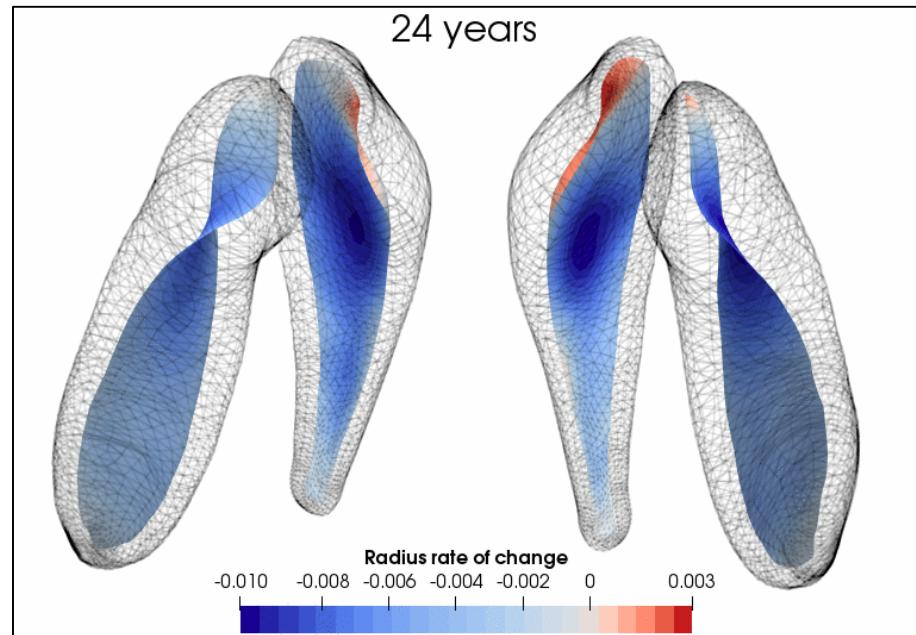




“Shaping up: Shape Analysis driven by Biomedical Applications: Past and Future

Guido Gerig
NYU Tandon School of Engineering
Departments of Computer Science and Engineering & Biomedical Engineering

Please see links to slides and videos of long lectures on next slide.



This talk is a short & updated version of two long lectures given at the MISS 2014 Summer School

Medical Imaging Summer School 2014

28 July - 1 Aug 2014 Favignana, Sicily

Medical Imaging meets Computer Vision

Programme and Slides:

<https://iplab.dmi.unict.it/miss14/programme.html>

[Guido Gerig \(slides\)](#)

Shaping up! Introduction into Shape Analysis

[Xavier Pennec \(slides\)](#)

Geometric Structures for Statistics on Shapes and Deformations in Computational Anatomy

[Guido Gerig \(slides\)](#)

Quantification of Object Dynamics by Spatiotemporal Shape Analysis

Video Lectures:

<https://iplab.dmi.unict.it/miss14/speakers&Syllabus.htm>

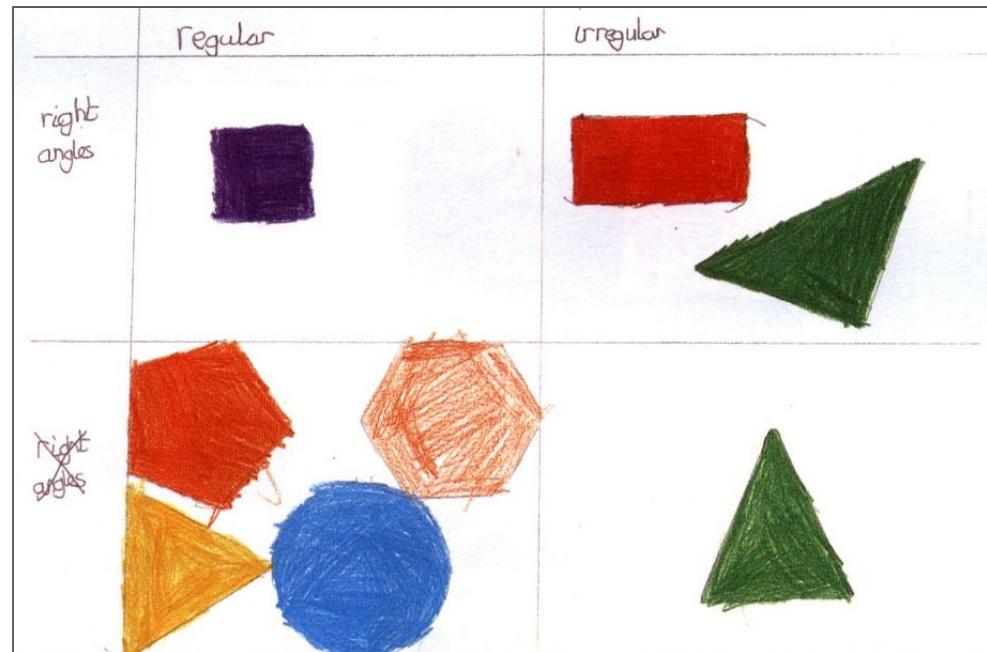
[Link to VIDEO LECTURES](#)

Contents

- Abridged History of Shape Analysis
 - Kendall Shape Space, SSM, ASM, AAM
 - Geometry Representations: Boundary, Skeleton, Harmonics, Graph Spectra, Distance Transform, ..
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 - Problems with Size Normalization
 - Longitudinal Shape Analysis
 - Shapes come with covariates (sex, age, diagnosis, test scores, ...)

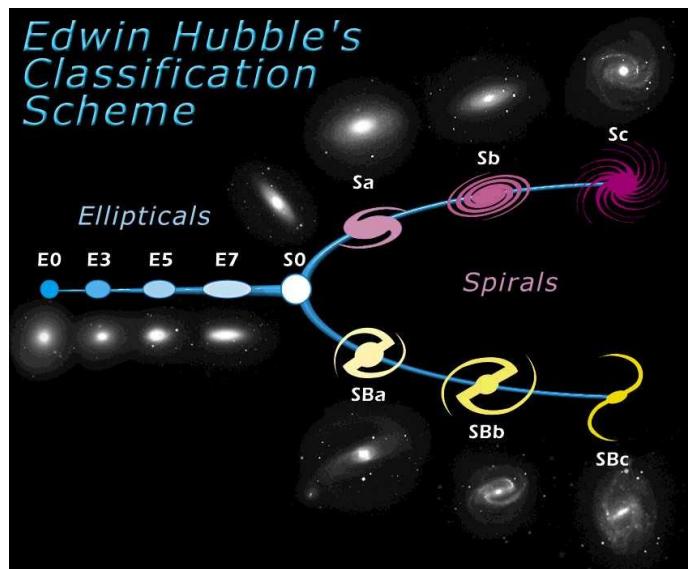
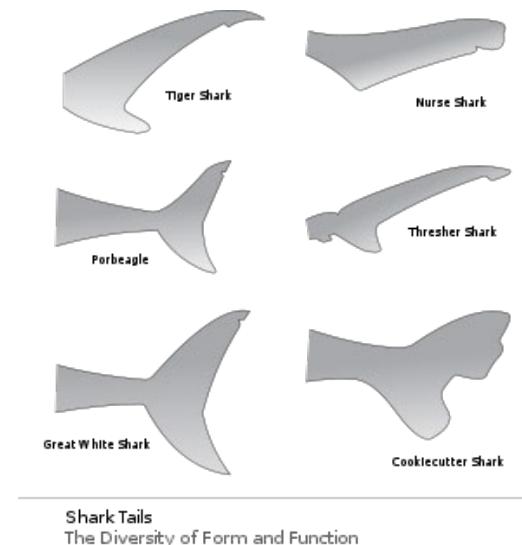
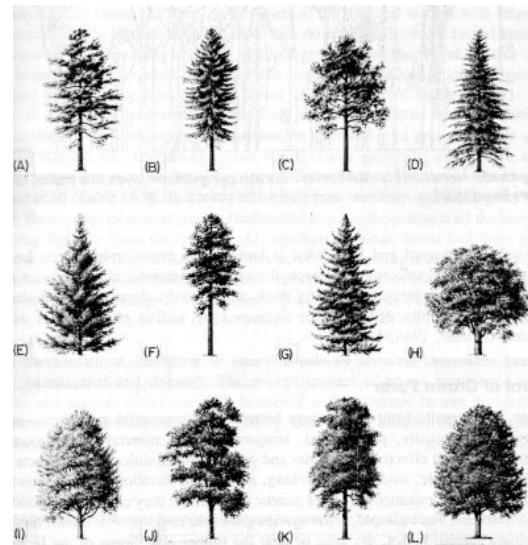
Shape

The word “shape” is commonly used in everyday language, usually referring to the geometry of an object.



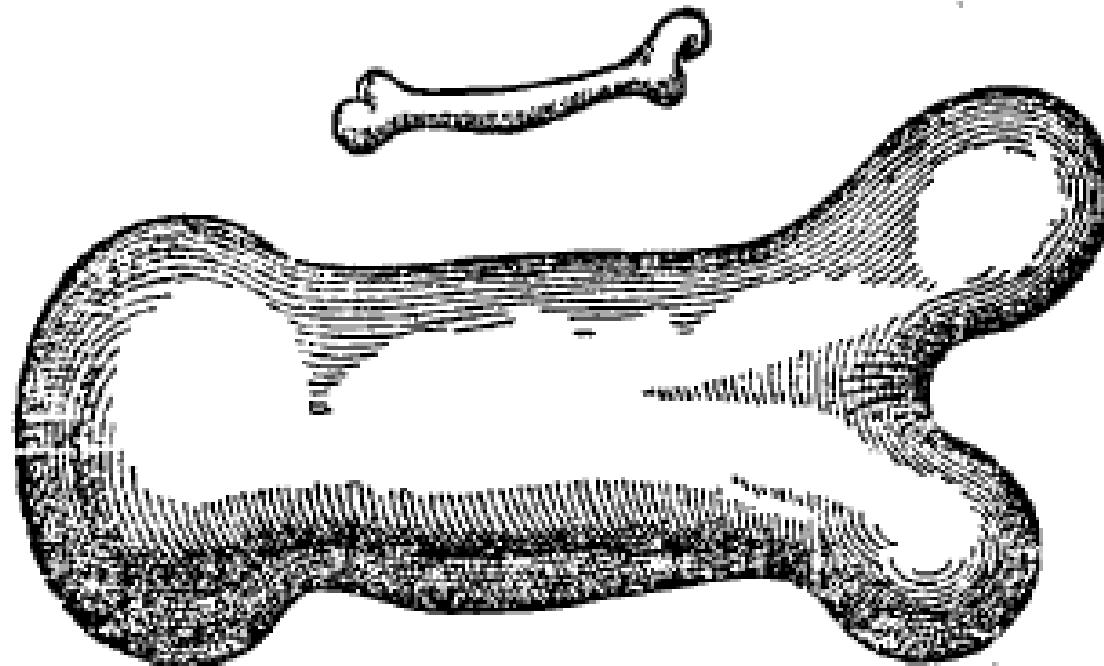
School performance test

Application Domains



Term	Shape
Cylindrical	
Discoidal	
Spherical	
Tabular	
Ellipsoidal	
Equant	
Irregular	

Concept of shape is not new



From Galileo (1638) illustrating the differences in shapes of the bones of small and large animals.

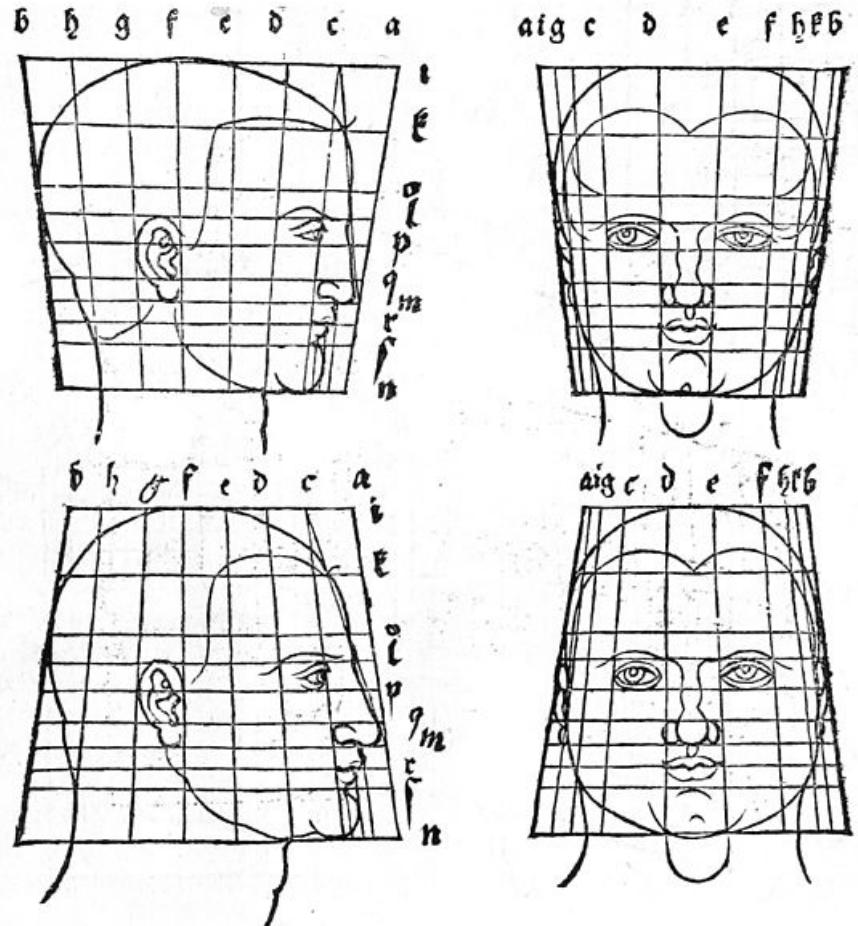
Even earlier...



Albrecht Dürer (1471-1528):
German painter, printmaker,
engraver and mathematician.

Studies of human proportions.

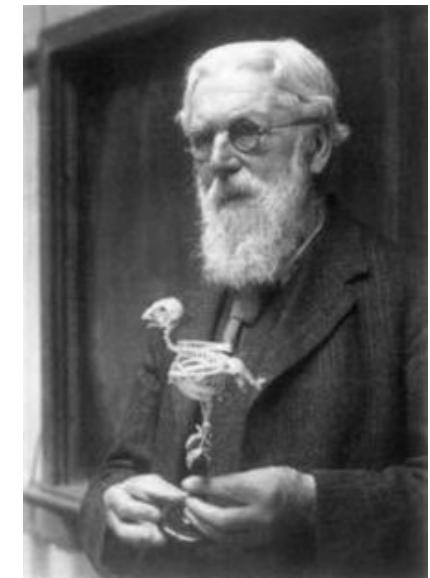
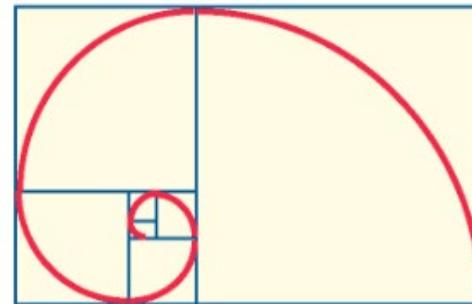
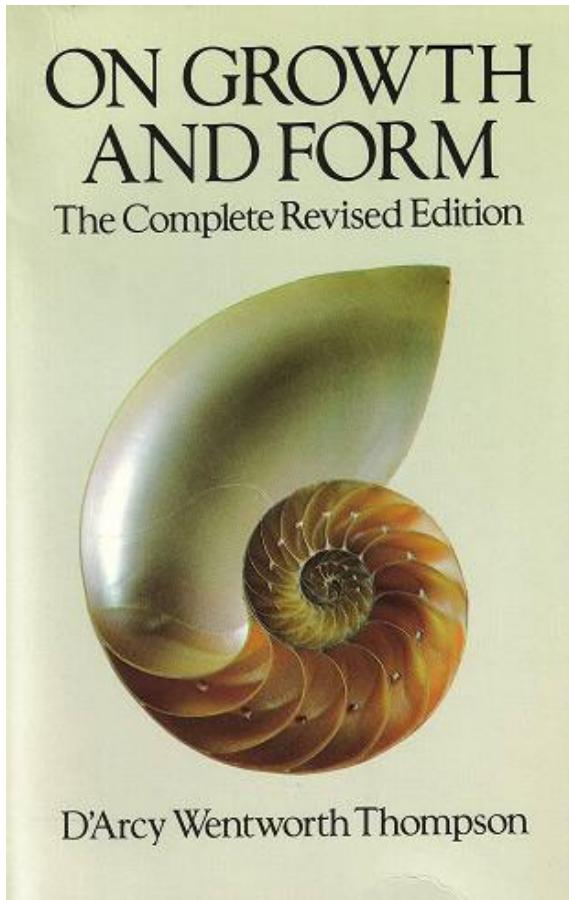
http://commons.wikimedia.org/wiki/Albrecht_Durer



Face transformations by Albrecht Dürer

http://commons.wikimedia.org/wiki/File:Durer_face_transformations.jpg

Highly Recommended Reading

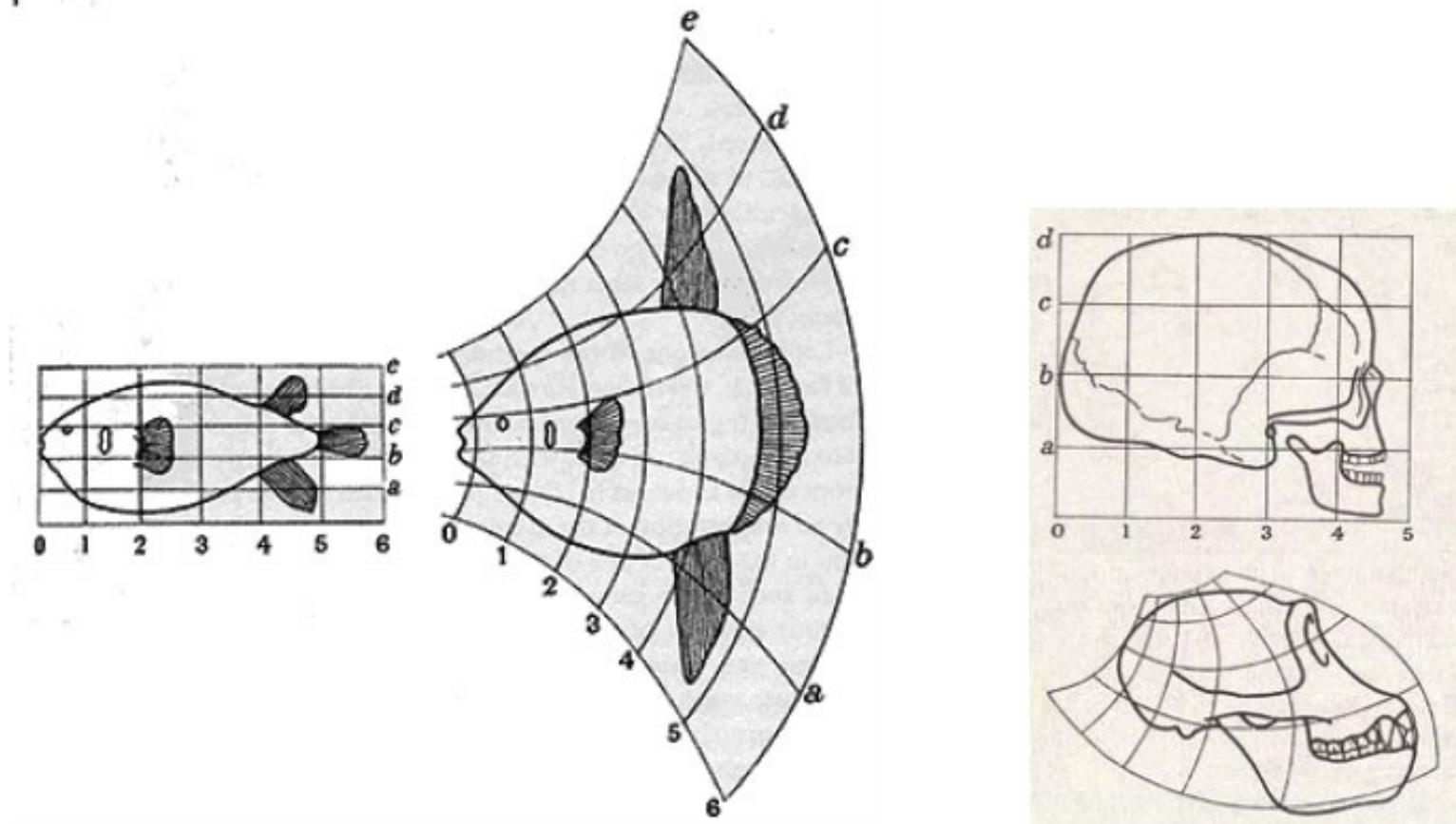


<http://archive.org/download/ongrowthform00thom/ongrowthform00thom.pdf>

<http://ia700301.us.archive.org/10/items/ongrowthform00thom/ongrowthform00thom.pdf>

D'Arcy Wentworth Thompson, *On Growth and Form* (1917, mathematics and biology)

Transformation Models



From D'Arcy Thompson, *On Growth and Form*, 1917.

Later: Bookstein's work on Thin-Plate-Splines

“A transformation for extracting new descriptors of shape”, Harry Blum (1967).

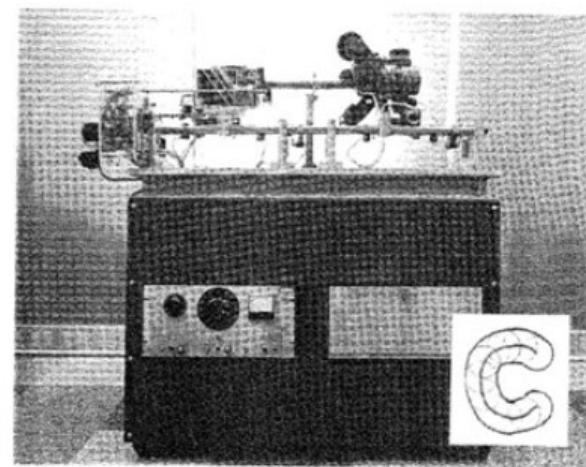
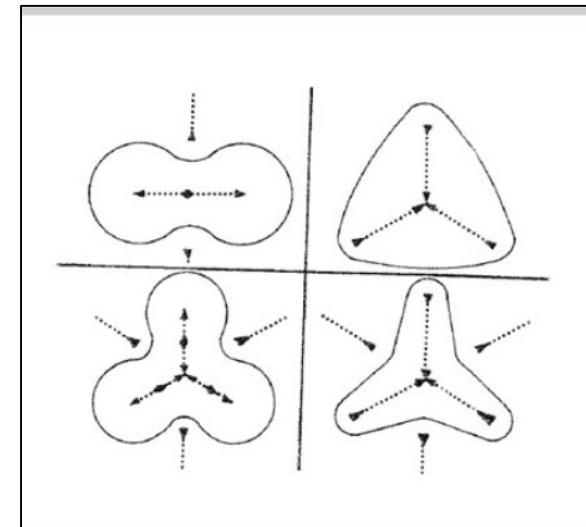
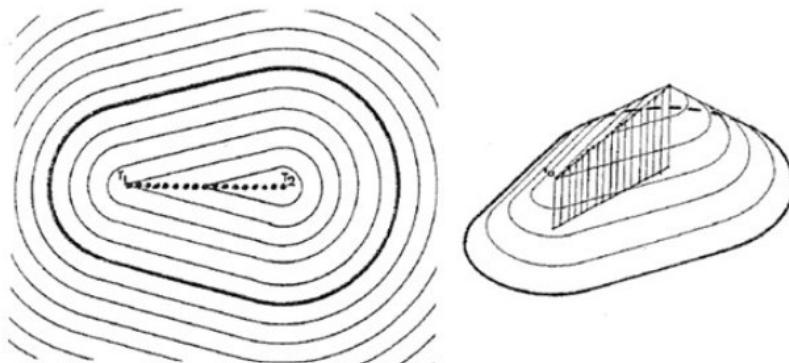
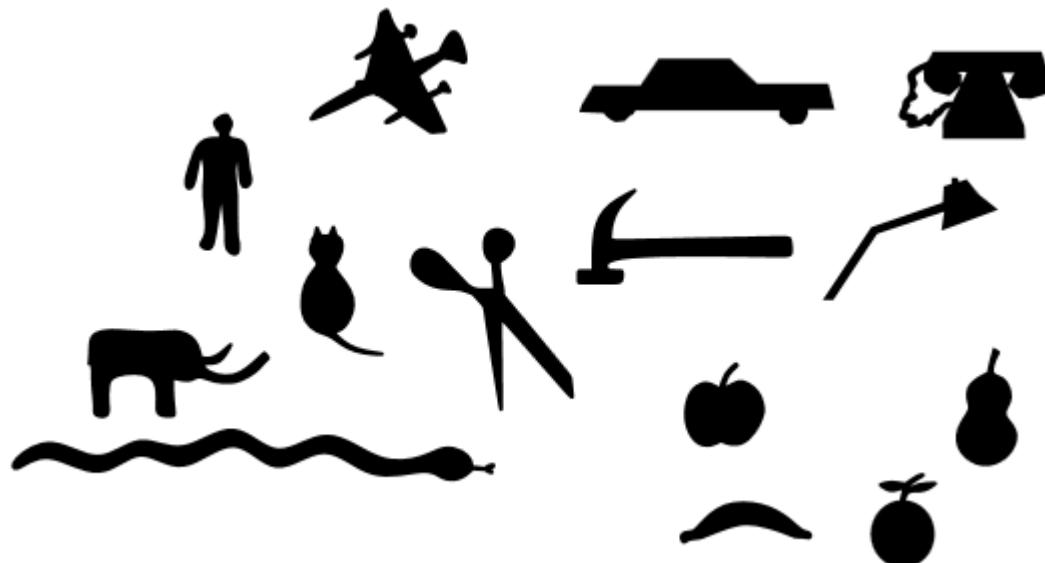


Fig. 19. An early optomechanical device for obtaining grassfire propagation using photographic defocussing.

