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# Introduction

# Experimental Setup

This chapter contains the general experimental setup for researches discussed in this dissertation. It introduces the setup for state excitation, laser cooling, pulse amplification, Tera Hertz (THz) pulse generation, data collection and etc. It also provides some suggestion for maintenance and daily operation. All experiments are performed on Newport RS 3000 optical tables to reduce mechanic vibrations, in a temperature controlled room to reduces external thermal fluctuation. Other than specifically mentioned, the repetition rate of all experiments is fifteen hertz. Before getting into experiments in the lab, people are supposed to have already taken the safety training.

## Magneto-Optical Trap

Since its invention in 1987 [1], Magneto-Optical Trap (MOT) has become a very important and popular technology in the area of atomic research and has been widely used to generate cold neutral atoms. This technology is a combination of magnetic field gradient and counter-propagating laser beams. Gradient magnetic field is used to generate position dependent energy levels of atoms so that atoms not in the trap center would absorb purposely prepared photons of fine-tuned lasers and be pushed back to the center by scattering photons in random directions.

Due to its ease of operation and low cost, MOT has been used to trap cold atoms in all the experiments described in this dissertation. We use as the atom source in all experiments. A MOT system contains high vacuum chambers, pump and repump lasers, pressure gauges, atomic beams generation and etc. Since Mary has already given a very good and detailed description about this system [3], a simplified version will be provided here.

### Principle of Magneto-Optical Trap

A simplified one-dimensional two-state system can help to understand the cooling process. As shown in Figure 2.1, suppose one atom has a spin S = 0 ground state and a spin S = 1 excited states. In the one-dimensional system with axis *z*, the weak inhomogeneous magnetic filed varies linearly so where *M* is a constant. Due to Zeeman effect , this field splits the degeneracy of the excited states. creating positon dependent energies for the atoms. In the system, a beam of light propagates in the direction and anther beam of light propagates in the counter direction. Both beams are red detuned from the zero-field resonance. On one hand, an atom with a position is more likely to absorb photons from to jump from ground spin state to state since state is closer to resonance. On the other hand, an atom with a position is more likely to absorb photons from the to jump from ground spin state to state. The atom absorbs a photon and is pushed back to the center. It will then scatter the absorbed photon. But as long as the field is weak, the scattering direction is random, which generates a net force of pushing back on the atom. So the atom will be cooled and confined in the MOT.

detuning

MOT

Figure .: Simplified one dimensional model for MOT [2].

As shown in Figure 2.2, a three dimensional Magneto-Optical Trap is similar to one dimensional model. The gradient magnetic field in a three dimensional MOT is provided by anti-Helmholtz coils. Other than the two main anti-Helmholtz coils as shown in this schematic, there are six more shim coils which enable fine tuning of the magnetic field inside the MOT. The six laser beams are from the same laser head. We use three beam splitters to split the main beam into six with equal strength. These three beams are then reflected by mirrors to generate six counter-propagating beams in total. Before reflected back, the beams polarization are rotated by waveplates placed in front of the mirrors.

Figure .: Schematic for MOT design. It’s a combination of anti-Helmholtz coils and six counter-propagating beams.

The above discussion gives an idea about trapping two-level atoms. When dealing with , there are more states involved, although the basic principal is as same as described above. As shown in Figure 2.3, there are complicated hyperfine levels involved in a realistic MOT. The ground state has been split into two hyperfine levels F = 2 and F = 3. The excited state has been split into four hyperfine levels F = 1, 2, 3, 4. Ideally, the trap laser intends to transfer atoms from F = 3 to F = 4. But because F = 3 and F = 4 are so close that the trap laser transfers a portion of atoms to F = 3. The atoms in F = 3 decay quickly back to F = 2 and escape from the MOT. To avoid such a loss, a second repump laser is introduced into the system. The repump laser transfers the atoms in F = 2 back to F = 3. These atoms can later decay back to F = 3 and be transferred again by the trap laser.

F=4

F=3

F=2

F=1

F=3

F=2

Trap 780.030 nm

Repump 780.024 nm

3 GHz

120 MHz

Figure .: Hyperfine energy structure of . Trap laser is driving transition from F = 3 to F = 4 and repump laser is driving transition from F = 2 to F = 3.

### Saturated Absorption Spectroscopy

To find the well defined frequency for the trap laser and repump laser, we utilize a method called Saturated Absorption Spectroscopy or SAS. The basic idea is this:

1. Split a small branch from the main beam of the laser. The function of this branch is to help find the right frequency. Pass it through a cell containing rubidium vapor. Call this branch “pump” beam. It’s strong enough to saturate the absorption along the path.
2. Reflect back the pump beam and pass it through the cell again. Call the coming back beam “probe” beam.
3. Detect the probe intensity using a photo detector. If the beam frequency is a little off the well defined frequency, due to Doppler effect, the atoms in the cell will absorb both pump and probe beams. If the beam frequency is right the well defined frequency, only zero-velocity atoms can absorb photons from the pump beam. The probe beam would not decrease its intensity since the pump beam has already saturated the absorption. Thus there will be an intensity increase of the probe beam when the frequency of the beam is the right Doppler-free frequency.

In the experiment, the real setup is a little more complicated than the above description. The beam passing through the cell has ben split further into two beams. One comes back as a probe and the other is detected directly. The detected signal from the probe is then subtracted by the signal from the other beam, which creates a push-pull configuration. In this way, we can get rid of the fluctuation of the main beam intensity and stabilize the absorption spectrum.

The right spectrum for trap and repump lasers are shown in Figure 2.4. It’s generated by sweeping the piezo voltage of the laser head with a tringle or sine wave. The pump beam from the trapping beam has been increased by 36 MHz using an acousto-optic modulator (AOM) before being sent to the cell. In this way, the main trapping beam sent into the MOT is detuned by 36 MHz.

Once the right spectrum pattern has been found, lock the lasers.

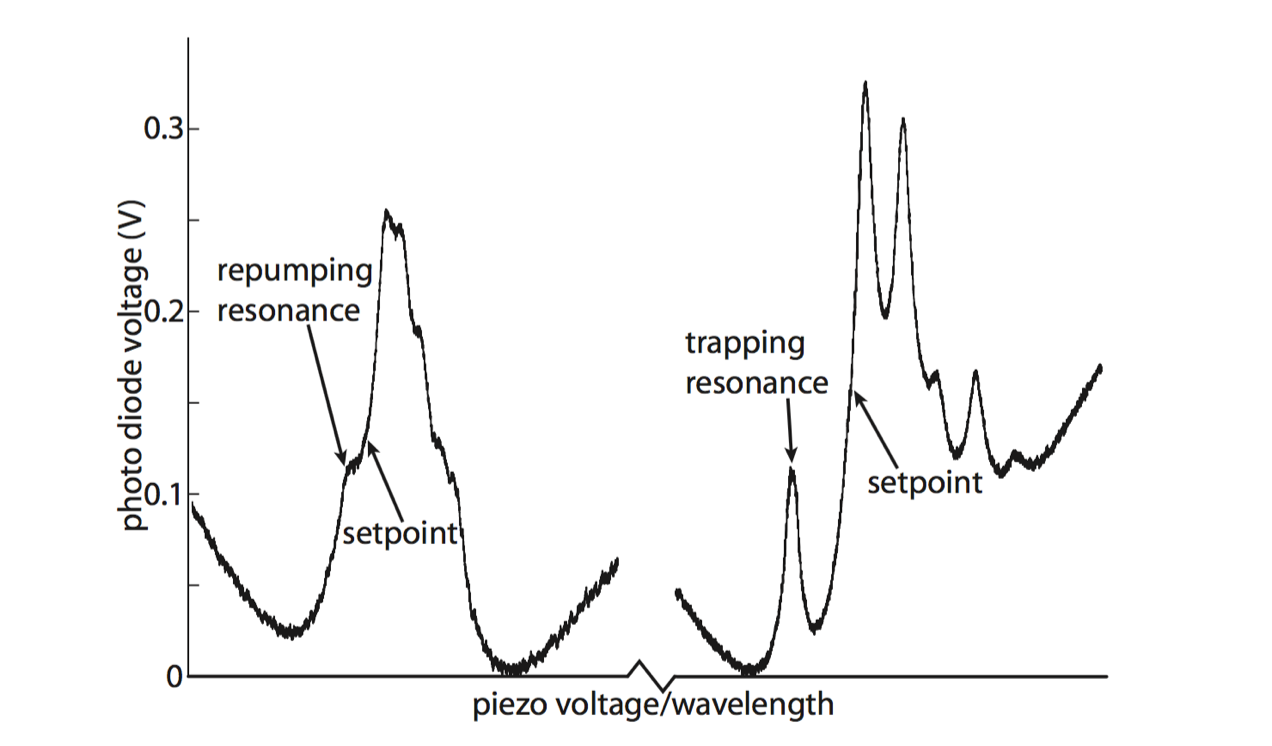


Figure .: Plots of the saturated absorption spectrum near the repumping and trapping resonances.[3]

### Ultra High Vacuum Chamber

The MOT is sitting inside an ultra high vacuum chamber. When pressure goes higher, because of the collision by high speed atoms, less atoms will stay in the trap. The usual operation pressure is between torr to torr. To achieve such a low pressure, a serial of pumps: rough pump, turbo pump and ion pump have been utilized. It does not need to use all of these pumps often, but if needed, Hyunwook has given a detailed description of the operation of these pumps for reference [4].

