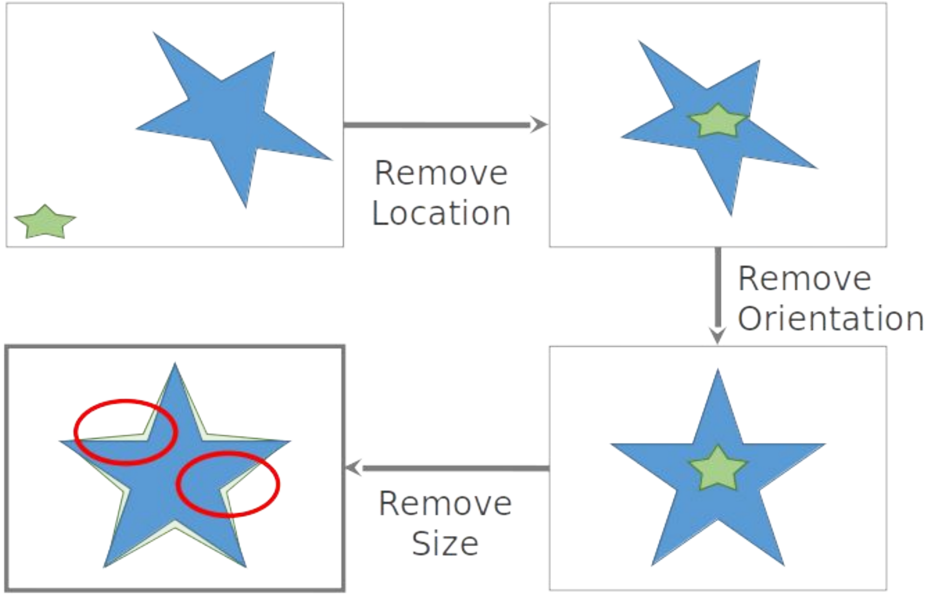
Particle-Based Shape Modeling for Arbitrary Regions-of-Interest

Hong Xu, Alan Morris, and Shireen Elhabian

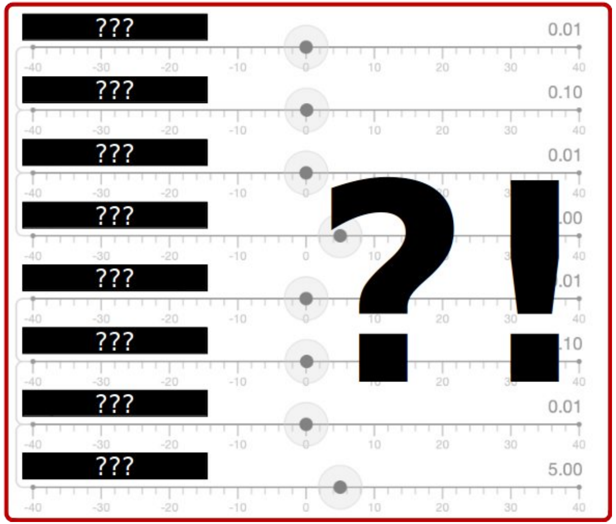
What is Shape?!



Shape = Object - Location - Orientation - Size (Optional)

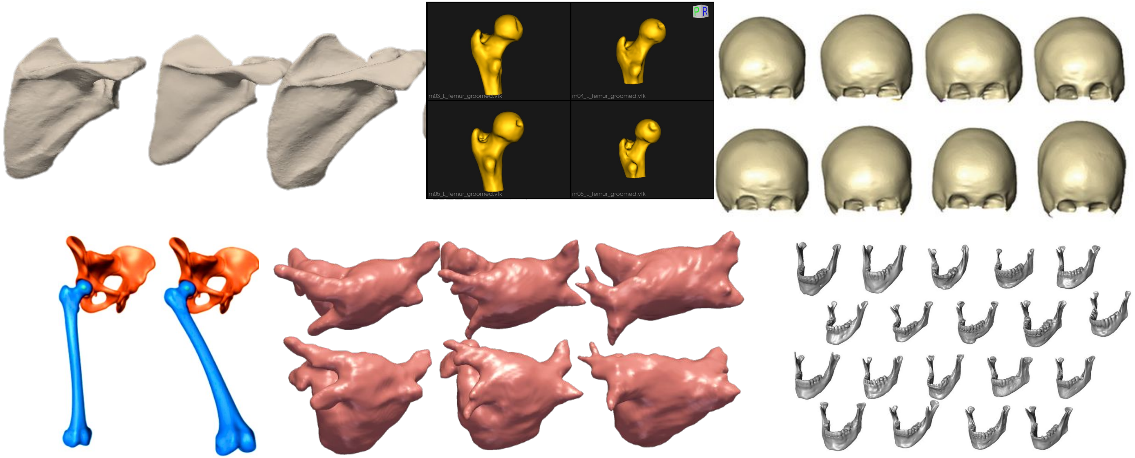
What is shape modeling?

Shapes with unknown parameters?



Shape modeling is about learning population-specific parameterization directly from data.

Given a population of shapes

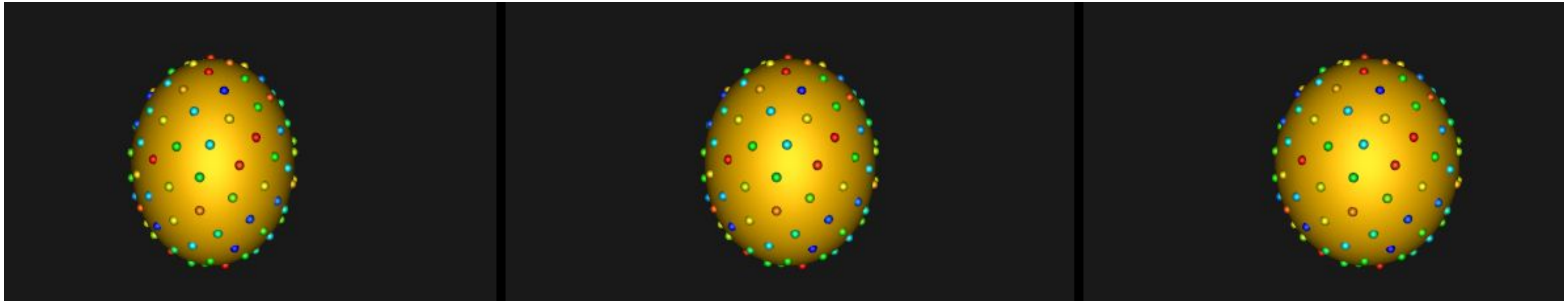


Can we estimate modes of variation?
I.e. In the above population, **what are the modes of variation and how do they interact?**

[*] Ambellan, Felix, et al. "Statistical shape models: understanding and mastering variation in anatomy." Biomedical Visualisation. Springer, Cham, 2019. 67-84.

Particle-based Shape Modeling (PSM)

Populates shape surfaces with particles that maintain configuration congruence across the population.



