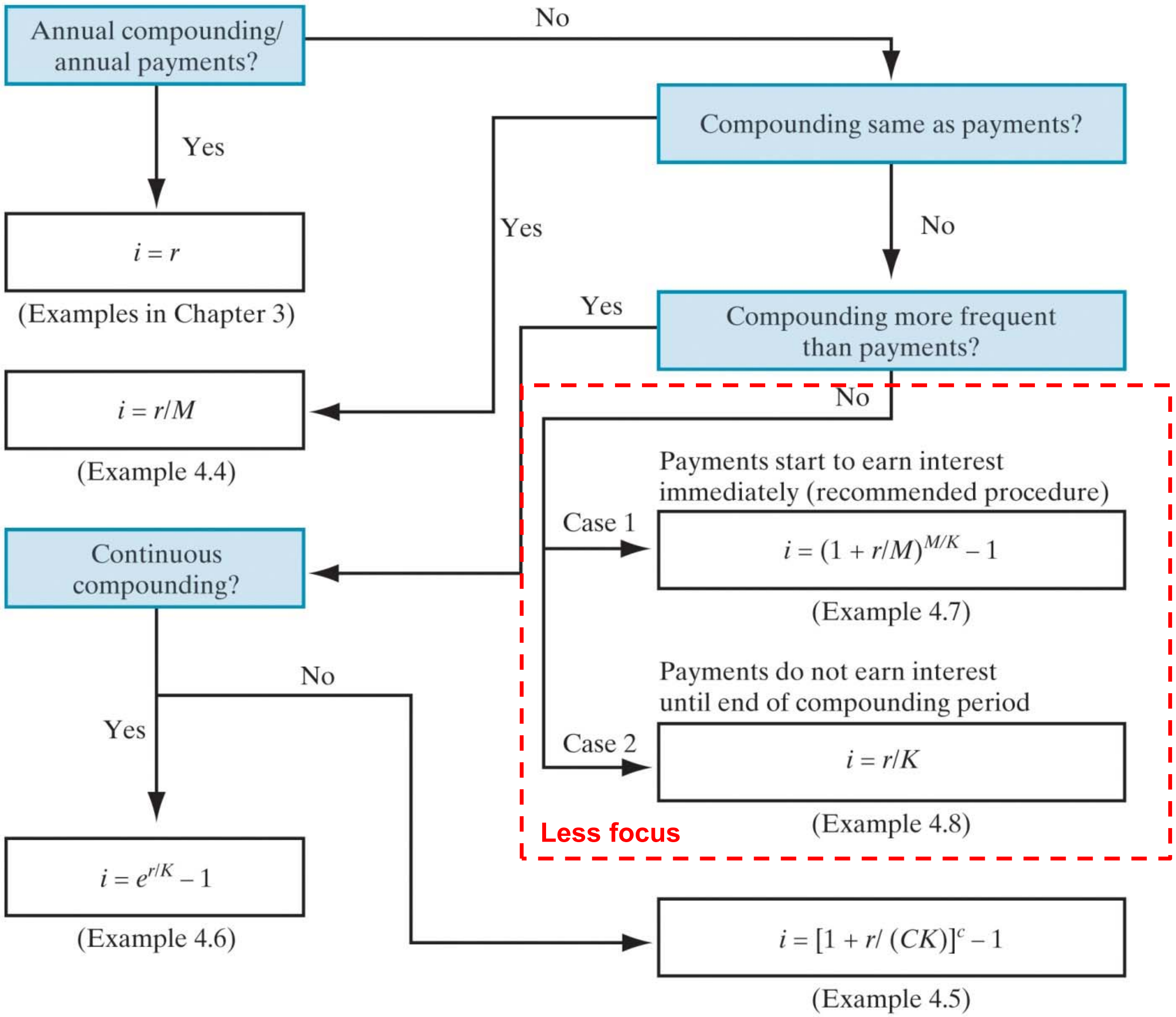
$$i = \left(1 + \frac{r}{M}\right)^{C} - 1 = \left(1 + \frac{r}{CK}\right)^{C} - 1$$

M = number of interest periods per year

C = number of interest periods per payment period

K = Number of payment periods per year

Figure 4.10 A decision flowchart demonstrating how to compute the effective interest rate *i* per payment period.



- Amortized Loans
  - Example 3: (Chan S. Park, example 4.12)
    - Using the tabular method find P_n, I_n and B_n for each n

TABLE 4.5 Creating a Loan Repayment Schedule with Excel (Example 4.12)

	A	B	C	D	E
1	Amount Borrowed		$5,000.00		
2	Interest Rate (%)		1%		
3	Loan Period (months)		24		
4					
5	Payment	Size of	Interest	Principal	Loan
6	Period	Payment	Payment	Payment	Balance
7					
8	0				$5,000.00
9	1	$235.37	$50.00	$185.37	$4,814.63
10	2	$235.37	$48.15	$187.22	$4,627.41
11	3	$235.37	$46.27	$189.09	$4,438.32
12	4	$235.37	$44.38	$190.98	$4,247.33
13	5	$235.37	$42.47	$192.89	$4,054.44

Step 1: Monthly installments, "A_n" as calculated, $235.37

Step 2: I_n = i × balance at end of previous period

$I_1 = 0.01 \times \$5000 = \$50$

$I_2 = 0.01 \times \$4814.63 = \$48.15$

Step 3:

$P_n = A_n - I_n$

Step 4:

$B_n = B_{n-1} - P_n$

$B_n = A(P/A, i, N-n)$

$I_n = (B_{n-1})i = A(P/A, i, N-n+1)i$

$P_n = A - I_n = A - A(P/A, i, N-n+1)i = A(P/F, i, N-n+1)$

Adding an addition "Extra" dollar per month to pay off the loan in S years

$B_n = (A + Extra)(P/A, i, S)$

- Present Worth Analysis:
  - Summary of Key Equations:

$$PW = -P + A(P/A,i,N) + F(P/F,i,N)$$

$$PW = -P + \sum_{n=0}^{N} A_n(P/F,i,n) + F(P/F,i,N)$$

$$PW = AE(P/A,i,N)$$

$$PW = FW(P/F,i,N)$$

where,

$P =$ initial investment (n = 0)

$A =$ annual cost / revenue (n = 1, 2, …, N)

$F =$ future costs, salvage value or expected income from sale of the item (n = N)

$PW =$ present worth of the investment taking A, F, i and N into account

$AE =$ annual equivalence / worth of the investment taking P, F, i and N into account

$FW =$ future worth of the investment taking P, A, i and N into account

$i =$ interest rate, MARR

$N =$ project life (n = 1, 2, …, N)

If PW > 0 ➔ ACCEPT
If PW = 0 ➔ INDIFFERENT
If PW < 0 ➔ REJECT

- Capitalized Equivalent Method
  - Uses
    - Perpetual project service life
      - N ➔ ∞
    - Extremely long project service life
      - N ≥ 50 years
  - Capitalized Cost Equation:
    - $CE = \frac{A}{i}$ * it comes from: $\lim_{N\to\infty}(P/A,i,N) = \lim_{N\to\infty}\left[\frac{(1+i)^N-1}{i(1+i)^N}\right] = \frac{1}{i}$
    - e.g.
      A = \$120K,
      i = 8%,
      N = 50 years

$$CE = \frac{A}{i} = A(P/A,i,N)$$

$$CE = \frac{\$120K}{8\%} = \$120K(P/A,8\%,50)$$

$$CE = \$1500K = \$1468K$$ *. . . Minimal difference*

- # Future Worth Analysis:
  - Summary of Key Equations:

$$FW = -P(F/P,i,N) + A(F/A,i,N) + F$$

$$FW = -P(F/P,i,N) + \sum_{n=0}^{N} A_n (F/P,i,N-n) + F$$

$$FW = AE(F/A,i,N)$$

$$FW = PW(F/P,i,N)$$

where,

$P =$ initial investment (n = 0)

$A =$ annual cost / revenue (n = 1, 2, . . . , N)

$F =$ future costs, salvage value or expected income from sale of the item (n = N)

$PW =$ present worth of the investment taking A, F, i and N into account

$AE =$ annual equivalence / worth of the investment taking P, F, i and N into account

$FW =$ future worth of the investment taking P, A, i and N into account

$i =$ interest rate, MARR

$N =$ project life (n = 1, 2, . . . , N)

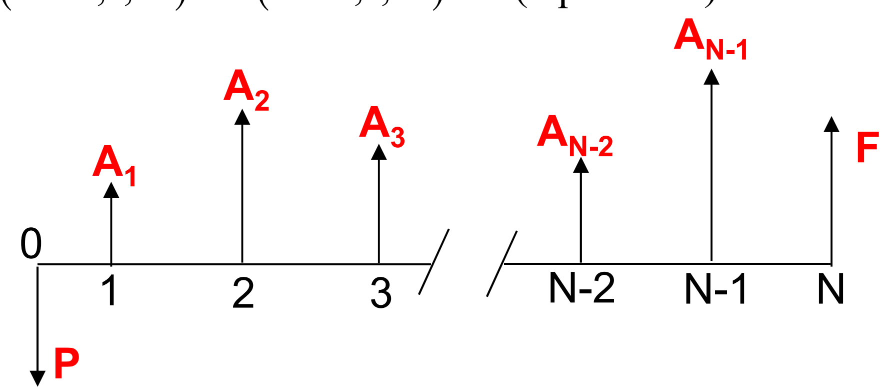
• Annual Equivalent Worth Analysis:

– Using the Key Equations:

$$AE = -P(A/P,i,N) + \sum_{n=1}^{N} A_n (P/F,i,N)(A/P,i,N) + F(A/F,i,N) \qquad \text{(equation 1)}$$

$$AE = -P(A/P,i,N) + \sum_{n=1}^{N} A_n (F/P,i,N-n)(A/F,i,N) + F(A/F,i,N) \qquad \text{(equation 2)}$$

- Used to calculate the AE from first principles.
- Assumes the net annual costs / revenues are NOT constant throughout the life of the project.



- **Equation 2:** Each net annual cost / revenue value is treated individually as a “present” value and is and is compounded to a future value (over “N-n” years) using the compound amount factor for single payments (F/P, i, N). This is then equated to an annual equivalence (AE) using the sinking fund factor (A/F,I,N).

- # Example 7:

A utility company is considering adding a second feedwater heater to its existing system unit to increase the efficiency of the system and thereby reduce fuel costs. The second feedwater heater to go with the 150-MW system will cost $1,650,000 and has a service life of 25 years. The expected salvage value of the unit is considered negligible. With the second unit installed, the efficiency of the system will improve from 55% to 56%. The fuel cost to run the feedwater is estimated at $0.05 kWh. The system unit will have a load factor of 85%, meaning that the system will run 85% of the year.

(a) Determine the equivalent annual worth of adding the second unit with an interest rate of 12%.

(b) If the fuel cost increases at the annual rate of 4% after first year, what is the equivalent annual worth of having the second feedwater unit at i=12%?

