University of Wisconsin - Madison

Model Solution

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adapted from KTH ACM Contest Template Library

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Contest (1)

template.cpp

15 lines

```
#include <bits/stdc++.h>
using namespace std;

#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define trav(a, x) for(auto& a : x)
#define all(x) x.begin(), x.end()
#define sz(x) (int)(x).size()
typedef long long ll;
typedef pair<int, int> pii;
typedef vector<int> vi;

int main() {
  cin.sync_with_stdio(0); cin.tie(0);
  cin.exceptions(cin.failbit);
}
```

hash.sh

1 lines

```
tr -d '[:space:]' | md5sum
```

hash-cpp.sh

1 lines

```
cpp -dD -P -fpreprocessed | tr -d '[:space:]' | md5sum
```

Makefile

25 lines

```
CXX = g++
CXXFLAGS = -O2 -std=gnu++14 -Wall -Wextra -Wno-unused-
   ↪result -pedantic -Wshadow -Wformat=2 -Wfloat-equal -
   ↪Wconversion -Wlogical-op -Wshift-overflow=2 -
   ↪Wduplicated-cond -Wcast-qual -Wcast-align
# pause:#pragma GCC diagnostic {ignored|warning} "-Wshadow"
DEBUGFLAGS = -D_GLIBCXX_DEBUG -D_GLIBCXX_DEBUG_PEDANTIC -
   ↪fsanitize=address -fsanitize=undefined -fno-sanitize-
   ↪recover=all -fstack-protector -D_FORTIFY_SOURCE=2
CXXFLAGS += $(DEBUGFLAGS) # flags with speed penalty
TARGET := $(notdir $(CURDIR))
EXECUTE := ./$(TARGET)
CASES := $(sort $(basename $(wildcard *.in)))
TESTS := $(sort $(basename $(wildcard *.out)))
all: $(TARGET)
clean:
  -rm -rf $(TARGET) *.res
%: %.cpp
  $(LINK.cpp) $< $(LOADLIBES) $(LDLIBS) -o $@
run: $(TARGET)
  time $(EXECUTE)
%.res: $(TARGET) %.in
  time $(EXECUTE) < $*.in > $*.res
%.out: %
test_%: %.res %.out
  diff $*.res $*.out
runs: $(patsubst %,%.res,$(CASES))
test: $(patsubst %,test_%,$(TESTS))
.PHONY: all clean run test test_% runs
.PRECIOUS: %.res
```

vimrc

8 lines

```
set nocp ai bs=2 hls ic is lbr ls=2 mouse=a nu ru sc scs
   ↪smd so=3 sw=4 ts=4
filetype plugin indent on
syn on
map gA m'ggVG"+y''

com -range=% -nargs=1 P exe "<line1>,<line2>!".<q-args> |y|
   ↪sil u|echom @"
com -range=% Hash <line1>,<line2>P tr -d '[:space:]' |
   ↪md5sum
au FileType cpp com! -buffer -range=% Hash <line1>,<line2>P
   ↪ cpp -dD -P -fpreprocessed | tr -d '[:space:]' |
   ↪md5sum
```

nanorc

3 lines

```
set tabsize 4
set const
set autoindent
```

MinorThings.cpp

41 lines

```
#include <bits/stdc++.h>
using namespace std;

// Define Hash Function for hash map
struct HASH{
    size_t operator()(const P &x)const{
        return hash<ll>()((x.first)^((x.second)<<32));
    }
};
// multiply numbers up to 1e18 under some modulo
ll big_mul(ll a, ll b)
{
    ll q = (ll)((ld) a * (ld) b / (ld) mod);
    ll r = a * b - q * mod;

    return (r + mod) % mod;
}

int main(){
  //random number
  mt19937 rng(chrono::steady_clock::now().time_since_epoch
     ↪().count());
  cout << rng() % 5 << endl;
      std::shuffle(v.begin(), v.end(), rng);

    //calculating sum of floor(n/i) in O(sqrt(n))
    for (int i = 1, j = 0; i <= n; i = j + 1)  j = n/(n/i),
       ↪ ans += 1ll*(j-i+1)*(n/i);

    // Iterate every submask
    for(int mask = 0; mask < (1 << n); mask++) {
        for(int sub = mask; ; sub = (sub - 1) & mask) {
            //...
            if(sub == 0) break;
        }
    }

    //Better hash map
    unordered_map<int, int> mp;
    mp.reserve(32768);
    mp.max_load_factor(0.25);
}
```

clion.txt

3 lines

```
set(CMAKE_CXX_STANDARD 17)
set(GCC_COVERAGE_COMPILE_FLAGS "-g -O2 -std=gnu++17 -static
   ↪ -Wall -Werror")
set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} ${
   ↪GCC_COVERAGE_COMPILE_FLAGS}" )
```

troubleshoot.txt

52 lines

```
Pre-submit:
Write a few simple test cases, if sample is not enough.
Are time limits close? If so, generate max cases.
Is the memory usage fine?
Could anything overflow?
Make sure to submit the right file.

Wrong answer:
Print your solution! Print debug output, as well.
Are you clearing all datastructures between test cases?
Can your algorithm handle the whole range of input?
Read the full problem statement again.
Do you handle all corner cases correctly?
Have you understood the problem correctly?
Any uninitialized variables?
Any overflows?
Confusing N and M, i and j, etc.?
Are you sure your algorithm works?
What special cases have you not thought of?
Are you sure the STL functions you use work as you think?
Add some assertions, maybe resubmit.
Create some testcases to run your algorithm on.
Go through the algorithm for a simple case.
Go through this list again.
Explain your algorithm to a team mate.
Ask the team mate to look at your code.
Go for a small walk, e.g. to the toilet.
Is your output format correct? (including whitespace)
Rewrite your solution from the start or let a team mate do
   ↪it.

Runtime error:
Have you tested all corner cases locally?
Any uninitialized variables?
Are you reading or writing outside the range of any vector?
Any assertions that might fail?
Any possible division by 0? (mod 0 for example)
Any possible infinite recursion?
Invalidated pointers or iterators?
Are you using too much memory?
Debug with resubmits (e.g. remapped signals, see Various).

Time limit exceeded:
Do you have any possible infinite loops?
What is the complexity of your algorithm?
Are you copying a lot of unnecessary data? (References)
How big is the input and output? (consider scanf)
Avoid vector, map. (use arrays/unordered_map)
What do your team mates think about your algorithm?

Memory limit exceeded:
What is the max amount of memory your algorithm should need
   ↪?
Are you clearing all datastructures between test cases?
```

Data structures (2)

1DBIT.cpp

Description: 0-indexed BIT

12 lines

```
int n;
```

```cpp
int bit[N];

void add(int x, int val) {
    for(int i = x; i < n; i |= i + 1) bit[i] += val;
}

ll get(int x) {
    ll res = 0;
    for(int i = x; i >= 0; i = (i & (i + 1)) - 1)  res +=
       ↪bit[i];
    return res;
} // hash-cpp-all = e3a683942a154d5b6f649fa4539e93e8
```

2DBIT.cpp

29 lines

```cpp
vector<int> vals[N], f[N];

void addupd(int x, int y) {
    for (int i = x; i < N; i |= i + 1) vals[i].push_back(y)
       ↪;
}

void addget(int x, int y) {
    if (x < 0 || y < 0) return;
    for (int i = x; i >= 0; i = (i & (i + 1)) - 1) vals[i].
       ↪push_back(y);
}

void upd(int x, int y, int v) {
    for (int i = x; i < N; i |= i + 1) {
        for (int j = lower_bound(vals[i].begin(), vals[i].
           ↪end(), y) - vals[i].begin();
             j < (int) f[i].size(); j |= j + 1) {
            f[i][j] += v;
        }
    }
}

int get(int x, int y) {
    if (x < 0 || y < 0) return 0;
    int res = 0;
    for (int i = x; i >= 0; i = (i & (i + 1)) - 1)
        for (int j = lower_bound(vals[i].begin(), vals[i].
           ↪end(), y) - vals[i].begin(); j >= 0;
             j = (j & (j + 1)) - 1)
            res += f[i][j];
    return res;
} // hash-cpp-all = a5676421701a8edc43b837632d70b2d2
```

DynamicConvexHullTrick.cpp

<bits/stdc++.h>

32 lines

```cpp
typedef long long ll;

struct Line {
  mutable ll k, m, p;
  bool operator<(const Line& o) const { return k < o.k; }
  bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
  // (for doubles, use inf = 1/.0, div(a,b) = a/b)
  const ll inf = LLONG_MAX;
  ll div(ll a, ll b) { // floored division
    return a / b - ((a ^ b) < 0 && a % b); }
  bool isect(iterator x, iterator y) {
    if (y == end()) { x->p = inf; return false; }
    if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
    else x->p = div(y->m - x->m, x->k - y->k);
    return x->p >= y->p;
  }
  void add(ll k, ll m) {
    auto z = insert({k, m, 0}), y = z++, x = y;
    while (isect(y, z)) z = erase(z);
    if (x != begin() && isect(--x, y)) isect(x, y = erase(y
       ↪));
    while ((y = x) != begin() && (--x)->p >= y->p)
      isect(x, erase(y));
  }
  ll query(ll x) {
    assert(!empty());
    auto l = *lower_bound(x);
    return l.k * x + l.m;
  }
}; // hash-cpp-all = 78fbd6d5c3d5351b8f2712bd6c535ce9
```

StaticConvexHullTrick.cpp

Description: Maintaining upper convex hull, querying the maximum. Need to put in lines in strictly increasing order of slope.

<bits/stdc++.h>

49 lines

```cpp
struct Line {
    ll k, m;
    Line(ll _k, ll _m) {
        k = _k, m = _m;
    }
    Pll inter(Line o) {
        return {m - o.m, o.k - k};
    }
};

struct Hull {
    deque<Line> que;

    bool leq(Pll a, Pll b) {
        return a.first * b.second <= a.second * b.first;
    }
    // k needs to be strictly increasing!
    void add(ll k, ll m) {
        while(que.size() > 1) {
            int ls = que.size() - 1;
            if(leq(que[ls].inter(Line(k, m)), que[ls-1].
               ↪inter(que[ls]))) que.pop_back();
            else break;
        }
        que.push_back({k, m});
    }
    // Arbitrary x.
    ll query_bin(ll x) {
      if(que.empty()) return -INF;
        int l = 0, r = que.size() - 1;
        while(l < r) {
            int mi = (l + r) / 2;
            if(que[mi].k * x + que[mi].m < que[mi+1].k * x
               ↪+ que[mi+1].m) l = mi + 1;
            else r = mi;
        }
        return que[l].k * x + que[l].m;
    }
    // If querying increasing x.
    ll query(ll x) {
      if(que.empty()) return -INF;
        while(que.size() > 1) {
            if(que[0].k * x + que[0].m < que[1].k * x + que
               ↪[1].m) que.pop_front();
            else break;
        }
        return que[0].k * x + que[0].m;
    }

} hull;
// hash-cpp-all = 817ff0175d9f4dd826f400423d205fc4
```

MonotonousDeque.cpp

Description: Monotonous Interval Min Queries

17 lines

```cpp
struct MonQueue {
    deque<P> que;

    void clear() {
        que.clear();
    }
    void add(P p) {
        while(!que.empty() && que.back().first >= p.first)
           ↪que.pop_back();
        que.push_back(p);
    }

    int get(int l, int r) {
        while(!que.empty() && (que.front().second < l ||
           ↪que.front().second > r)) que.pop_front();
        return que.empty() ? -INF : que.front().first;
    }
} cque;
// hash-cpp-all = dcfed26f0df829fcc97aa066b87bedff
```

PersistentSegmentTreePointUpdate.cpp

45 lines

```cpp
int ncnt = 1; // Need to initialize before every test case!

struct node{
    int ls, rs, sum;
} ns[N * 30];

int newnode(int val){
    ns[ncnt].ls = ns[ncnt].rs = 0;
    ns[ncnt].sum = val;
    return ncnt++;
}

int newnode(int ls, int rs){
    ns[ncnt].ls = ls;
    ns[ncnt].rs = rs;
    ns[ncnt].sum = (ls ? ns[ls].sum : 0) + (rs ? ns[rs].sum
       ↪ : 0);
    return ncnt++;
}

int n, q;
int num[N];
int x[N], zeros[N];
int vs[N];

int build(int a[], int tl = 0, int tr = n-1){
    if(tl == tr) return newnode(a[tl]);
    int mid = (tl + tr) / 2;
    return newnode(build(a, tl, mid), build(a, mid + 1, tr)
       ↪);
}

int get_sum(int v, int l, int r, int tl = 0, int tr = n-1){
    if(tr < l || tl > r) return 0;
    if(l <= tl && tr <= r) return ns[v].sum;
```

```
    int tm = (tl + tr) / 2;

    return get_sum(ns[v].ls, l, r, tl, tm)
           + get_sum(ns[v].rs, l, r, tm + 1, tr);
}

int update(int v, int pos, int tl = 0, int tr = n-1){
    if(tl == tr) return newnode(ns[v].sum + 1);
    int tm = (tl + tr) / 2;
    if(pos <= tm) return newnode(update(ns[v].ls, pos, tl,
       ↪tm), ns[v].rs);
    else return newnode(ns[v].ls, update(ns[v].rs, pos, tm
       ↪+1, tr));
} // hash-cpp-all = 942ce4b7625a9496966519b6af9abf8b
```

PersistentSegmentTreeRangeUpdate.cpp

69 lines

```
int ncnt = 1 // Need to initialize before every test case!

struct node{
    int ls, rs, lazy;
    ll sum;
} ns[N * 100];

int newnode(int val){
    ns[ncnt].ls = ns[ncnt].rs = 0;
    ns[ncnt].sum = val;
    ns[ncnt].lazy = 0;
    return ncnt++;
}

int newnode(int ls, int rs){
    ns[ncnt].ls = ls;
    ns[ncnt].rs = rs;
    ns[ncnt].sum = (ls ? ns[ls].sum : 0) + (rs ? ns[rs].sum
       ↪ : 0);
    ns[ncnt].lazy = 0;
    return ncnt++;
}

int n, q;
int num[N];
int vs[N];
int tim = 0;

int newlazynode(int v, int val, int l, int r){
    ns[ncnt].ls = ns[v].ls;
    ns[ncnt].rs = ns[v].rs;
    ns[ncnt].lazy = ns[v].lazy + val;
    ns[ncnt].sum = ns[v].sum + (r - l + 1) * val;
    return ncnt++;
}

void push_down(int v, int tl, int tr){
    if(ns[v].lazy){
        if(tl != tr){
            int mid = (tl + tr) / 2;
            ns[v].ls = newlazynode(ns[v].ls, ns[v].lazy, tl
               ↪, mid);
            ns[v].rs = newlazynode(ns[v].rs, ns[v].lazy,
               ↪mid + 1, tr);
        }
        ns[v].lazy = 0;
    }
}

int build(int a[], int tl = 0, int tr = n-1){
    if(tl == tr) return newnode(a[tl]);
    int mid = (tl + tr) / 2;
    return newnode(build(a, tl, mid), build(a, mid + 1, tr)
       ↪);
}

ll get_sum(int v, int l, int r, int tl = 0, int tr = n-1){
    if(tr < l || tl > r) return 0;
    if(l <= tl && tr <= r) return ns[v].sum;
    push_down(v, tl, tr);
    int tm = (tl + tr) / 2;

    return get_sum(ns[v].ls, l, r, tl, tm)
           + get_sum(ns[v].rs, l, r, tm + 1, tr);
}

int update(int v, int l, int r, int val, int tl = 0, int tr
   ↪ = n-1){
    if(tr < l || tl > r) return v;
    if(l <= tl && tr <= r) return newlazynode(v, val, tl,
       ↪tr);
    push_down(v, tl, tr);
    int tm = (tl + tr) / 2;
    return newnode(update(ns[v].ls, l, r, val, tl, tm),
       ↪update(ns[v].rs, l, r, val, tm+1, tr));
} // hash-cpp-all = cfcdd348217ece5628f563d25ece156d
```

SegmentTreePointUpdate.cpp

<bits/stdc++.h> 60 lines

```
#define lson(x) 2*x+1
#define rson(x) 2*x+2

typedef long long ll;
typedef pair<int, int> P;
const int N = (int)2e5 + 500, mod = (int)1e9 + 7;

int n;
P p[N];
int rs[N];

struct node {
    int mn;
    int cnt;

    void merge(node &LHS, node &RHS) {
        mn = min(LHS.mn, RHS.mn);
        cnt = (LHS.mn == mn ? LHS.cnt : 0) + (RHS.mn == mn
           ↪? RHS.cnt : 0);
        cnt %= mod;
    }
};

struct Tree {
    node dat[N * 4];

    void init_dat(int l, int r, int x){
        if(l == r){dat[x].mn = p[l].first; dat[x].cnt = 1;
           ↪return ;}

        int mid = (l + r) / 2;
        init_dat(l, mid, lson(x));
        init_dat(mid+1, r, rson(x));
        dat[x].merge(dat[lson(x)], dat[rson(x)]);
    }

    void update(int pos, int x, int l, int r, int val, int
       ↪cnt){
        int mid = (l + r) / 2;
        if(l == r) {
            dat[x].mn = val;
            dat[x].cnt = cnt;
            return ;
        }
        if(pos <= mid) update(pos, lson(x), l, mid, val,
           ↪cnt);
        else update(pos, rson(x), mid+1, r, val, cnt);
        dat[x].merge(dat[lson(x)], dat[rson(x)]);
    }

    node query(int a, int b, int x, int l, int r){
        if(r < a || b < l) return {mod + 5, 0};

        int mid = (l + r) / 2;
        if(a <= l && r <= b) return dat[x];

        node res;
        node LHS = query(a, b, lson(x), l, mid);
        node RHS = query(a, b, rson(x), mid+1, r);
        res.merge(LHS, RHS);
        return res;
    }
} tree;
// hash-cpp-all = 307ccdb73b985795d794b5de82a02c27
```

SegmentTreeRangeUpdate.cpp

<bits/stdc++.h> 103 lines

```
#define ls(x) x * 2 + 1
#define rs(x) x * 2 + 2

typedef long long ll;
const int N = (int)1e6 + 50;
int INF = (int)1e9 + 50;

int n,m,q;
int a[N], b[N];
int num[N];

struct node {
    int mn, add;

    void add_val(int x) {
        mn += x;
        add += x;
    }

    void merge(node &ls, node &rs) {
        mn = min(ls.mn, rs.mn);
    }
};

struct Tree {
    node dat[4 * N];

    void push_down(int x, int l, int r) {
        if(dat[x].add) {
            if(l < r) {
                dat[ls(x)].add_val(dat[x].add);
                dat[rs(x)].add_val(dat[x].add);
            }
            dat[x].add = 0;
        }
    }

    void init(int x = 0, int l = 0, int r = n-1) {
```

```
        if(l == r) {
            dat[x].mn = num[l];
            dat[x].add = 0;
            return ;
        }
        int mid = (l + r) / 2;
        init(ls(x), l, mid);
        init(rs(x), mid + 1, r);
        dat[x].add = 0;
        dat[x].merge(dat[ls(x)], dat[rs(x)]);
    }

    node query(int a, int b, int x = 0, int l = 0, int r =
       ↪N-1) {
        int mid = (l + r) / 2;
        if(r < a || l > b) return {INF, 0};

        push_down(x, l, r);

        if(l >= a && r <= b) return dat[x];

        node LHS = query(a, b, ls(x), l, mid);
        node RHS = query(a, b, rs(x), mid+1, r);
        node res;
        res.merge(LHS, RHS);
        return res;
    }

    void update(int a, int b, int x, int l, int r, int
       ↪delta) {
        int mid = (l + r) / 2;
        if(r < a || l > b) return ;

        push_down(x, l, r);

        if(l >= a && r <= b) {
            dat[x].add_val(delta);
            return ;
        }

        update(a, b, ls(x), l, mid, delta);
        update(a, b, rs(x), mid+1, r, delta);

        dat[x].merge(dat[ls(x)], dat[rs(x)]);
    }

    void update(int a, int b, int delta) {
        update(a, b, 0, 0, N - 1, delta);
    }

    int find(int x, int l, int r) {
        if(l == r) return l;
        int mid = (l + r) / 2;
        push_down(x, l, r);

        if(dat[rs(x)].mn < 0) return find(rs(x), mid + 1, r
           ↪);
        else return find(ls(x), l, mid);
    }

    int find() {
        if(dat[0].mn >= 0) return -1;
        else return find(0, 0, N-1);
    }

} tree;
// hash-cpp-all = 8e290db8ea0801e20d078a3886a3acdb
```

SparseTable.cpp

<bits/stdc++.h> — 26 lines

```
typedef long long ll;
const int N = (int)3e5 + 50, LOGN = 19;

struct RMQ {
    int n;
    mm[N];

    void build() {
        mm[0]=-1;
        for(int i = 1; i <= n; i++) mm[i]= (i & (i-1)) == 0
           ↪ ? mm[i-1] + 1 : mm[i-1];
        for(int i = 0; i < n; i++){
            st[0][i] = x[i];
        }
        for(int lg = 1; lg < LOGN; lg++){
            for(int j = 0; j + (1 << lg) - 1 < n; j++){
                st[lg][j] = min(st[lg-1][j], st[lg-1][j
                   ↪+(1<<(lg-1))]);
            }
        }
    }

    int rmq(int l, int r){
        int k = mm[r - l + 1];
        return min(st[k][l], st[k][r-(1<<k)+1]);
    }
} rmq;
// hash-cpp-all = 58a1ba63eb165b67e821731f83538d02
```

HashMap.h

Description: Hash map with the same API as unordered_map, but ~3x faster. Initial capacity must be a power of 2 (if provided).

