

# CpE 690: Introduction to VLSI Design

## Lecture 8

### Transient Response and Delay

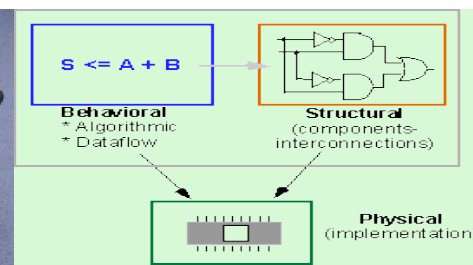
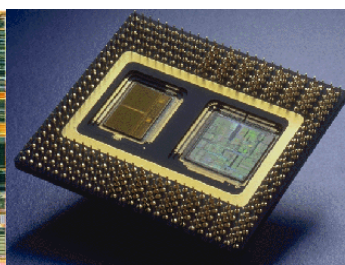
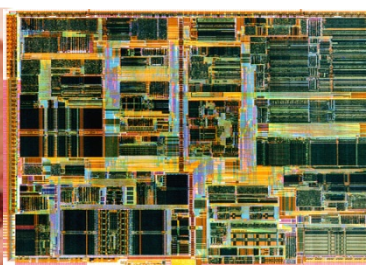
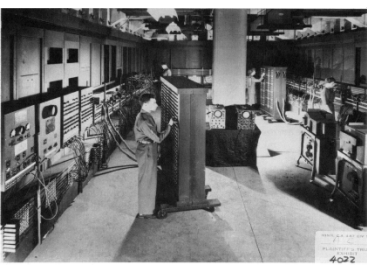
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Adapted from Lecture Notes, David Mahoney Harris CMOS VLSI Design

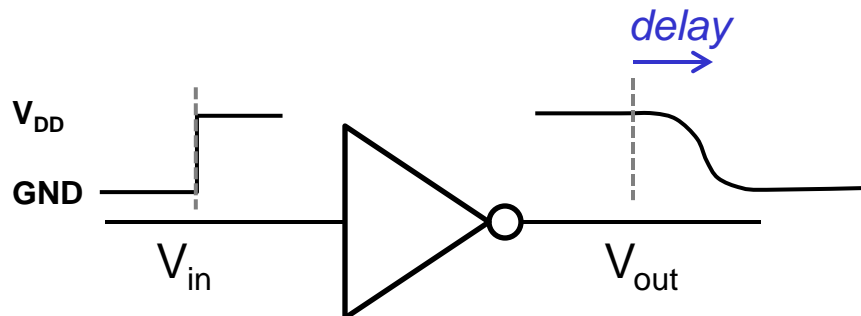


# Activity

- 1) If the width of a transistor increases, the current will  
increase                      decrease                      not change
- 2) If the length of a transistor increases, the current will  
increase                      decrease                      not change
- 3) If the supply voltage of a chip increases, the maximum transistor current will  
increase                      decrease                      not change
- 4) If the width of a transistor increases, its gate capacitance will  
increase                      decrease                      not change
- 5) If the length of a transistor increases, its gate capacitance will  
increase                      decrease                      not change
- 6) If the supply voltage of a chip increases, the gate capacitance of each transistor will  
increase                      decrease                      not change

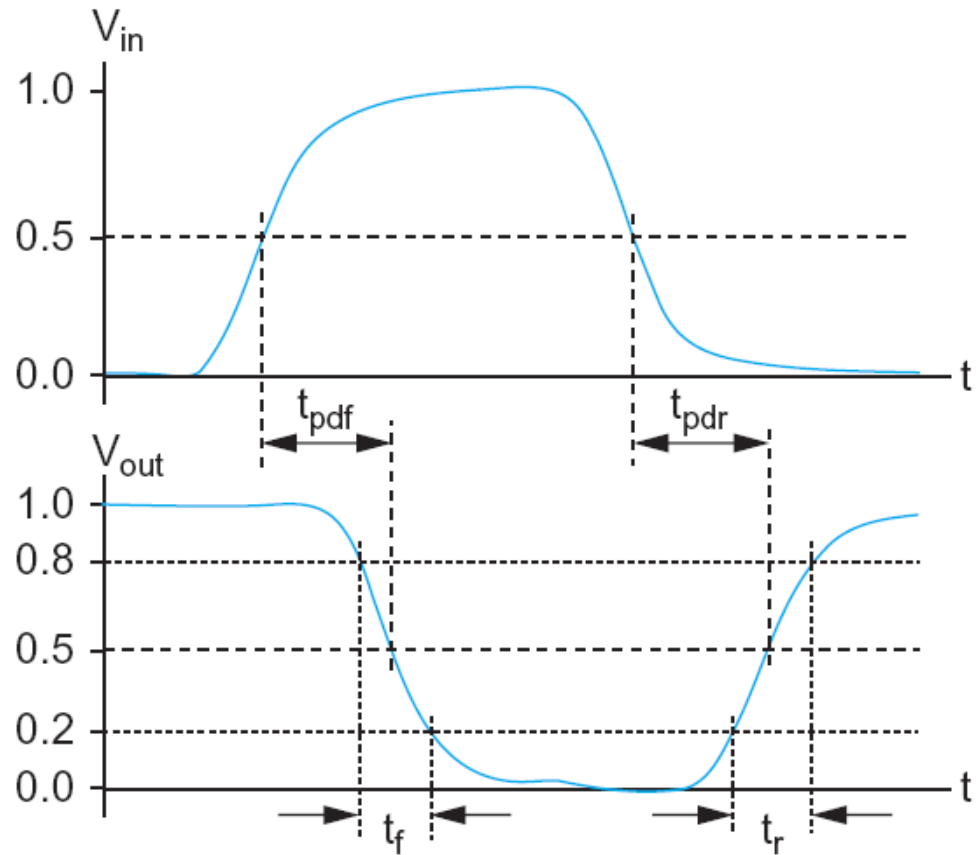
# Transient Response

- **DC analysis** tells us  $V_{out}$  if  $V_{in}$  is **constant**
- **Transient analysis** tells us  $V_{out}(t)$  in response to a **change** in  $V_{in}$
- Requires solving differential equations
- Input is usually considered to be a step or ramp
  - From GND to  $V_{DD}$  or vice versa