Example 6.1 (Chan S. Park)

- Example 1:
  - Install a 150MW feedwater heater, given:
    - Initial cost, P = $1,650,000
    - Service life, N = 25 years
    - Salvage value, S = 0
    - Expected improvement in fuel efficiency = 1%
      - Pre installation: 55%
      - Post installation: 56%
    - Fuel cost = $0.05kWh
    - Load factor = 85%
  - a. Determine the annual worth for installing the unit at i = 12% ➔ constant fuel price, uniform A
  - b. If the fuel cost increases at the annual rate of 4%, what is AE at i = 12%? ➔ varying fuel price (%), varying A, treat as Geometric Gradient Series

Example 6.1 (Chan S. Park)

- # Example 1: part (a)
  - Reduction in energy consumption = **4,870kW**
    - $$\left.\begin{aligned}&\text{Before adding the second unit, } \frac{150MW}{55\%} = 272,727kW\\&\text{After adding the second unit, } \frac{150MW}{56\%} = 267,857kW\end{aligned}\right\} 272,727kW - 267,857kW = 4,870kW$$
  - Annual operating hours @ 85% = **7,446 hours/year**
    - $(365)(24)(85\%) = 7,446 \text{ hours/year}$
  - Assuming constant fuel cost over the service life of the second heater,
    - $$A_{\text{fuel savings}} = (\text{reduction in energy consumption}) \times (\text{fuel cost}) \times (\text{operating hours per year})$$
      $$A_{\text{fuel savings}} = (4,870kW) \times (\$0.05/kWh) \times (7,446 hours/year)$$
      $$A_{\text{fuel savings}} = \boxed{\$1,813,149 \text{ per year}}$$
      $$\text{AE} = \text{-P}(\text{A/P,i,N}) + \text{A}$$
      $$AE = -\$1,650,000(\text{A/P},12\%,25) + \$1,813,149$$
      $$AE = -\$1,650,000(0.1275) + \$1,813,149$$
      $$\boxed{AE = \$1,602,774}$$

Excel:

32	AE	
33	P	$1,650,000
34	(A/P,12%,25)	0.1275
35	A	$1,813,149
36	AE	=PMT(12%,25,B33,,)+B35

Example 6.1 (Chan S. Park)

- Example 1: part (b)
  - From part (a), A_1 = \$1,813,149
  - Calculate present worth (PW)
    - $$PW = -P + A_1\left[\frac{1-(1+g)^N(1+i)^{-N}}{i-g}\right]$$

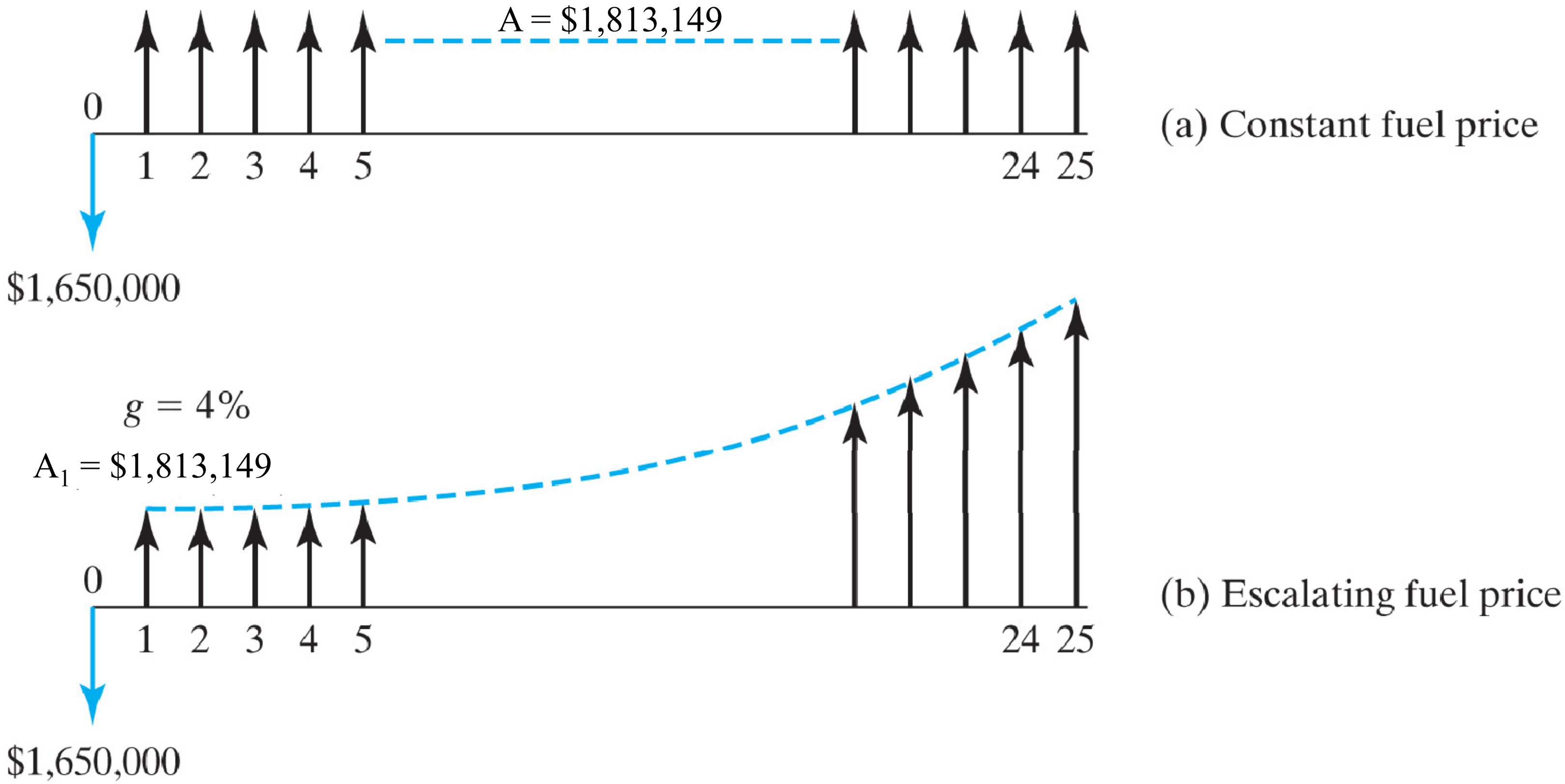
      $$PW = -\$1{,}650{,}000 + \$1{,}813{,}149\left[\frac{1-(1+4\%)^{25}(1+12\%)^{-25}}{12\% - 4\%}\right]$$

      $$PW = \$17{,}460{,}293$$
  - Calculate annual worth (AW)
    - $$AE = PW(A/P, i, N)$$

      $$AE = \$17{,}460{,}293(A/P, 12\%, 25)$$

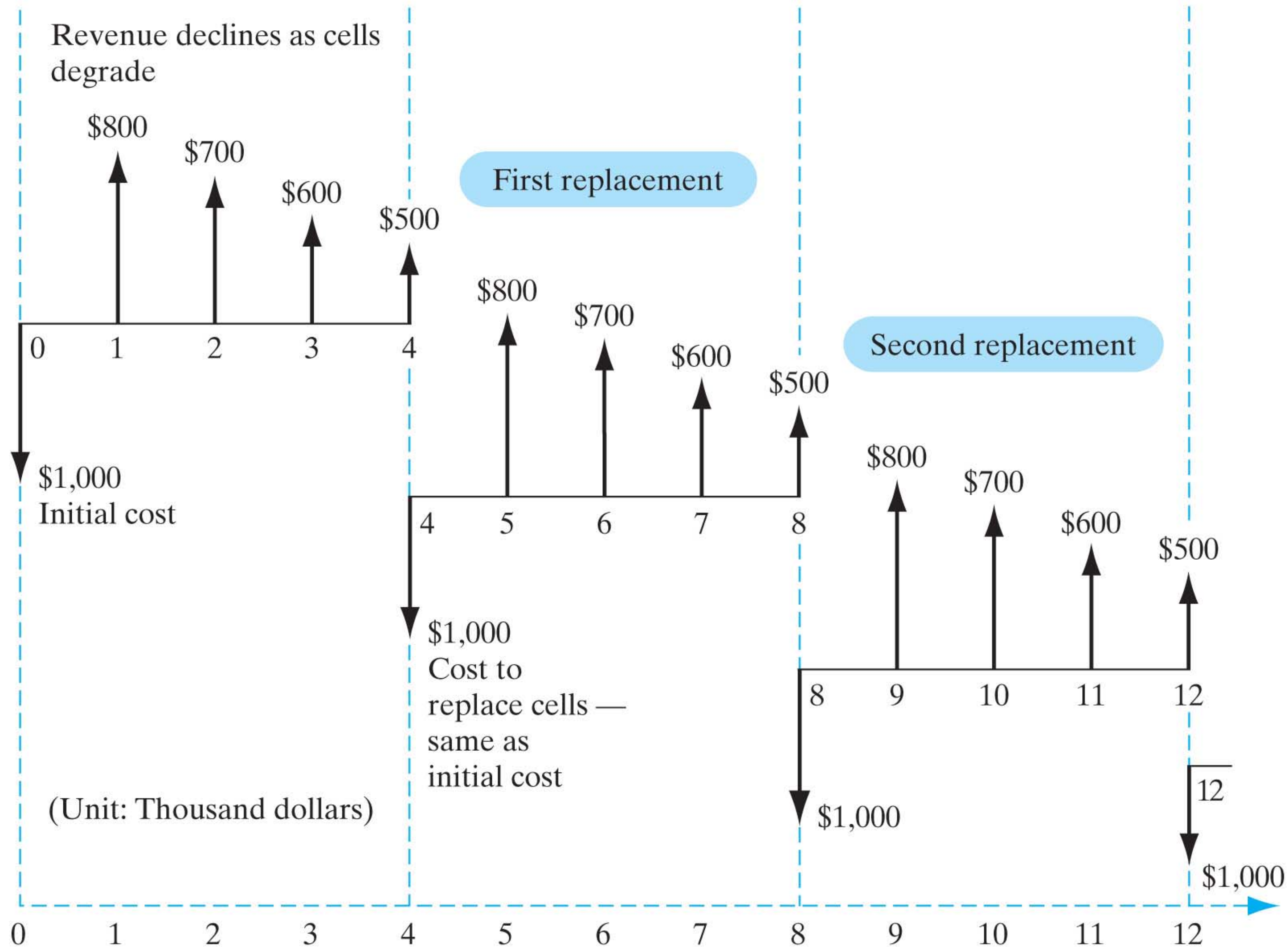
      $$AE = \$17{,}460{,}293(0.1275)$$

      $$AE = \$2{,}226{,}187$$



Example 6.1 (Chan S. Park, Figure 6.1)

- Repeating Cash Flow Cycles:
  - e.g. equipment, solar cells for example, may need to be replaced periodically as they will degrade over time compromising efficiency.
  - AE is calculated by examining the first cash flow cycle -
    - Calculate PW for the first cash flow cycle
    - Calculate AE for the first cash flow cycle
  - This yields the same solution as when the entire project is examined.