Harry Blum (January 30, 1924^[1] – April 19, 1987) was a researcher at the [National Institutes of Health](#),^[2] known for his work in 1967 introducing the [medial axis](#) and [grassfire transform](#) of a shape,^[3] and more generally for his work on [shape analysis](#), [topological skeletonization](#),^[4] biological form, and [visual perception](#).

Shape and Human Vision

“Our Visual world contains a vast arrangement of objects, yet we are amazingly robust in recognizing them. This includes objects projected from novel viewpoints, or partially occluded objects. We are even able to describe totally unfamiliar objects, or to recognize unexpected ones out of context.”



What aspect of the geometry should be computed to allow robust recognition?

**Formal Definition?
Theory of Shape?**

What is Shape?

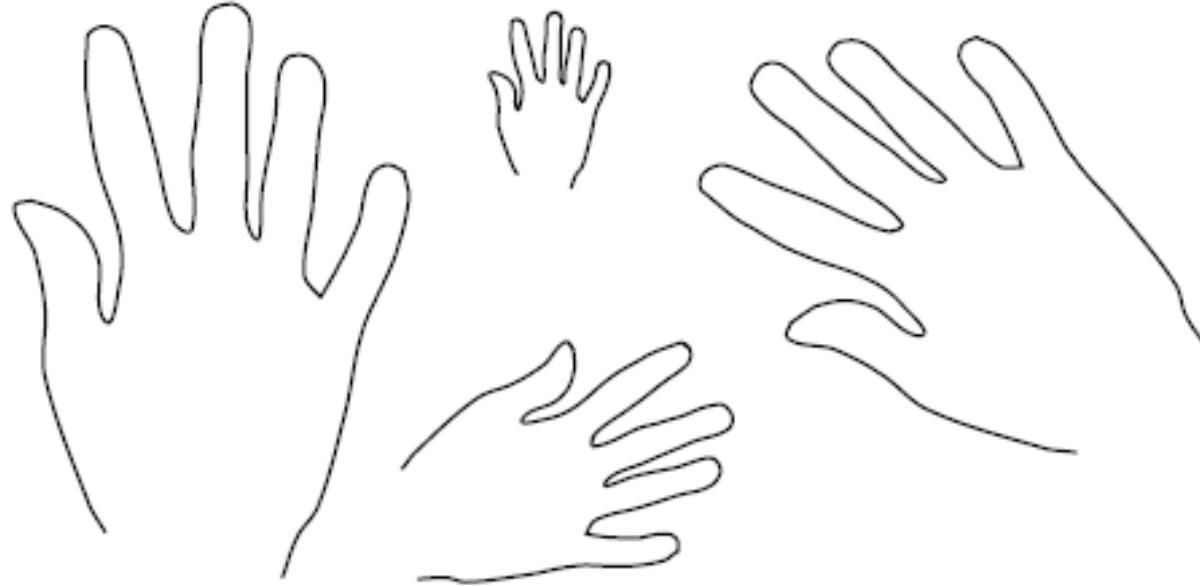


Figure 1: Four copies of the same shape, but under different Euclidean transformations.

Shape is the geometry of an object **modulo**
position, orientation, and size.

Kendall '77, Dryden, Mardia

From: Stegmann and Gomez, [Kendall, 1977]

Shape Statistics: Variability



Box of Phrenological Heads

Made and sold by William Bally, Dublin, 1831.

The 60 model heads in this box illustrate a wide range of human characteristics which phrenologists believed could be discovered by measuring the shape of the skull.

One of the initiators of the study of phrenology, Johann Caspar Spurzheim (1776-1832), wrote a pamphlet which accompanied the set, describing the qualities to be expected from each head shape. Number 54, for example, is the bust of a scientist.

science museum

Brought to Life

Exploring the History of Medicine

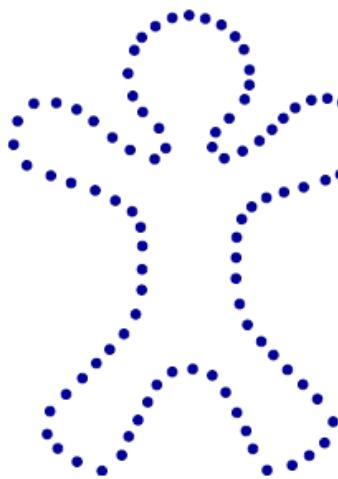


[link](#)

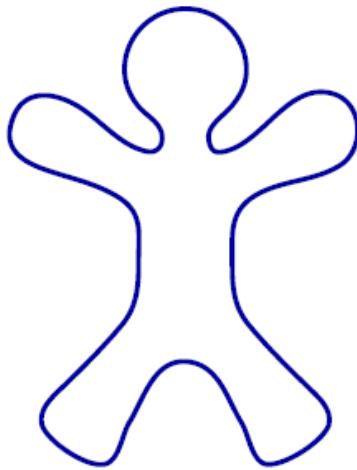
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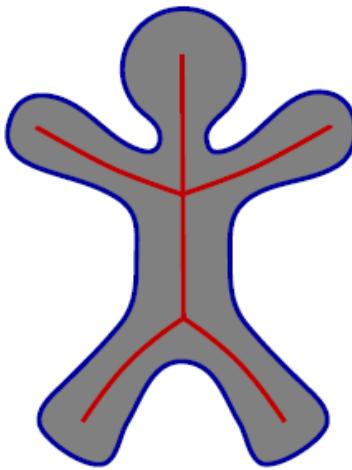
Shape Representations



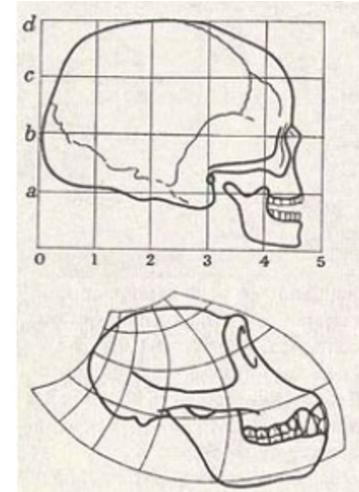
Dense Boundary
Points



Continuous Boundary
(Fourier, splines)

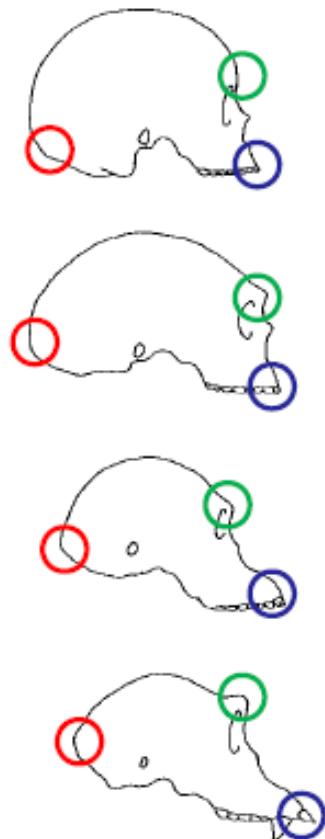


Medial Axis
(solid interior)



Deformation Field

Landmark Correspondence



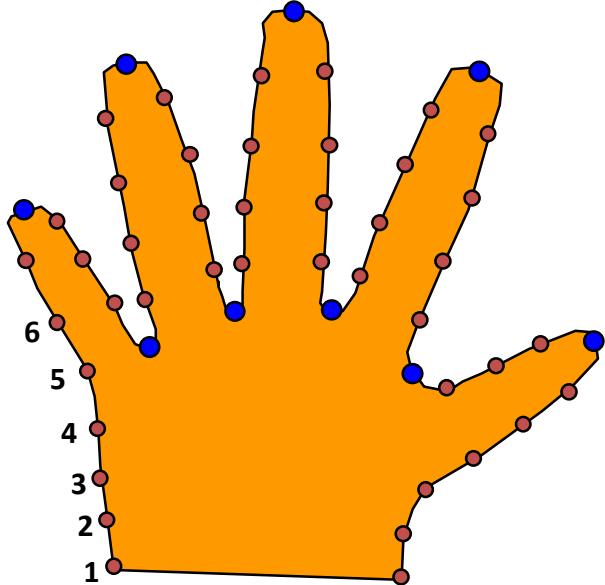
Homology:

Corresponding (homologous) features on skull images.

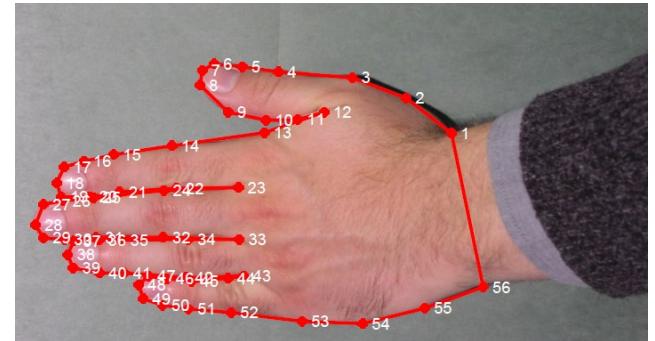
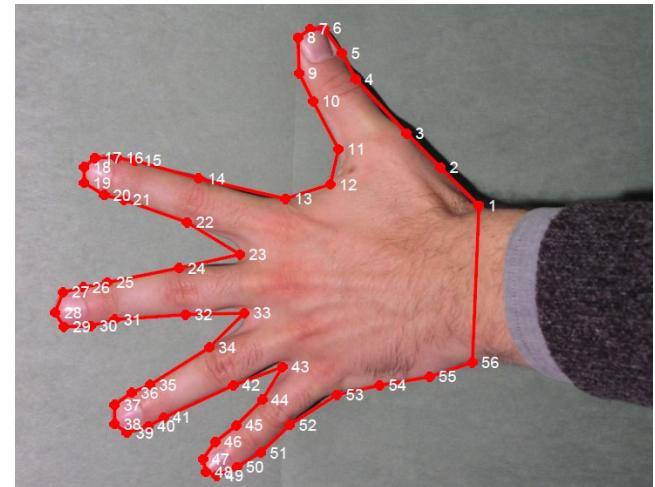
Modelling Shape

- Define each example using points
- Each (aligned) example is a vector

$$\mathbf{x}_i = \{x_{i1}, y_{i1}, x_{i2}, y_{i2} \dots x_{in}, y_{in}\}$$



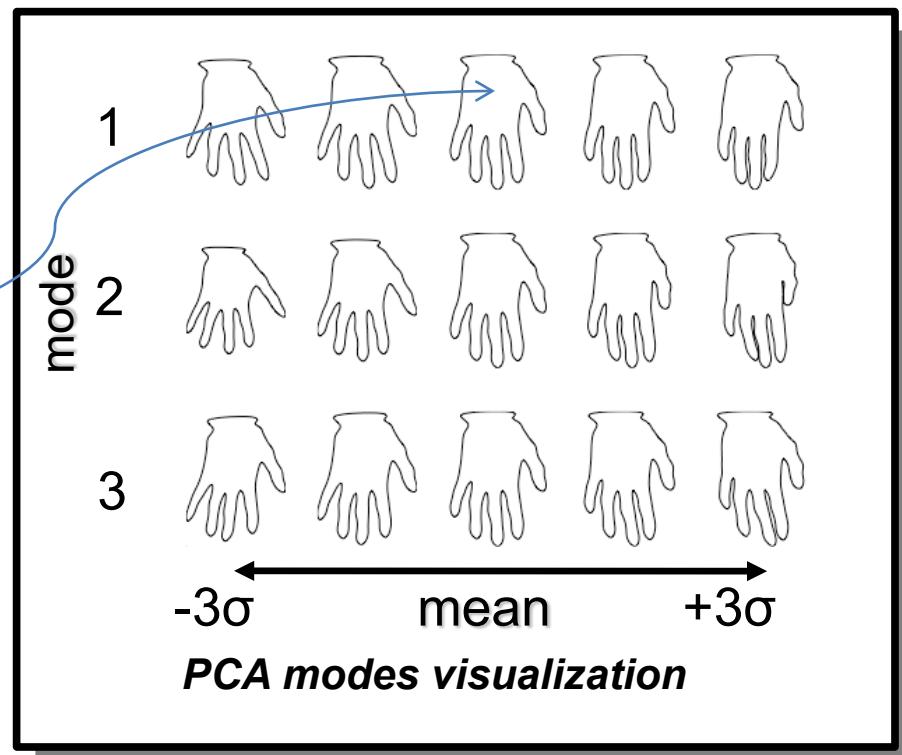
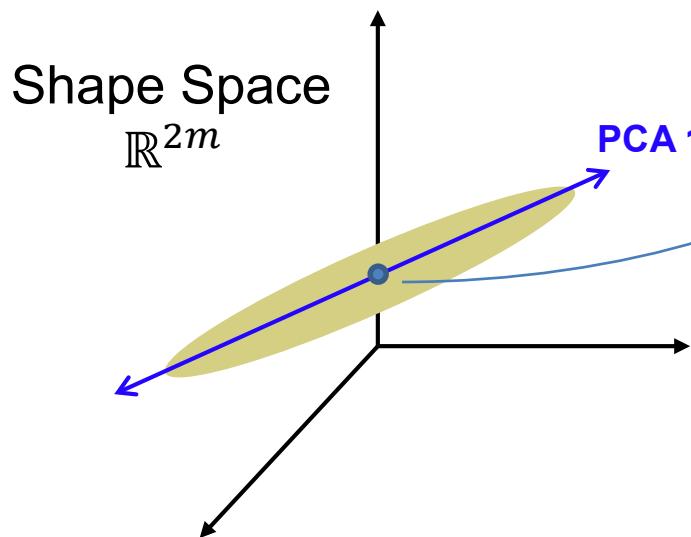
Cootes, Taylor 1993



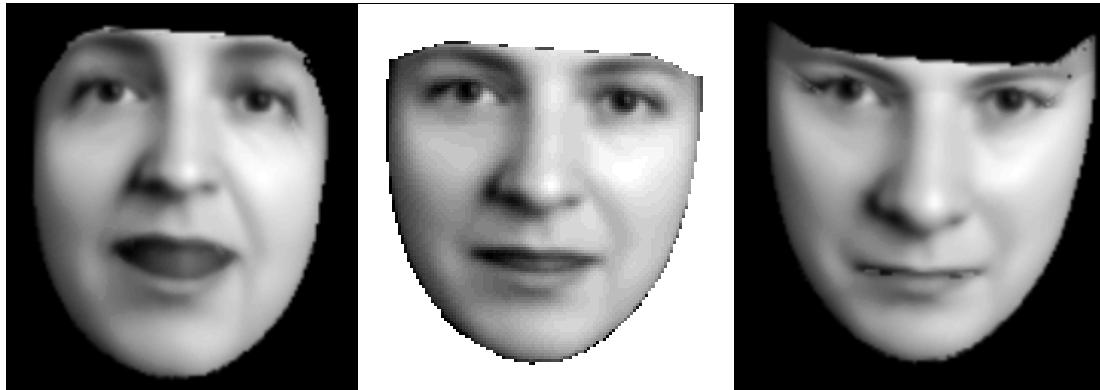
Stegmann and Gomez, 2002

Statistics in Shape Space

- Manual/automatic correspondences
- Gaussian models
- PCA for dimensionality reduction in shape space



Shape and Texture Modes: Active Shape and Appearance Model ASM



Shape variation (texture fixed)



Texture variation (shape fixed)

Courtesy of Chris Taylor, 1995



Eigenfaces: Sirovich & Kirby
87, Turk & Pentland 91

“Good” and “Bad” Correspondence

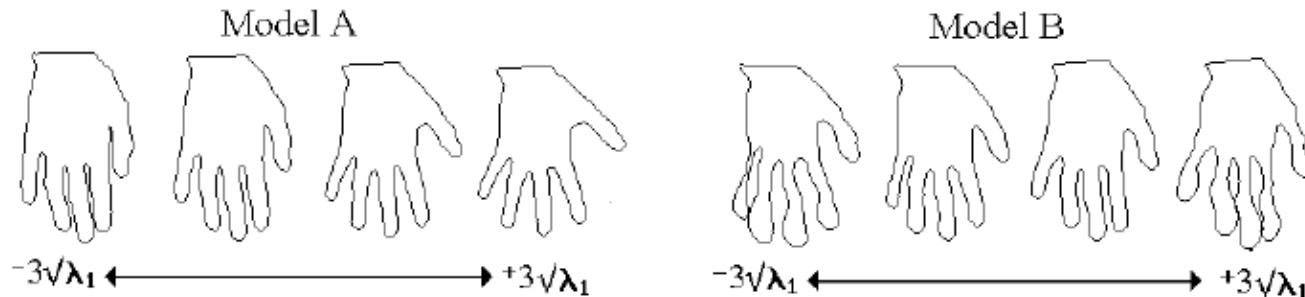
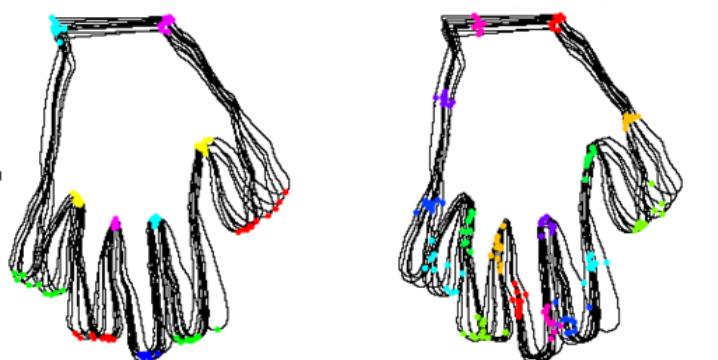


Figure 3.4. The first mode of variation of models *A* and *B*. The first parameter (b_1) is varied by $\pm 3\sqrt{\lambda_m}$.



A: Manual placement of corresponding landmarks

B: Arc-length parametrization

“Good” placement:

- R. Davis: Simplest Model has minimum stochastic complexity → Information Theory, Minimum Description Length (MDL)
- May lead to better, more compact statistical shape models.

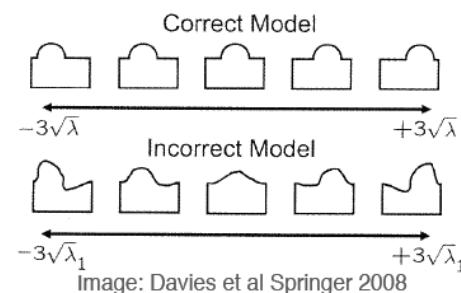
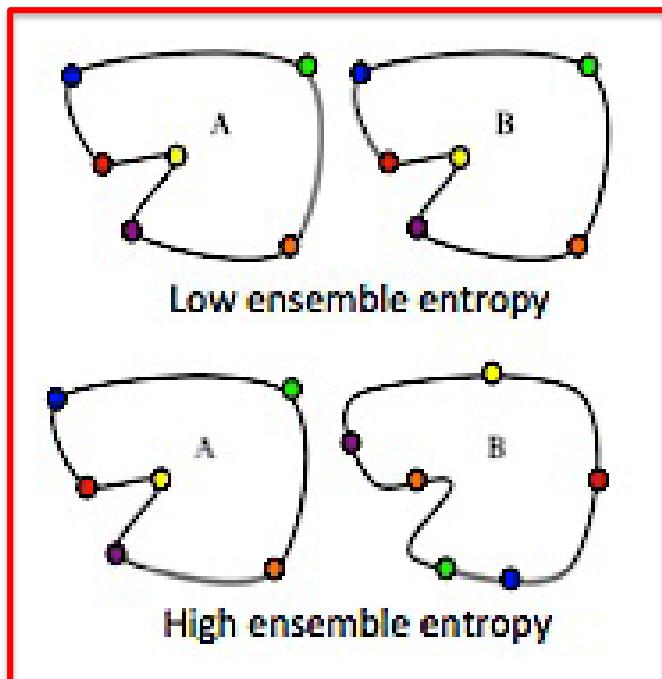


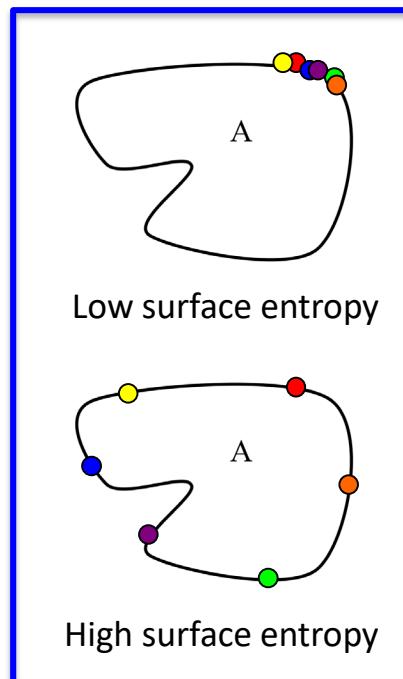
Image: Davies et al Springer 2008

Entropy-based Particle Systems

- Contours/Surfaces as discrete point sets, no parameterization
- Dynamic particles, positions optimize the information of the system: ensemble entropy, surface entropy