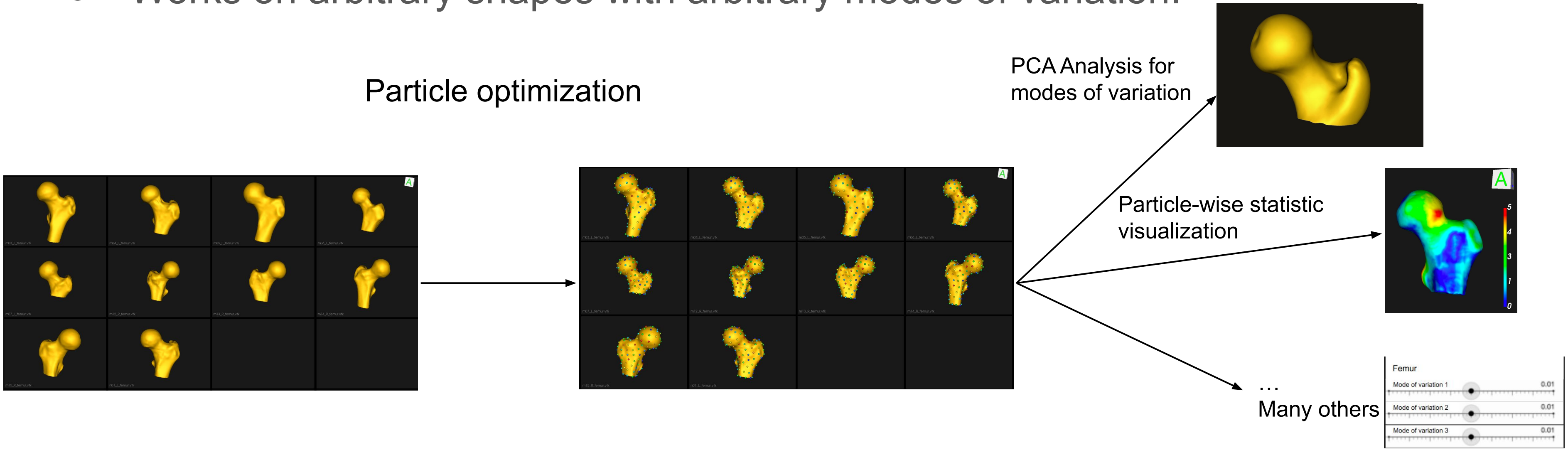
ShapeWorks

An Integrated Suite for Shape Representation and Analysis & more...

www.shapeworks.sci.utah

Particle-based Shape Modeling (PSM)

- Works on arbitrary shapes with arbitrary modes of variation.



- It populates shape surfaces with particles that maintain correspondence across shapes and the statistical analyses can be subsequently applied on these particles.

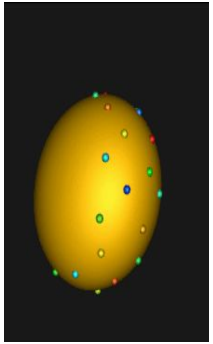
The PSM Optimization

Correspondence Sampling

$$f(\mathcal{P}) = H(\mathcal{P}) - \boxed{\sum_{j=1}^{J} H(\mathbf{P}_i)}$$

The sampling term assures that for each shape, particles spread uniformly

Shape k example



The PSM Optimization

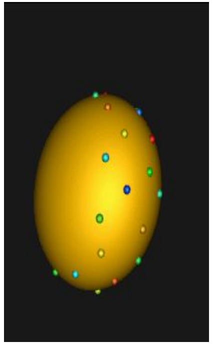
The correspondence term acts over particles on all shapes at once, maintaining the same ordering of particles on every surface:

Correspondence   Sampling

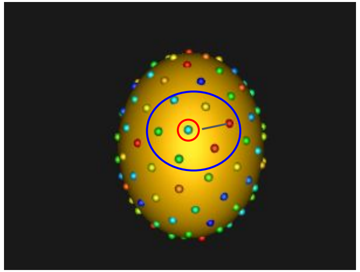
$$f(\mathcal{P}) = \boxed{H(\mathcal{P})} - \sum_{j=1}^{J} H(\mathbf{P}_i)$$

The sampling term assures that for each shape, particles spread uniformly

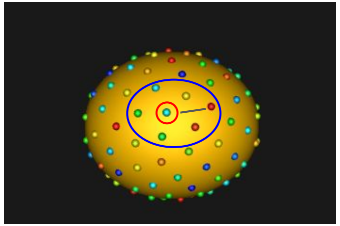
Shape k example



Shape 1



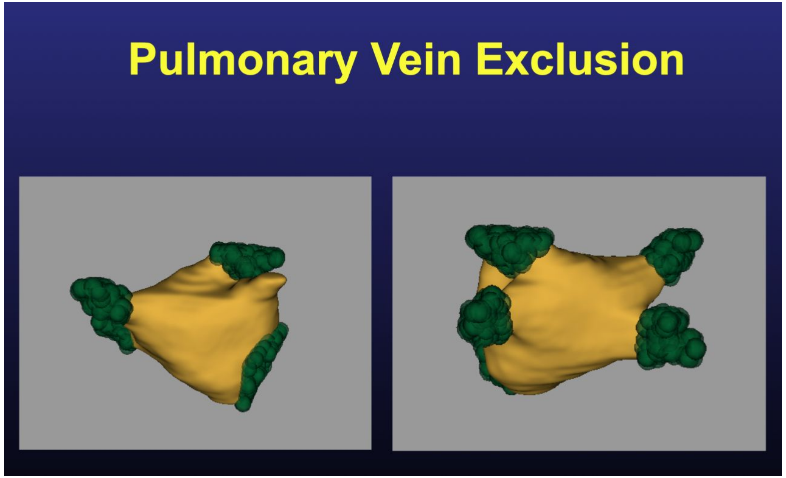
Shape 2

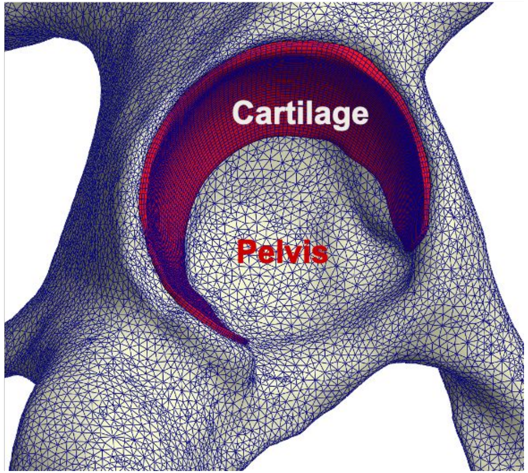


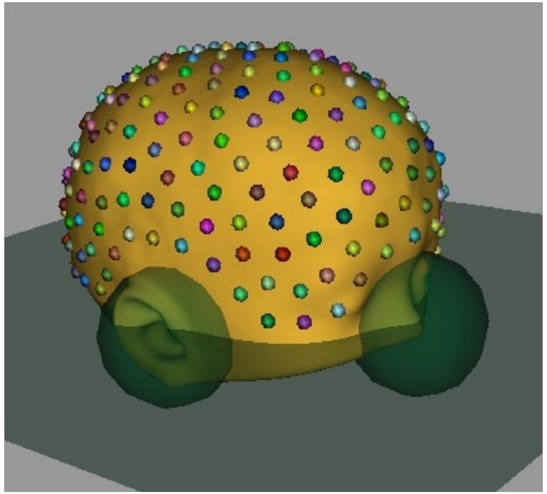
Correspondence means that every particle's neighbors will find a similar configuration around it on every shape. We can see that the teal particle is in correspondence since all its neighbors have a similar configuration in both shapes.

