RESEARCH OF QUANTUM GENETIC ALGORITH AND ITS APPLICATION IN BLIND SOURCE SEPARATION[1]

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Abstract This letter proposes two algorithms: a novel Quantum Genetic Algorithm (QGA) based on the improvement of Han's Genetic Quantum Algorithm (GQA) and a new Blind Source Separation (BSS) method based on QGA and Independent Component Analysis (ICA). The simulation result shows that the efficiency of the new BSS method is obviously higher than that of the Conventional Genetic Algorithm (CGA).

Key words Quantum computation; Genetic algorithm; Quantum genetic algorithm; Independent component analysis, Blind source separation

I. Introduction

In the early 1980s, Benioff and Feynman proposed the concept of quantum computing[1,2]. The quantum computing, using the superposition, entanglement and coherence characters of quantum states, is likely to resolve the NP problem in classic computing. Since then, the quantum computing has attracted wide attention and soon become the hot topic of research, especially after Grover's random database search algorithm[3] and Shor's quantum prime factoring algorithms[4] were proposed.

In 2000, Han proposed the Genetic Quantum Algorithm (GQA)[5]. The algorithm used the quantum state vector description and the quantum rotation gate operation. Its main contribution is to apply qubit's probabilistic amplitude representation to the coding of the chromosome. However, since this algorithm is mainly used to resolve the 0-1 knapsack problem, it has low adaptability and its efficiency leaves much to be improved. In this letter, a novel Quantum Genetic Algorithm (QGA) is proposed based on the improvement of the GQA. By adopting multi-state gene coding method and general quantum rotation gate strategy, dynamic adjusting rotation angle mechanism, and introducing quantum crossover operation, QGA has achieved higher efficiency and applicability. And a new Blind Source Separation (BSS) method based on the QGA and Independent Component Analysis (ICA) is also proposed. Compared with the BSS method adopting the Conventional Genetic Algorithm (CGA)[6], the computing efficiency of QGA is obviously superior over CGA.

II. Quantum Genetic Algorithm

The QGA is based on the representation of the quantum state vector. It applies the probabilistic amplitude representation of qubit to the coding of chromosome, which makes one chromosome represent the superposition of many states, and uses quantum rotation gates to realize the update operation, to conquer the premature convergence by applying quantum crossover and eventually reaches the optimum resolution of the goal.

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1. Qubit encode

In quantum computer, the smallest information location is a two-state quantum system, called qubit. The difference between the qubit and the classic bit is that it can stay in the superposition of the two quantum states simultaneously, e.g.

$$|\psi>=\alpha|0>+\beta|1> \tag{1}$$

where α and β are complex numbers, satisfying

$$|\alpha|^2+|\beta|^2=1 \tag{2}$$

$|0>$ represents the state of spin up, while $|1>$ represents the state of spin down, so one qubit can represent two state information ($|0>$ and $|1>$) simultaneously.

In QGA, qubit is used to store and represent one gene. This gene may be in the '1' state, the '0' state, or any superposition of the two. That is to say, the information represented by this gene is not stable, but probable; therefore, when an operation is carried out on this gene, it may be done to all probable information simultaneously.

In the knapsack problem[5], because each gene has only two states ('0' and '1'), it is much easier. But in other problems, each gene may have multi-states, so the coding method in Ref.[5] is not general. The solution is to design a unitary transformation, which can be operated to multi-states simultaneously. This coding method is simple and efficient, but the computing is very complex. Moreover, the multi-dimensional unitary transform is very difficult to design. The other solution is to adopt the binary coding method in genetic algorithm to encode these qubits of multi-states, using one qubit to represent two states, two qubits to represent four states, etc. The latter method has better adaptability, and is easier to realize. In this letter, the multi-qubits are applied to represent the multi-state gene, as follows:

$$q_j^t=\left(\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}\alpha_{11}^t & \alpha_{12}^t & \cdots & \alpha_{1k}^t & \alpha_{21}^t & \alpha_{22}^t & \cdots & \alpha_{2k}^t & \alpha_{m1}^t & \alpha_{m2}^t & \cdots & \alpha_{mk}^t \\ \beta_{11}^t & \beta_{12}^t & \cdots & \beta_{1k}^t & \beta_{21}^t & \beta_{22}^t & \cdots & \beta_{2k}^t & \beta_{m1}^t & \beta_{m2}^t & \cdots & \beta_{mk}^t\end{array}\right) \tag{3}$$

where q_j^t represents the t-th generation and the j-th individual chromosome, k is the qubit number of every coding state, m is the state number in each chromosome.

The adoption of qubit coding enables one chromosome to represent the superposition of multi-states simultaneously, making the QGA superior in diversity to the classic genetic algorithm. Convergence can be also obtained with the qubit representation. As $|\alpha|^2$ or $|\beta|^2$ approaches to 0 or1, the qubit chromosome converges to one single state.