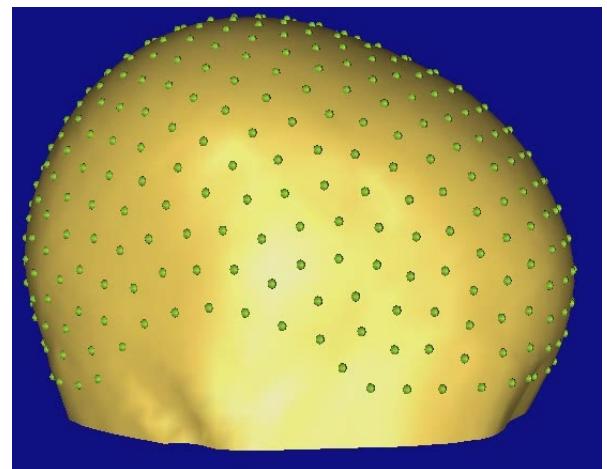


low is better

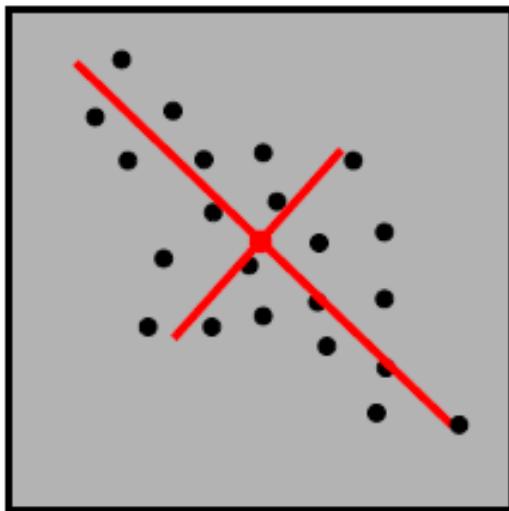


high is better

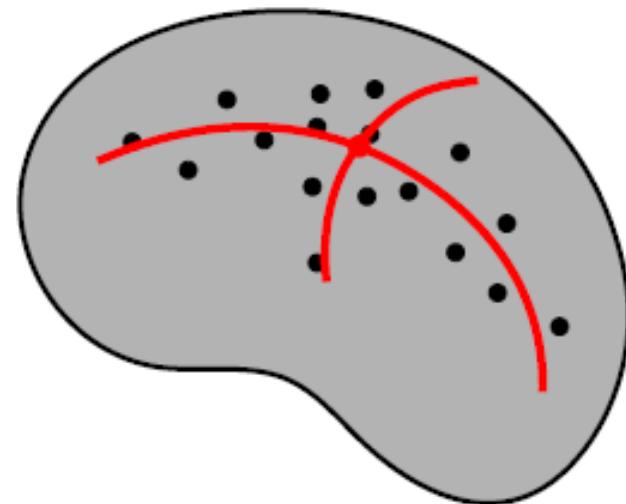
$$Q = H(Z) - \sum_k H(P^k)$$



Generalizing PCA: Principal Geodesic Analysis

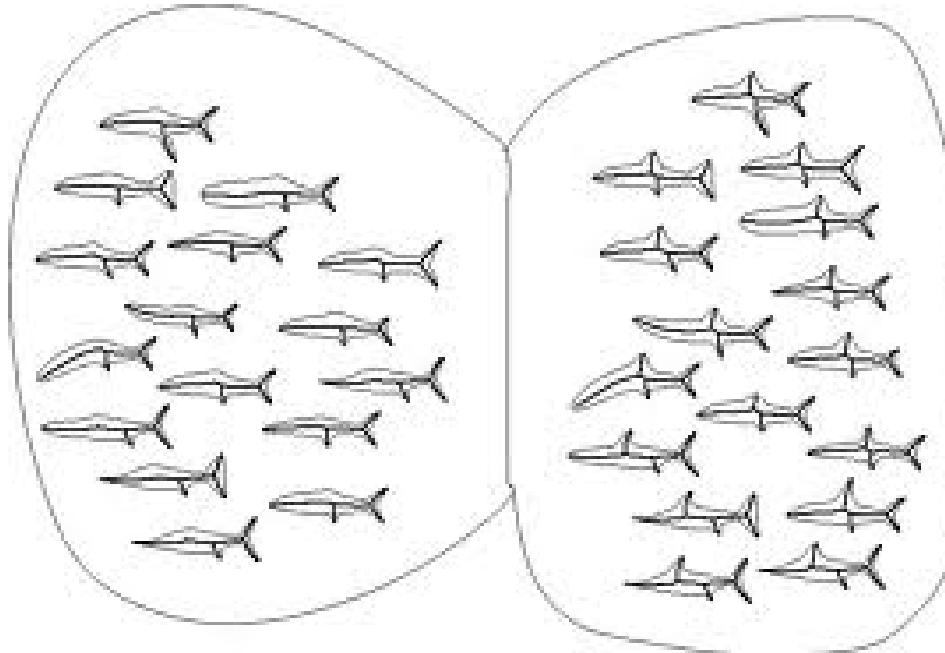


Linear Statistics
Principal Components Analysis
(PCA)



Curved Statistics
Principal Geodesics Analysis
(PGA)

Shock Graph: Shape Transformation



Invariance of shock graph to flexibly deformable objects.

Matching dog to cat via shock graph editing

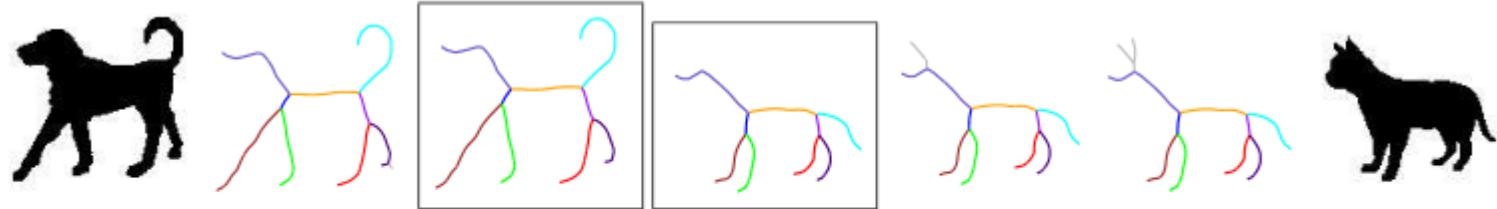
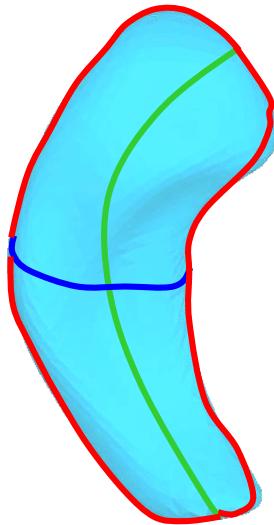


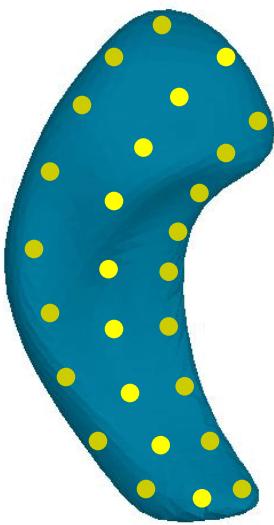
Figure 5: This figure from [43] intermediate shock graphs resulting from applying the edits in the optimal edit sequence for matching a cat and dog. The boxed shock graphs have the same topology. The distance between the shapes is the sum of all edit costs.

3D Shape Representations



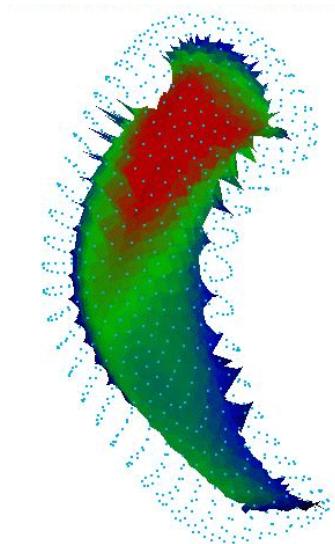
SPHARM

Boundary, fine scale, parametric



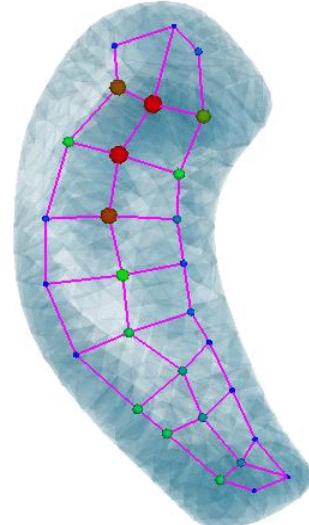
PDM

Boundary, fine scale, sampled



Skeleton

Medial, fine scale, sampled



S-rep

Medial, coarse scale, sampled

Skeleton from boundary points

Implied Surface

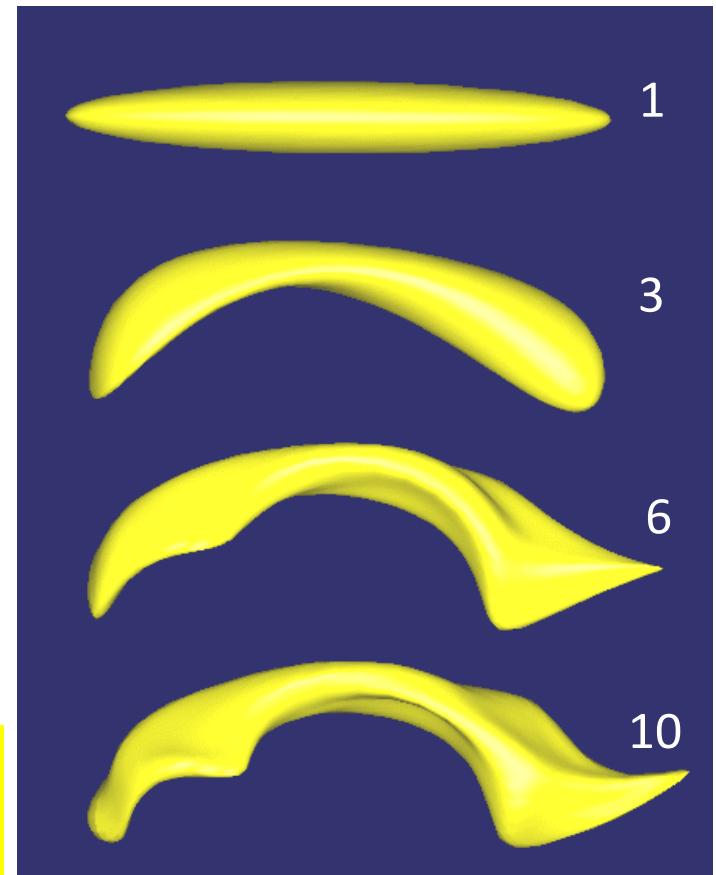
$$m = (\underline{x}, r, F, \theta)$$

$$r(\theta, \phi) = \sum_{k=0}^{\infty} \sum_{m=-k}^k c_k^m Y_k^m(\theta, \phi)$$

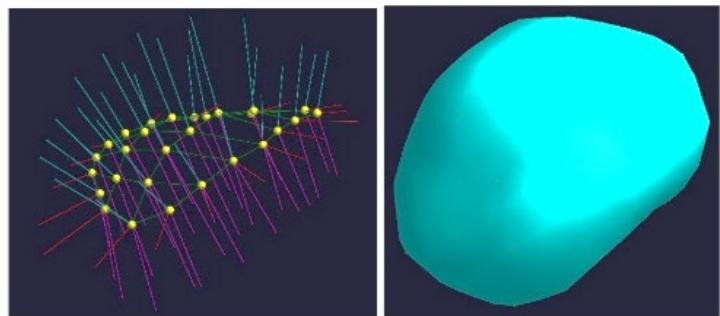
Spherical Harmonics (SPHARM)

1. Extract voxel surface
2. Area preserving parameterization
3. First order ellipsoid alignment
4. Fit SPHARM to coordinates
5. Sample parameterization and reconstruct object

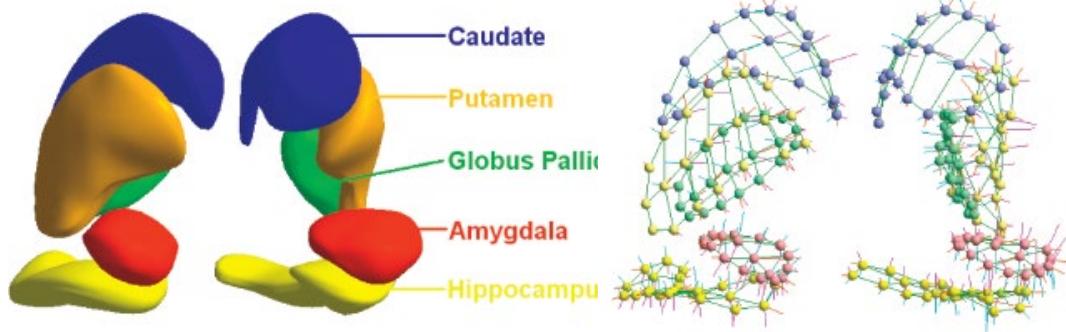
$$\begin{array}{c} x \\ y \\ z \end{array} \rightarrow \mathbf{r}(\theta, \phi) = \begin{pmatrix} x(\theta, \phi) \\ y(\theta, \phi) \\ z(\theta, \phi) \end{pmatrix}$$
$$\mathbf{r}(\theta, \phi) = \sum_{k=0}^K \sum_{m=-k}^k \mathbf{c}_k^m Y_k^m(\theta, \phi) \rightarrow \mathbf{c}_k^m = \begin{pmatrix} c_{xk}^m \\ c_{yk}^m \\ c_{zk}^m \end{pmatrix}$$



Medial Axis / Skeletal Representation: Intrinsic Shape Model

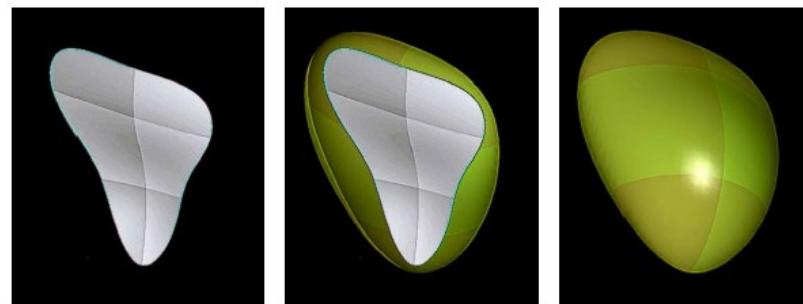
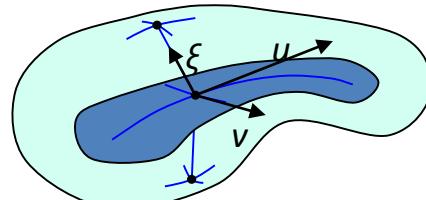


S-rep: Prostate s-rep and
implied boundary: Pizer et al.
(discrete)

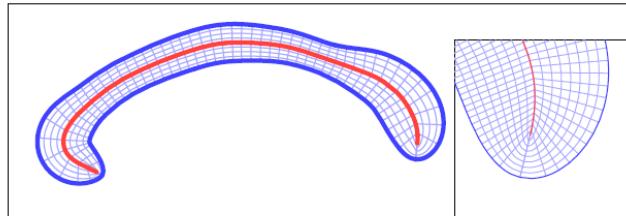


Gorczowski , Pizer, Styner, Gerig et al.,
T-PAMI 2010, Stats on deformations
vs. thickness

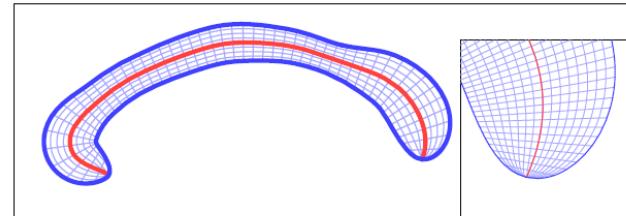
CM-rep: Yushkevich
(continuous, parametric)
Yushkevich et al., TMI 2006



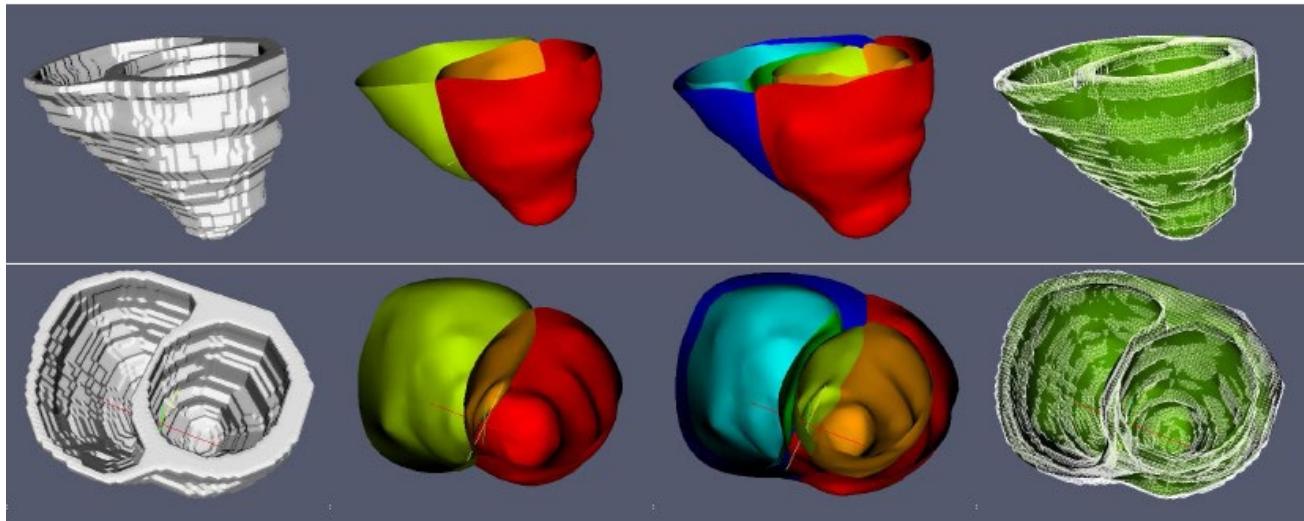
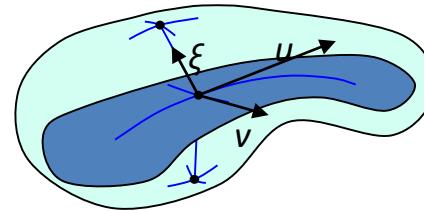
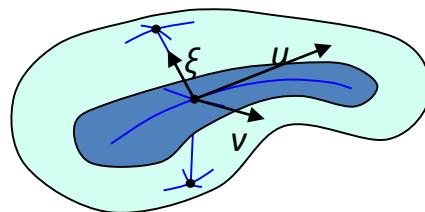
CM-rep



(a) Blum Skeleton Based parametrization



(b) SLS Based parametrization



Level-Set Formulation: Shapes as signed distance functions

- Embed shape contour as 0-level set
- Calculate Euclidean distance transform.
- Contour represented as image with embedded set of signed distance functions.

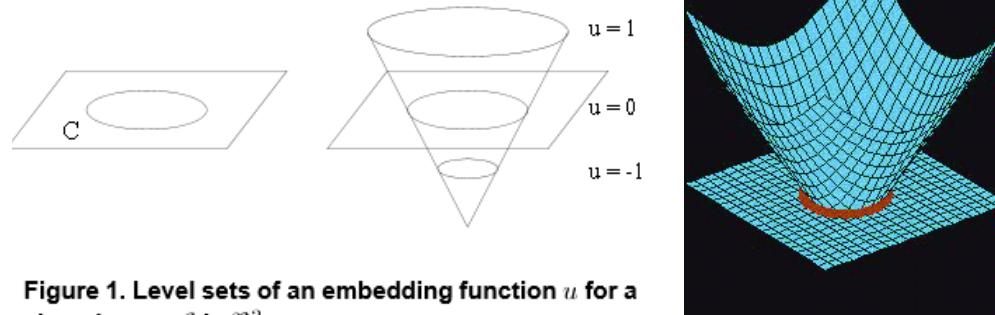
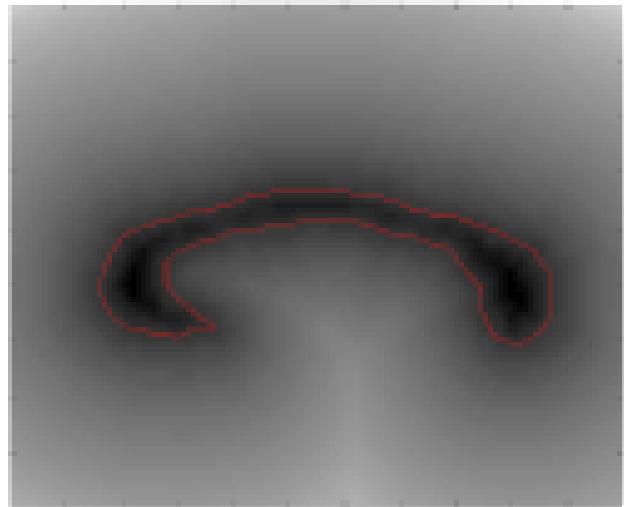
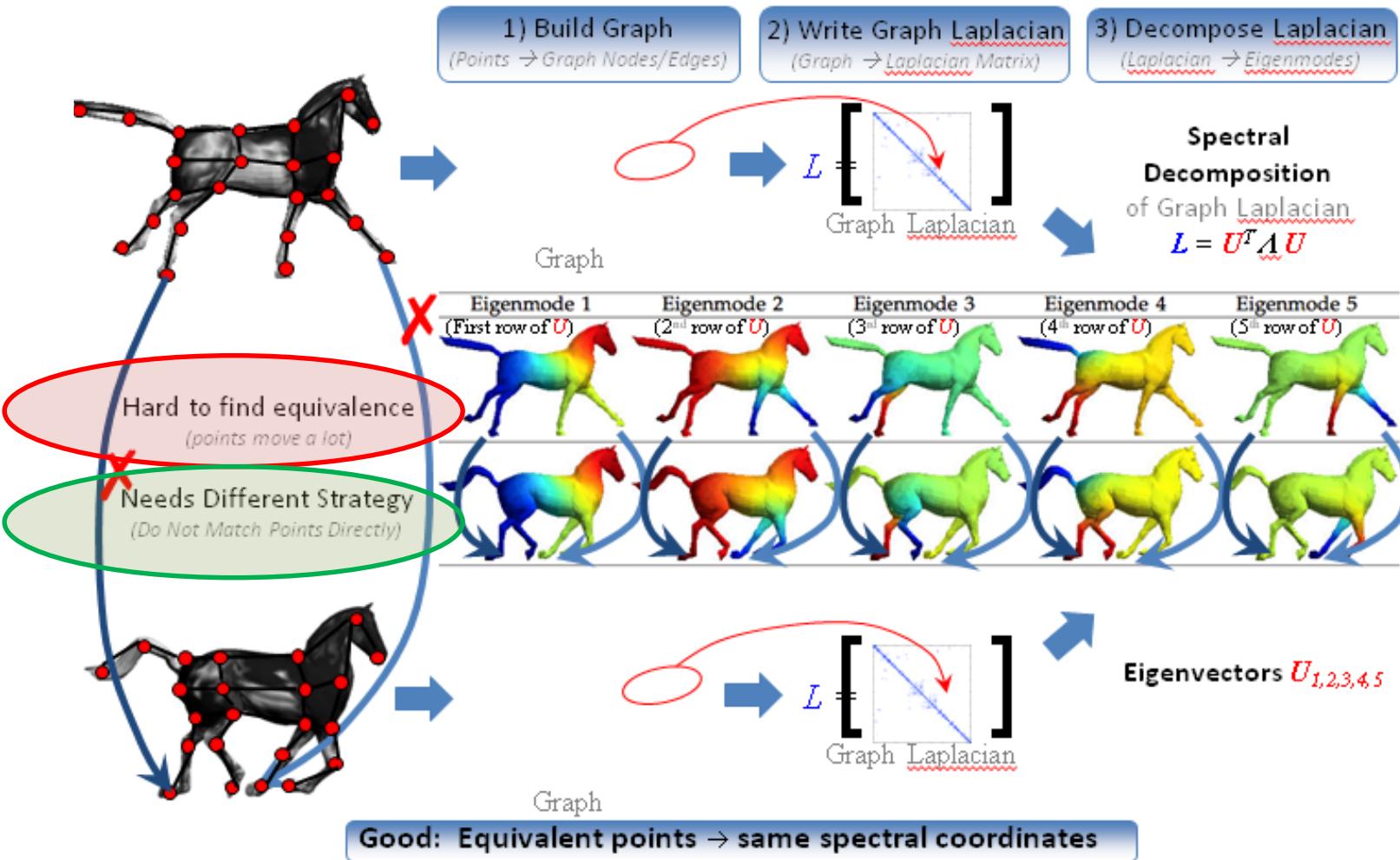


Figure 1. Level sets of an embedding function u for a closed curve \mathcal{C} in \mathbb{R}^2 .