2 lines

```
#include <bits/extc++.h>
__gnu_pbds::gp_hash_table<ll, int> h({},{},{},{}, {1 <<
   ↪16}); // hash-cpp-all =
   ↪d4f7c9a985615ad5fd0981e66d468825
```

OrderStatisticTree.h

Description: A set (not multiset!) with support for finding the n'th element, and finding the index of an element.
Time: $\mathcal{O}(\log N)$

16 lines

```
#include <bits/extc++.h>
using namespace __gnu_pbds;

template<class T>
using Tree = tree<T, null_type, less<T>, rb_tree_tag,
    tree_order_statistics_node_update>;

void example() {
  Tree<int> t, t2; t.insert(8);
  auto it = t.insert(10).first;
  assert(it == t.lower_bound(9));
  assert(t.order_of_key(10) == 1);
  assert(t.order_of_key(11) == 2);
  assert(*t.find_by_order(0) == 8);
  t.join(t2); // assuming T < T2 or T > T2, merge t2 into t
} // hash-cpp-all = 782797f91ca134bf996558987dbf1924
```

LiChao.cpp

36 lines

```
struct node {
    point val;
    node *l, *r;
    node(point _val = {0,inf}) {
        val = _val;
        l = r = nullptr;
    }
};

ll f(point val, ll x) {
    return val.X*x + val.Y;
}

void update(node *&root, int l, int r, point val) {
    if(val.Y==inf) return ;
    if(!root) {
        root = new node(val);
        return ;
    }
    int mid = (l+r)/2;
    int a = (f(val, l) < f(root->val, l));
    int b = (f(val, mid) < f(root->val, mid));
    int c = (f(val, r) < f(root->val, r));
    if(b) swap(root->val, val);
    if(l==r) return ;
    if(a != b) update(root->l, l, mid, val);
    else if(b != c) update(root->r, mid+1, r, val);
}

ll query(node *root, int l, int r, ll x) {
    if(!root) return inf;
    if(l==r) return f(root->val, x);
    int mid = (l+r)/2;
    if(x<=mid) return min(f(root->val, x), query(root->l,l,
       ↪mid,x));
    return min(f(root->val, x), query(root->r,mid+1,r,x));
} // hash-cpp-all = 0c1ab53613ea5a13a4579e61616dced1
```

Treap.cpp

56 lines

```
struct node {
    char key;
    ll prior;
    int sz, rev;
    node *l, *r;
    node(char _key = 0) {
        key = _key;
        prior = (rand()<<16) + rand();
        sz = 1;
        rev = 0;
    }
};
int sz(node *x) {
    return x ? x->sz : 0;
}
void pushdown(node *root) {
    if(root && root->rev) {
        swap(root->l, root->r);
        if(root->l) root->l->rev ^= 1;
        if(root->r) root->r->rev ^= 1;
        root->rev = 0;
    }
}
void update(node *root) {
    if(root) {
        pushdown(root->l); pushdown(root->r);
        root->sz = sz(root->l) + sz(root->r) + 1;
    }
}
void split(node *root, int pos, node *&l, node *&r) {
```

```
    pushdown(root);
    if(!root) l = r = nullptr;
    else if(sz(root->l)+1<=pos) l = root, split(root->r,
      ↪pos-sz(root->l)-1, l->r, r);
    else r = root, split(root->l, pos, l, r->l);
    update(root);
}
void merge(node *&root, node *l, node *r) {
    pushdown(l); pushdown(r);
    if(!l || !r) root = l ? l : r;
    else if(l->prior > r->prior) root = l, merge(root->r, l
      ↪->r, r);
    else root = r, merge(root->l, l, r->l);
    update(root);
}
int main() {
    node *root = nullptr;
    for(int i=1;i<=n;i++) merge(root, root, new node(s[i]))
      ↪;
    node *s1, *s2;
    split(root, k, s1, s2); // split root to s1 = [1..k]
      ↪and s2 = [k+1..n]
    merge(root, s1, s2); // merge s1 and s2 to get new root
    // reverse [i, j]
    split(root, j, s2, s3);
    split(s2, i-1, s1, s2);
    s2->rev ^= 1; pushdown(s2);
    merge(root, s1, s2);
    merge(root, root, s3);
} // hash-cpp-all = 06c048e5cdc7f8e301bdba948cc198e8
```

Numerical (3)

GoldenSectionSearch.h

Description: Finds the argument minimizing the function f in the interval $[a, b]$ assuming f is unimodal on the interval, i.e. has only one local minimum. The maximum error in the result is eps. Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.
Usage: `double func(double x) { return 4+x+.3*x*x; }`
`double xmin = gss(-1000,1000,func);`
Time: $\mathcal{O}\left(\log((b-a)/\epsilon)\right)$

14 lines

```
double gss(double a, double b, double (*f)(double)) {
  double r = (sqrt(5)-1)/2, eps = 1e-7;
  double x1 = b - r*(b-a), x2 = a + r*(b-a);
  double f1 = f(x1), f2 = f(x2);
  while (b-a > eps)
    if (f1 < f2) { //change to > to find maximum
      b = x2; x2 = x1; f2 = f1;
      x1 = b - r*(b-a); f1 = f(x1);
    } else {
      a = x1; x1 = x2; f1 = f2;
      x2 = a + r*(b-a); f2 = f(x2);
    }
  return a;
} // hash-cpp-all = 31d45b514727a298955001a74bb9b9fa
```

Polynomial.h

17 lines

```
struct Poly {
  vector<double> a;
  double operator()(double x) const {
    double val = 0;
    for(int i = sz(a); i--;) (val *= x) += a[i];
    return val;
  }
  void diff() {
    rep(i,1,sz(a)) a[i-1] = i*a[i];
    a.pop_back();
  }
  void divroot(double x0) {
    double b = a.back(), c; a.back() = 0;
    for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b,
      ↪b=c;
    a.pop_back();
  }
}; // hash-cpp-all = c9b7b07a5aae7b0a6df1b8cdb046375f
```

PolyRoots.h

Description: Finds the real roots to a polynomial.
Usage: `poly_roots({{2,-3,1}},-1e9,1e9) // solve x^2-3x+2 = 0`
Time: $\mathcal{O}\left(n^2 \log(1/\epsilon)\right)$

"Polynomial.h" 23 lines

```
vector<double> poly_roots(Poly p, double xmin, double xmax)
  ↪ {
  if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
  vector<double> ret;
  Poly der = p;
  der.diff();
  auto dr = poly_roots(der, xmin, xmax);
  dr.push_back(xmin-1);
  dr.push_back(xmax+1);
  sort(all(dr));
  rep(i,0,sz(dr)-1) {
    double l = dr[i], h = dr[i+1];
    bool sign = p(l) > 0;
    if (sign ^ (p(h) > 0)) {
      rep(it,0,60) { // while (h - l > 1e-8)
        double m = (l + h) / 2, f = p(m);
        if ((f <= 0) ^ sign) l = m;
        else h = m;
      }
      ret.push_back((l + h) / 2);
    }
  }
  return ret;
} // hash-cpp-all = 2cf1903cf3e930ecc5ea0059a9b7fce5
```

PolyInterpolate.h

Description: Given n points (x[i], y[i]), computes an n-1-degree polynomial p that passes through them: $p(x) = a[0]*x^0 + ... + a[n-1]*x^{n-1}$. For numerical precision, pick $x[k] = c*\cos(k/(n-1)*\pi), k = 0 \ldots n-1$.
Time: $\mathcal{O}\left(n^2\right)$

13 lines

```
typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
  vd res(n), temp(n);
  rep(k,0,n-1) rep(i,k+1,n)
    y[i] = (y[i] - y[k]) / (x[i] - x[k]);
  double last = 0; temp[0] = 1;
  rep(k,0,n) rep(i,0,n) {
    res[i] += y[k] * temp[i];
    swap(last, temp[i]);
    temp[i] -= last * x[k];
  }
  return res;
} // hash-cpp-all = 08bf48c9301c849dfc6064b6450af6f3
```

BerlekampMassey.h

Description: Recovers any n-order linear recurrence relation from the first $2n$ terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size $\leq n$.
Usage: `BerlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}`

"../number-theory/ModPow.h" 20 lines

```
vector<ll> BerlekampMassey(vector<ll> s) {
  int n = sz(s), L = 0, m = 0;
  vector<ll> C(n), B(n), T;
  C[0] = B[0] = 1;

  ll b = 1;
  rep(i,0,n) { ++m;
    ll d = s[i] % mod;
    rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
    if (!d) continue;
    T = C; ll coef = d * modpow(b, mod-2) % mod;
    rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
    if (2 * L > i) continue;
    L = i + 1 - L; B = T; b = d; m = 0;
  }

  C.resize(L + 1); C.erase(C.begin());
  trav(x, C) x = (mod - x) % mod;
  return C;
} // hash-cpp-all = 40387d9fed31766a705d6b2206790deb
```

LinearRecurrence.h

Description: Generates the k'th term of an n-order linear recurrence $S[i] = \sum_j S[i-j-1]tr[j]$, given $S[0 \ldots n-1]$ and $tr[0 \ldots n-1]$. Faster than matrix multiplication. Useful together with Berlekamp–Massey.
Usage: `linearRec({0, 1}, {1, 1}, k) // k'th Fibonacci number`
Time: $\mathcal{O}\left(n^2 \log k\right)$

26 lines

```
typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) { // hash-cpp-1
  int n = sz(S);

  auto combine = [&](Poly a, Poly b) {
    Poly res(n * 2 + 1);
    rep(i,0,n+1) rep(j,0,n+1)
      res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
    for (int i = 2 * n; i > n; --i) rep(j,0,n)
      res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) %
        ↪mod;
    res.resize(n + 1);
    return res;
  };

  Poly pol(n + 1), e(pol);
  pol[0] = e[1] = 1;

  for (++k; k; k /= 2) {
    if (k % 2) pol = combine(pol, e);
    e = combine(e, e);
  }

  ll res = 0;
  rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
  return res;
} // hash-cpp-1 = 261dd85251df2df60ee444e087e8ffc2
```

Integrate.h

Description: Simple integration of a function over an interval using Simpson's rule. The error should be proportional to h^4, although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

8 lines

```
double quad(double (*f)(double), double a, double b) {
  const int n = 1000;
  double h = (b - a) / 2 / n;
  double v = f(a) + f(b);
  rep(i,1,n*2)
    v += f(a + i*h) * (i&1 ? 4 : 2);
  return v * h / 3;
} // hash-cpp-all = 65e2375b3152c23048b469eb414fe6b6
```

IntegrateAdaptive.h

Description: Fast integration using an adaptive Simpson's rule.
Usage: double z, y;
double h(double x) { return x*x + y*y + z*z <= 1; }
double g(double y) { ::y = y; return quad(h, -1, 1); }
double f(double z) { ::z = z; return quad(g, -1, 1); }
double sphereVol = quad(f, -1, 1), pi = sphereVol*3/4;

16 lines

```
typedef double d;
d simpson(d (*f)(d), d a, d b) {
  d c = (a+b) / 2;
  return (f(a) + 4*f(c) + f(b)) * (b-a) / 6;
}
d rec(d (*f)(d), d a, d b, d eps, d S) {
  d c = (a+b) / 2;
  d S1 = simpson(f, a, c);
  d S2 = simpson(f, c, b), T = S1 + S2;
  if (abs (T - S) <= 15*eps || b-a < 1e-10)
    return T + (T - S) / 15;
  return rec(f, a, c, eps/2, S1) + rec(f, c, b, eps/2, S2);
}
d quad(d (*f)(d), d a, d b, d eps = 1e-8) {
  return rec(f, a, b, eps, simpson(f, a, b));
} // hash-cpp-all = ad8a754372ce74e5a3d07ce46c2fe0ca
```

Determinant.h

Description: Calculates determinant of a matrix. Destroys the matrix.
Time: $\mathcal{O}(N^3)$

15 lines

```
double det(vector<vector<double>>& a) {
  int n = sz(a); double res = 1;
  rep(i,0,n) {
    int b = i;
    rep(j,i+1,n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;
    if (i != b) swap(a[i], a[b]), res *= -1;
    res *= a[i][i];
    if (res == 0) return 0;
    rep(j,i+1,n) {
      double v = a[j][i] / a[i][i];
      if (v != 0) rep(k,i+1,n) a[j][k] -= v * a[i][k];
    }
  }
  return res;
} // hash-cpp-all = bd5cec161e6ad4c483e662c34eae2d08
```

IntDeterminant.h

Description: Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.
Time: $\mathcal{O}(N^3)$

18 lines

```
const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
  int n = sz(a); ll ans = 1;
  rep(i,0,n) {
    rep(j,i+1,n) {
      while (a[j][i] != 0) { // gcd step
        ll t = a[i][i] / a[j][i];
        if (t) rep(k,i,n)
          a[i][k] = (a[i][k] - a[j][k] * t) % mod;
        swap(a[i], a[j]);
        ans *= -1;
      }
    }
    ans = ans * a[i][i] % mod;
    if (!ans) return 0;
  }
  return (ans + mod) % mod;
} // hash-cpp-all = 3313dc3b38059fdf9f41220b469cfd13
```

Simplex.h

Description: Solves a general linear maximization problem: maximize $c^T x$ subject to $Ax \leq b$, $x \geq 0$. Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of $c^T x$ otherwise. The input vector is set to an optimal x (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that $x = 0$ is viable.
Usage: vvd A = {{1,-1}, {-1,1}, {-1,-2}};
vd b = {1,1,-4}, c = {-1,-1}, x;
T val = LPSolver(A, b, c).solve(x);
Time: $\mathcal{O}(NM * \#pivots)$, where a pivot may be e.g. an edge relaxation. $\mathcal{O}(2^n)$ in the general case.

68 lines

```
typedef double T; // long double, Rational, double + mod<P
  ↪>...
typedef vector<T> vd;
typedef vector<vd> vvd;

const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s],N[s]))
  ↪ s=j

struct LPSolver {
  int m, n;
  vi N, B;
  vvd D;

  LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) { //
      ↪hash-cpp-1
      rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
      rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[
        ↪i];}
      rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
      N[n] = -1; D[m+1][n] = 1;
    } // hash-cpp-1 = 6ff8e92a6bb47fbd6606c75a07178914

  void pivot(int r, int s) { // hash-cpp-2
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
      T *b = D[i].data(), inv2 = b[s] * inv;
      rep(j,0,n+2) b[j] -= a[j] * inv2;
      b[s] = a[s] * inv2;
    }
    rep(j,0,n+2) if (j != s) D[r][j] *= inv;
    rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
  } // hash-cpp-2 = 9cd0a84b89fb678b2888e0defa688de2

  bool simplex(int phase) { // hash-cpp-3
    int x = m + phase - 1;
    for (;;) {
      int s = -1;
      rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
      if (D[x][s] >= -eps) return true;
      int r = -1;
      rep(i,0,m) {
        if (D[i][s] <= eps) continue;
        if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
                     < MP(D[r][n+1] / D[r][s], B[r])) r = i
                       ↪;
      }
      if (r == -1) return false;
      pivot(r, s);
    }
  } // hash-cpp-3 = f156440bce4f5370ea43b0efa7de25ed

  T solve(vd &x) { // hash-cpp-4
    int r = 0;
    rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
      pivot(r, n);
      if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
      rep(i,0,m) if (B[i] == -1) {
        int s = 0;
        rep(j,1,n+1) ltj(D[i]);
        pivot(i, s);
      }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
  } // hash-cpp-4 = 396a95621f5e196bb87eb95518560dfb
};
```

math-simplex.cpp

Description: Simplex algorithm. WARNING- segfaults on empty (size 0) max cx st Ax<=b, x>=0 do 2 phases; 1st check feasibility; 2nd check boundedness and ans

