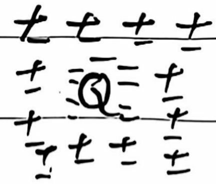
Mar. 3rd. Static Screening

Jellium Model 在一种边界处能带斜率无穷，需要修正。
原因，假定 Jellium 时，阳离子不动，Actually，阳离子有两个效应，一：离子振动，声子；二：离子吸引电子。
在此先解决二，由于阳离子千倍质量于电子，其周围会产生阴电子膜，用以下模型假定，正负离子平衡系统中引入一阳离子Q，从经典角度求其周围介电常数。

$\varepsilon = 1$ $\quad \rho_0 = 0 \quad \varphi_0 = 0$ 原本



由于引入 ρ_{ext} 诱导阴离子 ρ_{induce}

产生 φ_{ext} $\qquad$ φ_{induce}

定义介电常数：$\varphi_{ext}(q) = \varepsilon(q)\,\varphi_{tot}(q)$

$$\varphi_{tot} = \varphi_{ind} + \varphi_0 + \varphi_{ext}$$

Maxwell Equation Fourier：
$$q^2 \varphi_{ext}(q) = 4\pi \rho_{ext}(q)$$
$$q^2 \varphi_{ind}(q) = 4\pi \rho_{ind}(q)$$

$$\Rightarrow \varepsilon(q) = 1 - \frac{4\pi}{q^2} \cdot \frac{\rho_{ind}(q)}{\varphi_{tot}(q)}$$

Thomas-Fermi Screening：增加的外电场非常均匀

uniform $\rho_{ext}(r)$

$\Rightarrow$ uniform $\rho_{ind}(r)$

$\Rightarrow$ uniform $\rho_{tot}(r)$

$\Rightarrow$ uniform $\varphi_{tot}(r)$

We call it a semi-classical approxiamation, since electron here have both precise k and r, so that we can write out their precise energy, $E[(r)(k)]$ for uniform $\varphi_{ext}(r)$. in Fermi-Dirac

$$f_k^{\mu} = \{\exp[\beta(\varepsilon_k - \mu - \underbrace{e\varphi_{tot}(r)}_{const})]+1\}^{-1} \quad \text{(electron)}$$

$$n(r) = \frac{\sum_{\sigma,k} f_k^{\mu}}{V}$$

$$= \frac{1\times 2}{V}\int d_k^3 \frac{1}{(2\pi)^3} \cdot f_k^{\mu}$$

$$n(r) = \frac{2}{V}\int d^3_k \frac{1}{(2\pi)^3} f_k^{\mu}(\varphi_{tot}=0)$$ (ion：离子仍不动. φ_{tot} 无影响)

$$\Rightarrow \rho_{ind}(r) = e\,n_{ele}(r) + (-e)\,n_{ion}(r)$$ at $\varphi_{tot}(\infty)=\varphi_{tot}(-\infty)\approx 0$

$\Rightarrow \varphi_{tot}\simeq 0$ Taylor Expand φ_{tot}

$$= -e^2\frac{\partial n_0}{\partial\mu}\varphi_{tot}(r)$$

$$\Rightarrow \varepsilon(q) = 1 + \frac{4\pi}{q^2}\Big[\underbrace{\int_k \partial_\mu f_k^{\mu}}\Big] \rightarrow \text{屏蔽衰减长度}$$

so that for a given Q as $\varphi_{ext}(q) = \frac{4\pi}{q^2}\cdot Q$.

$$\varphi_{tot}(r) = \int e^{iqr} d^3q \frac{1}{(2\pi)^3}\varphi_{tot}(q) \rightarrow \frac{\varphi_{ext}(q)}{\varepsilon(q)}$$

$$= \frac{Q}{r}\exp\left(-\frac{r}{\lambda}\right)$$

where: $\lambda = \sqrt{\dfrac{1}{4\pi^2 e^2 \int_k \partial_\mu f_k^{\mu}}}$ $\quad\lambda\uparrow \Rightarrow \varphi_{tot}$ 因Q屏蔽越差.

