University of Wisconsin - Madison

# Model Solution

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adapted from KTH ACM Contest Template Library

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# Contest (1)

## template.cpp

15 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
#define all(x) x.begin(), x.end()
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.sync_with_stdio(0); cin.tie(0);
  cin.exceptions(cin.failbit);
}
```

## hash.sh

1 lines

```
tr -d '[:space:]' | md5sum
```

## hash-cpp.sh

1 lines

```
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum
```

## Makefile

25 lines

```
CXX = g++
CXXFLAGS = -O2 -std=gnu++14 -Wall -Wextra -Wno-unused-
    ↪result -pedantic -Wshadow -Wformat=2 -Wfloat-equal -
    ↪Wconversion -Wlogical-op -Wshift-overflow=2 -
    ↪Wduplicated-cond -Wcast-qual -Wcast-align
# pause:#pragma GCC diagnostic {ignored|warning} "-Wshadow"
DEBUGFLAGS = -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -
    ↪fsanitize=address -fsanitize=undefined -fno-sanitize-
    ↪recover=all -fstack-protector -D_FORTIFY_SOURCE=2
CXXFLAGS += $(DEBUGFLAGS) # flags with speed penalty
TARGET := $(notdir $(CURDIR))
EXECUTE := ./$(TARGET)
CASES := $(sort $(basename $(wildcard *.in)))
TESTS := $(sort $(basename $(wildcard *.out)))
all: $(TARGET)
clean:
  -rm -rf $(TARGET) *.res
%: %.cpp
  $(LINK.cpp) $< $(LOADLIBES) $(LDLIBS) -o $@
run: $(TARGET)
  time $(EXECUTE)
%.res: $(TARGET) %.in
  time $(EXECUTE) < $*.in > $*.res
%.out: %
test_%: %.res %.out
  diff $*.res $*.out
runs: $(patsubst %,%.res,$(CASES))
test: $(patsubst %,test_%,$(TESTS))
.PHONY: all clean run test test_% runs
.PRECIOUS: %.res
```

## vimrc

8 lines

```
set nocp ai bs=2 hls ic is lbr ls=2 mouse=a nu ru sc scs
    ↪smd so=3 sw=4 ts=4
filetype plugin indent on
syn on
map gA m'ggVG"+y''
com -range=% -nargs=1 P exe "<line1>,<line2>!".<q-args> |y|
    ↪sil u|echom @"
com -range=% Hash <line1>,<line2>P tr -d '[:space:]' |
    ↪md5sum
au FileType cpp com! -buffer -range=% Hash <line1>,<line2>P
    ↪ cpp -dD -P -fpreprocessed | tr -d '[:space:]' |
    ↪md5sum
```

## nanorc

3 lines

```
set tabsize 4
set const
set autoindent
```

# Data structures (2)

## 1DBIT.cpp

**Description:** 0-indexed BIT

12 lines

```
int n;
int bit[N];

void add(int x, int val) {
    for(int i = x; i < n; i |= i + 1) bit[i] += val;
}

ll get(int x) {
    ll res = 0;
    for(int i = x; i >= 0; i = (i & (i + 1)) - 1)  res +=
        ↪bit[i];
    return res;
} // hash-cpp-all = e3a683942a154d5b6f649fa4539e93e8
```

## 2DBIT.cpp

29 lines

```
vector<int> vals[N], f[N];

void addupd(int x, int y) {
    for (int i = x; i < N; i |= i + 1) vals[i].push_back(y)
        ↪;
}

void addget(int x, int y) {
    if (x < 0 || y < 0) return;
    for (int i = x; i >= 0; i = (i & (i + 1)) - 1) vals[i].
        ↪push_back(y);
}

void upd(int x, int y, int v) {
    for (int i = x; i < N; i |= i + 1) {
        for (int j = lower_bound(vals[i].begin(), vals[i].
            ↪end(), y) - vals[i].begin();
             j < (int) f[i].size(); j |= j + 1) {
            f[i][j] += v;
        }
    }
}

int get(int x, int y) {
    if (x < 0 || y < 0) return 0;
    int res = 0;
    for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
        for (int j = lower_bound(vals[i].begin(), vals[i].
            ↪end(), y) - vals[i].begin(); j >= 0;
             j = (j & (j + 1)) - 1)
            res += f[i][j];
    return res;
} // hash-cpp-all = a5676421701a8edc43b837632d70b2d2
```

## DynamicConvexHullTrick.cpp

<bits/stdc++.h>

32 lines

```
typedef long long ll;

struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y
        ↪));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
}; // hash-cpp-all = 78fbd6d5c3d5351b8f2712bd6c535ce9
```

## StaticConvexHullTrick.cpp

**Description:** Maintaining upper convex hull, querying the maximum. Need to put in lines in strictly increasing order of slope.

<bits/stdc++.h>

49 lines

```
struct Line {
    ll k, m;
    Line(ll _k, ll _m) {
        k = _k, m = _m;
    }
    Pll inter(Line o) {
        return {m - o.m, o.k - k};
    }
};

struct Hull {
    deque<Line> que;

    bool leq(Pll a, Pll b) {
        return a.first * b.second <= a.second * b.first;
    }
    // k needs to be strictly increasing!
    void add(ll k, ll m) {
        while(que.size() > 1) {
            int ls = que.size() - 1;
```

```cpp
            if(leq(que[ls].inter(Line(k, m)), que[ls-1].inter(que[ls]))) que.pop_back();
            else break;
        }
        que.push_back({k, m});
    }
    // Arbitrary x.
    ll query_bin(ll x) {
      if(que.empty()) return -INF;
        int l = 0, r = que.size() - 1;
        while(l < r) {
            int mi = (l + r) / 2;
            if(que[mi].k * x + que[mi].m < que[mi+1].k * x + que[mi+1].m) l = mi + 1;
            else r = mi;
        }
        return que[l].k * x + que[l].m;
    }
    // If querying increasing x.
    ll query(ll x) {
      if(que.empty()) return -INF;
        while(que.size() > 1) {
            if(que[0].k * x + que[0].m < que[1].k * x + que[1].m) que.pop_front();
            else break;
        }
        return que[0].k * x + que[0].m;
    }

} hull;
// hash-cpp-all = 817ff0175d9f4dd826f400423d205fc4
```

## MediumDivideTree.cpp

**Description:** I unofficially named it medium divide tree. HDU 2665 / POJ 2104 O(logn) query for kth number in certain interval

<iostream>, <cstdio>, <algorithm> 62 lines

```cpp
const int N = (int)1e5 + 500, LOGN = 20;

int n,q;
int num[N], sorted[N];
int tree[LOGN][N], toleft[LOGN][N];

void build(int dep, int l, int r){
    if(l == r) return ;
    int mid = (l + r) / 2;
    int same = mid - l + 1;
    for(int i = l; i <= r; i++) if(tree[dep][i] < sorted[mid]) same--;
    int lpos = l, rpos = mid + 1;
    for(int i = l; i <= r; i++){
        if(tree[dep][i] < sorted[mid]){
            tree[dep+1][lpos++] = tree[dep][i];
        }
        else if(tree[dep][i] == sorted[mid] && same > 0){
            tree[dep+1][lpos++] = tree[dep][i]; same--;
        }
        else tree[dep+1][rpos++] = tree[dep][i];
        toleft[dep][i] = toleft[dep][l-1] + lpos - l;
    }
    build(dep + 1, l, mid);
    build(dep + 1, mid + 1, r);
}

void init(){
    for(int i = 1; i <= n; i++) sorted[i] = tree[0][i] = num[i];
    sort(sorted + 1, sorted + n + 1);
    build(0, 1, n);
}

int query(int dep, int l, int r, int ql, int qr, int k){
    if(ql == qr) return tree[dep][ql];
    int mid = (l + r) / 2;
    int cnt = toleft[dep][qr] - toleft[dep][ql - 1];
    if(cnt >= k){
        int newl = l + toleft[dep][ql-1] - toleft[dep][l-1];
        int newr = newl + cnt - 1;
        return query(dep + 1, l, mid, newl, newr, k);
    }
    else{
        int newr = qr + toleft[dep][r] - toleft[dep][qr];
        int newl = newr - (qr - ql - cnt);
        return query(dep + 1, mid + 1, r, newl, newr, k - cnt);
    }
}

int main(){
    int T;
    scanf("%d", &T);
    while(T--){
        scanf("%d%d", &n, &q);
        for(int i = 1; i <= n; i++) scanf("%d", &num[i]);
        init();
        while(q--){
            int a, b, k;
            scanf("%d%d%d", &a, &b, &k);
            printf("%d\n", query(0, 1, n, a, b, k));
        }
    }
} // hash-cpp-all = d4734a2a63915dbee574fd7fa50ed251
```

## MonotonousDeque.cpp

**Description:** Monotonous Interval Min Queries 17 lines

```cpp
struct MonQueue {
    deque<P> que;

    void clear() {
        que.clear();
    }
    void add(P p) {
        while(!que.empty() && que.back().first >= p.first) que.pop_back();
        que.push_back(p);
    }

    int get(int l, int r) {
        while(!que.empty() && (que.front().second < l || que.front().second > r)) que.pop_front();
        return que.empty() ? -INF : que.front().first;
    }
} cque;
// hash-cpp-all = dcfed26f0df829fcc97aa066b87bedff
```

## PersistentSegmentTreePointUpdate.cpp

45 lines

```cpp
int ncnt = 1; // Need to initialize before every test case!

struct node{
    int ls, rs, sum;
} ns[N * 30];

int newnode(int val){
    ns[ncnt].ls = ns[ncnt].rs = 0;
    ns[ncnt].sum = val;
    return ncnt++;
}

int newnode(int ls, int rs){
    ns[ncnt].ls = ls;
    ns[ncnt].rs = rs;
    ns[ncnt].sum = (ls ? ns[ls].sum : 0) + (rs ? ns[rs].sum : 0);
    return ncnt++;
}

int n, q;
int num[N];
int x[N], zeros[N];
int vs[N];

int build(int a[], int tl = 0, int tr = n-1){
    if(tl == tr) return newnode(a[tl]);
    int mid = (tl + tr) / 2;
    return newnode(build(a, tl, mid), build(a, mid + 1, tr));
}

int get_sum(int v, int l, int r, int tl = 0, int tr = n-1){
    if(tr < l || tl > r) return 0;
    if(l <= tl && tr <= r) return ns[v].sum;
    int tm = (tl + tr) / 2;

    return get_sum(ns[v].ls, l, r, tl, tm)
          + get_sum(ns[v].rs, l, r, tm + 1, tr);
}

int update(int v, int pos, int tl = 0, int tr = n-1){
    if(tl == tr) return newnode(ns[v].sum + 1);
    int tm = (tl + tr) / 2;
    if(pos <= tm) return newnode(update(ns[v].ls, pos, tl, tm), ns[v].rs);
    else return newnode(ns[v].ls, update(ns[v].rs, pos, tm+1, tr));
} // hash-cpp-all = 942ce4b7625a9496966519b6af9abf8b
```

## PersistentSegmentTreeRangeUpdate.cpp

69 lines

```cpp
int ncnt = 1 // Need to initialize before every test case!

struct node{
    int ls, rs, lazy;
    ll sum;
} ns[N * 100];

int newnode(int val){
    ns[ncnt].ls = ns[ncnt].rs = 0;
    ns[ncnt].sum = val;
    ns[ncnt].lazy = 0;
    return ncnt++;
}

int newnode(int ls, int rs){
    ns[ncnt].ls = ls;
    ns[ncnt].rs = rs;
    ns[ncnt].sum = (ls ? ns[ls].sum : 0) + (rs ? ns[rs].sum : 0);
    ns[ncnt].lazy = 0;
    return ncnt++;
```

```cpp
}

int n, q;
int num[N];
int vs[N];
int tim = 0;

int newlazynode(int v, int val, int l, int r){
    ns[ncnt].ls = ns[v].ls;
    ns[ncnt].rs = ns[v].rs;
    ns[ncnt].lazy = ns[v].lazy + val;
    ns[ncnt].sum = ns[v].sum + (r - l + 1) * val;
    return ncnt++;
}

void push_down(int v, int tl, int tr){
    if(ns[v].lazy){
        if(tl != tr){
            int mid = (tl + tr) / 2;
            ns[v].ls = newlazynode(ns[v].ls, ns[v].lazy, tl, mid);
            ns[v].rs = newlazynode(ns[v].rs, ns[v].lazy, mid + 1, tr);
        }
        ns[v].lazy = 0;
    }
}

int build(int a[], int tl = 0, int tr = n-1){
    if(tl == tr) return newnode(a[tl]);
    int mid = (tl + tr) / 2;
    return newnode(build(a, tl, mid), build(a, mid + 1, tr));
}

ll get_sum(int v, int l, int r, int tl = 0, int tr = n-1){
    if(tr < l || tl > r) return 0;
    if(l <= tl && tr <= r) return ns[v].sum;
    push_down(v, tl, tr);
    int tm = (tl + tr) / 2;

    return get_sum(ns[v].ls, l, r, tl, tm)
          + get_sum(ns[v].rs, l, r, tm + 1, tr);
}

int update(int v, int l, int r, int val, int tl = 0, int tr = n-1){
    if(tr < l || tl > r) return v;
    if(l <= tl && tr <= r) return newlazynode(v, val, tl, tr);
    push_down(v, tl, tr);
    int tm = (tl + tr) / 2;
    return newnode(update(ns[v].ls, l, r, val, tl, tm), update(ns[v].rs, l, r, val, tm+1, tr));
} // hash-cpp-all = cfcdd348217ece5628f563d25ece156d
```

## SegmentTreePointUpdate.cpp

<bits/stdc++.h> 60 lines

```cpp
#define lson(x) 2*x+1
#define rson(x) 2*x+2

typedef long long ll;
typedef pair<int, int> P;
const int N = (int)2e5 + 500, mod = (int)1e9 + 7;

int n;
P p[N];
int rs[N];

struct node {
    int mn;
    int cnt;

    void merge(node &LHS, node &RHS) {
        mn = min(LHS.mn, RHS.mn);
        cnt = (LHS.mn == mn ? LHS.cnt : 0) + (RHS.mn == mn ? RHS.cnt : 0);
        cnt %= mod;
    }
};

struct Tree {
    node dat[N * 4];

    void init_dat(int l, int r, int x){
        if(l == r){dat[x].mn = p[l].first; dat[x].cnt = 1; return ;}

        int mid = (l + r) / 2;
        init_dat(l, mid, lson(x));
        init_dat(mid+1, r, rson(x));
        dat[x].merge(dat[lson(x)], dat[rson(x)]);
    }

    void update(int pos, int x, int l, int r, int val, int cnt){
        int mid = (l + r) / 2;
        if(l == r) {
            dat[x].mn = val;
            dat[x].cnt = cnt;
            return ;
        }
        if(pos <= mid) update(pos, lson(x), l, mid, val, cnt);
        else update(pos, rson(x), mid+1, r, val, cnt);
        dat[x].merge(dat[lson(x)], dat[rson(x)]);
    }

    node query(int a, int b, int x, int l, int r){
        if(r < a || b < l) return {mod + 5, 0};

        int mid = (l + r) / 2;
        if(a <= l && r <= b) return dat[x];

        node res;
        node LHS = query(a, b, lson(x), l, mid);
        node RHS = query(a, b, rson(x), mid+1, r);
        res.merge(LHS, RHS);
        return res;
    }
} tree;
// hash-cpp-all = 307ccdb73b985795d794b5de82a02c27
```

## SegmentTreeRangeUpdate.cpp

<bits/stdc++.h> 103 lines