Why Regions of Interest?

Instances exist where regions of interest must be isolated.

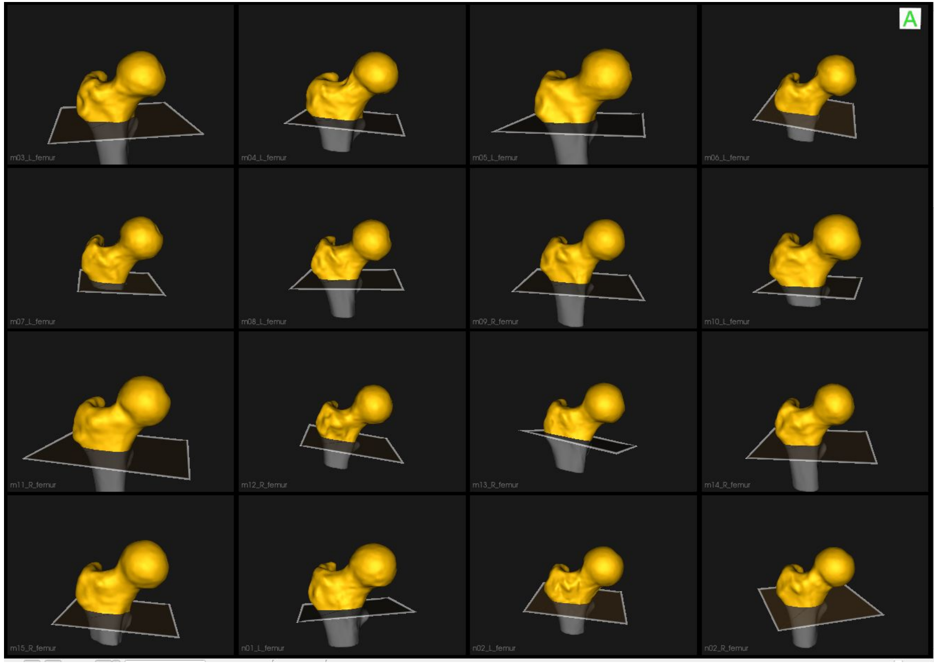






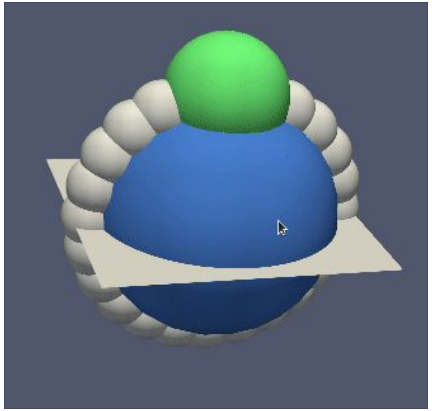
Regions of Interest

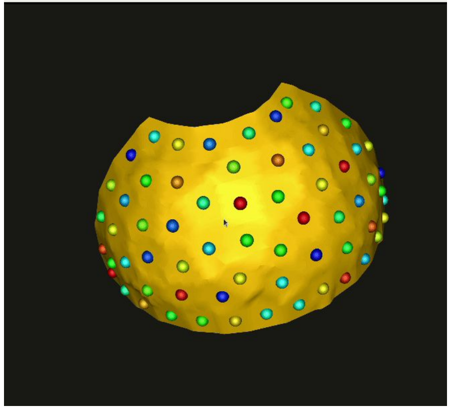
- We propose a solution to isolate areas during the optimization.
- Without the need to alter the input.



Region Exclusion

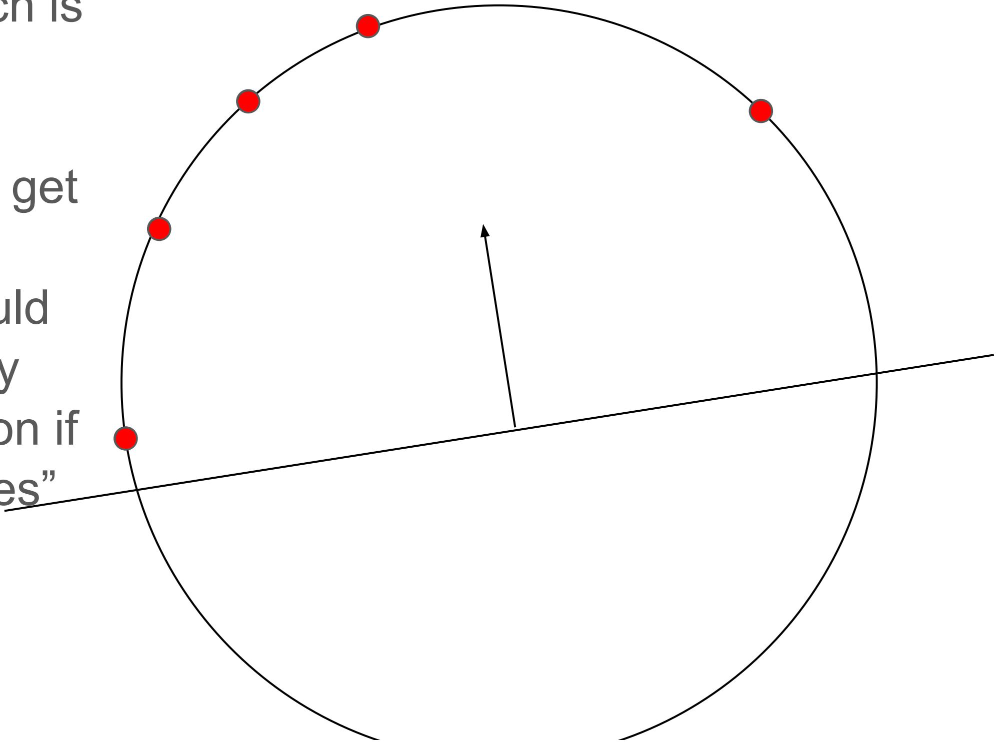
- We add constraints to the optimization to define regions where particles are disallowed from entering.