The pressure in the UHV chamber is measured by two types of gauges: thermocouple and Bayard-Alpert ionization gauges, both of which are monitored by Varian senTorr gauge. The thermocouple gauge measures pressure from to torr, while Varian senTor gauge can measure as low as torr. The ion pump can also display the pressure and we usually refer to that reading for daily operation.

### Performance of the MOT

The temperature of the atoms trapped in the MOT can go down as low as about 70 K. Mary discussed the details to measure the atom’s temperature and calibrate the density of atoms trapped in the MOT [3]. A brief introduction of the method is provided below.

Before getting temperature or density, we need to get the diameter of the atom cloud. This is done by producing a thin horizontal “film” of excitation laser beam which will excite all the atoms in the film from to and then to . The ion signal is correlated with the atoms distributed in the vertical direction. Using the same method and we can then find the atoms’ horizontal distribution, which should be the same as the vertical distribution if the MOT is ideally round. Using Gaussian curve to fit the distribution and we can get the diameter of the MOT. A typical MOT has a diameter from 0.4 to 1 mm.

The basic idea for temperature measurement is to turn off the trapping beam and observe the expansion of the atom cloud, since there is correlation between temperature and expansion rate according to Boltzmann distribution: .

The basic idea for measuring atom number in the could is to measure the total power scattered from the cloud or the fluorescence . The scattered power contributed by an individual atom, which decays from to once every two natural lifetime, is calculated by

( .)

where is the wavelength of the scattered photon and is the natural lifetime. Then we can get the total atom number:

. ( .)

## Lasers and Amplifiers

### Nd:YAG Lasers

Nd:YAG lasers are one kind of solid state lasers. The lasing medium in this laser is neodymium-doped yttrium aluminum garnet (). Such a medium is pumped by flash lamps and absorbs mostly in the bands between 730–760 nm and 790–820 nm [5]. It then emits light which mostly centered at 1064 nm. When the laser pulses fire is controlled by flash lamps and Q-switch [refer to Figure 2.6]. The infrared output is not very useful either for directly pumping dye laser or exciting atoms in our experiments, but it can be used to generate other frequencies. For the experiments described in later chapters, Potassium Dihydrogen Phosphate (KDP) crystals are used to generate 2nd or 3rd harmonics of the source frequency. 2nd and 3rd harmonic lasers are centered at 532 nm and 355 nm respectively. The green light at 532 nm is used to pump Regenerative amplifier, linear amplifier and dye laser. The ultraviolet light at 355 nm is commonly used to pump dye amplifiers.

pump

igure 2.5n Figure 2.5. ong the gain medium h. r and a polarizer, , not larger than the normal pump level. Dipole interaction.ump

1064 nm

Figure .: Schematic of Nd:YAG transition [6]. It is a typical Four-level Transition Scheme.



Figure .: Layout of Continuum Surelite Nd:YAG laser. It can output beams of 4 different frequencies.

One Nd:YAG laser combined with KDP can generated 2nd and 3rd harmonics at the same time. But some experiments require more 2nd and 3rd harmonics at different times. There are two Nd:YAG lasers used in the lab. One is Spectra-Physics GCR-100 Series. Its function is to generate 532 nm green light. This green light is the pump light for Regenerative Amplifier and Linear Amplifier, both of which will be discussed in following content. The other one is Continuum Surelite and it’s used to generate ultraviolet light. It’s used to pump dye lasers and dye amplifiers in the experiments.

### Diode Lasers

Diode lasers are lasers using a *p-n* junction or a *p-i-n* structure to generate gain. Semiconductor components are usually compact so diode lasers are commonly used in space-limited cases. Another advantage of diode lasers is that their output frequency is tunable. The cavity of a diode laser is controlling by a small grating in the diode laser head and the grating is usually attached to a piezo. By changing the voltage applied on the piezo, it is convenient to tune the output frequency. In the experiments, following diode lasers are used:

* Vortex tunable diode lasers from New Focus. Continuous Wave or CW laser. Typical output frequency is 780 nm and output power 40 mW. They are used as trap and repump lasers for the Magneto-Optical Trap.
* Millennia Vs diode laser from Spectra-Physics. CW laser. Typical output frequency is 532 nm and output power around 300 mW. It’s used as the pump of seed light.
* TA-SHG pro High Power Frequency-Doubled Tunable Diode Laser System. CW laser. Typical output frequency is 480 nm and output power 150 mW. It’s used to generate Rydberg excitation pulses.

### Mode Lock Lasers

Mode locked lasers are commonly used to generate ultra short laser pulses. A mode locked laser is a laser to which the technology of mode locking is applied. A bunch of different independent oscillations with different frequency components in a cavity could not compose a pulse, since there is no fixed phase between each other. But if the phase between each oscillation is fixed, these oscillations could generate intense bursts periodically or a train of pulses consistently. Such phase fixing process is the so called “mode locking” process, and there are two major ways to achieve the mode locking: active mode locking and passive mode locking.

The mode lock laser used in the experiments is model MTS mini Ti:Sapphire laser kit from Kapteyn-Muranen. It uses Kerr-lens mode locking technology which is one of passive mode locking technologies to mode lock laser. Its diagram is shown in Figure 2.6. When the CW pump beam going through the Ti:Sapphire crystal is not stable, because higher intensity light can pass the crystal easier than low intensity light, the cavity is in favor of high intensity light pulses. So the routine operation is to touch the 2nd prism to produce disturbance to generate pulses. The outcome are pulses of light as short as sub 15 femtoseconds at a repetition rate about 90 MHz. The output pulse spectrum is monitored using a spectrometer. If the output is not mode locked, it is a CW beam and the spectrum is a line with no bandwidth. For well mode locked pulses, the spectrum is very stable and has a bandwidth. The narrowness of the output pulses is enough for our experiment but the power is too small. To get narrow pulses with large enough power, we use the pulses from the mode lock laser as “seed light” and amplify them. The amplification process is discussed in later content.



Figure .: Basic layout of the mode lock laser [9]. Dashed line is the pump light from Millennia Vs diode laser and solid line is the oscillation in the cavity which is centered at 780-800 nm. By tapping the 2nd prism, we can produce a temporary unstable beam. Stronger intensity part in this beam will be enhanced thus produce pulsed outputs.

### Chirped Pulse Amplification

As mentioned above, the output from the mode locked laser has very short duration but its amplitude is not large enough for the experiments. So the output beam from mode lock laser or the so called “seed light” has to be amplified. This is achieved through a popular technology called “Chirped Pulse Amplification”. The basic idea is this:

1. Stretch the short pulses to a broad duration so that the peak energy is not very high. It’s easier to amplify low energy peaks than high energy peaks. As shown in Figure 2.7, the combination of reflecting mirrors and a grating in the stretcher acts as a pair of gratings and disperses the seed light’s spectrum. By stretching the seed light pulse, the energy in each pulse is much smaller and it’s much easier to amplify the pulse.
2. Amplify the stretched pulses using amplifiers such as regenerative amplifiers and linear amplifiers. How the regenerative amplifier and linear amplifier amplify the pulse is discussion in the following content.
3. Compress the amplified stretched pulses to high intensity short pulses back using a compressor. Compressor acts as an opponent of a stretcher, but it also utilizes a grating. In the experiments, the compressor is adjusted to find the best performance of the Tera Hertz generation.



initial short pulse

pulse stretcher

regenerative amplifier

linear amplifier

pulse compressor

Figure .: Schematic of chirped pulse amplification system. Seed light is at first stretched using stretcher. Then the stretched pulse gets amplified. At last the pulse is compressed to be very short pulse with high energy.

### Regenerative Amplifier

The first amplifier in the chirped pulse amplification is a regenerative amplifier. It use a solid-state medium Ti:Sapphire as the gain medium. Pulses are switched into the optical resonator by an optical switch realized with an electro-optical modulator and a polarizer, multiply pass through the gain medium in an optical resonator being amplified, and finally are switched out by another optical switch. This schematic is shown in Figure 2.8. The input beam has a vertical polarization to the paper surface and is reflected by the first polarizer to the switch-in pockels cell. When the switch-in pockels cell is triggered, it works as a quarter wave plate and rotates the beam’s polarization from vertical to horizontal before the beam comes back to the first polarizer. The beam with horizontal polarization goes through the first polarizer and comes into the gain medium to get amplified. After a few runs in the cavity (usually 5 to 6 runs) to gain maximum intensity, the beam will be switched out by the switch-out pockels cell with a vertical polarization.

switch-out pockels cell

gain medium

pump pulse

polarizer 1

switch-in pockels cell

polarizer 2

input pulse

output pulse

Figure .: Schematic of regenerative amplifier. Switch in pockels cell controls when the pulse comes into the resonator and switch out pockels cell controls when the pulse comes out.