40 lines

```
vector<double> simplex(vector<vector<double> > A, vector<
  ↪double> b, vector<double> c) {
  int n = (int) A.size(), m = (int) A[0].size()+1, r = n, s
    ↪ = m-1;
  vector<vector<double> > D = vector<vector<double> > (n+2,
    ↪ vector<double>(m+1));
  vector<int> ix = vector<int> (n+m);
  for (int i=0; i<n+m; i++) ix[i] = i;
  for (int i=0; i<n; i++) {
    for (int j=0; j<m-1; j++)D[i][j]=-A[i][j];
    D[i][m-1] = 1;
    D[i][m] = b[i];
    if (D[r][m] > D[i][m]) r = i;
  }
  for (int j=0; j<m-1; j++) D[n][j]=c[j];
  D[n+1][m-1] = -1; int z = 0;
  for (double d;;) {
    if (r < n) {
      swap(ix[s], ix[r+m]);
      D[r][s] = 1.0/D[r][s];
      for (int j=0; j<=m; j++) if (j!=s) D[r][j] *= -D[r][s
        ↪];
      for(int i=0; i<=n+1; i++)if(i!=r) {
        for (int j=0; j<=m; j++) if(j!=s) D[i][j] += D[r][j
          ↪] * D[i][s];
```

```
        D[i][s] *= D[r][s];
      }
    }
    r = -1; s = -1;
    for (int j=0; j <m; j++) if (s<0 || ix[s]>ix[j]) {
      if (D[n+1][j]>eps || D[n+1][j]>-eps && D[n][j]>eps) s
        ↪ = j;
    }
    if (s < 0) break;
    for (int i=0; i<n; i++) if(D[i][s]<-eps) {
      if (r < 0 || (d = D[r][m]/D[r][s]-D[i][m]/D[i][s]) <
        ↪-eps
        || d < eps && ix[r+m] > ix[i+m]) r=i;
    }
    if (r < 0) return vector<double>(); // unbounded
  }
  if (D[n+1][m] < -eps) return vector<double>(); //
    ↪infeasible
  vector<double> x(m-1);
  for (int i = m; i < n+m; i ++) if (ix[i] < m-1) x[ix[i]]
    ↪= D[i-m][m];
  printf("%.2lf\n", D[n][m]);
  return x; // ans: D[n][m]
} // hash-cpp-all = 70201709abdff05eff90d9393c756b95
```

SolveLinear.h

Description: Solves $A * x = b$. If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in A and b is lost.
Time: $\mathcal{O}\left(n^2m\right)$

38 lines

```
typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vector<vd>& A, vd& b, vd& x) {
  int n = sz(A), m = sz(x), rank = 0, br, bc;
  if (n) assert(sz(A[0]) == m);
  vi col(m); iota(all(col), 0);

  rep(i,0,n) {
    double v, bv = 0;
    rep(r,i,n) rep(c,i,m)
      if ((v = fabs(A[r][c])) > bv)
        br = r, bc = c, bv = v;
    if (bv <= eps) {
      rep(j,i,n) if (fabs(b[j]) > eps) return -1;
      break;
    }
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) swap(A[j][i], A[j][bc]);
    bv = 1/A[i][i];
    rep(j,i+1,n) {
      double fac = A[j][i] * bv;
      b[j] -= fac * b[i];
      rep(k,i+1,m) A[j][k] -= fac*A[i][k];
    }
    rank++;
  }

  x.assign(m, 0);
  for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j,0,i) b[j] -= A[j][i] * b[i];
  }
  return rank; // (multiple solutions if rank < m)
} // hash-cpp-all = 44c9ab90319b30df6719c5b5394bc618
```

SolveLinear2.h

Description: To get all uniquely determined values of x back from SolveLinear, make the following changes:

"SolveLinear.h" 8 lines

```
rep(j,0,n) if (j != i) // instead of rep(j,i+1,n)
// ... then at the end:
x.assign(m, undefined);
rep(i,0,rank) {
  rep(j,rank,m) if (fabs(A[i][j]) > eps) goto fail;
  x[col[i]] = b[i] / A[i][i];
fail:; }
// hash-cpp-all = 08e495d9d51e80a183ccd030e3bf6700
```

SolveLinearBinary.h

Description: Solves $Ax = b$ over $\mathbb{F}_2$. If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys A and b.
Time: $\mathcal{O}\left(n^2m\right)$

34 lines

```
typedef bitset<1000> bs;

int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
  int n = sz(A), rank = 0, br;
  assert(m <= sz(x));
  vi col(m); iota(all(col), 0);
  rep(i,0,n) {
    for (br=i; br<n; ++br) if (A[br].any()) break;
    if (br == n) {
      rep(j,i,n) if(b[j]) return -1;
      break;
    }
    int bc = (int)A[br]._Find_next(i-1);
    swap(A[i], A[br]);
    swap(b[i], b[br]);
    swap(col[i], col[bc]);
    rep(j,0,n) if (A[j][i] != A[j][bc]) {
      A[j].flip(i); A[j].flip(bc);
    }
    rep(j,i+1,n) if (A[j][i]) {
      b[j] ^= b[i];
      A[j] ^= A[i];
    }
    rank++;
  }

  x = bs();
  for (int i = rank; i--;) {
    if (!b[i]) continue;
    x[col[i]] = 1;
    rep(j,0,i) b[j] ^= A[j][i];
  }
  return rank; // (multiple solutions if rank < m)
} // hash-cpp-all = fa2d7a3e3a84d8fb47610cc474e77b4e
```

MatrixInverse.h

Description: Invert matrix A. Returns rank; result is stored in A unless singular (rank < n). Can easily be extended to prime moduli; for prime powers, repeatedly set $A^{-1} = A^{-1}(2I - AA^{-1})$ (mod p^k) where A^{-1} starts as the inverse of A mod p, and k is doubled in each step.
Time: $\mathcal{O}\left(n^3\right)$

35 lines

```
int matInv(vector<vector<double>>& A) {
  int n = sz(A); vi col(n);
  vector<vector<double>> tmp(n, vector<double>(n));
  rep(i,0,n) tmp[i][i] = 1, col[i] = i;

  rep(i,0,n) {
    int r = i, c = i;
    rep(j,i,n) rep(k,i,n)
      if (fabs(A[j][k]) > fabs(A[r][c]))
        r = j, c = k;
    if (fabs(A[r][c]) < 1e-12) return i;
    A[i].swap(A[r]); tmp[i].swap(tmp[r]);
    rep(j,0,n)
      swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
    swap(col[i], col[c]);
    double v = A[i][i];
    rep(j,i+1,n) {
      double f = A[j][i] / v;
      A[j][i] = 0;
      rep(k,i+1,n) A[j][k] -= f*A[i][k];
      rep(k,0,n) tmp[j][k] -= f*tmp[i][k];
    }
    rep(j,i+1,n) A[i][j] /= v;
    rep(j,0,n) tmp[i][j] /= v;
    A[i][i] = 1;
  }

  for (int i = n-1; i > 0; --i) rep(j,0,i) {
    double v = A[j][i];
    rep(k,0,n) tmp[j][k] -= v*tmp[i][k];
  }

  rep(i,0,n) rep(j,0,n) A[col[i]][col[j]] = tmp[i][j];
  return n;
} // hash-cpp-all = ebfff64122d6372fde3a086c95e2cfc7
```

Tridiagonal.h

Description: $x = \text{tridiagonal}(d, p, q, b)$ solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i,\ 1 \le i \le n,$$

where a_0, a_{n+1}, b_i, c_i and d_i are known. a can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, ..., -1, 1\}, \{0, c_1, c_2, \ldots, c_n\}, \{b_1, b_2, \ldots, b_n, 0\}, \{a_0, d_1, d_2, \ldots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.
If $|d_i| > |p_i| + |q_{i-1}|$ for all i, or $|d_i| > |p_{i-1}| + |q_i|$, or the matrix is positive definite, the algorithm is numerically stable and neither `tr` nor the check for `diag[i] == 0` is needed.
Time: $\mathcal{O}\left(N\right)$

26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>&
  ↪super,
  const vector<T>& sub, vector<T> b) {
  int n = sz(b); vi tr(n);
  rep(i,0,n-1) {
```

```
    if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i]
      ↪== 0
      b[i+1] -= b[i] * diag[i+1] / super[i];
      if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
      diag[i+1] = sub[i]; tr[++i] = 1;
    } else {
      diag[i+1] -= super[i]*sub[i]/diag[i];
      b[i+1] -= b[i]*sub[i]/diag[i];
    }
  }
  for (int i = n; i--;) {
    if (tr[i]) {
      swap(b[i], b[i-1]);
      diag[i-1] = diag[i];
      b[i] /= super[i-1];
    } else {
      b[i] /= diag[i];
      if (i) b[i-1] -= b[i]*super[i-1];
    }
  }
  return b;
} // hash-cpp-all = 8f9fa8b1e5e82731da914aed0632312f
```

3.1 Fourier transforms

fft.cpp

Description: FFT/NTT, polynomial mod/log/exp

303 lines

```
namespace fft {
#if FFT
// FFT
using dbl = double;
struct num { // hash-cpp-1
  dbl x, y;
  num(dbl x_ = 0, dbl y_ = 0) : x(x_), y(y_) { }
};
inline num operator+(num a, num b) { return num(a.x + b.x,
  ↪a.y + b.y); }
inline num operator-(num a, num b) { return num(a.x - b.x,
  ↪a.y - b.y); }
inline num operator*(num a, num b) { return num(a.x * b.x -
  ↪ a.y * b.y, a.x * b.y + a.y * b.x); }
inline num conj(num a) { return num(a.x, -a.y); }
inline num inv(num a) { dbl n = (a.x*a.x+a.y*a.y); return
  ↪num(a.x/n,-a.y/n); }
// hash-cpp-1 = d2cc70ff17fe23dbfe608d8bce4d827b
#else
// NTT
const int mod = 998244353, g = 3;
// For p < 2^30 there is also (5 << 25, 3), (7 << 26, 3),
// (479 << 21, 3) and (483 << 21, 5). Last two are > 10^9.
struct num { // hash-cpp-2
  int v;
  num(ll v_ = 0) : v(int(v_ % mod)) { if (v<0) v+=mod; }
  explicit operator int() const { return v; }
};
inline num operator+(num a,num b){return num(a.v+b.v);}
inline num operator-(num a,num b){return num(a.v+mod-b.v);}
inline num operator*(num a,num b){return num(1ll*a.v*b.v);}
inline num pow(num a, int b) {
  num r = 1;
  do{if(b&1)r=r*a;a=a*a;}while(b>>=1);
  return r;
}
inline num inv(num a) { return pow(a, mod-2); }
// hash-cpp-2 = 62f50e0b94ea4486de6fbc07e826040a
#endif

using vn = vector<num>;
vi rev({0, 1});
vn rt(2, num(1)), fa, fb;

inline void init(int n) { // hash-cpp-3
  if (n <= sz(rt)) return;
  rev.resize(n);
  rep(i,0,n) rev[i] = (rev[i>>1] | ((i&1)*n)) >> 1;
  rt.reserve(n);
  for (int k = sz(rt); k < n; k *= 2) {
    rt.resize(2*k);
#if FFT
    double a=M_PI/k; num z(cos(a),sin(a)); // FFT
#else
    num z = pow(num(g), (mod-1)/(2*k)); // NTT
#endif
    rep(i,k/2,k) rt[2*i] = rt[i], rt[2*i+1] = rt[i]*z;
  }
} // hash-cpp-3 = 408005a3c0a4559a884205d5d7db44e9

inline void fft(vector<num> &a, int n) { // hash-cpp-4
  init(n);
  int s = __builtin_ctz(sz(rev)/n);
  rep(i,0,n) if (i < rev[i]>>s) swap(a[i], a[rev[i]>>s]);
  for (int k = 1; k < n; k *= 2)
    for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
      num t = rt[j+k] * a[i+j+k];
      a[i+j+k] = a[i+j] - t;
      a[i+j] = a[i+j] + t;
    }
} // hash-cpp-4 = 1f0820b04997ddca9b78742df352d419

// Complex/NTT
vn multiply(vn a, vn b) { // hash-cpp-5
  int s = sz(a) + sz(b) - 1;
  if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s-1) : 0, n = 1 << L;
  a.resize(n), b.resize(n);
  fft(a, n);
  fft(b, n);
  num d = inv(num(n));
  rep(i,0,n) a[i] = a[i] * b[i] * d;
  reverse(a.begin()+1, a.end());
  fft(a, n);
  a.resize(s);
  return a;
} // hash-cpp-5 = 7a20264754593de4eb7963d8fc3d8a15

// Complex/NTT power-series inverse
// Doubles b as b[:n] = (2 - a[:n] * b[:n/2]) * b[:n/2]
vn inverse(const vn& a) { // hash-cpp-6
  if (a.empty()) return {};
  vn b({inv(a[0])});
  b.reserve(2*a.size());
  while (sz(b) < sz(a)) {
    int n = 2*sz(b);
    b.resize(2*n, 0);
    if (sz(fa) < 2*n) fa.resize(2*n);
    fill(fa.begin(), fa.begin()+2*n, 0);
    copy(a.begin(), a.begin()+min(n,sz(a)), fa.begin());
    fft(b, 2*n);
    fft(fa, 2*n);
    num d = inv(num(2*n));
    rep(i, 0, 2*n) b[i] = b[i] * (2 - fa[i] * b[i]) * d;
    reverse(b.begin()+1, b.end());
    fft(b, 2*n);
    b.resize(n);
  }
  b.resize(a.size());
  return b;
} // hash-cpp-6 = 61660c4b2c75faa72062368a381f059f

#if FFT
// Double multiply (num = complex)
using vd = vector<double>;
vd multiply(const vd& a, const vd& b) { // hash-cpp-7
  int s = sz(a) + sz(b) - 1;
  if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s-1) : 0, n = 1 << L;
  if (sz(fa) < n) fa.resize(n);
  if (sz(fb) < n) fb.resize(n);

  fill(fa.begin(), fa.begin() + n, 0);
  rep(i,0,sz(a)) fa[i].x = a[i];
  rep(i,0,sz(b)) fa[i].y = b[i];
  fft(fa, n);
  trav(x, fa) x = x * x;
  rep(i,0,n) fb[i] = fa[(n-i)&(n-1)] - conj(fa[i]);
  fft(fb, n);
  vd r(s);
  rep(i,0,s) r[i] = fb[i].y / (4*n);
  return r;
} // hash-cpp-7 = c2431bc9cb89b2ad565db6fba6a21a32

// Integer multiply mod m (num = complex) // hash-cpp-8
vi multiply_mod(const vi& a, const vi& b, int m) {
  int s = sz(a) + sz(b) - 1;
  if (s <= 0) return {};
  int L = s > 1 ? 32 - __builtin_clz(s-1) : 0, n = 1 << L;
  if (sz(fa) < n) fa.resize(n);
  if (sz(fb) < n) fb.resize(n);

  rep(i,0,sz(a)) fa[i] = num(a[i] & ((1<<15)-1), a[i] >>
    ↪15);
  fill(fa.begin()+sz(a), fa.begin() + n, 0);
  rep(i,0,sz(b)) fb[i] = num(b[i] & ((1<<15)-1), b[i] >>
    ↪15);
  fill(fb.begin()+sz(b), fb.begin() + n, 0);

  fft(fa, n);
  fft(fb, n);
  double r0 = 0.5 / n; // 1/2n
  rep(i,0,n/2+1) {
    int j = (n-i)&(n-1);
    num g0 = (fb[i] + conj(fb[j])) * r0;
    num g1 = (fb[i] - conj(fb[j])) * r0;
    swap(g1.x, g1.y); g1.y *= -1;
    if (j != i) {
      swap(fa[j], fa[i]);
      fb[j] = fa[j] * g1;
      fa[j] = fa[j] * g0;
    }
    fb[i] = fa[i] * conj(g1);
    fa[i] = fa[i] * conj(g0);
  }
  fft(fa, n);
  fft(fb, n);
  vi r(s);
  rep(i,0,s) r[i] = int((ll(fa[i].x+0.5)
        + (ll(fa[i].y+0.5) % m << 15)
        + (ll(fb[i].x+0.5) % m << 15)
        + (ll(fb[i].y+0.5) % m << 30)) % m);
  return r;
} // hash-cpp-8 = e8c5f6755ad1e5a976d6c6ffd37b3b22
```

```
#endif

} // namespace fft

// For multiply_mod, use num = modnum, poly = vector<num>
using fft::num;
using poly = fft::vn;
using fft::multiply;
using fft::inverse;
// hash-cpp-9
poly& operator+=(poly& a, const poly& b) {
  if (sz(a) < sz(b)) a.resize(b.size());
  rep(i,0,sz(b)) a[i]=a[i]+b[i];
  return a;
}
poly operator+(const poly& a, const poly& b) { poly r=a; r
  ↪+=b; return r; }
poly& operator-=(poly& a, const poly& b) {
  if (sz(a) < sz(b)) a.resize(b.size());
  rep(i,0,sz(b)) a[i]=a[i]-b[i];
  return a;
}
poly operator-(const poly& a, const poly& b) { poly r=a; r
  ↪-=b; return r; }
poly operator*(const poly& a, const poly& b) {
  // TODO: small-case?
  return multiply(a, b);
}
poly& operator*=(poly& a, const poly& b) {return a = a*b;}
// hash-cpp-9 = 61b8743c2b07beed0e7ca857081e1bd4
poly& operator*=(poly& a, const num& b) { // Optional
  trav(x, a) x = x * b;
  return a;
}
poly operator*(const poly& a, const num& b) { poly r=a; r*=
  ↪b; return r; }

// Polynomial floor division; no leading 0's plz
poly operator/(poly a, poly b) { // hash-cpp-10
  if (sz(a) < sz(b)) return {};
  int s = sz(a)-sz(b)+1;
  reverse(a.begin(), a.end());
  reverse(b.begin(), b.end());
  a.resize(s);
  b.resize(s);
  a = a * inverse(move(b));
  a.resize(s);
  reverse(a.begin(), a.end());
  return a;
} // hash-cpp-10 = a6589ce8fcf1e33df3b42ee703a7fe60
poly& operator/=(poly& a, const poly& b) {return a = a/b;}
poly& operator%=(poly& a, const poly& b) { // hash-cpp-11
  if (sz(a) >= sz(b)) {
    poly c = (a / b) * b;
    a.resize(sz(b)-1);
    rep(i,0,sz(a)) a[i] = a[i]-c[i];
  }
  return a;
} // hash-cpp-11 = 9af255f48abbeafd8acde353357b84fd
poly operator%(const poly& a, const poly& b) { poly r=a; r
  ↪%=b; return r; }

// Log/exp/pow
poly deriv(const poly& a) { // hash-cpp-12
  if (a.empty()) return {};
  poly b(sz(a)-1);
  rep(i,1,sz(a)) b[i-1]=a[i]*i;
  return b;
} // hash-cpp-12 = 94aa209b3e956051e6b3131bf1faafd1
poly integ(const poly& a) { // hash-cpp-13
  poly b(sz(a)+1);
  b[1]=1; // mod p
  rep(i,2,sz(b)) b[i]=b[fft::mod%i]*(-fft::mod/i); // mod p
  rep(i,1,sz(b)) b[i]=a[i-1]*b[i]; // mod p
  //rep(i,1,sz(b)) b[i]=a[i-1]*inv(num(i)); // else
  return b;
} // hash-cpp-13 = 6f13f6a43b2716a116d347000820f0bd
poly log(const poly& a) { // a[0] == 1 // hash-cpp-14
  poly b = integ(deriv(a)*inverse(a));
  b.resize(a.size());
  return b;
} // hash-cpp-14 = ce1533264298c5382f72a2a1b0947045
poly exp(const poly& a) { // a[0] == 0 // hash-cpp-15
  poly b(1,num(1));
  if (a.empty()) return b;
  while (sz(b) < sz(a)) {
    int n = min(sz(b) * 2, sz(a));
    b.resize(n);
    poly v = poly(a.begin(), a.begin() + n) - log(b);
    v[0] = v[0]+num(1);
    b *= v;
    b.resize(n);
  }
  return b;
} // hash-cpp-15 = f645d091e4ae3ee3dc2aa095d4aa699a
poly pow(const poly& a, int m) { // m >= 0 // hash-cpp-16
  poly b(a.size());
  if (!m) { b[0] = 1; return b; }
  int p = 0;
  while (p<sz(a) && a[p].v==0) ++p;
  if (1ll*m*p >= sz(a)) return b;
  num mu = pow(a[p], m), di = inv(a[p]);
  poly c(sz(a) - m*p);
  rep(i,0,sz(c)) c[i] = a[i+p] * di;
  c = log(c);
  trav(v,c) v = v * m;
  c = exp(c);
  rep(i,0,sz(c)) b[i+m*p] = c[i] * mu;
  return b;
} // hash-cpp-16 = 0f4830b9de34c26d39f170069827121f

