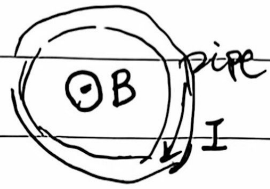
DATE NOTES

Superconductivity:

Meisner Effect Ring



Experiment 1:

①$T>T_c$ 制备 ② $T<T_c$. $\int B dl$ ↑↑，产生I ③ I越来越大不消失 10^5

Experiment 2:

①$T>T_c$ 制备 ③加缓慢 $\int B dL$ 至Max，无I产生 ③ $T<T_c$ 产生I 在 $O(1)$ 量级.
为一种非平衡效应（平衡，能最低. $I=0$）.

环状（周期边值）Schrodinger Equation

$$\mathcal{H}\Phi = E\Phi \quad \mathcal{H} = \frac{\left(-i\frac{\partial}{\partial\varphi} - \frac{\phi}{2\pi}\right)^2}{2I}$$

环中磁通 $\frac{\phi}{2\pi} = \hat{A}_\varphi$

$$\Phi(\varphi+2\pi) = \Phi(\varphi)$$

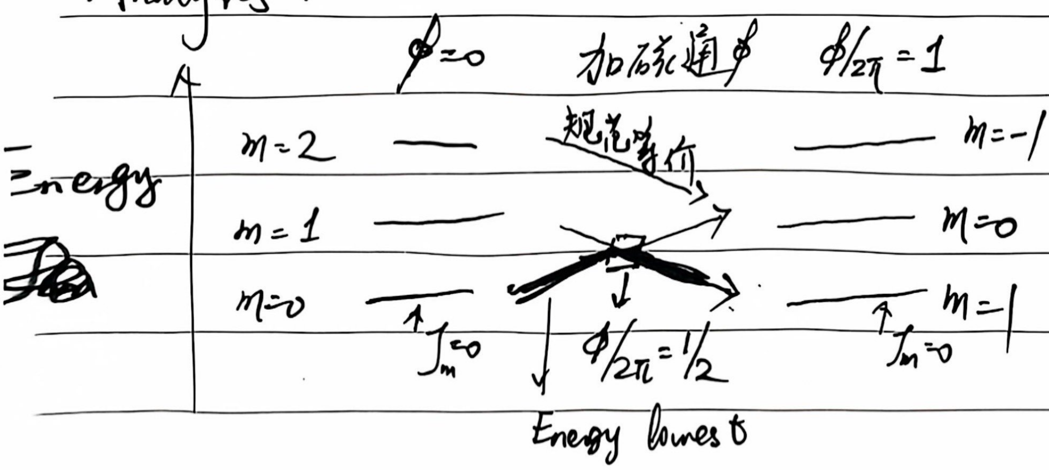
$\Rightarrow \Phi_m(\varphi) = e^{im\varphi}\frac{1}{\sqrt{2\pi}}$ 本征解

$\Rightarrow E_m = \frac{1}{2I}\left(m - \frac{\phi}{2\pi}\right)^2$

$\Rightarrow J_m(电流) = v(e) = \frac{P-qA}{m} = \frac{q_e}{I}\left(m - \frac{\phi}{2\pi}\right)$ 不是质量!

为m号本征态在宏观电流I.外场 ϕ 下所携电流量

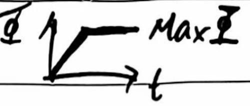
Analysis:

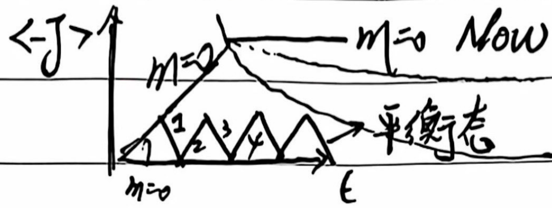


产生10^5电流. 若Fermion相消. Boson大量处同一态上 叠至10^5. 猜测为Boson.

随ϕ↑, 系统若一直缓变(平衡态)将先于 $m=0$, 后于 $m=1$ 态上. (min Energy)

现在非平衡. 如 $\phi(t)/2\pi = t$ 线性变化ϕ.

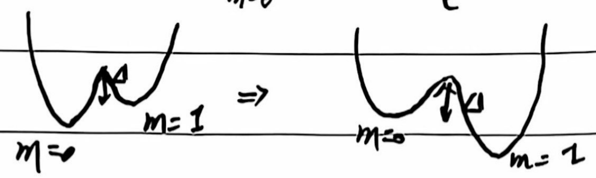




超导此衰变在10^{15}(s)量级

正常导体至Φ_{max}后$\langle J\rangle$衰减

Key is



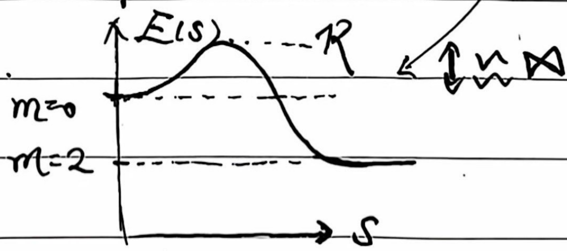
是否能穿越此Δ至$m=1$态.

Δ决定了跃迁至它本应去的态($m=1$)的时间

在超导体中Δ过大. 产生一直10^{15}量级效应.

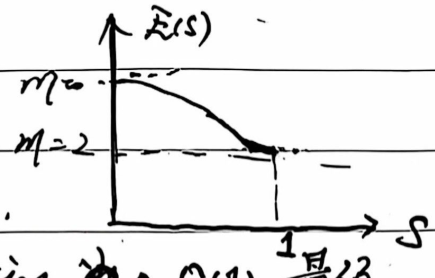
在开放系统中, High/Low Level Energy 终会Decay 在有限时间. Setting
$\Psi(s) = a(s)|m=0\rangle + b(s)|m=2\rangle$ $E = \langle H\rangle + K\int|\Psi|^2 d\Psi$ 引入K的环境干扰项, 在此非平衡体系中, 假定 $b(s)$ $S\in[0,1]$ 为增函数 由始-终可得 $E(s) = const + N_b\{-4\pi K|b(s)|^4 + [4\pi K - \underbrace{(E_0-E_2)}_{>0}]|b(s)|^2\}$

即尽管需10^{15}(s)量级, 但衰减 $\tau=\infty$ 情况不可能发生, 尽管实验中未见衰减. 但理论上必如此.



① 能垒在N_b(10^{24})量级, 因此10^{15}(s)才完成跃迁 ($K > \frac{E_0-E_2}{4\pi}$)

② ($K < \frac{E_0-E_2}{4\pi}$), 衰减在$O(1)$量级



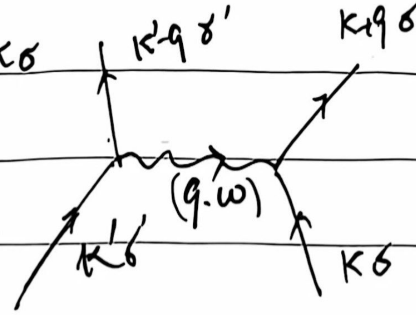
对环境K相同. 降温减至态尽为基态

$K>K_c$ 10^{15}(s). 称superconductor state.