Conversion of repeating cash flow cycles into an equivalent annual payment. (Chan S. Park, Figure 6.2)

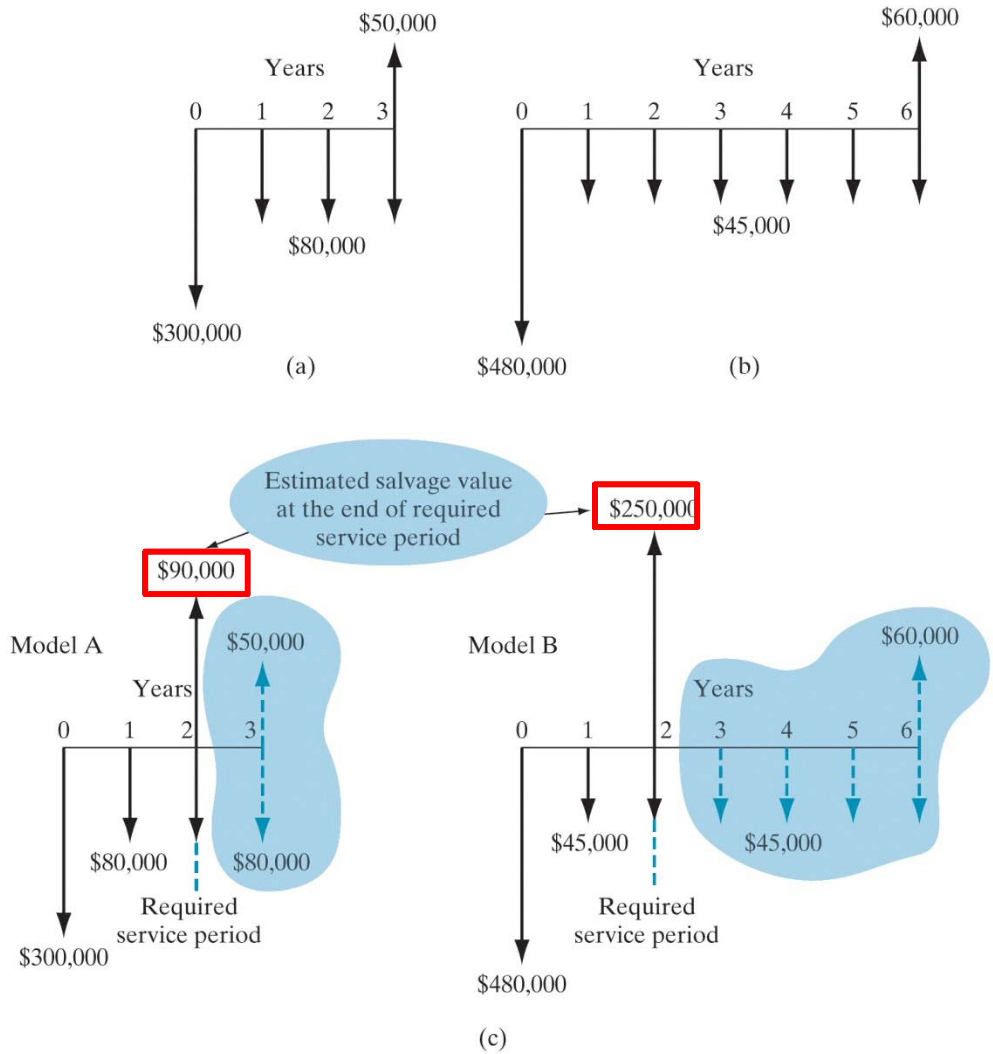
- Mutually Exclusive Alternatives
  - Two main considerations:
    - The analysis period equals the project life
      - Calculate as before.
    - The analysis period does not equal the project life
      - Lowest common multiple approach can be avoided if:
        - » The service of the selected alternative is required on a continuous basis.
        - » Each alternative will be replaced by an identical asset that has the same costs and performance
        - » Treat as a repeating cash flow in this case

• Example 2:

(project's life is longer than analysis period: salvage value)

- Model A costs $150,000 and has a life of 6,000 hours before it will require any major overhaul. Two units of model A would be required to remove the material within two years, and the operating cost for each unit would run to $40,000/year for 2,000 hours of operation. At this operational rate, the model would be operable for three years, at the end of which time it is estimated that the salvage value will be $25,000 for each machine.

- A more efficient model B costs $240,000 each, has a life of 12,000 hours without any major overhaul, and costs $22,500 to operate for 2,000 hours per year to complete the job within two years. The estimated salvage value of model B at the end of six years is $30,000. Once again, two units of model B would be required to remove the material within two years.



Example 5.11 and Figure 5.15 (Chan S. Park)

- Example 2:

(project's life is shorter than analysis period)

  - Given i =15%:

n	Model A	Model B
0	-$12,500	-$15,000
1	-$5,000	-$4,000
2	-$5,000	-$4,000
3	-$5,000 + $2,000	-$4,000
4		-$4,000 + $1,500

The Smith Novelty Company, a mail-order firm, wants to install an automatic mailing system to handle product announcements and invoices. The $12,500 semiautomatic model A will last three years, while the fully automatic model B will cost $15,000 and last four years. As business grows to a certain level, neither of the models may be able to handle the expanded volume at the end of year 5. Since both models have a shorter life than the required service period of 5 years, we need to make an explicit assumption of how the service requirement is to be met. Suppose that the company considers leasing equipment comparable to model A at an annual payment of $6,000 (after taxes) and with an annual operating cost of $5,000. ***Example 5.11 (Chan S. Park)***

Example 3.1:

(project's life is shorter than analysis period: lease/subcontract)

n	Model A	Model B
0	-$12,500	-$15,000
1	-$5,000	-$4,000
2	-$5,000	-$4,000
3	-$5,000 + $2,000	-$4,000
4	-$5,000 - $6,000	-$4,000 + $1,500
5	-$5,000 - $6,000	-$5,000 - $6,000

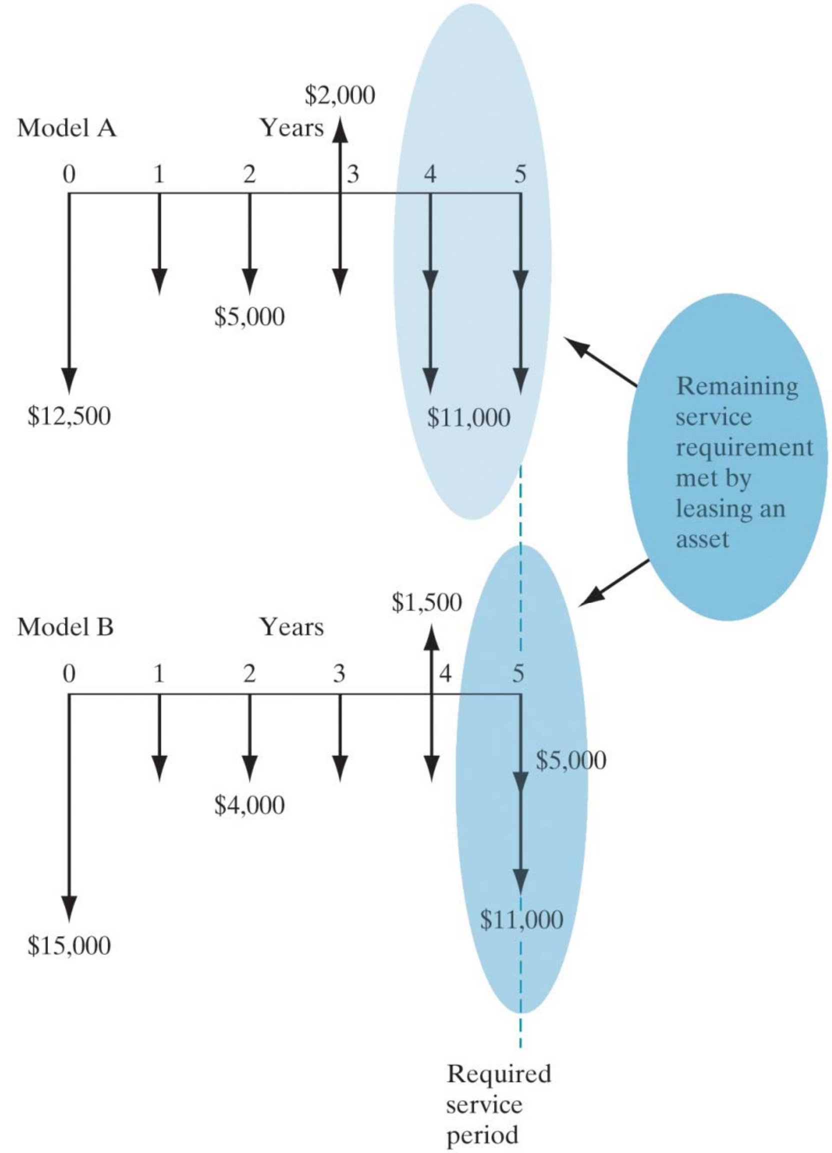


Figure 5.16
Comparison for service projects with unequal lives when the required service period is longer than the individual project life (Example 5.11).

- Example 3.2:

(Analysis period is not specified)

– Given:

n	Model A	Model B
0	-$12,500	-$15,000
1	-$5,000	-$4,000
2	-$5,000	-$4,000
3	-$5,000 + $2,000	-$4,000
4		-$4,000 + $1,500

i = 15%

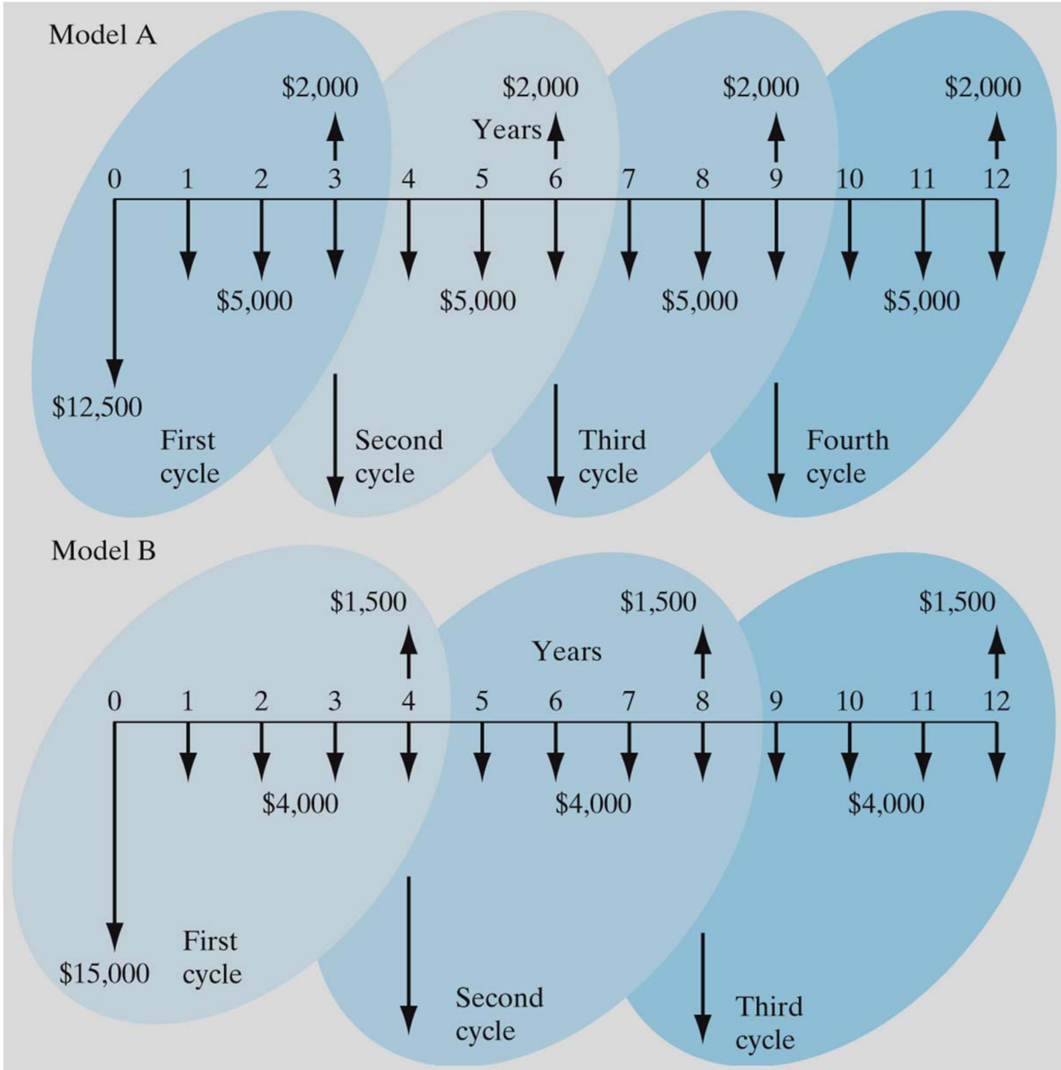
Example 5.11 (Chan S. Park)

