

Mathematics for Machine Learning

Linear Algebra

Linear Algebra

System of linear equation

- vector - obj - moves around space
- vector space - mapping in between vector space

Operations - Vector

Geometric Object

List of attributes

Properties

Attributes

Represent Polar Coordinate

In terms of basis vector - Co-ordinate System

Associative

Distributive

Commutative

Distributive

$$\text{Cosine Rule } (1) \quad c^2 = a^2 + b^2 + 2a \cdot b \cdot \cos \theta$$

$$(2) \quad 18-51^2 = 11^2 + 15^2 - 2 \cdot 11 \cdot 15$$

$$0 \Rightarrow \theta \Rightarrow r \cdot s = ab \cos \theta$$

$$= |r| |s| \cos \theta$$

$$r \cdot y = |r|^2$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Reference Frame

Vector - describe space -> Co-ordinate System

basis vector

angle

different

independent from co-ordinate system

Transform - dot product

Projection - regular & equally spaced grid

map

wrap / fold space

Linearly independent

span space

easy - n vectors

Orthogonal

Unit Length

Matrix - object that operates on vector

Simultaneous equation -> matrix

Matrix Transformation

Properties

Type

$$Ar = r'$$

$$A(nr) = nr'$$

$$A(r+s) = Ar + As$$

$$\text{Identity Matrix } \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{stretching } \begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{squeezing } \begin{bmatrix} k & 0 \\ 0 & 1/k \end{bmatrix}$$

Linear Algebra

notation - mathematical object

vector

system of manipulating

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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Matrix makes linear mapping

Matrix Product

represent

Einstein Convention

coating

none square matrix

matrix product

Geometrically similar

similar

similar

similar

similar

similar

similar

$$(ab)_{ik} = \sum_j a_{ij} b_{jk} = a_{ij} b_{jk}$$

$$u \cdot v = u^T v$$

$$u \cdot v = v \cdot u$$

$$u \cdot v = v \cdot u$$

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changes in basis

world

my

Bear's basis vector

in my world

* Bear's = My vector

* (Bv) = (Mv) ->

B^-1 * Mv = Bv

My Vector -> Bear Vector

Basis Vectors -> Orthogonal

-> Dot product -> Projection

Transformation in Bear's world

B^-1 * transformation * B * (x y)

vector in my world

Transformed vector in my world

Transformed vector in bears world

Orthogonal Matrix A = [a1 a2 ... an]

unit length

orthogonal to each other

a_i \cdot a_j = 0 \quad i \neq j

a_i \cdot a_i = 1 \quad i=j

orthonormal matrix

Transpose A^T = A^j_i -> A^T A = I

A^T = A^-1

AA^T = I

orthonormal basis set

orthonormal basis set

Convert to orthonormal basis vector

linearly independent -> Determinant

Gram Schmidt process

v = v1, v2, ... vn

v1 -> u1 = v1 / ||v1|| - unit vector

v2 -> u2 = v2 - (v2 \cdot u1) u1

u2 = v2 / ||v2||

u3 = v3 - (v3 \cdot u1) u1 - (v3 \cdot u2) u2

u3 = v3 / ||v3||

orthogonal unit basis set

orthonormal basis set

orthonormal basis set

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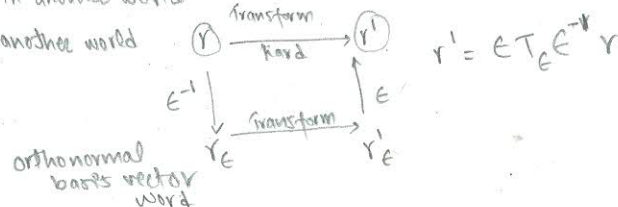
orthonormal basis set

orthonormal basis set

orthonormal basis set

Transformation in another world

another world



Eigen Vector & Eigen Value

↳ German - charakteristisch

Geometry

Linear transformation - vector

every vector in space \rightarrow shape distort
After transformation, some vector may lie in same line

Characteristic vector } eigen vector
Impact } length \rightarrow eigen value
direction \rightarrow eigen value

2D

- Uniform scale - any vector
- Rotation $\begin{cases} \theta \neq 180^\circ - \text{no vector} \\ \theta = 180^\circ - \text{any vector } \lambda = -1 \end{cases}$ } eigen vector
- Horizontal shear & vertical scaling - 2

3D Rotation axis of rotation = eigen vector

Linear Algebra

definition - $Ax = \lambda x$

$$(A - \lambda I)x = 0$$

$$\Rightarrow A - \lambda I = 0 \quad |A - \lambda I| = 0$$

x - vector
 A - transformation
 λ = eigen value

Changing eigen basis \rightarrow efficient matrix manipulation + many diagonalization

$$V_n = T(T(T(\dots(T \cdot v_0))))$$

$$= T^n v_0$$

↳ computationally expensive if T is not diagonal matrix
Change basis where $T \rightarrow$ diagonal matrix
eigen basis

matrix using eigen vectors / value

$$T = C D C^{-1}$$

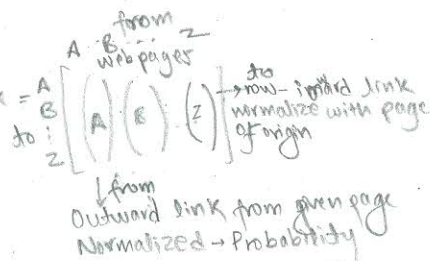
$$T^2 = C D^2 C^{-1}$$

$$T^n = C D^n C^{-1}$$

- Undiagonalizable matrix
- complex eigen vector

PageRank Algorithm

Link Matrix =



Rank of A

- rank of all pages
- link to page A?
- outgoing link from page A

$$r_A = \sum_{j=1}^n L_{Aj} r_j$$

weight from link matrix

$$r = Lr$$

- initial guess for r
- iterate until convergence

Power Method

- only interested eigenvalue = 1
- sparse matrix

Damping factor

$$r^{i+1} = d(Lr^{(i)}) + \frac{1-d}{n}$$

$d \in (0, 1]$

$d = 1 - p \rightarrow$ random

Stability vs speed

d - probability to follow link