在 $K<K_c$ 在此衰减为 Normal situation 为 $O(1)$ 量级

BCS Hamiltonian

$$\hat V = \frac{e^2}{2V}\sum_{kk'\sigma\sigma' q} a^+_{k+q\sigma} a^+_{k'-q\sigma'} V_{q\omega} a_{k'\sigma'} a_{k\sigma}$$



$$V_{q\omega} = \frac{4\pi}{q^2+\lambda_{TF}^{-2}} \times \frac{\omega^2}{\omega^2-\omega^2_{phonon}(q)}$$

$$\omega_{phonon}(q) = C_s|q|$$

$\omega = E_{k'} - E_{k'-q} = E_{k+q} - E_k = \omega$ $\omega < \omega_{phonon}$ 吸引作用

① 从上式推论 when: $-k = +k' \Rightarrow \omega = 0$ 必然吸引

Define $k'_{new} = k+q$. we have.

$$\hat V = \sum_{kk'\sigma'\sigma} V_{kk'\sigma'\sigma} a^+_{k'\sigma} a^+_{-k'\sigma'} a_{-k\sigma'} a_{k\sigma}$$ 吸引作用.费米面不稳, pair(k.k') 向外动

② And those: $\sum_{k.k'}$ 求和在 $|\varepsilon_k|\,|\varepsilon_{k'}| \le \omega_D$ 范围内近似. $V = \begin{cases} -\frac{1}{2}V_0 & \text{inside} \\ 0 & \text{outside} \end{cases}$

$\varepsilon_k = E_k - \mu$ ω_D 为费米球ω 代表仅能激发费米海附近,

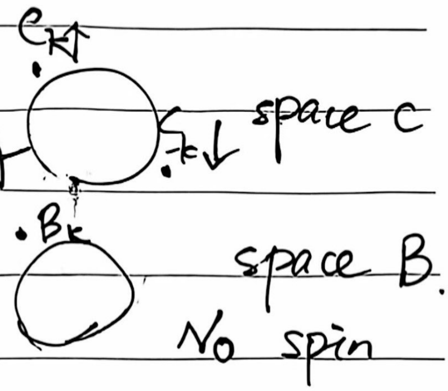
③ ⇒ 仅考虑自旋相反. 称 s-wave superconductor.

Finally, we arrive.

$$\hat{\mathcal{H}}_{BCS} = \sum \varepsilon_k a^+_{k\sigma} a_{k\sigma} \underset{\Delta}{-} V_0 \sum_{kk'}^{\text{inside}} a^+_{k'\uparrow} a^+_{-k'\downarrow} a_{-k\downarrow} a_{k\uparrow}$$

Instable $|\Omega\rangle$ ↑

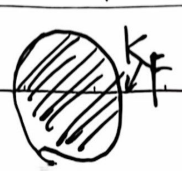
Now, define $B^+_k = C^+_{k\uparrow} C^+_{-k\downarrow}$ 产生一对费米子



$$|\Omega\rangle = \prod_\sigma \prod_{|k|\le k_F} C^+_{k\sigma}|\text{empty}\rangle$$

$$= \prod_{|k|\le k_F} B^+_k |\text{empty}\rangle$$

=



to prove $|\Omega\rangle$ = (///) unstable in $\hat{\mathcal{H}}_{BCS}$. we prove $\exists \Phi$, $\langle H\rangle_\Phi \le \langle H\rangle_\Omega$

$$|\Phi\rangle = \Big(\overset{B_K}{|\Omega\rangle} + \overset{B_{K'}}{|\Omega\rangle} \Big) \frac{1}{\sqrt{2}}$$ ~~对~~ (k, k' 额外对总能降低作用 $(-V_0)$.

$$= \frac{1}{\sqrt{2}}\left(B_K^+|\Omega\rangle + B_{K'}^+|\Omega\rangle \right)$$

$$\hat{\mathcal{H}}_{BCS} = \hat{\mathcal{H}}_0 - V_0 \sum B_{K'}^+ B_K$$

计算 $\langle \hat{\mathcal{H}}_{BCS}\rangle_\Phi = -V_0 < 0$ 必 $\le \langle H\rangle_\Omega$

Generally. $|\psi_{pair}\rangle = \sum_{\substack{k>k_F \\ k, \varepsilon_k>0}} \alpha_k B_k^+ |\Omega\rangle$; $\sum_{k>k_F} |\alpha_k|^2 = 1$

How to choose α_k. So that $\langle \hat{\mathcal{H}}_{BCS}\rangle_{\psi_{pair}}$ Lowest ?

一个带条件的极值问题, using Langrange 乘子.

$$E(\alpha_k) = \langle \psi_{pair}(\alpha_k) | \hat{\mathcal{H}}_{BCS} | \psi_{pair}(\alpha_k) \rangle$$

$$0 = \frac{\partial}{\partial \alpha_{k_1}} \Big[E(\alpha_k) - \lambda \Big(\sum_{k>k_F} |\alpha_k|^2 - 1 \Big) \Big] \quad (\forall k.)$$

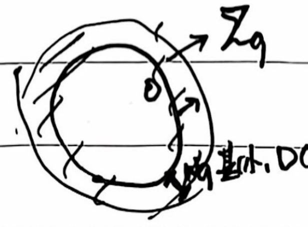
So that we solve $\alpha_k(\lambda)$.

Like a miracle. $\alpha_k(\lambda)$ 代入 $E(\alpha_k)$ 式中

$$\Rightarrow E_{min} = \lambda$$

$\lambda = E_{min}$ 代回原方程 ~~参~~ k) 消去 $\alpha_k . \alpha_q$.

$$1 = -V_0 \sum_{0<\varepsilon_q\le w_D} \frac{1}{E_{min} - \langle H_{BCS}\rangle_\Omega - 2\varepsilon_q}$$



So that we can solve E_{min}

$$\sum_{0<\varepsilon_q\le w_D} \longrightarrow \int_0^{w_0} d\varepsilon_q\, D(\varepsilon_q) \approx \int_0^{w_D} d\varepsilon_q\, D(w_D)$$

so that:

$$E_{min}-E_{\Omega} = \frac{-2W_D}{e^{2/V_0D_0}-1} \simeq \underset{\text{费米能量}}{(-2W_D)}\cdot \exp\Big(-\underset{\text{费米面态密度}}{\frac{2}{V_0D_0}}\Big)$$

说明 $|\psi_{pair}\rangle$ 中加一对减 $(-2W_D)$ 能量，且 exp 项无法通过 $\hat{H}_{BCS}$ 对 V_0 微扰展开得到，(对 exp 项 Taylor 展开后任意 $O(V_0D_0)$ 系数为零)。

对 $V_0>0$ (库伦吸引，而对 $V_0<0$ 排斥作用费米面稳定(更强 Pauli 不相容不变本质)，而 $V_0>0$ 与 Pauli 不相容互斥)

Boson + $V<0$ superfluid   Fermion + $V_0>0$ superconductor.