### Linear Amplifier

Linear amplifier is used when the pulse intensity from the regenerative amplifier is still not large enough. A pulse also achieves the amplification by multiply passing through the gain medium Ti:Sapphire crystal, but it’s relatively simpler than regenerative amplifier. The highest output from linear amplifier in our lab is over 600 mw. Its structure is shown in Figure 2.9.

pump pulse

input pulse

output pulse

Figure .: Schematic of linear amplifier. Beam passes the gain medium multiple times and gets amplified.

### Dye Laser and Dye Amplifier

Different from Nd:YAG laser, which is a solid state laser, a dye laser is a laser which uses an organic dye as the lasing medium, usually as a liquid solution. Its advantage, compared to solid state lasers, is that it can be tuned for a much wider range of wavenumbers. The wide bandwidth makes it particularly suitable for tunable lasers and pulsed lasers. (At the same time, its disadvantage is the frequency instability.)

Organic dye is dissolved in solvent and circulated through a dye cell which is shot by pulsed pump light. When it’s excited by pump light, it fluoresces over a range of wavelengths. Certain wavelengths will be stimulated when the dye cell is placed in a cavity and thus a laser will be generated. By changing the cavity, the frequency which is to be stimulated, is tunable. This is the basic idea of dye lasers.

There are two main styles of dye lasers. One is Hansch-style and the other Littman-style. In our experiments, only Hansch-style dye laser [7] has been used for Rydberg excitations.



Nd:YAG light

tuning grating

telescope

dye cell

coupler

output

doubling crystal

Figure .: Schematic for a Hansch dye laser and 2nd harmonic generation. The angle of the tuning grating determines the output frequency.

This dye laser is used to generate 25*s* Rydberg atoms. The proper laser dye is LDS 925, which is dissolved in methanol solvent, with a concentration of 250 mg/L. This solution is pumped by 2nd harmonic from Continuum Surelite. The pump light has been focused about a millimeter into the dye cell by a cylindrical lens, creating a line of gain medium across the face of the cell. The dye cell works as a fluorescence generator, as well as an amplifier. The telescope expands the beam to reduce the intensity of light on the tuning grating. The grating is rotatable, which determines the frequency of the light diffracted back to the cavity. The light then comes back to the dye amplifier, being amplified and escapes from the cavity. Its infrared output laser is then frequency doubled to generate blue pulses which frequency is centered at 486 nm. For most of the time, a pulse from the dye laser does not only contains the frequency we want but a broad range of wavelengths. A typical line width for this kind of dye laser is on the order of 1 . To reduce the line width, we usually put a bandwidth filter or an etalon before sending the beam into the MOT chamber. In the chamber, the beam drives Rb atoms from 5*p* state to 25*s* state.

The dye cell can also work separately as an amplifier in Figure 2.11. This double amplifier can be used to amplify seed light from other lasers. We do not know exactly the output power of light coming from the dye amplifier, but we make sure the state transition is saturated by the amplified laser beam. If there is no observable reduce of the population on a state such as 25*s* when inserting a 20% beam reducer in the path, we are confident that the state transition is saturated and the power of the beam is large enough.

dye cell 1

dye cell 2

pump beam 1

pump beam 2

seed light

amplified light

Figure .: Schematic for double cell dye amplifier used in the lab.

## Tera Hertz Pulses

### Tera Hertz Generation

Tera Hertz (or THz) pulse generated in our lab are pulses with a frequency of the order of Tera Hertz and the duration a few ps.

The THz pulses work as pump and probe tools.

## Detection and Data Collection

### Selective Field Ionization

In some experiments, we want to detect wavepackets. But wavepackets cannot be detected directly. Instead, state distribution has been detected to reveal the wavepacket dynamics. As an efficient state distribution detection technology, Selective Field Ionization has been used to widely [8].

The highest electron is trapped in a 1/r potential trap in alkali atoms. When an offset field is applied to the atom, the trap will be tipped as shown in Figure 2.12, which lowers the barriers trapping the electron. When barrier is low enough, the electron would be able to escape from the trap. During the tipping process, higher state electrons tend to be ionized earlier than low state electrons.

In the MOT chamber, there are four metal rods. Two of them are connected to high voltage pulse supply and the other two connected to ground or low static voltage. These 4 rods create a strong electric field which gets maximum in 1 s (slow ionization field) or 500 *n*s (fast ionization field). Atoms in this field will be ionized and the ions will fly to a detector composed of micro-channel plates (MCP). Atoms in different states are ionized at different times, so the electric signal have different arrival time thus the population of states can be distinguished.

Figure .: Schematic of the tipping of electron potential. Solid line is the 1/r potential when there is no external field applied to the atom. Dashed line shows the tip of potential when a filed is applied to the atom. When such a field is strong enough, electrons are able to escape from the trap.

### Synchronization System

Before taking any measurement, we have to make sure the timing is right. The timing is controlled by a synchronization system in the lab. It’s a combination of clocks, delay generators and synchronization boxes. The principal behind such a system is that it’s flexible enough to adapt to changes. Figure 2.13 shows the synchronization system working in the experiment described in chapter 6. A little changes need for this system to work for experiments described in chapter 4 & chapter 5. “Master” is a divider which divides the commercial 60 Hz electricity supply by 4 and provides a 15 Hz source to trigger a digital delay/pulse generator, Model DG535 from Stanford Research Systems, INC. One channel of this delay generator will be the trigger of another DG535 which controls the time of firing the lamps inside Surelite Nd:YAG laser. Another channel will trigger the lamp of GCR-100 Nd:YAG laser. The GCR-100 will provide a “ready” signal when the lamp is outputting power. This signal is synchronized in the SM-1 synchronization box with one pulse from the seed light. SM-1 outputs trigger signals for DG645 and another DG535. DG535 controls the Q-switch of GCR-100 Nd:YAG and pockels cells in the Regenerative Amplifier to get seed light amplified. DG645 controls the Q-switch of Surelite Nd:YAG, the ionization filed, scopes and etc. The delays are easy to change on these delay generators so this system can handle different timing for different experiments.



Figure .: Schematic of the Synchronization System. White cycles are inputs and dark cycles are outputs.

### Measurement Operation

The electric signal from MCP is collected using oscilloscopes. And the oscilloscopes transfer the data on the screen to computers which a using programs written in Labview. A typical electric signal representing a state population is a peak with some width. Usually, the larger a state population is, the higher the peak is. But the height is not an accurate value to measure the population. Instead, the area of peak is proportional to the state population. As show in Figure 2.13, the main peak crossed by a gate is the ionization signal of states. When there is no ionization signal, the peak will disappear and there is only background left. Using the program written in Labview, we can easily measure the area under the peak in the gate. After subtracting this area by the area when there is only background, we can get the real area representing the excitation population. As the population changes, the integrated value in the gate changes accordingly.



Figure .: A typical ionization signal shown on an oscilloscope. The central peak representing the population of state 32*s* + 32*p*. The measurement program puts a gate across the peak and integrates the area under the peak in the gate.

## Maintenance and Daily Operation

Before doing experiments, people should have finished the safety training.

Before turning on lasers, internal lock switch has to be flipped on. It controls the interlock of most lasers in the lab. If any laser is on, a red bulb outside the lab will be on to give a warning signal.

### Daily Examination

1. Check the MOT chamber pressure. The reading from the ion pump should be no more than orders of torr. If the pressure is higher than this reading, there might be some leakage.
2. Check pressure of canned nitrogen which is used to keep GCR-100 laser head clean and dry. The inner pressure should be higher than zero and the output pressure should be around 5 psi. From previous experience, the nitrogen needs to be replaced every two to three weeks.
3. Check the room temperature readings. The readings should be from 72 to 74 . For some extreme climates, the temperature may be out of this range.
4. Check temperature of cooling water from external sources. The supply water should have a temperature around 60 .
5. Check cooling water level for each laser before turning on the laser. The water level should be in the proper range marked in the box.
6. Check the fume hoods to make sure they are working properly.

### Operation of Regenerative Amplifier

Turn on the seed light pump laser power switch. When the temperature is stabilized, turn on the laser. The pump should be in mode “power” and the setup for power is “3.75 W” shown on the display screen.

1. Let the pump laser warm for at least half an hour. Then lock the mode. If the mode lock is not very stable, usually it’s because the alignment of the seed light is off and it needs adjustment.
2. Turn on the regenerative pump GCR-100 Nd:YAG laser. Slowly increase the power of the pumping lamp until it heats the maximum. It usually takes several seconds or minutes for the dimmer light to turn on. If it takes too long, it’s probably because there are two many ions in the cooling system and the charge of lamps is not working properly. Reflush the cooling system using deionized water and try again.
3. Let the Nd:YAG laser warm for at least one hour to acquire thermal balance.
4. Change the output pockels cell’s timing to be the long timing set, which is 4us longer than the short timing set (which should be almost the same every day). This is to enable self lasing of the amplifier. Switch on all the pockels cells in the setup. Increase the Nd:YAG pump light to be a little higher than the threshold. (The threshold may vary a little bit every day. The recent value should be marked down in the log book.)
5. There should be a bright spot showing in the TV monitor, which means the regenerative amplifier is now lasing itself. If there is no bright spot, increase the pump light a little higher but not larger than the normal pump level. Adjust the coupling mirrors to make sure the threshold is minimized.
6. Block the pump light. Change back the output pockels cell’s timing to the short timing set. Increase the pump light to the ordinary operation level (which is also written down in the log book). Unblock the pump light. Now on the scope, there should be a stably increasing pulse train.