```cpp
#define ls(x) x * 2 + 1
#define rs(x) x * 2 + 2

typedef long long ll;
const int N = (int)1e6 + 50;
int INF = (int)1e9 + 50;

int n,m,q;
int a[N], b[N];
int num[N];

struct node {
    int mn, add;

    void add_val(int x) {
        mn += x;
        add += x;
    }

    void merge(node &ls, node &rs) {
        mn = min(ls.mn, rs.mn);
    }
};

struct Tree {
    node dat[4 * N];

    void push_down(int x, int l, int r) {
        if(dat[x].add) {
            if(l < r) {
                dat[ls(x)].add_val(dat[x].add);
                dat[rs(x)].add_val(dat[x].add);
            }
            dat[x].add = 0;
        }
    }

    void init(int x = 0, int l = 0, int r = n-1) {
        if(l == r) {
            dat[x].mn = num[l];
            dat[x].add = 0;
            return ;
        }
        int mid = (l + r) / 2;
        init(ls(x), l, mid);
        init(rs(x), mid + 1, r);
        dat[x].add = 0;
        dat[x].merge(dat[ls(x)], dat[rs(x)]);
    }

    node query(int a, int b, int x = 0, int l = 0, int r = N-1) {
        int mid = (l + r) / 2;
        if(r < a || l > b) return {INF, 0};

        push_down(x, l, r);

        if(l >= a && r <= b) return dat[x];

        node LHS = query(a, b, ls(x), l, mid);
        node RHS = query(a, b, rs(x), mid+1, r);
        node res;
        res.merge(LHS, RHS);
        return res;
    }

    void update(int a, int b, int x, int l, int r, int delta) {
        int mid = (l + r) / 2;
        if(r < a || l > b) return ;

        push_down(x, l, r);

        if(l >= a && r <= b) {
            dat[x].add_val(delta);
            return ;
        }
```

```
        update(a, b, ls(x), l, mid, delta);
        update(a, b, rs(x), mid+1, r, delta);

        dat[x].merge(dat[ls(x)], dat[rs(x)]);
    }

    void update(int a, int b, int delta) {
        update(a, b, 0, 0, N - 1, delta);
    }

    int find(int x, int l, int r) {
        if(l == r) return l;
        int mid = (l + r) / 2;
        push_down(x, l, r);

        if(dat[rs(x)].mn < 0) return find(rs(x), mid + 1, r
            ↪);
        else return find(ls(x), l, mid);
    }

    int find() {
        if(dat[0].mn >= 0) return -1;
        else return find(0, 0, N-1);
    }

} tree;
// hash-cpp-all = 8e290db8ea0801e20d078a3886a3acdb
```

### SparseTable.cpp

<bits/stdc++.h> 26 lines

```
typedef long long ll;
const int N = (int)3e5 + 50, LOGN = 19;

struct RMQ {
    int n;
    mm[N];

    void build() {
        mm[0]=-1;
        for(int i = 1; i <= n; i++) mm[i]= (i & (i-1)) == 0
            ↪ ? mm[i-1] + 1 : mm[i-1];
        for(int i = 0; i < n; i++){
            st[0][i] = x[i];
        }
        for(int lg = 1; lg < LOGN; lg++){
            for(int j = 0; j + (1 << lg) - 1 < n; j++){
                st[lg][j] = min(st[lg-1][j], st[lg-1][j
                    ↪+(1<<(lg-1))]);
            }
        }
    }

    int rmq(int l, int r){
        int k = mm[r - l + 1];
        return min(st[k][l], st[k][r-(1<<k)+1]);
    }
} rmq;
// hash-cpp-all = 58a1ba63eb165b67e821731f83538d02
```

### HashMap.h

**Description:** Hash map with the same API as unordered_map, but ~3x faster. Initial capacity must be a power of 2 (if provided).

2 lines

```
#include <bits/extc++.h>
__gnu_pbds::gp_hash_table<ll, int> h({},{},{},{}, {1 <<
    ↪16}); // hash-cpp-all =
    ↪d4f7c9a985615ad5fd0981e66d468825
```

### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the n'th element, and finding the index of an element.
**Time:** $\mathcal{O}(\log N)$

16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
} // hash-cpp-all = 782797f91ca134bf996558987dbf1924
```

# Numerical (3)

### GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function $f$ in the interval $[a, b]$ assuming $f$ is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is *eps*. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
**Usage:** `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
**Time:** $\mathcal{O}(\log((b-a)/\epsilon))$

14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
} // hash-cpp-all = 31d45b514727a298955001a74bb9b9fa
```

### Polynomial.h

17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
      ↪b=c;
    a.pop_back();
  }
}; // hash-cpp-all = c9b7b07a5aae7b0a6df1b8cdb046375f
```

### PolyRoots.h

**Description:** Finds the real roots to a polynomial.
**Usage:** `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
**Time:** $\mathcal{O}(n^2 \log(1/\epsilon))$

"Polynomial.h" 23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax)
  ↪ {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
} // hash-cpp-all = 2cf1903cf3e930ecc5ea0059a9b7fce5
```

### PolyInterpolate.h

**Description:** Given $n$ points (x[i], y[i]), computes an n-1-degree polynomial $p$ that passes through them: $p(x) = a[0]*x^0 + ... + a[n-1]*x^{n-1}$. For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \dots n-1$.
**Time:** $\mathcal{O}(n^2)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
} // hash-cpp-all = 08bf48c9301c849dfc6064b6450af6f3
```

### BerlekampMassey.h

**Description:** Recovers any $n$-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
**Usage:** `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`

"../number-theory/ModPow.h" 20 lines

```
vector<ll> BerlekampMassey(vector<ll> s) {
```

```
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  trav(x, C) x = (mod - x) % mod;
  return C;
} // hash-cpp-all = 40387d9fed31766a705d6b2206790deb
```

## LinearRecurrence.h

**Description:** Generates the $k$'th term of an $n$-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \dots n-1]$ and $tr[0 \dots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
**Usage:** linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number
**Time:** $\mathcal{O}\left(n^2 \log k\right)$ 26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) { // hash-cpp-1
  int n = sz(S);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
        ↪mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
} // hash-cpp-1 = 261dd85251df2df60ee444e087e8ffc2
```

## Integrate.h

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to $h^4$, although in practice you will want to verify that the result is stable to desired precision when epsilon changes. 8 lines

```
double quad(double (*f)(double), double a, double b) {
  const int n = 1000;
  double h = (b - a) / 2 / n;
  double v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
} // hash-cpp-all = 65e2375b3152c23048b469eb414fe6b6
```

## IntegrateAdaptive.h

**Description:** Fast integration using an adaptive Simpson's rule.
**Usage:**
```
double z, y;
double h(double x) { return x*x + y*y + z*z <= 1; }
double g(double y) { ::y = y; return quad(h, -1, 1); }
double f(double z) { ::z = z; return quad(g, -1, 1); }
double sphereVol = quad(f, -1, 1), pi = sphereVol*3/4;
```
16 lines

```
typedef double d;
d simpson(d (*f)(d), d a, d b) {
  d c = (a+b) / 2;
  return (f(a) + 4*f(c) + f(b)) * (b-a) / 6;
}
d rec(d (*f)(d), d a, d b, d eps, d S) {
  d c = (a+b) / 2;
  d S1 = simpson(f, a, c);
  d S2 = simpson(f, c, b), T = S1 + S2;
  if (abs (T - S) <= 15*eps || b-a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps/2, S1) + rec(f, c, b, eps/2, S2);
}
d quad(d (*f)(d), d a, d b, d eps = 1e-8) {
  return rec(f, a, b, eps, simpson(f, a, b));
} // hash-cpp-all = ad8a754372ce74e5a3d07ce46c2fe0ca
```

## Determinant.h

**Description:** Calculates determinant of a matrix. Destroys the matrix.
**Time:** $\mathcal{O}\left(N^3\right)$ 15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
} // hash-cpp-all = bd5cec161e6ad4c483e662c34eae2d08
```

## IntDeterminant.h

**Description:** Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
**Time:** $\mathcal{O}\left(N^3\right)$ 18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
} // hash-cpp-all = 3313dc3b38059fdf9f41220b469cfd13
```

## Simplex.h

**Description:** Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \le b$, $x \ge 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal $x$ (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
**Usage:**
```
vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
```
**Time:** $\mathcal{O}\left(NM * \#pivots\right)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}\left(2^n\right)$ in the general case. 68 lines

```
typedef double T; // long double, Rational, double + mod<P
  ↪>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
  ↪ s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) { //
      ↪hash-cpp-1
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[
        ↪i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    } // hash-cpp-1 = 6ff8e92a6bb47fbd6606c75a07178914

  void pivot(int r, int s) { // hash-cpp-2
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  } // hash-cpp-2 = 9cd0a84b89fb678b2888e0defa688de2

  bool simplex(int phase) { // hash-cpp-3
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
```

```
      if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                   < MP(D[r][n+1] / D[r][s], B[r])) r = i;
    }
    if (r == -1) return false;
    pivot(r, s);
  }
} // hash-cpp-3 = f156440bce4f5370ea43b0efa7de25ed

T solve(vd &x) { // hash-cpp-4
  int r = 0;
  rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
  if (D[r][n+1] < -eps) {
    pivot(r, n);
    if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
    rep(i,0,m) if (B[i] == -1) {
      int s = 0;
      rep(j,1,n+1) ltj(D[i]);
      pivot(i, s);
    }
  }
  bool ok = simplex(1); x = vd(n);
  rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
  return ok ? D[m][n+1] : inf;
} // hash-cpp-4 = 396a95621f5e196bb87eb95518560dfb
};
```

## math-simplex.cpp

**Description:** Simplex algorithm. WARNING- segfaults on empty (size 0) max cx st Ax<=b, x>=0 do 2 phases; 1st check feasibility; 2nd check boundedness and ans

40 lines

```
vector<double> simplex(vector<vector<double> > A, vector<double> b, vector<double> c) {
  int n = (int) A.size(), m = (int) A[0].size()+1, r = n, s = m-1;
  vector<vector<double> > D = vector<vector<double> > (n+2, vector<double>(m+1));
  vector<int> ix = vector<int> (n+m);
  for (int i=0; i<n+m; i++) ix[i] = i;
  for (int i=0; i<n; i++) {
    for (int j=0; j<m-1; j++)D[i][j]=-A[i][j];
    D[i][m-1] = 1;
    D[i][m] = b[i];
    if (D[r][m] > D[i][m]) r = i;
  }
  for (int j=0; j<m-1; j++) D[n][j]=c[j];
  D[n+1][m-1] = -1; int z = 0;
  for (double d;;) {
    if (r < n) {
      swap(ix[s], ix[r+m]);
      D[r][s] = 1.0/D[r][s];
      for (int j=0; j<=m; j++) if (j!=s) D[r][j] *= -D[r][s];
      for(int i=0; i<=n+1; i++)if(i!=r) {
        for (int j=0; j<=m; j++) if(j!=s) D[i][j] += D[r][j] * D[i][s];
        D[i][s] *= D[r][s];
      }
    }
    r = -1; s = -1;
    for (int j=0; j <m; j++) if (s<0 || ix[s]>ix[j]) {
      if (D[n+1][j]>eps || D[n+1][j]>-eps && D[n][j]>eps) s = j;
    }
    if (s < 0) break;
    for (int i=0; i<n; i++) if(D[i][s]<-eps) {
      if (r < 0 || (d = D[r][m]/D[r][s]-D[i][m]/D[i][s]) < -eps
        || d < eps && ix[r+m] > ix[i+m]) r=i;
    }
    if (r < 0) return vector<double>(); // unbounded
  }
  if (D[n+1][m] < -eps) return vector<double>(); // infeasible
  vector<double> x(m-1);
  for (int i = m; i < n+m; i ++) if (ix[i] < m-1) x[ix[i]] = D[i-m][m];
  printf("%.2lf\n", D[n][m]);
  return x; // ans: D[n][m]
} // hash-cpp-all = 70201709abdff05eff90d9393c756b95
```

## SolveLinear.h

**Description:** Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in $A$ and $b$ is lost.

**Time:** $\mathcal{O}\left(n^2 m\right)$

38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
} // hash-cpp-all = 44c9ab90319b30df6719c5b5394bc618
```

## SolveLinear2.h

**Description:** To get all uniquely determined values of $x$ back from SolveLinear, make the following changes:

"SolveLinear.h" 8 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
// hash-cpp-all = 08e495d9d51e80a183ccd030e3bf6700
```

## SolveLinearBinary.h

**Description:** Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys $A$ and $b$.

**Time:** $\mathcal{O}\left(n^2 m\right)$

34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
} // hash-cpp-all = fa2d7a3e3a84d8fb47610cc474e77b4e
```

## MatrixInverse.h

**Description:** Invert matrix $A$. Returns rank; result is stored in $A$ unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$ where $A^{-1}$ starts as the inverse of A mod p, and k is doubled in each step.

**Time:** $\mathcal{O}\left(n^3\right)$

35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
```

```
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
} // hash-cpp-all = ebfff64122d6372fde3a086c95e2cfc7
```

## Tridiagonal.h

**Description:** $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \; 1 \le i \le n,$$

where $a_0$, $a_{n+1}$, $b_i$, $c_i$ and $d_i$ are known. $a$ can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \\ \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If $|d_i| > |p_i| + |q_{i-1}|$ for all $i$, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.

**Time:** $\mathcal{O}(N)$

26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>&
   ↪super,
    const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i]
       ↪== 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
} // hash-cpp-all = 8f9fa8b1e5e82731da914aed0632312f
```