$|\psi_{pair}\rangle$ 代表了 $|\Omega\rangle(1+\alpha_k B_k^\dagger)$ 在对一阶 $B_k^\dagger$ 做的 α_k 优化，真正的基态为 $|\Omega\rangle(1+\alpha_k B_k^\dagger+\beta_{kk'}B_k^\dagger B_{k'}^\dagger+\gamma_{kk'k''}B_k^\dagger B_{k'}^\dagger B_{k''}^\dagger)\cdots\cdots$ 无限之优化

Mean-Field:

$$\underbrace{c^\dagger_{k'\uparrow}c^\dagger_{-k'\downarrow}}_{A}\ \underbrace{c_{-k\downarrow}c_{k\uparrow}}_{B}$$

$$\hat{\mathcal{H}}_{BdG} = \sum_{k\sigma}\varepsilon_k c^\dagger_{k\sigma}c_{k\sigma} + (-V_0)\sum_{k}{}'\left(\Delta_T c_{-k\downarrow}c_{k\uparrow} + \Delta_T^* c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\right)$$

$$\Delta_T = \sum_k{}' \langle c^\dagger_{k\uparrow}c^\dagger_{-k\downarrow}\rangle_T$$

Δ_T 复数不太好，Define: $c^{New}_{k\sigma} = e^{i\theta/2}c_{k\sigma} \Rightarrow \Delta_T\in\mathbb{R}$

or

$$h_{BdG} = \frac{1}{2}\begin{pmatrix}c^\dagger_{p\uparrow} & c_{-p\downarrow}\end{pmatrix}\begin{pmatrix}\varepsilon_p & -\Delta\\ -\Delta & -\varepsilon_p\end{pmatrix}\begin{pmatrix}c_{p\uparrow}\\ c^\dagger_{-p\downarrow}\end{pmatrix} \quad \Psi^\dagger_p = \begin{pmatrix}c_{p\uparrow}\\ c^\dagger_{-p\downarrow}\end{pmatrix}$$

$$\mathcal{H}_{BdG} = \sum_p \Psi_p^\dagger h_{BdG}\Psi_p + \text{const} \quad \{(\Psi_p)_\alpha, (\Psi_{p'})_{\beta'}\} = \delta_{pp'}\delta_{\alpha}$$

Calculate $|\Psi_{pair}\rangle$ in detail:

(a) $E_\Omega = \langle \Omega | \sum_k C^\dagger_{k\sigma} C_{k\sigma} | \Omega \rangle - V_0 \sum_{\le w_D}^{k,k'} \langle \Omega | C^\dagger_{k'\uparrow} C^\dagger_{-k'\downarrow} C_{-k\downarrow} C_{k\uparrow} | \Omega \rangle$.

$\Downarrow$ Ω inside w_D $\times 2(\uparrow$ and $\downarrow)$. $\qquad \Downarrow$ $\delta_{k,k'}$ Ω inside w_D.

$$= 2\sum_{\varepsilon_k \le 0}^{k} \varepsilon_k - V_0 \sum_{-w_D \le \varepsilon_k \le 0}^{k} 1$$

(b)(i) $E(\alpha_k) = \langle \Psi_{pair}(\alpha_k) | \hat{H}_{BCS} | \Psi_{pair}(\alpha_k) \rangle = \sum_{kq} \alpha_k^* h_{kq} \alpha_q$

$$0 = \partial/\partial(\alpha_k) \left[E(\alpha_k) - \lambda\left(\sum_{k>k_F} |\alpha_k|^2 - 1\right)\right] \quad \forall_k.$$

$$\Rightarrow 0 = \frac{\partial}{\partial \alpha_q^*}\left[\sum_{k,q} \alpha_k^* h_{kq}\alpha_q - \lambda\left(\sum_{k>k_F} \underbrace{|\alpha_k|^2}_{(\alpha_k^*\alpha_k)} - 1\right)\right]$$

$$\Rightarrow 0 = \left(\sum_{k>k_F} h_{kq} \alpha_q^*\right) - \lambda \alpha_k$$

$$\Rightarrow E_{min} = \sum_k \left(\sum_{q} h_{kq} \alpha_{q}\right) \alpha_k^*$$

$$= \sum_k (\lambda \alpha_k) \alpha_k^* = \lambda \sum_k |\alpha_k|^2 = \lambda$$

(ii) $\Rightarrow$ $0 = \frac{\partial}{\partial \alpha_q}\left[\sum_{kq} \alpha_k^* h_{kq} \alpha_q - E_{min}\left(\sum |\alpha_k|^2 - 1\right)\right]$

其中 E_{min} 又是 $\langle \Psi_{pair}(\alpha_k) | \hat{H}_{BCS} | \Psi_{pair}(\alpha_k) \rangle$

$= \langle \Omega | \left(\sum_k \alpha_k B_k^\dagger\right)^\dagger \hat{H}_{BCS} \left(\sum_q \alpha_q B_q^\dagger\right) | \Omega \rangle$ 因此

故 $h_{kq} = \langle \Omega | B_k \hat{H}_{BCS} B_q^\dagger | \Omega \rangle$

$$= \underbrace{\langle \Omega | B_k \hat{H}_0 | B_q^\dagger | \Omega \rangle}_{\delta_{kq} \cdot 2\varepsilon_k} + \langle \Omega | B_k (-V_0) \sum_r B_r^\dagger B_r B_q^\dagger | \Omega \rangle$$

对第二项 k=q 的项：$\cancel{\sum_k \alpha_k^+ \alpha_k h_{k.k}}$ - 代入

$$\Rightarrow E_{min}\alpha_k = \sum_{0\sim w_D} h_{kq}\alpha_q$$

h_{kq} 第一项 $\sum_q \delta_{k.q}\langle B_k^+\Omega|\hat{H}_0|B_q^+\Omega\rangle \alpha_q^+\alpha_q\alpha_q$

$= [E_{\Omega\text{前半部分}} + \underset{\text{outside}}{\varepsilon_k}(\sum_q \alpha_q^+\alpha_q)]\alpha_k$ $\underset{(a)\text{问}}{2\varepsilon_k}$ 能量 + 在费米面在外的k能量 ε_k.

$= [E_{\Omega\text{前半部分}} + \underset{\text{outside}}{2\varepsilon_k}]\alpha_k$

② h_{kq} 第二项

~~① $\delta_{kq} \Rightarrow E_\Omega$ 后半部分 同①~~

② $k\neq q$ $(-V_0)\left[\langle ◐, k'|\sum_r B_r^+B_r|◐, k''\rangle\right]$

对 r inside 与 ◐ 作用，为 E_Ω 后半部分内积，

对 r outside，为 $k.k''$ 为 1，

对 r inside outside 不提供，

$$= E_{\Omega\text{后半部分}}\alpha_k + (-V_0)\sum_{\text{outside}}\alpha_{k'}$$

$$\Rightarrow E_{min}\alpha_k = [E_{\Omega\text{前半}} + \underset{\text{outside}}{2\varepsilon_k} + E_{\Omega\text{后半}}]\alpha_k + (-V_0)\sum_{\text{outside}}\alpha_{k'}$$

$$\Rightarrow (E_{min} - E_\Omega - 2\varepsilon_k)\alpha_k = -V_0\sum_{0<\varepsilon_k'<w_D}\alpha_{k'}$$

( ) $\boxed{(c)}$ $$\alpha_k = -\frac{V_0}{E_{min}-E_\Omega-2\varepsilon_k}\sum\alpha_{k'}$$

$$\cancel{\sum_{\text{outside}}\alpha_k} = \sum_{\text{outside}}\frac{V_0}{E_{min}-E_\Omega-2\varepsilon_k}\cancel{\sum_{\text{outside}}\alpha_{k'}}$$

$$\Rightarrow 1 = V_0\sum_{\text{outside}}\frac{1}{2\varepsilon_k+E_\Omega-E_{min}}.$$

(d) $\Sigma_{outside} = \int_0^{W_D} d\varepsilon_q\, D(W_q) \approx \int_0^{W_D} d\varepsilon_q\, D(W_D) \nearrow D_0$

$\Rightarrow 1 = \int_0^{W_D} d\varepsilon_q \frac{1}{2\varepsilon_q + E_\Omega - E_{min}} D_0 V_0$

$= \frac{1}{2} D_0 V_0 \ln \frac{2W_D + E_\Omega - E_{min}}{E_\Omega - E_{min}} \Rightarrow \left(1 + \ln \frac{2W_D}{E_\Omega - E_{min}}\right)$

$e^{\left(\frac{1}{D_0V_0} - 1\right)} = \frac{2W_D}{E_\Omega - E_{min}}$; $E_{min} - E_\Omega = \frac{-2W_D}{e^{\frac{2}{D_0V_0}} - 1}$

when: $V_0 D_0 \ll 1$

$\Rightarrow E_{min} - E_\Omega \approx (-2W_D) \cdot \exp\left(-\frac{2}{V_0 D_0}\right)$.