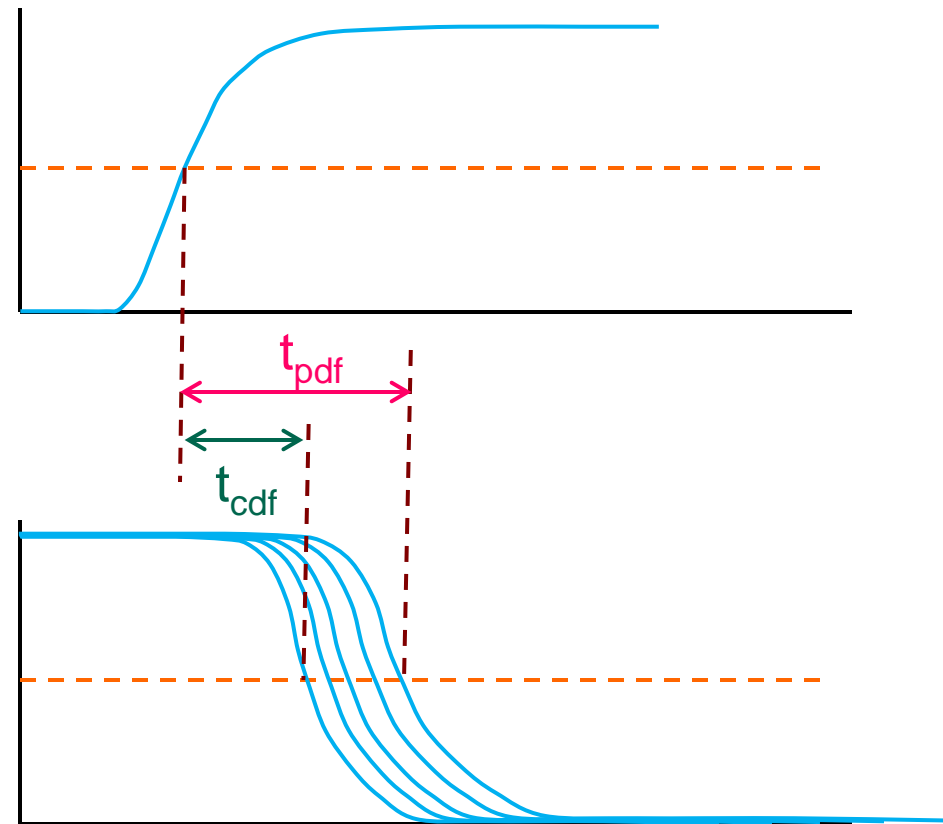
# Delay Definitions

- $t_{pdr}$ : *rising propagation delay*
  - maximum time from input crossing  $V_{DD}/2$  to rising output crossing  $V_{DD}/2$
- $t_{pdf}$ : *falling propagation delay*
  - maximum time from input crossing  $V_{DD}/2$  to falling output crossing  $V_{DD}/2$
- $t_{pd}$ : *average propagation delay*
  - $t_{pd} = (t_{pdr} + t_{pdf})/2$
- $t_r$ : *rise time*
  - from output crossing  $0.2 V_{DD}$  to  $0.8 V_{DD}$
- $t_f$ : *fall time*
  - from output crossing  $0.8 V_{DD}$  to  $0.2 V_{DD}$



# Delay Definitions (cont.)

- $t_{cdf}$  : *falling contamination delay*
  - minimum time from input crossing  $V_{DD}/2$  to falling output crossing  $V_{DD}/2$
- $t_{cdr}$  : *rising contamination delay*
  - minimum time from input crossing  $V_{DD}/2$  to rising output crossing  $V_{DD}/2$
- $t_{cd}$  : *avg. contamination delay*
  - $t_{pd} = (t_{cdr} + t_{cdf})/2$

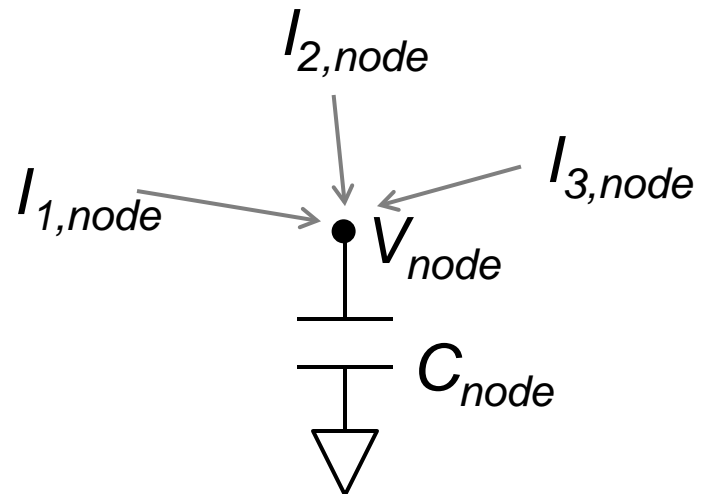


# Delay in CMOS Circuits

- Switching CMOS gate generates output current in response to changing input voltages
- All nodes have some finite capacitance (to ground)
  - gate capacitance
  - parasitic source/drain (diode) capacitance
  - parasitic wiring capacitance
- Transient waveforms found by solving:

$$C_{node} \cdot (dV_{node}/dt) = \sum_k I_{k,node}$$

for each node in circuit



# Inverter Step Response

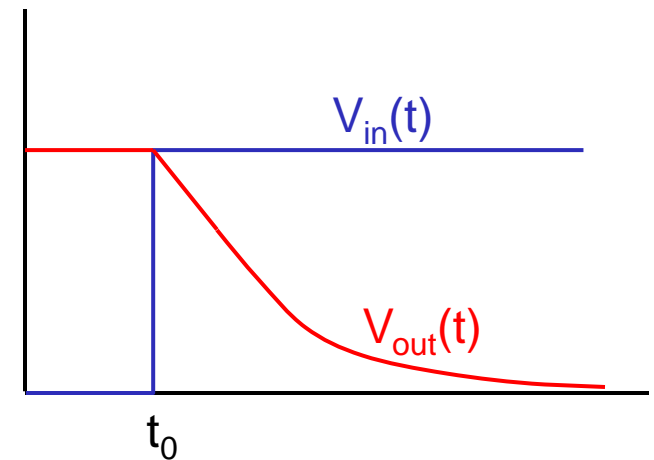
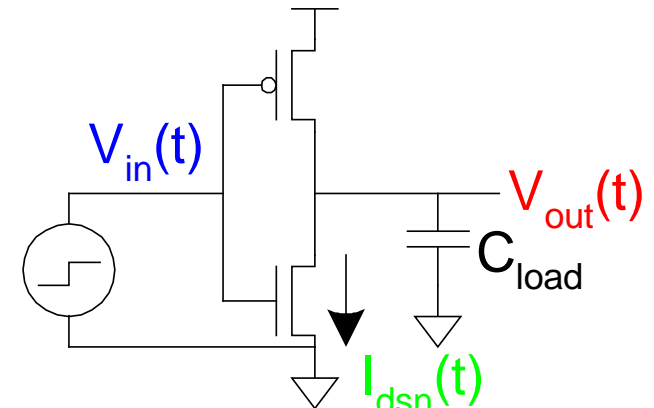
- Find step response of inverter driving  $C_{load}$

$$V_{in}(t) = u(t - t_0) \cdot V_{DD}$$

$$V_{out}(t < t_0) = V_{DD}$$

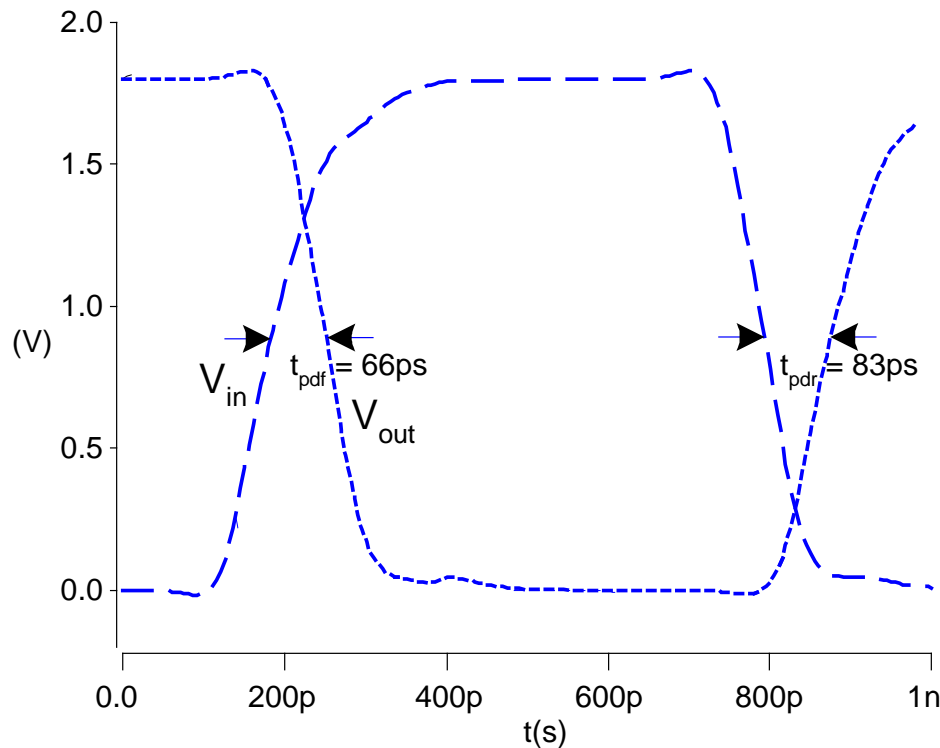
$$dV_{out}(t)/dt = -I_{dsn}(t)/C_{load}$$

$$I_{dsn}(t) = \begin{cases} 0 & \text{for } t < t_0 \\ (\beta/2) \cdot (V_{DD} - V_t)^2 & \text{for } V_{out} > V_{DD} - V_t \\ \beta \cdot (V_{DD} - V_t - V_{out}(t)/2) \cdot V_{out}(t) & \text{for } V_{out} < V_{DD} - V_t \end{cases}$$