– Find:
  - NPW of each alternative
  - AE of each alternative

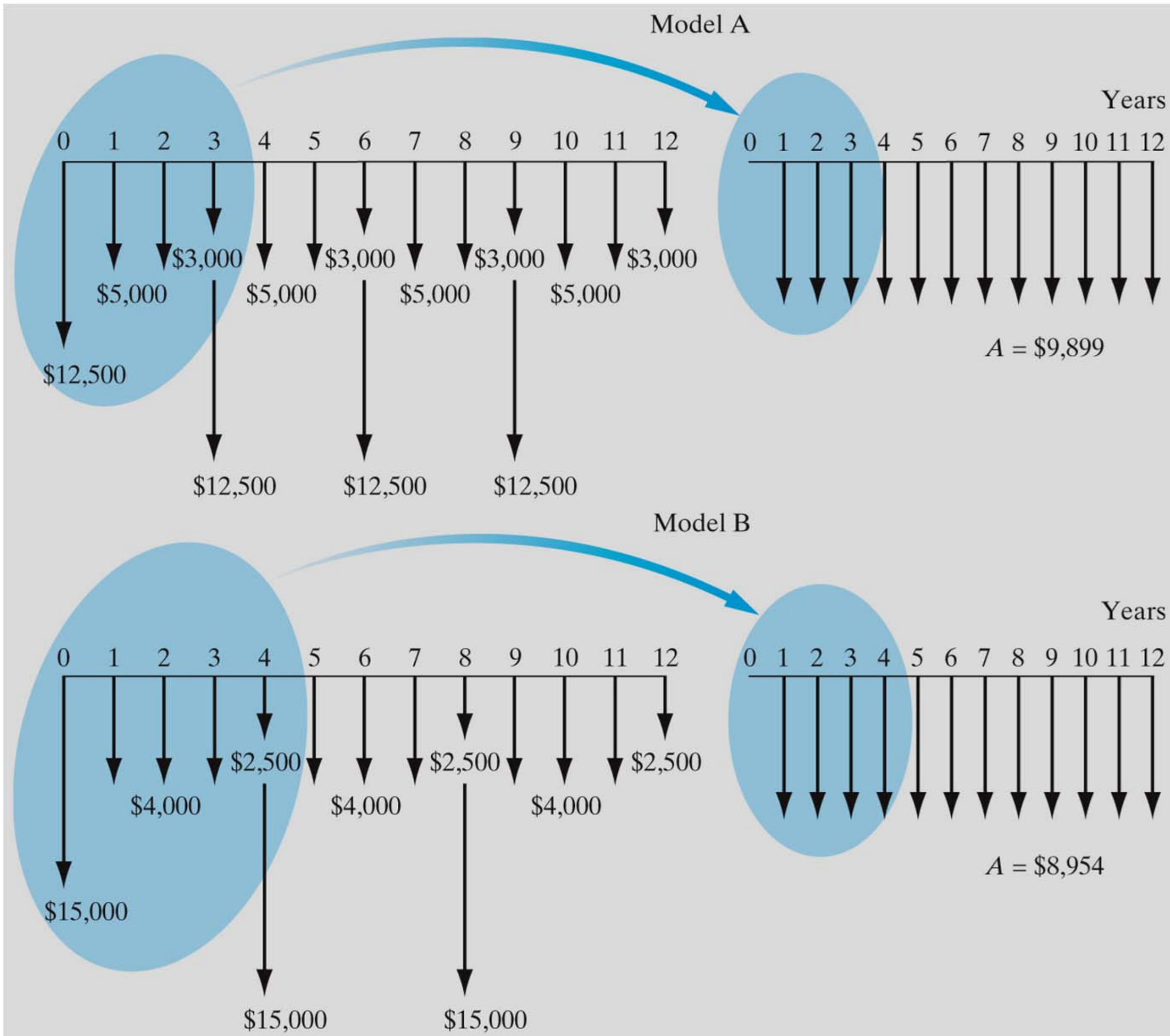
– Select the best alternative

- Example 3.2:
  - Method 1: lowest common multiple
    - The projects must be comparable in terms of TIME.
    - Identify the lowest common multiple or LCM (LCM = 12 years)
    - Any cash flow difference between the alternatives will be revealed during the first 12 years
    - The same cash flow patterns will then repeat within each 12 year cycle for an indefinite period.

Example 5.13 (Chan S. Park)



Example 5.13 (Chan S. Park, Figure 5.18)



Example 6.3 (Chan S. Park, Figure 6.3)

- Example 3.2: Model A
  - N = 3, therefore 4 replacements occur in a 12 year period.
  - PW for 1st investment cycle, PW_1:
    - $PW = -P + A(P/A,i,N) + F(P/F,i,N)$

      $PW = -\$12{,}500 - \$5{,}000(P/A,15\%,3) + \$2{,}000(P/F,15\%,3)$

      $PW = -\$12{,}500 - \$5{,}000(2.2832) + \$2{,}000(0.6575)$

      $\boxed{PW_1 = -\$22{,}601}$

Method 2 → $\boxed{AE_1 = PW_1(A/P,i,N) = -\$22{,}601(A/P,15\%,3) = -\$22{,}601(0.4380) = -\$9{,}899}$

  - Total PW for 4 replacement cycles:
    - $PW = -PW_1[1 + (P/F,i,N) + (P/F,i,2N) + (P/F,i,3N)]$

      $PW = -\$22{,}601[1 + (P/F,15\%,3) + (P/F,15\%,6) + (P/F,15\%,9)]$

      $PW = -\$22{,}601[1 + 0.6575 + 0.4323 + 0.2843]$

Method 1 →

$$\boxed{\begin{aligned}&PW = -\$53{,}657\\ &AE = PW(A/P,i,N) = -\$53{,}657(A/P,15\%,12) = -\$53{,}657(0.1845) = -\$9{,}900\end{aligned}}$$

- Example 3.2: Model B
  - N = 4, therefore 3 replacements occur in a 12 year period.
  - PW for 1st investment cycle, PW_1:
    - $PW = -P + A(P/A,i,N) + F(P/F,i,N)$

      $PW = -\$15{,}000 - \$4{,}000(P/A,15\%,4) + \$1{,}500(P/F,15\%,4)$

      $PW = -\$15{,}000 - \$4{,}000(2.8550) + \$1{,}500(0.5718)$

      $\boxed{PW_1 = -\$25{,}562}$

      Method 2 → $\boxed{AE_1 = PW_1(A/P,i,N) = -\$25{,}562(A/P,15\%,4) = -\$25{,}562(0.3503) = -\$8{,}954}$
  - Total PW for 3 replacement cycles:
    - $PW = -PW_1[1 + (P/F,i,N) + (P/F,i,2N)]$

      $PW = -\$25{,}562[1 + (P/F,15\%,4) + (P/F,15\%,8)]$

      $PW = -\$25{,}562[1 + 0.5718 + 0.3269]$

      Method 1 → $\boxed{\begin{array}{l} PW = -\$48{,}535 \\ AE = PW(A/P,i,N) = -\$48{,}535(A/P,15\%,12) = -\$48{,}535(0.1845) = -\$8{,}955 \end{array}}$

- Key Definitions: (Chan S. Park)
  - Internal Rate of Return (IRR):
    - *The internal rate of return is the interest rate charges on the unrecovered balance of the investment such that, when the project terminates, the unrecovered project balance will be zero.*
      - *The IRR equates the present worth, future worth and annual equivalence worth of the entire series of cash flows to zero.*

$$\boxed{\sum PW = 0; \qquad \sum AE = 0; \qquad \sum FW = 0}$$

    - *This internal rate of return is the return that a company would earn if it invested in itself rather than investing the money elsewhere.*

- Rate of Return Computational Methods:
  - Direct solution method
    - Two-flow transaction project (an investment followed by a single payment).

      OR
    - Project with a service life of two years of return.
    - Set up PW or FW equation using cash flows given and set equation equal to zero.
  - Trial-and-error method
    - Complicated cash flows.
    - Linear Interpolation.
  - Computer solution method
    - Complicated cash flows.
    - Solve graphically.
    - Use Excel IRR function.

- Direct solution method:
  - Example 4: (Chan S. Park, example 7.3)
    - Consider two investment projects with the following cash flow transactions:

n	Project 1	Project 2
0	-$2,000	-$2,000
1	0	$1,300
2	0	$1,500
3	0	-
4	$3,500	-

    - Calculate the IRR, i*, for each project.

- Direct solution method:
  - Example 4: (Chan S. Park, example 7.3)
    - Project 1:
      - **Present Worth (PW) Method:**

$$PW(i^*) = \sum_{n=1}^{N} F(1+i^*)^{-N} + P = 0$$

$$PW(i^*) = \$3{,}500(1+i^*)^{-4} - \$2{,}000 = 0$$

$$\$2{,}000 = \$3{,}500(1+i^*)^{-4} \Rightarrow 0.5714 = \frac{1}{(1+i^*)^4} \Rightarrow (1+i^*)^4 = \frac{1}{0.5714} \Rightarrow (1+i^*)^4 = 1.75$$

$$i^* = (1.75)^{-1/4} - 1$$

$$i^* = 0.1502 = 15.02\%$$

      - **Future Worth (FW) Method:**

$$FW(i^*) = \sum_{n=1}^{N} P(1+i^*)^{N} + F = 0$$

$$FW(i^*) = -\$2{,}000(1+i^*)^4 + \$3{,}500 = 0$$

$$\$3{,}500 = \$2{,}000(1+i^*)^4$$

$$1.75 = (1+i^*)^4$$

$$i^* = (1.75)^{1/4} - 1$$

$$i^* = 0.1502 = 15.02\%$$

- Direct solution method:
  - Example 4: (Chan S. Park, example 7.3)
    - Project 2:
      - **Present Worth (PW) Method:**

$$PW(i^*) = \sum_{n=1}^{N} F(1+i^*)^{-N} + P = 0$$

$$PW(i^*) = \$1,300(1+i^*)^{-1} + \$1,500(1+i^*)^{-2} - \$2,000 = 0$$

Transform into a quadratic equation where, $x = (1+i^*)^{-1}$

$$\Rightarrow PW(X) = \$1,500x^2 + \$1,300x - \$2,000 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1,300 \pm \sqrt{1,300^2 - 4(1,500)(2,000)}}{2(1,500)}$$

$$\Rightarrow x = 0.8 \ \text{ OR } \ x = -1.667$$

Solve for i* using the values for x

$$\Rightarrow x = 0.8 \ \therefore \ i^* = \frac{1}{0.8} - 1 = 25\%$$

$$\Rightarrow x = -1.667 \ \therefore \ i^* = \frac{1}{-1.667} - 1 = -160\% \ \ldots \text{ no economic significance}$$

      - **Future Worth (FW) Method yields the same answer.**

- Trial-and-Error Method:
  - Key Steps:
    - Estimate an IRR value for which PW > 0
    - Estimate an IRR value for which PW < 0
    - Interpolate to find where PW = 0
  - Linear Interpolation: The relationship is not truly linear, therefore the closer the PW values (calculated using the estimated IRR values) are to zero, the more accurate the answer will be.
    - Multiple answers can therefore exist.