(e) $\exp\left(-\frac{2}{V_0D_0}\right)(-2W_D)\Big|_{V_0D_0 \to 0} = 0$

$\exp\left(-\frac{2}{V_0D_0}\right)(-2W_D) \cdot (-2/D_0) \cdot \frac{1}{V_0^2}\Big|_{V_0 \to 0}$.

指数 0 × 级数 ∞ = 0

⋮ ⋮

余下级数皆如此. $\forall$ Taylor Expand at V_0^2 系数 0.

(f). 因此. 微扰 H_{BCS} =

一阶 $\langle H_1 \rangle$ at V_0^1 级同 Taylor V_0^1 为 0

二阶. $\frac{\langle V_0^2 \rangle}{E' - E}$ at V_0^2 级同 Taylor V_0^2 为 0

⋮ ⋮

$\forall$ 阶皆 0. perbubation 不可用

(g). $-V_0 > 0$ 排斥力, 即 $E_{min} - E_\Omega = \frac{-2W_D}{0-1} = 2W_D > 0$

$\Rightarrow E_{min} > E_\Omega$ 原本全在费米面内 $|\Omega\rangle$ 为稳定态.

$\Rightarrow$

Bogoliubons.

$$\mathcal{H}_{BdG} = \sum_p \Psi_p^+ \underbrace{\begin{pmatrix} \varepsilon_p & -\Delta_T \\ -\Delta_T & -\varepsilon_p \end{pmatrix}}_{\rightsquigarrow h_{bdg}} \Psi_p \qquad \Psi_p = \begin{pmatrix} C_{p\uparrow} \\ C^+_{-p\downarrow} \end{pmatrix}$$

$$= \varepsilon_p \tau_z + (-\Delta_T)\tau_x$$

solve h_{bdg} 本征值.向量.

$E_{p.1} = \sqrt{\varepsilon_p^2+\Delta^2}$ $\quad W_{p.1} = \begin{pmatrix} u_p \\ -v_p \end{pmatrix}$ $\quad u_p = \sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_p}{\sqrt{\varepsilon_p^2+\Delta^2}}\right)}$

$E_{p.2} = -E_{p.1}$. $\quad W_{p.2} = \begin{pmatrix} v_p \\ u_p \end{pmatrix}$ $\quad v_p = \sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_p}{\sqrt{\varepsilon_p^2+\Delta^2}}\right)}$

$h_{bdg} = E_{p1} W_{p1} W_{p1}^+ + E_{p2} W_{p2} W_{p2}^+$

$\hat{\mathcal{H}}_{bdg} = \sum_{p.n=1.2.} E_{pn} \Psi_p^+ W_{pn} W_{pn}^+ \Psi_p$

Define: $\mathcal{I}_{pn} = W_{pn}^+ \Psi_p \quad (1\times 1)$ we create "$\mathcal{I}_p^+$" like quasiparticle.

$\mathcal{H}_{BdG} = \sum_{pn} E_{pn} \mathcal{I}_{pn}^+ \mathcal{I}_{pn} \rightsquigarrow$

$|\Omega\rangle = \Pi_{p.n.} (\mathcal{I}_{p.n}^+)^{N_{p.n}} |G.S\rangle$

$\cancel{E = E_{G.S} + \sum_{p.n=1.} N_{p.n} E_{p.n.}}$

$\hookrightarrow$ Is $|\Omega\rangle$ All Excitation? $\mathcal{I}$如同 c 为 "Fermion" $\mathcal{H}_{free}$为费米球$|G.S\rangle$ $\mathcal{H}_{bdG}$类似,而$\mathcal{H}_{free}$的Excitation有限制①不能费米面内产生;②可在费米面内挖去,如 $(C^+_{out} C_{in})|G.S\rangle$ 如果挖去inside 反而提升能量,亦为Excitation.故包含一切的$\mathcal{H}_{free}$的Excitation为

$$= \underbrace{\prod_{\varepsilon_p>0} (C^+_{p\sigma})^{N_{p\sigma}}}_{填入外E\uparrow} \underbrace{\prod_{\varepsilon_p<0} (C_{p\sigma})^{N_p}}_{挖去内E\uparrow} |\text{●}\rangle$$

~~类似地做$\mathcal{H}_{BDG}$的Excitation~~.更简洁表述上式. $\{b^+.b\} = 1$

Def: $b_{k\sigma} = \begin{cases} C^+_{k\sigma} & \varepsilon_k > 0 \\ C_{-k,-\sigma} & \varepsilon_k < 0 \end{cases}$ $\quad$ Excitation: $\prod_{k\sigma} (b^+_{k\sigma})^{N_{k\sigma}} |\text{●}\rangle$

$\mathcal{H}_{free} = \sum_\sigma \left[\sum_{k,\ \varepsilon_k>0} |\varepsilon_k| b^+_{k\sigma} b_{k\sigma} + \sum_{k,\ \varepsilon_k<0} -|\varepsilon_k| b_{k\sigma} b^+_{k\sigma} \right]$

so that $b_{k\sigma} |\text{●}\rangle = 0$

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$\mathcal{H}_{free} = \sum_{\sigma k} |\varepsilon_k| b_{k\sigma}^{+} b_{k\sigma} + const$

So that Excitation Energy 从 $\varepsilon_k \to |\varepsilon_k| \geq 0$

True Excitation $E = E_{G.S} + \sum N_{k\sigma} |\varepsilon_k|$

那末，H_{BDG} 的 All Excitation 为何？

$\mathcal{H}_{BDG} = \sum_{\substack{n=1.2 \\ p}} E_{pn} I_{pn}^{+} I_{pn}$ 类似 $\sum_{k\sigma} \varepsilon_k C_{k\sigma}^{+} C_{k\sigma}$

也要新定义算符 因为 $E_{p2}<0$ 反而降低能量

$I_{p.2}^{+\,New} = I_{-p.2}^{+} \Rightarrow E_{p.2}^{New} = -E_{-p.2} > 0$

而类似 H_{free} H_{BDG} 亦可写作：

$\mathcal{H}_{BDG} = \sum_{p.n} E_{p.n} I_{pn}^{+} I_{pn} + \underline{\underline{Const}}$ where $E_{p1} = E_{p2}^{(New)} > 0$ $\sqrt{\varepsilon_p^2+\Delta^2}$ (一激二重简并 有 Gap Δ) 与基态