# Simulated Inverter Delay

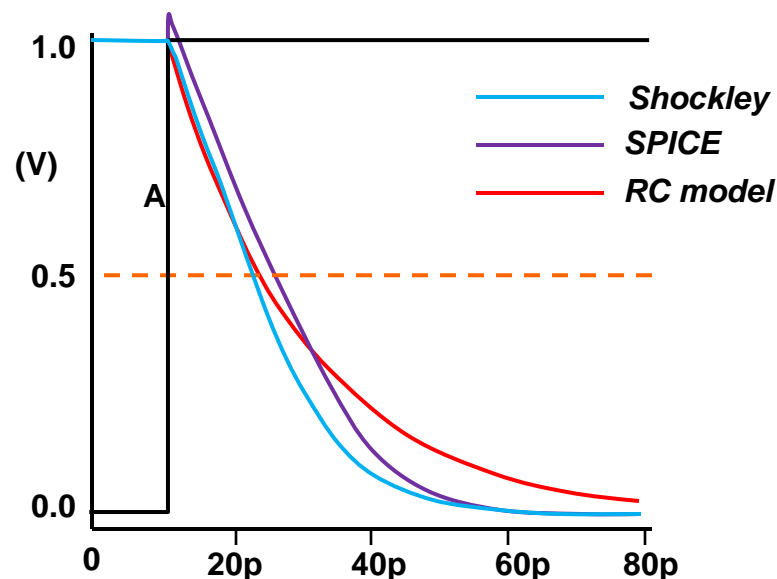
- Solving differential equations by hand is too hard
- SPICE simulator solves the equations numerically
- Uses more accurate I-V models too!
- But simulations take time to write!





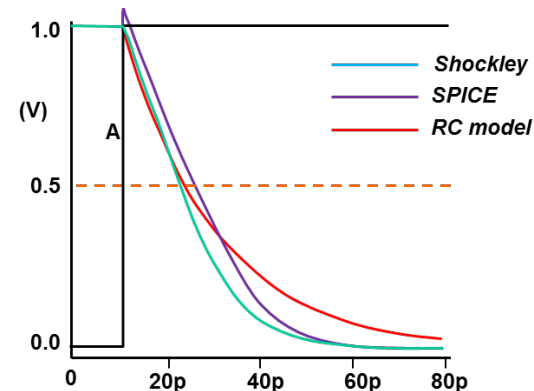
# Delay Estimation

- We would like to be able to easily estimate delay
  - For exploration of design space, don't need to be as accurate as simulation
  - Want a technique where its easier to ask “What if?”
- The step response usually looks like a 1<sup>st</sup> order RC response with a decaying exponential.
- Can we model conducting transistor as effective resistance?



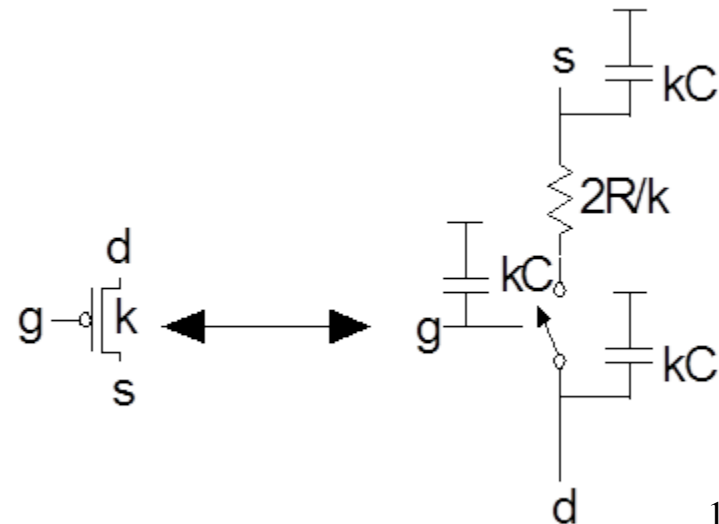
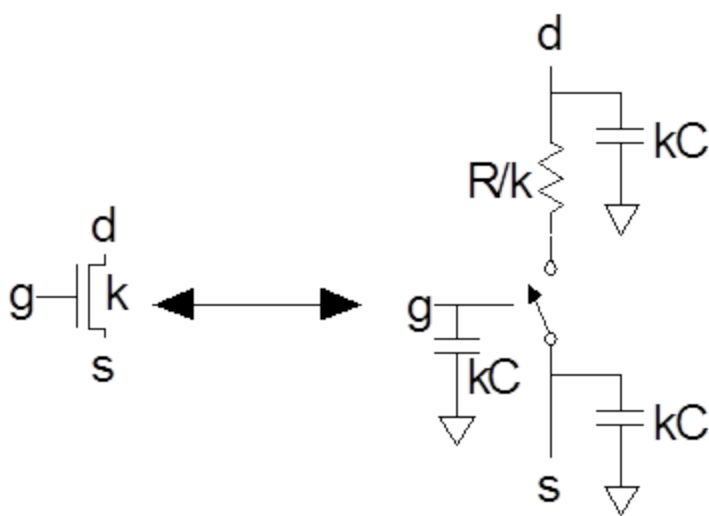
# Effective Resistance

- Simplification: treat transistor as resistor
  - Replace  $I_{ds}(V_{ds}, V_{gs})$  with effective resistance  $R$
  - $I_{ds} = V_{ds}/R$  or 0 depending on gate voltage
- $R$  averaged across switching of digital gate
  - Too inaccurate to predict current at any given time
  - But good enough to predict gate delay



# RC Delay Model

- Use equivalent circuits for MOS transistors
  - Ideal switch + capacitance and ON resistance
  - Unit nMOS has resistance  $R$ , capacitance  $C$
  - Unit pMOS has resistance  $2R$ , capacitance  $C$
- Capacitance (gate & diffusion) proportional to width
- Resistance inversely proportional to width