# 3.1 Fourier transforms

## fft.cpp

**Description:** FFT/NTT, polynomial mod/log/exp

303 lines

```
namespace fft {
#if FFT
// FFT
using dbl = double;
struct num { // hash-cpp-1
  dbl x, y;
  num(dbl x_ = 0, dbl y_ = 0) : x(x_), y(y_) { }
};
inline num operator+(num a, num b) { return num(a.x + b.x,
   ↪a.y + b.y); }
inline num operator-(num a, num b) { return num(a.x - b.x,
   ↪a.y - b.y); }
inline num operator*(num a, num b) { return num(a.x * b.x -
   ↪ a.y * b.y, a.x * b.y + a.y * b.x); }
inline num conj(num a) { return num(a.x, -a.y); }
inline num inv(num a) { dbl n = (a.x*a.x+a.y*a.y); return
   ↪num(a.x/n,-a.y/n); }
// hash-cpp-1 = d2cc70ff17fe23dbfe608d8bce4d827b
#else
// NTT
const int mod = 998244353, g = 3;
// For p < 2^30 there is also (5 << 25, 3), (7 << 26, 3),
// (479 << 21, 3) and (483 << 21, 5). Last two are > 10^9.
struct num { // hash-cpp-2
  int v;
  num(ll v_ = 0) : v(int(v_ % mod)) { if (v<0) v+=mod; }
  explicit operator int() const { return v; }
};
inline num operator+(num a,num b){return num(a.v+b.v);}
inline num operator-(num a,num b){return num(a.v+mod-b.v);}
inline num operator*(num a,num b){return num(1ll*a.v*b.v);}
inline num pow(num a, int b) {
  num r = 1;
  do{if(b&1)r=r*a;a=a*a;}while(b>>=1);
  return r;
}
inline num inv(num a) { return pow(a, mod-2); }
// hash-cpp-2 = 62f50e0b94ea4486de6fbc07e826040a
#endif

using vn = vector<num>;
vi rev({0, 1});
vn rt(2, num(1)), fa, fb;

inline void init(int n) { // hash-cpp-3
  if (n <= sz(rt)) return;
  rev.resize(n);
  rep(i,0,n) rev[i] = (rev[i>>1] | ((i&1)*n)) >> 1;
  rt.reserve(n);
  for (int k = sz(rt); k < n; k *= 2) {
    rt.resize(2*k);
#if FFT
    double a=M_PI/k; num z(cos(a),sin(a)); // FFT
#else
    num z = pow(num(g), (mod-1)/(2*k)); // NTT
#endif
    rep(i,k/2,k) rt[2*i] = rt[i], rt[2*i+1] = rt[i]*z;
  }
} // hash-cpp-3 = 408005a3c0a4559a884205d5d7db44e9

inline void fft(vector<num> &a, int n) { // hash-cpp-4
  init(n);
  int s = __builtin_ctz(sz(rev)/n);
  rep(i,0,n) if (i < rev[i]>>s) swap(a[i], a[rev[i]>>s]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      num t = rt[j+k] * a[i+j+k];
      a[i+j+k] = a[i+j] - t;
      a[i+j] = a[i+j] + t;
    }
} // hash-cpp-4 = 1f0820b04997ddca9b78742df352d419

// Complex/NTT
vn multiply(vn a, vn b) { // hash-cpp-5
  int s = sz(a) + sz(b) - 1;
  if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s-1) : 0, n = 1 << L;
  a.resize(n), b.resize(n);
  fft(a, n);
  fft(b, n);
  num d = inv(num(n));
  rep(i,0,n) a[i] = a[i] * b[i] * d;
  reverse(a.begin()+1, a.end());
  fft(a, n);
  a.resize(s);
  return a;
} // hash-cpp-5 = 7a20264754593de4eb7963d8fc3d8a15

// Complex/NTT power-series inverse
// Doubles b as b[:n] = (2 - a[:n] * b[:n/2]) * b[:n/2]
vn inverse(const vn& a) { // hash-cpp-6
  if (a.empty()) return {};
  vn b({inv(a[0])});
  b.reserve(2*a.size());
  while (sz(b) < sz(a)) {
    int n = 2*sz(b);
    b.resize(2*n, 0);
    if (sz(fa) < 2*n) fa.resize(2*n);
    fill(fa.begin(), fa.begin()+2*n, 0);
    copy(a.begin(), a.begin()+min(n,sz(a)), fa.begin());
    fft(b, 2*n);
    fft(fa, 2*n);
    num d = inv(num(2*n));
    rep(i, 0, 2*n) b[i] = b[i] * (2 - fa[i] * b[i]) * d;
    reverse(b.begin()+1, b.end());
    fft(b, 2*n);
    b.resize(n);
  }
  b.resize(a.size());
  return b;
} // hash-cpp-6 = 61660c4b2c75faa72062368a381f059f

#if FFT
// Double multiply (num = complex)
using vd = vector<double>;
vd multiply(const vd& a, const vd& b) { // hash-cpp-7
  int s = sz(a) + sz(b) - 1;
```

```
  if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s-1) : 0, n = 1 << L;
  if (sz(fa) < n) fa.resize(n);
  if (sz(fb) < n) fb.resize(n);

  fill(fa.begin(), fa.begin() + n, 0);
  rep(i,0,sz(a)) fa[i].x = a[i];
  rep(i,0,sz(b)) fa[i].y = b[i];
  fft(fa, n);
  trav(x, fa) x = x * x;
  rep(i,0,n) fb[i] = fa[(n-i)&(n-1)] - conj(fa[i]);
  fft(fb, n);
  vd r(s);
  rep(i,0,s) r[i] = fb[i].y / (4*n);
  return r;
} // hash-cpp-7 = c2431bc9cb89b2ad565db6fba6a21a32

// Integer multiply mod m (num = complex) // hash-cpp-8
vi multiply_mod(const vi& a, const vi& b, int m) {
  int s = sz(a) + sz(b) - 1;
  if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s-1) : 0, n = 1 << L;
  if (sz(fa) < n) fa.resize(n);
  if (sz(fb) < n) fb.resize(n);

  rep(i,0,sz(a)) fa[i] = num(a[i] & ((1<<15)-1), a[i] >>
    ↪15);
  fill(fa.begin()+sz(a), fa.begin() + n, 0);
  rep(i,0,sz(b)) fb[i] = num(b[i] & ((1<<15)-1), b[i] >>
    ↪15);
  fill(fb.begin()+sz(b), fb.begin() + n, 0);

  fft(fa, n);
  fft(fb, n);
  double r0 = 0.5 / n; // 1/2n
  rep(i,0,n/2+1) {
    int j = (n-i)&(n-1);
    num g0 = (fb[i] + conj(fb[j])) * r0;
    num g1 = (fb[i] - conj(fb[j])) * r0;
    swap(g1.x, g1.y); g1.y *= -1;
    if (j != i) {
      swap(fa[j], fa[i]);
      fb[j] = fa[j] * g1;
      fa[j] = fa[j] * g0;
    }
    fb[i] = fa[i] * conj(g1);
    fa[i] = fa[i] * conj(g0);
  }
  fft(fa, n);
  fft(fb, n);
  vi r(s);
  rep(i,0,s) r[i] = int((ll(fa[i].x+0.5)
        + (ll(fa[i].y+0.5) % m << 15)
        + (ll(fb[i].x+0.5) % m << 15)
        + (ll(fb[i].y+0.5) % m << 30)) % m);
  return r;
} // hash-cpp-8 = e8c5f6755ad1e5a976d6c6ffd37b3b22
#endif

} // namespace fft

// For multiply_mod, use num = modnum, poly = vector<num>
using fft::num;
using poly = fft::vn;
using fft::multiply;
using fft::inverse;
// hash-cpp-9
poly& operator+=(poly& a, const poly& b) {
  if (sz(a) < sz(b)) a.resize(b.size());
  rep(i,0,sz(b)) a[i]=a[i]+b[i];
  return a;
}
poly operator+(const poly& a, const poly& b) { poly r=a; r
  ↪+=b; return r; }
poly& operator-=(poly& a, const poly& b) {
  if (sz(a) < sz(b)) a.resize(b.size());
  rep(i,0,sz(b)) a[i]=a[i]-b[i];
  return a;
}
poly operator-(const poly& a, const poly& b) { poly r=a; r
  ↪-=b; return r; }
poly operator*(const poly& a, const poly& b) {
  // TODO: small-case?
  return multiply(a, b);
}
poly& operator*=(poly& a, const poly& b) {return a = a*b;}
// hash-cpp-9 = 61b8743c2b07beed0e7ca857081e1bd4
poly& operator*=(poly& a, const num& b) { // Optional
  trav(x, a) x = x * b;
  return a;
}
poly operator*(const poly& a, const num& b) { poly r=a; r*=
  ↪b; return r; }

// Polynomial floor division; no leading 0's plz
poly operator/(poly a, poly b) { // hash-cpp-10
  if (sz(a) < sz(b)) return {};
  int s = sz(a)-sz(b)+1;
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
  a.resize(s);
  b.resize(s);
  a = a * inverse(move(b));
  a.resize(s);
  reverse(a.begin(), a.end());
  return a;
} // hash-cpp-10 = a6589ce8fcf1e33df3b42ee703a7fe60
poly& operator/=(poly& a, const poly& b) {return a = a/b;}
poly& operator%=(poly& a, const poly& b) { // hash-cpp-11
  if (sz(a) >= sz(b)) {
    poly c = (a / b) * b;
    a.resize(sz(b)-1);
    rep(i,0,sz(a)) a[i] = a[i]-c[i];
  }
  return a;
} // hash-cpp-11 = 9af255f48abbeafd8acde353357b84fd
poly operator%(const poly& a, const poly& b) { poly r=a; r
  ↪%=b; return r; }

// Log/exp/pow
poly deriv(const poly& a) { // hash-cpp-12
  if (a.empty()) return {};
  poly b(sz(a)-1);
  rep(i,1,sz(a)) b[i-1]=a[i]*i;
  return b;
} // hash-cpp-12 = 94aa209b3e956051e6b3131bf1faafd1
poly integ(const poly& a) { // hash-cpp-13
  poly b(sz(a)+1);
  b[1]=1; // mod p
  rep(i,2,sz(b)) b[i]=b[fft::mod%i]*(-fft::mod/i); // mod p
  rep(i,1,sz(b)) b[i]=a[i-1]*b[i]; // mod p
  //rep(i,1,sz(b)) b[i]=a[i-1]*inv(num(i)); // else
  return b;
} // hash-cpp-13 = 6f13f6a43b2716a116d347000820f0bd
poly log(const poly& a) { // a[0] == 1 // hash-cpp-14
  poly b = integ(deriv(a)*inverse(a));
  b.resize(a.size());
  return b;
} // hash-cpp-14 = ce1533264298c5382f72a2a1b0947045
poly exp(const poly& a) { // a[0] == 0 // hash-cpp-15
  poly b(1,num(1));
  if (a.empty()) return b;
  while (sz(b) < sz(a)) {
    int n = min(sz(b) * 2, sz(a));
    b.resize(n);
    poly v = poly(a.begin(), a.begin() + n) - log(b);
    v[0] = v[0]+num(1);
    b *= v;
    b.resize(n);
  }
  return b;
} // hash-cpp-15 = f645d091e4ae3ee3dc2aa095d4aa699a
poly pow(const poly& a, int m) { // m >= 0 // hash-cpp-16
  poly b(a.size());
  if (!m) { b[0] = 1; return b; }
  int p = 0;
  while (p<sz(a) && a[p].v==0) ++p;
  if (1ll*m*p >= sz(a)) return b;
  num mu = pow(a[p], m), di = inv(a[p]);
  poly c(sz(a) - m*p);
  rep(i,0,sz(c)) c[i] = a[i+p] * di;
  c = log(c);
  trav(v,c) v = v * m;
  c = exp(c);
  rep(i,0,sz(c)) b[i+m*p] = c[i] * mu;
  return b;
} // hash-cpp-16 = 0f4830b9de34c26d39f170069827121f

// Multipoint evaluation/interpolation
// hash-cpp-17
vector<num> eval(const poly& a, const vector<num>& x) {
  int n=sz(x);
  if (!n) return {};
  vector<poly> up(2*n);
  rep(i,0,n) up[i+n] = poly({0-x[i], 1});
  per(i,1,n) up[i] = up[2*i]*up[2*i+1];
  vector<poly> down(2*n);
  down[1] = a % up[1];
  rep(i,2,2*n) down[i] = down[i/2] % up[i];
  vector<num> y(n);
  rep(i,0,n) y[i] = down[i+n][0];
  return y;
} // hash-cpp-17 = a079eba46c3110851ec6b0490b439931
// hash-cpp-18
poly interp(const vector<num>& x, const vector<num>& y) {
  int n=sz(x);
  assert(n);
  vector<poly> up(n*2);
  rep(i,0,n) up[i+n] = poly({0-x[i], 1});
  per(i,1,n) up[i] = up[2*i]*up[2*i+1];
  vector<num> a = eval(deriv(up[1]), x);
  vector<poly> down(2*n);
  rep(i,0,n) down[i+n] = poly({y[i]*inv(a[i])});
  per(i,1,n) down[i] = down[i*2] * up[i*2+1] + down[i*2+1]
    ↪* up[i*2];
  return down[1];
} // hash-cpp-18 = 74f15e1e82d51e852b321a1ff75ba1fd
```

## FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x\oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of $a$ must be a power of two.

**Time:** $\mathcal{O}(N \log N)$ 16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) trav(x, a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
} // hash-cpp-all = 3de473e2c1de97e6e9ff0f13542cf3fb
```

# Number theory (4)

## 4.1 Modular arithmetic

### ModularArithmetic.h

**Description:** Operators for modular arithmetic. You need to set mod to some number first and then you can use the structure.

"euclid.h" 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod);
    ↪ }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
}; // hash-cpp-all = 35bfea8c111cb24c4ce84c658446961b
```

### ModInverse.h

**Description:** Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

4 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
// hash-cpp-all = 6f684f0b9ae6c69f42de68f023a81de5
```

### ModPow.h

6 lines

```
const ll mod = 1000000007; // faster if const
ll modpow(ll a, ll e) {
  if (e == 0) return 1;
  ll x = modpow(a * a % mod, e >> 1);
  return e & 1 ? x * a % mod : x;
} // hash-cpp-all = 2fa6d9ccac4586cba0618aad18cdc9de
```

### ModSum.h

**Description:** Sums of mod'ed arithmetic progressions.
modsum(to, c, k, m) $= \sum_{i=0}^{to-1} (ki + c)\%m$. divsum is similar but for floored division.

**Time:** $\log(m)$, with a large constant. 19 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (k) {
    ull to2 = (to * k + c) / m;
    res += to * to2;
    res -= divsum(to2, m-1 - c, m, k) + to2;
  }
  return res;
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
} // hash-cpp-all = 8d6e082e0ea6be867eaea12670d08dcc
```

### ModMulLL.h

**Description:** Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for large $c$.

**Time:** $\mathcal{O}(64/bits \cdot \log b)$, where $bits = 64 - k$, if we want to deal with $k$-bit numbers. 19 lines

```
typedef unsigned long long ull;
const int bits = 10;
// if all numbers are less than 2^k, set bits = 64-k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
  ull x = a * (b & (po - 1)) % c;
  while ((b >>= bits) > 0) {
    a = (a << bits) % c;
    x += (a * (b & (po - 1))) % c;
  }
  return x % c;
}
ull mod_pow(ull a, ull b, ull mod) {
  if (b == 0) return 1;
  ull res = mod_pow(a, b / 2, mod);
  res = mod_mul(res, res, mod);
  if (b & 1) return mod_mul(res, a, mod);
  return res;
} // hash-cpp-all = 40cd743544228d297c803154525107ab
```

### ModSqrt.h

**Description:** Tonelli-Shanks algorithm for modular square roots.

**Time:** $\mathcal{O}(\log^2 p)$ worst case, often $\mathcal{O}(\log p)$

"ModPow.h" 30 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1);
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1;
  int r = 0;
  while (s % 2 == 0)
    ++r, s /= 2;
  ll n = 2; // find a non-square mod p
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p);
  ll g = modpow(n, s, p);
  for (;;) {
    ll t = b;
    int m = 0;
    for (; m < r; ++m) {
      if (t == 1) break;
      t = t * t % p;
    }
    if (m == 0) return x;
    ll gs = modpow(g, 1 << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
    r = m;
  }
} // hash-cpp-all = 83e24bd39c8c93946ad3021b8ca6c3c4
```

## 4.2 Primality

### eratosthenes.h

**Description:** Prime sieve for generating all primes up to a certain limit. isprime$[i]$ is true iff $i$ is a prime.

**Time:** lim=100'000'000 $\approx$ 0.8 s. Runs 30% faster if only odd indices are stored. 11 lines

```
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
  isprime.set(); isprime[0] = isprime[1] = 0;
  for (int i = 4; i < lim; i += 2) isprime[i] = 0;
  for (int i = 3; i*i < lim; i += 2) if (isprime[i])
    for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
  vi pr;
  rep(i,2,lim) if (isprime[i]) pr.push_back(i);
  return pr;
} // hash-cpp-all = 0564a3337fb69c0b87dfd3c56cdfe2e3
```

### MillerRabin.h

**Description:** Miller-Rabin primality probabilistic test. Probability of failing one iteration is at most 1/4. 15 iterations should be enough for 50-bit numbers.

**Time:** 15 times the complexity of $a^b \bmod c$.

"ModMulLL.h" 16 lines

```
bool prime(ull p) {
  if (p == 2) return true;
  if (p == 1 || p % 2 == 0) return false;
  ull s = p - 1;
  while (s % 2 == 0) s /= 2;
  rep(i,0,15) {
    ull a = rand() % (p - 1) + 1, tmp = s;
    ull mod = mod_pow(a, tmp, p);
    while (tmp != p - 1 && mod != 1 && mod != p - 1) {
      mod = mod_mul(mod, mod, p);
      tmp *= 2;
    }
    if (mod != p - 1 && tmp % 2 == 0) return false;
  }
  return true;
} // hash-cpp-all = ccddf18bab60a654ff4af45e95dd60b6
```

### factor.h

**Description:** Pollard's rho algorithm. It is a probabilistic factorisation algorithm, whose expected time complexity is good. Before you start using it, run `init(bits)`, where bits is the length of the numbers you use. Returns factors of the input without duplicates.
**Time:** Expected running time should be good enough for 50-bit numbers.

"ModMulLL.h", "MillerRabin.h", "eratosthenes.h" 35 lines