此时 E_{p1} 为 I_{p1} 准粒子能量 >0 故 $I_{p1}|G.S\rangle = 0$

$(u_p C_{p\uparrow} - v_p C_{-p\downarrow}^{+})|G.S\rangle = 0$

猜出 $|G.S\rangle = \prod_p (u_p + v_p C_{p\uparrow}^{+} C_{-p\downarrow}^{+})|empty\rangle$

ie $|BCS\rangle$ state

可见 $|G.S\rangle$ 中有 0.2.4… 个粒子 $\hat{N}|BCS\rangle \neq N_{ele}|BCS\rangle$

DATE

Exercise: p-wave superconductor

$$H_{p+ip} = \sum_p \varepsilon_p C_p^\dagger C_p + \left(\Delta_p C_p^\dagger C_{-p}^\dagger + \Delta_p^* C_{-p} C_p\right)$$

$$\varepsilon_p = \frac{P_x^2 + P_y^2}{2m} - \mu$$

$$\Delta_p = (P_x + iP_y)\Delta$$

Set: $\Psi_p^\dagger = (C_p^\dagger, C_{-p})$

For: $H_{p+ip} = \sum_p \Psi_p^\dagger \begin{pmatrix} A & B \\ C & D \end{pmatrix} \Psi_p + \text{const}$

$$= \sum_p A \cdot C_p^\dagger C_p + B \cdot C_p^\dagger C_p^\dagger + C \cdot C_{-p} C_p + D\, C_{-p} C_{-p}^\dagger$$

-- 对应，则: $A = \varepsilon_p$

$$D C_{-p} C_{-p}^\dagger = -D C_{-p}^\dagger C_{-p} + \underbrace{1 \cdot D}_{\text{const}}$$

$$D = -\varepsilon_{-p}$$

$$B C_p^\dagger C_{-p}^\dagger = -B C_{-p}^\dagger C_{-p}^\dagger$$

$$B = -\Delta_p$$

$$C \cdot C_{-p} C_p = \Delta_p^* C_{-p} C_p$$

$$C = \Delta_p^*$$

$$\therefore h'_{Bdg} = \begin{pmatrix} \varepsilon_p & -\Delta(P_x + iP_y) \\ \Delta(P_x - iP_y) & -\varepsilon_{-p} \end{pmatrix}$$

Solving: Eigenvalue:

$$E_{p,1} = \sqrt{\varepsilon_p^2 - |\Delta_p|^2} \qquad E_{p,2} = -\sqrt{\varepsilon_p^2 - |\Delta_p|^2}$$

即然如此，那么之后所有讨论全同 S-wave 不过 $\Delta^2 \to -|\Delta_p|^2$

$$u_p = \sqrt{\frac{1}{2}\left(1+\frac{\varepsilon_p}{\sqrt{\varepsilon_p^2-|\Delta_p|^2}}\right)} \qquad W_{p1} = \begin{pmatrix} u_p \\ -v_p \end{pmatrix}$$

$$v_p = \sqrt{\frac{1}{2}\left(1-\frac{\varepsilon_p}{\sqrt{\varepsilon_p^2-|\Delta_p|^2}}\right)} \qquad W_{p2} = \begin{pmatrix} v_p \\ u_p \end{pmatrix}$$

令 $\gamma_{pn} = W_{pn}^{+} \Psi_p$. $\qquad \Psi_p^{+} = (C_p^{+}, C_{-p})$

$$\mathcal{H}_{BDG} = \sum_{pn} E_{pn} \gamma_{pn}^{*} \gamma_{pn} \qquad E_{pn} = \sqrt{\varepsilon_p^2 - |\Delta_p|^2}$$

Using "Real Excitation" $E_{p.2}^{New} = -E_{-p.\tilde{2}}$

$$\gamma_{pn} = u_p C_p - v_p C_{-p}^{+}$$

Finally.

$$|G.S\rangle = \prod_p \left(u_p + v_p C_p^{+} C_{-p}^{+}\right) |\text{empty}\rangle$$

同Exercise 仅可验证 $\gamma_p |G.S\rangle = 0$

不过是 $u_p. v_p$ 形式有改变而已.

$$\varepsilon_p = \frac{p^2}{2m} - \mu$$

$$\Delta_p = (p_x + i p_y)\Delta$$

如课上步骤 $\frac{1}{V}\sum_p' u_p v_p \exp(i\vec{p}\cdot\vec{r})$

$$= \frac{1}{V}\sum_p' -\frac{\sqrt{|\Delta_p|^2}}{\sqrt{\varepsilon_p^2-|\Delta_p|^2}} \exp(i\vec{p}\cdot\vec{r}). \qquad |\Delta_p|^2 = p^2\Delta^2$$

$\approx$ 积出为 $\quad r\exp\left(-\frac{r}{a}\right)$ 形式

如同 H 原子 $l=1$ wave function. 应 $-\frac{i\partial}{\partial\phi}\Psi = 1\Psi$

因此，其 wave Function 应交换反对称，如同 $l=1$ 对应之 P 轨道一样的 wave-Function 故称 p-wave superconductor

DATE

Mar. 20th.

Gap is $\Delta_T = V_0 \sum_{K}' \langle C^{+}_{K\uparrow} C^{+}_{-K\downarrow} \rangle_{T.\,BCS}$ ($K.\ \xi_K \in (-w_D, w_D)$)

($T=0$)

$$\Delta_{T=0} = V_0 \sum_{K}' \langle empty \Big| \prod_{p} (u_p + v_p C_{-p\downarrow} C_{p\uparrow}) \Big| C^{+}_{K\uparrow} C^{+}_{-K\downarrow} \Big|$$

$$\prod_{p'} (u_p + v_p C^{+}_{p\uparrow} C^{+}_{-p\downarrow} \Big| empty \rangle$$

$$\Delta_{T=0} = V_0 \sum_{K}' v_K u_K = \frac{V_0}{2} \sum_{K}' \frac{\Delta_{T=0}}{\sqrt{\xi_K^2 + \Delta^2_{T=0}}}$$

solve 此方程. 即得出 $\Delta_{T=0}$ 自洽场方程

$$1 = \frac{V_0}{2} \cdot \sum_{K}' \cdot \frac{1}{\sqrt{\xi_K^2 + \Delta_0^2}}$$

$$\sum_{K}' = \int_{-w_D}^{w_D} d\xi_K \cdot D_0$$

$$\Rightarrow \quad 1 = D_0 V_0 \sinh^{-1} (w_D / \Delta_0)$$

when: $w_D \gg \Delta_0$.

$$\Delta_0 \approx 2 w_D \exp\left(-\frac{1}{D_0 V_0}\right)$$

Gap between First Excited state and Ground state

Can't Taylor Expand, 微扰 Failed.

Cooper Pair Wave Function.

$\Phi(x_c, r) = C^+_{k\uparrow} C^+_{-k\downarrow} |\Omega\rangle$ $\qquad C_x = \sum_k e^{ikx} C_k$

or $\quad C^+_{x+\frac{r}{2}\uparrow} C^+_{x-\frac{r}{2}\downarrow} |\Omega\rangle$

$\langle \Phi(x_c, r) | \Omega \rangle = \frac{1}{V} \sum'_p u_p v_p \exp(ipr) \Rightarrow$

为p空间波函数 — Fourier 变换.

$$\sum_p \frac{\Delta}{\sqrt{\varepsilon_p^2 + \Delta^2}} \exp(i\vec{p}\vec{r}).$$

一个与p方向无关、仅与p大小有关函数 Fourier后亦仅与r大小有关.

$$\Downarrow$$

$$\exp(i k_F |r|) \cdot \exp\left(\frac{-|r|}{\xi_{BCS}}\right)$$

which means $L_z = -i\frac{\partial}{\partial\varphi}$ $\quad L_z \cdot \Phi = 0$ 球状角量子零

$\Rightarrow$ Cooperpair S-wave Function 为交换对称.