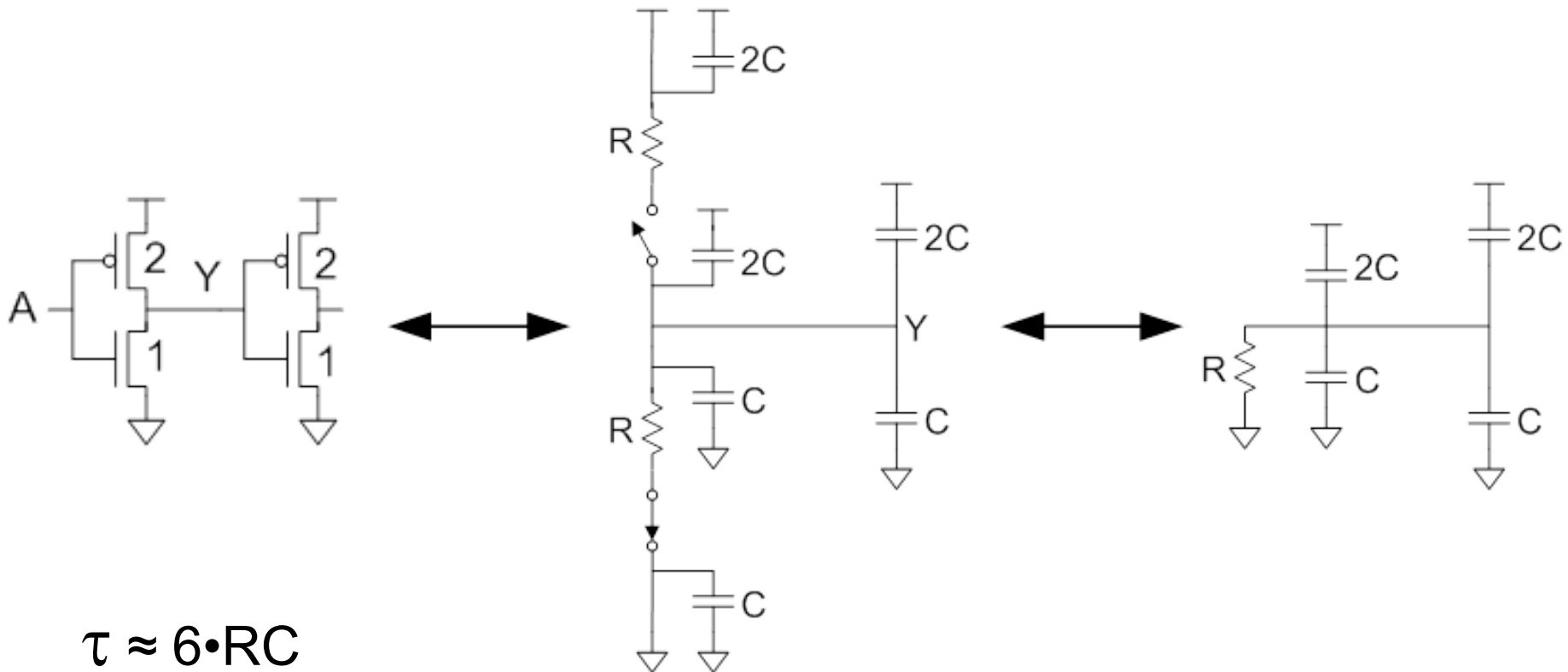
# RC Values

- Capacitance
  - $C = C_g = C_s = C_d = 2 \text{ fF}/\mu\text{m}$  of gate width in  $0.6 \mu\text{m}$
  - Gradually decline to  $1 \text{ fF}/\mu\text{m}$  in nanometer techs.
- Resistance
  - $R \approx 5\text{-}10 \text{ K}\Omega \cdot \mu\text{m}$  in  $0.6 \mu\text{m}$  process
  - Improves with shorter channel lengths
- Unit transistors
  - May refer to minimum contacted device ( $4/2 \lambda$ )
  - Or maybe  $W=1 \mu\text{m}$  device (doesn't matter as long as you are consistent)

	AMI 0.6 $\mu\text{m}$	TSMC 250nm	TSMC 180nm	IBM 130nm	IBM 65nm
$R_n$ ( $\text{k}\Omega \cdot \mu\text{m}$ )	9.2	4.0	2.7	2.5	1.3
$R_n$ ( $\text{k}\Omega \cdot 4\lambda$ )	7.7	8.0	7.5	9.6	10
$R_p$ ( $\text{k}\Omega \cdot \mu\text{m}$ )	19.9	8.9	6.5	6.4	2.9
$R_p$ ( $\text{k}\Omega \cdot 4\lambda$ )	16.6	17.8	18.1	24.7	22.3

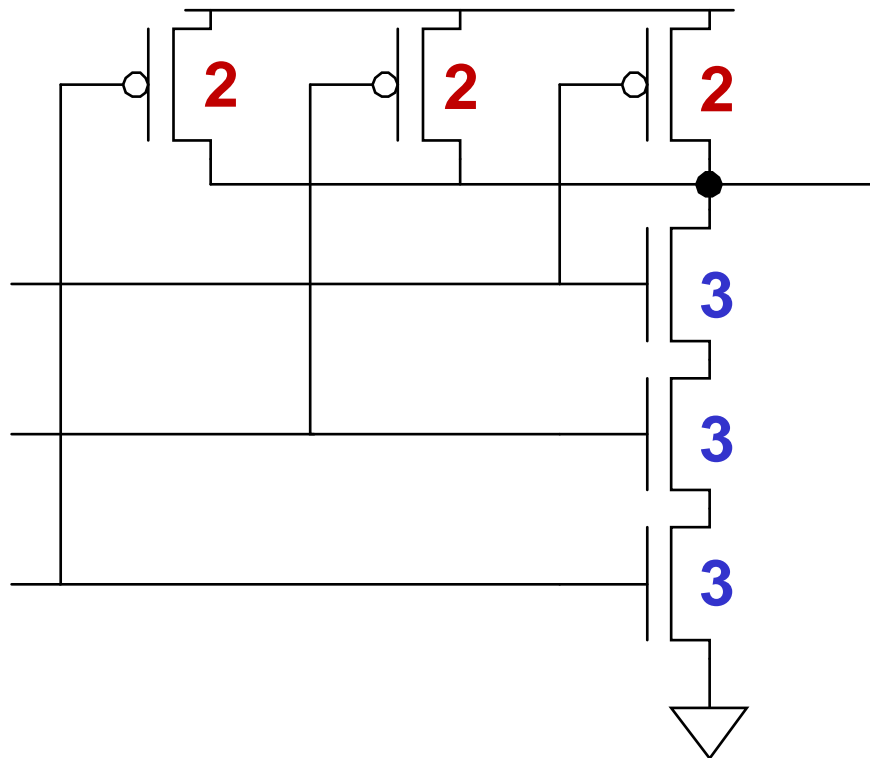
# RC Values

- Estimate the delay of a fanout-of-1 inverter



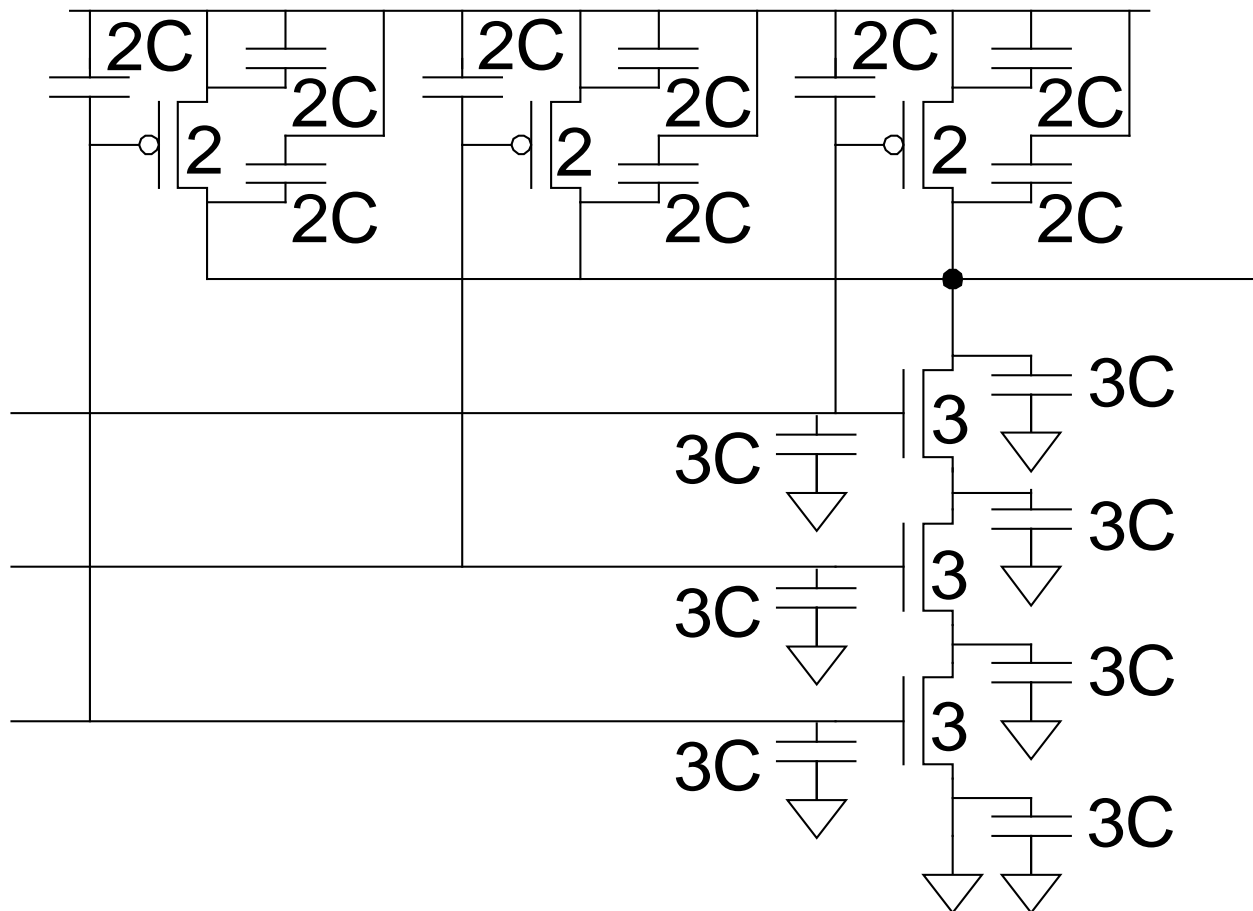
## Example: 3-input NAND

- Sketch a 3-input NAND with transistor widths chosen to achieve (worst case) effective rise and fall resistances equal to a unit inverter ( $R$ ).