2. The structure of QGA

The structure of QGA is described as follows:

(1) initialize colony $Q(t_0)$;

(2) make $P(t_0)$ by measuring $Q(t_0)$ states;

(3) evaluate $P(t_0)$;

(4) store the best individual among $P(t_0)$ and its fitness;

(5) while (not termination condition) do

begin

{

$t=t+1$;

make $P(t)$ by measuring $Q(t)$ states;

evaluate $P(t)$;

update $Q(t)$ using quantum crossover and quantum gates $U(t)$, get son colony $Q(t+1)$;
store the best individual among $P(t)$ and its fitness;
}
end

The first step of QGA is to initialize $Q(t_0)$, and the entire chromosome in the population are initialized to $(1/\sqrt{2}, 1/\sqrt{2})$. It means that one qubit chromosome may represent the superposition of all possible states with the same probability.

$$\left|\psi_{q_j^0}\right\rangle = \sum_{k=1}^{2^m} \frac{1}{\sqrt{2^m}} |S_k\rangle \tag{4}$$

where S_k is the k-th state of this chromosome represented by a length m binary string $(x_1, x_2, \cdots, x_m)$, $x_i (i = 1, 2, \cdots, m)$ is either 0 or 1.

The second step is to measure all the individuals of the initial colony and obtain a group of definite solution $P(t) = (p_1^t, p_2^t, \cdots, p_n^t)$, where p_j^t is the j-th solution in the t-th generation (the j-th individual measurement), represented as a length m binary string, where every bit '0' or '1' is selected by the probability of the qubit ($|\alpha_{ij}^t|^2$ or $|\beta_{ij}^t|^2$, $i = 1, 2, \cdots, m$). The measuring procedure is as follows: Generate a random number r between 0 and 1. If $r > |\alpha_{ij}^t|^2$, the measuring result is 1; otherwise 0. Then evaluate the group of solution with its fitness, the initial best individual and its fitness among the binary solution $P(t)$ is then selected and stored as the aim of the later evolution.

In the "while" loop step, the solution of the generation is converged little by little to the optimum solution. In each iteration, get a group of solution $P(t)$ through measuring $Q(t)$, calculate the fitness of every solution, and then, according to the present evolutionary aim and the adjust strategy set beforehand, carry out the quantum crossover on the individuals of the generation, update them by applying the quantum gates to get $Q(t+1)$, store the update optimum solution and compare it with the present evolutionary aim. If the optimum solution is bigger than the evolutionary aim, the present evolutionary aim is replaced by the optimum solution; otherwise the present evolutionary aim remains unchanged.

3. Qubit crossover

In the genetic algorithm, the function of crossover is to realize the exchange of each individual's structural information. Through the crossover, the mode of low rank, short distance and high mean fitness can unite to generate high rank and high fitness individual. Ref.[5] says there is no need to do quantum crossover, for the chromosome's qubit representation has enough diversity. But it is probable that the result will become locally optimal without crossover, because every individual will update towards one aim. The combined crossover[7] among every individual, i.e. qubit crossover, is proposed in this letter by the use of the quantum coherent character. The concrete procedure is as follows:

(1) Sorting all the individuals by the random probability;

(2) Selecting the first gene of the first individual as the first gene of the new individual, selecting the second gene of its contiguous individual as the second gene of the same individual, and looping until the new individual has the same amount of the genes.

(3) Doing the others in the same way until the new colony has the same size.

4. Qubit rotation gate strategy

As the updating execution mechanism, the quantum gate $U(t)$ can be designed in compliance with the practical problems. Quantum rotation gate can be used according to the calculation character of QGA. The updating operation of the quantum gate is shown as follows:

$$\begin{pmatrix} \alpha_i' \\ \beta_i' \end{pmatrix} = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_i \end{pmatrix} \tag{6}$$

where (α_i, β_i) is the i-th qubit, θ_i is the rotation angle.

An updating strategy is designed to solve the knapsack problem in Ref.[5], but it is designed for a special task, thus it is not universal. In this letter, a general updating strategy irrelative with the concrete problem is proposed, as in Tab.1.