Non-static Case

$$\cancel{E_{ext}(r', t') = \int_{t', r'} dt' dr' \, \varepsilon(r.t\, r'.t') \, E_{tot}(r' t')}$$

传播子 G

$$\varepsilon(r.t.r'.t') \Longleftrightarrow \varepsilon(r-r'.t-t') \quad \text{Normally}.$$

$$E(q.\omega) = \varepsilon(q.\omega) E_{tot}(q.\omega).$$

Fourier: $\varepsilon(r.t) = \int d^3q \, d\omega \, \exp(-i\omega t + iqr) \, \tilde{\varepsilon}(q.\omega)$

$$\tilde{\varepsilon}(q.\omega) = \int d^3r \, dt \, \exp(i\omega t - iqr) \exp(-0^+ t) \, \varepsilon(r.t)$$

之所以有 0^+t 项，在二式中 st: $t \to \infty$ 时，积分收敛。

在一式中，二式积出 $\tilde{\varepsilon}(q.\omega)$ 得 $\varepsilon(t)$ 有 $\theta(t)$ 形式。

即 $t<0$ 不会传播

using formula. $J = \tilde{\sigma} E$ $\quad E_{ext} = \varepsilon E_{tot}$

$E = -\nabla \varphi$ $\quad J_{tot} = J_{ind} \; (J_0 = 0) \; (J_{ext} = 0)$

$$\cancel{\Rightarrow J_{ind}(q.\omega) = \sigma(q.\omega)(-iq)\varphi_{tot}(q.\omega)} \quad \sigma(q.\omega) = \frac{\tilde{\sigma}(q.\omega)}{\varepsilon(q.\omega)}$$

流守恒: $\quad -i\omega \tilde{\rho}_{ind}(q.\omega) + iq \tilde{J}_{ind} = 0$

so that: $\boxed{\varepsilon(q.\omega) = 1 + i \dfrac{4\pi \tilde{\sigma}(q.\omega)}{\omega}}$

回顾 Bruus. 我们又得到了电导与介电常数关系. 即 $\boxed{\text{涨落-耗散定理}}$

$$\begin{cases} J = \tilde{\sigma} E \\ E = \varepsilon E_{tot} \end{cases}$$

$\overset{ext}{}$

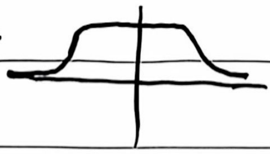
而由外电场产生 J 产生焦耳热 eg. $E_{ext} = |E| \cos\omega t$

$J_{ind}(t) = |E| \frac{1}{2}(\sigma e^{i\omega t} + \sigma^* e^{-i\omega t})$ 则: $|T \quad \cdots T$

$= \frac{1}{2}|E|^2(\mathrm{Re}\,\sigma)$ σ实部描述耗散 Dissipation

φ_{ext} 扰动产生 φ_{ind} 反映涨落.

Dissipation - Fluctuation Theory:

所谓涨落 ε. 其描述外电场诱导电场之强度. 比如原本. 我有一个分布 Fermi-Dirac  Now. 外电场 $E_{ext} \to \varphi_{ext}$ 改变此分布, 即对此分布产生一压缩.  New. 整体分布之改变由 $\mathrm{Im}\,\varepsilon$ 反映之涨落(响应). 若要从新态, 去除 φ_{ext} 回为原态. 则需花费能量. 比如新态 n. 要跃迁回原态 m. 概率为 $I_{n,m}$. n.m. 对应能量为 E_n. E_m. 总能改变为 $\sum_{n,m}(I_{n\to m})(E_n-E_m)\exp(i(E_n-E_m))$ 其能量 $\propto (\mathrm{Re}\,\sigma)$ (Bruss 中可看出), 即反映态之改变(响应), 所应消耗之能量 $(\mathrm{Re}\,\sigma)$.

因外界改变(ε)

Mar. 5 Linear-Response Theory at first-quantization.

$$\mathcal{H}_{eff} = \frac{\hat{P}^2}{2m} + \hat{\Phi}_{tot}(t) \qquad \hat{\Phi}_{tot}(t) = (-e)\int d\vec{r}\, |r\rangle \varphi_{tot}(t,r)\langle r|$$

At $t(-\infty)$ $\quad \hat{H}(t) = \frac{\hat{P}^2}{2m} \qquad f_k = f_{\text{Fermi-Dirac}}$.