// Multipoint evaluation/interpolation
// hash-cpp-17
vector<num> eval(const poly& a, const vector<num>& x) {
  int n=sz(x);
  if (!n) return {};
  vector<poly> up(2*n);
  rep(i,0,n) up[i+n] = poly({0-x[i], 1});
  per(i,1,n) up[i] = up[2*i]*up[2*i+1];
  vector<poly> down(2*n);
  down[1] = a % up[1];
  rep(i,2,2*n) down[i] = down[i/2] % up[i];
  vector<num> y(n);
  rep(i,0,n) y[i] = down[i+n][0];
  return y;
} // hash-cpp-17 = a079eba46c3110851ec6b0490b439931
// hash-cpp-18
poly interp(const vector<num>& x, const vector<num>& y) {
  int n=sz(x);
  assert(n);
  vector<poly> up(n*2);
  rep(i,0,n) up[i+n] = poly({0-x[i], 1});
  per(i,1,n) up[i] = up[2*i]*up[2*i+1];
  vector<num> a = eval(deriv(up[1]), x);
  vector<poly> down(2*n);
  rep(i,0,n) down[i+n] = poly({y[i]*inv(a[i])});
  per(i,1,n) down[i] = down[i*2] * up[i*2+1] + down[i*2+1]
    ↪* up[i*2];
  return down[1];
} // hash-cpp-18 = 74f15e1e82d51e852b321a1ff75ba1fd
```

FastSubsetTransform.h

Description: Transform to a basis with fast convolutions of the form $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$, where $\oplus$ is one of AND, OR, XOR. The size of a must be a power of two.
Time: $\mathcal{O}(N \log N)$

16 lines

```
void FST(vi& a, bool inv) {
  for (int n = sz(a), step = 1; step < n; step *= 2) {
    for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
      int &u = a[j], &v = a[j + step]; tie(u, v) =
        inv ? pii(v - u, u) : pii(v, u + v); // AND
        inv ? pii(v, u - v) : pii(u + v, u); // OR
        pii(u + v, u - v);                   // XOR
    }
  }
  if (inv) trav(x, a) x /= sz(a); // XOR only
}
vi conv(vi a, vi b) {
  FST(a, 0); FST(b, 0);
  rep(i,0,sz(a)) a[i] *= b[i];
  FST(a, 1); return a;
} // hash-cpp-all = 3de473e2c1de97e6e9ff0f13542cf3fb
```

Number theory (4)

4.1 Modular arithmetic

ModularArithmetic.h

Description: Operators for modular arithmetic. You need to set `mod` to some number first and then you can use the structure.

"euclid.h" 18 lines

```
const ll mod = 17; // change to something else
struct Mod {
  ll x;
  Mod(ll xx) : x(xx) {}
  Mod operator+(Mod b) { return Mod((x + b.x) % mod); }
  Mod operator-(Mod b) { return Mod((x - b.x + mod) % mod);
    ↪ }
  Mod operator*(Mod b) { return Mod((x * b.x) % mod); }
  Mod operator/(Mod b) { return *this * invert(b); }
  Mod invert(Mod a) {
    ll x, y, g = euclid(a.x, mod, x, y);
    assert(g == 1); return Mod((x + mod) % mod);
  }
  Mod operator^(ll e) {
    if (!e) return Mod(1);
    Mod r = *this ^ (e / 2); r = r * r;
    return e&1 ? *this * r : r;
  }
}; // hash-cpp-all = 35bfea8c111cb24c4ce84c658446961b
```

ModInverse.h

Description: Pre-computation of modular inverses. Assumes LIM $\leq$ mod and that mod is a prime.

4 lines

```
const ll mod = 1000000007, LIM = 200000;
ll* inv = new ll[LIM] - 1; inv[1] = 1;
rep(i,2,LIM) inv[i] = mod - (mod / i) * inv[mod % i] % mod;
// hash-cpp-all = 6f684f0b9ae6c69f42de68f023a81de5
```

ModPow.h

6 lines

```
const ll mod = 1000000007; // faster if const
ll modpow(ll a, ll e) {
  if (e == 0) return 1;
  ll x = modpow(a * a % mod, e >> 1);
  return e & 1 ? x * a % mod : x;
} // hash-cpp-all = 2fa6d9ccac4586cba0618aad18cdc9de
```

ModSum.h

Description: Sums of mod'ed arithmetic progressions.
`modsum(to, c, k, m)` $= \sum_{i=0}^{to-1} (ki + c)\%m$. `divsum` is similar but for floored division.
Time: $\log(m)$, with a large constant.

19 lines

```
typedef unsigned long long ull;
ull sumsq(ull to) { return to / 2 * ((to-1) | 1); }

ull divsum(ull to, ull c, ull k, ull m) {
  ull res = k / m * sumsq(to) + c / m * to;
  k %= m; c %= m;
  if (k) {
    ull to2 = (to * k + c) / m;
    res += to * to2;
    res -= divsum(to2, m-1 - c, m, k) + to2;
  }
  return res;
}

ll modsum(ull to, ll c, ll k, ll m) {
  c = ((c % m) + m) % m;
  k = ((k % m) + m) % m;
  return to * c + k * sumsq(to) - m * divsum(to, c, k, m);
} // hash-cpp-all = 8d6e082e0ea6be867eaea12670d08dcc
```

ModMulLL.h

Description: Calculate $a \cdot b \bmod c$ (or $a^b \bmod c$) for large c.
Time: $\mathcal{O}\left(64/bits \cdot \log b\right)$, where $bits = 64 - k$, if we want to deal with k-bit numbers.

19 lines

```
typedef unsigned long long ull;
const int bits = 10;
// if all numbers are less than 2^k, set bits = 64-k
const ull po = 1 << bits;
ull mod_mul(ull a, ull b, ull &c) {
  ull x = a * (b & (po - 1)) % c;
  while ((b >>= bits) > 0) {
    a = (a << bits) % c;
    x += (a * (b & (po - 1))) % c;
  }
  return x % c;
}
ull mod_pow(ull a, ull b, ull mod) {
  if (b == 0) return 1;
  ull res = mod_pow(a, b / 2, mod);
  res = mod_mul(res, res, mod);
  if (b & 1) return mod_mul(res, a, mod);
  return res;
} // hash-cpp-all = 40cd743544228d297c803154525107ab
```

ModSqrt.h

Description: Tonelli-Shanks algorithm for modular square roots.
Time: $\mathcal{O}\left(\log^2 p\right)$ worst case, often $\mathcal{O}\left(\log p\right)$

"ModPow.h" 30 lines

```
ll sqrt(ll a, ll p) {
  a %= p; if (a < 0) a += p;
  if (a == 0) return 0;
  assert(modpow(a, (p-1)/2, p) == 1);
  if (p % 4 == 3) return modpow(a, (p+1)/4, p);
  // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
  ll s = p - 1;
  int r = 0;
  while (s % 2 == 0)
    ++r, s /= 2;
  ll n = 2; // find a non-square mod p
  while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
  ll x = modpow(a, (s + 1) / 2, p);
  ll b = modpow(a, s, p);
  ll g = modpow(n, s, p);
  for (;;) {
    ll t = b;
    int m = 0;
    for (; m < r; ++m) {
      if (t == 1) break;
      t = t * t % p;
    }
    if (m == 0) return x;
    ll gs = modpow(g, 1 << (r - m - 1), p);
    g = gs * gs % p;
    x = x * gs % p;
    b = b * g % p;
    r = m;
  }
} // hash-cpp-all = 83e24bd39c8c93946ad3021b8ca6c3c4
```

4.2 Primality

eratosthenes.h

Description: Prime sieve for generating all primes up to a certain limit. isprime[i] is true iff i is a prime.
Time: lim=100'000'000 $\approx$ 0.8 s. Runs 30% faster if only odd indices are stored.

11 lines

```
const int MAX_PR = 5000000;
bitset<MAX_PR> isprime;
vi eratosthenes_sieve(int lim) {
  isprime.set(); isprime[0] = isprime[1] = 0;
  for (int i = 4; i < lim; i += 2) isprime[i] = 0;
  for (int i = 3; i*i < lim; i += 2) if (isprime[i])
    for (int j = i*i; j < lim; j += i*2) isprime[j] = 0;
  vi pr;
  rep(i,2,lim) if (isprime[i]) pr.push_back(i);
  return pr;
} // hash-cpp-all = 0564a3337fb69c0b87dfd3c56cdfe2e3
```

MillerRabin.h

Description: Miller-Rabin primality probabilistic test. Probability of failing one iteration is at most 1/4. 15 iterations should be enough for 50-bit numbers.
Time: 15 times the complexity of $a^b \mod c$.

"ModMulLL.h" 16 lines

```
bool prime(ull p) {
  if (p == 2) return true;
  if (p == 1 || p % 2 == 0) return false;
  ull s = p - 1;
  while (s % 2 == 0) s /= 2;
  rep(i,0,15) {
    ull a = rand() % (p - 1) + 1, tmp = s;
    ull mod = mod_pow(a, tmp, p);
    while (tmp != p - 1 && mod != 1 && mod != p - 1) {
      mod = mod_mul(mod, mod, p);
      tmp *= 2;
    }
    if (mod != p - 1 && tmp % 2 == 0) return false;
  }
  return true;
} // hash-cpp-all = ccddf18bab60a654ff4af45e95dd60b6
```

factor.h

Description: Pollard's rho algorithm. It is a probabilistic factorisation algorithm, whose expected time complexity is good. Before you start using it, run `init(bits)`, where bits is the length of the numbers you use. Returns factors of the input without duplicates.
Time: Expected running time should be good enough for 50-bit numbers.

"ModMulLL.h", "MillerRabin.h", "eratosthenes.h" 35 lines

```
vector<ull> pr;
ull f(ull a, ull n, ull &has) {
  return (mod_mul(a, a, n) + has) % n;
}
vector<ull> factor(ull d) {
  vector<ull> res;
  for (int i = 0; i < sz(pr) && pr[i]*pr[i] <= d; i++)
    if (d % pr[i] == 0) {
      while (d % pr[i] == 0) d /= pr[i];
      res.push_back(pr[i]);
    }
  //d is now a product of at most 2 primes.
  if (d > 1) {
    if (prime(d))
      res.push_back(d);
    else while (true) {
      ull has = rand() % 2321 + 47;
      ull x = 2, y = 2, c = 1;
      for (; c==1; c = __gcd((y > x ? y - x : x - y), d)) {
        x = f(x, d, has);
        y = f(f(y, d, has), d, has);
      }
      if (c != d) {
        res.push_back(c); d /= c;
        if (d != c) res.push_back(d);
        break;
      }
    }
  }
  return res;
}
void init(int bits) {//how many bits do we use?
  vi p = eratosthenes_sieve(1 << ((bits + 2) / 3));
  pr.assign(all(p));
} // hash-cpp-all = 67b304bd690b2a8445a7b4dbf93996d7
```

4.3 Divisibility

euclid.h

Description: Finds the Greatest Common Divisor to the integers a and b. Euclid also finds two integers x and y, such that $ax + by = \gcd(a, b)$. If a and b are coprime, then x is the inverse of $a \pmod b$.

7 lines

```
ll gcd(ll a, ll b) { return __gcd(a, b); }

ll euclid(ll a, ll b, ll &x, ll &y) {
  if (b) { ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d; }
  return x = 1, y = 0, a;
} // hash-cpp-all = 63e6f8d2f560b27cb800273d63d2102c
```

Euclid.java

Description: Finds {x, y, d} s.t. ax + by = d = gcd(a, b). 11 lines

```
static BigInteger[] euclid(BigInteger a, BigInteger b) {
  BigInteger x = BigInteger.ONE, yy = x;
  BigInteger y = BigInteger.ZERO, xx = y;
  while (b.signum() != 0) {
    BigInteger q = a.divide(b), t = b;
    b = a.mod(b); a = t;
    t = xx; xx = x.subtract(q.multiply(xx)); x = t;
    t = yy; yy = y.subtract(q.multiply(yy)); y = t;
  }
  return new BigInteger[]{x, y, a};
}
```

4.4 Fractions

ContinuedFractions.h

Description: Given N and a real number $x \geq 0$, finds the closest rational approximation p/q with $p, q \leq N$. It will obey $|p/q - x| \leq 1/qN$. For consecutive convergents, $p_{k+1}q_k - q_{k+1}p_k = (-1)^k$. ($p_k/q_k$ alternates between $> x$ and $< x$.) If x is rational, y eventually becomes ∞; if x is the root of a degree 2 polynomial the a's eventually become cyclic.

Time: $\mathcal{O}(\log N)$ 21 lines

```
typedef double d; // for N ~ 1e7; long double for N ~ 1e9
pair<ll, ll> approximate(d x, ll N) {
  ll LP = 0, LQ = 1, P = 1, Q = 0, inf = LLONG_MAX; d y = x
    ↪;
  for (;;) {
    ll lim = min(P ? (N-LP) / P : inf, Q ? (N-LQ) / Q : inf
      ↪),
      a = (ll)floor(y), b = min(a, lim),
      NP = b*P + LP, NQ = b*Q + LQ;
    if (a > b) {
      // If b > a/2, we have a semi-convergent that gives
        ↪us a
      // better approximation; if b = a/2, we *may* have
        ↪one.
      // Return {P, Q} here for a more canonical
        ↪approximation.
      return (abs(x - (d)NP / (d)NQ) < abs(x - (d)P / (d)Q)
        ↪) ?
        make_pair(NP, NQ) : make_pair(P, Q);
    }
    if (abs(y = 1/(y - (d)a)) > 3*N) {
      return {NP, NQ};
    }
    LP = P; P = NP;
    LQ = Q; Q = NQ;
  }
} // hash-cpp-all = dd6c5e1084a26365dc6321bd935975d9
```

FracBinarySearch.h

Description: Given f and N, finds the smallest fraction $p/q \in [0, 1]$ such that $f(p/q)$ is true, and $p, q \leq N$. You may want to throw an exception from f if it finds an exact solution, in which case N can be removed.

Usage: `fracBS([](Frac f) { return f.p>=3*f.q; }, 10); // {1,3}`

Time: $\mathcal{O}(\log(N))$ 24 lines

```
struct Frac { ll p, q; };

template<class F>
Frac fracBS(F f, ll N) {
  bool dir = 1, A = 1, B = 1;
  Frac lo{0, 1}, hi{1, 1}; // Set hi to 1/0 to search (0, N
    ↪]
  assert(!f(lo)); assert(f(hi));
  while (A || B) {
    ll adv = 0, step = 1; // move hi if dir, else lo
    for (int si = 0; step; (step *= 2) >>= si) {
      adv += step;
      Frac mid{lo.p * adv + hi.p, lo.q * adv + hi.q};
      if (abs(mid.p) > N || mid.q > N || dir == !f(mid)) {
        adv -= step; si = 2;
      }
    }
    hi.p += lo.p * adv;
    hi.q += lo.q * adv;
    dir = !dir;
    swap(lo, hi);
    A = B; B = !!adv;
  }
  return dir ? hi : lo;
} // hash-cpp-all = 214844f17d0c347ff436141729e0c829
```

4.5 Chinese remainder theorem

chinese.h

Description: Chinese Remainder Theorem.

`chinese(a, m, b, n)` returns a number x, such that $x \equiv a \pmod m$ and $x \equiv b \pmod n$. For not coprime n, m, use `chinese_common`. Note that all numbers must be less than 2^{31} if you have Z = unsigned long long.

Time: $\log(m + n)$

"euclid.h" 13 lines

```
template<class Z> Z chinese(Z a, Z m, Z b, Z n) {
  Z x, y; euclid(m, n, x, y);
  Z ret = a * (y + m) % m * n + b * (x + n) % n * m;
  if (ret >= m * n) ret -= m * n;
  return ret;
}

template<class Z> Z chinese_common(Z a, Z m, Z b, Z n) {
  Z d = gcd(m, n);
  if (((b -= a) %= n) < 0) b += n;
  if (b % d) return -1; // No solution
  return d * chinese(Z(0), m/d, b/d, n/d) + a;
} // hash-cpp-all = da3099704e14964aa045c152bb478c14
```

4.6 Pythagorean Triples

The Pythagorean triples are uniquely generated by

$$a = k \cdot (m^2 - n^2), \;\; b = k \cdot (2mn), \;\; c = k \cdot (m^2 + n^2),$$

with $m > n > 0$, $k > 0$, $m \perp n$, and either m or n even.

4.7 Primes

$p = 962592769$ is such that $2^{21} \mid p - 1$, which may be useful. For hashing use 970592641 (31-bit number), 31443539979727 (45-bit), 3006703054056749 (52-bit). There are 78498 primes less than 1 000 000.

Primitive roots exist modulo any prime power p^a, except for $p = 2, a > 2$, and there are $\phi(\phi(p^a))$ many. For $p = 2, a > 2$, the group $\mathbb{Z}_{2^a}^\times$ is instead isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_{2^{a-2}}$.

4.8 Estimates

$\sum_{d|n} d = O(n \log \log n)$.

The number of divisors of n is at most around 100 for $n < 5e4$, 500 for $n < 1e7$, 2000 for $n < 1e10$, 200 000 for $n < 1e19$.

Combinatorial (5)

5.1 Permutations

5.1.1 Factorial

n	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
n	11	12	13	14	15	16	17			
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
n	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

IntPerm.h

Description: Permutation -> integer conversion. (Not order preserving.)

Time: $\mathcal{O}(n)$ 6 lines