Previous Approach

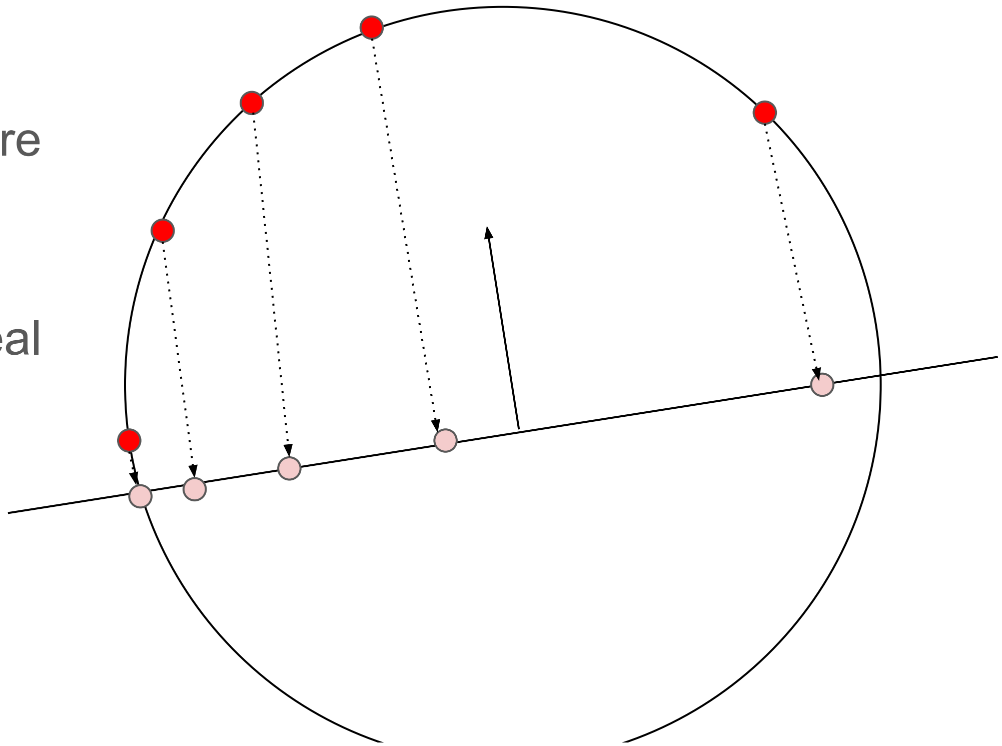
- An existing approach is to have “shadow particles” repel real particles when they get too close to a constraint. This would be done naturally by the sampling function if the “shadow particles” are added to the optimization.



Datar M, Cates J, Fletcher PT, Gouttard S, Gerig G, Whitaker R. Particle based shape regression of open surfaces with applications to developmental neuroimaging. Med Image Comput Comput Assist Interv. 2009;12(Pt 2):167-74. doi: 10.1007/978-3-642-04271-3_21. PMID: 20426109; PMCID: PMC3138541.

Previous Approach

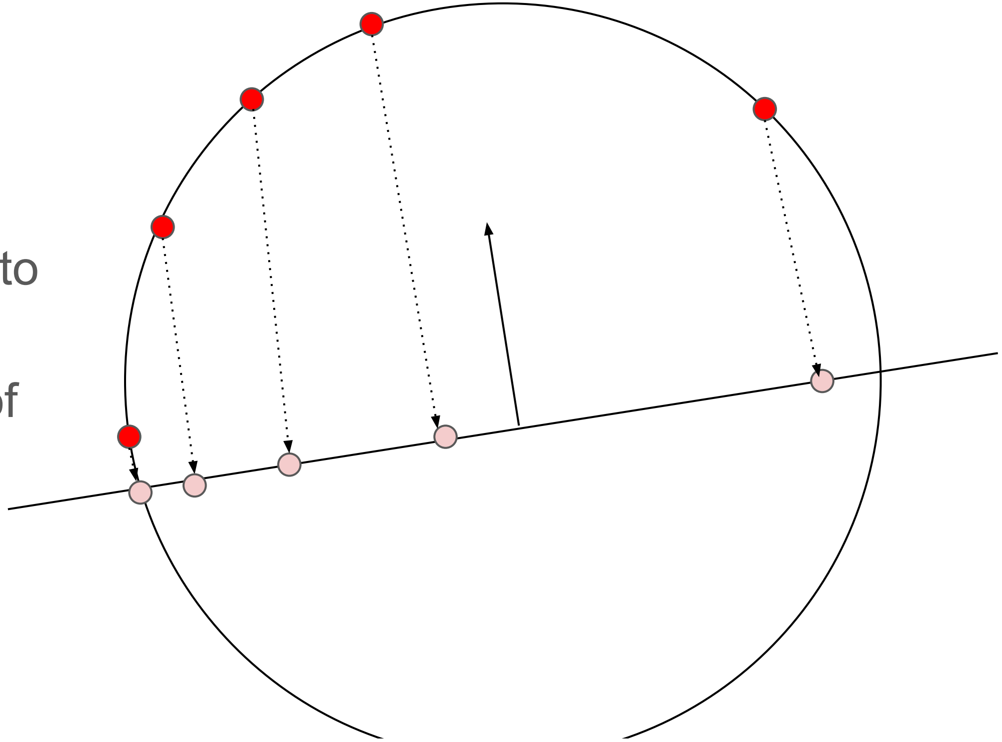
- Each particle is projected onto the surface of the sphere or cutting plane, creating a “shadow particle” for each real particle.



Datar M, Cates J, Fletcher PT, Gouttard S, Gerig G, Whitaker R. Particle based shape regression of open surfaces with applications to developmental neuroimaging. Med Image Comput Comput Assist Interv. 2009;12(Pt 2):167-74. doi: 10.1007/978-3-642-04271-3_21. PMID: 20426109; PMCID: PMC3138541.

Previous Approach

- This works well but scales poorly since every added constraint will add another *n* particles to the optimization, *n* being the number of particles the optimization starts with.



Datar M, Cates J, Fletcher PT, Gouttard S, Gerig G, Whitaker R. Particle based shape regression of open surfaces with applications to developmental neuroimaging. Med Image Comput Comput Assist Interv. 2009;12(Pt 2):167-74. doi: 10.1007/978-3-642-04271-3_21. PMID: 20426109; PMCID: PMC3138541.

Solution: Quadratic Penalty for Inequality Constraints

- Quadratic penalty is a traditional optimization that solves constrained optimization problems by rewriting them as unconstrained ones.
- Given an optimization problem with inequality constraints

Minimize $f(x)$

Subject to $g_1(x) \leq 0,\ g_2(x) \leq 0,\ \ldots,\ g_r(x) \leq 0$

Solution: Quadratic Penalty for Inequality Constraints

- We can write out the unconstrained optimization in the form

$$g_{i,m}(\mathbf{p}) \leq 0 \quad \Longrightarrow \quad g^{+}_{i,m}(\mathbf{p}) = \max(0, g_{i,m}(\mathbf{p}))$$