At t: 加入了外场. 响应开始.

原本 $t(-\infty)$ 时 $|\psi_{k_1}(-\infty)\rangle \xrightarrow{U} |\psi_{k_1}(t)\rangle$

$|\psi_{k_2}(-\infty)\rangle \xrightarrow{U} |\psi_{k_2}(t)\rangle$.

在t时刻 k可能变. 但 $\psi(t)$ 的标签 k_1, k_2 不变. 即t态仍然、只依赖于初态. 因为演化U确定. 可逆.

$$\rho_{ind}(x,t) = (-e)\cdot 2\cdot \int_K \left(|\langle x|\psi_k(t)\rangle|^2 - |\langle x|\psi_k(0)\rangle|^2\right) f_k^{u}$$

t时刻f变，而计ρ_{ind}时电子密度$|\ |^2$标签k因仅依赖初态仍用f_k^{u}

$$|\psi_k(t)\rangle = e^{iHt}|k\rangle \qquad H = H_0 + \Phi_{tot}(t).$$

求和展开.

$$= (1-iH_0\Delta s)(1-i\hat{\Phi}_{tot}(t-\Delta s)\Delta s)\cdots\cdots(1-iH_0\Delta s)(1-i\hat{\Phi}_{tot}(t_{-\infty})\Delta s)|k\rangle$$

Linear Response ⇒ 提出1次项.

$$(\hat{\Phi}_{tot})^0 = (1-iH_0\Delta s)\cdots(1-iH_0\Delta s)|k\rangle = \exp[i\overbrace{H_0}^{\frac{k^2}{2m}}(t-t_{-\infty})]|k\rangle.$$

$$(\hat{\Phi}_{tot})^1 = \sum_s(1-iH_0\Delta s)\cdots(1-iH_0\Delta s)(\cancel{1}-i\hat{\Phi}_{tot}(s)\Delta s)\cdots(1-iH_0\Delta s)|k\rangle$$

$$= \int ds\, \exp[-iH_0(t-s)](-i\hat{\Phi}_{tot}(s))\exp[-iH_0(s-t_{-\infty})]ds\ |k\rangle$$

$$\exp(-iH_0(t-t_{-\infty}+s-t))$$

$$(\hat{\Phi}_{tot})^0|k\rangle = |\psi_{k,H_0}(t)\rangle$$

$$(\hat{\Phi}_{tot})^1|k\rangle = \int ds\, \exp[-iH_0(t-s)](-i\hat{\Phi}_{tot}(s))\exp[-iH_0(t-s)]|\psi_{k,H_0}(t)$$

此二项相加即$|\psi_k(t)\rangle$

$$\langle x|p\rangle = e^{ipx}$$

calculate $\rho_{ind}(x,t)$ 代入$|\psi_k(t)\rangle$ $\quad \hat{I} = \int|x\rangle\langle x| = \int|p\rangle\langle p|\frac{dp}{(2\pi)^3} = |p\rangle\int\langle p|$

$$\int_s(-i)ds\langle x|p\rangle\int_p\langle p|e^{-i\frac{p^2}{2m}(t-s)}|y\rangle\int_y dy\langle y|\underbrace{\hat{\Phi}_{tot}(s)e^{-ip^2/2m(t-s)}|\psi_k^0(t)}$$

$$\int_y dy\langle y|(-e)\int d\vec{r}|r\rangle\varphi_{tot}(r,s)\langle r|e^{-ip^2/2m(t}\psi_k^0(t)$$

$$\Rightarrow \int_y dy\, \varphi_{total}(y,s)\langle y|e^{-ip^2/2m(t-s)}\psi_k^0(t)\rangle \qquad r=y$$

$$= \langle x | \psi_k^0(t)\rangle + (-i)\int_s ds \int_p e^{ipx} e^{-i\frac{p^2}{2m}(t-s)} \int_y e^{-ipy}(-e)$$

$$\varphi_{tot}(y.s)\, e^{i\frac{p^2}{2m}(t-s)} \langle y | \psi_k^0(t)\rangle$$

则 $\rho_{ind} = \int_k (|\langle x|\psi_k(t)\rangle|^2 - |\langle x|k\rangle|^2) f_k^u (-2e)$ 做 Fourier

$$= \int_k \int (-堆)\,\varphi_{tot} + O(\varphi_{tot}^2) \quad f_k^u \, dk \, (-2e)$$