Tab.1 Rotation angle selection strategy

x_i	b_i	$f(x_i) > f(b_i)$	$\Delta\theta_i$	$s(\alpha_i, \beta_i)$			
				$\alpha_i \cdot \beta_i > 0$	$\alpha_i \cdot \beta_i < 0$	$\alpha_i = 0$	$\beta_i = 0$
0	0	False	0	–	–	–	–
0	0	True	0	–	–	–	–
0	1	False	δ	+1	−1	0	±1
0	1	True	δ	−1	+1	±1	0
1	0	False	δ	−1	+1	±1	0
1	0	True	δ	+1	−1	0	±1
1	1	False	0	–	–	–	–
1	1	True	0	–	–	–	–

Rotation angle $\theta_i = s(\alpha_i, \beta_i)\Delta\theta_i$, where $s(\alpha_i, \beta_i)$ and $\Delta\theta_i$ represent the direction and the value of the rotation, respectively. The parameters are shown in Tab.1. The updating strategy is to compare the fitness $f(x_i)$ of the current measured value of the individual q_j^t with the present evolutionary aim's fitness $f(b_i)$. If $f(x_i) > f(b_i)$, then adjust the qubit of the corresponding bit ($x_i \neq b_i$) to make the probability amplitude evolve toward the direction benefiting the appearance of x_i. On the contrary, if $f(x_i) < f(b_i)$, then adjust the qubit of the corresponding bit to make the probability amplitude evolve toward the direction benefiting the appearance of b_i.

In Tab.1, δ is the angle step of every updating. The value of δ has an effect on the convergence speed; if the value is too big, the solution may diverge or have a premature convergence to a local optimum. In this letter, we adopt the dynamic adjust strategy of δ, that is to say, set the value of δ between 0.1π and 0.005π by dynamic adjustment according to the difference of the genetic generations. The experimental results demonstrate that the convergence speed of the dynamic adjustment of δ is higher than that of the fixed rotation angle.

In this letter, the QGA is applied to BSS research. The results demonstrate that the effectiveness of the QGA is superior to that of the CGA when being used in BSS.

III. Blind Source Separation Algorithm Based on QGA

The principle of BSS algorithm is referred to Ref.[6]. The key to realize the BSS based on the QGA is: (1) the coding method of chromosomes' qubit; (2) the selection of fitness function; (3) the evolutionary mechanism of chromosomes by the quantum crossover and rotation gate. The concrete separation method is described as follows.

1. The selection of fitness function

Just the same as in Ref.[6], the sum of kurtosis's absolute value is used as the fitness function in this letter.

2. The formation of the initial colony

The initial colony composed of several separation matrices is generated. In this letter, in order to compare with the algorithm based on CGA in Ref.[6], the size of the initial group is assumed 6, 10, 20 matrices, respectively.

3. The qubit coding pattern of chromosome

We encode the separation matrix $\boldsymbol{W}$ with qubit. For example, we want to separate four source signals, and the separation matrix is a 4×4 matrix. In QGA, we adopt multi-qubits to encode the multi-state gene, and every number is represented with a binary value of 16 bit, so the chromosome is encoded as a 16 bit ×16 qubit code, as is shown in Fig.1. Each qubit in every chromosome is initialized with $1/\sqrt{2}$.

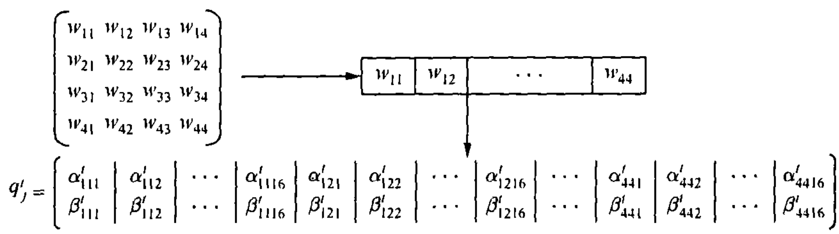

Fig.1 Multi-qubit coding pattern of chromosome

4. Evolutionary mechanism with quantum crossover and rotation gate

The CGA adopts the selection, crossover and mutation operations to make the colony approach to the global optimum solution. Because of the random character of the genetic operation, it is essential to have enough individuals and genetic operations among each individual to reach optimum. While in QGA, the qubit coding is a probability representation, every chromosome can represent the superposition of multi-states simultaneously, so it has enough diversity itself. The quantum rotation gate is employed to search the optimum solution and quantum crossover is used to utilize all chromosomes' information to avoid the premature convergence. So QGA not only possess rapid convergence, but also has good global search capability.

5. Initialization and the restrictions

The prerequisite of adopting the kurtosis as the criterion of the nongausssianity is the premise of zero-mean and the constraint $E\{\boldsymbol{YY}^T\} = \boldsymbol{I}$. In this letter, we adopt the same initialization and restrictions as in Ref.[6].

Procedure of the separation operation is as follows:

(1) Reading the source signal. We select the image and voice signal in this letter;

(2) Centering and whiting the signals. It is necessary for the signal pre-processing;

(3) Generating n separation matrices as initial individuals and encoding them with qubit, so as to get initial colony $Q(t_0)$;

(4) Measuring the initial colony. The procedure is: Make $P(t_0)$ by measuring $Q(t_0)$ states; convert $P(t_0)$ to the separation matrices; get the separated signals; center and white the separated signals; calculate the fitness function; find the optimum individual and its solution as the later evolutionary aim;

(5) Going to the loop step. Measure colony $Q(t)$ once again, get the solution $P(t)$, convert $P(t)$ to the separation matrices; get the separated signals; center and white the

separated signals; calculate the fitness function; perform the qubit crossover and quantum rotation gate as evolutionary method; find the optimum individual and its solution as the next evolutionary aim; go to the next loop;

(6) Getting the separation matrix from the best individual's solution. Separate the signals; center and white the separated signals; draw the images; go to end.