```
vector<ull> pr;
ull f(ull a, ull n, ull &has) {
  return (mod_mul(a, a, n) + has) % n;
}
vector<ull> factor(ull d) {
  vector<ull> res;
  for (int i = 0; i < sz(pr) && pr[i]*pr[i] <= d; i++)
    if (d % pr[i] == 0) {
      while (d % pr[i] == 0) d /= pr[i];
      res.push_back(pr[i]);
    }
  //d is now a product of at most 2 primes.
  if (d > 1) {
    if (prime(d))
      res.push_back(d);
    else while (true) {
      ull has = rand() % 2321 + 47;
      ull x = 2, y = 2, c = 1;
      for (; c==1; c = __gcd((y > x ? y - x : x - y), d)) {
        x = f(x, d, has);
        y = f(f(y, d, has), d, has);
      }
      if (c != d) {
        res.push_back(c); d /= c;
        if (d != c) res.push_back(d);
        break;
      }
    }
  }
  return res;
}
void init(int bits) {//how many bits do we use?
  vi p = eratosthenes_sieve(1 << ((bits + 2) / 3));
  pr.assign(all(p));
} // hash-cpp-all = 67b304bd690b2a8445a7b4dbf93996d7
```

## 4.3 Divisibility

### euclid.h

**Description:** Finds the Greatest Common Divisor to the integers $a$ and $b$. Euclid also finds two integers $x$ and $y$, such that $ax + by = \gcd(a, b)$. If $a$ and $b$ are coprime, then $x$ is the inverse of $a \pmod b$.

7 lines

```
ll gcd(ll a, ll b) { return __gcd(a, b); }

ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) { ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d; }
  return x = 1, y = 0, a;
} // hash-cpp-all = 63e6f8d2f560b27cb800273d63d2102c
```

### Euclid.java

**Description:** Finds {x, y, d} s.t. ax + by = d = gcd(a, b).

11 lines

```
static BigInteger[] euclid(BigInteger a, BigInteger b) {
  BigInteger x = BigInteger.ONE, yy = x;
  BigInteger y = BigInteger.ZERO, xx = y;
  while (b.signum() != 0) {
    BigInteger q = a.divide(b), t = b;
    b = a.mod(b); a = t;
    t = xx; xx = x.subtract(q.multiply(xx)); x = t;
    t = yy; yy = y.subtract(q.multiply(yy)); y = t;
  }
  return new BigInteger[]{x, y, a};
}
```

## 4.4 Fractions

### ContinuedFractions.h

**Description:** Given $N$ and a real number $x \ge 0$, finds the closest rational approximation $p/q$ with $p, q \le N$. It will obey $|p/q - x| \le 1/qN$. For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If $x$ is rational, $y$ eventually becomes $\infty$; if $x$ is the root of a degree 2 polynomial the $a$'s eventually become cyclic.
**Time:** $\mathcal{O}(\log N)$

21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x
    ↪;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf
      ↪),
       a = (ll)floor(y), b = min(a, lim),
       NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives
        ↪us a
      // better approximation; if b = a/2, we *may* have
        ↪one.
      // Return {P, Q} here for a more canonical
        ↪approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)
        ↪) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
} // hash-cpp-all = dd6c5e1084a26365dc6321bd935975d9
```

### FracBinarySearch.h

**Description:** Given $f$ and $N$, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \le N$. You may want to throw an exception from $f$ if it finds an exact solution, in which case $N$ can be removed.
**Usage:** `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`
**Time:** $\mathcal{O}(\log(N))$

24 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N
    ↪]
  assert(!f(lo)); assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
} // hash-cpp-all = 214844f17d0c347ff436141729e0c829
```

## 4.5 Chinese remainder theorem

### chinese.h

**Description:** Chinese Remainder Theorem.
`chinese(a, m, b, n)` returns a number $x$, such that $x \equiv a \pmod m$ and $x \equiv b \pmod n$. For not coprime $n, m$, use `chinese_common`. Note that all numbers must be less than $2^{31}$ if you have Z = unsigned long long.
**Time:** $\log(m + n)$

"euclid.h" 13 lines

```
template<class Z> Z chinese(Z a, Z m, Z b, Z n) {
  Z x, y; euclid(m, n, x, y);
  Z ret = a * (y + m) % m * n + b * (x + n) % n * m;
  if (ret >= m * n) ret -= m * n;
  return ret;
}

template<class Z> Z chinese_common(Z a, Z m, Z b, Z n) {
  Z d = gcd(m, n);
  if (((b -= a) %= n) < 0) b += n;
  if (b % d) return -1; // No solution
  return d * chinese(Z(0), m/d, b/d, n/d) + a;
} // hash-cpp-all = da3099704e14964aa045c152bb478c14
```

## 4.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \quad b = k \cdot (2mn), \quad c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either $m$ or $n$ even.

## 4.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power $p^a$, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

## 4.8 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of $n$ is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

# Combinatorial (5)

## 5.1 Permutations

### 5.1.1 Factorial

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| $n!$ | 1 | 2 | 6 | 24 | 120 | 720 | 5040 | 40320 | 362880 | 3628800 |

| $n$ | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
|---|---|---|---|---|---|---|---|
| $n!$ | 4.0e7 | 4.8e8 | 6.2e9 | 8.7e10 | 1.3e12 | 2.1e13 | 3.6e14 |

| $n$ | 20 | 25 | 30 | 40 | 50 | 100 | 150 | 171 |
|---|---|---|---|---|---|---|---|---|
| $n!$ | 2e18 | 2e25 | 3e32 | 8e47 | 3e64 | 9e157 | 6e262 | >DBL_MAX |

IntPerm.h
**Description:** Permutation -> integer conversion. (Not order preserving.)
**Time:** $\mathcal{O}(n)$
6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  trav(x,v) r=r * ++i + __builtin_popcount(use & -(1 << x)),
    use |= 1 << x;                // (note: minus, not ~!)
  return r;
} // hash-cpp-all = e1b8eaea02324af14a3da94f409019b8
```

### 5.1.2 Cycles

Let $g_S(n)$ be the number of $n$-permutations whose cycle lengths all belong to the set $S$. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

### 5.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

### 5.1.4 Burnside's lemma

Given a group $G$ of symmetries and a set $X$, the number of elements of $X$ *up to symmetry* equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where $X^g$ are the elements fixed by $g$ ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length $n$, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

## 5.2 Partitions and subsets

### 5.2.1 Partition function

Number of ways of writing $n$ as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 20 | 50 | 100 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $p(n)$ | 1 | 1 | 2 | 3 | 5 | 7 | 11 | 15 | 22 | 30 | 627 | ~2e5 | ~2e8 |

### 5.2.2 Binomials

binomialModPrime.h
**Description:** Lucas' thm: Let $n, m$ be non-negative integers and $p$ a prime. Write $n = n_k p^k + ... + n_1 p + n_0$ and $m = m_k p^k + ... + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod{p}$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.
**Time:** $\mathcal{O}(\log_p n)$
10 lines

```
ll chooseModP(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] %
      ↪ p;
    n /= p; m /= p;
  }
  return c;
} // hash-cpp-all = 81845faa6ecd635c391e4f0134f0676c
```

multinomial.h
**Description:** Computes $\binom{k_1 + \cdots + k_n}{k_1, k_2, \ldots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \ldots k_n!}$.
6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
} // hash-cpp-all = a0a3128f6afa4721166feb182b82f130
```

## 5.3 General purpose numbers

### 5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t - 1}$ (FFT-able). $B[0, \ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1} \sum_{k=0}^{m} \binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^{\infty} f(x)dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m)$$

$$\approx \int_m^{\infty} f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

### 5.3.2 Stirling numbers of the first kind

Number of permutations on $n$ items with $k$ cycles.

$$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$$
$$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$$

$c(8,k) =$
$8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) =$
$0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

### 5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly $k$ elements are greater than the previous element. $k$ $j$:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ $j$:s s.t. $\pi(j) \geq j$, $k$ $j$:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$

$$E(n,0) = E(n,n-1) = 1$$

$$E(n,k) = \sum_{j=0}^{k} (-1)^j \binom{n+1}{j} (k+1-j)^n$$

#### 5.3.4 Stirling numbers of the second kind

Partitions of $n$ distinct elements into exactly $k$ groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$

$$S(n,1) = S(n,n) = 1$$

$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

#### 5.3.5 Bell numbers

Total number of partitions of $n$ distinct elements. $B(n) = 1,1,2,5,15,52,203,877,4140,21147,\ldots$. For $p$ prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

#### 5.3.6 Labeled unrooted trees

\# on $n$ vertices: $n^{n-2}$
\# on $k$ existing trees of size $n_i$: $n_1 n_2 \cdots n_k n^{k-2}$
\# with degrees $d_i$: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

#### 5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1,1,2,5,14,42,132,429,1430,4862,16796,58786,\ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with $n$ pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

## 5.4 Other

### nim-product.cpp

**Description:** Nim Product.

17 lines

```cpp
using ull = uint64_t;
ull _nimProd2[64][64];
ull nimProd2(int i, int j) {
  if (_nimProd2[i][j]) return _nimProd2[i][j];
  if ((i & j) == 0) return _nimProd2[i][j] = 1ull << (i|j);
  int a = (i&j) & -(i&j);
  return _nimProd2[i][j] = nimProd2(i ^ a, j) ^ nimProd2((i
    ↪ ^ a) | (a-1), (j ^ a) | (i & (a-1)));
}
ull nimProd(ull x, ull y) {
  ull res = 0;
  for (int i = 0; x >> i; i++)
    if ((x >> i) & 1)
      for (int j = 0; y >> j; j++)
        if ((y >> j) & 1)
          res ^= nimProd2(i, j);
  return res;
} // hash-cpp-all = e0411498c7a77d77ae793efab5500851
```

### schreier-sims.cpp

**Description:** Check group membership of permutation groups

52 lines

```cpp
struct Perm {
  int a[N];
  Perm() {
    for (int i = 1; i <= n; ++i) a[i] = i;
  }
  friend Perm operator* (const Perm &lhs, const Perm &rhs)
    ↪{
    static Perm res;
    for (int i = 1; i <= n; ++i) res.a[i] = lhs.a[rhs.a[i
      ↪]];
    return res;
  }
  friend Perm inv(const Perm &cur) {
    static Perm res;
    for (int i = 1; i <= n; ++i) res.a[cur.a[i]] = i;
    return res;
  }
};
class Group {
  bool flag[N];
  Perm w[N];
  std::vector<Perm> x;
public:
  void clear(int p) {
    memset(flag, 0, sizeof flag);
    for (int i = 1; i <= n; ++i) w[i] = Perm();
    flag[p] = true;
    x.clear();
  }
  friend bool check(const Perm&, int);
  friend void insert(const Perm&, int);
  friend void updateX(const Perm&, int);
} g[N];
bool check(const Perm &cur, int k) {
  if (!k) return true;
  int t = cur.a[k];
  return g[k].flag[t] ? check(g[k].w[t] * cur, k - 1) :
    ↪false;
}
void updateX(const Perm&, int);
void insert(const Perm &cur, int k) {
  if (check(cur, k)) return;
  g[k].x.push_back(cur);
  for (int i = 1; i <= n; ++i) if (g[k].flag[i]) updateX(
    ↪cur * inv(g[k].w[i]), k);
}
void updateX(const Perm &cur, int k) {
  int t = cur.a[k];
  if (g[k].flag[t]) {
    insert(g[k].w[t] * cur, k - 1);
  } else {
    g[k].w[t] = inv(cur);
    g[k].flag[t] = true;
    for (int i = 0; i < g[k].x.size(); ++i) updateX(g[k].x[
      ↪i] * cur, k);
  }
} // hash-cpp-all = 949a6e50dbdaea9cda09928c7eabedbc
```

# Graph (6)

## 6.1 Network flow

### PushRelabel.h

**Description:** Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

**Time:** $\mathcal{O}\left(V^2\sqrt{E}\right)$

51 lines

```cpp
typedef ll Flow;
struct Edge {
  int dest, back;
  Flow f, c;
};

struct PushRelabel {
  vector<vector<Edge>> g;
  vector<Flow> ec;
  vector<Edge*> cur;
  vector<vi> hs; vi H;
  PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n)
    ↪{}

  void add_edge(int s, int t, Flow cap, Flow rcap=0) {
    if (s == t) return;
    Edge a = {t, sz(g[t]), 0, cap};
    Edge b = {s, sz(g[s]), 0, rcap};
    g[s].push_back(a);
    g[t].push_back(b);
  }

  void add_flow(Edge& e, Flow f) {
    Edge &back = g[e.dest][e.back];
    if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
    e.f += f; e.c -= f; ec[e.dest] += f;
    back.f -= f; back.c += f; ec[back.dest] -= f;
  }
  Flow maxflow(int s, int t) {
    int v = sz(g); H[s] = v; ec[t] = 1;
    vi co(2*v); co[0] = v-1;
    rep(i,0,v) cur[i] = g[i].data();
    trav(e, g[s]) add_flow(e, e.c);

    for (int hi = 0;;) {
      while (hs[hi].empty()) if (!hi--) return -ec[s];
      int u = hs[hi].back(); hs[hi].pop_back();
```

```
      while (ec[u] > 0)  // discharge u
        if (cur[u] == g[u].data() + sz(g[u])) {
          H[u] = 1e9;
          trav(e, g[u]) if (e.c && H[u] > H[e.dest]+1)
            H[u] = H[e.dest]+1, cur[u] = &e;
          if (++co[H[u]], !--co[hi] && hi < v)
            rep(i,0,v) if (hi < H[i] && H[i] < v)
              --co[H[i]], H[i] = v + 1;
          hi = H[u];
        } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)
          add_flow(*cur[u], min(ec[u], cur[u]->c));
        else ++cur[u];
    }
  }
}; // hash-cpp-all = aaa2dd3fd7d9e6d994b295a959664c9a
```

## MinCostMaxFlow.h

**Description:** Min-cost max-flow. cap[i][j] != cap[j][i] is allowed; double edges are not. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.
**Time:** Approximately $\mathcal{O}\left(E^2\right)$ 81 lines

```
#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;
typedef vector<ll> VL;

struct MCMF {
  int N;
  vector<vi> ed, red;
  vector<VL> cap, flow, cost;
  vi seen;
  VL dist, pi;
  vector<pii> par;

  MCMF(int N) :
    N(N), ed(N), red(N), cap(N, VL(N)), flow(cap), cost(cap
      ↪),
    seen(N), dist(N), pi(N), par(N) {}

  void addEdge(int from, int to, ll cap, ll cost) {
    this->cap[from][to] = cap;
    this->cost[from][to] = cost;
    ed[from].push_back(to);
    red[to].push_back(from);
  }

  void path(int s) {
    fill(all(seen), 0);
    fill(all(dist), INF);
    dist[s] = 0; ll di;

    __gnu_pbds::priority_queue<pair<ll, int>> q;
    vector<decltype(q)::point_iterator> its(N);
    q.push({0, s});

    auto relax = [&](int i, ll cap, ll cost, int dir) {
      ll val = di - pi[i] + cost;
      if (cap && val < dist[i]) {
        dist[i] = val;
        par[i] = {s, dir};
        if (its[i] == q.end()) its[i] = q.push({-dist[i], i
          ↪});
        else q.modify(its[i], {-dist[i], i});
      }
    };

    while (!q.empty()) {
      s = q.top().second; q.pop();
      seen[s] = 1; di = dist[s] + pi[s];
      trav(i, ed[s]) if (!seen[i])
        relax(i, cap[s][i] - flow[s][i], cost[s][i], 1);
      trav(i, red[s]) if (!seen[i])
        relax(i, flow[i][s], -cost[i][s], 0);
    }
    rep(i,0,N) pi[i] = min(pi[i] + dist[i], INF);
  }

  pair<ll, ll> maxflow(int s, int t) {
    ll totflow = 0, totcost = 0;
    while (path(s), seen[t]) {
      ll fl = INF;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        fl = min(fl, r ? cap[p][x] - flow[p][x] : flow[x][p
          ↪]);
      totflow += fl;
      for (int p,r,x = t; tie(p,r) = par[x], x != s; x = p)
        if (r) flow[p][x] += fl;
        else flow[x][p] -= fl;
    }
    rep(i,0,N) rep(j,0,N) totcost += cost[i][j] * flow[i][j
      ↪];
    return {totflow, totcost};
  }