全部集中至 φ_{tot} 前面

而 $\varepsilon(q.w) = 1 - \frac{4\pi}{q^2} \cdot \frac{\rho_{ind}}{\varphi_{tot}} =$

$$= 1 - \frac{4\pi}{q^2} \cdot 2e^2 \int \frac{d\vec{k}}{(2\pi)^3} \cdot \frac{f^u_{k-q/2} - f^u_{k+q/2}}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + w + i0^+}$$

~~Kubo~~ Lindhard formula.

如. $\varepsilon(q.w) = 0$, 而 $\varphi_{tot} \neq 0$ 由 $\varphi_{tot} = \frac{1}{\varepsilon(q.w)} \varphi_{ext}$

即 $\varphi_{ext} = 0$ 即无外场下也有 φ_{tot}, φ_{tot} 是自发产生的. 不依赖于外场的 true Excitation. 本征模式.

如 $\varepsilon(q^0 . w(q_0)) = 0$ 即 $E_{ext} = 0$. $q = 0$ $w(0) = \Omega$ 时.

即无外场. 电荷密度不变. 仍有 $w(0)$.

$\Rightarrow$ ⊕离子以u集体右移. 负电荷以u集体左移.

$$m\ddot{u} = \underbrace{u n_{ele} (-e) \cdot 4\pi}_{(\text{高斯})} + eE_{ext}$$

$u = \cos(\frac{\Omega}{w(0)} t \cdots)$.

若 $E_{ext} \neq 0$ $\varepsilon(0.w) = 1 - \frac{\Omega^2}{w^2}$

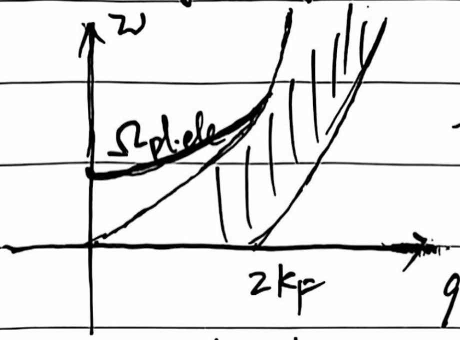
如: $\omega = 0$. $q \ll k_F$. $\varepsilon = 1 + \frac{1}{q^2\lambda}$ (之前 Tomas-Fermi Screening)

如: ω: finite; $q=0$ $\varepsilon = 1 - \Omega^2/\omega^2$

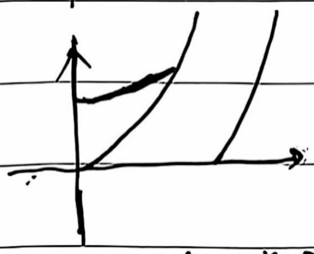
$$\Omega_{pl.ele.} \equiv \sqrt{\frac{4\pi e^2 k_F^3}{3\pi m}}$$ pl: plasma. 等离激元

如 $\varepsilon(q.\ \omega(q)) = 0$ 即 $\mathrm{Re}\,\varepsilon = 0$

$$\omega = \Omega_{pl.}\left(1 + \frac{9}{10}(q\lambda_{TF})^2 + o(q^2)\right)$$



一线 + 一区域的解.



$\mathrm{Im}\,\varepsilon = 0$.

三条线

$\varepsilon = 0 \Rightarrow$ 能够成为激发态仅此三条. 而 $\Omega_{pl.}$ 等激为真正的激发态. 余下的二线不稳定. 注意 q 为 3-D. 每个二线上 $(q.\omega)$ 总可找到 $q_1.\ q_2$ $\omega_1.\ \omega_2$ 满足 $\vec{q} = \vec{q}_1 + \vec{q}_2$ $\omega = \omega_1 + \omega_2$ 守恒条件, 则激发不稳. 而 1-D 下, 二线稳定, Bosonization.