Solution: Quadratic Penalty for Inequality Constraints

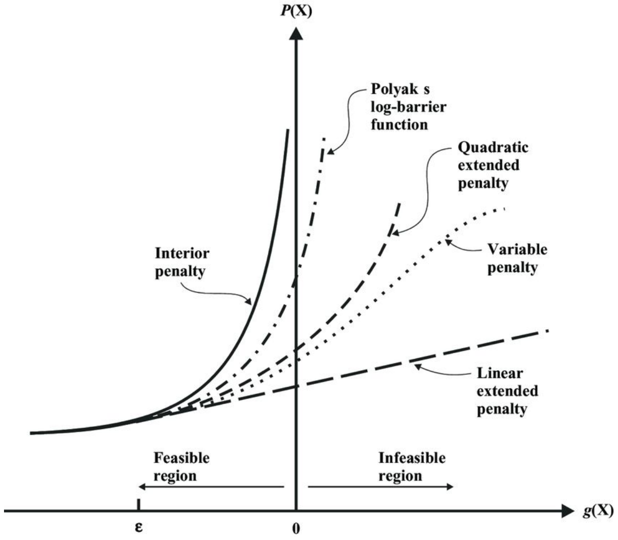
- We can write out the unconstrained optimization in the form

$$g_{i,m}(\mathbf{p}) \leq 0 \quad \Longrightarrow \quad g^{+}_{i,m}(\mathbf{p}) = \max(0, g_{i,m}(\mathbf{p}))$$

$$F(\mathcal{P}) = f(\mathcal{P}) + \sum_{i=1}^{I} \sum_{m=1}^{M_i} \sum_{j=1}^{J} g^{+}_{i,m}(\mathbf{p}_{i,j})$$

Solution: Quadratic Penalty for Inequality Constraints

- Notice that there is one linear and one quadratic element. The linear element excels at small violations while the quadratic corrects extreme violations well.

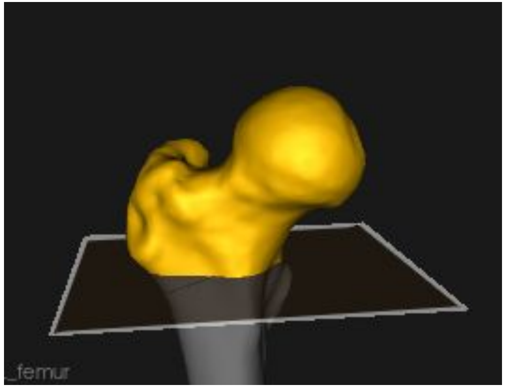


Solution: Quadratic Penalty for Inequality Constraints

$$F(\mathcal{P}) = f(\mathcal{P}) + \sum_{i=1}^{I} \sum_{m=1}^{M_i} \sum_{j=1}^{J} g_{i,m}^{+}(\mathbf{p}_{i,j})$$

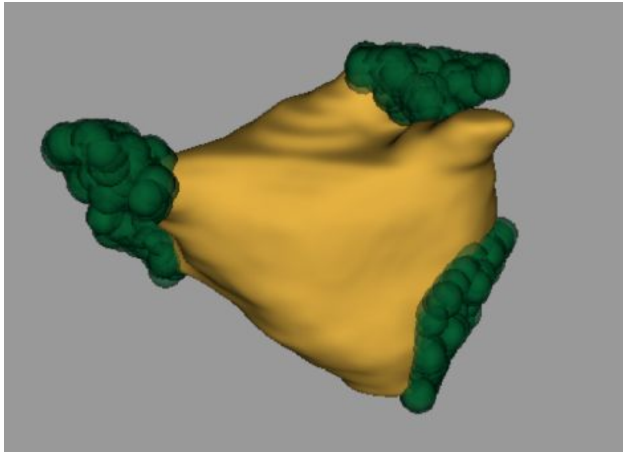
Cutting plane constraint

$g_j(P) = a\,x + b\,y + c\,z + d \leq 0$



Sphere constraint

$g_j(P) = (x - a)^2 + (y - b)^2 + (z - c)^2 - r^2 \leq 0$



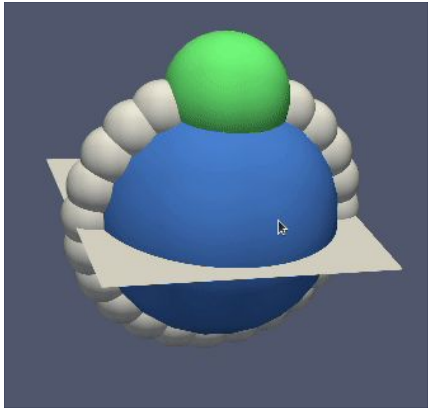
Solution: Quadratic Penalty for Inequality Constraints

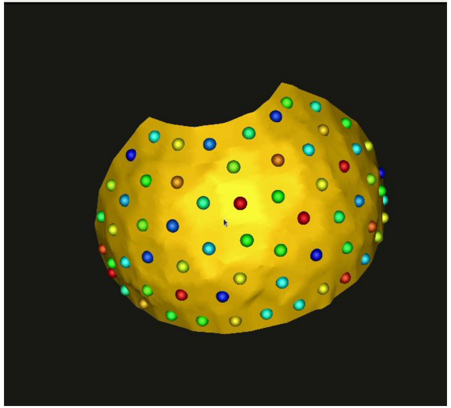
$$F(\mathcal{P}) = f(\mathcal{P}) + \sum_{i=1}^{I} \sum_{m=1}^{M_i} \sum_{j=1}^{J} g_{i,m}^{+}(\mathbf{p}_{i,j})$$

$g_j(P) = a\,x + b\,y + c\,z + d \leq 0$

$g_j(P) = (x - a)^2 + (y - b)^2 + (z - c)^2 - r^2 \leq 0$

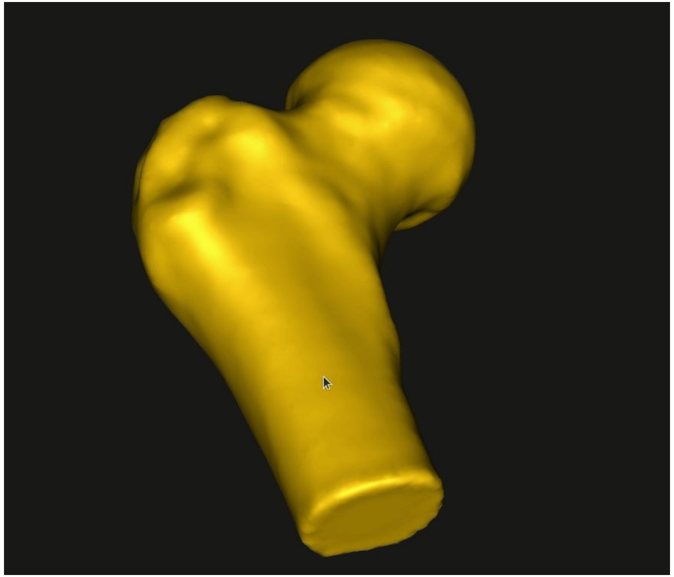
Combined constraint example





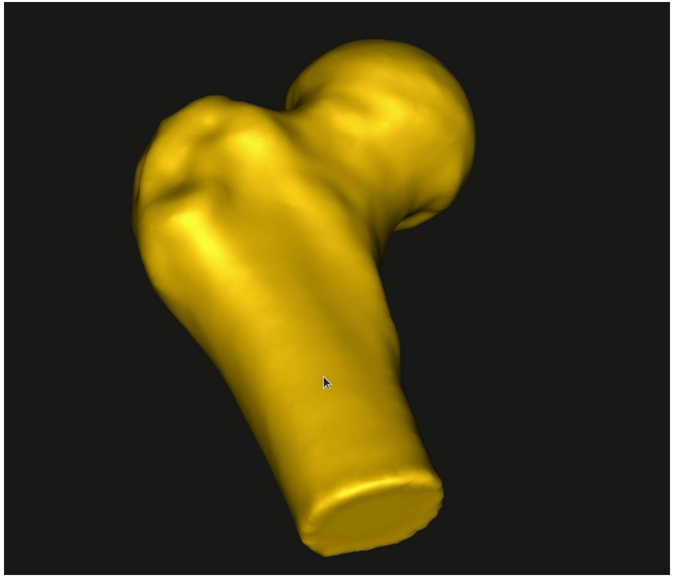
Free Form Constraints

- Geometric/parametric constraints gives the freedom to define constraints using collections of planes and/or spheres.
- Free form constraints allow us to select arbitrary regions to include or exclude.