```
int permToInt(vi& v) {
  int use = 0, i = 0, r = 0;
  trav(x,v)r=r * ++i + __builtin_popcount(use & -(1 << x)),
    use |= 1 << x;                // (note: minus, not ~!)
  return r;
} // hash-cpp-all = e1b8eaea02324af14a3da94f409019b8
```

5.1.2 Cycles

Let $g_S(n)$ be the number of n-permutations whose cycle lengths all belong to the set S. Then

$$\sum_{n=0}^{\infty} g_S(n) \frac{x^n}{n!} = \exp\left(\sum_{n \in S} \frac{x^n}{n}\right)$$

5.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1)+D(n-2)) = nD(n-1)+(-1)^n = \left\lfloor \frac{n!}{e} \right\rceil$$

5.1.4 Burnside's lemma

Given a group G of symmetries and a set X, the number of elements of X *up to symmetry* equals

$$\frac{1}{|G|}\sum_{g\in G}|X^g|,$$

where X^g are the elements fixed by g ($g.x = x$).

If $f(n)$ counts "configurations" (of some sort) of length n, we can ignore rotational symmetry using $G = \mathbb{Z}_n$ to get

$$g(n) = \frac{1}{n}\sum_{k=0}^{n-1} f(\gcd(n,k)) = \frac{1}{n}\sum_{k|n} f(k)\phi(n/k).$$

5.2 Partitions and subsets

5.2.1 Partition function

Number of ways of writing n as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1,\ p(n) = \sum_{k\in\mathbb{Z}\setminus\{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

n	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	∼2e5	∼2e8

5.2.2 Binomials

binomialModPrime.h

Description: Lucas' thm: Let n, m be non-negative integers and p a prime. Write $n = n_k p^k + \ldots + n_1 p + n_0$ and $m = m_k p^k + \ldots + m_1 p + m_0$. Then $\binom{n}{m} \equiv \prod_{i=0}^{k} \binom{n_i}{m_i} \pmod p$. fact and invfact must hold pre-computed factorials / inverse factorials, e.g. from ModInverse.h.
Time: $\mathcal{O}(\log_p n)$ 10 lines

```
ll chooseModP(ll n, ll m, int p, vi& fact, vi& invfact) {
  ll c = 1;
  while (n || m) {
    ll a = n % p, b = m % p;
    if (a < b) return 0;
    c = c * fact[a] % p * invfact[b] % p * invfact[a - b] %
      ↪ p;
    n /= p; m /= p;
  }
  return c;
} // hash-cpp-all = 81845faa6ecd635c391e4f0134f0676c
```

multinomial.h

Description: Computes $\binom{k_1+\cdots+k_n}{k_1,k_2,\ldots,k_n} = \frac{(\sum k_i)!}{k_1!k_2!\ldots k_n!}$. 6 lines

```
ll multinomial(vi& v) {
  ll c = 1, m = v.empty() ? 1 : v[0];
  rep(i,1,sz(v)) rep(j,0,v[i])
    c = c * ++m / (j+1);
  return c;
} // hash-cpp-all = a0a3128f6afa4721166feb182b82f130
```

5.3 General purpose numbers

5.3.1 Bernoulli numbers

EGF of Bernoulli numbers is $B(t) = \frac{t}{e^t-1}$ (FFT-able).
$B[0,\ldots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \ldots]$

Sums of powers:

$$\sum_{i=1}^{n} n^m = \frac{1}{m+1}\sum_{k=0}^{m}\binom{m+1}{k} B_k (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\sum_{i=m}^{\infty} f(i) = \int_m^\infty f(x)dx - \sum_{k=1}^{\infty}\frac{B_k}{k!} f^{(k-1)}(m)$$
$$\approx \int_m^\infty f(x)dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m))$$

5.3.2 Stirling numbers of the first kind

Number of permutations on n items with k cycles.

$c(n,k) = c(n-1,k-1) + (n-1)c(n-1,k),\ c(0,0) = 1$
$\sum_{k=0}^{n} c(n,k)x^k = x(x+1)\ldots(x+n-1)$
$c(8,k) =$
$8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$
$c(n,2) =$
$0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \ldots$

5.3.3 Eulerian numbers

Number of permutations $\pi \in S_n$ in which exactly k elements are greater than the previous element. k j:s s.t. $\pi(j) > \pi(j+1)$, $k+1$ j:s s.t. $\pi(j) \geq j$, k j:s s.t. $\pi(j) > j$.

$$E(n,k) = (n-k)E(n-1,k-1) + (k+1)E(n-1,k)$$
$$E(n,0) = E(n,n-1) = 1$$
$$E(n,k) = \sum_{j=0}^{k}(-1)^j\binom{n+1}{j}(k+1-j)^n$$

5.3.4 Stirling numbers of the second kind

Partitions of n distinct elements into exactly k groups.

$$S(n,k) = S(n-1,k-1) + kS(n-1,k)$$
$$S(n,1) = S(n,n) = 1$$
$$S(n,k) = \frac{1}{k!}\sum_{j=0}^{k}(-1)^{k-j}\binom{k}{j}j^n$$

5.3.5 Bell numbers

Total number of partitions of n distinct elements.
$B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \ldots.$ For p prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod p$$

5.3.6 Labeled unrooted trees

on n vertices: n^{n-2}
on k existing trees of size n_i: $n_1 n_2 \cdots n_k n^{k-2}$
with degrees d_i: $(n-2)!/((d_1-1)!\cdots(d_n-1)!)$

5.3.7 Catalan numbers

$$C_n = \frac{1}{n+1}\binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1,\ C_{n+1} = \frac{2(2n+1)}{n+2}C_n,\ C_{n+1} = \sum C_i C_{n-i}$$

$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \ldots$

- sub-diagonal monotone paths in an $n \times n$ grid.
- strings with n pairs of parenthesis, correctly nested.
- binary trees with with $n+1$ leaves (0 or 2 children).
- ordered trees with $n+1$ vertices.
- ways a convex polygon with $n+2$ sides can be cut into triangles by connecting vertices with straight lines.
- permutations of $[n]$ with no 3-term increasing subseq.

5.4 Other

nim-product.cpp
Description: Nim Product. 17 lines

```
using ull = uint64_t;
ull _nimProd2[64][64];
ull nimProd2(int i, int j) {
  if (_nimProd2[i][j]) return _nimProd2[i][j];
  if ((i & j) == 0) return _nimProd2[i][j] = 1ull << (i|j);
  int a = (i&j) & -(i&j);
  return _nimProd2[i][j] = nimProd2(i ^ a, j) ^ nimProd2((i
    ↪ ^ a) | (a-1), (j ^ a) | (i & (a-1)));
}
ull nimProd(ull x, ull y) {
  ull res = 0;
  for (int i = 0; x >> i; i++)
    if ((x >> i) & 1)
      for (int j = 0; y >> j; j++)
        if ((y >> j) & 1)
          res ^= nimProd2(i, j);
  return res;
} // hash-cpp-all = e0411498c7a77d77ae793efab5500851
```

schreier-sims.cpp
Description: Check group membership of permutation groups 52 lines

```
struct Perm {
  int a[N];
  Perm() {
    for (int i = 1; i <= n; ++i) a[i] = i;
  }
  friend Perm operator* (const Perm &lhs, const Perm &rhs)
    ↪{
    static Perm res;
    for (int i = 1; i <= n; ++i) res.a[i] = lhs.a[rhs.a[i
      ↪]];
    return res;
  }
  friend Perm inv(const Perm &cur) {
    static Perm res;
    for (int i = 1; i <= n; ++i) res.a[cur.a[i]] = i;
    return res;
  }
};
class Group {
  bool flag[N];
  Perm w[N];
  std::vector<Perm> x;
public:
  void clear(int p) {
    memset(flag, 0, sizeof flag);
    for (int i = 1; i <= n; ++i) w[i] = Perm();
    flag[p] = true;
    x.clear();
  }
  friend bool check(const Perm&, int);
  friend void insert(const Perm&, int);
  friend void updateX(const Perm&, int);
} g[N];
bool check(const Perm &cur, int k) {
  if (!k) return true;
  int t = cur.a[k];
  return g[k].flag[t] ? check(g[k].w[t] * cur, k - 1) :
    ↪false;
}
void updateX(const Perm&, int);
void insert(const Perm &cur, int k) {
  if (check(cur, k)) return;
  g[k].x.push_back(cur);
  for (int i = 1; i <= n; ++i) if (g[k].flag[i]) updateX(
    ↪cur * inv(g[k].w[i]), k);
}
void updateX(const Perm &cur, int k) {
  int t = cur.a[k];
  if (g[k].flag[t]) {
    insert(g[k].w[t] * cur, k - 1);
  } else {
    g[k].w[t] = inv(cur);
    g[k].flag[t] = true;
    for (int i = 0; i < g[k].x.size(); ++i) updateX(g[k].x[
      ↪i] * cur, k);
  }
} // hash-cpp-all = 949a6e50dbdaea9cda09928c7eabedbc
```

Graph (6)

6.1 Network flow

Dinic.cpp
<bits/stdc++.h> 71 lines

```
const int N = 100050;
const int INF = (int)1e9;

struct edge{
    int to, cap, rev;
    edge(int _to, int _cap, int _rev){
        to = _to, cap = _cap, rev = _rev;
    }
};

// Finding max flow in O(V^2 * E)
struct Dinic {
    vector<edge> G[N];
    int level[N], iter[N];

    void add_edge(int from, int to, int cap){
        G[from].push_back(edge(to, cap, G[to].size()));
        G[to].push_back(edge(from, 0, G[from].size() - 1));
    }

    void bfs(int s){
        memset(level, -1, sizeof(level));
        queue<int> que;
        level[s] = 0;
        que.push(s);
        while(!que.empty()){
            int v = que.front(); que.pop();
            for(int i = 0; i < G[v].size(); i++){
                edge &e = G[v][i];
                if(e.cap > 0 && level[e.to] < 0){
                    level[e.to] = level[v] + 1;
                    que.push(e.to);
                }
            }
        }
    }

    int dfs(int v, int t, int f){
        if(v == t) return f;

        for(int &i = iter[v]; i < G[v].size(); i++){
            edge &e = G[v][i];
            if(e.cap > 0 && level[v] < level[e.to]){
                int d = dfs(e.to, t, min(e.cap, f));
                if(d > 0){
                    e.cap -= d;
                    G[e.to][e.rev].cap += d;
                    return d;
                }
            }
        }
        return 0;
    }

    int max_flow(int s, int t){
        int flow = 0;
        for(;;){
            bfs(s);
            if(level[t] < 0) return flow;
            memset(iter, 0, sizeof(iter));
            int f;
            while((f = dfs(s, t, INF)) > 0){
                flow += f;
            }
        }
    }
} dinic;
// hash-cpp-all = 10b15611306a45ad6aff125e61930736
```

FordFulkerson.cpp
<cstdio>, <iostream>, <vector> 45 lines

```
const int N = 3100, INF = (int)1e9;

struct edge{
    int to, cap, rev;
    edge(int _to, int _cap, int _rev){
        to = _to, cap = _cap, rev = _rev;
    }
};

int n,m;
vector<edge> G[N];
bool used[N];

void add_edge(int from, int to, int cap){
    G[from].push_back(edge(to, cap, G[to].size()));
    G[to].push_back(edge(from, 0, G[from].size() - 1));
}

int dfs(int v, int t, int f){
    if(v == t) return f;
    used[v] = true;

    for(int i = 0; i < G[v].size(); i++){
        edge &e = G[v][i];
        if(!used[e.to] && e.cap > 0){
            int d = dfs(e.to, t, min(e.cap, f));
            if(d > 0){
                e.cap -= d;
                G[e.to][e.rev].cap += d;
                return d;
            }
        }
    }
    return 0;
}

int max_flow(int s, int t){
```

```
    int f = 0;
    for(;;){
        fill(used, used + N, false);
        int d = dfs(s, t, INF);
        if(d == 0) return f;
        f += d;
    }
} // hash-cpp-all = dfc076ce80c0411c05f82f3ae825453e
```

MaximumWeightMatching.cpp

<bits/stdc++.h> 48 lines

```
const int MAXN = 2010;
const int oo = 1000000007;

int dist[MAXN][MAXN];
// Finding the minimum weight prefect matching (of size n)
   ↪in O(N^3)
// The dist matrix is 1-indexed.
int hungarian(int n, int m){
  vector<int> u(n + 1), v(m + 1), p(m + 1), way(m + 1);
  for(int i = 1; i <= n; i++){
    p[0] = i;
    int j0 = 0;
    vector<int> minv(m + 1, oo);
    vector<char> used(m + 1, false);
    do{
      used[j0] = true;
      int i0 = p[j0], delta = oo, j1;
      for(int j = 1; j <= m; j++){
        if(!used[j]){
          int cur = dist[i0][j] - u[i0] - v[j];
          if(cur < minv[j]){
            minv[j] = cur;
            way[j] = j0;
          }
          if(minv[j] < delta){
            delta = minv[j];
            j1 = j;
          }
        }
      }
      for(int j = 0; j <= m; j++){
        if(used[j]){
          u[p[j]] += delta;
          v[j] -= delta;
        } else {
          minv[j] -= delta;
        }
      }
      j0 = j1;
    } while (p[j0] != 0);
    do{
      int j1 = way[j0];
      p[j0] = p[j1];
      j0 = j1;
    } while(j0);
  }
  return -v[0];
}
// hash-cpp-all = 7aa16f18feade9a42ebdb04feae4632b
```

MinimumCostMaxFlow.cpp

<iostream>, <cstdio>, <vector>, <queue> 59 lines

```
const int N = 55, MAX_V = 105, INF = (int)1e9;

typedef pair<int, int> P;

struct edge{int to, cap, cost, rev;};

// Finding Min Cost Max Flow in min(O(F * E * log(V), O(F *
   ↪ V * V));
struct MincostFlow {
    int V; //Please set V!!!!
    vector<edge> G[MAX_V];
    int h[MAX_V];
    int dist[MAX_V];
    int prevv[MAX_V], preve[MAX_V];

    void add_edge(int from, int to, int cap, int cost){
        G[from].push_back((edge){to, cap, cost, (int)G[to].
           ↪size()});
        G[to].push_back((edge){from, 0, -cost, (int)G[from
           ↪].size() - 1});
    }

    int min_cost_flow(int s, int t, int f){
        int res = 0;
        fill(h, h + V, 0);
        while(f > 0){
            priority_queue<P, vector<P>, greater<P> > que;
            fill(dist, dist + V, INF);
            dist[s] = 0;
            que.push(P(0, s));
            while(!que.empty()){
                P p = que.top(); que.pop();
                int v = p.second;
                if(dist[v] < p.first) continue;
                for(int i = 0; i < G[v].size(); i++){
                    edge &e = G[v][i];
                    if(e.cap > 0 && dist[e.to] > dist[v] +
                       ↪e.cost + h[v] - h[e.to]){
                        dist[e.to] = dist[v] + e.cost + h[v
                           ↪] - h[e.to];
                        prevv[e.to] = v;
                        preve[e.to] = i;
                        que.push(P(dist[e.to], e.to));
                    }
                }
            }
            if(dist[t] == INF) return -1;
            for(int v = 0; v < V; v++) h[v] += dist[v];

            int d = f;
            for(int v = t; v != s; v = prevv[v]){
                d = min(d, G[prevv[v]][preve[v]].cap);
            }
            f -= d;
            res += d * h[t];
            for(int v = t; v != s; v = prevv[v]){
                edge &e = G[prevv[v]][preve[v]];
                e.cap -= d;
                G[v][e.rev].cap += d;
            }
        }
        return res;
    }

} mf; // hash-cpp-all = e682f2794d1ab1c0a614ea4aa32a2425
```

6.2 DFS algorithms

SCC.h

Description: Finds strongly connected components in a directed graph. If vertices u, v belong to the same component, we can reach u from v and vice versa.
Usage: `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.
Time: $\mathcal{O}(E+V)$

24 lines

```
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F f) {
  int low = val[j] = ++Time, x; z.push_back(j);
  trav(e,g[j]) if (comp[e] < 0)
    low = min(low, val[e] ?: dfs(e,g,f));

  if (low == val[j]) {
    do {
      x = z.back(); z.pop_back();
      comp[x] = ncomps;
      cont.push_back(x);
    } while (x != j);
    f(cont); cont.clear();
    ncomps++;
  }
  return val[j] = low;
}
template<class G, class F> void scc(G& g, F f) {
  int n = sz(g);
  val.assign(n, 0); comp.assign(n, -1);
  Time = ncomps = 0;
  rep(i,0,n) if (comp[i] < 0) dfs(i, g, f);
} // hash-cpp-all = 2c7a153ddd31436517cf3ad28efa4ac5
```

BiconnectedComponents.h

Time: $\mathcal{O}(E+V)$

37 lines

```
int cur, num[maxn], low[maxn];
int sz;
vector<int> com[maxn];
void tarjan(int u, int last) {
    num[u] = low[u] = ++cur;
    st.push(u); // vertex
    for(auto tmp : way[u]) {
        int v = tmp.first, id = tmp.second;
        if(v == last) continue; // if(id == last)
        if(!num[v]) {
            // st.push(id);
            tarjan(v, u); // tarjan(v, id)
            low[u] = min(low[u], low[v]);
            /* if(low[v] >= num[u]) {
                sz++;
                while(1) {
                    int x = st.top(); st.pop();
                    com[sz].push_back(x);
                    if(x == id) break;
                }
            } */
        }
        else low[u] = min(low[u], num[v]);
        /* else if(num[v] < num[u]) {
            st.push(id);
            low[u] = min(low[u], num[v]);
```

```
        } */
    }
    if(num[u] == low[u]) { // vertex
        sz++;
        while(1) {
            int x = st.top(); st.pop();
            com[sz].push_back(x);
            if(x == u) break;
        }
    }
} // hash-cpp-all = 3eaf49bccded1e3dca97f788e75bea4b
```

2sat.h

Description: Calculates a valid assignment to boolean variables a, b, c,... to a 2-SAT problem, so that an expression of the type $(a||b)\&\&(!a||c)\&\&(d||!b)\&\&\ldots$ becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions (~x).
Usage: `TwoSat ts(number of boolean variables);`
`ts.either(0, ~3); // Var 0 is true or var 3 is false`
`ts.set_value(2); // Var 2 is true`
`ts.at_most_one({0,~1,2}); // <= 1 of vars 0, ~1 and 2 are true`
`ts.solve(); // Returns true iff it is solvable`
`ts.values[0..N-1] holds the assigned values to the vars`
Time: $\mathcal{O}(N+E)$, where N is the number of boolean variables, and E is the number of clauses.

57 lines

```
struct TwoSat {
  int N;
  vector<vi> gr;
  vi values; // 0 = false, 1 = true

  TwoSat(int n = 0) : N(n), gr(2*n) {}

  int add_var() { // (optional)
    gr.emplace_back();
    gr.emplace_back();
    return N++;
  }

  void either(int f, int j) {
    f = max(2*f, -1-2*f);
    j = max(2*j, -1-2*j);
    gr[f^1].push_back(j);
    gr[j^1].push_back(f);
  }
  void set_value(int x) { either(x, x); }

  void at_most_one(const vi& li) { // (optional)
    if (sz(li) <= 1) return;
    int cur = ~li[0];
    rep(i,2,sz(li)) {
      int next = add_var();
      either(cur, ~li[i]);
      either(cur, next);
      either(~li[i], next);
      cur = ~next;
    }
    either(cur, ~li[1]);
  }

  vi val, comp, z; int time = 0;
  int dfs(int i) {
    int low = val[i] = ++time, x; z.push_back(i);
    trav(e, gr[i]) if (!comp[e])
      low = min(low, val[e] ?: dfs(e));
    ++time;
    if (low == val[i]) do {
      x = z.back(); z.pop_back();
      comp[x] = time;
      if (values[x>>1] == -1)
        values[x>>1] = !(x&1);
    } while (x != i);
    return val[i] = low;
  }

  bool solve() {
    values.assign(N, -1);
    val.assign(2*N, 0); comp = val;
    rep(i,0,2*N) if (!comp[i]) dfs(i);
    rep(i,0,N) if (comp[2*i] == comp[2*i+1]) return 0;
    return 1;
  }
}; // hash-cpp-all = 288fb44b52e9016a30ce849e38390eb9
```

6.3 Trees

TreePower.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.
Time: construction $\mathcal{O}(N \log N)$, queries $\mathcal{O}(\log N)$

25 lines