At this point, Regenerative amplifier is ready. Fine tuning includes decreasing the threshold and making the pulse train more stable. The pump Nd:YAG laser needs to replace lamps every 700 hours under current repetition frequency. The normal output and the last replacement date are marked underneath the laser head on the optical table.

### Operation of MOT

1. First turn on the cooling water waives. Check the flow meter to make sure cold water is flowing through the coils’ cooling tubes. If there is no flow or the flow is too slow, power supplier of the main anti-Helmholtz coils should not be turned on. Check the water supply in and out pressure to make sure water can flow. The normal in pressure is 14 psi. Make sure there is no leakage of water from the cooling tubes.
2. Turn on diode lasers of trap and repump beams.
3. Turn on AOM driver, voltage ramp of the diode grating, scopes, TV monitors and power supplies of coils. Increase the output of the power supply of the main coils to 10V. The resistance of the main coils is 1 ohm, so the output current of the power supply should be around 10A.
4. Turn on the getter and slowly increase it to the operating value. A normal operating current is from 1.9 A to 2.5 A. When this value has to be as large as 3.5 A to generate an observable cloud of atoms shown on the TV monitor, it means the getter has been used up. Under ordinary usage, this process could take about 4 to 5 years. Once the getter has been used up, it should be replaced by a new one.
5. Let the trap and repump lasers warm for at least one hour to achieve thermal balance. Then adjust the piezo voltage of the lasers to find the right absorption signal. (If the absorption signal is no way similar to the proper pattern, use a spectrometer to check the output frequency. For the trap laser, the output frequency range should cover the value 384232.6 GHz and for the repump laser 384231.2 GHz. If a diode laser could not reach the required value, it is possible that the piezo in the laser head is damaged and needs to be replaced.) Lock lasers.

At this point, there should be a bright spot shown on the TV monitors. It’s the scattered infrared light from the cold atom source. A good MOT on the screen is a stable bright spot with a clear circular shape. If the spot is not stable or the shape is not round, the first step to try is to adjust the shim coils to make it good. If the shim coils do not do the work, more dedicated adjustments of the laser beams are necessary.

### 

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# Models in Simulation

This chapter introduces general knowledge about the physical models used in the research. These models include Rydberg Atoms, Two-Body Model, Dipole-Dipole Interaction Model, Blackbody Radiation and etc. They are also expressed in math language so that they compose a mathematical background for the simulation implemented in Chapter 4, Chapter 5 and Chapter 6. Other than specifically mentioned, all the units in this chapter are atomic units.

## Rydberg Atoms

Back to 1885, Balmer found the wavelengths of the visible series of atomic H is given by [1]:

|  |  |  |
| --- | --- | --- |
|  |  | (3.1) |

where b = 3464.6 . We now know equation (3.1) is the formula for the wavelengths of the Balmer series of transitions from the n = 2 states to the higher lying levels.

After quantitatively describing the wavelengths from H, people started to work on other atoms to unravel the mystery of atomic spectroscopy. Living and Dewar found that the observed spectral lines of Na could be grouped into different series [2]. Hartley found the significance of describing Balmer’s formula in terms of the wavenumber of the observed lines instead of the wavelength during his reach on spectra of Mg, Zn, and Cd [3]:

|  |  |  |
| --- | --- | --- |
|  | . | (3.2) |

Now it’s more clear what Balmer discovered reflects the energy difference between the *n* = 2 states and the higher lying levels.

Following those precedents’ work, Rydberg began to classify the spectra of other atoms, notably alkali atoms, into sharp, principal, and diffuse series of lines [4]. He found the wavenumbers of lines connoting the *s* and p series, for example, are given by:

|  |  |  |
| --- | --- | --- |
|  |  | (3.3) |

where + sign and constant n describe a sharp series of *s* states and the minus sign and a constant m describe a principal series of *p* states. If and m = 2 we can get Balmer’s formula for the H transition from n = 2.

Due to his significant contribution, people are now naming atoms in states of high principal quantum number “Rydberg Atoms”.

### Modern Model of Rydberg Atoms

If we consider Rydberg states of H and Na, as shown in Figure 3.1, they are essentially similar. The only difference is that Na atom has a core which is composed of 11 positive charges and 10 electrons. For most of the time, the highest external electron (Rydberg electron) is far from the core and the difference between Na, H and all Rydberg atoms is minimal. But when the Rydberg electron comes near the core, it can both polarize and penetrate the core, and change the wavefunctions and energies of Na Rydberg state from their hydrongenic counterparts.



Figure 3.1: Rydberg atoms of (a) H and (b) Na. In H the electron orbits around the point of charge of the proton. In Na it orbits around the +11 nuclear charge and ten inner shell electrons. In high l states Na behaves identically to H, but in low L states the Na electron penetrates and polarizes the inner shell electrons of the core [14].

We know how to calculate wavefunctions of H [15]. This process can be easily extended to generate wavefunctions for single valence electron atoms with spherical ionic cores. Such an approach is called Quantum Defect Theory [7]. Quantum Defect Theory (QDT) assumes that the core is spherically symmetric and frozen in place. So the effective potential, seen by the valence electron is spherically symmetric and only depends on *r*. This potential is lower than the coulomb -1/*r* potential only at small *r*, and the effect is to increase electron kinetic energy and decrease the wavelength of the radial oscillations relative to H. Suppose the phase shift is . The bound state radial wavefunctions are given by:

|  |  |  |
| --- | --- | --- |
|  |  | (3.4) |

where and are commonly termed the regular and irregular coulomb functions. This radial function will derive the the allowed eigen energies:

|  |  |  |
| --- | --- | --- |
|  |  | (3.5) |

where n is an integer. Equation (3.5) is the equation used in the simulation to calculate the energies of Rydberg atoms. Table 3.1 gives the 0th order approximation of the quantum defect for Rb.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |
|  | 3.13109 | 2.65456 | 2.64145 | 1.347157 | 0.016312 |

Table 3.1:Quantum defects for low-ℓ states of Rb [14].

### Properties of Rydberg Atoms

Table 3.2 summarizes the properties dependent on principal quantum number n of Rydberg atoms [8].

|  |  |
| --- | --- |
| property | n dependences |
| Binding energy  Energy between adjacent n states  Orbital radius  Geometric cross section  Dipole moment <ns|er|np>  Polarizability  Radiative lifetime  Fine-structure interval |  |

Table 3.2: Properties of Rydberg Atoms.

As introduced in later sections, dipole-dipole interaction between Rydberg atoms is proportional to . And the Rydberg electrons are far from cores, which makes the them easy to be affect by external forces. The very long enough lifetime of Rydberg atoms also reduces the threshold of detecting development of atoms. All these superior properties make them ideal objects for researching dipole-dipole interaction and electron dynamics.

## Two-Body Model

### What Is Two-Body Model

When talking about Dipole-Dipole interaction (which will be introduced in detail in later sections) between atoms, a simplified two-body model is often used. In this model, we suppose one atom can only be affected by its nearest neighbor. Such an assumption is not very accurate of course, because a nearest neighbor could never block the influence from other atoms. But compared to many-body model, two-body model provides a concise way of thinking dipole-dipole interaction between atoms [14]. Besides, two-body effect has been accepted widely to be the major effect between atoms [18][19]. So in this dissertation, all the calculation and simulation is the based on the two-body model.



Figure 3.2: Schematic of pairs in MOT. For an atom in a MOT, we only consider the effect from its nearest neighbor. One atom and its nearest neighbor is considered to be “a pair of atoms”.

For a two-body model or a pair of atoms, we can write their state, in non-interacting basis, as the combination of their individual states. For example, for a pair of atoms which are in ns state and np state respectively, we can write the state of this pair as nsnp. We call this state “pair state”. Such a convention is followed in all sections of this dissertation.

### Nearest Neighbor Distribution

For a pair of atoms, to get the effect of one atom on the other, we need to find the distance between them. We use the so called “Nearest Neighbor Distribution” theory to find the distance between an atom and its nearest neighbor. The nearest neighbor distribution function H(r) for a D-dimensional system is described by [20][21]:

|  |  |  |
| --- | --- | --- |
|  |  | (3.6) |

where is the density of the atoms and is the volume of the D-dimensional sphere. For a 3-D system, we get the distribution function:

|  |  |  |
| --- | --- | --- |
|  | . | (3.7) |

From Equation (3.7), we can find the most possible distance between in one pair is:

|  |  |  |
| --- | --- | --- |
|  | . | (3.8) |

For a MOT with density , temperature about 100 K, the most possible distance between one atom and its nearest neighbor is about 5 m and the velocity of the atoms is of the order of 10 cm/s. In 1 s, one atom can move about 0.1 m which is much smaller than the distance between two atoms. So we could consider the atoms as “frozen” or static atoms in the MOT for our experiments.