- Trial-and-Error Method:
  - **Overview:** Linear Interpolation
    - Linear interpolation equation:

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{(x_2 - x_1)}$$

where,

y_1 & y_2 = estimated values for the IRR

x_1 & x_2 = corresponding PW values for y_1 and y_2

y = actual IRR at PW = 0

x = PW associated with actual IRR = 0

- Trial-and-error method:
  - Example 5: (Chan S. Park, example 7.4)
    - You need to consider a new safety review project for the production plan of a new drug for your company:

n	Costs	Savings	Net Cash Flow
0	-$13,000	$0	-$13,000
1	-$2,300	$6,000	$3,700
2	-$2,300	$7,000	$4,700
3	-$2,300	$9,000	$6,700
4	-$2,300	$9,000	$6,700
5	-$2,300	$9,000	$6,700
6	-$2,300	$9,000	$6,700

    - Calculate i* for this project.

- Trial-and-error method:
  - Example 5: (Chan S. Park, example 7.4)
    - $PW = -P + \sum_{n=1}^{N} A_n (P/F, i, n) + F(P/F, i, N)$

      Guess 1: IRR = 30%

$$PW = -\$13{,}000 + \$3{,}700(P/F, 30\%, 1) + \$4{,}700(P/F, 30\%, 2) + \$6{,}700 \begin{bmatrix} (P/F, 30\%, 3) \\ +(P/F, 30\%, 4) \\ +(P/F, 30\%, 5) \\ +(P/F, 30\%, 6) \end{bmatrix}$$

$$PW = -\$13{,}000 + \$3{,}700(0.7692) + \$4{,}700(0.5917) + \$6{,}700 \begin{bmatrix} (0.4552) \\ +(0.3501) \\ +(0.2693) \\ +(0.2072) \end{bmatrix}$$

$$PW = \$1{,}215$$

- Trial-and-error method:
  - Example 5: (Chan S. Park, example 7.4)
    - $PW = -P + \sum_{n=1}^{N} A_n (P/F,i,N) + F(P/F,i,N)$

Guess 2 : IRR = 35%

$$PW = -\$13{,}000 + \$3{,}700(P/F,35\%,1) + \$4{,}700(P/F,35\%,2) + \$6{,}700\begin{bmatrix}(P/F,35\%,3)\\+(P/F,35\%,4)\\+(P/F,35\%,5)\\+(P/F,35\%,6)\end{bmatrix}$$

$$PW = -\$13{,}000 + \$3{,}700(0.7407) + \$4{,}700(0.5487) + \$6{,}700\begin{bmatrix}(0.4604)\\+(0.3011)\\+(0.2230)\\+(0.1652)\end{bmatrix}$$

$$PW = -\$339$$

- Trial-and-error method:
  - Example 5: (Chan S. Park, example 7.4)
    - Use linear interpolation to find where PW = 0

$$y = y_1 + \frac{(x - x_1)(y_2 - y_1)}{(x_2 - x_1)}$$

$$(x, y) = (0, ?); \quad (x_1, y_1) = (\$1215, 30\%); \quad (x_2, y_2) = (-\$339, 35\%)$$

$$y = 30\% - \frac{(0 - 1215)(35\% - 30\%)}{(-339 - 1215)}$$

$$y = 33.91\%$$

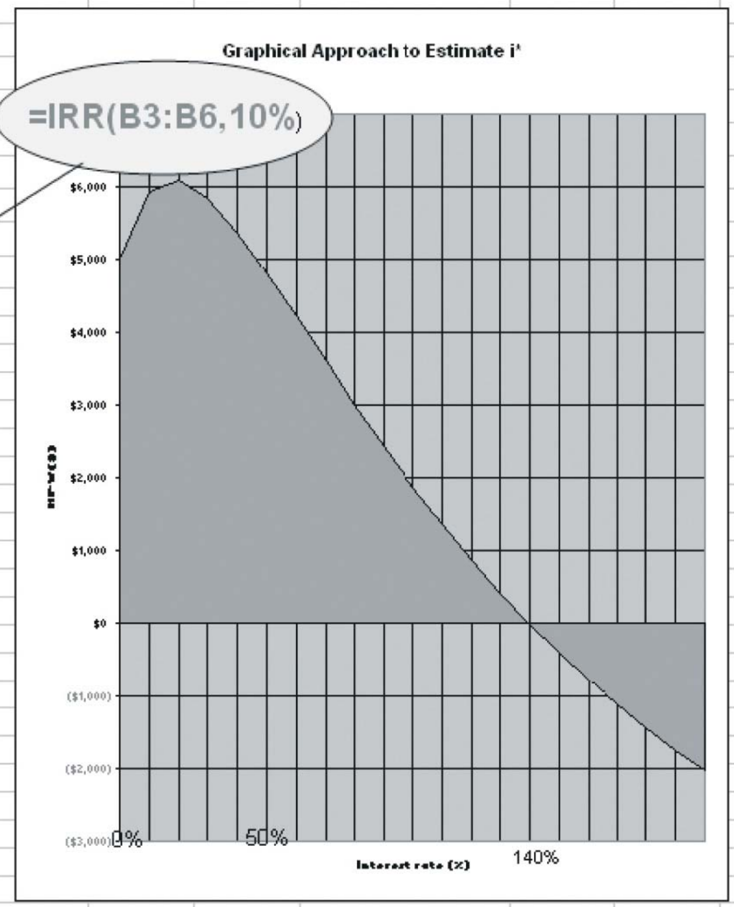
- Trial-and-error method:
  - Example 5: (Chan S. Park, example 7.4)
    - Remember:
      - The closer the PW values are to 0, the more accurate the linear interpolation will be.
        - » The relationship is not truly linear.
      - For example,
        - » If guesses of 33% and 34% are used instead of 30% and 35%,
        - » PW values of \$249 and -\$51 and an IRR of 33.83% results.
      - Multiple solutions therefore exist.

- Computer solution method:
  - Graphically
  - Excel IRR function

Graphical solution to rate-of-return problem for a typical non-simple investment (Example 7.5).
In fact, the project has two i*s — (i^*_1 = -15.95%, and i^*_2=140%).

Chan S. Park, example 7.5

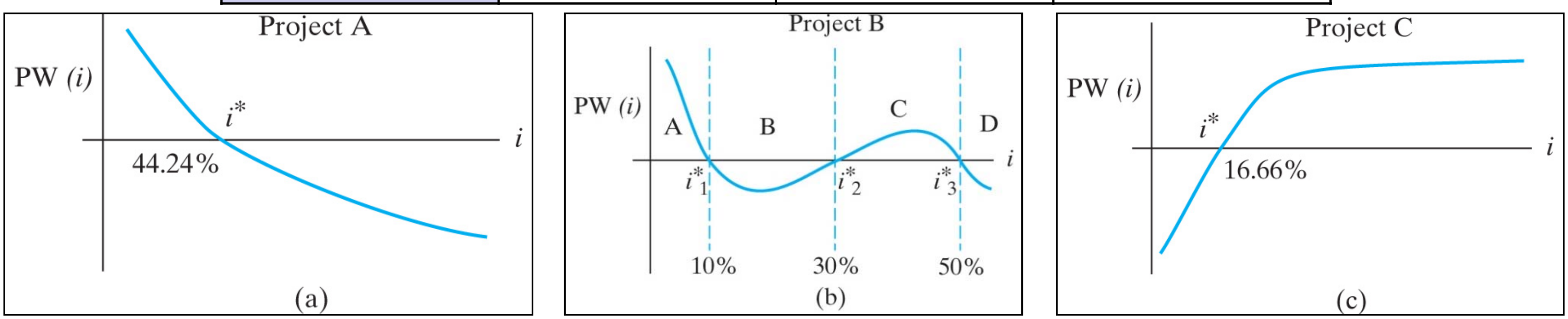
	A	B
1	Period	Cash Flow
2		
3	0	-10000
4	1	20000
5	2	20000
6	3	-25000
7		
8	IRR =	140%
9		
10	Interest (%)	NPW
11	0	$5,000
12	10	$5,928
13	20	$6,088
14	30	$5,840
15	40	$5,379
16	50	$4,815
17	60	$4,209
18	70	$3,597
19	80	$2,997
20	90	$2,422
21	100	$1,875
22	110	$1,359
23	120	$875
24	130	$422
25	140	($3)
26	150	($400)
27	160	($772)
28	170	($1,119)
29	180	($1,445)
30	190	($1,750)
31	200	($2,037)
32		



- Decision Rules and IRR:
  - Introduction:
    - Simple Investment:
      - An investment in which the initial cash flows are negative and only one sign change occurs in the remaining cash flow series.
    - Simple Borrowing:
      - An investment in which the initial cash flows are positive and only one sign change occurs in the remaining cash flow series.
    - Nonsimple Investment:
      - An investment in which one or more sign change occurs in the cash flow series.