At Finite T

$\Delta_T = V_0 \sum_K \langle C^+_{K\uparrow} C^+_{-K\downarrow} \rangle_T$ 代入: $\begin{pmatrix} C_{k\uparrow} \\ C^+_{-k\downarrow} \end{pmatrix} = \begin{pmatrix} u_k & v_k \\ -v_k & u_k \end{pmatrix} \begin{pmatrix} \tau_{k1} \\ \tau_{k2} \end{pmatrix}$.

代入为:

$V_0 \langle 1 - I^+_{k1} I_{k1} + I^+_{k2} I_{k2} \rangle_T$ 代入: $\langle I^+_{k1} I_{k1} \rangle_T = \dfrac{1}{1 + \exp\left(\frac{1}{kT} E_{k1}\right)}$

☆ 超导体平均场用巨正则系, 化学势已隐藏于 E_{k1} 中

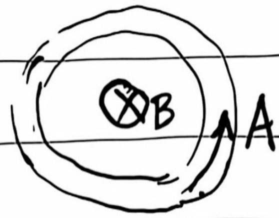
最终有一个复杂 Δ_T 形式. 得 $\Delta_T \overset{T=T_C}{=} 0$ $\quad T_C = 1.13\, W_D \exp\left(-\frac{1}{D_0 \ldots}\right)$

$$\Delta_T \propto \sqrt{1 - \frac{T}{T_C}}$$

Mar. 29th.

Now we can calculate Ring Meisner Effect via BCS theory

$$\oint \tilde{A}_z(-e) = \underbrace{e\tilde{F}}_{\text{磁通}}$$



$\vec{A}$ average $\quad A_z = \frac{F}{L_z}$

$\boxed{\text{Step 1}}$

we calculate $\langle J\rangle = -\frac{e}{V}\sum_{k\sigma}\frac{\vec{k}+e\vec{A}}{m}\langle n_{k\sigma}\rangle$

速度从电磁场 Hamiltonian.

这并不严谨，因为需满足平移对称性 $\langle a^\dagger_{k} a_{k'}\rangle = 0$ if $k' \neq k$
外场满足此才对称性才可. 严谨推导见上一本笔记 chapter 2.

$\boxed{\text{Step 2}}$

$$\mathcal{H}_{BDG,\tilde{F}} = \sum_K \tilde{\xi}_k c^\dagger_{k\sigma}c_{k\sigma} - \Delta_0\sum_k\left(c_{-k\downarrow}c_{k\uparrow} + c^\dagger_{k\uparrow}c_{-k\downarrow}\right)$$

$$\tilde{\xi}_k = \frac{k_x^2+k_y^2+(k_z+eA_z)^2}{2m} - \mu$$

$$h^{(p)}_{bdg} = \left[\xi_p + \frac{(eA_z)^2}{2m}\right](\text{Pauli}_z) - \Delta(\text{Pauli}_x) + \frac{p_z e A_z}{m}\hat{I}_{2\times 2}$$

solving Bogoliubons

$$E_{p1} = E^{New}_{p,2} = \sqrt{\left[\xi_p + \frac{(eA_z)^2}{2m}\right]^2 + \Delta^2} + \frac{p_z e A_z}{m}.$$

$$\begin{pmatrix} c_{p\uparrow} \\ c^\dagger_{-p\downarrow}\end{pmatrix} = \begin{pmatrix} \hat{u}_p & \hat{v}_p \\ -\hat{v}_p & \hat{u}_p \end{pmatrix}\begin{pmatrix} \tilde{I}_{p,1} \\ \tilde{I}^\dagger_{-p2}\end{pmatrix}$$

$$\hat{u}_p = \frac{1}{\sqrt{2}}\sqrt{1 + \frac{\xi_p + \frac{(eA_z)^2}{2m}}{\sqrt{\left[\xi_p + \frac{(eA_z)^2}{2m}\right]^2 + \Delta^2}}} \qquad \hat{v}_p = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{\xi_p + \frac{(eA_z)^2}{2m}}{\sqrt{\left[\xi_p + \frac{(eA_z)^2}{2m}\right]^2 + \Delta^2}}}$$

DATE . . NOTES

For some p. E_p might be negative now. to avoid ~~the~~ such situation that Enegy of Excitation is lower than Ground state, if $E_p>0$ $\Sigma_p|G.S\rangle=0$; if $E_p<0$ $\Sigma_p^+|G.S\rangle=0$. solving $|G.S\rangle$ similarly. we have

$$|G.S\rangle=\prod_{pE_p>0}(\hat{u}_p+v_pC^+_{p\uparrow}C^+_{-p\downarrow})\prod_{pE_p<0}(\hat{v}_p-\hat{u}_pC^+_{p\uparrow}C^+_{-p\downarrow})|empty\rangle$$

is $|G.s\rangle$ of H_{BDG} under the A_z External Field.

Step 3 我们看见了实验中的J，在此环中T<Tc逐渐增大。
现在 $\langle J\rangle=-e/_V\Sigma_{k\sigma}\frac{\vec{k}+e\vec{A}}{m}\langle n_{k\sigma}\rangle$ 即计算 $\langle n_{k\sigma}\rangle$ 利用step2.

$$\langle n_{k\sigma}\rangle=\langle G.S|n_{p\uparrow}|G.S\rangle+\langle G.S|n_{p\downarrow}|G.S\rangle$$

$(n_{-p\uparrow}, n_{-p\downarrow}$ 同 $n_{p\downarrow}, n_{p\uparrow})$

cooper pair 2 对.

At $T=0$. $\langle J\rangle=-\frac{e}{V}\sum_k\frac{k_z+eA_z}{m}\langle G.S|n_{k\uparrow}|G.S\rangle+\frac{k_z+eA_z}{m}\langle G.S|n_{k\downarrow}|G.S\rangle$

$$=-\frac{e}{V}\sum_k\frac{k_z+eA_z}{m}\langle G.S|n_{k\uparrow}|G.S\rangle+\frac{-k_z+eA_z}{m}\langle G.S|n_{-k\downarrow}|G.S\rangle$$

cooper pair

在 $T=0$. 所有电子均处于SC. 均为 cooper pair

$$=-e^2\frac{n_e l_e}{m}A_z$$

$$\Rightarrow\ \vec{J}=-e^2\frac{n_e l_e}{m}\vec{A}\quad(T=0)\qquad\boxed{\text{London Function}}$$

$\vec{A}$有Gauge. $\vec{J}$ why not? $\Rightarrow$ Nambu.

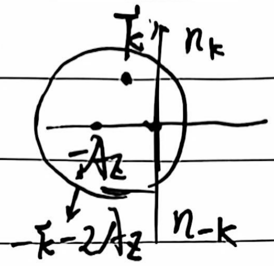
DATE . .

At Finite T: $T>T_c$. 同上做法.