### Förster Resonance Energy Transfer

Förster Resonance Energy Transfer (FRET) is a mechanism describing energy transfer between two atoms or molecules. It happens when two neighboring atoms are dipole-dipole coupled to higher and lower states with equal energy spacing. This mechanism could be described very easily using the two-body model (as shown in Figure 3.3). One atom is acting as a donor and the other one accepter. The atoms exchange energy as the donor is de-excited to a lower state and the accepter excited to a higher state. We utilize this mechanism a lot in our research described in this dissertation.

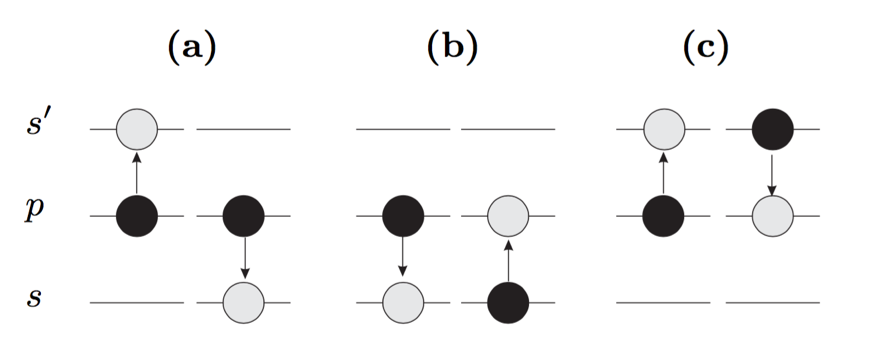


Figure 3.3: Schematic for typical FRETs. Black cycles represent the initial pair states and gray cycles the final pair states. (a) is , (b) and (c) [22].

## Dipole-Dipole Interaction

In the above discussion, dipole-dipole interaction has been mentioned several times. Now with the Rydberg Atoms model and two-body model, it makes easy to describe what exactly dipole-dipole interaction is.

### Dipole Moment

For Rydberg atoms, the most external electron spends most of the time far from the positive core. Although the atom as a whole is neutral, it has a polarity. We could simplify a Rydberg atom to be a system composed of a positive charge and a negative charge. Such a system is called a dipole and dipole moment is a measure of the polarity in this system. Its definition expressed in math is:

|  |  |  |
| --- | --- | --- |
|  |  | (3.9) |

where q is the charges’ amplitude and is the vector from the negative charge to the positive charge. This is a classic view of dipole moment and the dipole moment described in Equation (3.9) has a special name “permanent dipole moment”. This classic view will help us to understand the dipole-dipole interaction picture better. But another view from quantum physics is more commonly used.

In quantum physics, dipole moment is expressed as “transition dipole moment” and its definition is associated with the transition between state 1 and state 2:

|  |  |  |
| --- | --- | --- |
|  |  | (3.10) |

supposing charge is 1. Such a concept is introduced when calculating Schrodinger equation in quantum mechanics (see section 3.3.3). From Equation (3.10) we see if state 1 and state 2 are the same, then the transition dipole moment is 0, which means a transition dipole moment is always an off-diagonal matrix element.

This discussion about dipole moment has been divided into classic view and quantum view. Similar to that, dipole-dipole interaction model will also be explored from both views.

### Dipole-Dipole Interaction in Classic Picture

Although not very accurate, a classic dipole-dipole interaction picture has useful similarity to the interaction between quantum atoms and helps us to understand the quantum picture.

If there is only one dipole, the potential of this dipole is merely provided by the attractive force between the positive charge and negative charge. When two dipoles are getting close, the potential of such a system will be changed by the dipole-dipole interaction. As shown in Figure 3.4, two dipoles are interacting with each other. The total potential energy of this system does not only contain the inner coulomb potential of each individual dipole but also the potential caused by dipole-dipole interaction. The dipole-dipole interaction potential could be easily written as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.11) |

From reference [23], Equation (3.11) could be simplified to:

|  |  |  |
| --- | --- | --- |
|  |  | (3.12) |

Expressed in dipole moments, Equation (3.12) could also be written as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.13) |

where is the dipole moment of dipole 1 and is the dipole moment of dipole 2. From this expression, we see that whether the force between the two dipoles is attractive or repulsive is determined by the polarity of their dipole moments.

From Equation (3.13) we can also see the dipole-dipole interaction is strongly affected to the distance between the distance of dipoles or Rydberg atoms. This is the reason why two-body effect is the major effect in an ensemble of frozen atoms.

Suppose and q = 1, Equation (3.12) could also be written as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.14) |

where , , and specify the position of the Rydberg electron in the ith atom relative to the center of the atom.

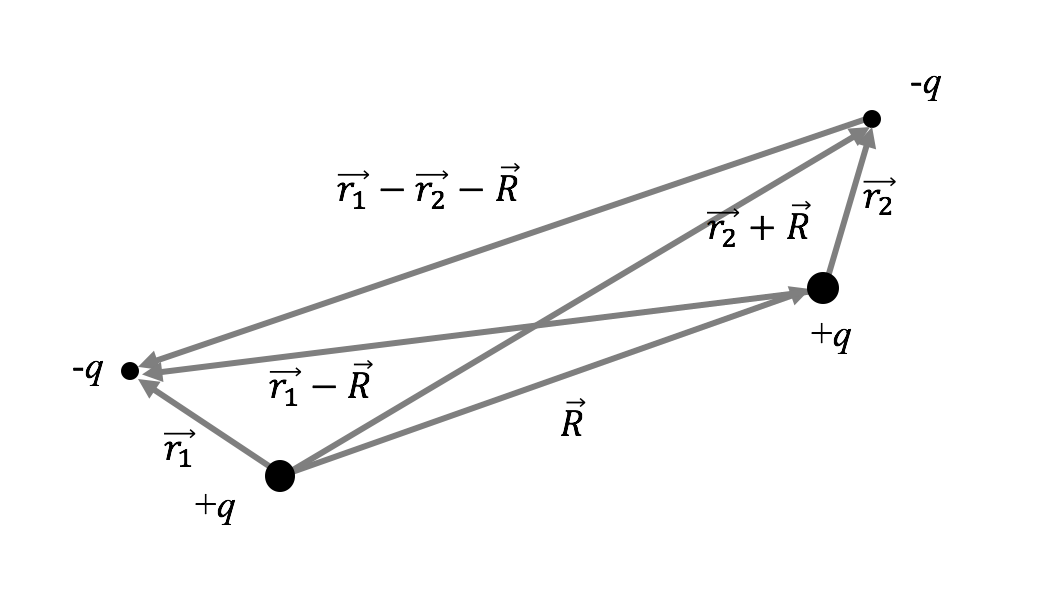


Figure .: Schematic for interaction between two dipoles in classic picture.

### Dipole-Dipole Interaction in Quantum Picture

The quantum dipole-dipole picture is based on the classic picture. Without loss of generality, we now continue with the resonance (a) in Figure 3.3. Suppose a system is composed of two dipoles starts from pair state *pp*, and we want to explore the evolution of the wavefunction in such a system. So we have:

|  |  |  |
| --- | --- | --- |
|  | . | (3.15) |

The total Hamiltonian of such a system is . *V* is the potential caused by dipole-dipole interaction as we discussion in the classic picture. The solution for time-dependent Schrodinger equation, is a combination of and :

|  |  |  |
| --- | --- | --- |
|  |  | (3.16) |

where all of the time dependence is contained in the coefficients and . And the Schrodinger equation can be written as:

|  |  |  |
| --- | --- | --- |
|  | ../../../../../Desktop/Screen%20Shot%202016-03-10%20at%202. | (3.17) |

where , with *p* and *q* representing pair states. And when in resonance . By solving Equation (3.14) we can get two eigen energy values:

|  |  |  |
| --- | --- | --- |
|  |  | (3.18) |

where *M* is expressed as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.19) |

This expression utilizes the concept of transition dipole moment mentioned in section 3.3.1.

We can also get two eigen states from Equation (3.14):

|  |  |  |
| --- | --- | --- |
|  | . | (3.20) |

So the complete solution of the Equation (3.14) is:

|  |  |  |
| --- | --- | --- |
|  |  | (3.21) |

in which and are constants determined by the initial condition.

The above discussion gives us a simple example of dealing with dipole-dipole interactions in a two-state simple system. To calculate the matrix elements for such a system and more complicated system, we need to go to deeper details. By separating the radial part and the angular part, the calculation is implemented as follows.

From Equation (3.14), using the relations between the Cartesian coordinates and the spherical harmonics (

|  |  |  |
| --- | --- | --- |
|  |  | (3.22) |

the dipole-dipole interaction can be written in terms of spherical harmonics as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.23) |

where is the spherical harmonics acting on the Rydberg electron of the *i*th atom.

To compute the matrix elements, Edmond’s C-tensors are introduced [24]. The relation between the spherical harmonic tensors and the C-tensors is:

|  |  |  |
| --- | --- | --- |
|  | . | (3.24) |

So the dipole-dipole interaction in form of C-tensors is:

|  |  |  |
| --- | --- | --- |
|  | . | (3.25) |

In Equation (3.25), a pair of C’s implies that the first C acts on the first atom and the second C on the second atoms.

Equation (3.25) is used in the simulation to calculation the matrix elements. As an example, the dipole-dipole matrix element between and is expressed as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.26) |

where and are the radial matrix elements and the single state expression has a format of . The radial matrix elements could be calculated sing a numerical Numerov integration algorithm [18].