Free Form Constraints Implementation

- Q: How do we express a free-form constraint in the form $g_j(x) \leq 0$?



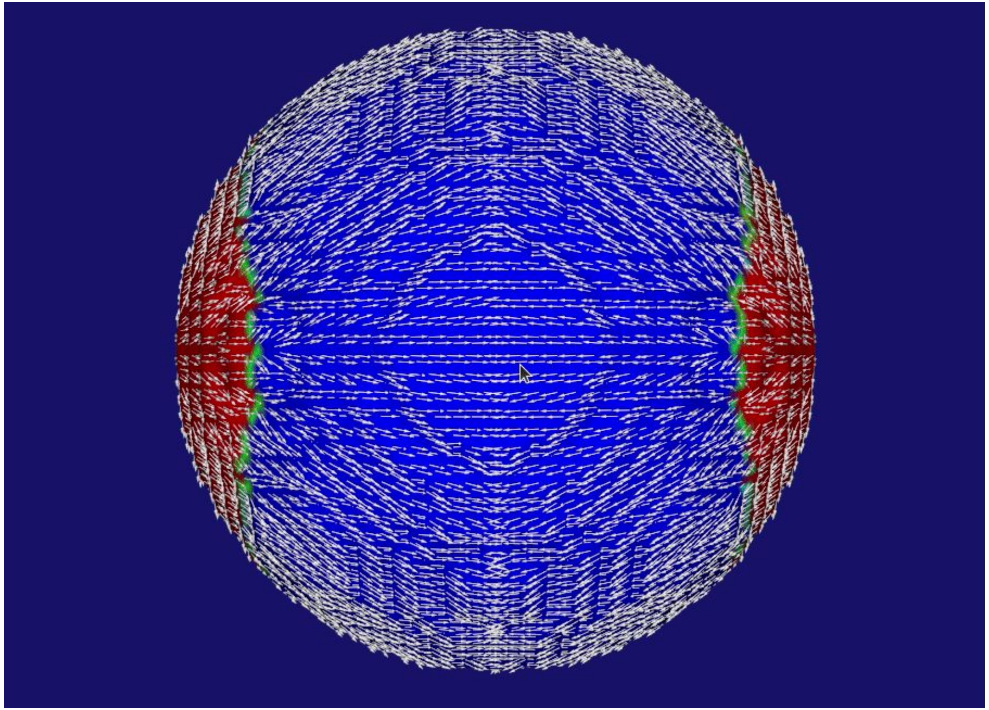
Free Form Constraints Implementation

- Q: How do we express a free-form constraint in the form $g_j(x) \leq 0$?

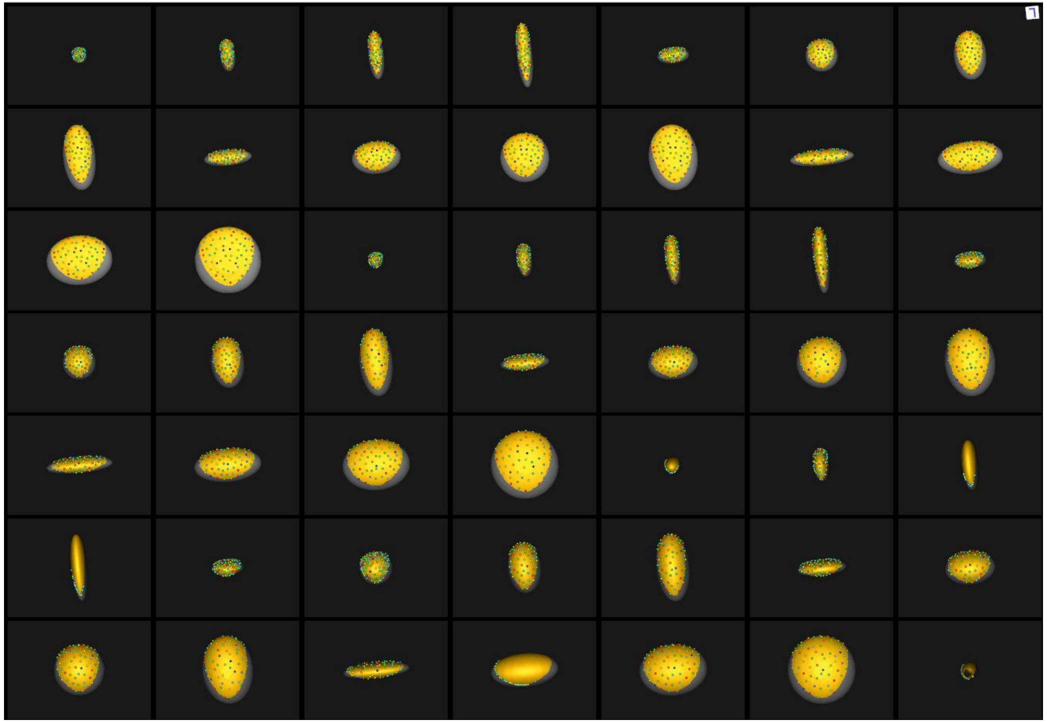
- A: Find a way to compute $g_j(x)$ *and* $g'_j(x)$ at any surface point in a manner in which $g_j(x)$ expresses the violation intensity and $g'_j(x)$ the direction to fix violation.

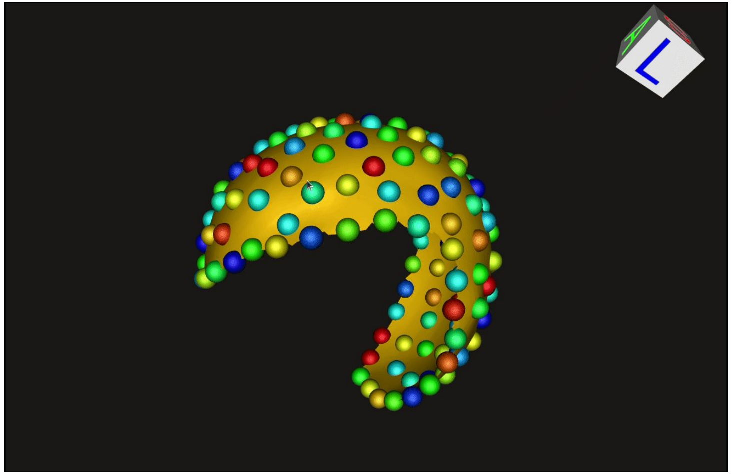
Free Form Constraints Implementation

- A signed geodesic distance field on a mesh. Each arrow indicates the direction to take at that place on the surface to avoid violation of constraints.
- Given these gradients, the quadratic penalty then handles the adjustment to any gradient update in the optimization whilst coordinating with any other constraint type (FFC, sphere or cutting plane).