Solving Lindhard Formula:

pre-proof: for $a \in \mathbb{C}$ $Re(a)>0$

$$\int_0^{+\infty} dx \exp(ax) = \frac{1}{a} \quad , \quad \int_0^{+\infty} dx \exp(-ax) = -\frac{1}{a}\left[\exp(-a\infty) - \exp(-a0)\right] = \frac{1}{a}$$

for a $Re(a)>0$ $\frac{1}{a} \neq 0$ $\exp(0^+\infty) = 0$

so that, $$\int_0^{+\infty} dx \exp[(-0^+ - ib)x] = \frac{1}{0^+ + ib}$$

Now $$\varepsilon(\omega, q) = 1 - \frac{4\pi}{q^2} \cdot \frac{\rho_{ind}}{\varphi_{tot}}$$

$$\rho_{ind} = (-e)\int_k \left[|\langle x|\psi_k(t)\rangle|^2 - |\langle x|\psi_k^0(t)\rangle|^2\right] f_k^{\mu} \cdot 2$$

$\psi_k(t) =$ Following Linear Response:

$$= \psi_k^0(t) + (-i)\int ds \exp[-iH_0(t-s)]\hat{\Phi}_{tot}(s) \exp(-iH_0(t-s))$$

$\psi_k^0(t)$

using: $\langle p|k\rangle = (2\pi)^3 \delta(p-k)$ $\quad \varepsilon_p = p^2/2m$

$\langle x|p\rangle = e^{ipx}$ $\quad \psi_k^0(t) = \left(e^{-i\frac{k^2}{2m}t}\right)\psi_k^0(0)$

$$1 = \int |x\rangle\langle x| d\vec{x} = |k\rangle \int_k \langle k| \frac{d^3k}{(2\pi)^3}$$

$$\psi_k(t) = \psi_k^0(t) + (-i)\int ds \int_p \underbrace{\exp[(-iH_0(t-s)]|p\rangle\langle p|}_{\text{代入} \int dp\, |p\rangle \exp(-i\frac{p^2}{2m}(t-s))\langle p|} \times \hat{\Phi}_{tot}(s) \times \int |k\rangle e[-iH_0(t-s)]\langle k| dk$$

一次量子化算符

代入 $\hat{\Phi}_{tot}(s) = (-e)\int d\vec{r}\, |r\rangle \varphi_{tot}(s.r)\langle r|$

$$\psi_k(t) = \psi_k^0(t)\left[1 + (-i)\int_s\int_p\int_{\vec{r}}\int_k |p\rangle \exp[-i\frac{p^2}{2m}(t-s)]\overset{e^{-ipr}}{\langle p|r\rangle} \varphi_{tot}(s.r)\overset{e^{irk}}{\langle r|k\rangle} \exp[-i\frac{k^2}{2m}(t-s)]\langle k| \right](-e)$$

$$\int_k -|\langle x|\psi_k(t)\rangle|-1|\langle x|\psi_k(0)\rangle|^2 f_k \qquad |\psi^0_k(t)|^2 = \left|e^{-i\frac{k^2}{2m}t}\psi^0_k(0)\right|^2 = (|k\rangle)^2 - 1$$

$$= \left(\langle x| \underset{x\langle x|k\rangle}{-\text{堆}} |k\rangle - \langle k| \underset{x\langle k|x\rangle}{-\text{堆}^\dagger} |x\rangle \right) f^{0i}_k$$ 保留 ψ_{tot} 一阶.