## Blackbody Radiation Model

Rydberg atoms are strongly affected by blackbody radiation, even at room temperature. The effect of thermal radiation is due two reasons [14]. First, the energy spacing between adjacent Rydberg levels are small (, so that at 300K. Second, the dipole matrix elements of transition between Rydberg states are large, providing coupling of the atoms to the thermal radiation. Because of the strong coupling between Rydberg atoms and the thermal radiation, population initially put into one state rapidly diffuse to other energetically nearby states by blackbody radiation.

### Blackbody Induced Transition

The spontaneous decay rate of the n state to the lower lying state is given by the Einstein A coefficient [25]. The thermal radiation induced stimulated emission rate is , which is the photon occupation number [26], times as large as the spontaneous rate:

|  |  |  |
| --- | --- | --- |
|  |  | (3.27) |

In terms of the average oscillator strength, Equation (3.27) is reexpressed as:

|  |  |  |
| --- | --- | --- |
|  |  | (3.28) |

where is the fine structure constant, is the energy difference , and

|  |  |  |
| --- | --- | --- |
|  |  | (3.29) |

where is the larger of and .

Equation (3.28) and Equation (3.29) are the equations used in Chapter 4 to calculate the blackbody radiation induced transition rate between nearby Rydberg levels.

### Radiation Model

When considering the decay of the population in one Rydberg state, as described in Chapter 4, radiation models similar to schematic in Figure 3.5 are used. There is blackbody induced transition between the starting state and its neighbors. There is also blackbody radiation between one neighbor and the neighbor’s neighbors. And so on. In calculation, a limited number of neighbors are chosen to make sure the calculation result converges.



Figure .: Decay model for atoms starting from state 40*s* as an example. The red dash lines between two states mean there is blackbody stimulated transition between these two states. The blue dash curves mean the spontaneous decay.

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# Absence of Collective Decay in a Cold Rydberg Gas

We have studied the decay of Rydberg excitations in a cold Rb gas. A 10 ns, pulsed-dye-amplified diode laser excites Rb atoms at 70 *μ*K in a magneto-optical trap to ns or np Rydberg states with principal quantum numbers . Time-delayed state-selective field ionization is used to directly monitor the population in the initial and neighboring Rydberg levels. The measured time dependence of the Rydberg population is well described by numerical simulations which consider only spontaneous emission and population transfer by blackbody radiation. No evidence for collective decay is found at atom densities up to 3 × . This result is in contrast to a previous study [Wang *et al.*, Phys. Rev. A **75**, 033802 (2007)], in which superradiant decay was theoretically predicted and experimentally inferred for atom density and laser focal volume conditions very similar to those considered here. Suppression of collective emission is likely due to variations in transition energies within the atom sample, dominated by inhomogeneities in dipole-dipole exchange interactions for initial *s* states, or by a combination of dipole-dipole and electric field inhomogeneities for the initial *p* states.

## Introduction

Atoms within cold Rydberg ensembles are coupled by strong long-range dipole-dipole (DD) interactions [1], making them interesting systems for exploring few- and many-body quantum dynamics in general and applications in quantum information in particular [2–35]. Of course, unlike in ground-state systems, finite Rydberg lifetimes limit the types of measurements and number of coherent manipulations that can be performed in a given experiment. Fortunately, isolated Rydberg atoms exhibit low spontaneous decay rates [1], potentially enabling processing over micro- to millisecond time scales. At first glance this stability against radiative decay might seem surprising given the large transition matrix elements between adjacent Rydberg states, which scale as n2. However, spontaneous decay to nearby levels via low-frequency emission is strongly suppressed by the dependence of the Einstein *A* coefficient. As a result, the predominant decay path for isolated Rydberg atoms in low-angular-momentum states is to the ground or low-lying excited levels, resulting in an scaling of the spontaneous emission rate [1].

That said, neighboring Rydberg levels can play a dominant role in the decay of a large number N of atoms which either are simultaneously excited in a volume with dimensions smaller than the wavelength λ of the emitted light, or are sequentially excited throughout a cylindrical volume with length L ≫ λ [36,37]. In his seminal paper [38], Dicke predicted that a dense collection of N radiators, either in very close proximity or in a properly phased extended distribution, could develop spontaneous correlations and collectively emit radiation at rates greatly exceeding (“superradiance”), or much smaller than (“subradiance”), those of individuals in the sample. For the two-level systems considered by Dicke, correlations between a large number of emitting atoms can initiate collective superradiant emission at a per atom rate up to N/4 times larger than that between the same two levels in an isolated atom [38]. Collective emission remains a subject of considerable interest in many different contexts, including Rydberg atoms [39–51].

The presence of blackbody radiation, the existence of multiple photo decay channels in a Rydberg ladder, and strong DD interactions between atoms, all explicitly neglected in Dicke’s 58 original paper (and in many subsequent treatments) [38,52,53], make it much more difficult to observe, characterize, and quantitatively predict collective decay phenomena in 61 Rydberg gases. In particular, DD interactions can suppress superradiance. In a thermal gas, this suppression results from DD-mediated collisions that homogeneously dephase the individual emitters in the ensemble at a rate greater than the superradiance rate [50]. In a frozen gas, DD exchange interactions couple pairs, or larger groups, of atoms leading to a variation in transition energies across the ensemble. Such inhomogeneities squelch the correlations that underlie superradiance, along with the collective emission [47]. That said, a clear signature of superradiance between Rydberg states, a fluorescence cascade from a Rydberg ladder proceeding at a rate much greater than spontaneous emission of isolated atoms, was first observed following pulsed-laser excitation of an elongated volume (L)λ) in a thermal cell [36]. More recently, direct evidence for superradiance was found in the millimeter-wave emission from a large cylindrical volume of Ca Rydberg atoms in a supersonic expansion, also with L ≫ λ [49,50]. As pointed out in the latter work, the rates for superradiant decay and DD dephasing within a given decay channel are essentially identical, up to a multiplicative geometric factor L/λ in the superradiance rate formula. Accordingly, it was suggested that collective decay should not play a major role in Rydberg depopulation unless L≫λ [50].

Still, despite competing DD effects, under certain conditions superradiance should play some role in cold ensembles where the dimensions of the excited volume are less than or comparable to λ. Indeed, evidence of reduced Rydberg lifetimes has been reported in several such experiments [39,48,51,54,55]. For example, Feng et al. observed 90 a density-dependent lifetime suppression of Cs Rydberg atoms in a magneto-optical trap (MOT) [54]. They attribute the suppression to a combination of neutral Rydberg atom collisions and superradiance. However, the evidence for superradiance appears tenuous as their calculations with and without superradiant contributions both fall within their measurement uncertainty (see their Fig. 2). In addition, if one applies their values for Rydberg collision velocity and cross section, the collisional depopulation rates are over three orders of magnitude too small to account for their observations. Han and Maeda attributed population transfer from initial to neighboring Rb Rydberg states to superradiance, but provided no evidence ruling out other possible population transfer mechanisms [56]. In other measurements, using fluorescence detection, Day et al. found Rydberg depopulation rates that were roughly twice that expected from single-atom spontaneous emission over a range of n states and at low densities, ρ ∼ 1 × [48]. The small variation of the lifetime suppression with principal quantum number coupled with trap loss measurements argued against collisional depopulation and blackbody ionization. Instead, the enhanced Rydberg decay rate was found to be qualitatively consistent with a simplified collective decay model. In other experiments, the inclusion of superradiant decay channels was found to improve the quality of model fits to electromagnetically induced transparency measurements in cold Rydberg gases [51,55].

Certainly, a substantial decrease in Rydberg lifetimes due to collective emission would have a significant impact on most cold Rydberg atom experiments. Perhaps more important, in the context of the exploration and control of few- or many-body Rydberg dynamics, are the influences of the spontaneous quantum correlations that are predicted to develop with the emission of the first photon from the sample and evolve as the Rydberg population descends through a ladder of Dicke states [38,53]. Interestingly, Wang et al. presented a sophisticated theoretical treatment of photo decay in a multilevel Rydberg system which makes definite predictions as to whether superradiance should occur for a given initial principal quantum number, atom density, and experimental volume [39]. The theory apparently reproduced the rapid decay, at a rate approximately 40 times greater than predicted from spontaneous emission alone, of an initial population of 43*p* atoms in a MOT at a density of ρ∼5×.

We have used pulsed-laser excitation of Rb Rydberg atoms in a MOT under conditions ostensibly similar to those used in Ref. [39] in an attempt to test the predictions of their Rydberg superradiance theory. We employ state-selective field ionization (SSFI) to measure the population in the initial and neighboring Rydberg states as a function of delay after the laser excitation. We find no evidence for the predicted collective decay over a range of principal quantum numbers , and atom densities ρ ∼ 3 × , despite the fact that, for these states, our highest density is more than two orders of magnitude above the predicted superradiance threshold [39]. Instead, our measurements are consistent with noncorrelated spontaneous decay combined with population redistribution via blackbody radiation.