  // If some costs can be negative, call this before
    ↪maxflow:
  void setpi(int s) { // (otherwise, leave this out)
    fill(all(pi), INF); pi[s] = 0;
    int it = N, ch = 1; ll v;
    while (ch-- && it--)
      rep(i,0,N) if (pi[i] != INF)
        trav(to, ed[i]) if (cap[i][to])
          if ((v = pi[i] + cost[i][to]) < pi[to])
            pi[to] = v, ch = 1;
    assert(it >= 0); // negative cost cycle
  }
}; // hash-cpp-all = 6915cee27314b77b2f5e256f1a96cdc0
```

## EdmondsKarp.h

**Description:** Flow algorithm with guaranteed complexity $O(VE^2)$. To get edge flow values, compare capacities before and after, and take the positive values only. 35 lines

```
template<class T> T edmondsKarp(vector<unordered_map<int, T
  ↪>>& graph, int source, int sink) {
  assert(source != sink);
  T flow = 0;
  vi par(sz(graph)), q = par;

  for (;;) {
    fill(all(par), -1);
    par[source] = 0;
    int ptr = 1;
    q[0] = source;

    rep(i,0,ptr) {
      int x = q[i];
      trav(e, graph[x]) {
        if (par[e.first] == -1 && e.second > 0) {
          par[e.first] = x;
          q[ptr++] = e.first;
          if (e.first == sink) goto out;
        }
      }
    }
    return flow;
out:
    T inc = numeric_limits<T>::max();
    for (int y = sink; y != source; y = par[y])
      inc = min(inc, graph[par[y]][y]);

    flow += inc;
    for (int y = sink; y != source; y = par[y]) {
      int p = par[y];
      if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
      graph[y][p] += inc;
    }
  }
} // hash-cpp-all = 979bb9ccc85090e328209bf565a2af26
```

## MinCut.h

**Description:** After running max-flow, the left side of a min-cut from $s$ to $t$ is given by all vertices reachable from $s$, only traversing edges with positive residual capacity. 1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

## GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.
**Time:** $\mathcal{O}\left(V^3\right)$ 31 lines

```
pair<int, vi> GetMinCut(vector<vi>& weights) {
  int N = sz(weights);
  vi used(N), cut, best_cut;
  int best_weight = -1;

  for (int phase = N-1; phase >= 0; phase--) {
    vi w = weights[0], added = used;
    int prev, k = 0;
    rep(i,0,phase){
      prev = k;
      k = -1;
      rep(j,1,N)
        if (!added[j] && (k == -1 || w[j] > w[k])) k = j;
      if (i == phase-1) {
        rep(j,0,N) weights[prev][j] += weights[k][j];
        rep(j,0,N) weights[j][prev] = weights[prev][j];
        used[k] = true;
        cut.push_back(k);
        if (best_weight == -1 || w[k] < best_weight) {
          best_cut = cut;
          best_weight = w[k];
        }
      } else {
        rep(j,0,N)
          w[j] += weights[k][j];
        added[k] = true;
      }
    }
  }
  return {best_weight, best_cut};
} // hash-cpp-all = 03261f13665169d285596975383c72b3
```

## 6.2 Matching

### hopcroftKarp.h

**Description:** Find a maximum matching in a bipartite graph.
**Usage:** `vi ba(m, -1); hopcroftKarp(g, ba);`
**Time:** $\mathcal{O}\left(\sqrt{V}E\right)$

45 lines

```
bool dfs(int a, int layer, const vector<vi>& g, vi& btoa,
      vi& A, vi& B) {
  if (A[a] != layer) return 0;
  A[a] = -1;
  trav(b, g[a]) if (B[b] == layer + 1) {
    B[b] = -1;
    if (btoa[b] == -1 || dfs(btoa[b], layer+2, g, btoa, A,
        ↪B))
      return btoa[b] = a, 1;
  }
  return 0;
}

int hopcroftKarp(const vector<vi>& g, vi& btoa) {
  int res = 0;
  vi A(g.size()), B(btoa.size()), cur, next;
  for (;;) {
    fill(all(A), 0);
    fill(all(B), -1);
    cur.clear();
    trav(a, btoa) if(a != -1) A[a] = -1;
    rep(a,0,sz(g)) if(A[a] == 0) cur.push_back(a);
    for (int lay = 1;; lay += 2) {
      bool islast = 0;
      next.clear();
      trav(a, cur) trav(b, g[a]) {
        if (btoa[b] == -1) {
          B[b] = lay;
          islast = 1;
        }
        else if (btoa[b] != a && B[b] == -1) {
          B[b] = lay;
          next.push_back(btoa[b]);
        }
      }
      if (islast) break;
      if (next.empty()) return res;
      trav(a, next) A[a] = lay+1;
      cur.swap(next);
    }
    rep(a,0,sz(g)) {
      if(dfs(a, 0, g, btoa, A, B))
        ++res;
    }
  }
} // hash-cpp-all = ee9fe891045fe156e995ef0276b80af6
```

### DFSMatching.h

**Description:** This is a simple matching algorithm but should be just fine in most cases. Graph $g$ should be a list of neighbours of the left partition. $n$ is the size of the left partition and $m$ is the size of the right partition. If you want to get the matched pairs, $match[i]$ contains match for vertex $i$ on the right side or $-1$ if it's not matched.
**Time:** $\mathcal{O}(EV)$ where $E$ is the number of edges and V is the number of vertices.

24 lines

```
vi match;
vector<bool> seen;
bool find(int j, const vector<vi>& g) {
  if (match[j] == -1) return 1;
  seen[j] = 1; int di = match[j];
  trav(e, g[di])
    if (!seen[e] && find(e, g)) {
      match[e] = di;
      return 1;
    }
  return 0;
}
int dfs_matching(const vector<vi>& g, int n, int m) {
  match.assign(m, -1);
  rep(i,0,n) {
    seen.assign(m, 0);
    trav(j,g[i])
      if (find(j, g)) {
        match[j] = i;
        break;
      }
  }
  return m - (int)count(all(match), -1);
} // hash-cpp-all = 178c94b6091dc009a15d348aef80dff0
```

### WeightedMatching.h

**Description:** Min cost bipartite matching. Negate costs for max cost.
**Time:** $\mathcal{O}(N^3)$

75 lines

```
typedef vector<double> vd;
bool zero(double x) { return fabs(x) < 1e-10; }
double MinCostMatching(const vector<vd>& cost, vi& L, vi& R
    ↪) {
  int n = sz(cost), mated = 0;
  vd dist(n), u(n), v(n);
  vi dad(n), seen(n);

  rep(i,0,n) {
    u[i] = cost[i][0];
    rep(j,1,n) u[i] = min(u[i], cost[i][j]);
  }
  rep(j,0,n) {
    v[j] = cost[0][j] - u[0];
    rep(i,1,n) v[j] = min(v[j], cost[i][j] - u[i]);
  }

  L = R = vi(n, -1);
  rep(i,0,n) rep(j,0,n) {
    if (R[j] != -1) continue;
    if (zero(cost[i][j] - u[i] - v[j])) {
      L[i] = j;
      R[j] = i;
      mated++;
      break;
    }
  }

  for (; mated < n; mated++) { // until solution is
      ↪feasible
    int s = 0;
    while (L[s] != -1) s++;
    fill(all(dad), -1);
    fill(all(seen), 0);
    rep(k,0,n)
      dist[k] = cost[s][k] - u[s] - v[k];

    int j = 0;
    for (;;) {
      j = -1;
      rep(k,0,n){
        if (seen[k]) continue;
        if (j == -1 || dist[k] < dist[j]) j = k;
      }
      seen[j] = 1;
      int i = R[j];
      if (i == -1) break;
      rep(k,0,n) {
        if (seen[k]) continue;
        auto new_dist = dist[j] + cost[i][k] - u[i] - v[k];
        if (dist[k] > new_dist) {
          dist[k] = new_dist;
          dad[k] = j;
        }
      }
    }

    rep(k,0,n) {
      if (k == j || !seen[k]) continue;
      auto w = dist[k] - dist[j];
      v[k] += w, u[R[k]] -= w;
    }
    u[s] += dist[j];

    while (dad[j] >= 0) {
      int d = dad[j];
      R[j] = R[d];
      L[R[j]] = j;
      j = d;
    }
    R[j] = s;
    L[s] = j;
  }
  auto value = vd(1)[0];
  rep(i,0,n) value += cost[i][L[i]];
  return value;
} // hash-cpp-all = 055ca9687f72b2dd5e2d2c6921f1c51d
```

### GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability $N/mod$.
**Time:** $\mathcal{O}(N^3)$

"../numerical/MatrixInverse-mod.h" 40 lines

```
vector<pii> generalMatching(int N, vector<pii>& ed) {
  vector<vector<ll>> mat(N, vector<ll>(N)), A;
  trav(pa, ed) {
    int a = pa.first, b = pa.second, r = rand() % mod;
    mat[a][b] = r, mat[b][a] = (mod - r) % mod;
  }

  int r = matInv(A = mat), M = 2*N - r, fi, fj;
  assert(r % 2 == 0);

  if (M != N) do {
    mat.resize(M, vector<ll>(M));
    rep(i,0,N) {
      mat[i].resize(M);
      rep(j,N,M) {
        int r = rand() % mod;
        mat[i][j] = r, mat[j][i] = (mod - r) % mod;
      }
    }
  } while (matInv(A = mat) != M);

  vi has(M, 1); vector<pii> ret;
  rep(it,0,M/2) {
    rep(i,0,M) if (has[i])
      rep(j,i+1,M) if (A[i][j] && mat[i][j]) {
        fi = i; fj = j; goto done;
```

```
    } assert(0); done:
    if (fj < N) ret.emplace_back(fi, fj);
    has[fi] = has[fj] = 0;
    rep(sw,0,2) {
      ll a = modpow(A[fi][fj], mod-2);
      rep(i,0,M) if (has[i] && A[i][fj]) {
        ll b = A[i][fj] * a % mod;
        rep(j,0,M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod
          ↪;
      }
      swap(fi,fj);
    }
  }
  return ret;
} // hash-cpp-all = bb8be4f4f83b4e4ccafaebf8534e4f82
```

### blossom.h

**Description:** O(EV) general matching

65 lines

```
// vertices 1~n, chd[x]=0 or y (x match y)
int n;
vector<int> g[N];
int chd[N],nex[N],fl[N],fa[N];
int gf(int x){return fa[x]==x?x:fa[x]=gf(fa[x]);}
void un(int x,int y){x=gf(x),y=gf(y);fa[x]=y;}
int qu[N],p,q;
int lca(int u,int v){
  static int t=0,x[N];
  t++;
  for(;; swap(u,v) )
    if(u){
      u=gf(u);
      if(x[u]==t)return u;
      x[u]=t;
      u= chd[u] ? nex[chd[u]] : 0;
    }
}
void lk(int a,int x){
  while(a!=x){
    int b=chd[a],c=nex[b];
    if(gf(c)!=x)nex[c]=b;
    if(fl[b]==2)fl[qu[q++]=b]=1;
    if(fl[c]==2)fl[qu[q++]=c]=1;
    un(a,b);un(b,c);
    a=c;
  }
}
void find(int rt){
  rep(i,1,n+1)nex[i]=fl[i]=0,fa[i]=i;
  p=q=0;qu[q++]=rt;fl[rt]=1;
  while(p!=q){
    int u=qu[p++];
    trav(v, g[u]) {
      if(gf(v)==gf(u) || fl[v]==2 || v==chd[u])continue;
      if(fl[v]==1){
        int x=lca(u,v);
        if(gf(u)!=x)nex[u]=v;
        if(gf(v)!=x)nex[v]=u;
        lk(u,x);
        lk(v,x);
      }else if(!chd[v]){
        nex[v]=u;
        while(v){
          u=nex[v];
          int t=chd[u];
          chd[v]=u;chd[u]=v;
          v=t;
        }
        return;
      }else{
        nex[v]=u;
        fl[v]=2;
        fl[qu[q++]=chd[v]]=1;
      }
    }
  }
}
int run_match(){
  memset(chd,0,sizeof(chd));
  rep(i,1,n+1)if(!chd[i])find(i);
  int cnt = 0;
  rep(i,1,n+1) cnt += bool(chd[i]);
  return cnt/2;
} // hash-cpp-all = 54d8d95b9dc2053ea903a35ce4928a11
```

### MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is an independent set.

"DFSMatching.h" 20 lines

```
vi cover(vector<vi>& g, int n, int m) {
  int res = dfs_matching(g, n, m);
  seen.assign(m, false);
  vector<bool> lfound(n, true);
  trav(it, match) if (it != -1) lfound[it] = false;
  vi q, cover;
  rep(i,0,n) if (lfound[i]) q.push_back(i);
  while (!q.empty()) {
    int i = q.back(); q.pop_back();
    lfound[i] = 1;
    trav(e, g[i]) if (!seen[e] && match[e] != -1) {
      seen[e] = true;
      q.push_back(match[e]);
    }
  }
  rep(i,0,n) if (!lfound[i]) cover.push_back(i);
  rep(i,0,m) if (seen[i]) cover.push_back(n+i);
  assert(sz(cover) == res);
  return cover;
} // hash-cpp-all = 9eeda105ef373dfc9bd11d0139e4fc82
```

## 6.3 DFS algorithms

### SCC.h

**Description:** Finds strongly connected components in a directed graph. If vertices $u, v$ belong to the same component, we can reach $u$ from $v$ and vice versa.

**Usage:** `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.

**Time:** $\mathcal{O}(E + V)$

24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  trav(e,g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));

  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;
  }
  return val[j] = low;
}
template<class G, class F> void scc(G& g, F f) {
  int n = sz(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
} // hash-cpp-all = 2c7a153ddd31436517cf3ad28efa4ac5
```

### BiconnectedComponents.h

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two distinct paths between any two nodes. Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

**Usage:** `int eid = 0; ed.resize(N);`
`for each edge (a,b) {`
`ed[a].emplace_back(b, eid);`
`ed[b].emplace_back(a, eid++); }`
`bicomps([&](const vi& edgelist) {...});`

**Time:** $\mathcal{O}(E + V)$

33 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F f) {
  int me = num[at] = ++Time, e, y, top = me;
  trav(pa, ed[at]) if (pa.second != par) {
    tie(y, e) = pa;
    if (num[y]) {
      top = min(top, num[y]);
      if (num[y] < me)
        st.push_back(e);
    } else {
      int si = sz(st);
      int up = dfs(y, e, f);
      top = min(top, up);
      if (up == me) {
        st.push_back(e);
        f(vi(st.begin() + si, st.end()));
        st.resize(si);
      }
      else if (up < me) st.push_back(e);
      else { /* e is a bridge */ }
    }
  }
  return top;
}

template<class F>
void bicomps(F f) {
  num.assign(sz(ed), 0);
  rep(i,0,sz(ed)) if (!num[i]) dfs(i, -1, f);
} // hash-cpp-all = e183ffd0266ca965525c2788c540f8f0
```

### 2sat.h

**Description:** Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a\|\|b)\&\&(!a\|\|c)\&\&(d\|\|!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).

**Usage:** `TwoSat ts(number of boolean variables);`
```
ts.either(0, ~3); // Var 0 is true or var 3 is false
ts.set_value(2); // Var 2 is true
ts.at_most_one({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true
ts.solve(); // Returns true iff it is solvable
ts.values[0..N-1] holds the assigned values to the vars
```

**Time:** $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

57 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int add_var() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f^1].push_back(j);
    gr[j^1].push_back(f);
  }
  void set_value(int x) { either(x, x); }

  void at_most_one(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = add_var();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    trav(e, gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    ++time;
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = time;
      if (values[x>>1] == -1)
        values[x>>1] = !(x&1);
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
}; // hash-cpp-all = 288fb44b52e9016a30ce849e38390eb9
```

## 6.4 Heuristics

### MaximalCliques.h

**Description:** Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Possible optimization: on the top-most recursion level, ignore 'cands', and go through nodes in order of increasing degree, where degrees go down as nodes are removed.