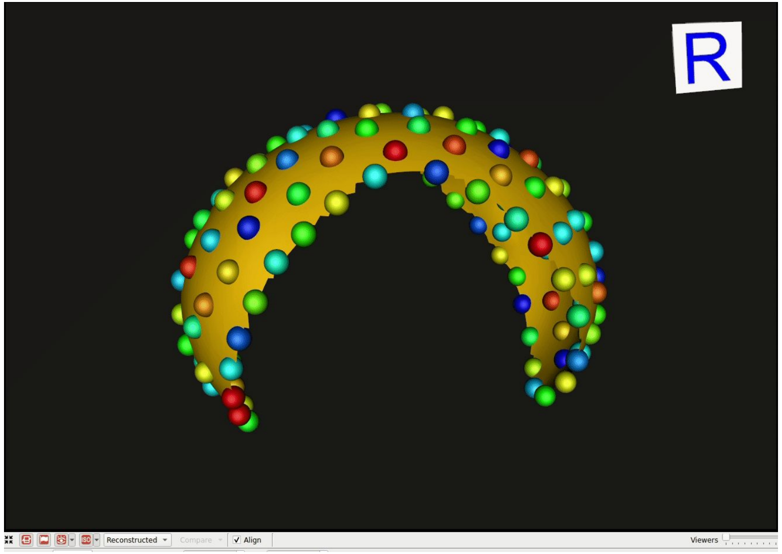


FFC on 64 Ellipsoids with 3 Modes of Variation

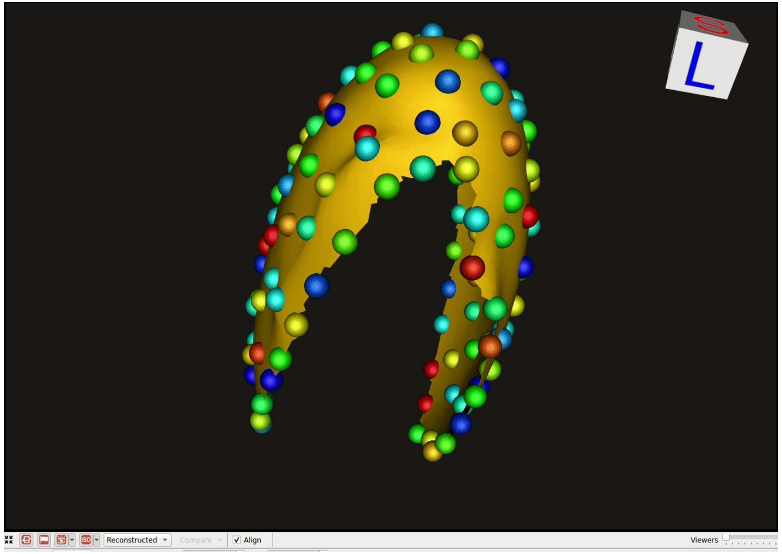




FFC on 64 Ellipsoids with 3 Modes of Variation

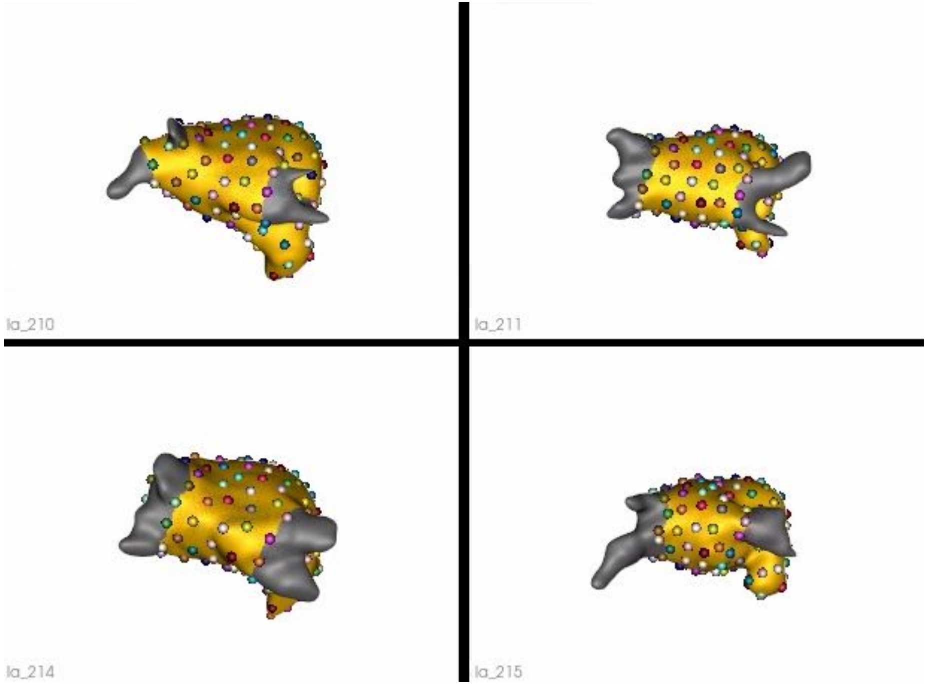


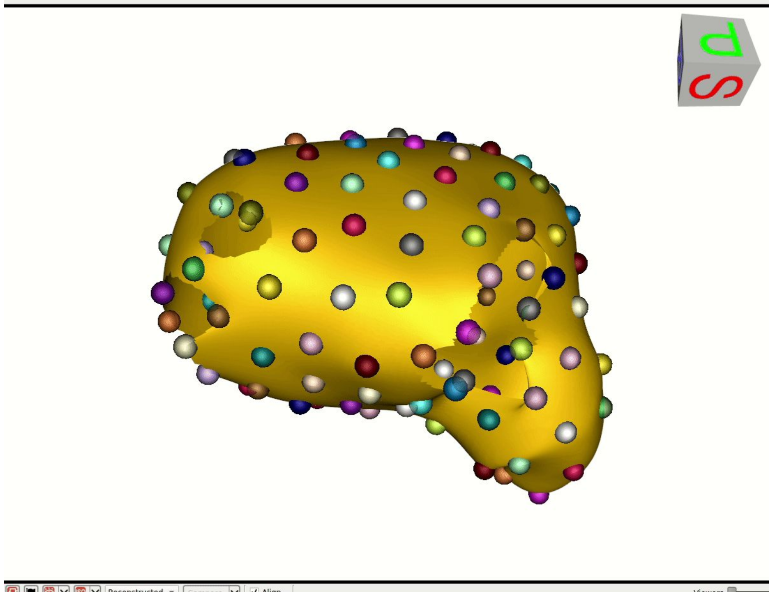
PCA first mode of variation



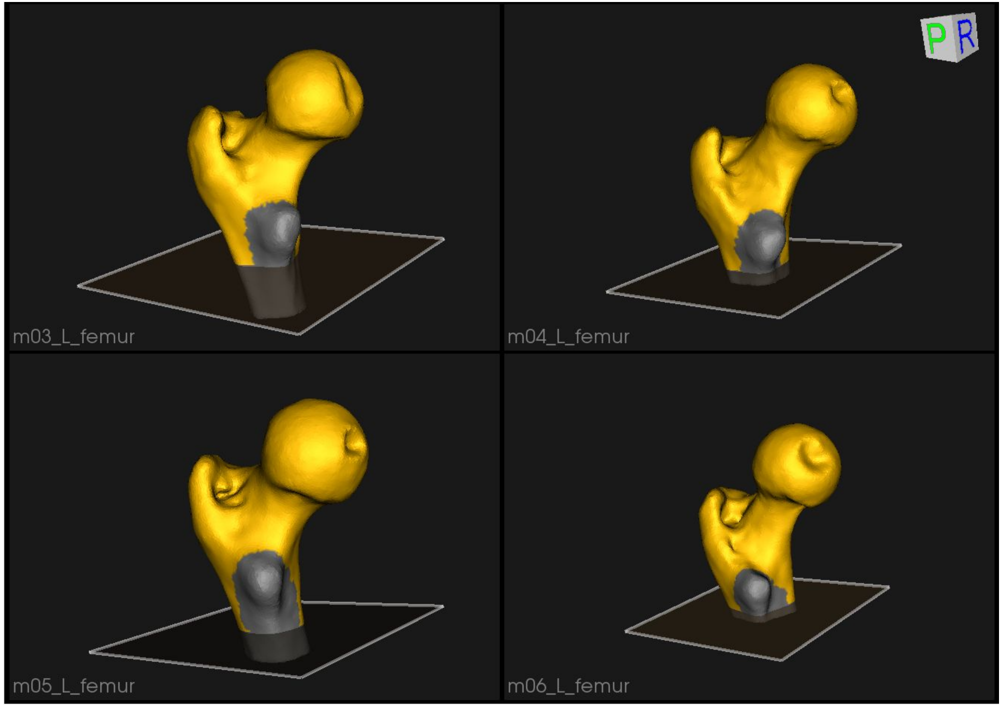
PCA second mode of variation