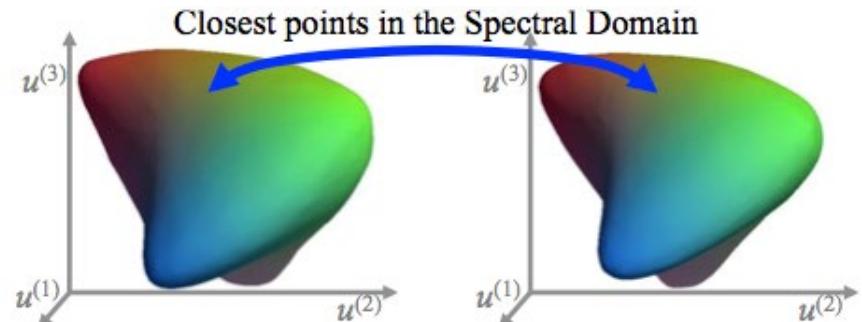
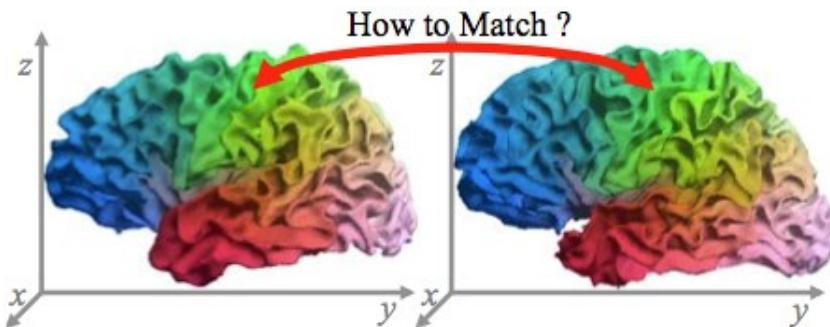


Graph Spectra/Laplacian

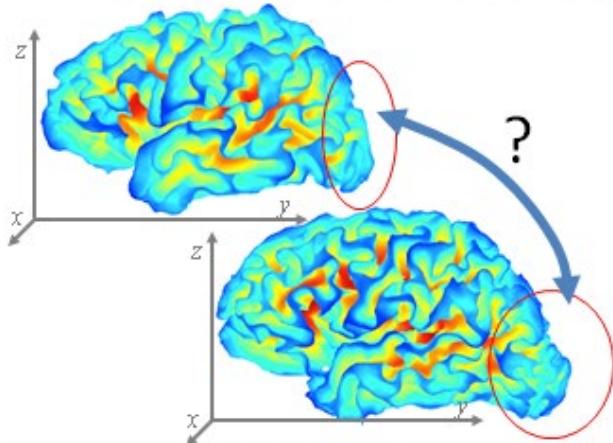
Computer Graphics: Compute and match graph spectra



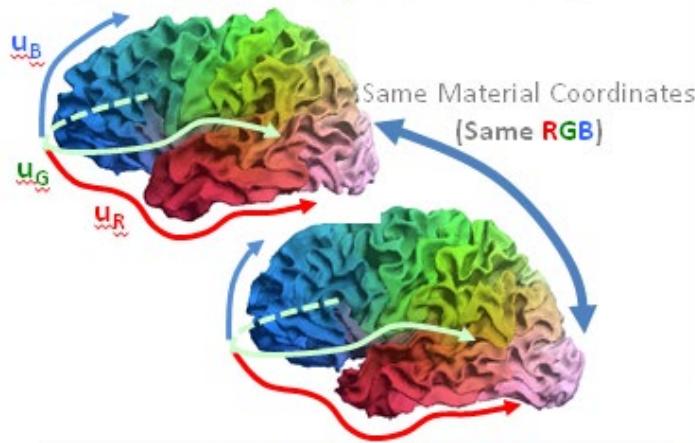
Graph Spectra/Laplacian



Cartesian Coordinates versus Material (Spectral) Coordinates



Cartesian Coordinates
Equivalent Points → May NOT Overlap in Space



Material/Shape Coordinates
Equivalent Points → Similar Shape Characteristics

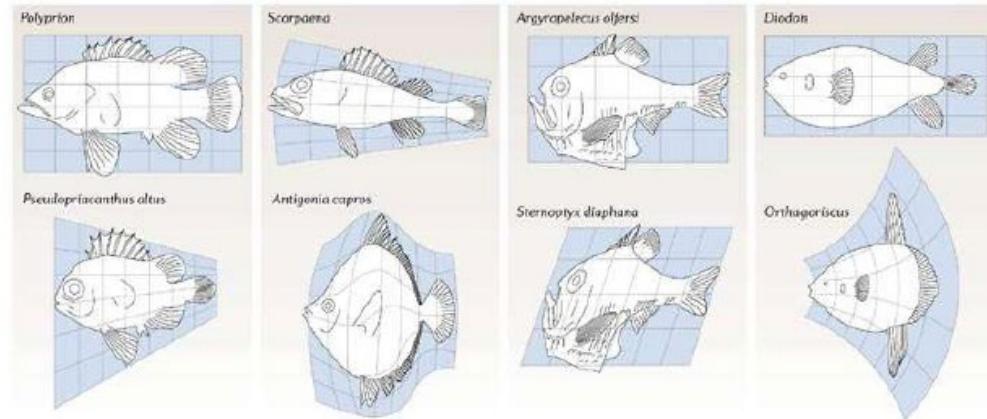
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Biological variation through mathematical transforms

D'Arcy Thompson laid out his vision in his treatise “On Growth and Form”. In 1917 he wrote:

In a very large part of morphology, our essential task lies [in the comparison](#) of related forms rather than in the precise definition of each; and the deformation of a complicated figure may be a phenomenon easy of comprehension, though the figure itself may be left unanalyzed and undefined.”



Shape Spaces: Kendall vs. Deformations

Kendall Shape Space:

- We are interested in the way the points of a shape move (or displace), but there is no general concept of a deformation -- analysis is based on the parameterization of the shape.
- Shape Space forms a complex projective space \mathbb{CP}^{k-2} .

Shape Spaces: Kendall vs. Deformations

Kendall Shape Space:

- We are interested in the way the points of a shape move (or displace), but there is no general concept of a deformation -- analysis is based on the parameterization of the shape.
- Shape Space forms a complex projective space \mathbb{CP}^{k-2} .

D'Arcy Thompson inspired deformation based analysis:

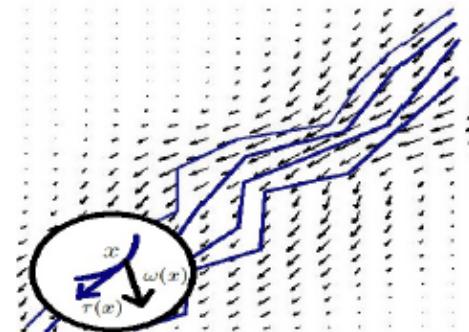
- Interested in the way the ambient space deforms.
- Statistical analysis is centered on the deformations of space, not movement & displacement of points on shapes.
- What is the Shape Space? Information via deformations.

“Correspondence-free” Registration: **Currents**

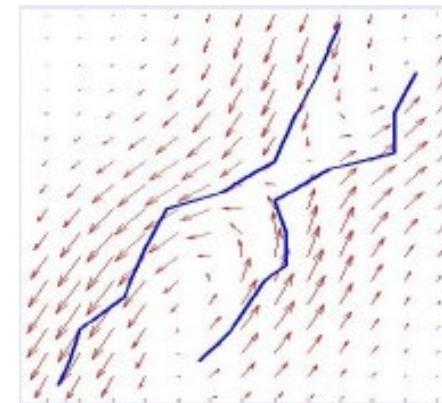
Topology and shape differences and noise can make point-to-point correspondence hard:

- **Currents**: Objects that integrate vector fields
- **Shape**: Oriented points = Set of normals (tangents)
- **Distance between curves**:

$$d(L_1, L_2)^2 = \int_{L_1} \omega_1(x)^t \tau_1(x) dx + \int_{L_2} \omega_2(x)^t \tau_2(x) dx - \int_{L_1} \omega_2(x)^t \tau_1(x) dx - \int_{L_2} \omega_1(x)^t \tau_2(x) dx$$



$$L(\omega) = \sum_{i=1\dots 4} \int_{L_i} \omega(x)^t \tau(x) d\lambda(x)$$



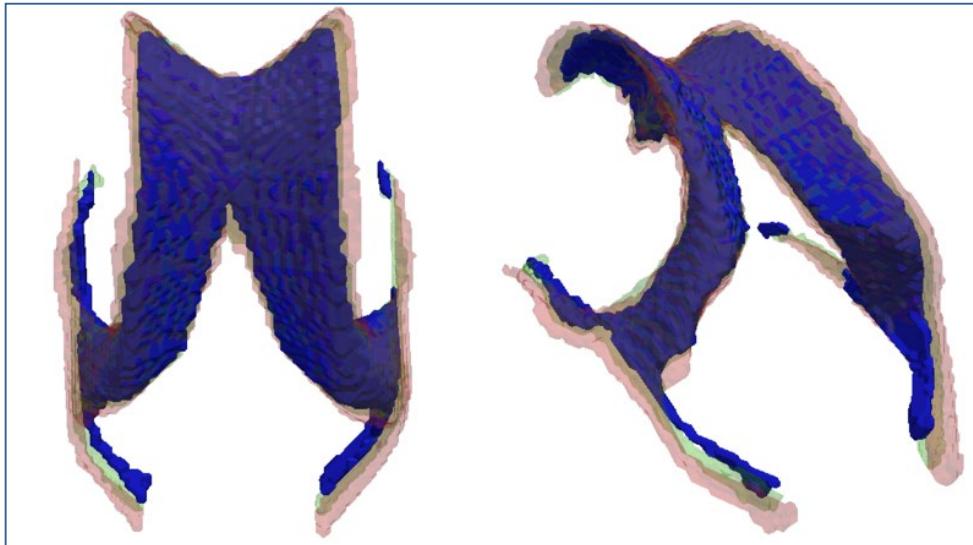
[Glaunes2004] Glaunes, J., Trouve, A., Younes, L. Diffeomorphic matching of distributions: a new approach, ... CVPR 2004.

[Durrleman2008] S. Durrleman, X. Pennec, A. Trouvé, P. Thompson, N. Ayache, Inferring Brain Variability from Diffeomorphic Deformations of Currents: an integrative approach, Medical Image Analysis 2008

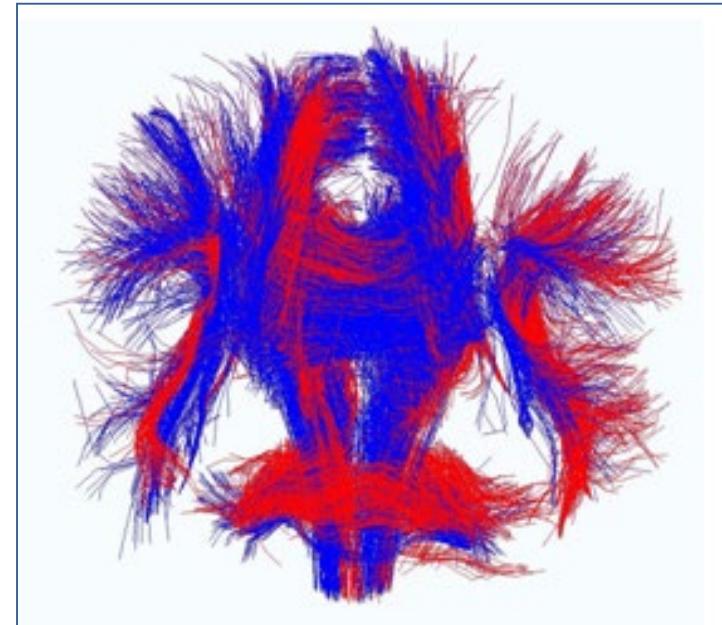
Correspondence-free Shape Analysis

Problems so far:

- Correspondence depends on shape parameterization
- Shapes with variable topology: correspondence undefined
- Now with **currents**:



Brain ventricles for infants 6mo to 2yrs



DTI Fiber Tracts from two subjects
[movie](#)

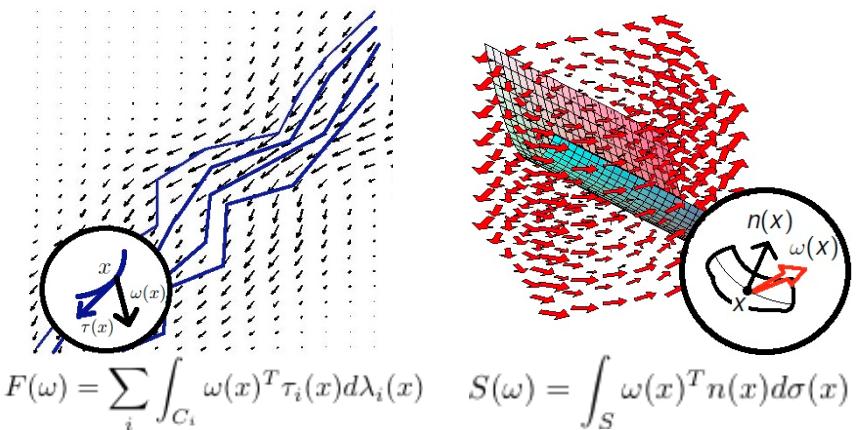
Correspondence-free similarity measures

- Depends on the kind of objects:
 - Images: sum of squared differences $\int |I(x) - I'(x)|^2 dx$
 - Landmarks: sum of squared differences $\sum_k |x_k - x'_k|^2$
 - Surface mesh and curves:
 - Currents [Glaunès'05]

$$\|F - F'\| = \sup_{\|\omega\| < 1} |F(\omega) - F'(\omega)|$$

$$\langle F, F' \rangle = \sum_p \sum_q \exp \left(-\frac{|x_p - x'_q|^2}{\sigma_W^2} \right) \tau_p^T \tau'_q$$

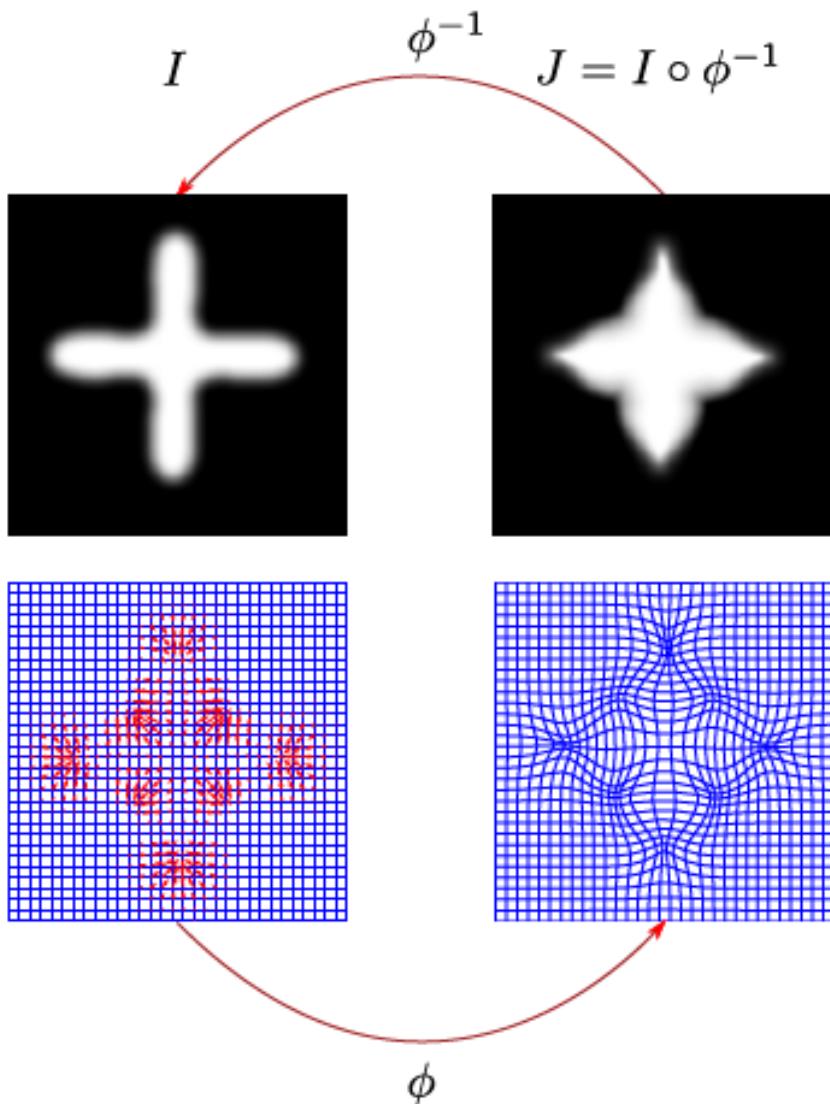
τ : tangents of curves/normals of surfaces



- No point correspondence needed
- Efficient numerical schemes (FFT)
- Robust to changes in topology
- Robust to differences in
 - mesh sampling
 - mesh imperfections...

→ **Usable routinely on large data sets**

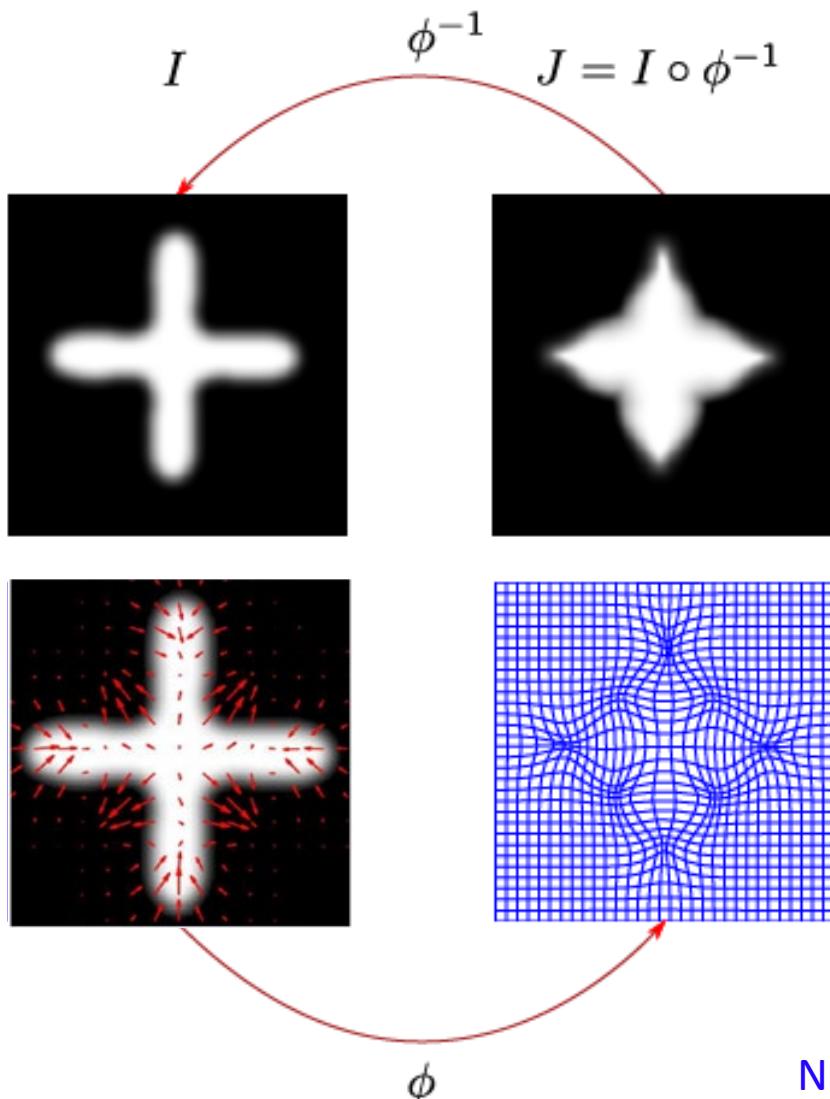
Ambient Space Deformation



Initial velocity* as a smooth vector field and the corresponding diffeomorphic flow that transforms the shape “plus” to “flower”.

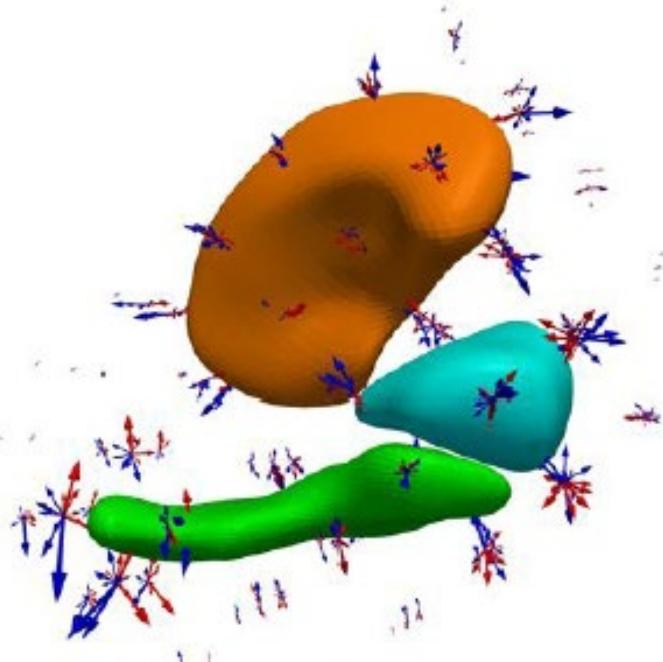
*velocity: momenta after convolution with kernel

Ambient Space Deformation



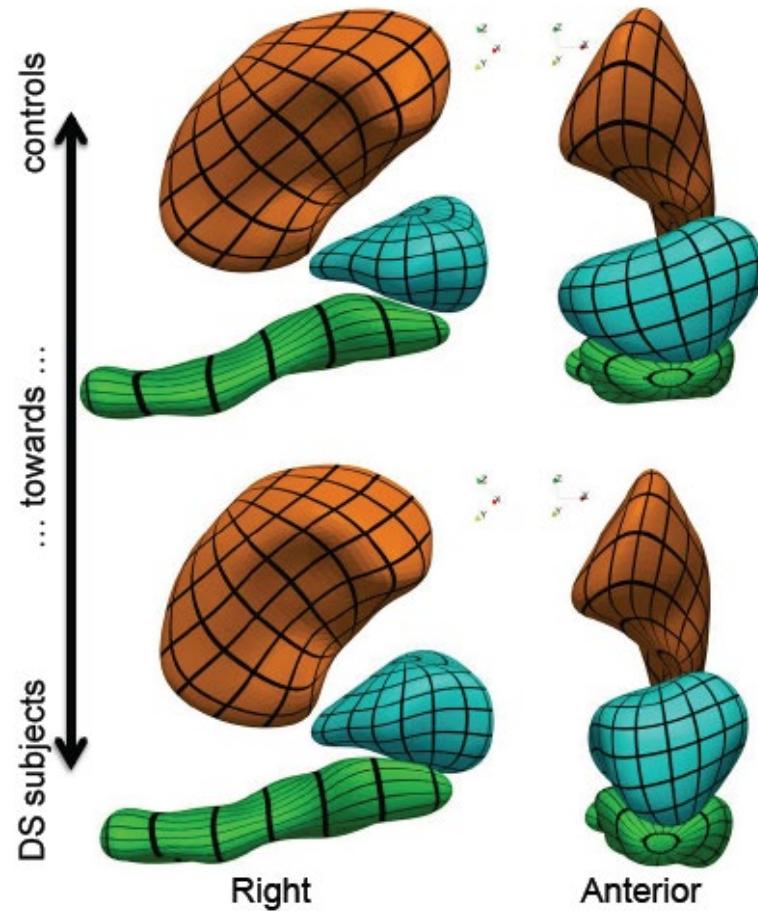
Momenta and the corresponding diffeomorphic flow that transforms the shape “plus” to “flower”.
Statistics on momenta.

Statistics on Deformations



Geodesics from atlas to each subject share the same tangent space.

Momentum vectors of DS subjects (red) and controls (blue) in atlas coordinate space.



Most discriminative deformation axis between Down's and Controls.

Statistics on Deformations



Most discriminative deformation axis between Down's Syndrome and Controls.