$$= \langle x| i(-e)\int_{sprk} e^{i\varepsilon_p(t-s)} e^{-ip(x-r)} \psi_{tot}(r,s)\, e^{-i\varepsilon_k(t-s)} e^{-ikr} |k\rangle$$

$$- \langle k|(-i)(-e)\int_{sprk} e^{-i\varepsilon_p(t-s)} e^{ip(x-r)} \psi_{tot}(r,s)\, e^{ikr} e^{i\varepsilon_k(t-s)} |x\rangle f$$

$$= (ie)\Big[\int_{sprk} e^{i(k-p)x} e^{i(\varepsilon_p-\varepsilon_k)(t-s)} e^{-i(k-p)r} \psi_{tot}(r,s)$$

$$+ (-)e^{i(p-k)x} e^{i(\varepsilon_k-\varepsilon_p)(t-s)} e^{-i(p-k)r} \psi_{tot}(r,s)\Big] f^u_k$$

$$\varphi(\omega,q) = \int_{x,t} \underset{\eta\to0^+}{e^{i\omega t-\eta t}} \exp[-iqx]\, \varphi(x,t)$$

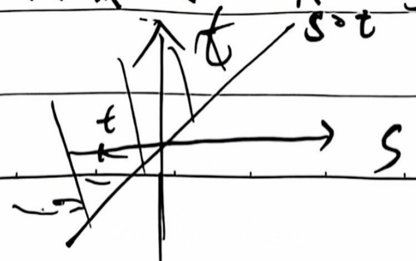
$$= (-2e)(ie)\int_{x,t,s,p,r,k} e^{(i\omega-\eta)(t-s)} e^{i(\varepsilon_p-\varepsilon_k)(t-s)} e^{(i\omega-\eta)s}$$

$$\times e^{-iqx} e^{i(k-p)x} e^{-i(k-p)r} \psi_{tot}(r,s) f^u_k$$

$$- e^{(i\omega-\eta)(t-s)} e^{i(\varepsilon_k-\varepsilon_p)(t-s)} e^{(i\omega-\eta)s}$$

$$\times e^{-iqx} e^{i(p-k)x} e^{-i(k-p)r} \psi_{tot}(r,s) f^u_k .$$

$$\Leftrightarrow \int_R dt \int_{-\infty}^{t} ds = \int_R ds \int_S^{\infty} dt = \int_R ds \int_{R^+} d(t-s)$$



$$\Rightarrow e^{[(i\omega-\eta)+(\varepsilon_k-\varepsilon_p)](t-s)} \int_R dt \int_{-\infty}^{t} ds$$

$$= \int_R ds \int_{R^+} d(t-s)\, e^{[(i\omega-\eta)+(\varepsilon_k-\varepsilon_p)](t-s)}$$

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$$= \delta\, [\eta - i\omega + i\varepsilon_k - i\varepsilon_p]^{-1}$$

for $e^{-iqx} e^{i(k-p)x}\, dx\, dp \Rightarrow \delta(k-p-q)\, dp \quad p = k-q$

$$= (2e^2) \int_{s.q.r.k} i \frac{f^u_k}{\eta - i\omega + i\varepsilon_k - i\varepsilon_{k-q}} \times e^{-i(k-(k-q))r} \varphi_{tot}(r.s) \times e^{(i\omega-\eta)s}$$

$$+ \int_{s.r.k.} (-i) \frac{f^u_k}{\eta - i\omega - i\varepsilon_k + i\varepsilon_{k+q}} \times e^{i(k-(k+q))r} \varphi_{tot}(r.s) \times e^{(i\omega-\eta)s}$$

由于 $\int_{r.s} e^{iqr} \varphi_{tot}(r.s)\, e^{(i\omega-\eta)s}$ 为 Fourier 为 $\varphi_{tot}(\omega.q)$.

$$\Rightarrow (2e^2)\left[\int_k \frac{i f^u_k}{\eta - i\omega + i\varepsilon_k - i\varepsilon_{k-q}} - \frac{i f^u_k}{\eta - i\omega - i\varepsilon_k + i\varepsilon_{k+q}}\, dk\right] \varphi_{tot}(\omega.q).$$

一项令 $k' = k - \frac{q}{2}$ $\quad \varepsilon_k \to \varepsilon_{k'+q/2} \quad \varepsilon_{k-q} \to \varepsilon_{k-q/2}. \quad f^u_k \to f^u_{k'-q/2}$