IV. Simulation Results and Analysis

In order to compare with the CGA in Ref.[6], we select the same signals as Ref.[6] in this letter. The run circumstance is PIII, 1.13GHz, 256MB SDRAM, the programming software is Matlab 5.3.

Tab.2 shows the comparison of the run time and fitness of the two algorithms. The generation is assumed as 200. The population size of CGA is 50, while the QGA's population size is selected as 6, 10, and 20, respectively. The table offers the mean value of the best fitness, the average fitness and the worst fitness within 200 generations over 20 runs and the elapsed time per generation.

Tab.2 Comparison of experimental results of CGA & QGA

	CGA	QGA		
Population size	50	6	10	20
The best fitness	8.2172	8.2162	8.2176	8.2178
The average fitness	8.2084	8.1705	8.2082	8.2096
The worst fitness	8.1395	8.0411	8.1386	8.1834
The elapsed time per generation(s)	72.2	4.8	7.6	14.4

Tab.2 shows that if we evaluate them by the best fitness, QGA with 6 individuals can reach the effect of CGA with 50 individuals, but QGA's elapsed time is only 1/15 of that of CGA; if we evaluate them by the mean fitness, QGA with 10 individuals can reach the effect of CGA with 50 individuals, but QGA's elapsed time is only 1/10 of that of CGA. When we use 20 individuals, the performance of QGA entirely exceeds CGA, with the elapsed time of only 1/5 of the latter.

Fig.2 shows the progress of the average fitness of the QGA and CGA. In the beginning, the performance of CGA is better than that of QGA due to its small population size. But with the increase of the number of generations, the convergence rate of QGA is faster than that of CGA. In particular, when we use 20 individuals, after 30 generations, the fitness of QGA is always better than that of CGA. Fig.3 shows the progress of the average fitness of QGA with 6 individuals using the fixed rotation angle and dynamic adjusting rotation angle. From the figure, we can see the superiority of the dynamic adjusting rotation angle.

Besides using the visible observation method to evaluate the separating effect, we can also adopt the SNR's and the PSNR's as quantitative criterion. QGA with 20 individuals after 50 generation operations, we select randomly one of the calculation results, the SNR of the separated signals are: PSNR(y_1)=69.76dB, PSNR(y_2)=69.48dB, SNR(y_3)=47.23dB, SNR(y_4)=49.79dB. In Ref.[6], the SNR of the separated signals are: PSNR(y_1)=62.16dB, PSNR(y_2)=65.24dB, SNR(y_3)=44.54dB, SNR(y_4)=46.27dB. The effect of separation is better than that of CGA.

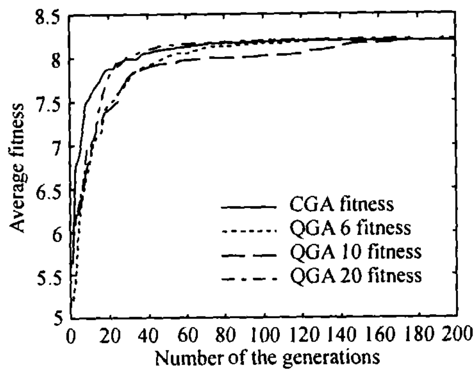

Fig.2 The progress of the average fitness of QGA and CGA

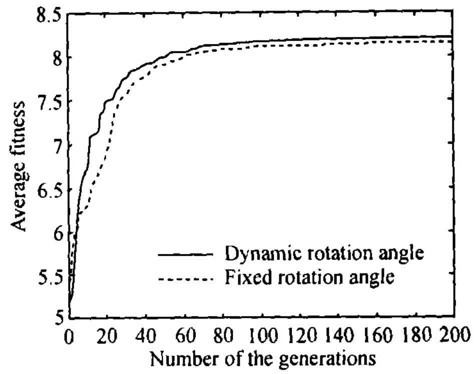

Fig.3 The progress of the average fitness of QGA with the fixed and dynamic rotation angle

V. Conclusions

This letter proposed a novel QGA based on the improvement of GQA. QGA adopts the multi-state gene qubit coding and quantum rotation gate strategy irrelative to the tasks, utilizes dynamic adjusting rotation angle mechanism, and introduces quantum crossover, make the algorithm more adaptable and efficient. By combining the QGA and ICA, we successfully resolve the BSS problem, and the result shows that the calculation efficiency of QGA is obviously better than that of CGA.

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