将 $n_{k\downarrow} \to n_{-k\downarrow}$ 然而 Finite T 下并不对称.

suppose. T s.t. $\Delta_T \to 0$ 费米面从原点0.由于 A_z 移至 $-A_z$.

$\therefore k \to -k-2A_z$



$\Rightarrow \underline{(-k-2A_z)+A_z} + k+A_z$

$=0 \Rightarrow \langle J\rangle = 0$

At Finite T. $T<T_c$ 一部分 cooper pair 被激发

$$\begin{pmatrix} c_{p\uparrow} \\ c^+_{-p\downarrow}\end{pmatrix} = \begin{pmatrix} u_p & v_p \\ -v_p & u_p \end{pmatrix}\begin{pmatrix} I_{p1} \\ I^+_{p2}\end{pmatrix}$$

$1-I^+_{p2}I_{p2}$

$$\Rightarrow \langle n_{p\uparrow}\rangle = \langle c^+_{p\uparrow}c_{p\uparrow}\rangle = \hat{u}_p^2\langle I^+_{p1}I_{p1}\rangle + \hat{v}_p^2\langle I_{-p2}I^+_{-p.2}\rangle$$

$$\langle n_{p\downarrow}\rangle = \quad = \hat{u}_p^2\langle I^+_{p2}I_{p2}\rangle + v_p^2\langle I_{p1}I^+_{-p.1}\rangle$$

$\langle J\rangle = -\frac{e}{V}\sum_{k\sigma}\frac{k_z+eA_z}{m}\langle \hat{n}_{k\sigma}\rangle$ $\langle \hat{n}_{k\sigma}\rangle$代上式.代上页抵消 k_z 一部分

$$\langle I^+_{p2}I_{p2}\rangle \overset{I的能量已知.}{=} \frac{1}{\exp(\beta E_{k2})+1}$$

$(E_{k2} = E_{\tilde{k}2})$

$$= -\frac{2e}{mV}\sum_k\left(\frac{k_z}{\exp(\beta\hat{E}_k)+1} + eA_z\left[\frac{\hat{u}_k^2-\hat{v}_k^2}{\exp(\beta\hat{E}_k)+1}+\hat{v}_k^2\right]\right)$$

$$= -\frac{2e^2A_z}{mV}\sum_k\left[\frac{2}{3}\cdot\frac{k^2}{2m}\cdot\partial_{n_F}\left(\sqrt{\xi_k^2+\Delta_T^2}\right)+\left(\frac{u_k^2-v_k^2}{\exp(\beta E_k)+1}+v_k^2\right)\right]$$

$= -e^2\frac{1}{m}n_{super\text{-}ele}(T)\cdot\vec{A}$ 其 $\propto|\Delta_T|^2$ when $T\approx T_c$

因此，cooper pair 减少的数目导致实验观测电流之降低
$(\langle J\rangle \propto |\Delta_T|^2 \quad \Delta_T = \langle n_{k\delta}\rangle \to$ number of pair
因此，可以认为 cooper pair 为 superconductor 中载流子.

Exercise:

[pf:] $$\mathcal{H}_{BDG.F} = \sum \hat{\varepsilon}_k C^{+}_{k\delta} C_{k\delta} - \sum \Delta_0 (C_{-k\downarrow}C_{k\uparrow} + C^{+}_{k\uparrow}C^{+}_{-k\downarrow}).$$
$$\hat{\varepsilon}_k = \frac{k_x^2 + k_y^2 + (k_z + eA_z)^2}{2m} - \mu$$

① Eigenvalue. 同 $\mathcal{H}_{BDG}$. 不过 ε_k 形式有变.
$$D C_{-p} C^{+}_{-p} = 1 \cdot D - D C^{+}_{-p} C_{-p}.$$
$$D \Rightarrow -\varepsilon_{-p} = \varepsilon_p - \frac{2k_z eA_z}{2m}.$$
$$\Rightarrow h_{bdg} = \left(\varepsilon_p + \frac{(eA_z)^2}{2m}\right) I_z - \Delta I_x + \frac{P_z eA_z}{m} I_0.$$

$\Rightarrow \lambda$ (1.二项) 为 $\sqrt{\varepsilon_p'^2 + \Delta^2} \quad \varepsilon_p' = \varepsilon_p + \frac{(eA_z)^2}{m}.$
λ 三项独走 $\hat{I}_{2\times2}$. $P_z eA_z/m$.

故 $$E_{1k} = E_{2.k} = \sqrt{\left[\varepsilon_p + \frac{(eA_z)^2}{m}\right]^2 + \Delta^2} + \frac{P_z \cdot eA_z}{m}.$$

因此: $$u_p^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_p + \frac{(eA_z)^2}{2m}}{\sqrt{\left(\varepsilon_p + \frac{(eA_z)^2}{2m}\right)^2 + \Delta^2}}\right)$$
类比 $$\hat{v}_p = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{\varepsilon_p + \frac{(eA_z)^2}{2m}}{\sqrt{\left(\varepsilon_p + \frac{(eA_z)^2}{2m}\right)^2 + \Delta^2}}}$$

Now. $$|G.S\rangle = \prod_{p.E>0} (\hat{u}_p + \hat{v}_p C^{+}_{p\uparrow} C^{+}_{-p\downarrow}) \prod_{p.E<0} (\hat{v}_p - \hat{u}_p C^{+}_{p\uparrow} C^{+}_{-p\downarrow}) |em\rangle$$

pf: $$I_p |G.S\rangle = \prod_{E_p>0} (u_p C_p - v_p C_p^{+}) \prod(m) |empty\rangle$$

这同 $\mathcal{H}_{BDG}$ 讲义上做法. Homework 4 已证.

DATE

$$\Gamma_p^\dagger |GS\rangle = \Big(\prod_{E_p>0} (w)\Big)\cdot\Big(\tilde{u}_p C_p^\dagger - \tilde{v}_p C_{-p}\Big)\Big(\prod_{p,\,E_p<0} \tilde{v}_p - \tilde{u}_p C_{p\uparrow}^\dagger C_{-p\downarrow}^\dagger\Big)|\text{em}$$

$E_p<0$ $\quad E_p<0 \quad E_p<0$

$$\overset{\text{certain } p}{=} \tilde{u}_p\tilde{v}_p C_p^\dagger \underbrace{|\text{empty}\rangle}_{\neq 0} - v_p^2 \underbrace{C_{-p}|\text{empty}\rangle}_{=0}$$

$$-\tilde{u}_p^2 \underbrace{C_p^\dagger C_p^\dagger}_{=0} C_{-p}^\dagger + \tilde{v}_p\tilde{u}_p C_{-p} C_p^\dagger C_{-p}^\dagger |\text{empty}\rangle$$

$$= (-1)\times \underbrace{C_{-p} C_{-p}^\dagger}_{=1} C_p^\dagger$$

$=0$ 综上. $\Gamma|GS\rangle = 0$.