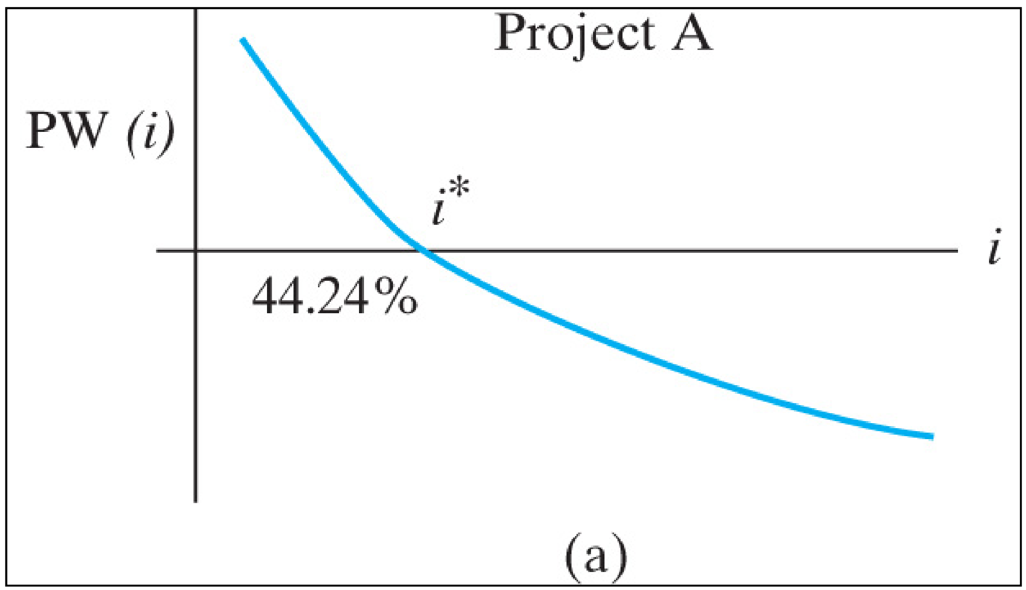
- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:

Period n	Net Cash Flow		
	Project A	Project B	Project C
0	-$1,000	-$1,000	$1,000
1	-$500	$3,900	-$450
2	$800	-$5,030	-$450
3	$1,500	$2,145	-$450
4	$2,000		



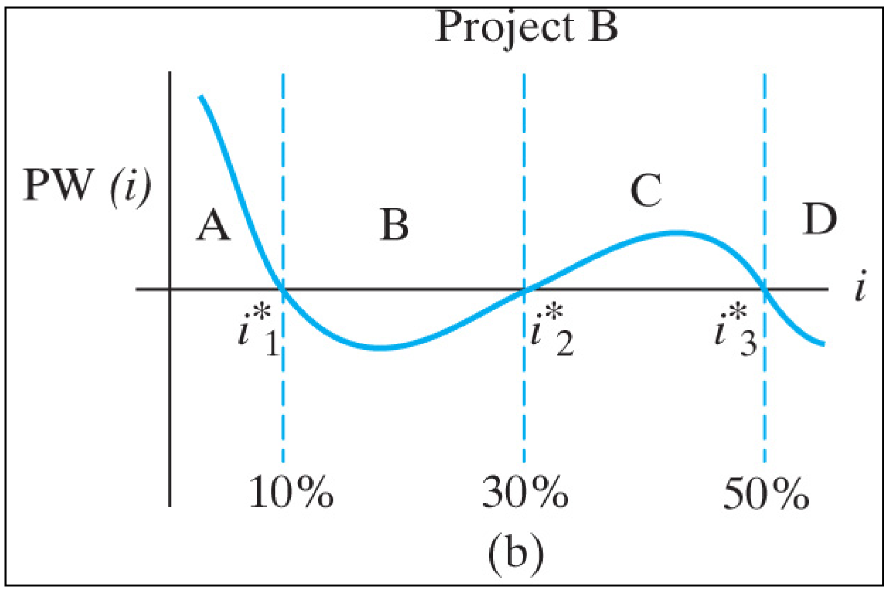
Present-Worth Profiles (Chan S. Park, Figure 7.1)

- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:
      - Project A represents many common simple investments.
      - The NPW profile (curve) crosses the x-axis only once.



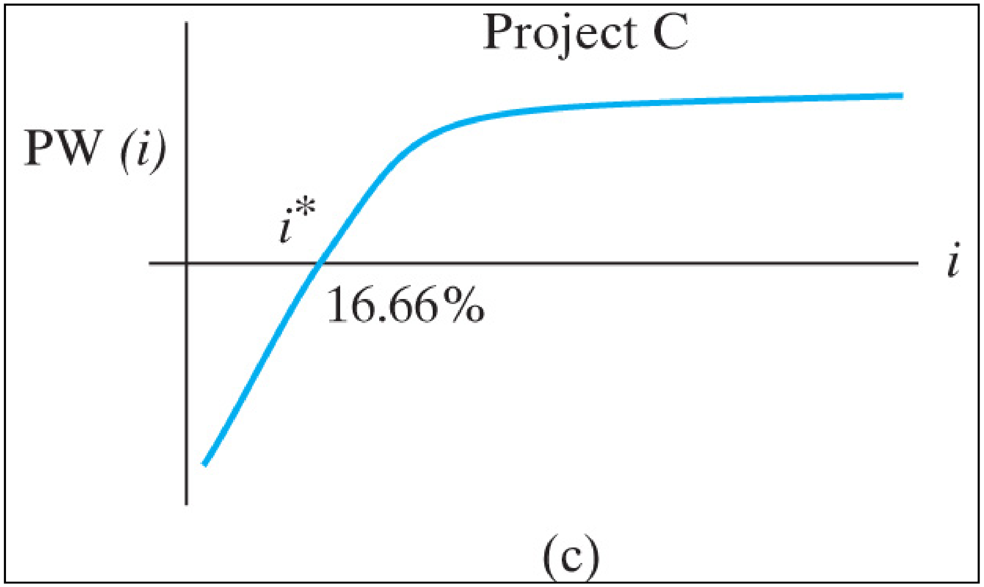
Present-Worth Profiles (Chan S. Park, Figure 7.1)

- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:
      - Project B represents a nonsimple investment.
      - The NPW profile (curve) crosses the x-axis at multiple points.



Present-Worth Profiles (Chan S. Park, Figure 7.1)

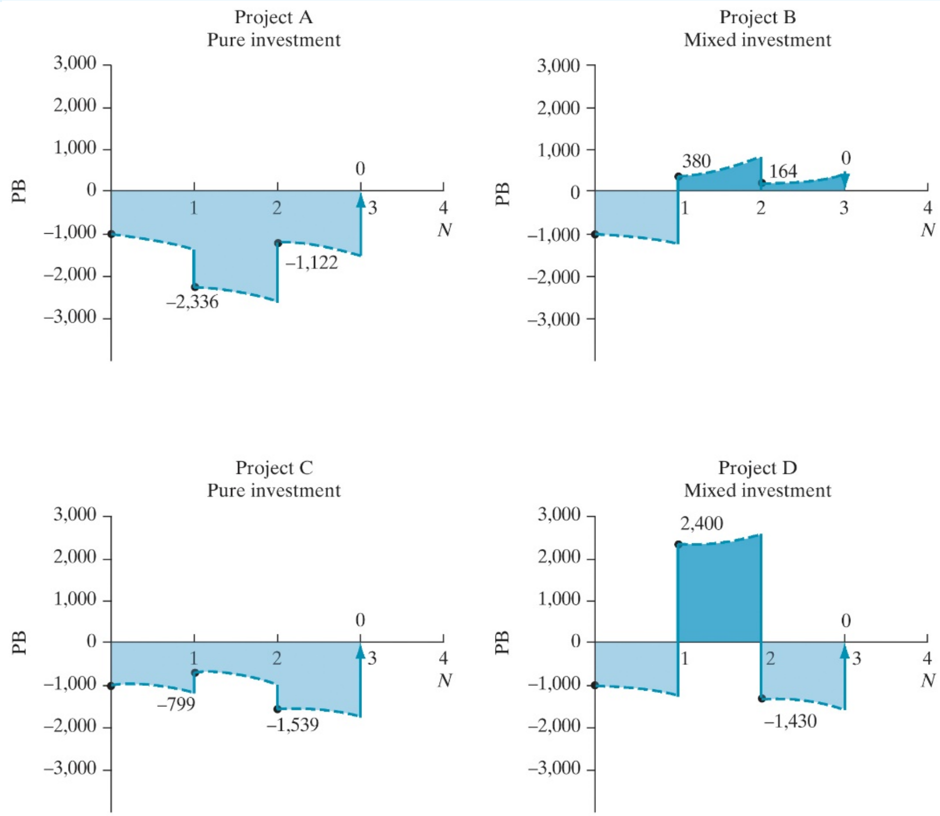
- Decision Rules and IRR:
  - Introduction: (Chan S. Park, example 7.1)
    - Simple and Nonsimple Investments:
      - Project C represents a simple borrowing cashflow.
      - There is only 1 sign change, however, the first cashflow is positive.



Present-Worth Profiles (Chan S. Park, Figure 7.1)

- Decision Rules and IRR:
  - Introduction:
    - Pure Investment:
      - An investment in which the firm never borrows money from the project.
      - Project balances (PB) are ≤ zero throughout the life of the investment with the first cashflow being negative.
      - Simple investments will always be pure investments.
    - Mixed investment:
      - An investment in which the firm borrows money from the project during the investment period.
      - PB > zero at some point during the life of the investment. Here the firms acts as a borrower, not an investor.

	Project Cash Flows			
n	**A**	**B**	**C**	**D**
0	−$1,000	−$1,000	−$1,000	−$1,000
1	1,000	1,600	500	3,900
2	2,000	−300	−500	−5,030
3	1,500	−200	2,000	2,145
i^*	33.64%	21.95%	29.95%	(10%, 30%, 50%)



Examples of Pure and Mixed investment projects (Chan S. Park, Figure 7.5)

- Recall the following important definitions:
  - MARR:
    - **M**inimum **A**ttractive **R**ate of **R**eturn
    - The minimum interest rate that the firm wants to earn on its investment.
  - IRR:
    - **I**nternal **R**ate of **R**eturn
    - The actual interest rate that the firm earns on its investment.
    - The IRR equates the present worth, future worth and annual equivalence worth of the entire series of cash flows to zero.

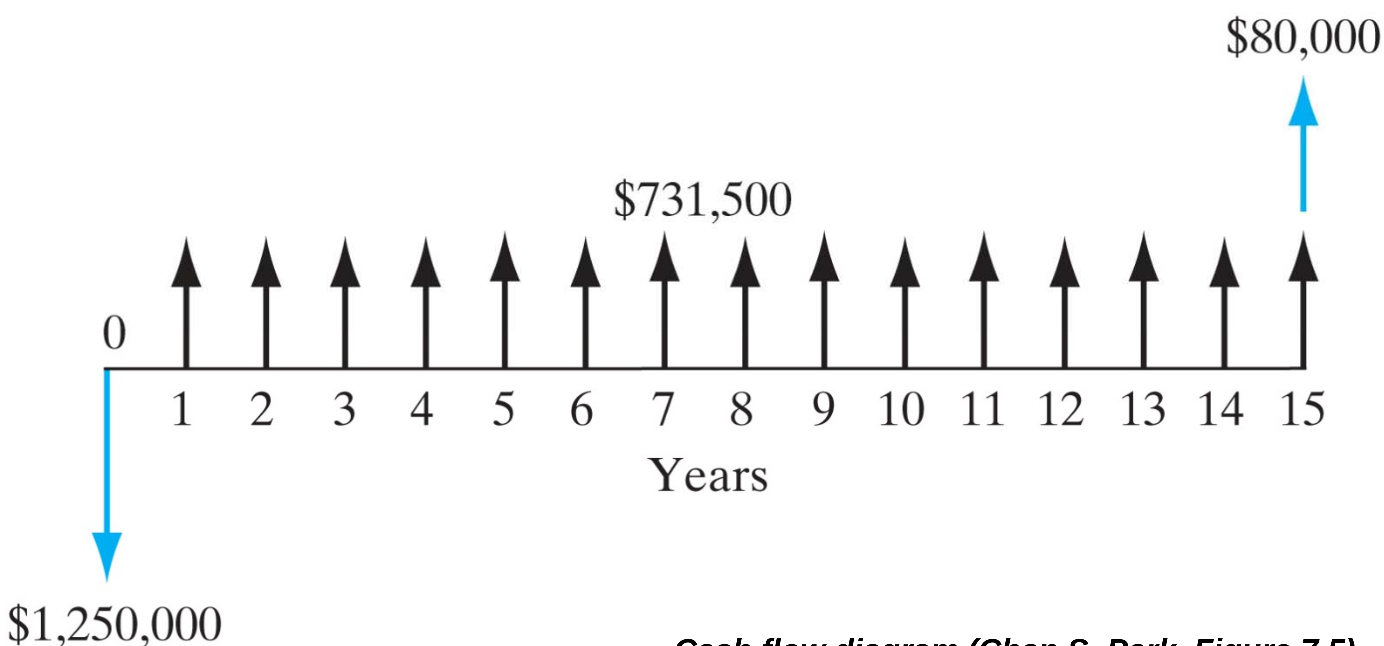
- Decision rules* for ***pure investment projects***:
  - If IRR > MARR ➔ ACCEPT
  - If IRR = MARR ➔ INDIFFERENT
  - If IRR < MARR ➔ REJECT

* **Note:**

Only applicable for single project evaluation.