## Experimental Procedure and Results

In the experiments, atoms at 70 *μ*K are held in a MOT. The full width at half maximum (FWHM) diameter of the atom cloud is 0.4 mm. The MOT is positioned at the center of four parallel rods which facilitate the application of static and pulsed electric fields in the y direction for exciting and detecting Rydberg atoms in the MOT. A 10 ns pulsed, tunable, dye-amplified, ∼480 nm diode laser propagating in the x direction is focused into the center of the MOT, creating a cylindrically shaped volume of cold Rydberg atoms with a FWHM diameter of ∼0.1 mm and a length of 0.4 mm. The MOT and Rydberg lasers are non collinear, preventing the excitation of Rydberg atoms throughout any extended volume from the lower-density background of thermal Rb atoms in the chamber. The ∼100 MHz bandwidth of the Rydberg excitation laser ensures that there is no excitation suppression via dipole blockade [2,3]. At a variable time τ after the laser excitation, a ramped voltage is applied to two of the rods, ionizing any Rydberg atoms in the interaction region and pushing the resulting ions toward a micro channel plate (MCP) detector. Ions originating from different Rydberg states arrive at the detector at different times. The integrated signals in different time bins are proportional to the populations in different Rydberg states and are recorded for each laser shot as a function of the ionization time τ. The experiment proceeds at the 15 Hz dye-laser repetition rate.

The diode laser is tuned to selectively excite atoms from the upper 5p trap level to ns and np Rydberg states with 26. Excitation of np states is facilitated by the application of a weak static electric field (from 30 V/cm 177 at *n*=26, 16 V/cm at *n*=32, to 7 V/cm at *n*=40). The density of 5*p* atoms in the MOT is determined, to within 30%, by combining measurements of the spatial dimensions of the atom cloud size via direct imaging with a CCD camera with measurements of the radiated fluorescence using an optical power meter. By saturating the Rydberg excitation using high laser fluence, we ensure that 50% of the 5p atoms within the interaction volume are excited to Rydberg states, enabling *u*s to determine the Rydberg atom density [57]. Subsidiary experiments on resonant energy transfer between Rydberg atoms are consistent with the Rydberg density determination [57,58]. The MOT fluorescence is monitored throughout the lifetime measurements, ensuring that the number of atoms in the MOT is constant to within a few percent as τ is scanned. Care is taken to minimize the amplified spontaneous emission from the dye-amplified laser pulse, eliminating direct photoionization of 5*p* atoms. For the *s*-state measurements, a small, ∼1.5 V/cm, residual electric field persists in the interaction region due to imperfect shielding of the high voltage biased MCP (a larger field is present for initial *p* states). This field is sufficient to eject any ions or electrons from the interaction region, eliminating extended interactions between charged particles and neutral Rydberg atoms, and preventing the spontaneous evolution of the Rydberg gas into a plasma [59,60]. Neither the small static field employed for the *p*-state measurements nor the smaller residual field present during the *s*-state measurements substantially alters the rates for spontaneous emission or population transfer induced by blackbody radiation. The potential influence of the field inhomogeneity on superradiant decay is considered in the Analysis and Discussion section below.

Figure 4.1 and Figure 4.2 show our principal experimental results. In Figure 4.1 (a) and (c), the probabilities for finding atoms in the 26*s* + 25*p*, 32*s*, and 40*s* states are plotted as functions of detection time τ for the maximum densities explored, ρ ∼ 3 × and ρ ∼ 1.5 × , respectively. Note that for the lowest initial *n* state the sum of the 26*s* and 25*p* populations is shown since their corresponding features could not be adequately separated in the time-resolved field-ionization signal. Within experimental uncertainties, the decays for the three initial s states are identical at the two densities shown. Additional measurements were made at Rydberg densities as low as ρ ∼ 2 × (for 32*s* initial states) and ρ ∼ 5 × (for 32*p* initial states), but no statistically significant differences were observed in the decays.

For spontaneous decay of isolated atoms at absolute zero, one would expect lifetimes of 28 and 58 *μ*s for the 32*s* and 40*s* atoms, respectively [61]. The measured lifetimes for the 32*s* and 40*s* states are substantially smaller, 19 and 38 *μ*s, respectively, due to population redistribution by blackbody radiation from the 300K environment surrounding the MOT. Indeed, redistributed population is detected in neighboring Rydberg levels. In particular, Figure 4.1 (b) and (d) show the delay-dependent population in the *p* states (26*p*, 32*p*, and 40*p*) that lie immediately above the respective initial s states. Although we would expect to find some atoms in the adjacent, lower-lying *p* states as well, small features reflecting that population in the time-of-ionization signal lie within the initial-state peak (for the case of 26*s*) or are masked by the tail of the larger, initial-state peak which precedes it.

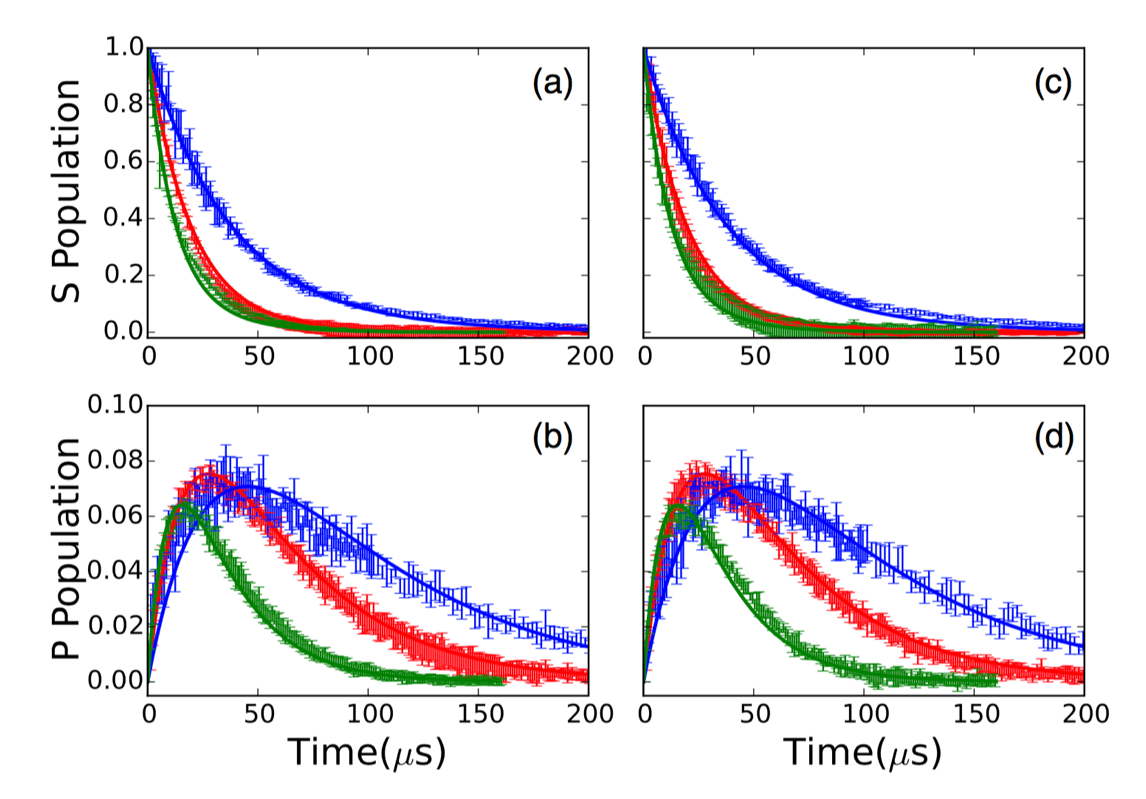


Figure .: (a), (c) Probabilities for finding atoms in 26*s* + 25*p* (green, fastest decay), 32*s* (red, intermediate decay), and 40*s* (blue, slowest decay) as a function of detection time τ for Rydberg densities of *ρ* ∼3× (a) and *ρ* ∼1.5× (c).Note that the sum of the 26*s* and 25*p* populations is shown since their corresponding features could not be adequately separated in the field-ionization signal. Vertical bars show the experimental data with uncertainties, and the solid curves are calculated as described in the text. Measurements and calculations for the 40*s* decay extend to 500 *μ*s where the remaining population is negligible. (b), (d) Probabilities for finding atoms in 26*p* (green, fastest rise and decay), 32*p* (red, intermediate rise and decay), and 40*p* (blue, slowest rise and decay) levels as a function of detection time τ. The states are populated by blackbody redistribution from the initial 26*s*, 32*s*, and 40*s* levels, respectively. The data were measured simultaneously with those shown in (a) and (c). Vertical bars show the experimental data with uncertainties, and the solid curves are calculated as described in the text. The measured *p*-state probabilities are normalized to the calculations as described in the text. The calculations have no free parameters and consider only the effects of spontaneous emission and blackbody radiation on isolated atoms.