```
vector<vi> treeJump(vi& P){
  int on = 1, d = 1;
  while(on < sz(P)) on *= 2, d++;
  vector<vi> jmp(d, P);
  rep(i,1,d) rep(j,0,sz(P))
    jmp[i][j] = jmp[i-1][jmp[i-1][j]];
  return jmp;
}

int jmp(vector<vi>& tbl, int nod, int steps){
  rep(i,0,sz(tbl))
    if(steps&(1<<i)) nod = tbl[i][nod];
  return nod;
}

int lca(vector<vi>& tbl, vi& depth, int a, int b) {
  if (depth[a] < depth[b]) swap(a, b);
  a = jmp(tbl, a, depth[a] - depth[b]);
  if (a == b) return a;
  for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
  }
  return tbl[0][a];
} // hash-cpp-all = bfce856c17ac46dca77ea0f43aaa359a
```

LCA.h

Description: Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected. Can also find the distance between two nodes.
Usage: `LCA lca(undirGraph);`
`lca.query(firstNode, secondNode);`
`lca.distance(firstNode, secondNode);`
Time: $\mathcal{O}(N \log N + Q)$

"../data-structures/RMQ.h" 37 lines

```
typedef vector<pii> vpi;
typedef vector<vpi> graph;

struct LCA {
  vi time;
  vector<ll> dist;
  RMQ<pii> rmq;

  LCA(graph& C) : time(sz(C), -99), dist(sz(C)), rmq(dfs(C)
    ↪) {}

  vpi dfs(graph& C) {
    vector<tuple<int, int, int, ll>> q(1);
    vpi ret;
    int T = 0, v, p, d; ll di;
    while (!q.empty()) {
      tie(v, p, d, di) = q.back();
      q.pop_back();
      if (d) ret.emplace_back(d, p);
      time[v] = T++;
      dist[v] = di;
      trav(e, C[v]) if (e.first != p)
        q.emplace_back(e.first, v, d+1, di + e.second);
    }
    return ret;
  }

  int query(int a, int b) {
    if (a == b) return a;
    a = time[a], b = time[b];
    return rmq.query(min(a, b), max(a, b)).second;
  }
  ll distance(int a, int b) {
    int lca = query(a, b);
    return dist[a] + dist[b] - 2 * dist[lca];
  }
}; // hash-cpp-all = aa0d4d6df33671856cc22ac596e7df06
```

CompressTree.h

Time: $\mathcal{O}(|S| \log |S|)$

23 lines

```
void build(vector<int> vec) {
    for(auto u : vir) // reset edge
    sort(vec.begin(),vec.end(),[](int u,int v) {return st[u
      ↪]<st[v];});
    int sz = 0; vector<int> stk(n+1);
    stk[++sz] = 1;
    for(auto u : vec) {
        int x = lca(u, stk[sz]);
        vir.push_back(u); vir.push_back(x);
        if(u == stk[sz]) continue;
        if(x != stk[sz]) {
            while(sz >= 2 && h[stk[sz-1]] >= h[x]) {
                add_edge(stk[sz-1], stk[sz]);
                sz--;
            }
            if(x != stk[sz]) {
                add_edge(x, stk[sz]);
                stk[sz] = x;
            }
        }
        stk[++sz] = u;
    }
    for(int i=1;i<=sz-1;i++) add_edge(stk[i], stk[i+1]);
} // hash-cpp-all = c9b8802cc4872ba0b7f8c56a031d4e2c
```

HLD.h

Description: Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most log(n) light edges. The function of the HLD can be changed by modifying T, LOW and f. f is assumed to be associative and commutative.

Usage: `HLD hld(G);`
`hld.update(index, value);`
`tie(value, lca) = hld.query(n1, n2);`

"../data-structures/SegmentTree.h" 93 lines

```
typedef vector<pii> vpi;

struct Node {
  int d, par, val, chain = -1, pos = -1;
};

struct Chain {
  int par, val;
  vector<int> nodes;
  Tree tree;
};

struct HLD {
  typedef int T;
  const T LOW = -(1<<29);
  void f(T& a, T b) { a = max(a, b); }

  vector<Node> V;
  vector<Chain> C;

  HLD(vector<vpi>& g) : V(sz(g)) {
    dfs(0, -1, g, 0);
    trav(c, C) {
      c.tree = {sz(c.nodes), 0};
      for (int ni : c.nodes)
        c.tree.update(V[ni].pos, V[ni].val);
    }
  }

  void update(int node, T val) {
    Node& n = V[node]; n.val = val;
    if (n.chain != -1) C[n.chain].tree.update(n.pos, val);
  }

  int pard(Node& nod) {
    if (nod.par == -1) return -1;
    return V[nod.chain == -1 ? nod.par : C[nod.chain].par].
      ↪d;
  }

  // query all *edges* between n1, n2
  pair<T, int> query(int i1, int i2) {
    T ans = LOW;
    while(i1 != i2) {
      Node n1 = V[i1], n2 = V[i2];
      if (n1.chain != -1 && n1.chain == n2.chain) {
        int lo = n1.pos, hi = n2.pos;
        if (lo > hi) swap(lo, hi);
        f(ans, C[n1.chain].tree.query(lo, hi));
        i1 = i2 = C[n1.chain].nodes[hi];
      } else {
        if (pard(n1) < pard(n2))
          n1 = n2, swap(i1, i2);
        if (n1.chain == -1)
          f(ans, n1.val), i1 = n1.par;
        else {
          Chain& c = C[n1.chain];
          f(ans, n1.pos ? c.tree.query(n1.pos, sz(c.nodes))
                        : c.tree.s[1]);
          i1 = c.par;
        }
      }
    }
    return make_pair(ans, i1);
  }

  // query all *nodes* between n1, n2
  pair<T, int> query2(int i1, int i2) {
    pair<T, int> ans = query(i1, i2);
    f(ans.first, V[ans.second].val);
    return ans;
  }

  pii dfs(int at, int par, vector<vpi>& g, int d) {
    V[at].d = d; V[at].par = par;
    int sum = 1, ch, nod, sz;
    tuple<int,int,int> mx(-1,-1,-1);
    trav(e, g[at]){
      if (e.first == par) continue;
      tie(sz, ch) = dfs(e.first, at, g, d+1);
      V[e.first].val = e.second;
      sum += sz;
      mx = max(mx, make_tuple(sz, e.first, ch));
    }
    tie(sz, nod, ch) = mx;
    if (2*sz < sum) return pii(sum, -1);
    if (ch == -1) { ch = sz(C); C.emplace_back(); }
    V[nod].pos = sz(C[ch].nodes);
    V[nod].chain = ch;
    C[ch].par = at;
    C[ch].nodes.push_back(nod);
    return pii(sum, ch);
  }
}; // hash-cpp-all = d952a915dfa34f93fbf32fe2755a651e
```

LinkCutTree.h

Description: Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.
Time: All operations take amortized $\mathcal{O}(\log N)$.

90 lines

```
struct Node { // Splay tree. Root's pp contains tree's
  ↪parent.
  Node *p = 0, *pp = 0, *c[2];
  bool flip = 0;
  Node() { c[0] = c[1] = 0; fix(); }
  void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
  }
  void push_flip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
  }
  int up() { return p ? p->c[1] == this : -1; }
  void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y :
      ↪ x;
    if ((y->p = p)) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
      x->c[h] = y->c[h ^ 1];
      z->c[h ^ 1] = b ? x : this;
    }
    y->c[i ^ 1] = b ? this : x;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
  }
  void splay() {
    for (push_flip(); p; ) {
      if (p->p) p->p->push_flip();
      p->push_flip(); push_flip();
      int c1 = up(), c2 = p->up();
      if (c2 == -1) p->rot(c1, 2);
      else p->p->rot(c2, c1 != c2);
    }
  }
  Node* first() {
    push_flip();
    return c[0] ? c[0]->first() : (splay(), this);
  }
};

struct LinkCut {
  vector<Node> node;
  LinkCut(int N) : node(N) {}

  void link(int u, int v) { // add an edge (u, v)
    assert(!connected(u, v));
    make_root(&node[u]);
    node[u].pp = &node[v];
  }
  void cut(int u, int v) { // remove an edge (u, v)
    Node *x = &node[u], *top = &node[v];
    make_root(top); x->splay();
    assert(top == (x->pp ?: x->c[0]));
    if (x->pp) x->pp = 0;
    else {
      x->c[0] = top->p = 0;
      x->fix();
    }
  }
  bool connected(int u, int v) { // are u, v in the same
    ↪tree?
    Node* nu = access(&node[u])->first();
    return nu == access(&node[v])->first();
  }
  void make_root(Node* u) {
    access(u);
    u->splay();
    if(u->c[0]) {
      u->c[0]->p = 0;
      u->c[0]->flip ^= 1;
      u->c[0]->pp = u;
      u->c[0] = 0;
      u->fix();
    }
  }
  Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
      pp->splay(); u->pp = 0;
      if (pp->c[1]) {
        pp->c[1]->p = 0; pp->c[1]->pp = pp; }
      pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
  }
}; // hash-cpp-all = 693483a825b93f7ad5f6ef219ba83e22
```

Centroid.cpp

28 lines

```
set<int> G[N];
int depth[N], par[N], sub[N];

int dfs1(int v, int p){
    sub[v] = 1;
    for(int nxt : G[v])
        if(nxt != p)
            sub[v] += dfs1(nxt, v);
    return sub[v];
}

int dfs2(int v, int p, int nn){
    for(int nxt : G[v]){
        if(nxt != p && sub[nxt] > nn/2) return dfs2(nxt, v,
            ↪ nn);
    }
    return v;
}

void decompose(int v, int p){
    dfs1(v, -1);
    int centroid = dfs2(v, -1, sub[v]);
    par[centroid] = p;
    for(int nxt : G[centroid]){
        G[nxt].erase(centroid);
        decompose(nxt, centroid);
    }
    G[centroid].clear();
} // hash-cpp-all = e5955539d9903a4299626c2df9ddc081
```

MatrixTree.h

Description: To count the number of spanning trees in an undirected graph G: create an $N \times N$ matrix `mat`, and for each edge $(a, b) \in G$, do `mat[a][a]++, mat[b][b]++, mat[a][b]--, mat[b][a]--`. Remove the last row and column, and take the determinant.

1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

6.4 Other

DSUWithRollbacks.cpp

50 lines

```
class DSU {
public:
    int n,com;
    int head[maxn], sz[maxn];
    stack<pii> stk;

    void init(int _n) {
        n = com = _n;
        for(int i=0;i<n;i++) {
            head[i] = i;
            sz[i] = 1;
        }
    }

    int findhead(int x) {
        if(x==head[x]) return x;
        return findhead(head[x]);
    }

    void add_edge(int u, int v) {
        int x = findhead(u), y = findhead(v);
        if(sz[x] > sz[y]) swap(x,y);
        stk.push({x,y});
        if(x!=y) {
            head[x] = y;
            sz[y] += sz[x];
            com--;
        }
    }

    void pop_edge() {
        int x = stk.top().X, y = stk.top().Y; stk.pop();
        if(x!=y) {
            head[x] = x;
            sz[y] -= sz[x];
            com++;
        }
    }

    int size() {
        return com;
    }
} dsu;

int main() {
    dsu.init(n);
    dsu.add_edge(u, v);
    dsu.pop_edge();
    ans = dsu.size();
} // hash-cpp-all = 6d15e7a74bae4c327fa48b036cc18f15
```

directed-MST.cpp

Description: Finds the minimum spanning arborescence from the root. (any more notes?)

73 lines

```
#define rep(i, n) for (int i = 0; i < n; i++)

#define N 110000
#define M 110000
#define inf 2000000000

struct edg {
    int u, v;
    int cost;
} E[M], E_copy[M];

int In[N], ID[N], vis[N], pre[N];

// edges pointed from root.
int Directed_MST(int root, int NV, int NE) {
  for (int i = 0; i < NE; i++)
    E_copy[i] = E[i];
    int ret = 0;
    int u, v;
    while (true) {
        rep(i, NV)   In[i] = inf;
        rep(i, NE) {
            u = E_copy[i].u;
            v = E_copy[i].v;
            if(E_copy[i].cost < In[v] && u != v) {
                In[v] = E_copy[i].cost;
                pre[v] = u;
            }
        }
        rep(i, NV) {
            if(i == root)   continue;
            if(In[i] == inf)    return -1; // no solution
        }

        int cnt = 0;
        rep(i, NV) {
          ID[i] = -1;
          vis[i] = -1;
        }
        In[root] = 0;

        rep(i, NV) {
            ret += In[i];
            int v = i;
            while(vis[v] != i && ID[v] == -1 && v != root)
                ↪{
                vis[v] = i;
                v = pre[v];
            }
            if(v != root && ID[v] == -1) {
                for(u = pre[v]; u != v; u = pre[u]) {
                    ID[u] = cnt;
                }
                ID[v] = cnt++;
            }
        }
        if(cnt == 0)    break;
        rep(i, NV) {
            if(ID[i] == -1) ID[i] = cnt++;
        }
        rep(i, NE) {
            v = E_copy[i].v;
            E_copy[i].u = ID[E_copy[i].u];
            E_copy[i].v = ID[E_copy[i].v];
            if(E_copy[i].u != E_copy[i].v) {
                E_copy[i].cost -= In[v];
            }
        }
        NV = cnt;
        root = ID[root];
    }
    return ret;
}
// hash-cpp-all = 84815c2bfececf3575ecf663c0703643
```

graph-dominator-tree.cpp

Description: Dominator Tree.

89 lines

```
struct Dominator{
    struct min_DSU{
        vector<int> par, val;
        vector<int> const&semi;
        min_DSU(int N, vector<int> const&semi):par(N, -1),
            ↪val(N), semi(semi){
            iota(val.begin(), val.end(), 0);
        }
        void comp(int x){
            if(par[par[x]]!=-1){
                comp(par[x]);
                if(semi[val[par[x]]]<semi[val[x]])
                    val[x] = val[par[x]];
                par[x]=par[par[x]];
            }
        }
        int f(int x){
            if(par[x]==-1) return x;
            comp(x);
            return val[x];
        }
        void link(int x, int p){
            par[x] = p;
        }
```

```
    };
    int N;
    vector<vector<int>> G, rG;
    vector<int> idom, order;
    Dominator(int _N):N(_N), G(N), rG(N){}
    void add_edge(int a, int b){
        G[a].emplace_back(b);
        rG[b].emplace_back(a);
    }
    vector<int> calc_dominators(int S){
        idom.assign(N, -1);
        vector<int> par(N, -1), semi(N, -1);
        vector<vector<int> > bu(N);
        stack<int> s;
        s.emplace(S);
        while(!s.empty()){
            int a=s.top();s.pop();
            if(semi[a]==-1){
                semi[a] = order.size();
                order.emplace_back(a);
                for(int i=0;i<(int)G[a].size();++i){
                    if(semi[G[a][i]]==-1){
                        par[G[a][i]]=a;
                        s.push(G[a][i]);
                    }
                }
            }
        }
        min_DSU uni(N, semi);
        for(int i=(int)order.size()-1;i>0;--i){
            int w=order[i];
            for(int f:rG[w]){
                int oval = semi[uni.f(f)];
                if(oval>=0 && semi[w]>oval) semi[w] = oval;
            }
            bu[order[semi[w]]].push_back(w);
            uni.link(w, par[w]);
            while(!bu[par[w]].empty()){
                int v = bu[par[w]].back(); bu[par[w]].
                    ↪pop_back();
                int u=uni.f(v);
                idom[v] = semi[u] < semi[v] ? u : par[w];
            }
        }
        for(int i=1;i<(int)order.size();++i){
            int w=order[i];
            if(idom[w] != order[semi[w]])
                idom[w] = idom[idom[w]];
        }
        idom[S]=-1;
        return idom;
    }
};

int main() {
    Dominator dom(3);
    add_edge(0,1); // 0 index
    add_edge(1,2);
    add_edge(0,2);
    vector<int> par = dom.calc_dominators(0); // input root
    // par[i] is dominator of i
    for(int i=0;i<n;i++) sz[i] = 1;
    for(int i=(int)dom.order.size()-1;i>=0;i--) { //
       ↪iterate with order
        int u = dom.order[i];
        sz[par[u]] += sz[u];
    }
} // hash-cpp-all = 7370cb493d52704785b9037314418d51
```

Geometry (7)

7.1 Geometric primitives

Point.h

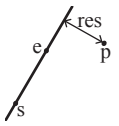
Description: Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

28 lines

```
template <class T> int sgn(T x) { return (x > 0) - (x < 0);
   ↪ }
template<class T>
struct Point {
  typedef Point P;
  T x, y;
  explicit Point(T x=0, T y=0) : x(x), y(y) {}
  bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y
     ↪); }
  bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y
     ↪); }
  P operator+(P p) const { return P(x+p.x, y+p.y); }
  P operator-(P p) const { return P(x-p.x, y-p.y); }
  P operator*(T d) const { return P(x*d, y*d); }
  P operator/(T d) const { return P(x/d, y/d); }
  T dot(P p) const { return x*p.x + y*p.y; }
  T cross(P p) const { return x*p.y - y*p.x; }
  T cross(P a, P b) const { return (a-*this).cross(b-*this)
     ↪; }
  T dist2() const { return x*x + y*y; }
  double dist() const { return sqrt((double)dist2()); }
  // angle to x-axis in interval [-pi, pi]
  double angle() const { return atan2(y, x); }
  P unit() const { return *this/dist(); } // makes dist()=1
  P perp() const { return P(-y, x); } // rotates +90
     ↪degrees
  P normal() const { return perp().unit(); }
  // returns point rotated 'a' radians ccw around the
     ↪origin
  P rotate(double a) const {
    return P(x*cos(a)-y*sin(a),x*sin(a)+y*cos(a)); }
  friend ostream& operator<<(ostream& os, P p) {
    return os << "(" << p.x << "," << p.y << ")"; }
}; // hash-cpp-all = 47ec0abfb6b604da9f744979e71bada0
```

lineDistance.h

Description:
Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

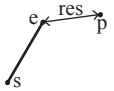


"Point.h" 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
  return (double)(b-a).cross(p-a)/(b-a).dist();
} // hash-cpp-all = f6bf6b556d99b09f42b86d28d1eaa86d
```

SegmentDistance.h

Description:
Returns the shortest distance between point p and the line segment from point s to e.