**Time:** $\mathcal{O}\left(3^{n/3}\right)$, much faster for sparse graphs

12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R
  ↪={}) {
  if (!P.any()) { if (!X.any()) f(R); return; }
  auto q = (P | X)._Find_first();
  auto cands = P & ~eds[q];
  rep(i,0,sz(eds)) if (cands[i]) {
    R[i] = 1;
    cliques(eds, f, P & eds[i], X & eds[i], R);
    R[i] = P[i] = 0; X[i] = 1;
  }
} // hash-cpp-all = b0d5b15b7ebdcde7ff57f0761c050583
```

### graph-clique.cpp

**Description:** Max clique N<64. Bit trick for speed. clique solver calculates both size and consitution of maximum clique uses bit operation to accelerate searching graph size limit is 63, the graph should be undirected can optimize to calculate on each component, and sort on vertex degrees can be used to solve maximum independent set

82 lines

```
class clique {
  public:
  static const long long ONE = 1;
  static const long long MASK = (1 << 21) - 1;
  char* bits;
  int n, size, cmax[63];
  long long mask[63], cons;
  // initiate lookup table
  clique() {
    bits = new char[1 << 21];
    bits[0] = 0;
    for (int i = 1; i < (1<<21); ++i)
      bits[i] = bits[i >> 1] + (i & 1);
  }
  ~clique() {
    delete bits;
  }
  // search routine
  bool search(int step,int siz,LL mor,LL con);
  // solve maximum clique and return size
  int sizeClique(vector<vector<int> >& mat);
  // solve maximum clique and return set
  vector<int>getClq(vector<vector<int> >&mat);
};
// step is node id, size is current sol., more is available
  ↪ mask, cons is constitution mask
bool clique::search(int step, int size,
                    LL more, LL cons) {
  if (step >= n) {
    if (size > this->size) {
      // a new solution reached
      this->size = size;
      this->cons = cons;
    }
    return true;
  }
  long long now = ONE << step;
  if ((now & more) > 0) {
    long long next = more & mask[step];
    if (size + bits[next & MASK] +
        bits[(next >> 21) & MASK] +
        bits[next >> 42] >= this->size
     && size + cmax[step] > this->size) {
      // the current node is in the clique
      if (search(step+1,size+1,next,cons|now))
        return true;
    }
  }
  long long next = more & ~now;
  if (size + bits[next & MASK] +
      bits[(next >> 21) & MASK] +
      bits[next >> 42] > this->size) {
    // the current node is not in the clique
    if (search(step + 1, size, next, cons))
      return true;
  }
  return false;
}
// solve maximum clique and return size
int clique::sizeClique(vector<vector<int> >& mat) {
  n = mat.size();
  // generate mask vectors
  for (int i = 0; i < n; ++i) {
    mask[i] = 0;
    for (int j = 0; j < n; ++j)
      if (mat[i][j] > 0) mask[i] |= ONE << j;
  }
  size = 0;
  for (int i = n - 1; i >= 0; --i) {
    search(i + 1, 1, mask[i], ONE << i);
    cmax[i] = size;
  }
  return size;
}
// calls sizeClique and restore cons
vector<int> clique::getClq(
    vector<vector<int> >& mat) {
  sizeClique(mat);
  vector<int> ret;
  for (int i = 0; i < n; ++i)
    if ((cons&(ONE<<i)) > 0) ret.push_back(i);
  return ret;
} // hash-cpp-all = 15b35db59a457782d2954fa526acf199
```

### cycle-counting.cpp

**Description:** Counts 3 and 4 cycles

<bits/stdc++.h> 62 lines

```
#define P 1000000007
#define N 110000

int n, m;
vector <int> go[N], lk[N];

int w[N];
int circle3(){ // hash-cpp-1
  int ans=0;
  for (int i = 1; i <= n; i++)
```

```
    w[i]=0;

  for (int x = 1; x <= n; x++) {
    for(int y:lk[x])w[y]=1;

    for(int y:lk[x])for(int z:lk[y])if(w[z]){
      ans=(ans+go[x].size()+go[y].size()+go[z].size()-6)%P;
    }

    for(int y:lk[x])w[y]=0;
  }
  return ans;
} // hash-cpp-1 = 719dcec935e20551fd984c12c3bfa3ba

int deg[N], pos[N], id[N];

int circle4(){ // hash-cpp-2
  for (int i = 1; i <= n; i++)
    w[i]=0;
  int ans=0;
  for (int x = 1; x <= n; x++) {
    for(int y:go[x])for(int z:lk[y])if(pos[z]>pos[x]){
      ans=(ans+w[z])%P;
      w[z]++;
    }
    for(int y:go[x])for(int z:lk[y])w[z]=0;
  }
  return ans;
} // hash-cpp-2 = 39b3aaf47e9fdc4dfff3fdfdf22d3a8e

inline bool cmp(const int &x,const int &y){
  return deg[x]<deg[y];
}

void init() {
  scanf("%d%d", &n, &m);
  for (int i = 1; i <= n; i++)
    deg[i] = 0, go[i].clear(), lk[i].clear();;
  while (m--) {
    int a,b;
    scanf("%d%d",&a,&b);
    deg[a]++;deg[b]++;
    go[a].push_back(b);go[b].push_back(a);
  }
  for (int i = 1; i <= n; i++)
    id[i] = i;
  sort(id+1,id+1+n,cmp);
  for (int i = 1; i <= n; i++) pos[id[i]]=i;
  for (int x = 1; x <= n; x++)
    for(int y:go[x])
      if(pos[y]>pos[x])lk[x].push_back(y);
}
```

## 6.5 Trees

### TreePower.h

**Description:** Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
**Time:** construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$ — 25 lines

```
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
} // hash-cpp-all = bfce856c17ac46dca77ea0f43aaa359a
```

### LCA.h

**Description:** Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected. Can also find the distance between two nodes.
**Usage:** `LCA lca(undirGraph);`
`lca.query(firstNode, secondNode);`
`lca.distance(firstNode, secondNode);`
**Time:** $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" — 37 lines

```
typedef vector<pii> vpi;
typedef vector<vpi> graph;

struct LCA {
  vi time;
  vector<ll> dist;
  RMQ<pii> rmq;

  LCA(graph& C) : time(sz(C), -99), dist(sz(C)), rmq(dfs(C)
     ↪) {}

  vpi dfs(graph& C) {
    vector<tuple<int, int, int, ll>> q(1);
    vpi ret;
    int T = 0, v, p, d; ll di;
    while (!q.empty()) {
      tie(v, p, d, di) = q.back();
      q.pop_back();
      if (d) ret.emplace_back(d, p);
      time[v] = T++;
      dist[v] = di;
      trav(e, C[v]) if (e.first != p)
        q.emplace_back(e.first, v, d+1, di + e.second);
    }
    return ret;
  }

  int query(int a, int b) {
    if (a == b) return a;
    a = time[a], b = time[b];
    return rmq.query(min(a, b), max(a, b)).second;
  }
  ll distance(int a, int b) {
    int lca = query(a, b);
    return dist[a] + dist[b] - 2 * dist[lca];
  }
}; // hash-cpp-all = aa0d4d6df33671856cc22ac596e7df06
```

### CompressTree.h

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most $|S| - 1$) pairwise LCA's and compressing edges. Returns a list of (par, orig_index) representing a tree rooted at 0. The root points to itself.
**Time:** $\mathcal{O}(|S| \log |S|)$

"LCA.h" — 20 lines

```
vpi compressTree(LCA& lca, const vi& subset) {
  static vi rev; rev.resize(sz(lca.dist));
  vi li = subset, &T = lca.time;
  auto cmp = [&](int a, int b) { return T[a] < T[b]; };
  sort(all(li), cmp);
  int m = sz(li)-1;
  rep(i,0,m) {
    int a = li[i], b = li[i+1];
    li.push_back(lca.query(a, b));
  }
  sort(all(li), cmp);
  li.erase(unique(all(li)), li.end());
  rep(i,0,sz(li)) rev[li[i]] = i;
  vpi ret = {pii(0, li[0])};
  rep(i,0,sz(li)-1) {
    int a = li[i], b = li[i+1];
    ret.emplace_back(rev[lca.query(a, b)], b);
  }
  return ret;
} // hash-cpp-all = dabd7520dba8306be5675979add23011
```

### HLD.h

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. The function of the HLD can be changed by modifying T, LOW and f. f is assumed to be associative and commutative.
**Usage:** `HLD hld(G);`
`hld.update(index, value);`
`tie(value, lca) = hld.query(n1, n2);`

"../data-structures/SegmentTree.h" — 93 lines

```
typedef vector<pii> vpi;

struct Node {
  int d, par, val, chain = -1, pos = -1;
};

struct Chain {
  int par, val;
  vector<int> nodes;
  Tree tree;
};

struct HLD {
  typedef int T;
  const T LOW = -(1<<29);
  void f(T& a, T b) { a = max(a, b); }

  vector<Node> V;
  vector<Chain> C;

  HLD(vector<vpi>& g) : V(sz(g)) {
    dfs(0, -1, g, 0);
    trav(c, C) {
      c.tree = {sz(c.nodes), 0};
      for (int ni : c.nodes)
        c.tree.update(V[ni].pos, V[ni].val);
    }
  }

  void update(int node, T val) {
```

```
    Node& n = V[node]; n.val = val;
    if (n.chain != -1) C[n.chain].tree.update(n.pos, val);
  }

  int pard(Node& nod) {
    if (nod.par == -1) return -1;
    return V[nod.chain == -1 ? nod.par : C[nod.chain].par].
       ↪d;
  }

  // query all *edges* between n1, n2
  pair<T, int> query(int i1, int i2) {
    T ans = LOW;
    while(i1 != i2) {
      Node n1 = V[i1], n2 = V[i2];
      if (n1.chain != -1 && n1.chain == n2.chain) {
        int lo = n1.pos, hi = n2.pos;
        if (lo > hi) swap(lo, hi);
        f(ans, C[n1.chain].tree.query(lo, hi));
        i1 = i2 = C[n1.chain].nodes[hi];
      } else {
        if (pard(n1) < pard(n2))
          n1 = n2, swap(i1, i2);
        if (n1.chain == -1)
          f(ans, n1.val), i1 = n1.par;
        else {
          Chain& c = C[n1.chain];
          f(ans, n1.pos ? c.tree.query(n1.pos, sz(c.nodes))
                        : c.tree.s[1]);
          i1 = c.par;
        }
      }
    }
    return make_pair(ans, i1);
  }

  // query all *nodes* between n1, n2
  pair<T, int> query2(int i1, int i2) {
    pair<T, int> ans = query(i1, i2);
    f(ans.first, V[ans.second].val);
    return ans;
  }

  pii dfs(int at, int par, vector<vpi>& g, int d) {
    V[at].d = d; V[at].par = par;
    int sum = 1, ch, nod, sz;
    tuple<int,int,int> mx(-1,-1,-1);
    trav(e, g[at]){
      if (e.first == par) continue;
      tie(sz, ch) = dfs(e.first, at, g, d+1);
      V[e.first].val = e.second;
      sum += sz;
      mx = max(mx, make_tuple(sz, e.first, ch));
    }
    tie(sz, nod, ch) = mx;
    if (2*sz < sum) return pii(sum, -1);
    if (ch == -1) { ch = sz(C); C.emplace_back(); }
    V[nod].pos = sz(C[ch].nodes);
    V[nod].chain = ch;
    C[ch].par = at;
    C[ch].nodes.push_back(nod);
    return pii(sum, ch);
  }
}; // hash-cpp-all = d952a915dfa34f93fbf32fe2755a651e
```

### LinkCutTree.h

**Description:** Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
**Time:** All operations take amortized $\mathcal{O}(\log N)$.
90 lines

```
struct Node { // Splay tree. Root's pp contains tree's
   ↪parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void push_flip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y :
       ↪ x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (push_flip(); p; ) {
      if (p->p) p->p->push_flip();
      p->push_flip(); push_flip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    push_flip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    make_root(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    make_root(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same
     ↪tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void make_root(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
}; // hash-cpp-all = 693483a825b93f7ad5f6ef219ba83e22
```

### MatrixTree.h

**Description:** To count the number of spanning trees in an undirected graph $G$: create an $N \times N$ matrix `mat`, and for each edge $(a, b) \in G$, do `mat[a][a]++, mat[b][b]++, mat[a][b]--, mat[b][a]--`. Remove the last row and column, and take the determinant.
1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

## 6.6 Other

### directed-MST.cpp

**Description:** Finds the minimum spanning arborescence from the root. (any more notes?)
73 lines

```
#define rep(i, n) for (int i = 0; i < n; i++)

#define N 110000
#define M 110000
#define inf 2000000000

struct edg {
    int u, v;
    int cost;
} E[M], E_copy[M];

int In[N], ID[N], vis[N], pre[N];

// edges pointed from root.
int Directed_MST(int root, int NV, int NE) {
  for (int i = 0; i < NE; i++)
    E_copy[i] = E[i];
    int ret = 0;
    int u, v;
    while (true) {
```

```
        rep(i, NV)   In[i] = inf;
        rep(i, NE) {
            u = E_copy[i].u;
            v = E_copy[i].v;
            if(E_copy[i].cost < In[v] && u != v) {
                In[v] = E_copy[i].cost;
                pre[v] = u;
            }
        }
        rep(i, NV) {
            if(i == root)   continue;
            if(In[i] == inf)    return -1; // no solution
        }

        int cnt = 0;
        rep(i, NV) {
          ID[i] = -1;
          vis[i] = -1;
        }
        In[root] = 0;

        rep(i, NV) {
            ret += In[i];
            int v = i;
            while(vis[v] != i && ID[v] == -1 && v != root) {
                vis[v] = i;
                v = pre[v];
            }
            if(v != root && ID[v] == -1) {
                for(u = pre[v]; u != v; u = pre[u]) {
                    ID[u] = cnt;
                }
                ID[v] = cnt++;
            }
        }
        if(cnt == 0)    break;
        rep(i, NV) {
            if(ID[i] == -1) ID[i] = cnt++;
        }
        rep(i, NE) {
            v = E_copy[i].v;
            E_copy[i].u = ID[E_copy[i].u];
            E_copy[i].v = ID[E_copy[i].v];
            if(E_copy[i].u != E_copy[i].v) {
                E_copy[i].cost -= In[v];
            }
        }
        NV = cnt;
        root = ID[root];
    }
    return ret;
}
// hash-cpp-all = 84815c2bfececf3575ecf663c0703643
```

## graph-dominator-tree.cpp

**Description:** Dominator Tree.

107 lines

```
#define N 110000 //max number of vertices

vector<int> succ[N], prod[N], bucket[N], dom_t[N];
int semi[N], anc[N], idom[N], best[N], fa[N], tmp_idom[N];
int dfn[N], redfn[N];
int child[N], size[N];
int timestamp;

void dfs(int now) { // hash-cpp-1
  dfn[now] = ++timestamp;
  redfn[timestamp] = now;
  anc[timestamp] = idom[timestamp] = child[timestamp] = size[timestamp] = 0;
  semi[timestamp] = best[timestamp] = timestamp;
  int sz = succ[now].size();
  for(int i = 0; i < sz; ++i) {
    if(dfn[succ[now][i]] == -1) {
      dfs(succ[now][i]);
      fa[dfn[succ[now][i]]] = dfn[now];
    }
    prod[dfn[succ[now][i]]].push_back(dfn[now]);
  }
} // hash-cpp-1 = 6412bfd6a0d21b66ddaa51ea79cbe7bd

void compress(int now) { // hash-cpp-2
  if(anc[anc[now]] != 0) {
    compress(anc[now]);
    if(semi[best[now]] > semi[best[anc[now]]])
      best[now] = best[anc[now]];
    anc[now] = anc[anc[now]];
  }
} // hash-cpp-2 = 1c9444eb3f768b7af8741fafbf3afb5a

inline int eval(int now) { // hash-cpp-3
  if(anc[now] == 0)
    return now;
  else {
    compress(now);
    return semi[best[anc[now]]] >= semi[best[now]] ? best[now]
    : best[anc[now]];
  }
} // hash-cpp-3 = 4e235f39666315b46dcd3455d5f866d1

inline void link(int v, int w) { // hash-cpp-4
  int s = w;
  while(semi[best[w]] < semi[best[child[w]]]) {
    if(size[s] + size[child[child[s]]] >= 2*size[child[s]]) {
      anc[child[s]] = s;
      child[s] = child[child[s]];
    } else {
      size[child[s]] = size[s];
      s = anc[s] = child[s];
    }
  }
  best[s] = best[w];
  size[v] += size[w];
  if(size[v] < 2*size[w])
    swap(s, child[v]);
  while(s != 0) {
    anc[s] = v;
    s = child[s];
  }
} // hash-cpp-4 = 270548fd021351ae21e97878f367b6f9