Mutually exclusive investment projects need the ***incremental analysis approach*.**

- Example 6: (Chan S. Park, example 7.7)
  - For the following cash flows:
    - Calculate the IRR for the investment.
    - Should the investment be accepted or rejected? (MARR = 18%)



Cash flow diagram (Chan S. Park, Figure 7.5)

- Example 6: (Chan S. Park, example 7.7)
  - Trial-and-error:

$$PW = -P + A(P/A,i,N) + F(P/F,i,N)$$

Guess 1: IRR = 55%

$$PW = -\$1{,}250{,}000 + \$731{,}500(P/A,55\%,15) + \$80{,}000(P/F,55\%,15)$$

$$PW = \$78{,}254$$

Guess 2: IRR = 60%

$$PW = -\$1{,}250{,}000 + \$731{,}500(P/A,60\%,15) + \$80{,}000(P/F,60\%,15)$$

$$PW = -\$31{,}821$$

Using linear interpolation as before:

$$\boxed{y = i^* = 58.55\%}$$

  - Excel IRR function:
    - i* = 58.47%

i* > MARR ➔ ACCEPT

n	Outflow	Inflow	Net Cash Flow
0	150	50	-100
1	100	360	260
2	218	50	-168

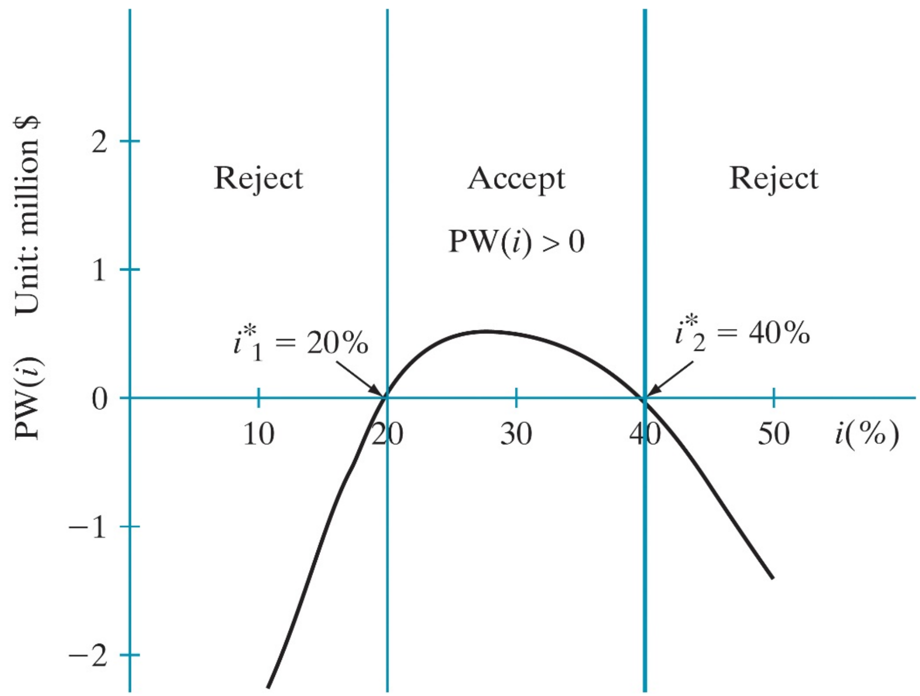


Figure 7.8 NPW plot for a nonsimple investment with multiple rates of return (Example 7.8).

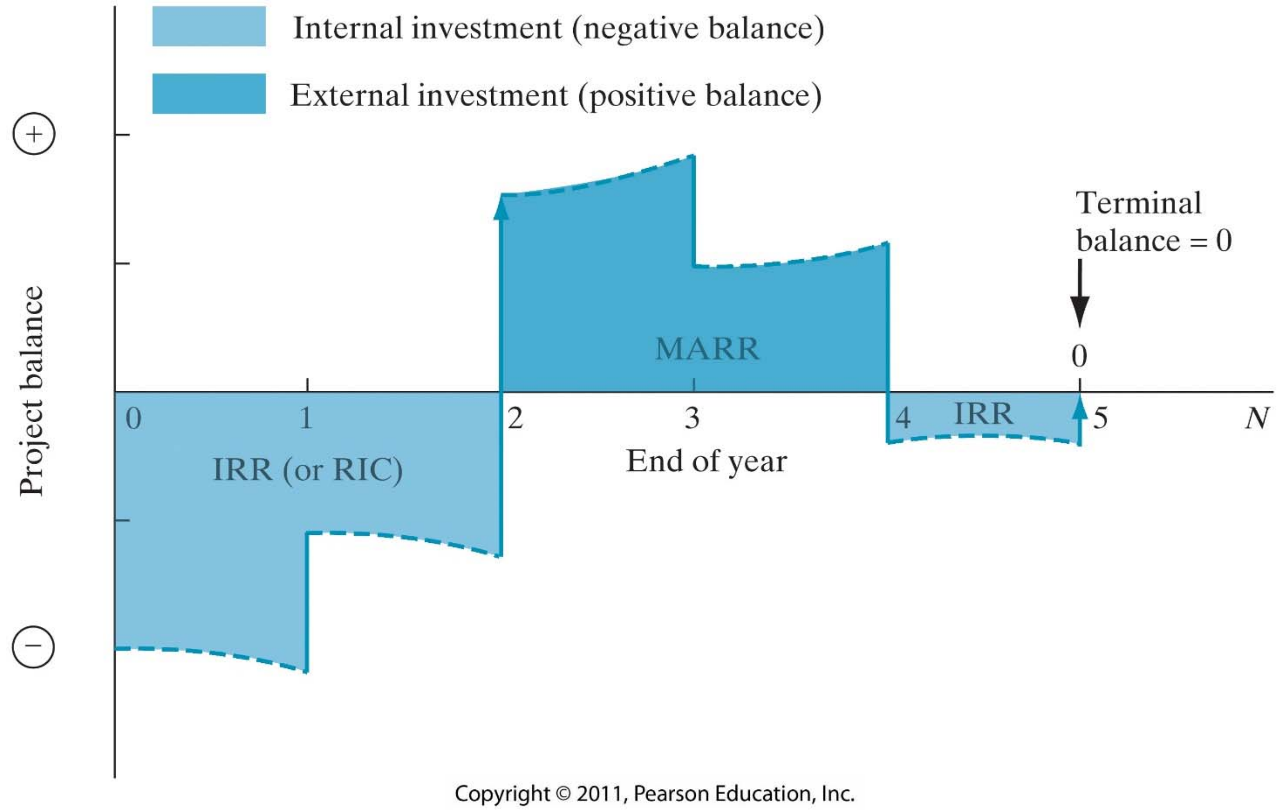


Figure 7.7 Computational logic for IRR (mixed investment).

Steps for determining the IRR for a mixed investment:

Step 1. Identify the MARR (or external interest rate).

Step 2. Calculate $\text{PB}(i, \text{MARR})_n$ (or simply PB_n) according to the rule

$$\text{PB}(i, \text{MARR})_0 = A_0.$$

$$\text{PB}(i, \text{MARR})_1 = \begin{cases} \text{PB}_0(1 + i) + A_1, \text{ if } \text{PB}_0 < 0 \\ \text{PB}_0(1 + \text{MARR}) + A_1, \text{ if } \text{PB}_0 > 0 \end{cases},$$

$$\vdots$$

$$\text{PB}(i, \text{MARR})_n = \begin{cases} \text{PB}_{n-1}(1 + i) + A_n, \text{ if } \text{PB}_{n-1} < 0 \\ \text{PB}_{n-1}(1 + \text{MARR}) + A_n, \text{ if } \text{PB}_{n-1} > 0 \end{cases}.$$

(As defined in the text, A_n stands for the net cash flow at the end of period n. Note that the terminal project balance must be zero.)

Step 3. Determine the value of i by solving the terminal project balance equation

$$\text{PB}(i, \text{MARR})_N = 0.$$

The interest rate i is the RIC (or IRR) for the mixed investment.

	A	B	C	D	E
1					
2	**Example 7.9 Calculating the Return on Invested Capital (RIC)**				
3					
4	**MARR =**	**6%**			
5	**Guess RIC**	**6.130%**			
6					
7		**Cash Flow**	**PB(RIC,MARR)**		
8					
9	**0**	**-1000**	**-1000**		
10	**1**	**3900**	**2839**	←	**:=IF(C9<0,C9*(1+$B$5)+B10,C9*(1+$B$4)+B10)**
11	**2**	**-5030**	**-2021**	←	**:=IF(C10<0,C10*(1+$B$5)+B11,C10*(1+$B$4)+B11)**
12	**3**	**2145**	**0**	←	**:=IF(C11<0,C11*(1+$B$5)+B12,C11*(1+$B$4)+B12)**
13					

Goal Seek

Set cell: $C$12

To value: 0

By changing cell: $B$5

OK Cancel

Figure 7.10 Calculating the return on invested capital (or true IRR) using Excel.