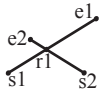
Usage:
```
Point<double> a, b(2,2), p(1,1);
bool onSegment = segDist(a,b,p) < 1e-10;
```

"Point.h" 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
  if (s==e) return (p-s).dist();
  auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)))
     ↪;
  return ((p-s)*d-(e-s)*t).dist()/d;
} // hash-cpp-all = 5c88f46fb14a05a4f47bbd23b8a9c427
```

SegmentIntersection.h

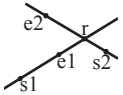
Description:
If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.



Usage:
```
vector<P> inter = segInter(s1,e1,s2,e2);
if (sz(inter)==1)
cout << "segments intersect at " << inter[0] << endl;
```
"Point.h", "OnSegment.h" 13 lines
```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
  auto oa = c.cross(d, a), ob = c.cross(d, b),
       oc = a.cross(b, c), od = a.cross(b, d);
  // Checks if intersection is single non-endpoint point.
  if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
    return {(a * ob - b * oa) / (ob - oa)};
  set<P> s;
  if (onSegment(c, d, a)) s.insert(a);
  if (onSegment(c, d, b)) s.insert(b);
  if (onSegment(a, b, c)) s.insert(c);
  if (onSegment(a, b, d)) s.insert(d);
  return {all(s)};
} // hash-cpp-all = 9d57f2f844788770022fbcd8bfc5b2f2
```

lineIntersection.h

Description:
If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



Usage:
```
auto res = lineInter(s1,e1,s2,e2);
if (res.first == 1)
cout << "intersection point at " << res.second << endl;
```
"Point.h" 8 lines
```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
  auto d = (e1 - s1).cross(e2 - s2);
  if (d == 0) // if parallel
    return {-(s1.cross(e1, s2) == 0), P(0, 0)};
  auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
  return {1, (s1 * p + e1 * q) / d};
} // hash-cpp-all = a01f815e2e60161e03879264c4826dd0
```

sideOf.h

Description: Returns where p is as seen from s towards e. 1/0/-1 $\Leftrightarrow$ left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

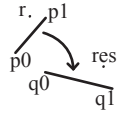
Usage:
```
bool left = sideOf(p1,p2,q)==1;
```
"Point.h" 9 lines
```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps)
    ↪{
  auto a = (e-s).cross(p-s);
  double l = (e-s).dist()*eps;
  return (a > l) - (a < -l);
} // hash-cpp-all = 3af81cc4f24d9d9fb109d930f3b9764c
```

OnSegment.h

Description: Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.
"Point.h" 3 lines
```
template<class P> bool onSegment(P s, P e, P p) {
  return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
} // hash-cpp-all = c597e8749250f940e4b0139f0dc3e8b9
```

linearTransformation.h

Description:
Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.



"Point.h" 6 lines
```
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
    const P& q0, const P& q1, const P& r) {
  P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
  return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.
    ↪dist2();
} // hash-cpp-all = 03a3061b3ef024b4e29ea06169932b21
```

Angle.h

Description: A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

Usage:
```
vector<Angle> v = {w[0], w[0].t360() ...}; // sorted
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }
// sweeps j such that (j-i) represents the number of
positively oriented triangles with vertices at 0 and i
```
35 lines
```
struct Angle {
  int x, y;
  int t;
  Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
  Angle operator-(Angle b) const { return {x-b.x, y-b.y, t
    ↪}; }
  int half() const {
    assert(x || y);
    return y < 0 || (y == 0 && x < 0);
  }
  Angle t90() const { return {-y, x, t + (half() && x >= 0)
    ↪}; }
  Angle t180() const { return {-x, -y, t + half()}; }
  Angle t360() const { return {x, y, t + 1}; }
};
bool operator<(Angle a, Angle b) {
  // add a.dist2() and b.dist2() to also compare distances
  return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
         make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle
  ↪between
// them, i.e., the angle that covers the defined line
  ↪segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
  if (b < a) swap(a, b);
  return (b < a.t180() ?
          make_pair(a, b) : make_pair(b, a.t360()));
}
Angle operator+(Angle a, Angle b) { // point a + vector b
  Angle r(a.x + b.x, a.y + b.y, a.t);
  if (a.t180() < r) r.t--;
  return r.t180() < a ? r.t360() : r;
}
Angle angleDiff(Angle a, Angle b) { // angle b - angle a
  int tu = b.t - a.t; a.t = b.t;
  return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a
    ↪)};
} // hash-cpp-all = 0f0602c74320456a8a8627737c1d3c64
```

7.2 Circles

CircleIntersection.h

Description: Computes the pair of points at which two circles intersect. Returns false in case of no intersection.
"Point.h" 11 lines
```
typedef Point<double> P;
bool circleInter(P a,P b,double r1,double r2,pair<P, P>*
    ↪out) {
  if (a == b) { assert(r1 != r2); return false; }
  P vec = b - a;
  double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
         p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*
           ↪d2;
  if (sum*sum < d2 || dif*dif > d2) return false;
  P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) /
    ↪d2);
  *out = {mid + per, mid - per};
  return true;
} // hash-cpp-all = 84d6d345ef48c336b811d1a54deda11d
```

CircleTangents.h

Description: Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.
"Point.h" 13 lines
```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double
    ↪r2) {
  P d = c2 - c1;
  double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
  if (d2 == 0 || h2 < 0)  return {};
  vector<pair<P, P>> out;
  for (double sign : {-1, 1}) {
    P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
    out.push_back({c1 + v * r1, c2 + v * r2});
  }
  if (h2 == 0) out.pop_back();
  return out;
} // hash-cpp-all = b0153d0ef1b8a6b1fa4d91480c4126e8
```

CirclePolygonIntersection.h

Description: Returns the area of the intersection of a circle with a ccw polygon.

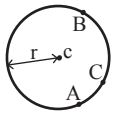
Time: $\mathcal{O}(n)$
"../../content/geometry/Point.h" 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
  auto tri = [&](P p, P q) {
    auto r2 = r * r / 2;
    P d = q - p;
    auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.
      ↪dist2();
    auto det = a * a - b;
    if (det <= 0) return arg(p, q) * r2;
    auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det
      ↪));
    if (t < 0 || 1 <= s) return arg(p, q) * r2;
    P u = p + d * s, v = p + d * t;
    return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
  };
  auto sum = 0.0;
  rep(i,0,sz(ps))
    sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
  return sum;
} // hash-cpp-all = a1ee63d6260cce862c677eb37a67efe5
```

circumcircle.h

Description:

The circumcirle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.



"Point.h" 9 lines

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
  return (B-A).dist()*(C-B).dist()*(A-C).dist()/
      abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
  P b = C-A, c = B-A;
  return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
} // hash-cpp-all = 1caa3aea364671cb961900d4811f0282
```

MinimumEnclosingCircle.h

Description: Computes the minimum circle that encloses a set of points.

Time: expected $\mathcal{O}(n)$

"circumcircle.h" 17 lines

```
pair<P, double> mec(vector<P> ps) {
  shuffle(all(ps), mt19937(time(0)));
  P o = ps[0];
  double r = 0, EPS = 1 + 1e-8;
  rep(i,0,sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
    o = ps[i], r = 0;
    rep(j,0,i) if ((o - ps[j]).dist() > r * EPS) {
      o = (ps[i] + ps[j]) / 2;
      r = (o - ps[i]).dist();
      rep(k,0,j) if ((o - ps[k]).dist() > r * EPS) {
        o = ccCenter(ps[i], ps[j], ps[k]);
        r = (o - ps[i]).dist();
      }
    }
  }
  return {o, r};
} // hash-cpp-all = 09dd0aa4515bb46bac3171f71d032132
```

7.3 Polygons

InsidePolygon.h

Description: Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

Usage: `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`
`bool in = inPolygon(v, P{3, 3}, false);`

Time: $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h" 11 lines

```
template<class P>
bool inPolygon(vector<P> &p, P a, bool strict = true) {
  int cnt = 0, n = sz(p);
  rep(i,0,n) {
    P q = p[(i + 1) % n];
    if (onSegment(p[i], q, a)) return !strict;
    //or: if (segDist(p[i], q, a) <= eps) return !strict;
    cnt ^= ((a.y<p[i].y) - (a.y<q.y)) * a.cross(p[i], q) >
      ↪0;
  }
  return cnt;
} // hash-cpp-all = 2bf504baea895948c3c332b0b474bd98
```

PolygonArea.h

Description: Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h" 6 lines

```
template<class T>
T polygonArea2(vector<Point<T>>& v) {
  T a = v.back().cross(v[0]);
  rep(i,0,sz(v)-1) a += v[i].cross(v[i+1]);
  return a;
} // hash-cpp-all = f123003799a972c1292eb0d8af7e37da
```

PolygonCenter.h

Description: Returns the center of mass for a polygon.

Time: $\mathcal{O}(n)$

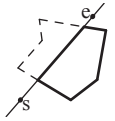
"Point.h" 9 lines

```
typedef Point<double> P;
P polygonCenter(const vector<P>& v) {
  P res(0, 0); double A = 0;
  for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
    res = res + (v[i] + v[j]) * v[j].cross(v[i]);
    A += v[j].cross(v[i]);
  }
  return res / A / 3;
} // hash-cpp-all = 9706dcc8eb8dae007d9a12070a93b128
```

PolygonCut.h

Description:

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.



Usage: `vector<P> p = ...;`
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h", "lineIntersection.h" 13 lines

```
typedef Point<double> P;
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
  vector<P> res;
  rep(i,0,sz(poly)) {
    P cur = poly[i], prev = i ? poly[i-1] : poly.back();
    bool side = s.cross(e, cur) < 0;
    if (side != (s.cross(e, prev) < 0))
      res.push_back(lineInter(s, e, cur, prev).second);
    if (side)
      res.push_back(cur);
  }
  return res;
} // hash-cpp-all = f2b7d494a6a577ade19ef0e9eed7f049
```

ConvexHull.h

Description:

Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

Time: $\mathcal{O}(n \log n)$

"Point.h" 13 lines

```
typedef Point<ll> P;
vector<P> convexHull(vector<P> pts) {
  if (sz(pts) <= 1) return pts;
  sort(all(pts));
  vector<P> h(sz(pts)+1);
  int s = 0, t = 0;
  for (int it = 2; it--; s = --t, reverse(all(pts)))
    trav(p, pts) {
      while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t
        ↪--;
      h[t++] = p;
    }
  return {h.begin(), h.begin() + t - (t == 2 && h[0] == h
    ↪[1])};
} // hash-cpp-all = 26a0a95e23f2183fa044ccdff1257cf8
```

HullDiameter.h

Description: Returns the two points with max distance on a convex hull (ccw, no duplicate/colinear points).

12 lines

```
typedef Point<ll> P;
array<P, 2> hullDiameter(vector<P> S) {
  int n = sz(S), j = n < 2 ? 0 : 1;
  pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
  rep(i,0,j)
    for (;; j = (j + 1) % n) {
      res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}})
        ↪;
      if ((S[(j + 1) % n] - S[j]).cross(S[i + 1] - S[i]) >=
        ↪ 0)
        break;
    }
  return res.second;
} // hash-cpp-all = c571b8edf5b751f996dc80283d32a92c
```

PointInsideHull.h

Description: Determine whether a point t lies inside a convex hull (CCW order, with no colinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

Time: $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h" 14 lines

```
typedef Point<ll> P;

bool inHull(const vector<P>& l, P p, bool strict = true) {
  int a = 1, b = sz(l) - 1, r = !strict;
  if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
  if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
  if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p)<=
    ↪ -r)
    return false;
  while (abs(a - b) > 1) {
    int c = (a + b) / 2;
```

```
    (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
  }
  return sgn(l[a].cross(l[b], p)) < r;
} // hash-cpp-all = 71446bda4e9e17eb03867d22daa5808e
```

LineHullIntersection.h

Description: Line-convex polygon intersection. The polygon must be ccw and have no colinear points. lineHull(line, poly) returns a pair describing the intersection of a line with the polygon: • $(-1,-1)$ if no collision, • $(i,-1)$ if touching the corner i, • (i,i) if along side $(i,i+1)$, • (i,j) if crossing sides $(i,i+1)$ and $(j,j+1)$. In the last case, if a corner i is crossed, this is treated as happening on side $(i,i+1)$. The points are returned in the same order as the line hits the polygon. extrVertex returns the point of a hull with the max projection onto a line.
Time: $\mathcal{O}(N+Q\log n)$

"Point.h" 39 lines

```
typedef array<P, 2> Line;
#define cmp(i,j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%
  ↪n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
int extrVertex(vector<P>& poly, P dir) {
  int n = sz(poly), lo = 0, hi = n;
  if (extr(0)) return 0;
  while (lo + 1 < hi) {
    int m = (lo + hi) / 2;
    if (extr(m)) return m;
    int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
    (ls < ms || (ls == ms && ls == cmp(lo, m)) ? hi : lo) =
      ↪ m;
  }
  return lo;
}

#define cmpL(i) sgn(line[0].cross(poly[i], line[1]))
array<int, 2> lineHull(Line line, vector<P> poly) {
  int endA = extrVertex(poly, (line[0] - line[1]).perp());
  int endB = extrVertex(poly, (line[1] - line[0]).perp());
  if (cmpL(endA) < 0 || cmpL(endB) > 0)
    return {-1, -1};
  array<int, 2> res;
  rep(i,0,2) {
    int lo = endB, hi = endA, n = sz(poly);
    while ((lo + 1) % n != hi) {
      int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
      (cmpL(m) == cmpL(endB) ? lo : hi) = m;
    }
    res[i] = (lo + !cmpL(hi)) % n;
    swap(endA, endB);
  }
  if (res[0] == res[1]) return {res[0], -1};
  if (!cmpL(res[0]) && !cmpL(res[1]))
    switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
      case 0: return {res[0], res[0]};
      case 2: return {res[1], res[1]};
    }
  return res;
} // hash-cpp-all = 758f2248384a18f71c84cf3d63a93ed7
```

7.4 Misc. Point Set Problems

ClosestPair.h

Description: Finds the closest pair of points.
Time: $\mathcal{O}(n\log n)$

"Point.h" 17 lines

```
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
  assert(sz(v) > 1);
  set<P> S;
  sort(all(v), [](P a, P b) { return a.y < b.y; });
  pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
  int j = 0;
  trav(p, v) {
    P d{1 + (ll)sqrt(ret.first), 0};
    while (v[j].y <= p.y - d.x) S.erase(v[j++]);
    auto lo = S.lower_bound(p - d), hi = S.upper_bound(p +
      ↪d);
    for (; lo != hi; ++lo)
      ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    S.insert(p);
  }
  return ret.second;
} // hash-cpp-all = d31bbf685a10a34219ab9bab86ee486e
```

Voronoi.h

"ConvexHull.h", "lineIntersection.h" 47 lines

```
vector<vector<P> > voronoi(const vector<P>& inp)
{
  vector<P> poly = inp;
  vector<vector<P>> res;
  vector<PP> inv;
  vector<pair<P,P>> ans;
  vector<PP> vor;
  vector<P> V;
  int n = poly.size();
  poly.push_back(P());
  poly.push_back(P());
  poly.push_back(P());
  poly.push_back(P());
  for (int i=0; i<n; i++)
  {
    poly[n] = P(-2*oo-poly[i].x,poly[i].y);
    poly[n+1] = (P(poly[i].x,2*oo-poly[i].y));
    poly[n+2] = (P(2*oo-poly[i].x,poly[i].y));
    poly[n+3] = (P(poly[i].x,-2*oo-poly[i].y));
    inv.clear();
    for (int j=0; j<n+4; j++)
    if (j!=i)
    {
      //int jj = j - (j>i);
      double tmp = (poly[j]-poly[i]).dist2();
      inv.push_back(make_pair((poly[j]-poly[i])/tmp,j));
      //cerr << inv[inv.size()-1].first << '\n';
    }
    vor = convexHull(inv);
    vor.push_back(*vor.begin());
    int vors = vor.size();
    ans.clear();
    for (int j=0; j<vors; j++)
    {
      P tmp = (poly[i]+poly[vor[j].second])/2.0;
      ans.push_back(make_pair(tmp,P(tmp.x-(poly[vor[j].
        ↪second].y-poly[i].y),tmp.y+(poly[vor[j].second].
        ↪x-poly[i].x))));
    }
    V.clear();
    for (int j=1; j<vors; j++)
    {
      pair<int,P> v = lineInter(ans[j-1].first,ans[j-1].
        ↪second,ans[j].first,ans[j].second);
      V.push_back(v.second);
    }
    res.push_back(V);
  }
  return res;
} // hash-cpp-all = 019d2df5df3a15722956d5875d4e3c82
```

7.5 3D

PolyhedronVolume.h

Description: Magic formula for the volume of a polyhedron. Faces should point outwards.

6 lines

```
template<class V, class L>
double signed_poly_volume(const V& p, const L& trilist) {
  double v = 0;
  trav(i, trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
  return v / 6;
} // hash-cpp-all = 1ec4d393ab307cedc3866534eaa83a0e
```

Point3D.h

Description: Class to handle points in 3D space. T can be e.g. double or long long.

32 lines

```
template<class T> struct Point3D {
  typedef Point3D P;
  typedef const P& R;
  T x, y, z;
  explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z)
    ↪{}
  bool operator<(R p) const {
    return tie(x, y, z) < tie(p.x, p.y, p.z); }
  bool operator==(R p) const {
    return tie(x, y, z) == tie(p.x, p.y, p.z); }
  P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
  P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
  P operator*(T d) const { return P(x*d, y*d, z*d); }
  P operator/(T d) const { return P(x/d, y/d, z/d); }
  T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
  P cross(R p) const {
    return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
  }
  T dist2() const { return x*x + y*y + z*z; }
  double dist() const { return sqrt((double)dist2()); }
  //Azimuthal angle (longitude) to x-axis in interval [-pi,
    ↪ pi]
  double phi() const { return atan2(y, x); }
  //Zenith angle (latitude) to the z-axis in interval [0,
    ↪pi]
  double theta() const { return atan2(sqrt(x*x+y*y),z); }
  P unit() const { return *this/(T)dist(); } //makes dist()
    ↪=1
  //returns unit vector normal to *this and p
  P normal(P p) const { return cross(p).unit(); }
  //returns point rotated 'angle' radians ccw around axis
  P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit
      ↪();
    return u*dot(u)*(1-c) + (*this)*c - cross(u)*s;
  }
}; // hash-cpp-all = 8058aeda36daf3cba079c7bb0b43dcea
```

3dHull.h

Description: Computes all faces of the 3-dimension hull of a point set. *No four points must be coplanar*, or else random results will be returned. All faces will point outwards.
Time: $\mathcal{O}(n^2)$

"Point3D.h" 49 lines

```
typedef Point3D<double> P3;
```

```
struct PR {
  void ins(int x) { (a == -1 ? a : b) = x; }
  void rem(int x) { (a == x ? a : b) = -1; }
  int cnt() { return (a != -1) + (b != -1); }
  int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
  assert(sz(A) >= 4);
  vector<vector<PR>> E(sz(A), vector<PR>(sz(A), {-1, -1}));
#define E(x,y) E[f.x][f.y]
  vector<F> FS;
  auto mf = [&](int i, int j, int k, int l) {
    P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
    if (q.dot(A[l]) > q.dot(A[i]))
      q = q * -1;
    F f{q, i, j, k};
    E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
    FS.push_back(f);
  };
  rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
    mf(i, j, k, 6 - i - j - k);

  rep(i,4,sz(A)) {
    rep(j,0,sz(FS)) {
      F f = FS[j];
      if(f.q.dot(A[i]) > f.q.dot(A[f.a])) {
        E(a,b).rem(f.c);
        E(a,c).rem(f.b);
        E(b,c).rem(f.a);
        swap(FS[j--], FS.back());
        FS.pop_back();
      }
    }
    int nw = sz(FS);
    rep(j,0,nw) {
      F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f
    ↪.c);
      C(a, b, c); C(a, c, b); C(b, c, a);
    }
  }
  trav(it, FS) if ((A[it.b] - A[it.a]).cross(
    A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
  return FS;
}; // hash-cpp-all = c172e9f2cb6b44ceca0c416fee81f1dc
```

sphericalDistance.h

Description: Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 (ϕ_1) and f2 (ϕ_2) from x axis and zenith angles (latitude) t1 (θ_1) and t2 (θ_2) from z axis. All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx*radius is then the difference between the two points in the x direction and d*radius is the total distance between the points.

8 lines