// idom[n] and other vertices that cannot be reached from n will be 0
void lengauer_tarjan(int n) { // n is the root's number //hash-cpp-5
  memset(dfn, -1, sizeof dfn);
  memset(fa, -1, sizeof fa);
  timestamp = 0;
  dfs(n);
  fa[1] = 0;
  for(int w = timestamp; w > 1; --w) {
    int sz = prod[w].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(prod[w][i]);
      if(semi[w] > semi[u])
        semi[w] = semi[u];
    }
    bucket[semi[w]].push_back(w);
    //anc[w] = fa[w]; link operation for o(mlogm) version
          link(fa[w], w);
    if(fa[w] == 0)
      continue;
    sz = bucket[fa[w]].size();
    for(int i = 0; i < sz; ++i) {
      int u = eval(bucket[fa[w]][i]);
      if(semi[u] < fa[w])
        idom[bucket[fa[w]][i]] = u;
      else
        idom[bucket[fa[w]][i]] = fa[w];
    }
    bucket[fa[w]].clear();
  }
  for(int w = 2; w <= timestamp; ++w) {
    if(idom[w] != semi[w])
      idom[w] = idom[idom[w]];
  }
  idom[1] = 0;
  for(int i = timestamp; i > 1; --i) {
    if(fa[i] == -1)
      continue;
    dom_t[idom[i]].push_back(i);
  }
  memset(tmp_idom, 0, sizeof tmp_idom);
  for (int i = 1; i <= timestamp; i++)
    tmp_idom[redfn[i]] = redfn[idom[i]];
  memcpy(idom, tmp_idom, sizeof idom);
} // hash-cpp-5 = f49c40461d92222d8d39b28b0de66828
```

## graph-negative-cycle.cpp

**Description:** negative cycle

31 lines

```
double b[N][N];

double dis[N];
int vis[N], pc[N];

bool dfs(int k) {
  vis[k] += 1; pc[k] = true;
  for (int i = 0; i < N; i++)
    if (dis[k] + b[k][i] < dis[i]) {
      dis[i] = dis[k] + b[k][i];
      if (!pc[i]) {
        if (dfs(i))
          return true;
      } else return true;
    }
  pc[k] = false;
  return false;
}

bool chk(double d) {
  for (int i = 0; i < N; i ++)
    for (int j = 0; j < N; j ++) {
      b[i][j] = -a[i][j] + d;
    }
  for (int i = 0; i < N; i++)
    vis[i] = false, dis[i] = 0, pc[i] = false;
  for (int i = 0; i < N; i++)
    if (!vis[i] && dfs(i))
```

```
    return true;
  return false;
} // hash-cpp-all = 9dca2d48b5f0f580f13d220e7ecdbf71
```

# Geometry (7)

## 7.1 Geometric primitives

### Point.h

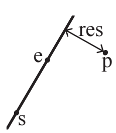
**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this); }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90 degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
}; // hash-cpp-all = 47ec0abfb6b604da9f744979e71bada0
```

### lineDistance.h

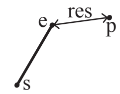
**Description:**
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.




"Point.h" 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
} // hash-cpp-all = f6bf6b556d99b09f42b86d28d1eaa86d
```

### SegmentDistance.h




**Description:**
Returns the shortest distance between point p and the line segment from point s to e.

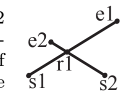
**Usage:** Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h" 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
  return ((p-s)*d-(e-s)*t).dist()/d;
} // hash-cpp-all = 5c88f46fb14a05a4f47bbd23b8a9c427
```

### SegmentIntersection.h

**Description:**
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



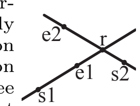

**Usage:** vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;

"Point.h", "OnSegment.h" 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
} // hash-cpp-all = 9d57f2f844788770022fbcd8bfc5b2f2
```

### lineIntersection.h

**Description:**
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.




**Usage:** auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;

"Point.h" 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
} // hash-cpp-all = a01f815e2e60161e03879264c4826dd0
```

### sideOf.h

**Description:** Returns where $p$ is as seen from $s$ towards $e$. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument *eps* is given 0 is returned if $p$ is within distance *eps* from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

**Usage:** bool left = sideOf(p1,p2,q)==1;

"Point.h" 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
```

```
int sideOf(const P& s, const P& e, const P& p, double eps)
   ↪{
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
} // hash-cpp-all = 3af81cc4f24d9d9fb109d930f3b9764c
```

## OnSegment.h

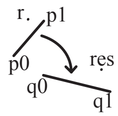
**Description:** Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h" 3 lines

```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
} // hash-cpp-all = c597e8749250f940e4b0139f0dc3e8b9
```

## linearTransformation.h

**Description:**




Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h" 6 lines

```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.
     ↪dist2();
} // hash-cpp-all = 03a3061b3ef024b4e29ea06169932b21
```

## Angle.h

**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

**Usage:** vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

35 lines

```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
     ↪}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)
     ↪}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
   ↪between
// them, i.e., the angle that covers the defined line
   ↪segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
     ↪)};
} // hash-cpp-all = 0f0602c74320456a8a8627737c1d3c64
```

# 7.2 Circles

## CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h" 11 lines

```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
   ↪out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
           ↪d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
     ↪d2);
  *out = {mid + per, mid - per};
  return true;
} // hash-cpp-all = 84d6d345ef48c336b811d1a54deda11d
```

## CircleTangents.h

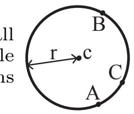
**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h" 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double
   ↪r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
} // hash-cpp-all = b0153d0ef1b8a6b1fa4d91480c4126e8
```

## circumcircle.h

**Description:**




The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h" 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} // hash-cpp-all = 1caa3aea364671cb961900d4811f0282
```

## MinimumEnclosingCircle.h

**Description:** Computes the minimum circle that encloses a set of points.

**Time:** expected $\mathcal{O}(n)$

"circumcircle.h" 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
} // hash-cpp-all = 09dd0aa4515bb46bac3171f71d032132
```

# 7.3 Polygons

## InsidePolygon.h

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

**Usage:** vector<P> v = {P{4,4}, P{1,2}, P{2,1}};
bool in = inPolygon(v, P{3, 3}, false);

**Time:** $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
       ↪0;
  }
  return cnt;
} // hash-cpp-all = 2bf504baea895948c3c332b0b474bd98
```

## PolygonArea.h

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
} // hash-cpp-all = f123003799a972c1292eb0d8af7e37da
```

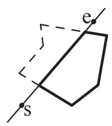
## PolygonCenter.h

**Description:** Returns the center of mass for a polygon.
**Time:** $\mathcal{O}(n)$

"Point.h" 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
} // hash-cpp-all = 9706dcc8eb8dae007d9a12070a93b128
```

## PolygonCut.h

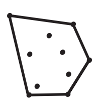



**Description:**
Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.
**Usage:** vector<P> p = ...;
p = polygonCut(p, P(0,0), P(1,0));

"Point.h", "lineIntersection.h" 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
} // hash-cpp-all = f2b7d494a6a577ade19ef0e9eed7f049
```

## ConvexHull.h



**Description:**
Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    trav(p, pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t
        ↪--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h
    ↪[1])};
} // hash-cpp-all = 26a0a95e23f2183fa044ccdff1257cf8
```

## HullDiameter.h

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/colinear points).

12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}})
        ↪;
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
        ↪ 0)
        break;
    }
  return res.second;
} // hash-cpp-all = c571b8edf5b751f996dc80283d32a92c
```

## PointInsideHull.h

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no colinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.
**Time:** $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<=
    ↪ -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
} // hash-cpp-all = 71446bda4e9e17eb03867d22daa5808e
```

## LineHullIntersection.h

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no colinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner $i$, • $(i,i)$ if along side $(i,i+1)$, • $(i,j)$ if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner $i$ is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.
**Time:** $\mathcal{O}(N + Q \log n)$

"Point.h" 39 lines

```
typedef array<P, 2> Line;
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%
  ↪n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) =
      ↪ m;
  }
  return lo;
}

#define cmpL(i) sgn(line[0].cross(poly[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> poly) {
  int endA = extrVertex(poly, (line[0] - line[1]).perp());
  int endB = extrVertex(poly, (line[1] - line[0]).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
} // hash-cpp-all = 758f2248384a18f71c84cf3d63a93ed7
```

# 7.4 Misc. Point Set Problems

## ClosestPair.h

**Description:** Finds the closest pair of points.
**Time:** $\mathcal{O}(n \log n)$

"Point.h" 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  trav(p, v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
      ↪d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
} // hash-cpp-all = d31bbf685a10a34219ab9bab86ee486e
```

## Voronoi.h

"ConvexHull.h", "lineIntersection.h" 47 lines

```
vector<vector<P> > voronoi(const vector<P>& inp)
{
  vector<P> poly = inp;
  vector<vector<P>> res;
```

```
  vector<PP> inv;
  vector<pair<P,P>> ans;
  vector<PP> vor;
  vector<P> V;
  int n = poly.size();
  poly.push_back(P());
  poly.push_back(P());
  poly.push_back(P());
  poly.push_back(P());
  for (int i=0; i<n; i++)
  {
    poly[n] = P(-2*oo-poly[i].x,poly[i].y);
    poly[n+1] = (P(poly[i].x,2*oo-poly[i].y));
    poly[n+2] = (P(2*oo-poly[i].x,poly[i].y));
    poly[n+3] = (P(poly[i].x,-2*oo-poly[i].y));
    inv.clear();
    for (int j=0; j<n+4; j++)
    if (j!=i)
    {
      //int jj = j - (j>i);
      double tmp = (poly[j]-poly[i]).dist2();
      inv.push_back(make_pair((poly[j]-poly[i])/tmp,j));
      //cerr << inv[inv.size()-1].first << '\n';
    }
    vor = convexHull(inv);
    vor.push_back(*vor.begin());
    int vors = vor.size();
    ans.clear();
    for (int j=0; j<vors; j++)
    {
      P tmp = (poly[i]+poly[vor[j].second])/2.0;
      ans.push_back(make_pair(tmp,P(tmp.x-(poly[vor[j].second].y-poly[i].y),tmp.y+(poly[vor[j].second].x-poly[i].x))));
    }
    V.clear();
    for (int j=1; j<vors; j++)
    {
      pair<int,P> v = lineInter(ans[j-1].first,ans[j-1].second,ans[j].first,ans[j].second);
      V.push_back(v.second);
    }
    res.push_back(V);
  }
  return res;
} // hash-cpp-all = 019d2df5df3a15722956d5875d4e3c82
```

## 7.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards. 6 lines

```
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilist) {
  double v = 0;
  trav(i, trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
} // hash-cpp-all = 1ec4d393ab307cedc3866534eaa83a0e
```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long. 32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z){}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0, pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
}; // hash-cpp-all = 8058aeda36daf3cba079c7bb0b43dcea
```

### 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
**Time:** $\mathcal{O}\left(n^2\right)$

"Point3D.h" 49 lines

```
typedef Point3D<double> P3;

struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
}; // hash-cpp-all = c172e9f2cb6b44ceca0c416fee81f1dc
```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 ($\phi_1$) and f2 ($\phi_2$) from x axis and zenith angles (latitude) t1 ($\theta_1$) and t2 ($\theta_2$) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points. 8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
} // hash-cpp-all = 611f0797307c583c66413c2dd5b3ba28
```

# Strings (8)

### AhoCorasick.cpp

**Description:** Init, and insert strings, and then build.

<bits/stdc++.h> 49 lines

```
const int M = (int)5e5 + 500;

struct Trie{
    static const int B = 26;

    int next[M][B], fail[M], end[M];
    int root, L;
    int newnode(){ // hash-cpp-1
        for(int i = 0; i < B; i++) next[L][i] = -1;
        end[L++] = 0;
        return L - 1;
    }
    // Please do initalize it !!!
    void init(){
        L = 0;
        root = newnode();
```

```
    }
    void insert(const string &buf){
        int len = buf.length();
        int now = root;
        for(int i = 0; i < len; i++){
            if(next[now][buf[i]-'a'] == -1) next[now][buf[i
              ↪]-'a'] = newnode();
            now = next[now][buf[i]-'a'];
        }
        end[now] ++;
    }
    void build(){
        queue<int> Q;
        fail[root] = root;
        for(int i = 0; i < B; i++){
            if(next[root][i] == -1) next[root][i] = root;
            else{
                fail[next[root][i]] = root;
                Q.push(next[root][i]);
            }
        }
        while(!Q.empty()){
            int now = Q.front();
            Q.pop();
            for(int i = 0; i < B; i++){
                if(next[now][i] == -1) next[now][i] = next[
                  ↪fail[now]][i];
                else{
                    fail[next[now][i]] = next[fail[now]][i
                      ↪];
                    Q.push(next[now][i]);
                }
            }
        }
    } // hash-cpp-1 = 825cf85504458ad07d9a0021d74cf1a6
} Aho;
```

## KMP.cpp

24 lines

```
string s, t;
int f[M];

void getnext(){
    int m = t.length();
    f[0] = 0; f[1] = 0;
    for(int i = 1; i < m; i++){
        int j = f[i];
        while(j && t[i] != t[j]) j = f[j];
        f[i+1] = t[i] == t[j] ? j + 1 : 0;
    }
}

int find(){
    int n = s.length(), m = t.length();
    int res = 0;
    int j = 0;
    for(int i = 0; i < n; i++){
        while(j && t[j] != s[i]) j = f[j];
        if(t[j] == s[i]) j++;
        if(j == m) res ++, j = f[j];
    }
    return res;
} // hash-cpp-all = 78587018eee67364b798a07b53e4f004
```

## Manacher.cpp

42 lines

```
struct Manacher {
    string s, sn;
    int p[2*N];

    int Init() {
        int len = s.length();
        sn = "$#";
        int j = 2;

        for (int i = 0; i < len; i++)
        {
            sn.push_back(s[i]);
            sn.push_back('#');
        }

        sn.push_back('\0');

        return sn.length();
    }

    int run() {
        int len = Init();
        int max_len = -1;

        int id = 0;
        int mx = 0;

        for (int i = 1; i < len; i++)
        {
            if (i < mx) p[i] = min(p[2 * id - i], mx - i);
            else p[i] = 1;

            while (sn[i - p[i]] == sn[i + p[i]]) p[i]++;

            if (mx < i + p[i]) id = i, mx = i + p[i];

            max_len = max(max_len, p[i] - 1);
        }
        return max_len;
    }
} mnc;
// hash-cpp-all = d0dff75e997690b1592e4b36c6e9aefc
```