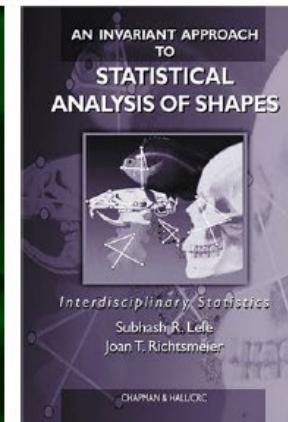
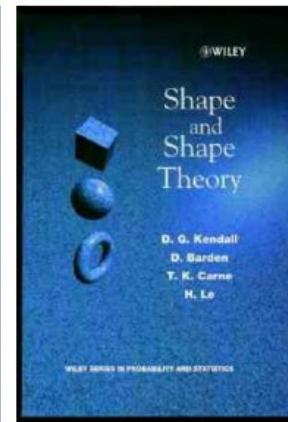
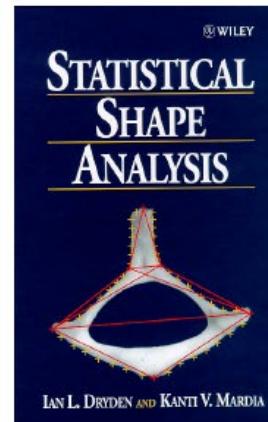
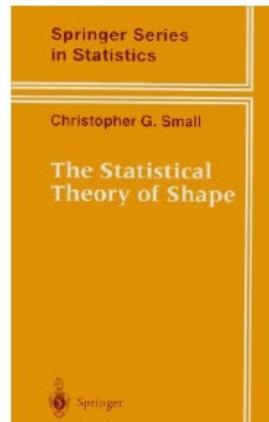
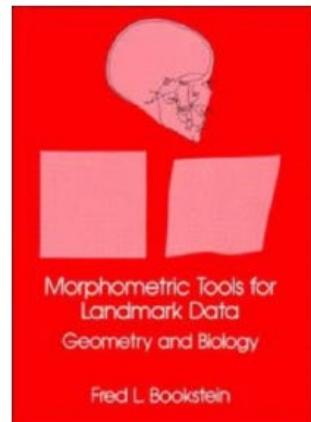
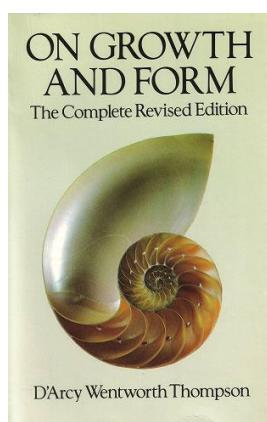
Durrleman et al, Neuroimage 2014

Main advantages:

- Shape space **independent** on shape representation.
- Natural way to handle **multiple shapes, topology variations, combinations of points, lines, contours, image intensity etc.**
- **Statistics on low #features rather than high-dimensional oversampled shape representation.**

Where to Learn More

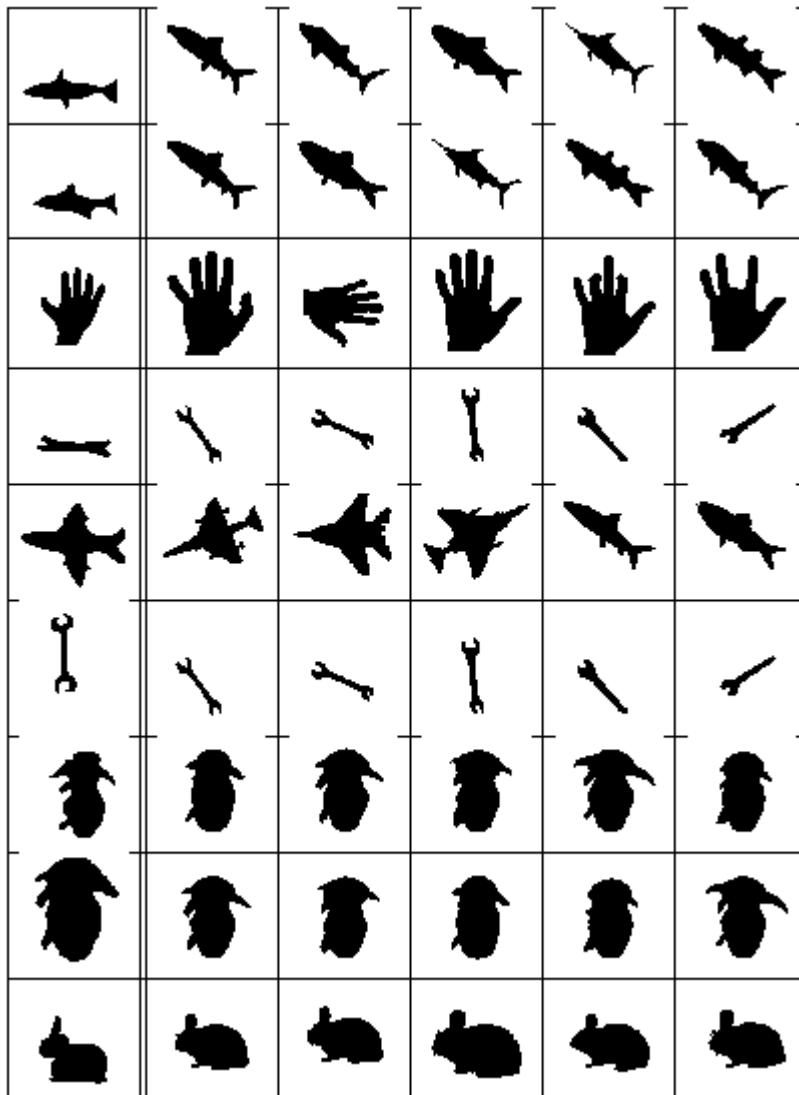
- Pioneers: d'Arcy Thompson, Fred Bookstein, and David Kendall.
- Bookstein (1991, Cambridge).
- Kendall, Barden and Carne, Shape and Shape Theory, Wiley, 1999.
- Dryden and Mardia, Statistical Shape Analysis, Wiley, 1998.
- Small, The Statistical Theory of Shape, Springer-Verlag, 1996.
- Grenander, HISTORY AS POINTS AND LINES, 1998-2003
- Lele and Richstmeier (2001, Chapman and Hall).
- Krim and Yezzi, Statistics and Analysis of Shapes, Birkhauser, 2006.



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 - Kendall Shape Space, SSM, ASM, AAM
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 - Shape Statistics via Deformations
- Current and Future
 - Problems with Size Normalization
 - Longitudinal Shape Analysis
 - Shapes come with covariates (sex, age, diagnosis, test scores, ...)

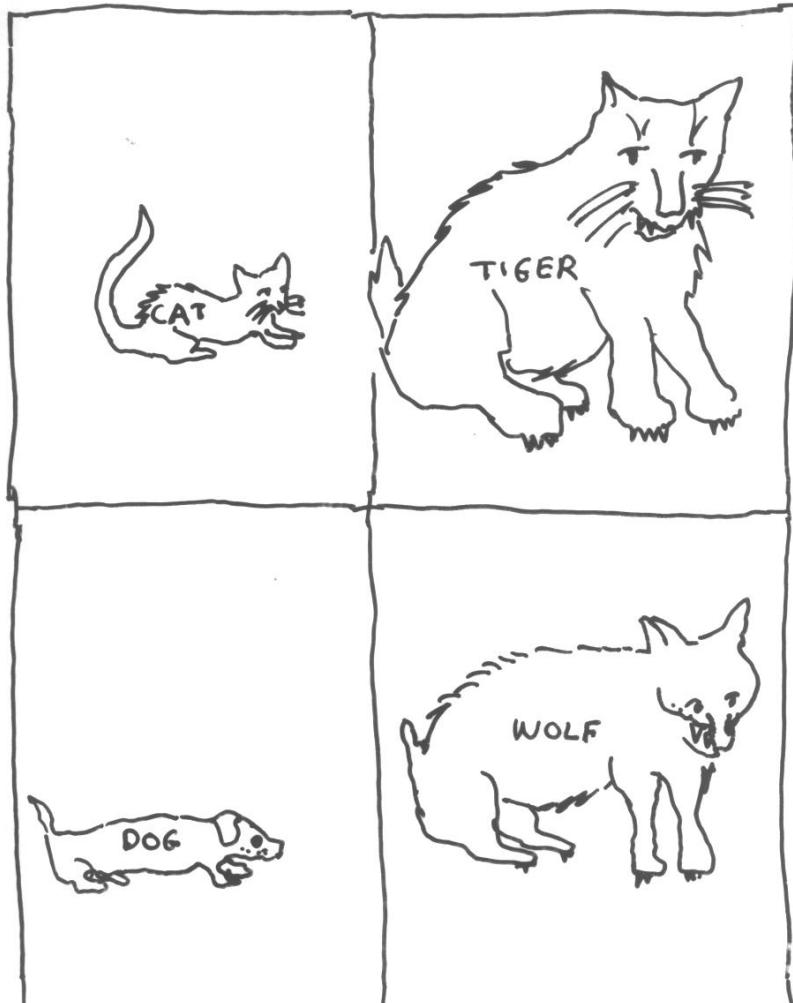
Shape Definition



Dryden/Mardia,
(Kendall 1977):

Shape is all the geometrical information that remains when **location, scale** and **rotational** effects are filtered out from an object.

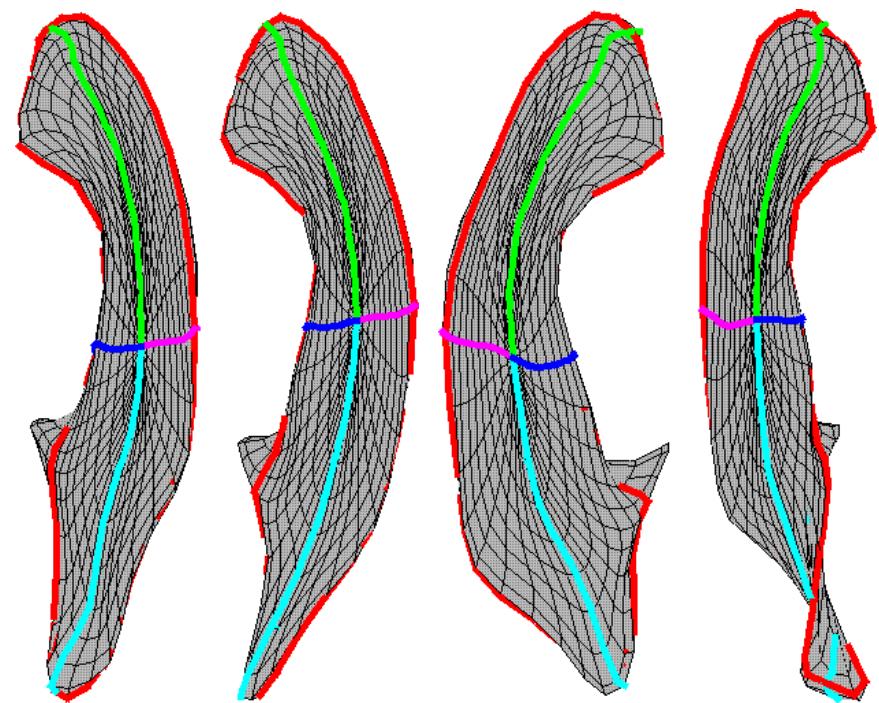
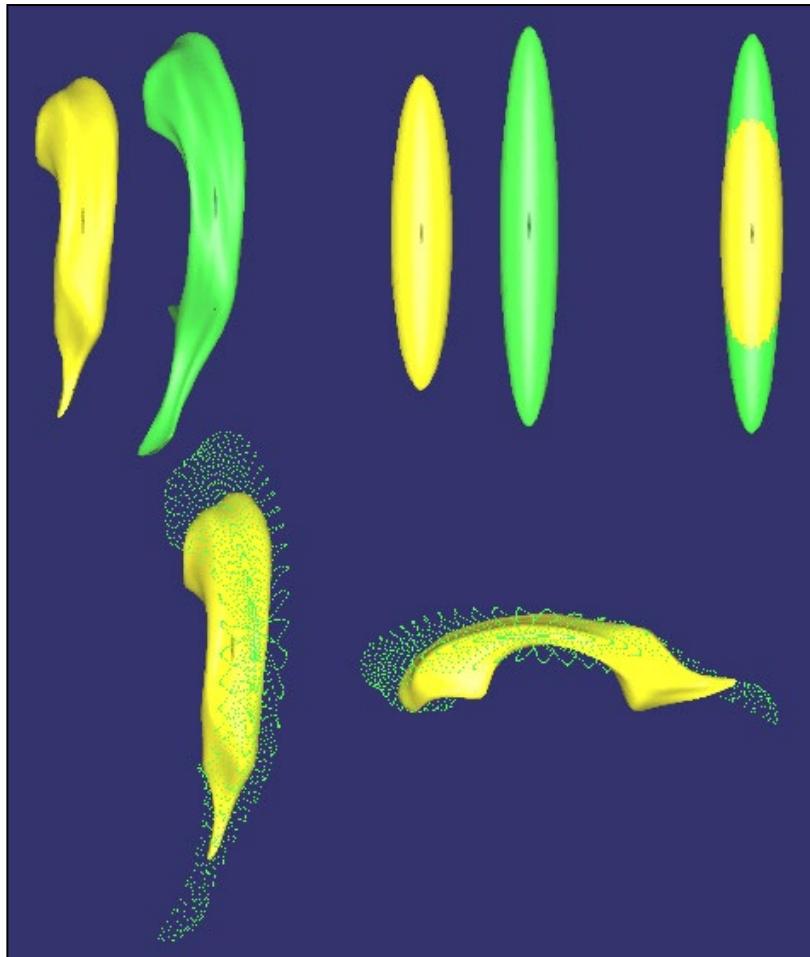
The Problem of Size and Shape



Dryden/Mardia (Kendall 1977): (*Sometimes we are also interested in retaining scale information as well as shape*)

Size-and-Shape is all the geometrical information that remains when location and rotational effects are filtered out from an object.

Object Alignment & Surface Homology with Biological Shapes



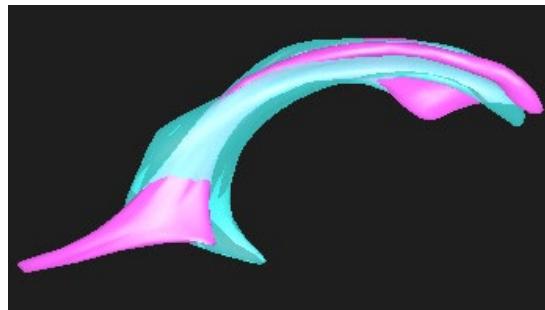
MZ pair

DZ pair

Surface Correspondence

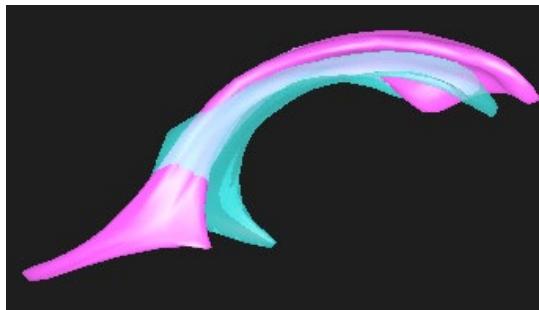
Object Alignment & Surface Homology with Biological Shapes

1stelli TR, no scal

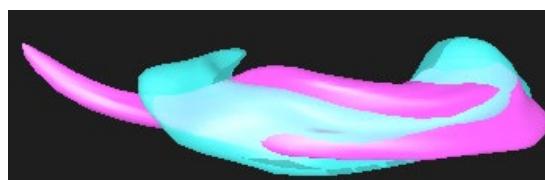


side

1stelli TR, vol scal



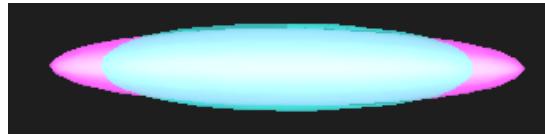
top



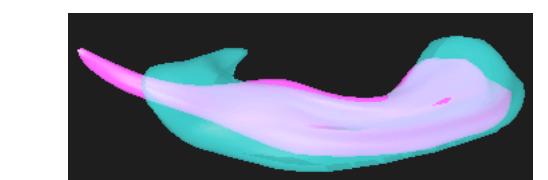
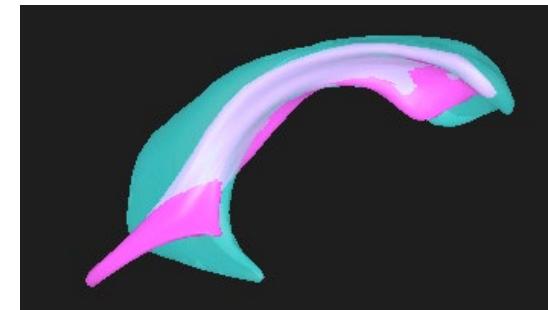
top



side



Procrustes TRS



Global scaling is mathematically appealing, but may not optimally represent biological shape difference or change.

Discussion: Normalization by Translation/Rotation/Scaling

- Biological/anatomical shapes are deformable.
- Different definitions of TRS will result in very different findings of shape differences:
 - shorter/longer
 - thinner/thicker
 - locality: differences at tail, or middle, or head:
Message for clinicians?
- Intrinsic versus extrinsic shape representations
- Choices to be tuned to problem at hand

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 - Shapes come with covariates (sex, age, diagnosis, test scores, ...)

Longitudinal Shape Analysis

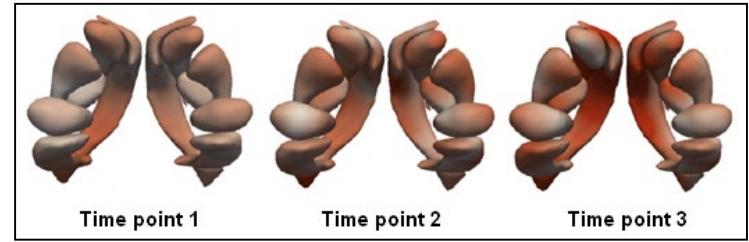
Clinicians most often study phenomena related to dynamic processes:

- Development, degeneration, monitoring recovery & efficacy of treatment
- Baseline to follow-up
- Disease progression
- Prediction of onset of clinical symptoms
- Personalized health care: Individual trajectories compared to expected “norm”
- Often: Longitudinal more powerful than cross-sectional analysis

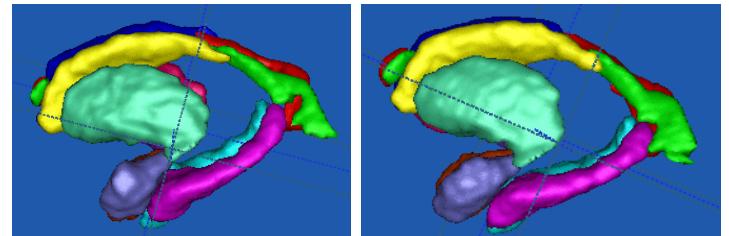
Pediatrics: (A)typical Brain Growth



Aging / Neurodegeneration



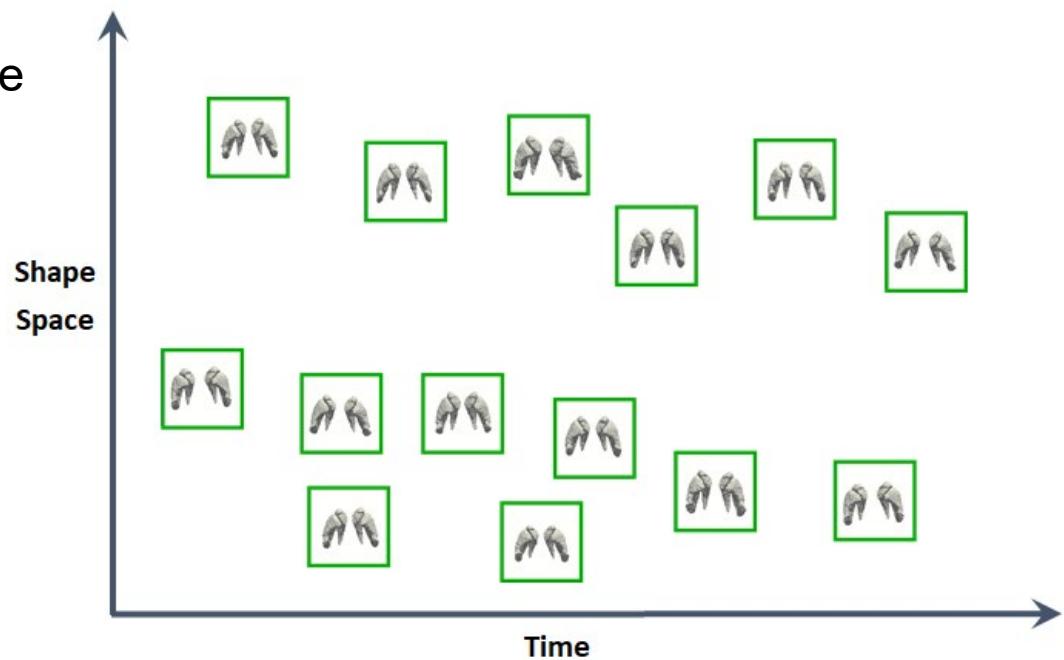
Ventricles & Nuclei (age 2 to 4)



Why Shape Regression?

Observed Anatomical Shapes: Aging caudate/putamen

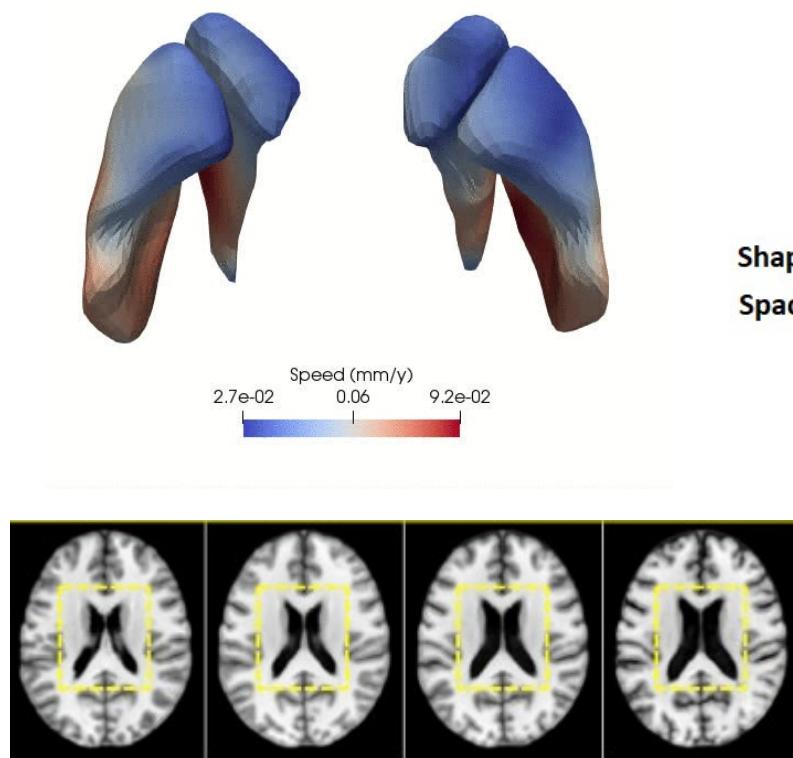
Sparsely observed shape sequence
as function of age 20 to 75



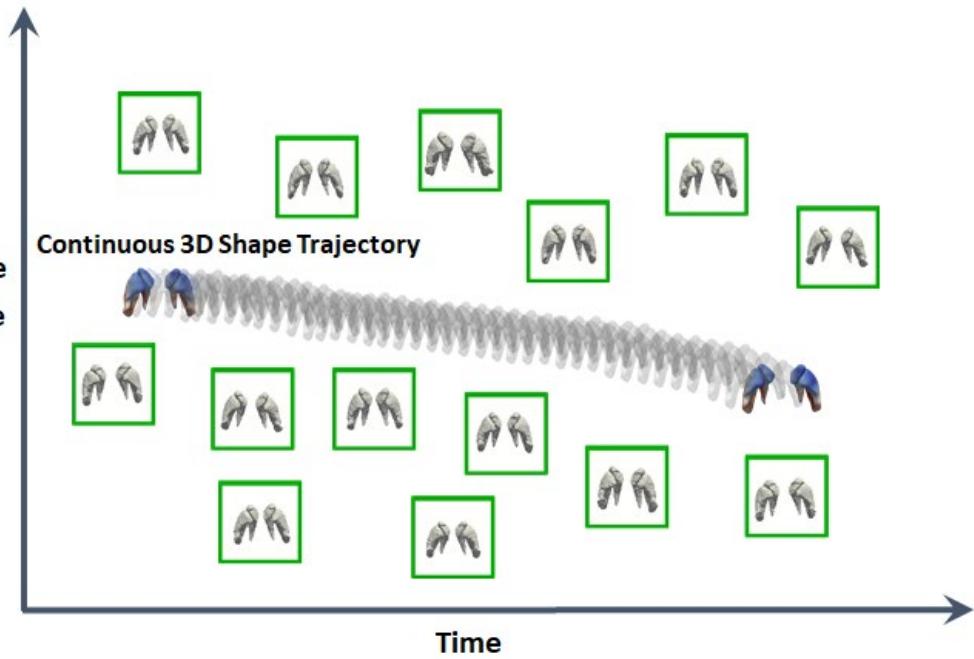
Why Shape Regression?