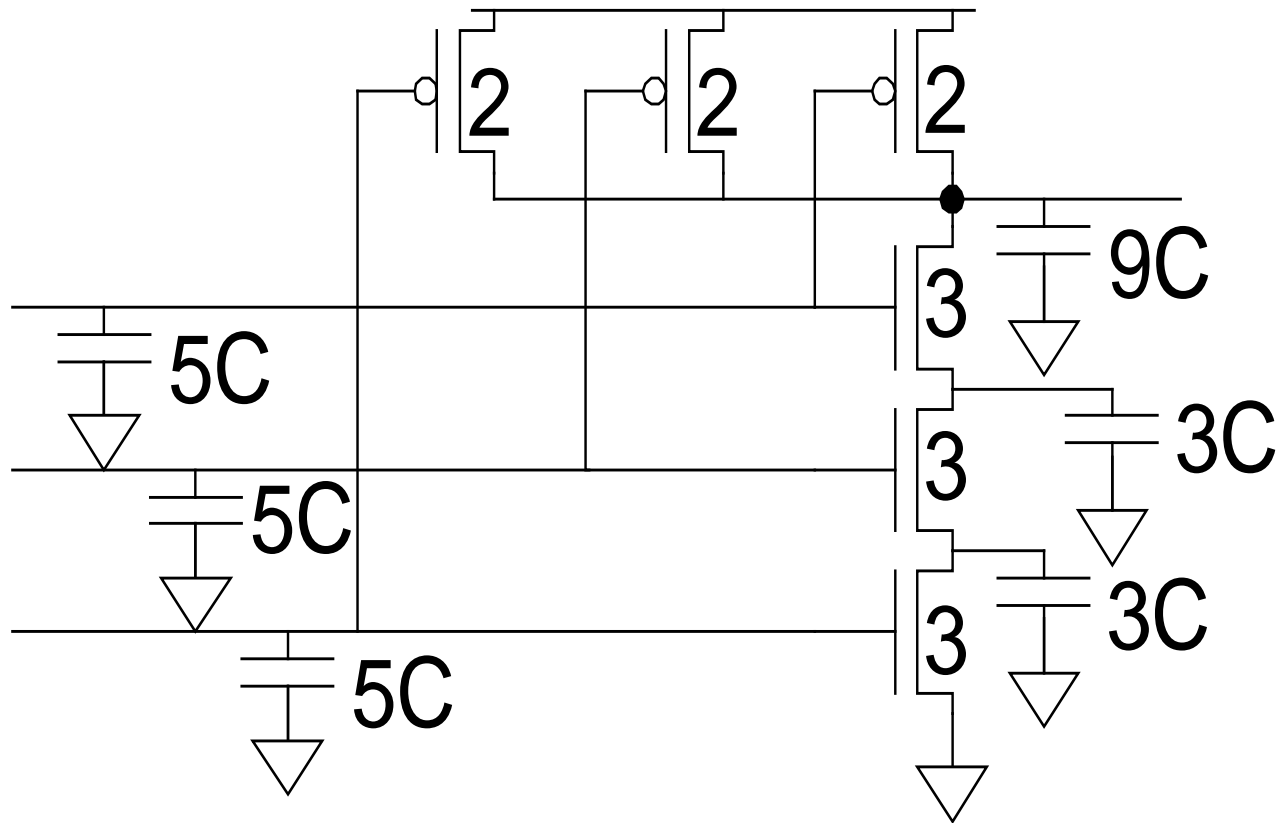
# 3-input NAND Capacitors

- Annotate the 3-input NAND gate with gate and diffusion capacitance.



# 3-input NAND Capacitors

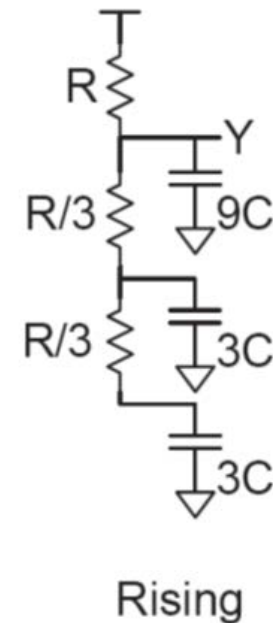
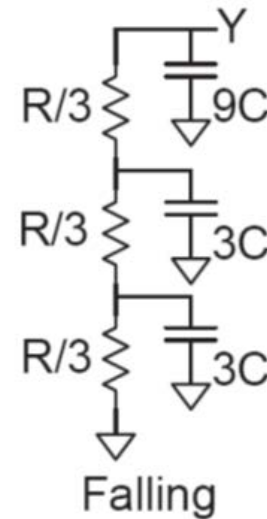
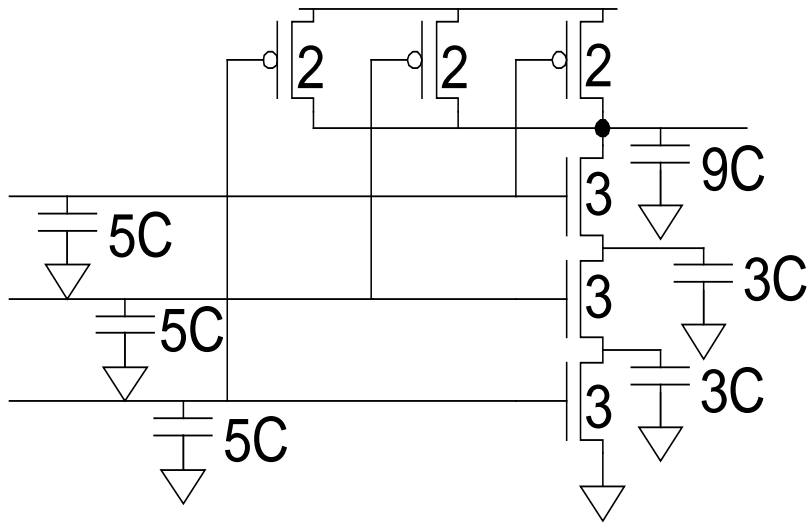
- Annotate the 3-input NAND gate with gate and diffusion capacitance.





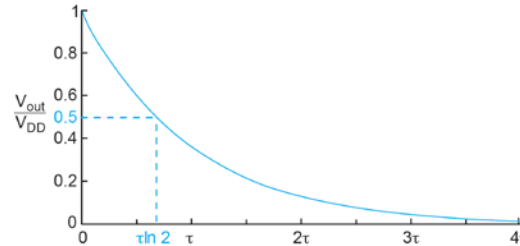
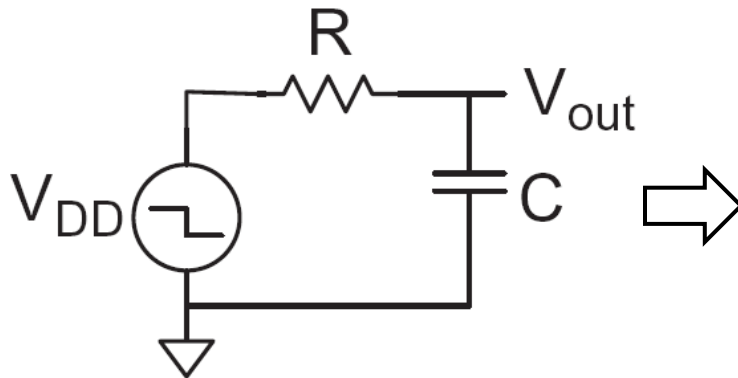
# Rise & Fall Delay

- What are worst-case rise and fall delays?