The measured lifetime for the combined 26*s* + 25*p* states is 14 *μ*s, the same as that expected from spontaneous decay of the 26*s* level alone [61]. Simulations (described in detail below) indicate that this apparent agreement is not due to the absence of blackbody transfer out of 26*s*. Rather, the small longer-lived 25*p* component of the signal masks much of the change in the 26*s* decay, with a predicted effective lifetime of 13 *μ*s for the 26*s* + 25*p* combination, similar to what we observe. We note that due to the slow rate of the ionizing field, there is a distribution of ionization times and, therefore, of detection efficiencies for atoms in different states. As a result, each of the measured *p*-state populations in Figure 4.1 (b) and (d) has been multiplied by a normalization factor to obtain the best agreement with the calculated decay curves that are shown in the figures and described in the next section.

Figure 4.2 shows analogous data for the decay of initially excited 26*p*, 32*p*, and 40*p* states. Again, due to blackbody redistribution, the lifetimes associated with these decays (18, 31, and 51 *μ*s), are considerably smaller than expected from spontaneous emission alone (37, 75, and 155 *μ*s) [61]. However, in this case, no substantial population is detected in the neighboring *s* or *d* levels. The analysis described in the next section indicates that the populations in these states are not detectable within our signal-to-noise ratio, remaining at or below the few percent level due to the relatively rapid spontaneous emission rate out of the s states, and smaller *p* → *s* and *p* → *d* blackbody transition rates.

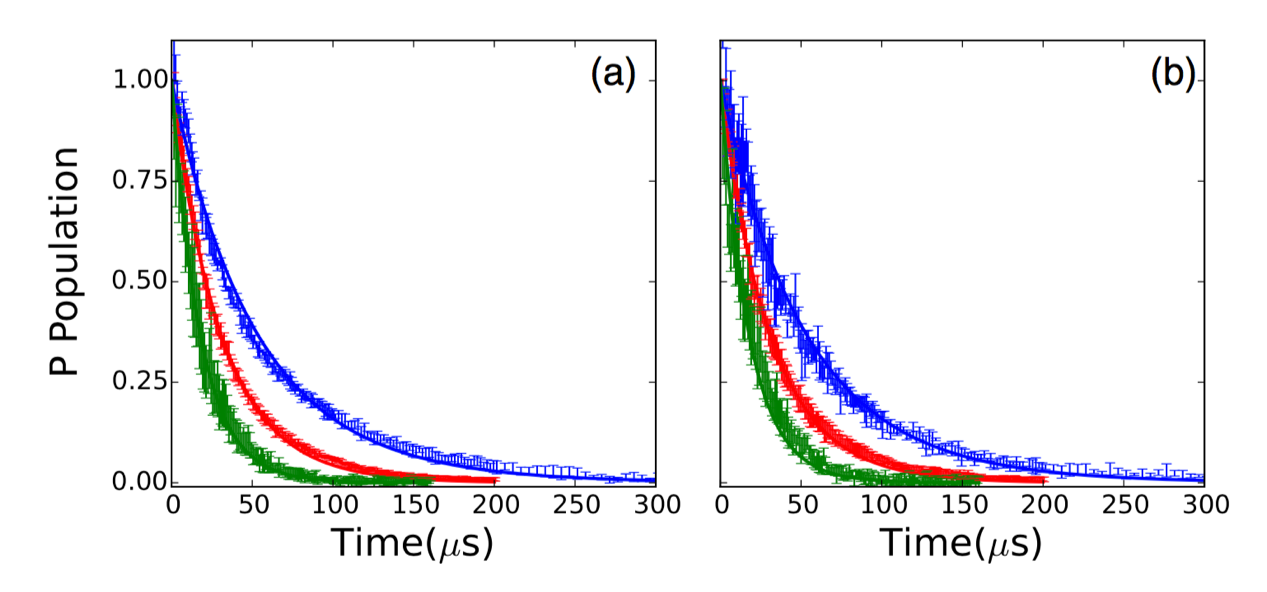


Figure .: Probabilities for finding atoms in 26*p* (green, fastest decay), 32*p* (red, intermediate decay), and 40*p* (blue, slowest decay) as functions of detection time τ for Rydberg densities of ρ∼3× (a) and ρ∼1.5× (b). Vertical bars show the experimental data with uncertainties, and the solid curves are calculated as described in the text. Measurements and calculations for the 40p decay extend to 500 *μ*s where the remaining population is negligible. The calculations have no free parameters and consider only the effects of spontaneous emission and blackbody radiation on isolated atoms.

## Analysis and Discussion

To determine if collective processes play any significant role in the decays we observe, we compare the measurements to the results of a simple rate equation model. The model includes population transfer via stimulated emission and absorption of blackbody radiation between an essential set of *s*, *p*, and *d* Rydberg states neighboring the initial level, as well as spontaneous emission out of those essential states to (undetected) lower-lying levels. We calculate the blackbody transition rates between the essential states [1] as well as the known total spontaneous emission rates of the *s*, *p*, and *d* Rydberg levels [1].

For example, for an initially excited 40*s* state, the rate equation describing the time-dependent population in the initial 40*s* level is:

|  |  |  |
| --- | --- | --- |
|  |  | (4.1) |

where is the population in state , is the 40*s* spontaneous decay rate, and is the blackbody transition rate from to [1]:

|  |  |  |
| --- | --- | --- |
|  |  | (4.2) |

which is mentioned in Chapter 3.

The populations in the secondary states 39*p* and 40*p* are computed using similar rate equations that include the total spontaneous decay rate out of those levels as well as blackbody transitions to and from pairs of *s* and *d* levels that lie immediately above and below each *p* state. We truncate the system of equations with rate equations that include spontaneous decay from the tertiary *s* and *d* levels and their blackbody couplings with the secondary states. Analogous systems of equations are used to compute the Rydberg population decay following initial *p*-state excitation. We note that, for initial or intermediate p states in particular, blackbody radiation redistributes a small, but non-negligible, fraction of the initial population beyond the nearest-neighbor *s* and *d* states. Therefore, an approximate expression [1]

|  |  |  |
| --- | --- | --- |
|  |  | (4.3) |

for the total blackbody decay rate from each p level is used to more accurately account for the net transfer out of these states.

The results of our calculation, which ignore any collective decay phenomena, are shown with the data in Figure 4.1. Overall, the agreement is reasonable. Aside from the previously noted renormalization of the experimental *p*-state population, no parameter adjustments have been made to obtain the level of agreement shown. The data provides no evidence of a significant reduction in the Rydberg lifetimes due to superradiance. This is true over a range of principal quantum numbers and atom densities where superradiant emission has been predicted to be the dominant decay path [39].

It is well established that superradiance is suppressed by inhomogeneities in transition energies across a sample of emitters [47], and we suspect that this is the case in our, and many other, cold atom experiments. In our experiments, three different effects contribute to such inhomogeneities. The first, and dominant mechanism for some of our measurements, is the DD exchange interaction. Consider a pair of identical atoms with two levels *s* and *p* and interatomic separation R. Spontaneous emission from the initial upper pair state *ss* results in the population of the bright configuration of the lower-energy pair state (*sp* + *ps*)/. However, due to the DD coupling between the atoms, ∝ /, the energy of this state is not the same as that for two atoms at infinite separation [1]. Accordingly, in a large ensemble of randomly spaced atoms, every possible configuration of *s* atoms and *p* atoms has a different energy, depending on the separation (and relative orientation) between the *p* atoms and their neighboring *s* atoms. As a result, any Dicke state, the bright linear combination of all possible configurations of *s* atoms and *p* atoms [38], is nonstationary. The phases of the constituent N-atom product states evolve at different rates, as determined by their DD energy shifts relative to their energies at infinite separation. The emission from these nonstationary Dicke states dephases at a rate comparable to the typical dipole-dipole energy shift for pairs of atoms in the ensemble. Superradiance cannot occur unless the system transitions down each step in the Dicke ladder more rapidly than this dephasing. A similar argument has been made by Gross and Haroche [62]. In the frequency domain, atoms with different transition energies at different locations in the ensemble do not collectively emit into the same field unless that emission occurs in a very short burst with a sufficiently broad, coherent bandwidth.

To determine the DD dephasing rate, we use the most probable nearest-neighbor separation in a random ensemble, *R* ≃ , and average over all orientations of the Rydberg states on any two neighboring atoms *ns* and to obtain [63,64]:

|  |  |  |
| --- | --- | --- |
|  |  | (4.4) |

Using a numerical Numerov integration algorithm to compute the relevant radial matrix elements [65], at the highest density studied (*ρ* = 3 × ) we obtain values for the DD exchange coupling between the ns and (*n* − 1)*p* states, = 2.4, 6.2, and 17 MHz, for *n* = 26, 32, and 40, respectively. These interaction strengths set effective lower limits for the rates at which collective emission from *ns* to (*n* − 1)*p* can occur. Similarly, for initial *np* states and *ρ* = 3 × , the relevant exchange coupling is to the nearest lower-lying *s* states, with , = 3.1, 7.8, and 20 MHz, for *n* = 26, 32, and 40, respectively.

The magnetic field gradient in the MOT is another source of energy inhomogeneities in our ensemble. As in Ref. [39], the magnetic field remains on during our measurements, resulting in a transition energy variation of approximately 1 MHz across the MOT. This inhomogeneity is smaller, or much smaller, than that due to dipole-dipole interactions at sufficiently high densities. It should not play a principal role in suppressing superradiance under the conditions used to produce Figure 4.1and Figure 4.2.

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