Modeling Spatio-Temporal Shape Change

Continuous shape sequence age 20 to 75

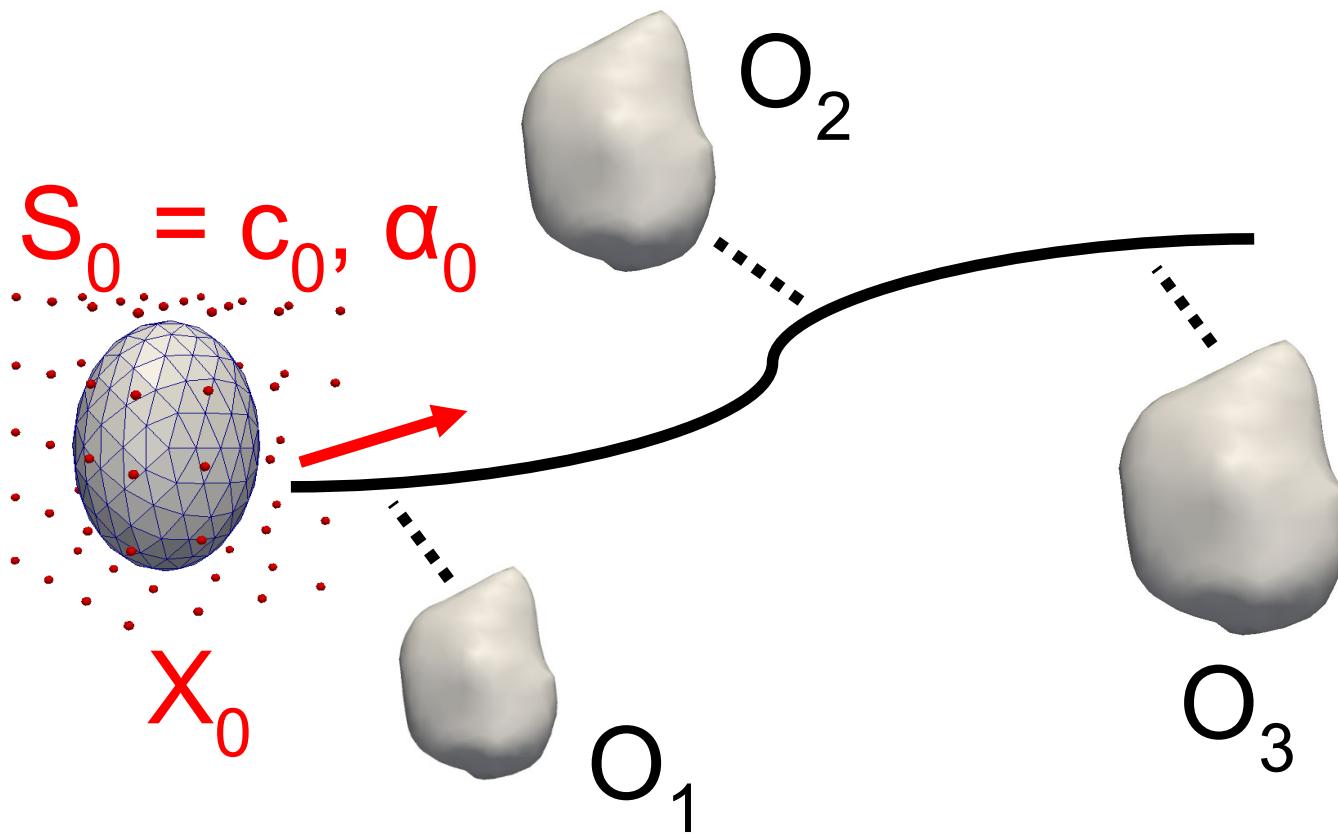


Ventricular enlargement due to brain atrophy



Sungmin Hong, MICCAI 2019

Geodesic Shape Regression



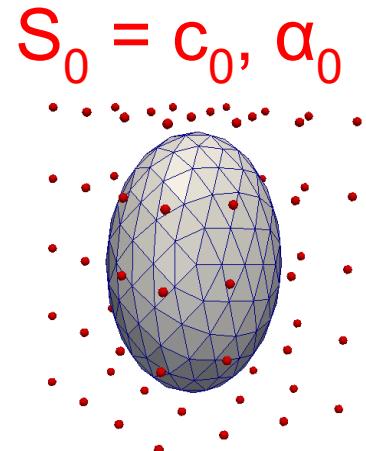
Geodesic shooting to evolve control points

Trajectory of control points defines **flow** of diffeomorphisms

Control Point Formulation

Geodesic shooting to evolve control points

$$\begin{cases} \dot{c}_i(t) = \sum_{p=1}^{N_c} K(c_i(t), c_p(t)) \alpha_p(t) \\ \dot{\alpha}_i(t) = - \sum_{p=1}^{N_c} \alpha_i(t)^t \alpha_p(t) \nabla_1 K(c_i(t), c_p(t)) \end{cases}$$

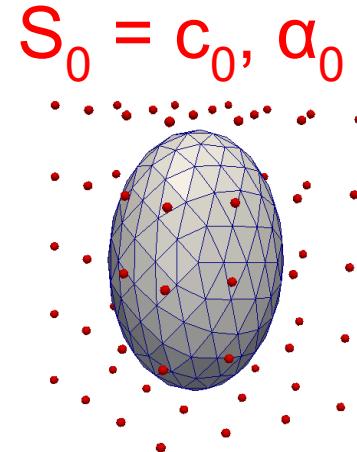


Trajectory of control points defines **flow** of diffeomorphisms

$$\dot{\phi}_t(x) = v(x, t) = \sum_{p=1}^{N_c} K(x, c_p(t)) \alpha_p(t)$$

Regression Criterion

$$E(\mathbf{X}_0, \mathbf{S}_0) = \sum_{i=1}^{N_{obs}} \frac{1}{2\lambda^2} D(\mathbf{X}(t_i), \mathbf{O}_{t_i}) + L(\mathbf{S}_0)$$



D is squared distance on currents

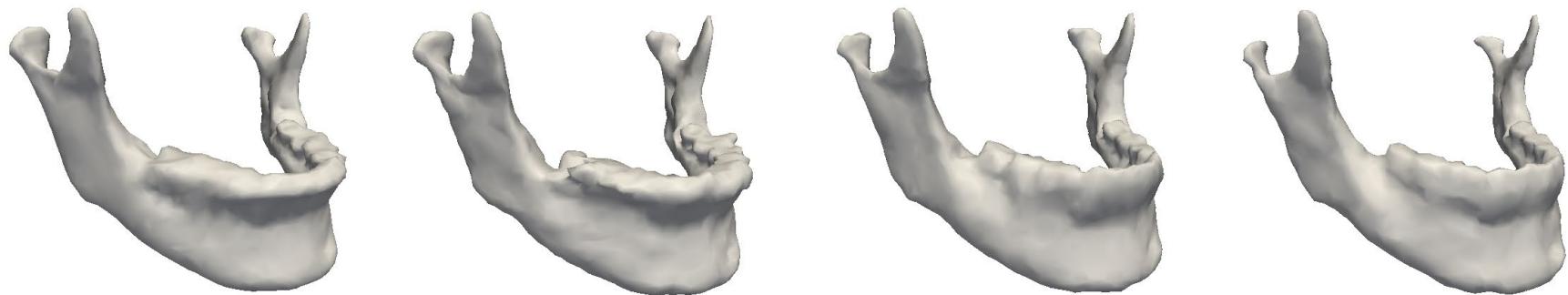
$$D(\mathbf{X}(t_i), \mathbf{O}_{t_i}) = \|(\phi_{t_i}(\mathbf{X}_0) - \mathbf{O}_{t_i})\|_{W^*}^2$$

Regularity defined by kinetic energy of control points

$$L(\mathbf{S}_0) = \sum_{p,q} \alpha_{0,p}^t K(c_{0,p}, c_{0,q}) \alpha_{0,q}$$

Longitudinal Shape Modeling

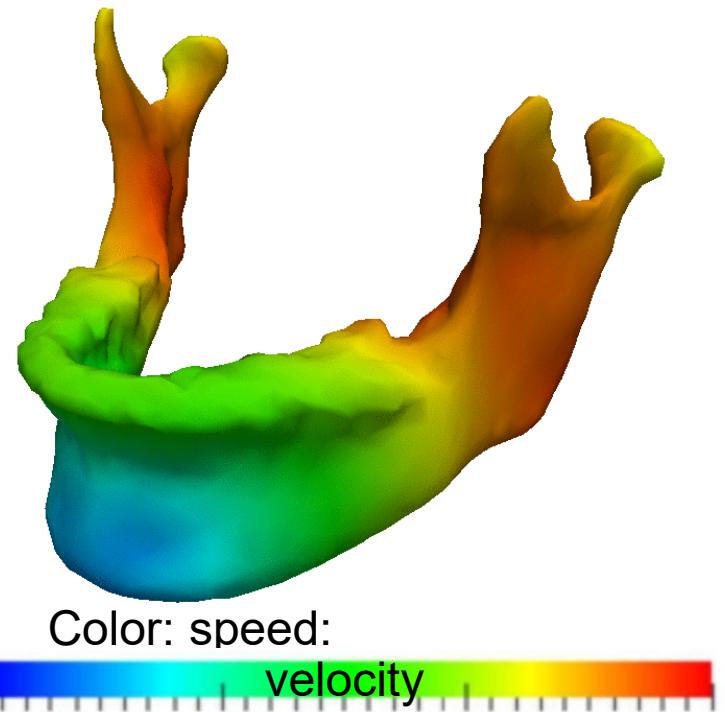
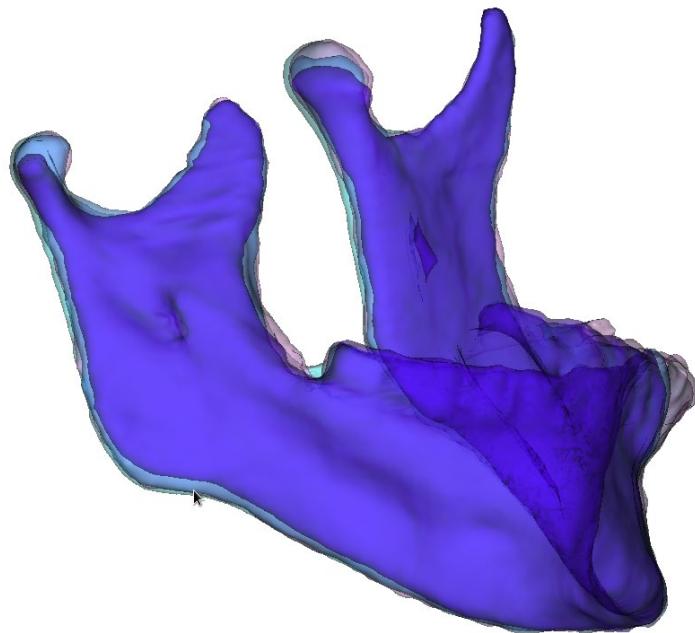
Example mandibular surgery case (Prof. L. Cividanes),
16-22 years post-surgery.



Follow-up scans in 2 years intervals.

Longitudinal Shape Modeling

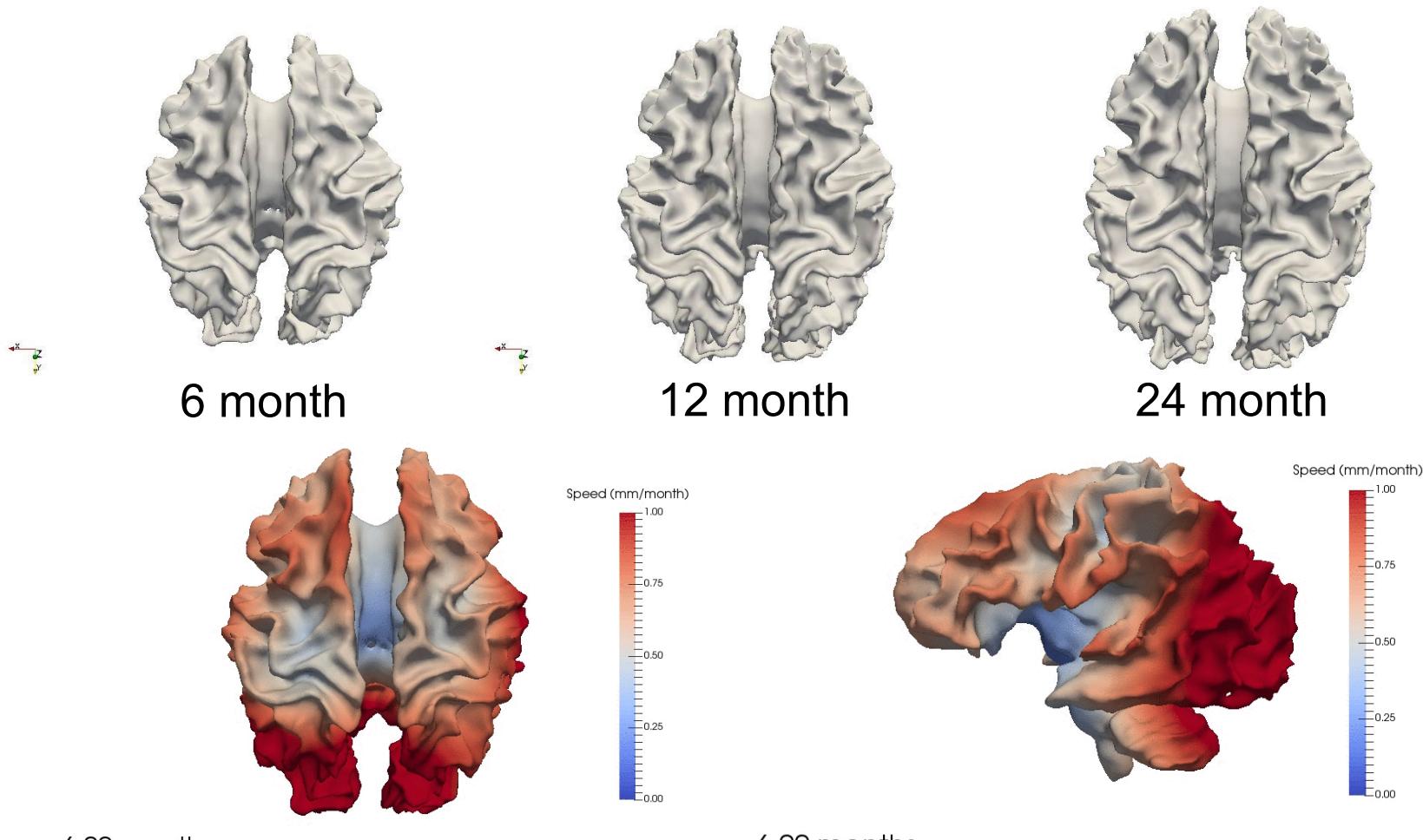
Example mandibular surgery case (Prof. L. Cividanes),
16-22 years post-surgery.



Qualitative analysis from overlay:
Do we understand remodeling?

Spatio-temporal shape
regression: Local and global.

Early Brain Development: 4D Shape Trajectories



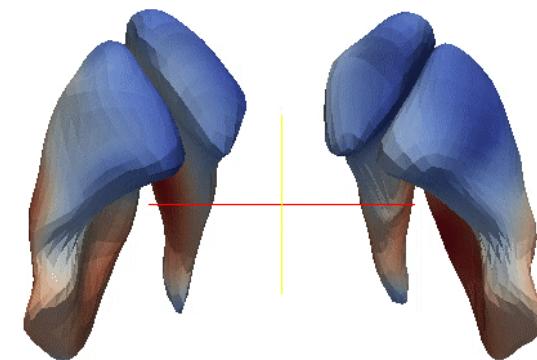
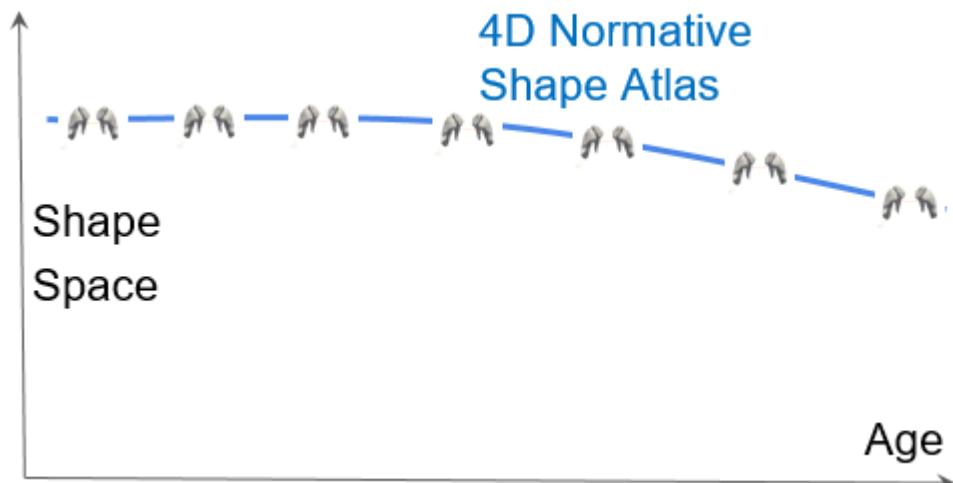
6.00 months

6.00 months

Geodesic Image Regression: Fishbaugh, Durrleman et al., 2015 / 2017 / 2019

Towards subject-specific analysis

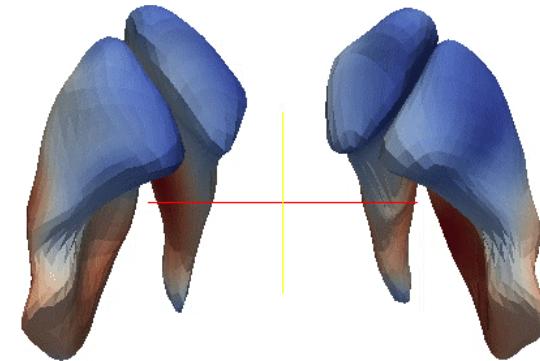
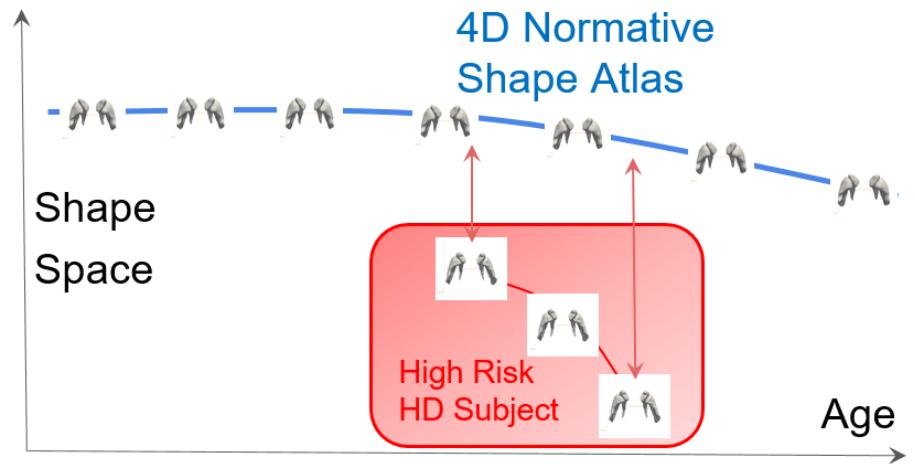
Question: Longitudinal Change in Normal versus Abnormal (Genetic Risk Huntington's)?



4D Normative Shape Atlas:
23 to 88 years, 243 subjects
Cross-sectional Regression

Towards subject-specific analysis

Question: Longitudinal Change in Normal versus Abnormal?