```
double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
  double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
  double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
  double dz = cos(t2) - cos(t1);
  double d = sqrt(dx*dx + dy*dy + dz*dz);
  return radius*2*asin(d/2);
} // hash-cpp-all = 611f0797307c583c66413c2dd5b3ba28
```

Strings (8)

AhoCorasick.cpp

Description: Init, and insert strings, and then build.

<bits/stdc++.h> 49 lines

```
const int M = (int)5e5 + 500;

struct Trie{
    static const int B = 26;

    int next[M][B], fail[M], end[M];
    int root, L;
    int newnode(){ // hash-cpp-1
        for(int i = 0; i < B; i++) next[L][i] = -1;
        end[L++] = 0;
        return L - 1;
    }
    // Please do initalize it !!!
    void init(){
        L = 0;
        root = newnode();
    }
    void insert(const string &buf){
        int len = buf.length();
        int now = root;
        for(int i = 0; i < len; i++){
            if(next[now][buf[i]-'a'] == -1) next[now][buf[i
            ↪]-'a'] = newnode();
            now = next[now][buf[i]-'a'];
        }
        end[now] ++;
    }
    void build(){
        queue<int> Q;
        fail[root] = root;
        for(int i = 0; i < B; i++){
            if(next[root][i] == -1) next[root][i] = root;
            else{
                fail[next[root][i]] = root;
                Q.push(next[root][i]);
            }
        }
        while(!Q.empty()){
            int now = Q.front();
            Q.pop();
            for(int i = 0; i < B; i++){
                if(next[now][i] == -1) next[now][i] = next[
                ↪fail[now]][i];
                else{
                    fail[next[now][i]] = next[fail[now]][i
                    ↪];
                    Q.push(next[now][i]);
                }
            }
        }
    } // hash-cpp-1 = 825cf85504458ad07d9a0021d74cf1a6
} Aho;
```

KMP.cpp

24 lines

```
string s, t;
int f[M];

void getnext(){
    int m = t.length();
    f[0] = 0; f[1] = 0;
    for(int i = 1; i < m; i++){
        int j = f[i];
        while(j && t[i] != t[j]) j = f[j];
        f[i+1] = t[i] == t[j] ? j + 1 : 0;
    }
}

int find(){
    int n = s.length(), m = t.length();
    int res = 0;
    int j = 0;
    for(int i = 0; i < n; i++){
        while(j && t[j] != s[i]) j = f[j];
        if(t[j] == s[i]) j++;
        if(j == m) res ++, j = f[j];
    }
    return res;
} // hash-cpp-all = 78587018eee67364b798a07b53e4f004
```

Manacher.cpp

42 lines

```
struct Manacher {
    string s, sn;
    int p[2*N];

    int Init() {
        int len = s.length();
        sn = "$#";
        int j = 2;

        for (int i = 0; i < len; i++)
        {
            sn.push_back(s[i]);
            sn.push_back('#');
        }

        sn.push_back('\0');

        return sn.length();
    }

    int run() {
        int len = Init();
        int max_len = -1;

        int id = 0;
        int mx = 0;

        for (int i = 1; i < len; i++)
        {
            if (i < mx) p[i] = min(p[2 * id - i], mx - i);
            else p[i] = 1;

            while (sn[i - p[i]] == sn[i + p[i]]) p[i]++;

            if (mx < i + p[i]) id = i, mx = i + p[i];

            max_len = max(max_len, p[i] - 1);
        }
        return max_len;
    }
} mnc;
// hash-cpp-all = d0dff75e997690b1592e4b36c6e9aefc
```

PolynomialHashing.cpp

<bits/stdc++.h> 62 lines

```
typedef long long ll;
typedef pair<int, int> P;
const int mods[4] = {(int)1e9 + 7, (int)1e9 + 9, (int)1e9 +
    ↪ 21, (int)1e9 + 33};
const int N = (int)2e5 + 50;

string s, t;
int p = 37;
ll pw[4][N];

struct hs {
    ll val[4];

    hs() { fill(val, val + 4, 0); }

    hs(ll a, ll b, ll c, ll d) {
        val[0] = a, val[1] = b, val[2] = c, val[3] = d;
    }

    bool operator<(const hs &other) const {
        for (int i = 0; i < 4; i++) if (val[i] != other.val
            ↪[i]) return val[i] < other.val[i];
        return false;
    }

    bool operator==(const hs &other) const {
        for (int i = 0; i < 4; i++) if (val[i] != other.val
            ↪[i]) return false;
        return true;
    }

    hs operator + (const hs &other) const{
        hs res;
        for(int i = 0; i < 4; i++) res.val[i] = ( val[i] +
            ↪other.val[i]) % mods[i];
        return res;
    }

    hs operator - (const hs &other) const{
        hs res;
        for(int i = 0; i < 4; i++) res.val[i] = (val[i] -
            ↪other.val[i] + mods[i]) % mods[i];
        return res;
    }

    hs operator ^ (const int pwi) const {
        hs res;
        for(int i = 0; i < 4; i++){
            res.val[i] = (val[i] * pw[i][pwi]) % mods[i];
        }
        return res;
    }

    void add(int x, int pwi){
        for(int i = 0; i < 4; i++) {
            val[i] = (val[i] + x * pw[i][pwi]) % mods[i];
            if(val[i] < 0) val[i] += mods[i];
        }
    }
};

void init() {
    for(int t = 0; t < 4; t++){
        pw[t][0] = 1;
        for(int i = 1; i < N; i++) pw[t][i] = pw[t][i-1] *
            ↪p % mods[t];
    }
} // hash-cpp-all = 442e4ad85b64b40bdb09978cbb6d3154
```

SAM.cpp

94 lines

```
struct state {
    int len, link;
    int next[B];
};

struct SAM {
    state st[MAXLEN * 2];
    int sz, last;
    int cnt[MAXLEN * 2];
    int anc[LOGN][MAXLEN * 2];

    void sam_init() { // hash-cpp-1
        st[0].len = 0;
        st[0].link = -1;
        memset(st[0].next, -1, sizeof(st[0].next));
        sz = 1;
        last = 0;
    } // hash-cpp-1 = 88458c9c46d0594815f25aa78c9c9a6f

    int sam_extend(int c, int nlast, int val) { // hash-cpp
        ↪-2
        last = nlast;
        int cur = sz++;
        cnt[cur] = val;
        st[cur].len = st[last].len + 1;
        memset(st[cur].next, -1, sizeof(st[cur].next));
        int p = last;
        while(p != -1 && st[p].next[c] == -1) {
            st[p].next[c] = cur;
            p = st[p].link;
        }
        if(p == -1) {
            st[cur].link = 0;
        } else {
            int q = st[p].next[c];
            if(st[p].len + 1 == st[q].len) {
                st[cur].link = q;
            } else {
                int clone = sz++;
                st[clone].len = st[p].len + 1;
                memcpy(st[clone].next, st[q].next, sizeof(
                    ↪st[q].next));
                st[clone].link = st[q].link;
                while(p != -1 && st[p].next[c] == q) {
                    st[p].next[c] = clone;
                    p = st[p].link;
                }
                st[q].link = st[cur].link = clone;
            }
        }
        last = cur;
        return last;
    } // hash-cpp-2 = a955371ff99c2fb0631535b05a18b044
} sam;

int n, m;
vector<int> tid[N];

struct Trie {
    int nxt[MAXLEN][B];
    int sz;
    int id[MAXLEN];

    void init() { // hash-cpp-3
        sz = 1;
        memset(nxt, -1, sizeof(nxt));
    } // hash-cpp-3 = 83a0e44461d7b8d82a1566fc38a1133c

    void add(string s, int idx) { // hash-cpp-4
        int cur = 0;
        for(char c : s) {
            int &nx = nxt[cur][c - 'a'];
            if(nx == -1) nx = sz++;
            cur = nx;
            tid[idx].push_back(cur);
        }
    } // hash-cpp-4 = 85a0d801ac069f16dbb1ce998394eebd

    void build_sam() { // hash-cpp-5
        sam.sam_init();
        queue<int> que;
        id[0] = 0;
        que.push(0);
        while(!que.empty()) {
            int v = que.front(); que.pop();
            for(int i = 0; i < B; i++) {
                if(nxt[v][i] != -1) {
                    id[nxt[v][i]] = sam.sam_extend(i, id[v
                        ↪], 1);
                    que.push(nxt[v][i]);
                }
            }
        }
        sam.build();
    } // hash-cpp-5 = 4366d5d3fee156021eaad3147606f8ff

} trie;
```

PAM.cpp

<bits/stdc++.h> 56 lines

```
const int mod = (int)1e9 + 7;

struct PAM {
    static const int N = (int) 1e6 + 50;
    int s[N], now;
    int nxt[N][26], fail[N], l[N], last, tot;
    int diff[N], anc[N];
    int ans[N], dp[N];

    void clear() { // hash-cpp-1
        s[0] = l[1] = -1;
        fail[0] = tot = now = 1;
        last = l[0] = 0;
        memset(nxt[0], -1, sizeof nxt[0]);
        memset(nxt[1], -1, sizeof nxt[1]);
    }

    PAM() { clear(); }

    int newnode(int len) {
        tot++;
        memset(nxt[tot], -1, sizeof nxt[tot]);
        fail[tot] = 0;
        l[tot] = len;
        return tot;
    }
```

```
    int get_fail(int x) {
        while (s[now - l[x] - 2] != s[now - 1])x = fail[x];
        return x;
    }

    void add(int ch) {
        s[now++] = ch;
        int cur = get_fail(last);
        if (nxt[cur][ch] == -1) {
            int tt = newnode(l[cur] + 2);
            fail[tt] = nxt[get_fail(fail[cur])][ch];
            if(fail[tt] == -1) fail[tt] = 0;
            nxt[cur][ch] = tt;
            diff[tt] = l[tt] - l[fail[tt]];
            anc[tt] = diff[tt] == diff[fail[tt]] ? anc[fail
               ↪[tt]] : fail[tt];
        }
        last = nxt[cur][ch];
    } // hash-cpp-1 = e98af9fdbd034392cf7c2342f36b059a

    void trans(int i) { // hash-cpp-2
        for(int p = last; p > 1; p = anc[p]) {
            dp[p] = ans[i - l[anc[p]] - diff[p]];
            if(diff[p] == diff[fail[p]]) {
                (dp[p] += dp[fail[p]]) %= mod;
            }
            (ans[i] += (i % 2 == 0) * dp[p]) %= mod;
        }
    } // hash-cpp-2 = 18f4e8a0c11c040383bfe58bbb139bce
} pam;
```

SuffixArray.cpp

Description: lcp[i] = lcp(sa[i], sa[i-1]). One indexed for everything.
Time: $\mathcal{O}(n \log n)$

25 lines

```
struct SuffixArray {
    vi sa, lcp, rk;
    SuffixArray(string& s, int lim=256) { // or
       ↪basic_string<int>
        int n = sz(s) + 1, k = 0, a, b;
        vi x(all(s)+1), y(n), ws(max(n, lim)), rank(n);
        sa = lcp = rk = y, iota(all(sa), 0);
        for (int j = 0, p = 0; p < n; j = max(1, j * 2),
           ↪lim = p) {
            p = j, iota(all(y), n - j);
            rep(i,0,n) if (sa[i] >= j) y[p++] = sa[i] - j;
            fill(all(ws), 0);
            rep(i,0,n) ws[x[i]]++;
            rep(i,1,lim) ws[i] += ws[i - 1];
            for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
            swap(x, y), p = 1, x[sa[0]] = 0;
            rep(i,1,n) a = sa[i - 1], b = sa[i], x[b] =
                    (y[a] == y[b] && y[a + j] == y[b + j])
                       ↪? p - 1 : p++;
        }
        rep(i,1,n) rank[sa[i]] = i;
        for (int i = 0, j; i < n - 1; lcp[rank[i++]] = k)
            for (k && k--, j = sa[rank[i] - 1];
                    s[i + k] == s[j + k]; k++);
        s = " " + s;
        for(int i = n - 1; i >= 1; i--) sa[i]++, rk[sa[i]]
           ↪= i;
    }
}; // hash-cpp-all = 01cdb651bc80da4ec63d9cc01d2cb14c
```

ZAlgorithm.cpp

Description: str = "aabaacd", z = (x, 1, 0, 2, 1, 0, 0)

<bits/stdc++.h> 18 lines

```
const int MAXN = (int)1e6 + 500;

string s;
int z[MAXN],cnt[MAXN];

void getZarr(string str) // hash-cpp-1
{
    memset(z, 0, sizeof(z));
    int n = str.length();
    for(int i = 1, l = 0, r = 0; i < n; ++i){
        if(i <= r)
            z[i] = min(r - i + 1, z[i - l]);
        while(i + z[i] < n && str[z[i]] == str[i + z[i]])
            ++z[i];
        if(i + z[i] - 1 > r)
            l = i,  r = i + z[i] - 1;
    }
} // hash-cpp-1 = 6d61faecf306b78847022e6fa5f26ea4
```

DigitDP.cpp

Description: 1

<bits/stdc++.h> 26 lines

```
const int B = 20;

int dp[10000][B];
int bit[B], b;
int pw2[B];
int A, B;

int get(int rem, int d, bool flag){
    if(rem < 0) return 0;
    if(d == -1) return rem >= 0;
    if(!flag && ~dp[rem][d]) return dp[rem][d];
    int lim = flag ? bit[d] : 9;
    int res = 0;
    for(int i = 0; i <= lim; i++){
        res += get(rem - i * pw2[d], d - 1, flag && lim ==
           ↪i);
    }
    return flag ? res : dp[rem][d] = res;
}

int solve(int x){
    b = 0;
    int t = x;
    while(t > 0){bit[b++] = t % 10; t /= 10;}
    return get(A, b-1, true);
}
// hash-cpp-all = d99f0e66d42b9dded6b9e05ba7a0ea4f
```

MinRotation.h

Description: Finds the lexicographically smallest rotation of a string.
Usage: rotate(v.begin(), v.begin()+min_rotation(v), v.end());
Time: $\mathcal{O}(N)$

8 lines

```
int min_rotation(string s) {
  int a=0, N=sz(s); s += s;
  rep(b,0,N) rep(k,0,N) {
    if (a+k == b || s[a+k] < s[b+k]) {b += max(0, k-1);
       ↪break;}
    if (s[a+k] > s[b+k]) { a = b; break; }
  }
  return a;
} // hash-cpp-all = 4bd552f76bcc771f09f5602c0914673a
```

Various (9)

9.1 Misc. algorithms

Karatsuba.h

Description: Faster-than-naive convolution of two sequences: $c[x] = \sum a[i]b[x-i]$. Uses the identity $(aX+b)(cX+d) = acX^2 + bd + ((a+c)(b+d) - ac - bd)X$. Doesn't handle sequences of very different length well. See also FFT, under the Numerical chapter.
Time: $\mathcal{O}\left(N^{1.6}\right)$

1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

9.2 Dynamic programming

digitDP.cpp

Description: 1

<bits/stdc++.h> 25 lines

```
const int B = 20;

int dp[10000][B];
int bit[B], b;
int pw2[B];

int get(int rem, int d, bool flag){
    if(rem < 0) return 0;
    if(d == -1) return rem >= 0;
    if(!flag && ~dp[rem][d]) return dp[rem][d];
    int lim = flag ? bit[d] : 9;
    int res = 0;
    for(int i = 0; i <= lim; i++){
        res += get(rem - i * pw2[d], d - 1, flag && lim ==
           ↪i);
    }
    return flag ? res : dp[rem][d] = res;
}

int solve(int x){
    b = 0;
    int t = x;
    while(t > 0){bit[b++] = t % 10; t /= 10;}
    return get(A, b-1, true);
}
// hash-cpp-all = 562b39c346c3e127832d23b153971125
```

KnuthDP.h

Description: When doing DP on intervals: $a[i][j] = \min_{i<k<j}(a[i][k] + a[k][j]) + f(i,j)$, where the (minimal) optimal k increases with both i and j, one can solve intervals in increasing order of length, and search $k = p[i][j]$ for $a[i][j]$ only between $p[i][j-1]$ and $p[i+1][j]$. This is known as Knuth DP. Sufficient criteria for this are if $f(b,c) \le f(a,d)$ and $f(a,c) + f(b,d) \le f(a,d) + f(b,c)$ for all $a \le b \le c \le d$. Consider also: LineContainer (ch. Data structures), monotone queues, ternary search.
Time: $\mathcal{O}\left(N^2\right)$

1 lines

```
// hash-cpp-all = d41d8cd98f00b204e9800998ecf8427e
```

9.3 Debugging tricks

- `signal(SIGSEGV, [](int) { _Exit(0); });` converts segfaults into Wrong Answers. Similarly one can catch SIGABRT (assertion failures) and SIGFPE (zero divisions). `_GLIBCXX_DEBUG` violations generate SIGABRT (or SIGSEGV on gcc 5.4.0 apparently).
- `feenableexcept(29);` kills the program on NaNs (`1`), 0-divs (`4`), infinities (`8`) and denormals (`16`).

9.4 Optimization tricks

9.4.1 Bit hacks

- `x & -x` is the least bit in `x`.
- `for (int x = m; x; ) { --x &= m; ...` } loops over all subset masks of `m` (except `m` itself).
- `c = x&-x, r = x+c; (((r^x) >> 2)/c) | r` is the next number after `x` with the same number of bits set.
- `rep(b,0,K) rep(i,0,(1 << K)) if (i & 1 << b) D[i] += D[i^(1 << b)];` computes all sums of subsets.

9.4.2 Pragmas

- `#pragma GCC optimize ("Ofast")` will make GCC auto-vectorize for loops and optimizes floating points better (assumes associativity and turns off denormals).
- `#pragma GCC target ("avx,avx2")` can double performance of vectorized code, but causes crashes on old machines.
- `#pragma GCC optimize ("trapv")` kills the program on integer overflows (but is really slow).

9.5 Other languages

Main.java

Description: Basic template/info for Java 14 lines

```
import java.util.*;
import java.math.*;
import java.io.*;
public class Main {
  public static void main(String[] args) throws Exception {
    BufferedReader br = new BufferedReader(new
       ↪InputStreamReader(System.in));
    PrintStream out = System.out;
    StringTokenizer st = new StringTokenizer(br.readLine())
       ↪;
    assert st.hasMoreTokens(); // enable with java -ea main
    out.println("v=" + Integer.parseInt(st.nextToken()));
    ArrayList<Integer> a = new ArrayList<>();
    a.add(1234); a.get(0); a.remove(a.size()-1); a.clear();
  }
}
```