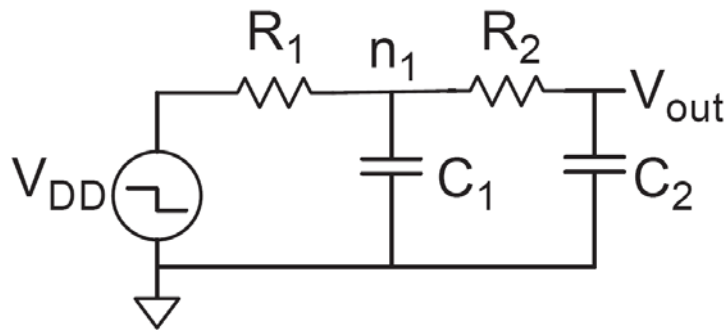


- How can we estimate delay of these networks?

# $\tau$ with multiple RC components



$$\tau = RC$$



$$H(s) = \frac{1}{1 + s[R_1C_1 + (R_1 + R_2) \cdot C_2] + s^2R_1C_1R_2C_2}$$

- Second order response is too complicated
  - defeats whole purpose of simplifying to an RC network
- Can approximate to:

$$\tau = \tau_1 + \tau_2 = R_1C_1 + (R_1 + R_2) \cdot C_2$$

# Elmore Delay

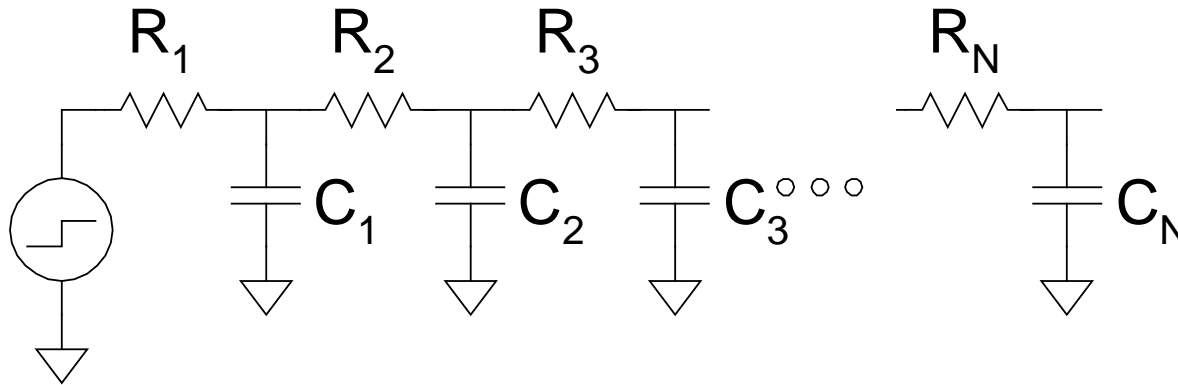
- ON transistors modeled as resistors
- Pullup or pulldown network represented as an RC tree
  - root of tree is driving voltage source (often VDD or GND)
  - leaves are capacitors at end of branches
- Elmore delay to any target (node j) in the branch:

$$t_{pdj} = \sum_i R_{sij} \cdot C_i$$

where:

- $i$  represents all the nodes in the branch
- $C_i$  is the capacitance at node  $i$
- $R_{sij}$  is the resistance of the shared path from the source to  $node_i$  and from the source to the target  $node_j$
- Elmore delay is conservative
  - over-estimates the delay

# Shared Path



- delay to node N is:

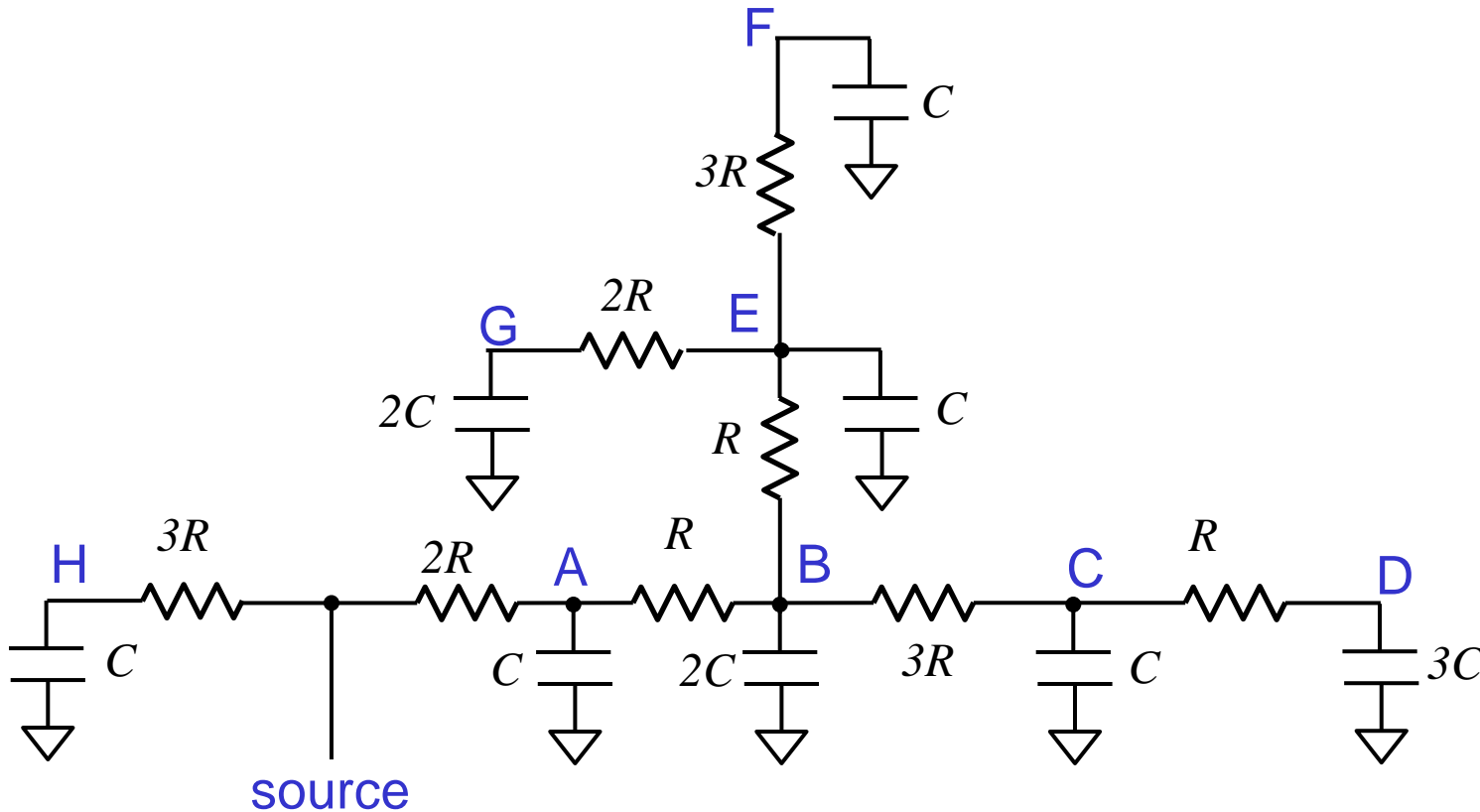
$$R_1 C_1 + (R_1 + R_2) \cdot C_2 + \dots + (R_1 + R_2 + \dots + R_n) \cdot C_N$$

- delay to node 2 is:

$$R_1 C_1 + (R_1 + R_2) \cdot C_2 + (R_1 + R_2) \cdot (C_3 + C_4 + \dots + C_N)$$