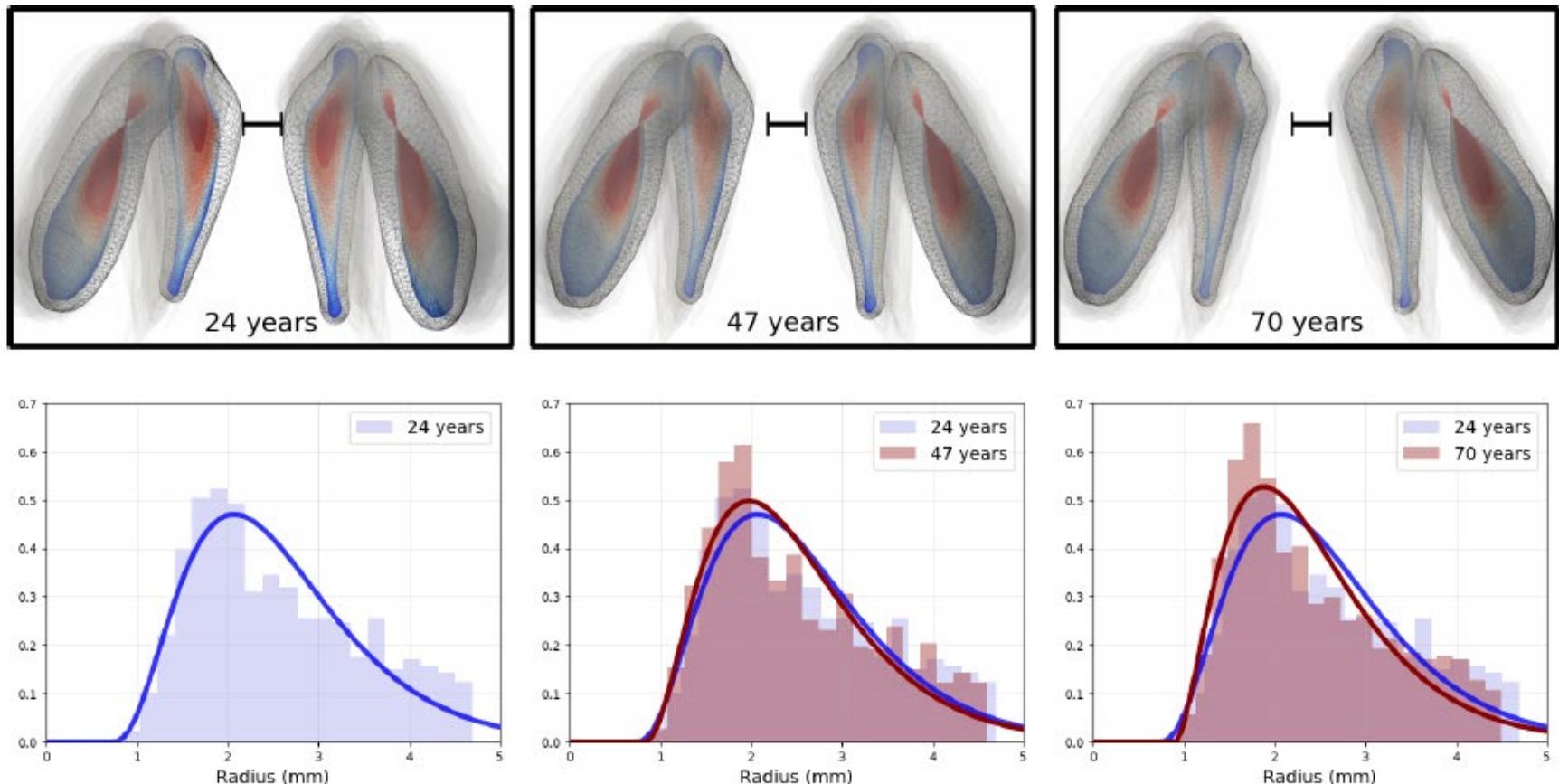
4D Normative Shape Atlas:
23 to 88 years, 243 subjects

- Map 4D Model of Individuals into 4D Normative Atlas
- Analysis of Shape Distance to Norm: **Decoupling of age effects versus pathological progression**
- Compare change trajectories

S. Hong et al, SPIE 2017, MICCAI 2019

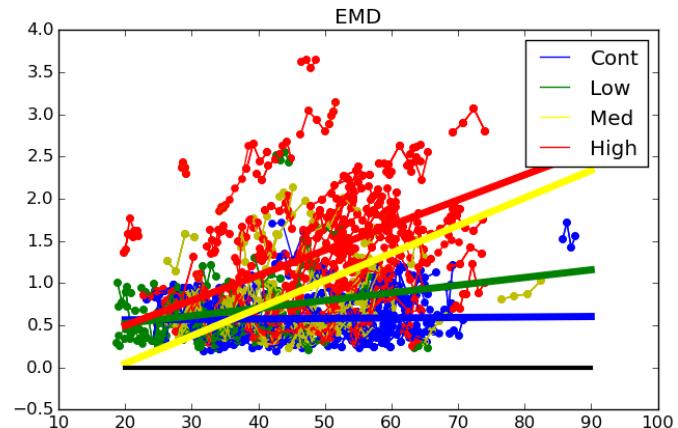
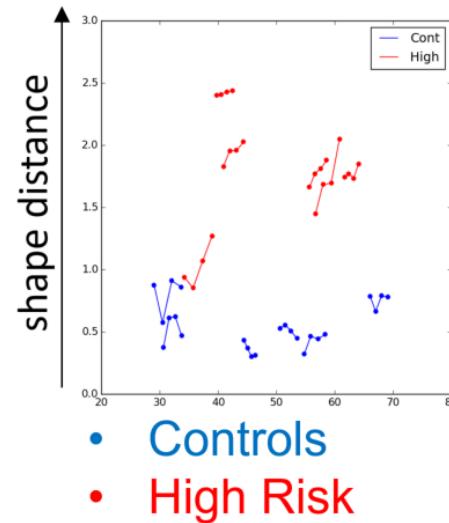
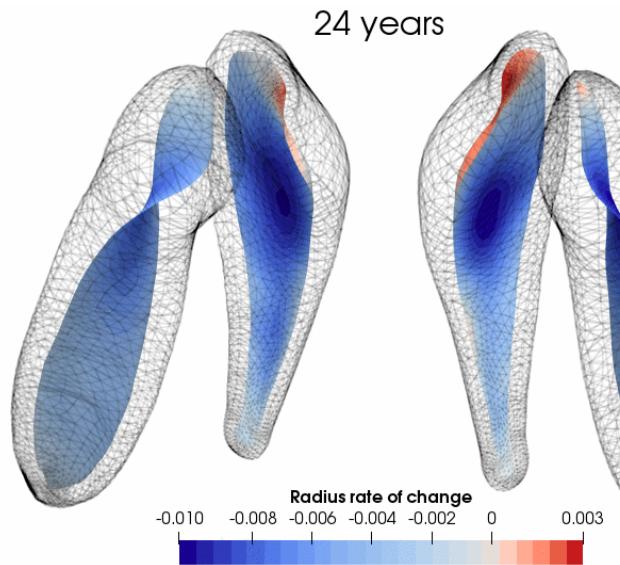
See also: Disentangling normal aging from Alzheimer's disease in structural magnetic resonance images, Lorenzi, Pennec, Frisoni, Ayache, ADNI, Neurobiology of Aging, 2015

Age-related changes: Longitudinal CMreps



Skeletal radius histogram: Thinning from 24 to 70 years

Preliminary Results Huntington's



Shape Thinning/
Degeneration:

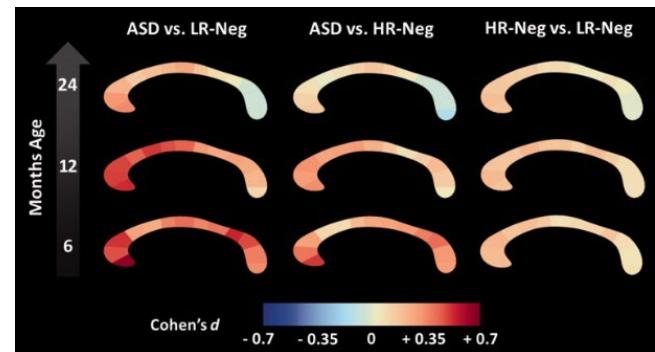
Shift to lower
values (left)

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Biological Shape Analysis

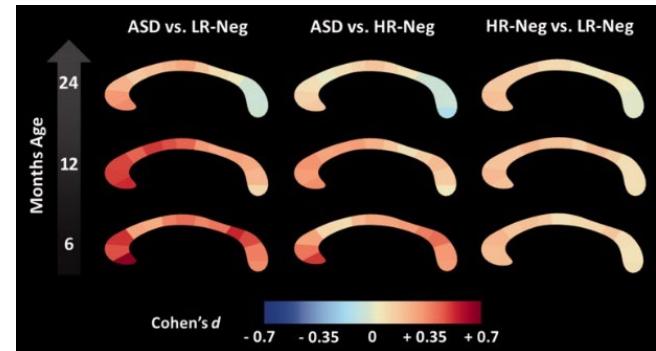
- Shape analysis is rarely just a two group classification problem.
- Biological shapes comes with covariates: Age, sex, group, disability score, etc.
- Traditional concept:
 - Extract shape parameters
 - Hand “numbers” over to biostatisticians



Statistical shape modeling followed by generalized linear modeling with covariates and group testing (e.g. Wolff, Gerig, et al., Brain 2015).

Biological Shape Analysis

- Shape analysis is rarely just a two group classification problem.
- Biological shapes comes with covariates: Age, sex, group, disability score, etc.
- Traditional concept:
 - Extract shape parameters
 - Hand “numbers” over to biostatisticians
- New: Integrate covariates into shape modeling & analysis step.
- Goal: Improved understanding of shape/covariate relationships.

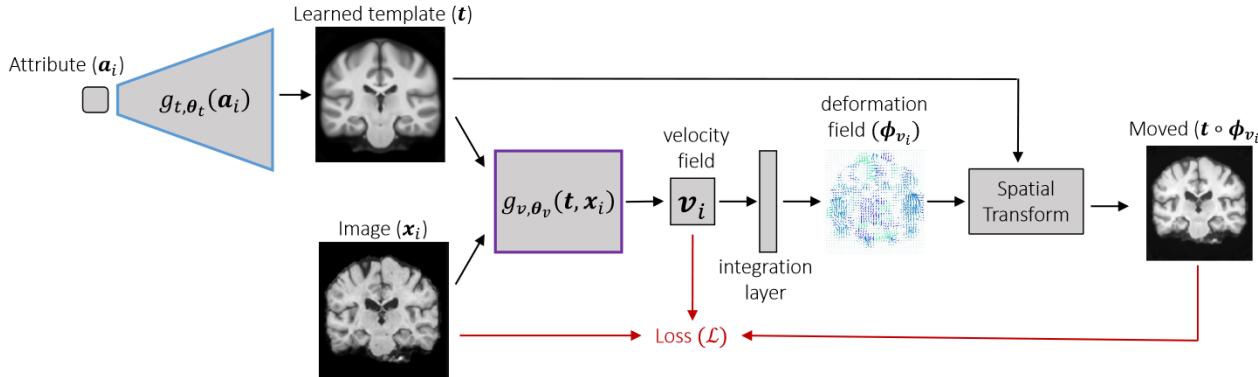


Statistical shape modeling followed by generalized linear modeling with covariates and group testing (e.g. Wolff, Gerig, et al., Brain 2015).



Voxelmorph

- Dalca et al., Learning Conditional Deformable Templates with Convolutional Networks, NeurIPS 2019



- The network takes as input an image **and an optional attribute vector**. The upper network $g_{t,\theta_t}(\cdot)$ outputs a template, which is then registered with the input image by the second network $g_{v,\theta_v}(\cdot)$. The loss function, derived from the negative log likelihood of the generative model, leverages the template warped into $\mathbf{t} \circ \phi_{\mathbf{v}_i}$.
- framework also enables learning a conditional template function given instance attributes, such as the age and sex of the subject in an MRI. Once learned, this function enables rapid synthesis of on-demand conditional templates. For example, it could construct a 3D brain MRI template for 35 year old women

<https://arxiv.org/abs/1908.02738>

Conditional Atlas Building: VoxelMorph: DL-Based Approach

- Use all study image data with multiple covariates.
- Conditional template estimation as function of continuous and/or categorical attributes (age, sex, diagnosis, etc.)

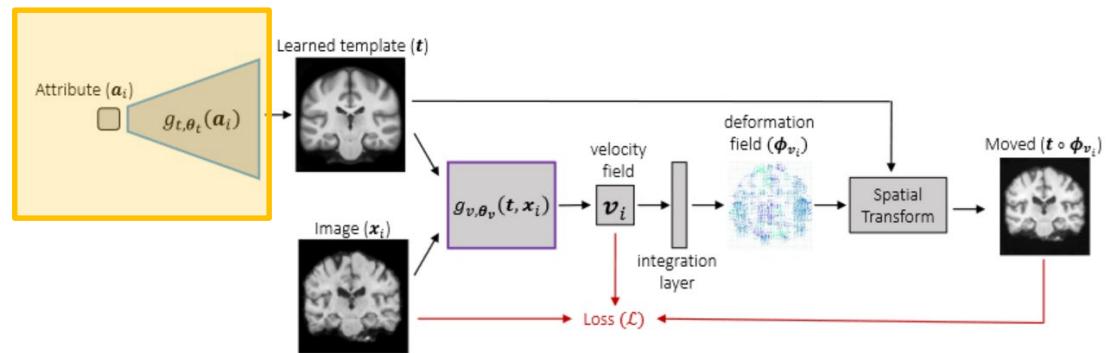


Figure 2: Overview. The network takes as input an image and an optional attribute vector. The upper network $g_{t, \theta_t}(\cdot)$ outputs a template, which is then registered with the input image by the second network $g_{v, \theta_v}(\cdot)$. The loss function, derived from the negative log likelihood of the generative model, leverages the template warped into $t \circ \phi_{v_i}$.

Adrian Dalca, et al.. Learning conditional deformable templates with convolutional networks. In Advances in neural information processing systems, pages 806–818, 2019. VoxelMorph

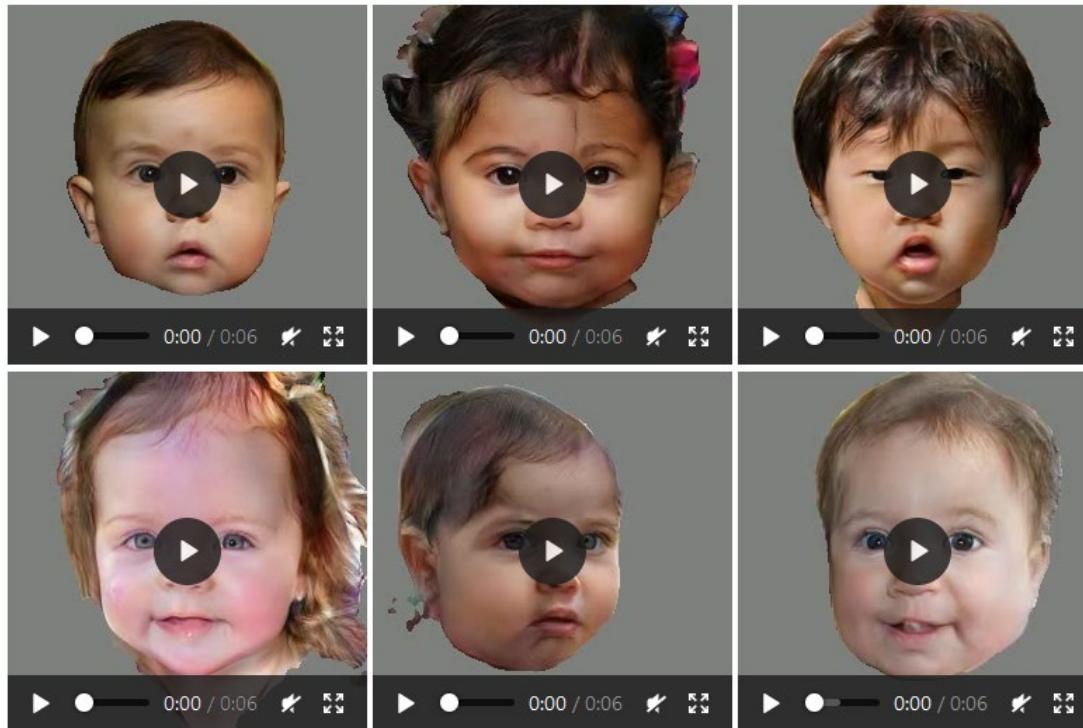
Dey, Ren, Dalca, Gerig. “Generative Adversarial Registration for Improved Conditional Deformable Templates”, ICCV, 2021.

Shapes/Images with Covariates

Lifespan Age Transformation Synthesis

Roy Or-El¹ Soumyadip Sengupta¹ Ohad Fried² Eli Shechtman³ Ira Kemelmacher-Shlizerman¹

¹University of Washington ²Stanford University ³Adobe Research



FFHQ Dataset

Lifespan Age Transformation Synthesis

Roy Or-El, et al., ECCV 2020

Dataset consists of 70,000 high-quality PNG images.

Life-span: Ages 0—70
Conditions: male/female,
Age, (sun)glasses

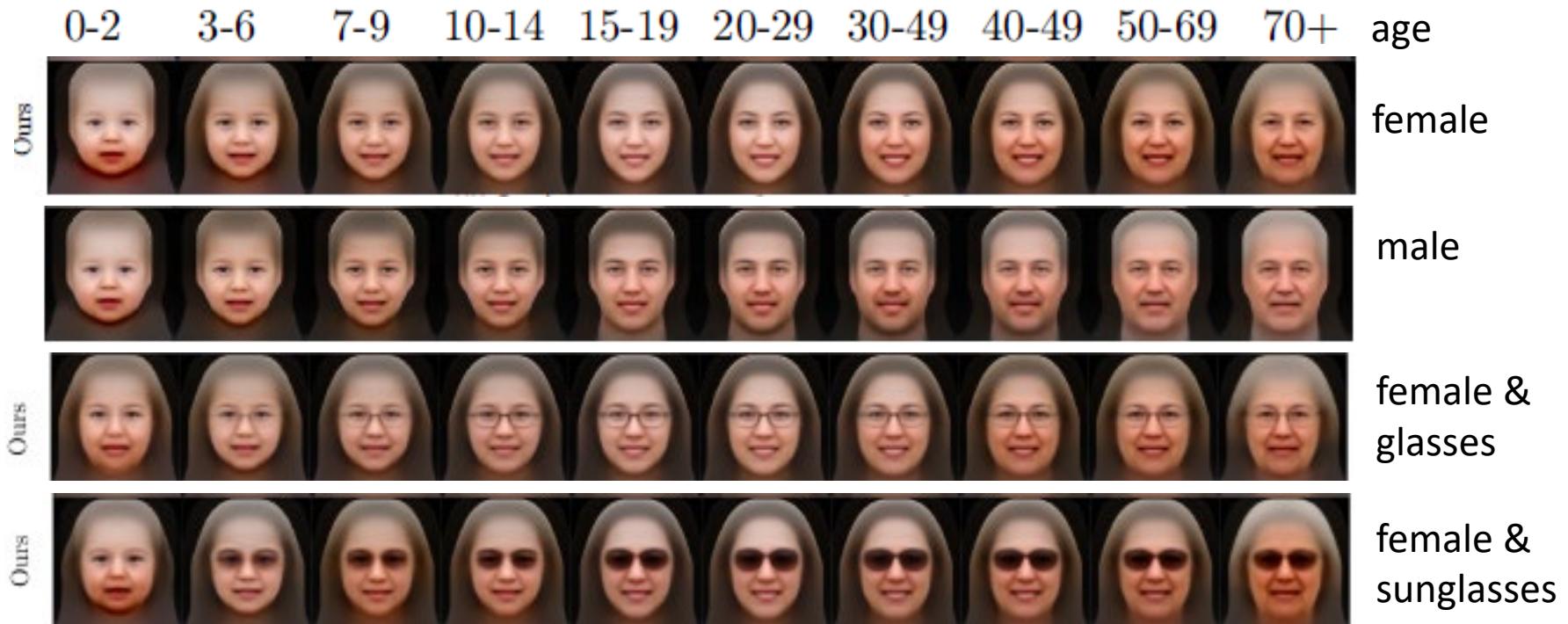
http://grail.cs.washington.edu/projects/lifespan_age_transformation_synthesis/
<https://arxiv.org/pdf/2003.09764.pdf>

Conditional Atlas Building: FFHQ

FFHQ Aging Dataset:



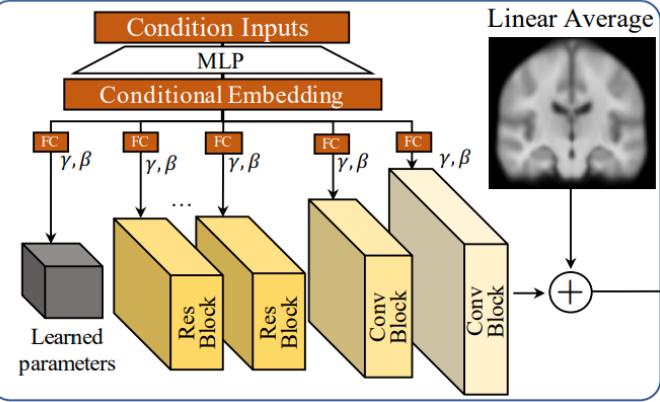
Fig. 3: FFHQ-Aging dataset. We label 70k images from FFHQ dataset [18] for gender and age via crowd-sourcing. In addition, for each image we extracted face semantic maps as well as head pose angles, glasses type and eye occlusion score.



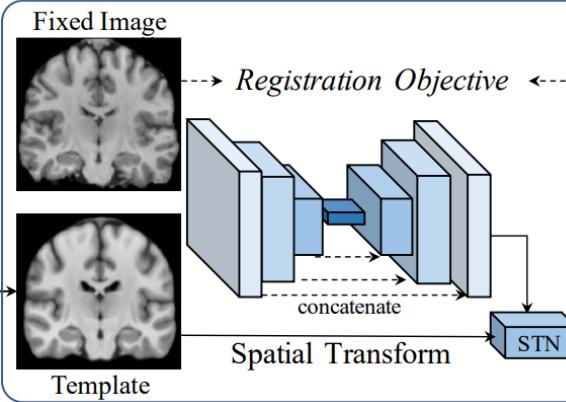
Framework Overview

(Dey, Dalca at al., ICCV 2021)

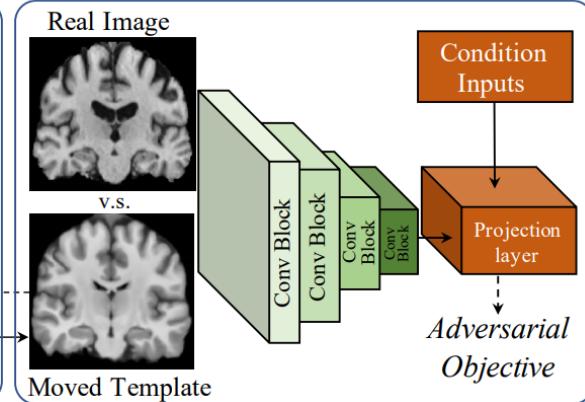
(a) Template generation sub-network



(b) Registration sub-network

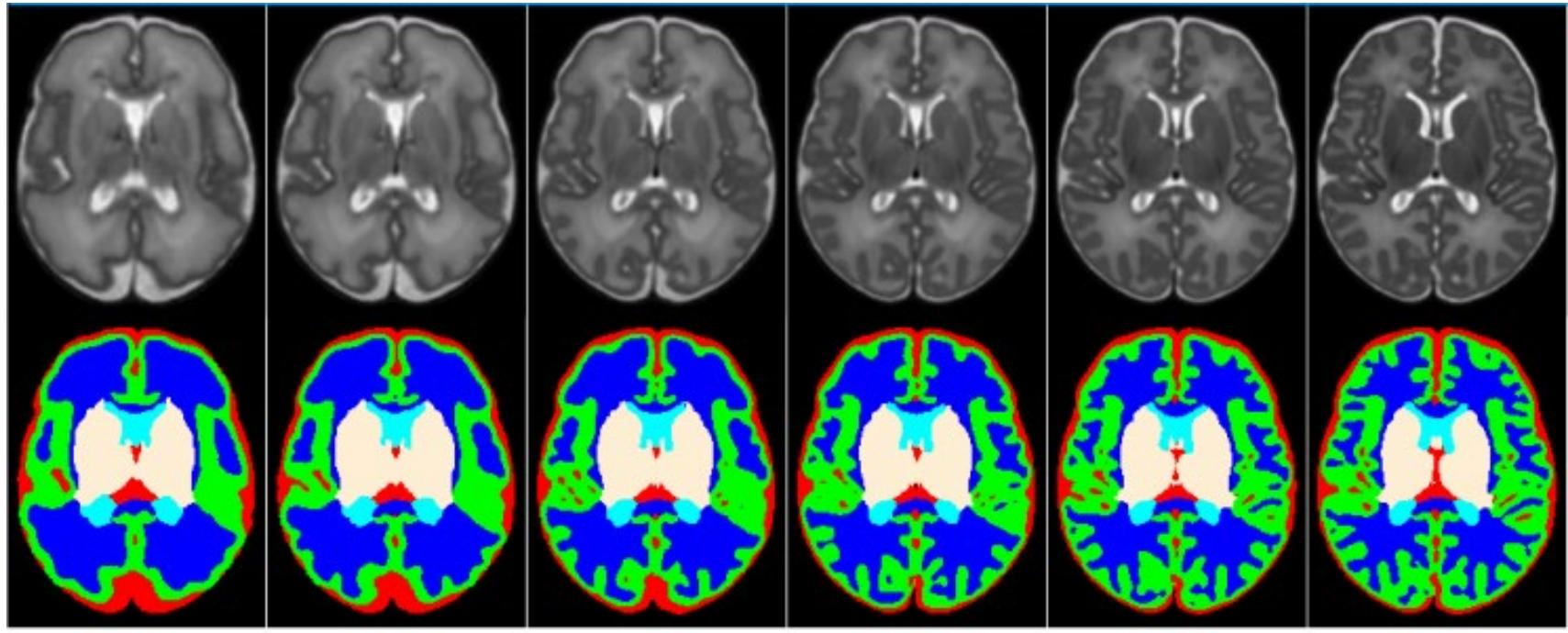


(c) Discriminator sub-network



- Feature-level conditioning
- High parameter-efficiency for several conditioning attributes
- Diffeomorphic Displacements via Stationary Velocity Fields
- Smooth & *central* deformations
- GAN conditioning for continuous *and* categorical attributes
- Extensive training stability pipeline

Conditional Atlas Building



558 T2w postnatal MR images from dHCP release 2, 20 to 44 weeks:

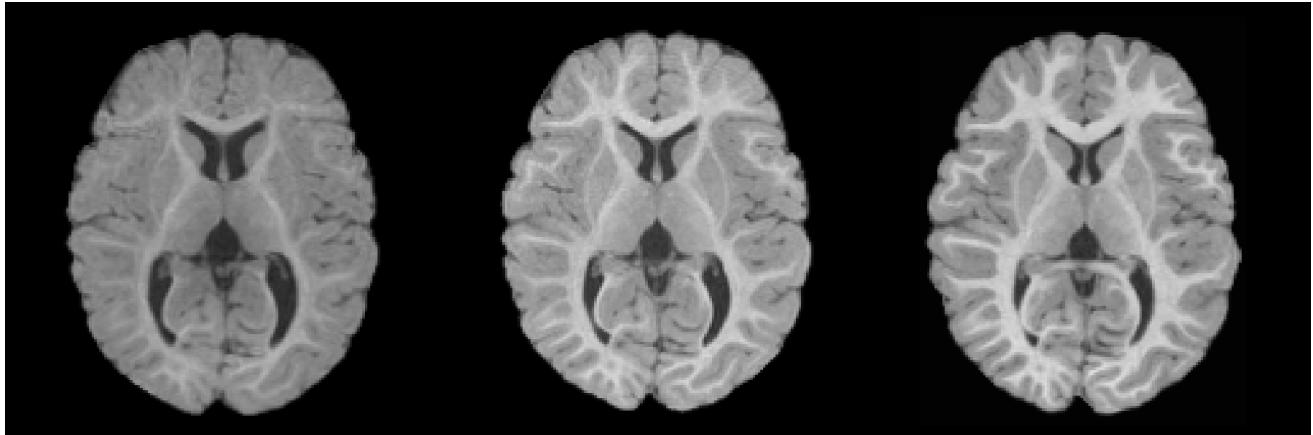
<http://www.developingconnectome.org/project/>

- Emer J Hughes, et al. A dedicated neonatal brain imaging system. Magnetic resonance in medicine, 78(2):794–804, 2017.
- Antonios Makropoulos et al. The developing human connectome project: A minimal processing pipeline for neonatal cortical surface reconstruction. Neuroimage, 173:88–112, 2018.