二项令 $k' = k - q/2 \quad \varepsilon_k \to \varepsilon_{k'+q/2} \quad \varepsilon_{k+q} = \varepsilon_{k'+q/2} \quad f^u_k \to f_{k'+q/2}.$

$$\Rightarrow 2e^2 \int_k \frac{i[f_{k'-q/2} + f_{k'+q/2}]}{\eta - i\omega + i\varepsilon_{k'-q/2} + i\varepsilon_{k'+q/2}}\, d^3k/(2\pi)^3\ \varphi_{tot}(\omega q).$$

Finally: $\varepsilon(q.\omega) = 1 - \frac{4\pi}{q^2} \cdot \frac{\rho_{ind}}{\varphi_{tot}}$

$$= 1 - \frac{4\pi}{q^2}(2e^2) \int \frac{d^3k}{(2\pi)^3} \cancel{\frac{f_{k-q/2} + f_{k+q/2}}{i\underset{i0^+}{\eta}\ \underset{+\omega}{}\ + \varepsilon_{k-q/2} + \varepsilon_{k+q/2}.}}\ \frac{f^u_{k-q/2} - f^u_{k+q/2}}{\omega + i\underset{0^+}{\eta} + \varepsilon_{k-q/2} + \varepsilon_k}$$

Generally，考虑外场条件下电子、离子都运动之情况，共有5部分.

外场：φ_{ext}

离子：φ^0_{ion} $\delta\varphi_{ion}$ ⟩因外场、电/离子变化之变化

电子 φ^0_{ele} $\delta\varphi_{ele}$

求介电常数φ.

在Jellium Model中，假设离子不动得ε_1(Lindhard formula)，

实际上，即假设离子受扰动$\delta\varphi_{ion}$亦算作外场 $\varphi^1_{ext}=\varphi_{ext}+\delta\varphi_{ion}$

且离子不动：$\delta\varphi_{ion}=0$ $\varphi^1_{ext}=\varphi_{ext}$ $\varepsilon_1\varphi_{tot}=\varphi_{ext}$，电子受

扰动故(φ_{ele}.$\delta\varphi_{ele}$.φ^0_{ion})介质中皆在。 在我们把δ_{ion}划分从

介质中划分至外场中Jellium将$\delta\varphi_{ion}$设为0.

另一方面，等离激元$\Omega(q\to 0)$假设电子不动，外场$\varphi^2_{ext}=\varphi_{ext}+\delta\varphi_{ele}$

影响下离子重排$\delta\varphi_{ion}$，$\Omega(q\to 0)$假设$\delta\varphi_{ele}=0$ $\varepsilon_2\varphi^2_{ext}=\varphi_{ext}$.

φ_{tot}中有$\delta\varphi_{ion}$，φ^0_{ele}，φ^0_{ion}

因此有公式：

$$\begin{cases}\varepsilon\varphi_{tot}=\varphi_{ext} \\ \varepsilon_1\varphi_{tot}=\varphi'_{ext} \\ \varepsilon_2\varphi_{tot}=\varphi''_{ext} \\ \varphi^0_{ion}+\varphi^0_{ele}=\text{const}\ (\text{静电场唯一性})\end{cases}$$

$$\varphi_{tot}=\varphi_{ext}+\varphi_{ele}+\underbrace{\varphi^0_{ion}}_{\text{const}}+\delta\varphi_{ion}+\delta\varphi_{ele}$$

$$\cancel{\varphi'_{tot}=\varphi_{ext}+\text{const}}+\quad \varphi'_{ext}=\varphi_{ext}+\delta\varphi_{ion}$$

$$\varphi''_{ext}=\varphi_{ext}+\delta\varphi_{ele}$$

(φ_{ext},ε.φ_{tot}.$\delta\varphi_{ion}$ $\delta\varphi_{ele}$).未知. 4方程. 可解关系.

✓ $\varepsilon=\varepsilon_1+\varepsilon_2-1$

$$\begin{cases}\varepsilon_1: \text{Linhard Formula. 上文公式} \\ \varepsilon_2: \Omega(\text{plasma})\end{cases}$$