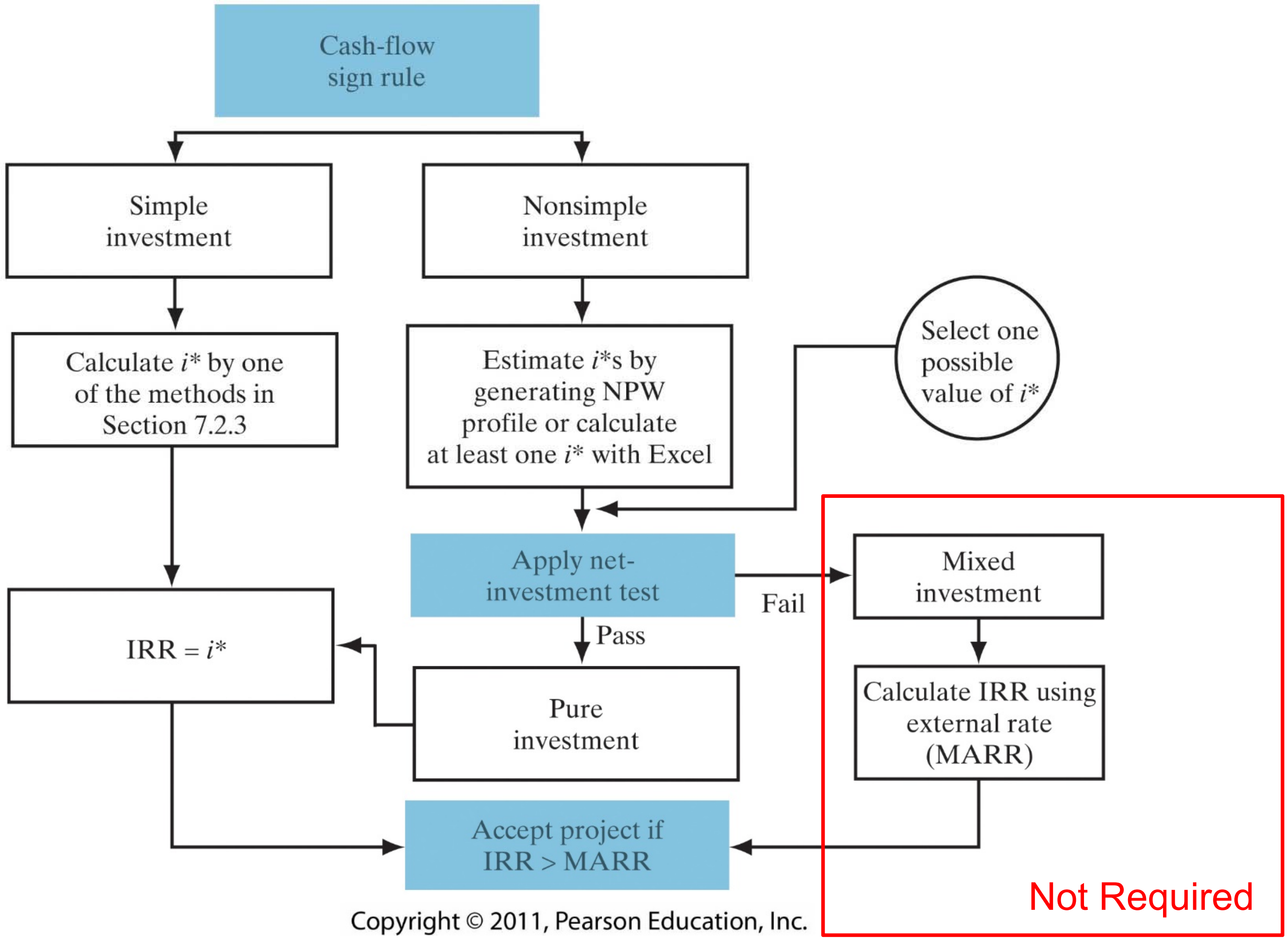


Figure 7.11 Summary of IRR criterion: A flowchart that summarizes how you may proceed to apply the net cash-flow sign rule and net-investment test to calculate IRR for a pure as well as a mixed investment.

- Mutually Exclusive Alternatives:
  - Two situations:
    - Alternatives with the same economic service life.
    - Alternatives that have unequal service lives.
  - Incremental IRR analysis is required because:
    - IRR is a relative (percentage) measure.
    - IRR ignores the scale of the investment
    - IRR cannot be analyzed in the same way as the 3 worths.

TABLE 7.6 Flaws in Project Ranking by IRR

n	A1	A2
0	−$1,000	−$5,000
1	2,000	7,000
IRR	100%	40%
PW(10%)	$818	$1,364

- Incremental IRR:
  - What is Incremental IRR?
    - The internal rate of return is calculated based on the incremental investment.
    - The incremental investment is calculated based on choosing a large project over a smaller project.
  - Incremental IRR considers ***increments of investment,*** therefore:

$$\boxed{\text{Cash Flow Difference} = \text{higher Investment Cost Project B} - \text{lower investment cost project A}}$$

- This ranking is IMPORTANT
- Investment cost = P

- Incremental IRR:
  - Decision rules:
    - If $IRR_{B\text{-}A} > MARR \rightarrow$ select B
    - If $IRR_{B\text{-}A} = MARR \rightarrow$ select either project
    - If $IRR_{B\text{-}A} < MARR \rightarrow$ select A

- Incremental IRR
  - Initial investments are equal?
    - Set up the increment so that the first non zero flow is negative.
    - e.g.

n	A	B	A - B
0	-$10,000	-$10,000	$0
1	$650	$6,740	-$6,090
2	$4,125	$3,350	$775
3	$6,950	$2,200	$4,750
4	$3,880	$1,470	$2,410
IRR	**17%**	**19%**	**12.88%**

- Incremental IRR
  - More than 2 mutually exclusive alternatives?
    - Compare in pairs by successive examination.
      - Consider 3 projects A, B, C
      - Verify IRR_A and IRR_B and IRR_C are each > MARR
        - » Any project whose IRR < MARR can be ruled out
      - Compare each incremental pair:
        - » A and B
        - » A and C
        - » B and C
      - Find the best alternative.

- Incremental IRR
  - More than 2 mutually exclusive alternatives?
    - e.g. MARR = 15%, select best alternative

n	A	B	C	A - B	C - A	C - B
0	($2,500)	($1,500)	($4,000)	($1,000)	($1,500)	($2,500)
1	$2,000	$1,300	$2,500	$700	$500	$1,200
2	$1,500	$1,000	$2,500	$500	$1,000	$1,500
3	$1,300	$1,000	$1,500	$300	$200	$500
IRR	**45.69%**	**56.49%**	**31.63%**	**27.61%**	**7.16%**	**15.17%**

- **Select alternative A.**

Select A **Select A** **Select C**

Note - A already selected over C

- Incremental IRR
  - Example 7: (Chan S. Park, example 7.13)
    - Based on the following data and a MARR of 15% (N = 6), using IRR, which manufacturing option should be chosen?

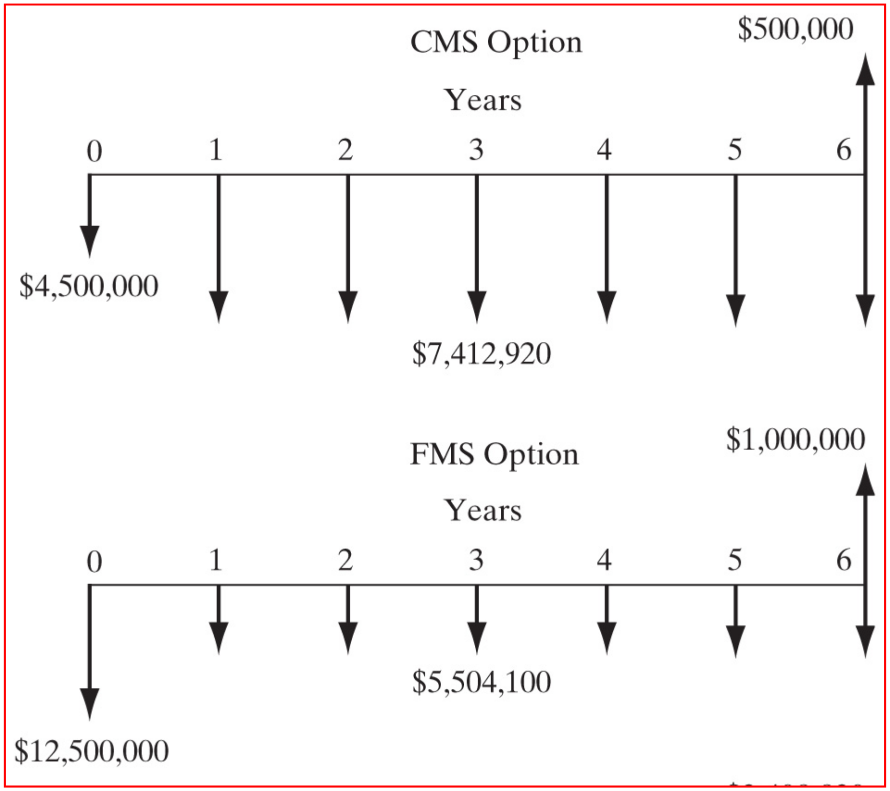
Items	**CMS Option**	**FMS Option**
Annual O&M costs:		
Annual labor cost	$1,169,600	$707,200
Annual material cost	$832,320	$598,400
Annual overhead cost	$3,150,000	$1,950,000
Annual tooling cost	$470,000	$300,000
Annual inventory cost	$141,000	$31,500
Annual income taxes	$1,650,000	$1,917,000
Total annual costs	$7,412,920	$5,504,100
Investment	$4,500,000	$12,500,000
Net salvage value	$500,000	$1,000,000

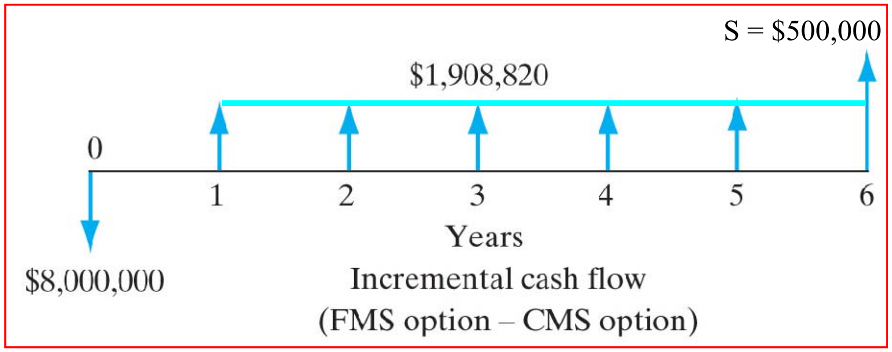
- Incremental IRR
  - Example 7: (Chan S. Park, example 7.13)
    - Assumption:
      - Both manufacturing systems would yield the same level of revenues over the analysis period.
    - Comparison basis:
      - Cost only as the revenues are equal.
    - Approach:
      - Calculate IRR based on the incremental cash flows.
      - $I_{FMS} > I_{CMS}$
      - Therefore the incremental cash flow is FMS – CMS.

- Incremental IRR
  - Example 7: (Chan S. Park, example 7.13)

n	CMS Option	FMS Option	Incremental (FMS-CMS)
0	-$4,500,000	-$12,500,000	-$8,000,000
1	-7,412,920	-5,504,100	1,908,820
2	-7,412,920	-5,504,100	1,908,820
3	-7,412,920	-5,504,100	1,908,820
4	-7,412,920	-5,504,100	1,908,820
5	-7,412,920	-5,504,100	1,908,820
6	-7,412,920	-5,504,100	1,908,820
Salvage	$500,000	$1,000,000	$500,000

- Incremental IRR
  - Example 7: (Chan S. Park, example 7.13)





Comparison of mutually exclusive alternatives on a cost only basis (Chan S. Park, Figure 7.11)

- Incremental IRR
  - Example 7: (Chan S. Park, example 7.13)
    - **To calculate the incremental IRR, calculate *i* where PW = 0**

$$\text{Objective: Calculate } i, \text{ where } PW_{FMS\text{-}CMS} = 0$$

$$PW_{FMS-CMS} = -P + A(P/A,i,N) + S(P/A,i,N)$$
$$PW_{FMS-CMS} = -\$8{,}000{,}000 + \$1{,}908{,}820(P/A,i,6) + \$500{,}000(P/F,i,6)$$

Solving for i using trial-and-error yields,

$i = 12.43\%$

- **Conclusion:**
  - **$IRR_{FMS\text{-}CMS}$ = 12.43%**
  - **MARR = 15%**
  - **$IRR_{FMS\text{-}CMS}$ < MARR**
  - **Therefore, select CMS**

Questions?