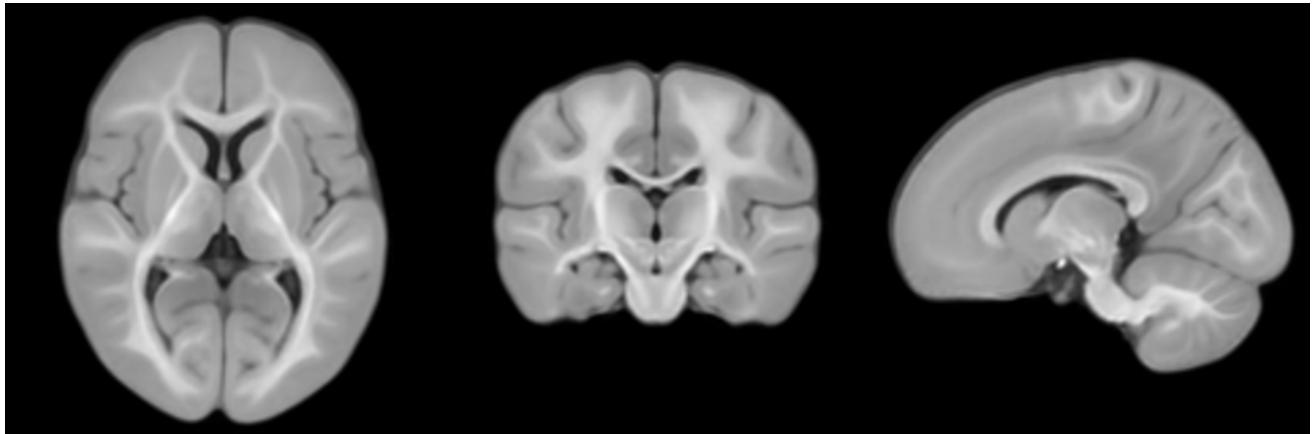
Dey, Ren, Dalca, Gerig. “Generative Adversarial Registration for Improved ConditionalDeformable Templates”, ICCV, 2021.

ACE-IBIS: Continuous Growth Templates

All 6/12/24 month T1w/T2w images used (>1200 image data).



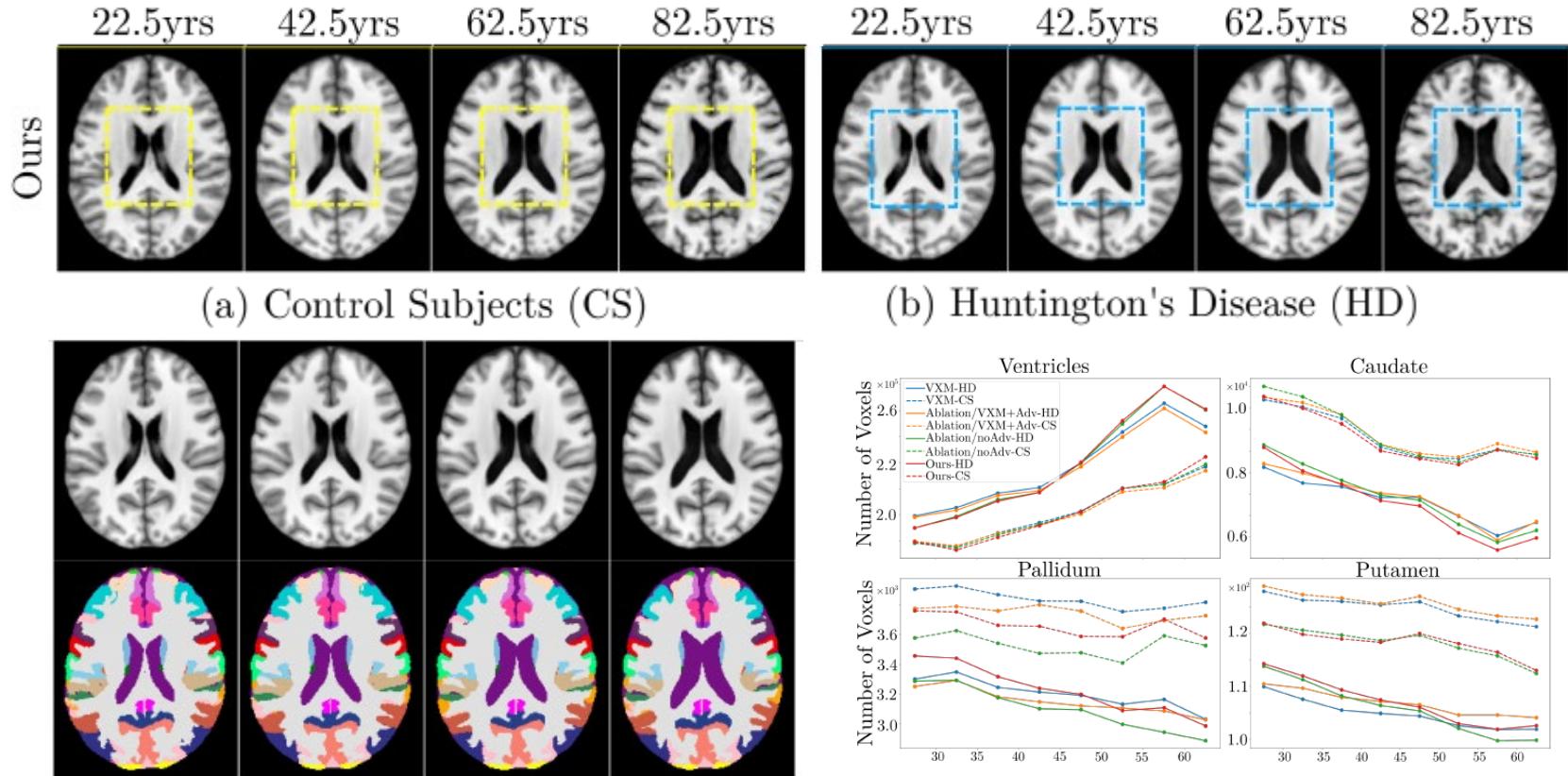
Conditionally, the synthesized templates faithfully capture the process of neuro-development.



Neel Dey, Mengwei Ren, Adrian V. Dalca, Guido Gerig; “Generative Adversarial Registration for Improved Conditional Deformable Templates”, Proc. ICCV’21

Conditional Atlas Building

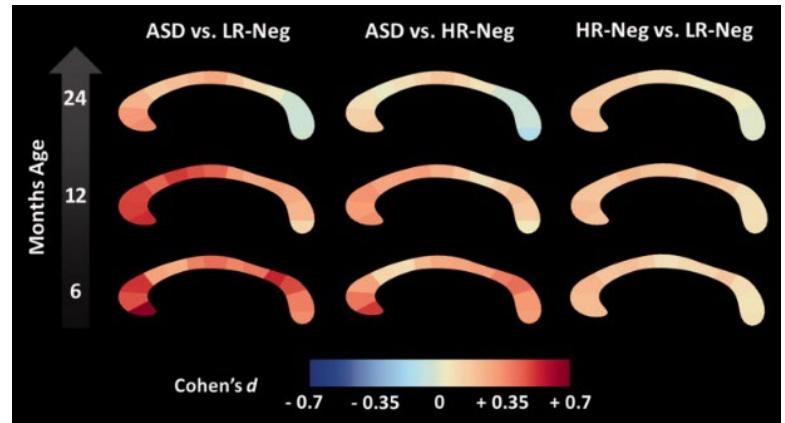
HD study: Controls vs. Diagnosed



Work in Progress: Analysis of longitudinal image changes as functions of covariates (here age and group).

Folding Statistical Modeling into Shape Analysis

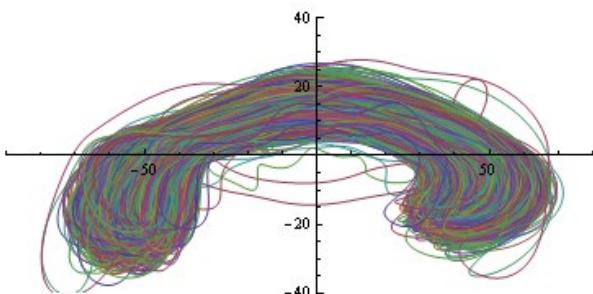
- **Approaches so far:**
 - YOU do the shape modeling
 - Statistical shape modeling followed by generalized linear modeling with covariates and group testing (e.g. Wolff, Gerig, et al., Brain 2015).
- **New:**
 - Novel hierarchical multi-geodesic model that can account for the effect of **subject-specific covariates** to the development (slope) of a shape change.
 - **Hierarchical Multi-Geodesic Model for Longitudinal Analysis of Temporal Trajectories of Anatomical Shape and Covariates**
- **Goal:**
 - Improved understanding of shape/covariate relationship.



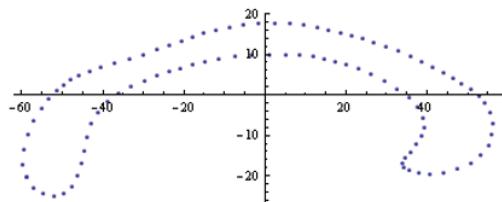
Statistical shape modeling followed by generalized linear modeling with covariates and group testing (e.g. Wolff, Gerig, et al., Brain 2015).

Joint Shape & Covariate Analysis

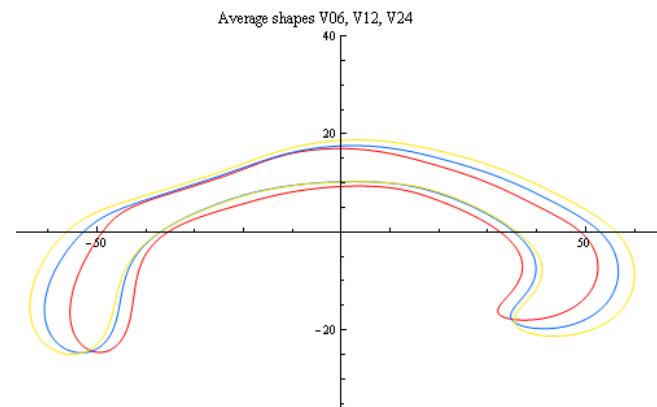
Shape space is non-Euclidean: Geodesic model on Riemannian manifold.



1040 corpus callosum shapes, ACE-IBIS, sampled by boundary points.

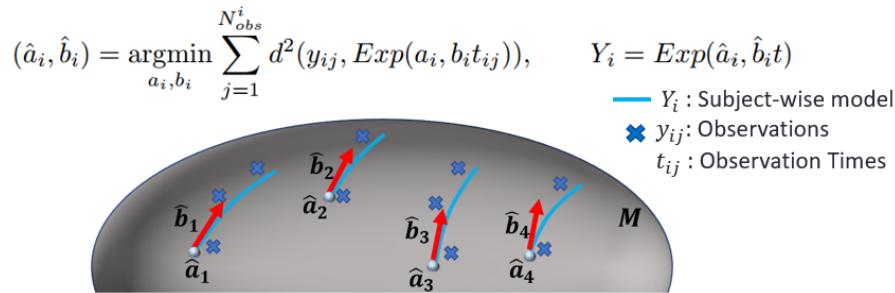


Longitudinal Shape Change per Subject



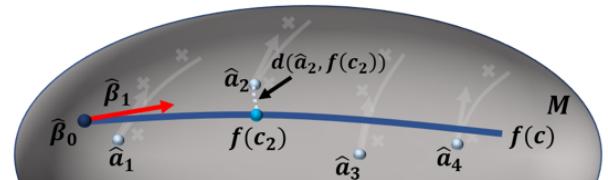
Hierarchical Model (“mixed effects model”), but with covariates and time

Estimate a subject-wise trend as a univariate geodesic model [1].



The intercept model $f(\eta)$ formulated as a multivariate geodesic model [2].
 Models the effects of covariates to the baselines of subject-specific trends.

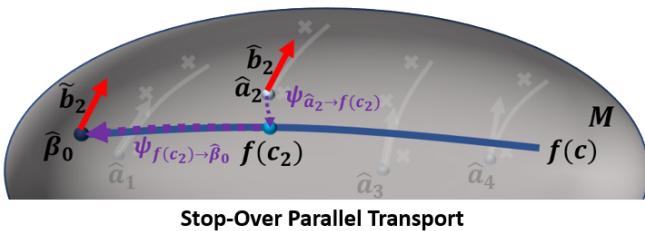
$$f(\boldsymbol{\eta}) = \operatorname{Exp}(\beta_0, \beta_1 \eta_1 + \dots + \beta_{N_\eta} \eta_{N_\eta})$$



Population Level $Y = \operatorname{Exp}(\operatorname{Exp}(f(\boldsymbol{\eta}), g(\boldsymbol{\theta})t), \epsilon)$

- Estimate a multi-geodesic model with a set of subject-specific geodesic models

Slope Model $g_0(\boldsymbol{\theta}) = \gamma_0 + \gamma_1 \theta_1 + \dots + \gamma_{N_\theta} \theta_{N_\theta}$ $\hat{g}(\boldsymbol{\theta}) = \psi_{\hat{\beta}_0 \rightarrow \hat{f}(\boldsymbol{\eta})}(\hat{g}_0(\boldsymbol{\theta}))$

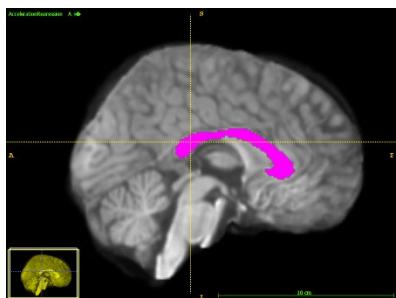


40/61

Study on ACE-IBIS Data (72 shapes, 24 ASD* subjects)

Towards Interpretability of Shape Analysis

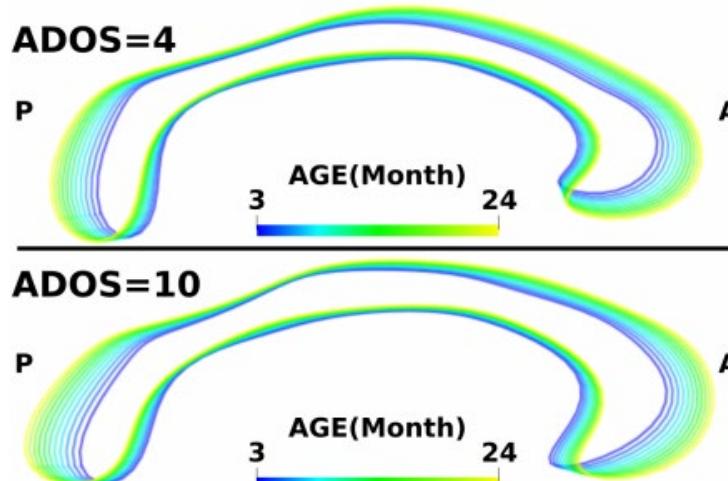
Corpus Callosum



Autism Diagnostic Observation Schedule (ADOS) Score

- Clinical test score on the behavioral and cognitive abilities
- 4 : Low ~ 10 : High

Shape Space : Scale x Kendall Shape - $\mathbf{M} = \mathbf{R} \times \mathbf{CP}^{k-2}$



Finding: Increased expansion of the anterior CC (genu and rostral body) over time for subjects with higher ADOS scores confirms expansion of CC in ASD.

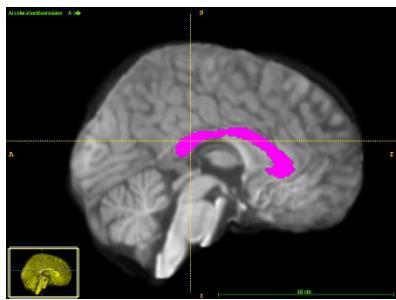
*Autism Spectrum Disorder

Sungmin Hong et al., MICCAI 2019

Study on ACE-IBIS Data (72 shapes, 24 ASD* subjects)

Towards Interpretability of Shape Analysis

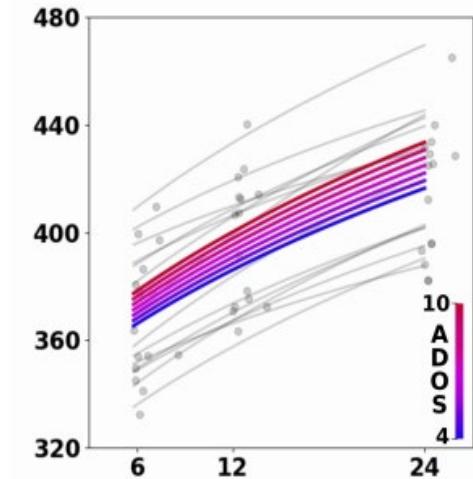
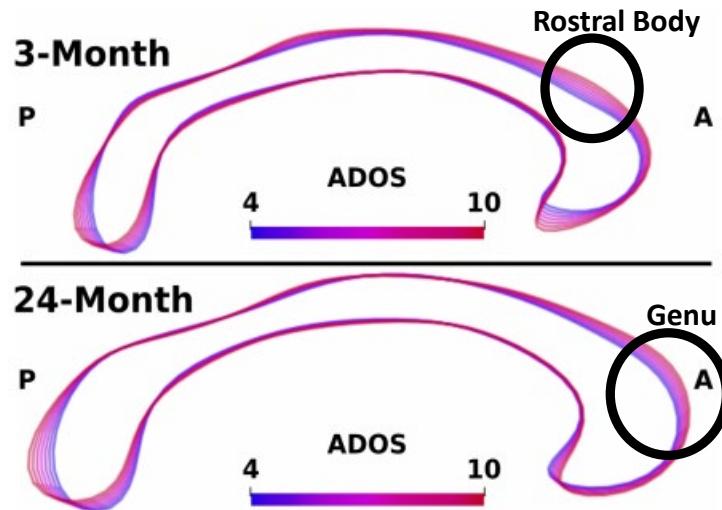
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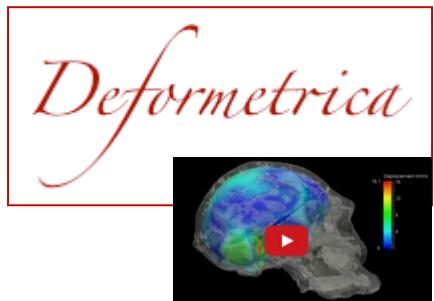
Shape Space : Scale x Kendall Shape - $\mathbf{M} = \mathbf{R} \times \mathbf{C}^{P^{k-2}}$



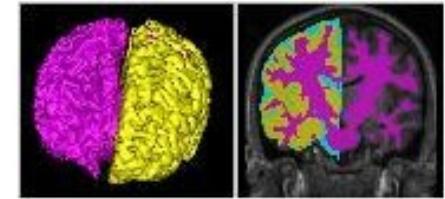
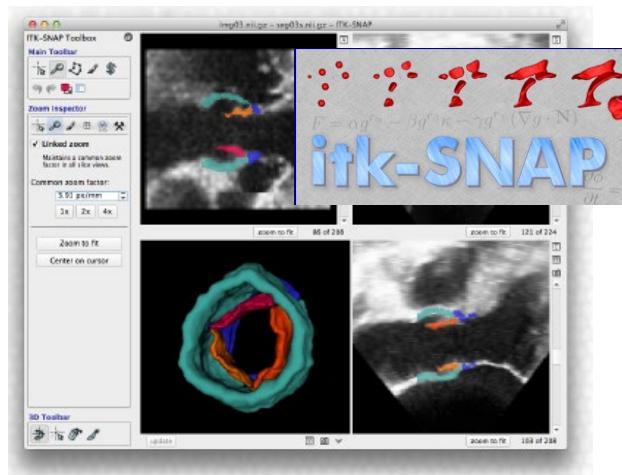
Finding: Increased expansion of the anterior CC (genu and rostral body) over time for subjects with higher ADOS scores confirms expansion of CC in ASD.

Software Engineering & Sharing

- Open-source/open-platform
- Industrial collaborations



neel-dey / Atlas-GAN



Acknowledgements

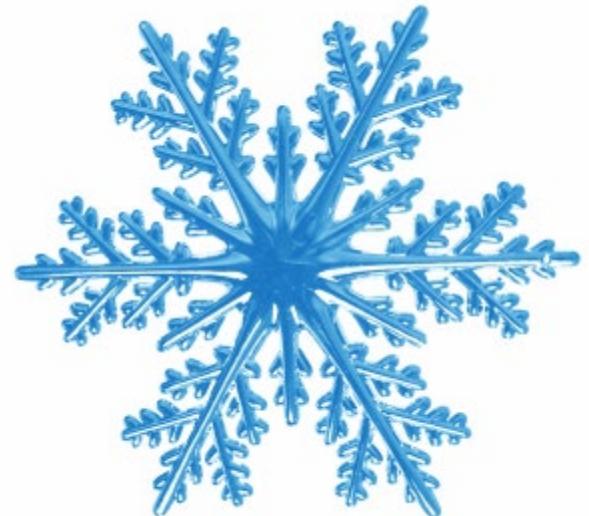
- NIH-NIBIB: SlicerSALT, **Shape** Analysis Toolbox
- NIH-NICHD: ACE-IBIS (**Autism** Center)
- NIH NINDS: **Down's** Syndrome
- NIH-NIDA: Brain dev. of offsprings of **cocaine abusing** mothers
- NIH-NEI: AMD: Age-related **Macular Degeneration**
- NIH-NEI: Novel **Glaucoma** Diagnostics for Structure & Function
- NYC CATT: NY Center for Advanced Technology in Telecomm.
- Kitware Inc.



Quotation of the Day

*“The perfection of mathematical beauty is such
..... that whatsoever is most beautiful and
regular is also found to be the most useful and
excellent.”*

D'Arcy Wentworth Thompson



Are you still in “Good Shape”?



Bob Dylan & the Band: The Shape I'm In



Go out yonder, peace in the valley
Come downtown, have to rumble in the alley

Oh, you don't know the shape I'm in

Has anybody seen my lady
This living alone will drive me crazy

Oh, you don't know the shape I'm in

I'm gonna go down by the wa - ter
But I ain't gonna jump in, no, no
I'll just be looking for my mak - er
And I hear that that's where she's been? oh
Out of nine lives, I spent seven
Now, how in the world do you get to heaven

Oh, you don't know the shape I'm in

Conclusions

- “Shape” is a fundamental concept of human perception.
- “Shape” is an essential concept for Medical Image Analysis.
- Many routine applications exist, e.g. selection of optimal implants.
- Foundational mathematical & statistical concepts help to make the field much more mature, but:
 - Need to bridge the gap between the **“Beauty of Math”** and **“Biological Shape”**.
 - Biological shapes come with sets of **covariates**, not just two groups.
 - Biological shapes are **deformable** and show **changes over time**.
 - **P-values** are common, but we need to explain shape changes to clinical researchers → **Interpretability**.