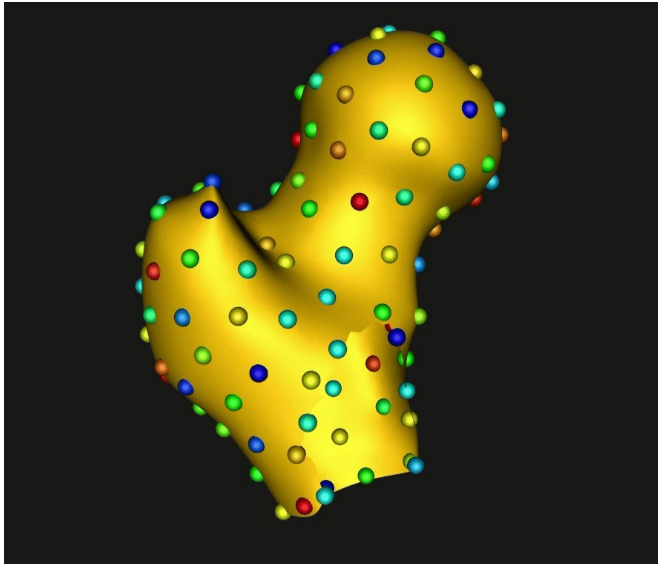
FFC on Left Atria





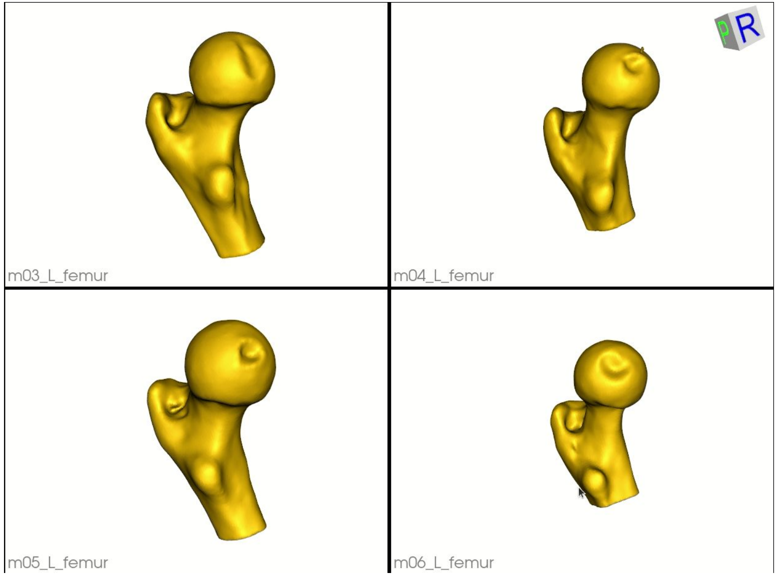
FFC and Cutting Planes on Femurs



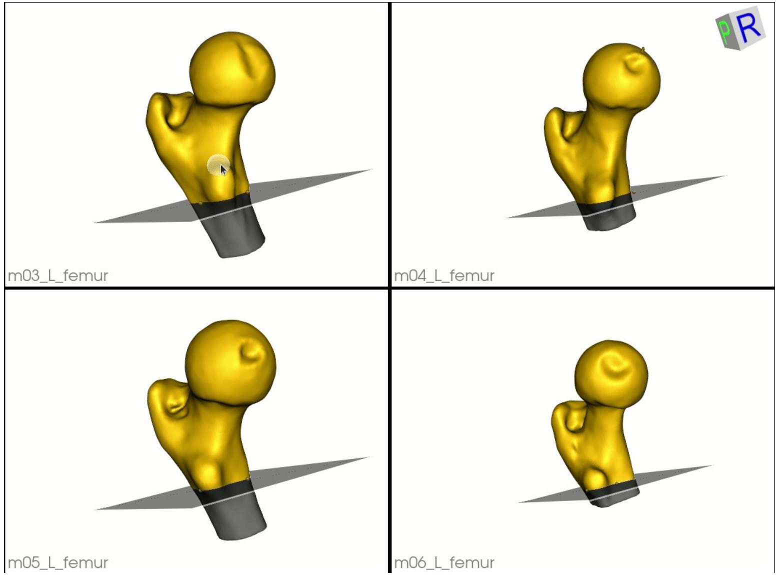


GUI Tool in ShapeWorks Studio

Defining cutting planes



Painting FFCs



Particle-Based Shape Modeling for Arbitrary Regions-of-Interest

Hong Xu, Alan Morris, and Shireen Elhabian

Thank You!