2. Bulk Meisner Effect

$$\vec{J}(T) = -e^2 \frac{n_{super\text{-}ele}(T)}{m}\vec{A}$$

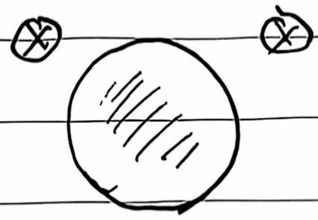
(i) $\nabla\times\vec{B} = 4\pi\vec{J}$

$$\Rightarrow \nabla\times\vec{B}\cdot\frac{1}{4\pi} = -e^2\frac{n_{super\text{-}ele}(T)}{m}\vec{A}$$

$$\Rightarrow \nabla\times(\nabla\times\vec{B}) = \nabla(\underbrace{\nabla\cdot\vec{B}}_{=0}) - \Delta\vec{B} = -e^2\frac{n_{super\text{-}ele}(T)}{m}\nabla\times\vec{A}$$

$$= -e^2\frac{n_{super\text{-}ele}(T)}{m}\vec{B}$$

$$\Rightarrow \Delta\vec{B} = \frac{4\pi e^2 n_{super\text{-}ele}}{m}\vec{B}$$

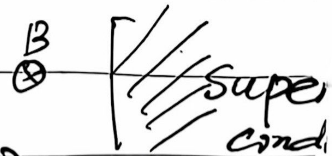


(ii) Assume $\vec{B} = B_z\cdot\hat{z}$, superconductor in $[x, y, z \quad x\in[0,+\infty), y, z\in$

$B(x, y)$ is uniform.

$\partial/\partial y^2 = \partial/\partial z^2 (B) = 0 \qquad \partial/\partial x^2\,\vec{B} = \frac{\partial^2 B_z}{\partial x^2}\cdot\vec{z}$

$= M\cdot B_z\cdot\vec{z}$ $\qquad B_x = B_y = 0$



$\Rightarrow$ 通解 $\ddot{B}_z(x) = \overset{大于0}{M} B_z(x)$ in Boundary condition: $B(x) = \begin{cases} B_0\hat{z} & x=0 \\ 0 & x\to+\infty\end{cases}$

衰减 $B_z = Ae^{-\sqrt{M}x}$ $\qquad B_z(\infty) = 0$ ✓

$B_z(0) = e^{-\sqrt{M}\cdot 0}\cdot A = B_0 \Rightarrow A = B_0$

Therefore $B_z = e^{-\sqrt{M}x} \cdot B_0$

$\therefore B = \hat{z} \cdot B_0 \cdot e^{-\sqrt{M}x}$ $\qquad M = \frac{4\pi e^2 n_{super\text{-}ele}(T)}{m}$

(iii) $e^{-\sqrt{M}\cdot\lambda} = 1/e \qquad \lambda = \frac{1}{\sqrt{M}}$

Apr. 2nd.

Landau-Ginzburg theory for superconductor

we should minimize free energy

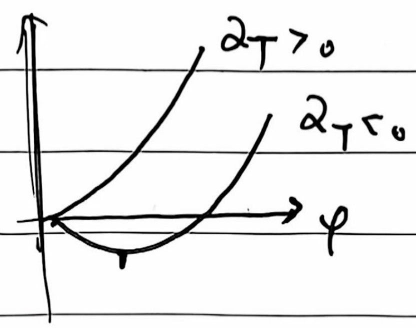
$$F_0[\Phi] = \int d^3x \ \ \alpha_T|\Phi(x)|^2 + \frac{1}{2}\beta_T|\Phi(x)|^4 + \gamma|\nabla\Phi|^2$$

conserve $F[e^{i\Lambda}\Phi(x)] = F[\Phi(x)]$

require ~~rotation~~ invariant Φ of 平移对称

using 鞍点近似 $\Phi(x) = \Phi_0$ set $|\Phi_0|^2 = \rho_0$

$V(\alpha_T\rho_0 + \frac{1}{2}\beta_T\rho_0^2)$



设 set $\alpha_T = (T-T_c) \times a$

when: $T > T_c$ $\rho = 0$ $F_0(min)$

when $T < T_c$ $\rho = \left|\frac{a(T-T_c)}{b}\right|$ $F_0(min)$.

Input. a. b. γ by experiment

Output. many predictions ~~of~~

DATE · ·

To conserve $\Phi(x) \to e^{i\Lambda(x)}\Phi(x)$，$\nabla\Phi$与$\Lambda$有关，复杂！

we set gauge $A(x) \to A(x) - \nabla\Lambda(x)$

$$[-i\nabla + eA(x)]\Phi(x) = e^{ie\Lambda(x)}(-i\nabla + e^{*}A(x) - \nabla\Lambda(x))\Phi(x)$$

$$F[\Phi, A] = F[\Phi e^{ie\Lambda(x)}, A - \nabla\Lambda(x)]$$

因此，F中$|\nabla\Phi|^2$项可以不用$|\nabla(\Phi(x)e^{i\Lambda(x)})|^2$

而写作：

$$F = \int \alpha_T|\Phi|^2 + \beta_T|\Phi|^4 + \frac{1}{2m^*}|(-i\nabla + e^{*}A)\Phi|^2 + \frac{B^2}{8\pi}\, d^3x$$

minimize F：　　　　仅磁场，中性体系 $E \simeq 0$

$$dF = -SdT - pdV$$

实验中，可做到等温，打入磁场(H)等，用 $dG = -SdT - dH\cdot B$ 更好

$G = F - BH/4\pi$ 极小化G在(T, H)等时比较现实 $G[\Phi, T, H] \to G$

$$G\left[\frac{\Phi}{A}\right] \rightsquigarrow G(\rho, A) \quad \rho\text{海}$$

$$G\left[\frac{\Phi}{A}\right] = \int d^3x \cdot \frac{1}{2m^*}|[-i\nabla + e^{*}A(x)]\Phi(x)|^2 + \alpha_T|\Phi(x)|^2 + \frac{1}{2}\beta_T|\Phi$$

$$+ \frac{(\nabla\times\vec{A} - \vec{H})^2}{8\pi} - \frac{\vec{H}^2}{8\pi}$$

$\Rightarrow$ $\Phi(x)$定义$= \underbrace{\sqrt{\rho(x)}}_{\text{模长}}\ \underbrace{e^{i\phi(x)}}_{\text{幅角}}$　$\rho(x) \to \rho(x)$　$\phi(x) \to \phi(x) + e^{*}\Lambda(x)$

$$A' = A + \frac{1}{2e}\nabla\phi(x)$$

$G[\rho, A']$ 替代 $G[\Phi, A], \Lambda]$ 因此规范Λ下不变，为：

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NOTES

$$G[\rho, A'] = \int \alpha_T \rho + \beta_T \rho^2 \cdot \frac{1}{2} + \frac{1}{4m} |\nabla \sqrt{\rho}|^2 + \frac{e^2}{m} A'^2 \cdot \rho + \frac{(\nabla \times A - \mathcal{H})^2}{8\pi} + \frac{\mathcal{H}^2}{8\pi}$$

minimize

$$\frac{\partial G}{\partial A'} = 0 \Rightarrow \nabla \times B = 4\pi \left(-\frac{2e^2}{m} \rho A' \right) + \nabla \times \mathcal{H}$$

$$\Rightarrow \nabla B = \lambda^{-2} B \quad \lambda = \sqrt{\frac{m}{8\pi e^2 \bar{\rho}}}$$ called London penetration length

$$B = \mathcal{H} \exp(-z/\lambda)$$ 磁场在超导体内快速衰减

$$\frac{\partial G}{\partial \sqrt{\rho}} = 0 \Rightarrow \alpha_T \sqrt{\rho} + \beta_T (\sqrt{\rho})^3 - \frac{1}{4m} \Delta (\sqrt{\rho}) + \frac{e^2}{m} (A')^2 \sqrt{\rho} = 0$$