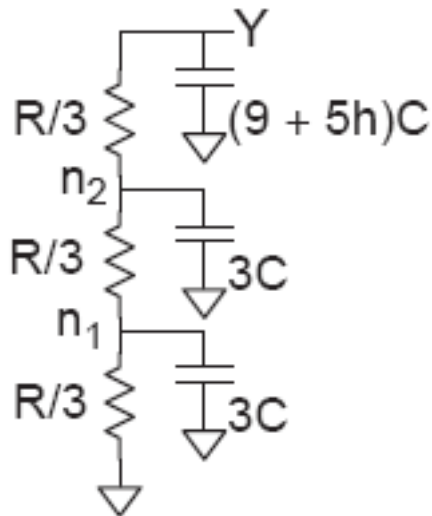
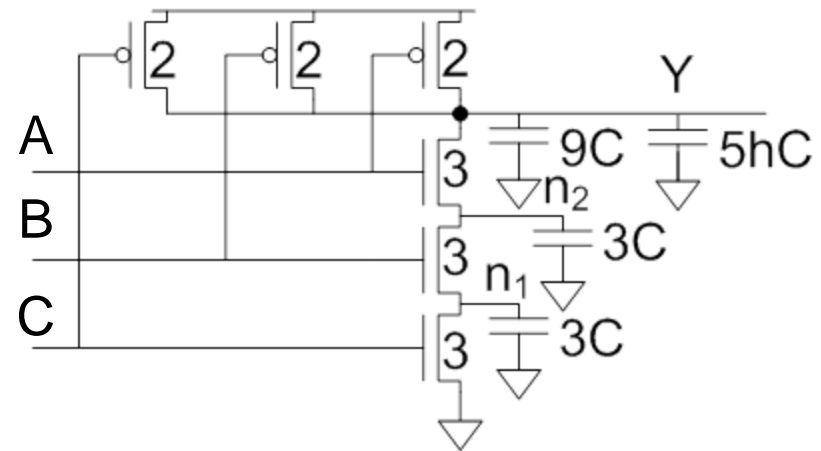
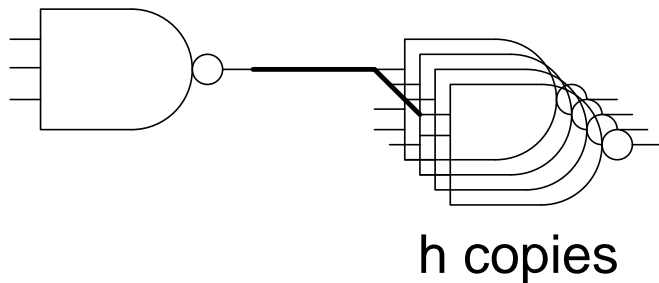
# Example: Elmore Delay

- Calculate delay from source to all nodes in circuit:



# 3-input NAND: pull-down delay

- Estimate worst-case rising and falling delay of 3-input NAND driving  $h$  identical gates.



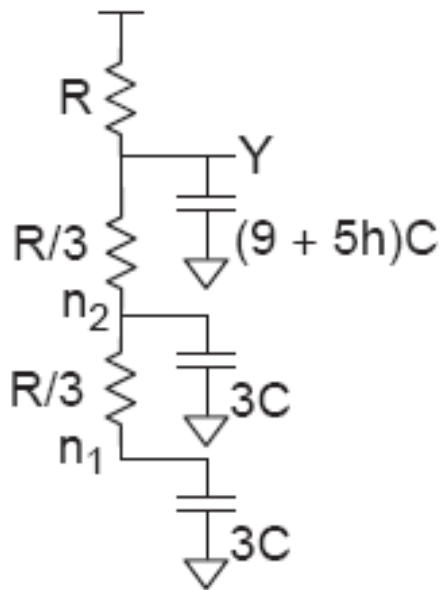
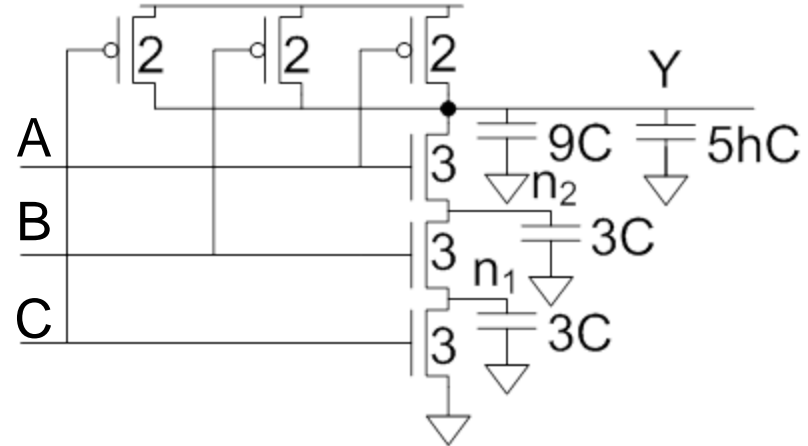
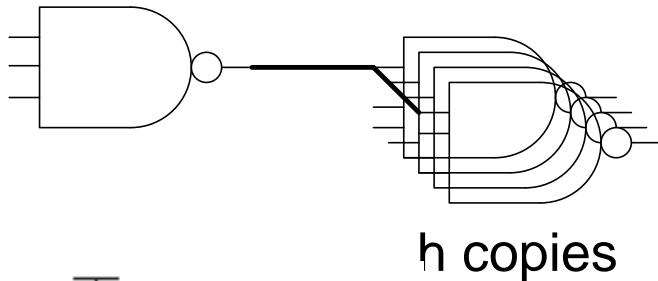
Worst case pull-down delay occurs when ABC goes from (110) to (111)

$$t_{pdf} = (3C)\left(\frac{R}{3}\right) + (3C)\left(\frac{R}{3} + \frac{R}{3}\right) + [(9 + 5h)C]\left(\frac{R}{3} + \frac{R}{3} + \frac{R}{3}\right)$$

$$t_{pdf} = (12 + 5h)RC$$

# 3-input NAND: pull-up delay

- Estimate worst-case rising and falling delay of 3-input NAND driving  $h$  identical gates.



Worst case pull-up delay occurs when ABC goes from (111) to (110)

$$t_{pdr} = [(9 + 5h)C](R) + (3C)(R) + (3C)(R)$$

$$t_{pdr} = (15 + 5h)RC$$

# Delay Components

$$t_{pdf} = (12 + 5h)RC$$

$$t_{pdr} = (15 + 5h)RC$$

- Delay has two parts
  - *Parasitic delay*
    - 15 or 12 RC
    - Independent of load
  - *Effort delay*
    - 5h RC
    - Proportional to load capacitance

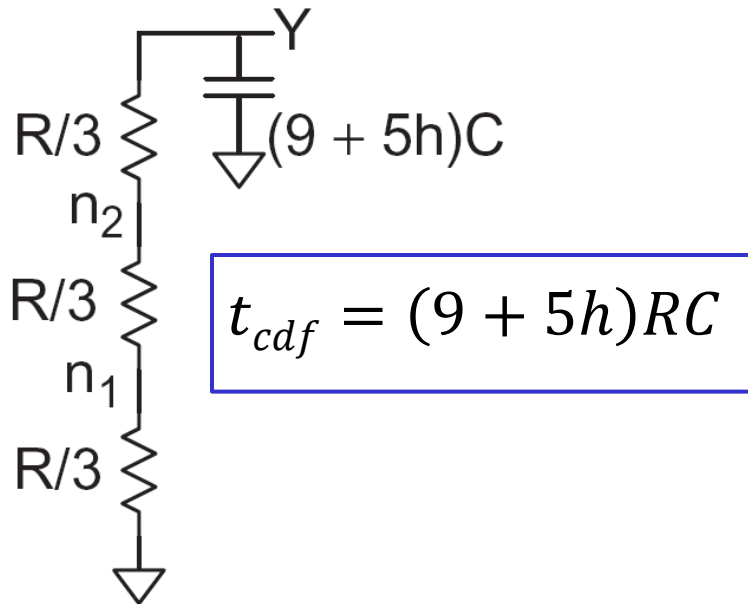


# Contamination Delay

- Best-case (contamination) delay can be substantially less than propagation delay:

*if top nMOS is last to turn on:*

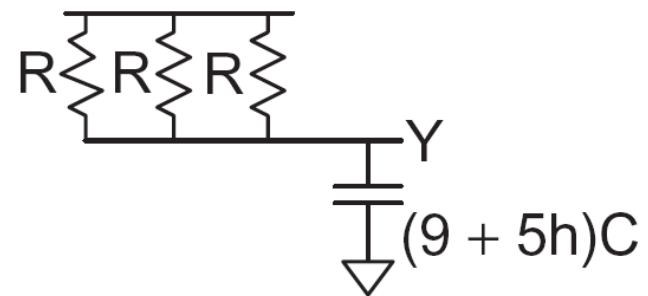
i.e. ABC goes from (011) to (111)



compare to:  $t_{pdf} = (12 + 5h)RC$

*if all pMOS turn on simultaneously:*

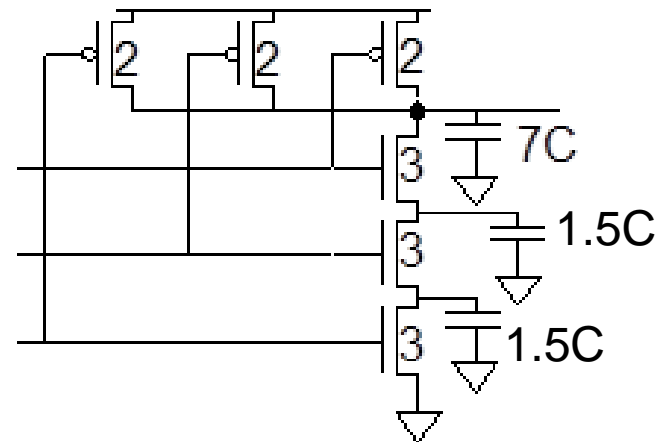
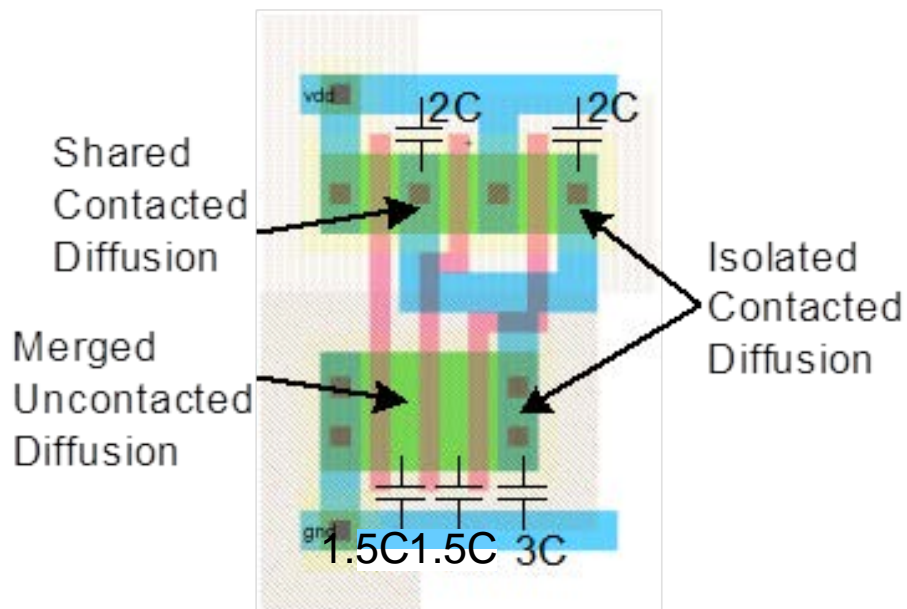
i.e. ABC goes from (111) to (000)



$t_{pdr} = (15 + 5h)RC$

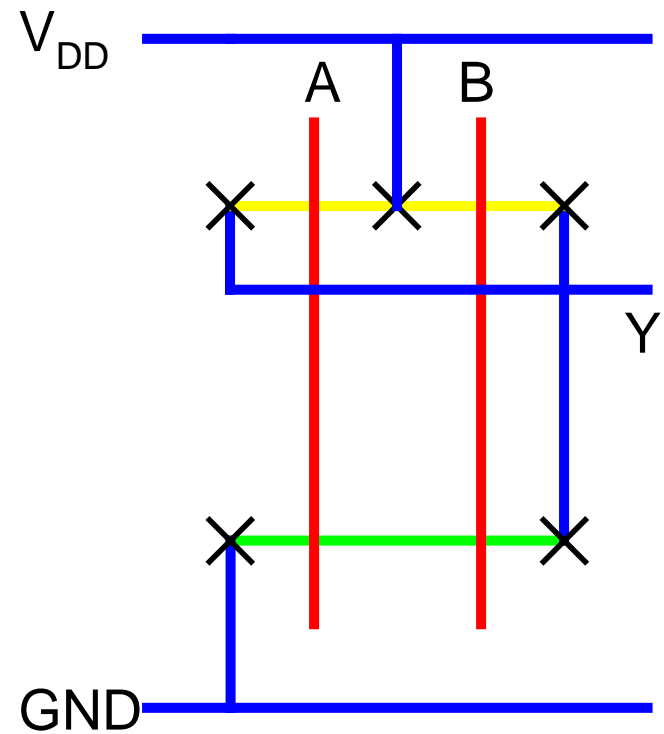
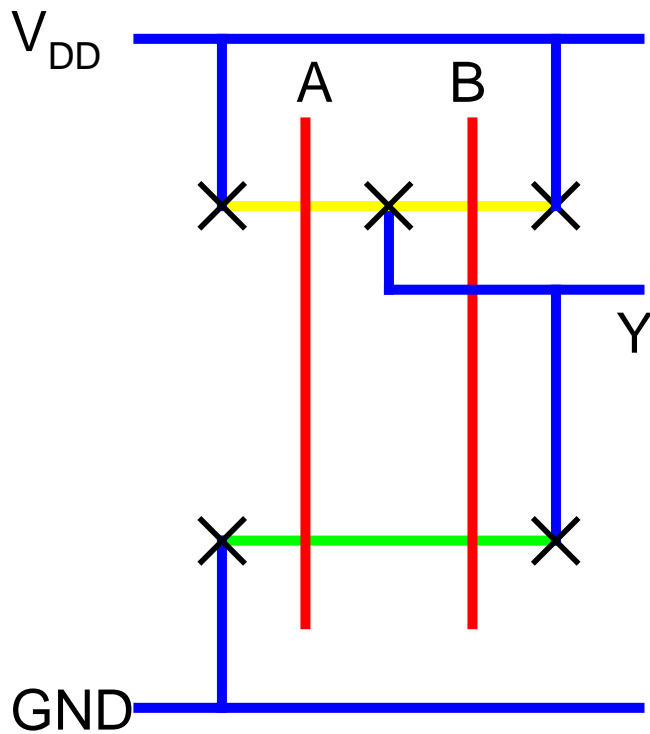
# Diffusion Capacitance

- We assumed contacted diffusion on every s / d.
  - but shared on series nMOS chain
- Good layout minimizes diffusion area
- Good NAND3 layout shares one diffusion contact
  - Reduces output capacitance by  $2C$
- Merged un-contacted diffusion also helps



# Layout Comparison

- Which layout is better?



## Example: Gate delays

For the gate  $Y = \overline{A + B.C}$

- a) Draw the schematic
- b) Size the transistors to give pullup and pulldown strength equal to unit size inverter
- c) Annotate with effective R of each transistor and C of each node
- d) Calculate worst case rising & falling propagation delay while driving  $h$  similar gates (via input B)
- e) Calculate best case rising & falling contamination delay while driving  $h$  similar gates (via input B)