## PolynomialHashing.cpp

<bits/stdc++.h> 62 lines

```
typedef long long ll;
typedef pair<int, int> P;
const int mods[4] = {(int)1e9 + 7, (int)1e9 + 9, (int)1e9 +
   ↪ 21, (int)1e9 + 33};
const int N = (int)2e5 + 50;

string s, t;
int p = 37;
ll pw[4][N];

struct hs {
    ll val[4];

    hs() { fill(val, val + 4, 0); }

    hs(ll a, ll b, ll c, ll d) {
        val[0] = a, val[1] = b, val[2] = c, val[3] = d;
    }

    bool operator<(const hs &other) const {
        for (int i = 0; i < 4; i++) if (val[i] != other.val
          ↪[i]) return val[i] < other.val[i];
        return false;
    }

    bool operator==(const hs &other) const {
        for (int i = 0; i < 4; i++) if (val[i] != other.val
          ↪[i]) return false;
        return true;
    }

    hs operator + (const hs &other) const{
        hs res;
        for(int i = 0; i < 4; i++) res.val[i] = ( val[i] +
          ↪other.val[i]) % mods[i];
        return res;
    }

    hs operator - (const hs &other) const{
        hs res;
        for(int i = 0; i < 4; i++) res.val[i] = (val[i] -
          ↪other.val[i] + mods[i]) % mods[i];
        return res;
    }

    hs operator ^ (const int pwi) const {
        hs res;
        for(int i = 0; i < 4; i++){
            res.val[i] = (val[i] * pw[i][pwi]) % mods[i];
        }
        return res;
    }

    void add(int x, int pwi){
        for(int i = 0; i < 4; i++) {
            val[i] = (val[i] + x * pw[i][pwi]) % mods[i];
            if(val[i] < 0) val[i] += mods[i];
        }
    }
};

void init() {
    for(int t = 0; t < 4; t++){
        pw[t][0] = 1;
        for(int i = 1; i < N; i++) pw[t][i] = pw[t][i-1] *
          ↪p % mods[t];
    }
} // hash-cpp-all = 442e4ad85b64b40bdb09978cbb6d3154
```

## SAM.cpp

94 lines

```
struct state {
    int len, link;
    int next[B];
};

struct SAM {
    state st[MAXLEN * 2];
    int sz, last;
    int cnt[MAXLEN * 2];
    int anc[LOGN][MAXLEN * 2];

    void sam_init() { // hash-cpp-1
        st[0].len = 0;
        st[0].link = -1;
        memset(st[0].next, -1, sizeof(st[0].next));
        sz = 1;
```

```
        last = 0;
    } // hash-cpp-1 = 88458c9c46d0594815f25aa78c9c9a6f

    int sam_extend(int c, int nlast, int val) { // hash-cpp
        ↪-2
        last = nlast;
        int cur = sz++;
        cnt[cur] = val;
        st[cur].len = st[last].len + 1;
        memset(st[cur].next, -1, sizeof(st[cur].next));
        int p = last;
        while(p != -1 && st[p].next[c] == -1) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if(p == -1) {
            st[cur].link = 0;
        } else {
            int q = st[p].next[c];
            if(st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            } else {
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                memcpy(st[clone].next, st[q].next, sizeof(
                    ↪st[q].next));
                st[clone].link = st[q].link;
                while(p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
        return last;
    } // hash-cpp-2 = a955371ff99c2fb0631535b05a18b044
} sam;

int n, m;
vector<int> tid[N];

struct Trie {
    int nxt[MAXLEN][B];
    int sz;
    int id[MAXLEN];

    void init() { // hash-cpp-3
        sz = 1;
        memset(nxt, -1, sizeof(nxt));
    } // hash-cpp-3 = 83a0e44461d7b8d82a1566fc38a1133c

    void add(string s, int idx) { // hash-cpp-4
        int cur = 0;
        for(char c : s) {
            int &nx = nxt[cur][c - 'a'];
            if(nx == -1) nx = sz++;
            cur = nx;
            tid[idx].push_back(cur);
        }
    } // hash-cpp-4 = 85a0d801ac069f16dbb1ce998394eebd

    void build_sam() { // hash-cpp-5
        sam.sam_init();
        queue<int> que;
        id[0] = 0;
        que.push(0);
        while(!que.empty()) {
            int v = que.front(); que.pop();
            for(int i = 0; i < B; i++) {
                if(nxt[v][i] != -1) {
                    id[nxt[v][i]] = sam.sam_extend(i, id[v
                        ↪], 1);
                    que.push(nxt[v][i]);
                }
            }
        }
        sam.build();
    } // hash-cpp-5 = 4366d5d3fee156021eaad3147606f8ff

} trie;
```

## PAM.cpp

<bits/stdc++.h> 56 lines

```
const int mod = (int)1e9 + 7;

struct PAM {
    static const int N = (int) 1e6 + 50;
    int s[N], now;
    int nxt[N][26], fail[N], l[N], last, tot;
    int diff[N], anc[N];
    int ans[N], dp[N];

    void clear() { // hash-cpp-1
        s[0] = l[1] = -1;
        fail[0] = tot = now = 1;
        last = l[0] = 0;
        memset(nxt[0], -1, sizeof nxt[0]);
        memset(nxt[1], -1, sizeof nxt[1]);
    }

    PAM() { clear(); }

    int newnode(int len) {
        tot++;
        memset(nxt[tot], -1, sizeof nxt[tot]);
        fail[tot] = 0;
        l[tot] = len;
        return tot;
    }

    int get_fail(int x) {
        while (s[now - l[x] - 2] != s[now - 1])x = fail[x];
        return x;
    }

    void add(int ch) {
        s[now++] = ch;
        int cur = get_fail(last);
        if (nxt[cur][ch] == -1) {
            int tt = newnode(l[cur] + 2);
            fail[tt] = nxt[get_fail(fail[cur])][ch];
            if(fail[tt] == -1) fail[tt] = 0;
            nxt[cur][ch] = tt;
            diff[tt] = l[tt] - l[fail[tt]];
            anc[tt] = diff[tt] == diff[fail[tt]] ? anc[fail
                ↪[tt]] : fail[tt];
        }
        last = nxt[cur][ch];
    } // hash-cpp-1 = e98af9fdbd034392cf7c2342f36b059a

    void trans(int i) { // hash-cpp-2
        for(int p = last; p > 1; p = anc[p]) {
            dp[p] = ans[i - l[anc[p]] - diff[p]];
            if(diff[p] == diff[fail[p]]) {
                (dp[p] += dp[fail[p]]) %= mod;
            }
            (ans[i] += (i % 2 == 0) * dp[p]) %= mod;
        }
    } // hash-cpp-2 = 18f4e8a0c11c040383bfe58bbb139bce
} pam;
```

## SuffixArray.cpp

**Description:** `lcp[i] = lcp(sa[i], sa[i-1])`. One indexed for everything.

**Time:** $\mathcal{O}(n \log n)$

25 lines

```
struct SuffixArray {
    vi sa, lcp, rk;
    SuffixArray(string& s, int lim=256) { // or
        ↪basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = rk = y, iota(all(sa), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2),
            ↪lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++;
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                (y[a] == y[b] && y[a + j] == y[b + j])
                    ↪? p - 1 : p++;
        }
        rep(i,1,n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                    s[i + k] == s[j + k]; k++);
        s = " " + s;
        for(int i = n - 1; i >= 1; i--) sa[i]++, rk[sa[i]]
            ↪= i;
    }
}; // hash-cpp-all = 01cdb651bc80da4ec63d9cc01d2cb14c
```

## ZAlgorithm.cpp

**Description:** str = "aabaacd", z = (x, 1, 0, 2, 1, 0, 0)

<bits/stdc++.h> 18 lines

```
const int MAXN = (int)1e6 + 500;

string s;
int z[MAXN],cnt[MAXN];

void getZarr(string str) // hash-cpp-1
{
    memset(z, 0, sizeof(z));
    int n = str.length();
    for(int i = 1, l = 0, r = 0; i < n; ++i){
        if(i <= r)
            z[i] = min(r - i + 1, z[i - 1]);
        while(i + z[i] < n && str[z[i]] == str[i + z[i]])
            ++z[i];
        if(i + z[i] - 1 > r)
            l = i,  r = i + z[i] - 1;
    }
} // hash-cpp-1 = 6d61faecf306b78847022e6fa5f26ea4
```

### DigitDP.cpp

**Description:** 1

<bits/stdc++.h> 26 lines

```
const int B = 20;

int dp[10000][B];
int bit[B], b;
int pw2[B];
int A, B;

int get(int rem, int d, bool flag){
    if(rem < 0) return 0;
    if(d == -1) return rem >= 0;
    if(!flag && ~dp[rem][d]) return dp[rem][d];
    int lim = flag ? bit[d] : 9;
    int res = 0;
    for(int i = 0; i <= lim; i++){
        res += get(rem - i * pw2[d], d - 1, flag && lim == i);
    }
    return flag ? res : dp[rem][d] = res;
}

int solve(int x){
    b = 0;
    int t = x;
    while(t > 0){bit[b++] = t % 10; t /= 10;}
    return get(A, b-1, true);
}
// hash-cpp-all = d99f0e66d42b9dded6b9e05ba7a0ea4f
```

### MinRotation.h

**Description:** Finds the lexicographically smallest rotation of a string.
**Usage:** `rotate(v.begin(), v.begin()+min_rotation(v), v.end());`
**Time:** $\mathcal{O}(N)$

8 lines

```
int min_rotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1); break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
} // hash-cpp-all = 4bd552f76bcc771f09f5602c0914673a
```

# Various (9)

## 9.1 Misc. algorithms

### Karatsuba.h

**Description:** Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX+b)(cX+d) = acX^2 + bd + ((a+c)(b+d) - ac - bd)X$. Doesn't handle sequences of very different length well. See also FFT, under the Numerical chapter.
**Time:** $\mathcal{O}(N^{1.6})$

1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

## 9.2 Dynamic programming

### KnuthDP.h

**Description:** When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal $k$ increases with both $i$ and $j$, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
**Time:** $\mathcal{O}(N^2)$

1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

## 9.3 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

## 9.4 Optimization tricks

### 9.4.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ... }` loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

### 9.4.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

### BumpAllocator.h

**Description:** When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

9 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
  static size_t i = sizeof buf;
  assert(s < i);
  return (void*)&buf[i -= s];
}
void operator delete(void*) {}
// hash-cpp-all = 745db225903de8f3cdfa051660956100
```

### SmallPtr.h

**Description:** A 32-bit pointer that points into BumpAllocator memory.

"BumpAllocator.h" 10 lines

```
template<class T> struct ptr {
  unsigned ind;
  ptr(T* p = 0) : ind(p ? unsigned((char*)p - buf) : 0) {
    assert(ind < sizeof buf);
  }
  T& operator*() const { return *(T*)(buf + ind); }
  T* operator->() const { return &**this; }
  T& operator[](int a) const { return (&**this)[a]; }
  explicit operator bool() const { return ind; }
}; // hash-cpp-all = 2dd6c9773f202bd47422e255099f4829
```

### BumpAllocatorSTL.h

**Description:** BumpAllocator for STL containers.
**Usage:** `vector<vector<int, small<int>>> ed(N);`

14 lines

```
char buf[450 << 20] alignas(16);
size_t buf_ind = sizeof buf;

template<class T> struct small {
  typedef T value_type;
  small() {}
  template<class U> small(const U&) {}
  T* allocate(size_t n) {
    buf_ind -= n * sizeof(T);
    buf_ind &= 0 - alignof(T);
    return (T*)(buf + buf_ind);
  }
  void deallocate(T*, size_t) {}
}; // hash-cpp-all = bb66d4225a1941b85228ee92b9779d4b
```

### Unrolling.h

6 lines

```
#define F {...; ++i;}
int i = from;
while (i&3 && i < to) F // for alignment, if needed
while (i + 4 <= to) { F F F F }
while (i < to) F
// hash-cpp-all = 520e76d6182da81d99aa0e67b36a0b3d
```

### SIMD.h

**Description:** Cheat sheet of SSE/AVX intrinsics, for doing arithmetic on several numbers at once. Can provide a constant factor improvement of about 4, orthogonal to loop unrolling. Operations follow the pattern "_mm(256)?_name_(si(128|256)|epi(8|16|32|64)|pd|ps)". Not all are described here; grep for _mm_ in /usr/lib/gcc/*/4.9/include/ for more. If AVX is unsupported, try 128-bit operations, "emmintrin.h" and #define __SSE__ and __MMX__ before including it. For aligned memory use _mm_malloc(size, 32) or int buf[N] alignas(32), but prefer loadu/storeu.

43 lines

```
#pragma GCC target ("avx2") // or sse4.1
#include "immintrin.h"

typedef __m256i mi;
#define L(x) _mm256_loadu_si256((mi*)&(x))

// High-level/specific methods:
// load(u)?_si256, store(u)?_si256, setzero_si256,
   ↪_mm_malloc
// blendv_(epi8|ps|pd) (z?y:x), movemask_epi8 (hibits of
   ↪bytes)
// i32gather_epi32(addr, x, 4): map addr[] over 32-b parts
   ↪of x
// sad_epu8: sum of absolute differences of u8, outputs 4
   ↪xi64
// maddubs_epi16: dot product of unsigned i7's, outputs 16
   ↪xi15
// madd_epi16: dot product of signed i16's, outputs 8xi32
// extractf128_si256(, i) (256->128), cvtsi128_si32 (128->
   ↪lo32)
// permute2f128_si256(x,x,1) swaps 128-bit lanes
// shuffle_epi32(x, 3*64+2*16+1*4+0) == x for each lane
// shuffle_epi8(x, y) takes a vector instead of an imm

// Methods that work with most data types (append e.g.
   ↪_epi32):
// set1, blend (i8?x:y), add, adds (sat.), mullo, sub, and/
   ↪or,
// andnot, abs, min, max, sign(1,x), cmp(gt|eq), unpack(lo|
   ↪hi)

int sumi32(mi m) { union {int v[8]; mi m;} u; u.m = m;
  int ret = 0; rep(i,0,8) ret += u.v[i]; return ret; }
mi zero() { return _mm256_setzero_si256(); }
mi one() { return _mm256_set1_epi32(-1); }
bool all_zero(mi m) { return _mm256_testz_si256(m, m); }
bool all_one(mi m) { return _mm256_testc_si256(m, one()); }

ll example_filteredDotProduct(int n, short* a, short* b) {
  int i = 0; ll r = 0;
  mi zero = _mm256_setzero_si256(), acc = zero;
  while (i + 16 <= n) {
    mi va = L(a[i]), vb = L(b[i]); i += 16;
    va = _mm256_and_si256(_mm256_cmpgt_epi16(vb, va), va);
    mi vp = _mm256_madd_epi16(va, vb);
    acc = _mm256_add_epi64(_mm256_unpacklo_epi32(vp, zero),
      _mm256_add_epi64(acc, _mm256_unpackhi_epi32(vp, zero)
        ↪));
  }
  union {ll v[4]; mi m;} u; u.m = acc; rep(i,0,4) r += u.v[
    ↪i];
  for (;i<n;++i) if (a[i] < b[i]) r += a[i]*b[i]; // <-
    ↪equiv
  return r;
} // hash-cpp-all = 551b820442570276f239d9d7e0800c65
```

### Hashmap.h

**Description:** Faster/better hash maps, taken from CF

14 lines

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
gp_hash_table<int, int> table;

struct custom_hash {
  size_t operator()(uint64_t x) const {
    x += 48;
    x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
    x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
    return x ^ (x >> 31);
  }
};
gp_hash_table<int, int, custom_hash> safe_table;
// hash-cpp-all = e62eb2668aee2263b6d72043f3652fb2
```

## 9.5 Other languages

### Main.java

**Description:** Basic template/info for Java

14 lines

```
import java.util.*;
import java.math.*;
import java.io.*;
public class Main {
  public static void main(String[] args) throws Exception {
    BufferedReader br = new BufferedReader(new
      ↪InputStreamReader(System.in));
    PrintStream out = System.out;
    StringTokenizer st = new StringTokenizer(br.readLine())
      ↪;
    assert st.hasMoreTokens(); // enable with java -ea main
    out.println("v=" + Integer.parseInt(st.nextToken()));
    ArrayList<Integer> a = new ArrayList<>();
    a.add(1234); a.get(0); a.remove(a.size()-1); a.clear();
  }
}
```