Comparing two paired means

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Read in the data and calculate the diffence in zinc concentration

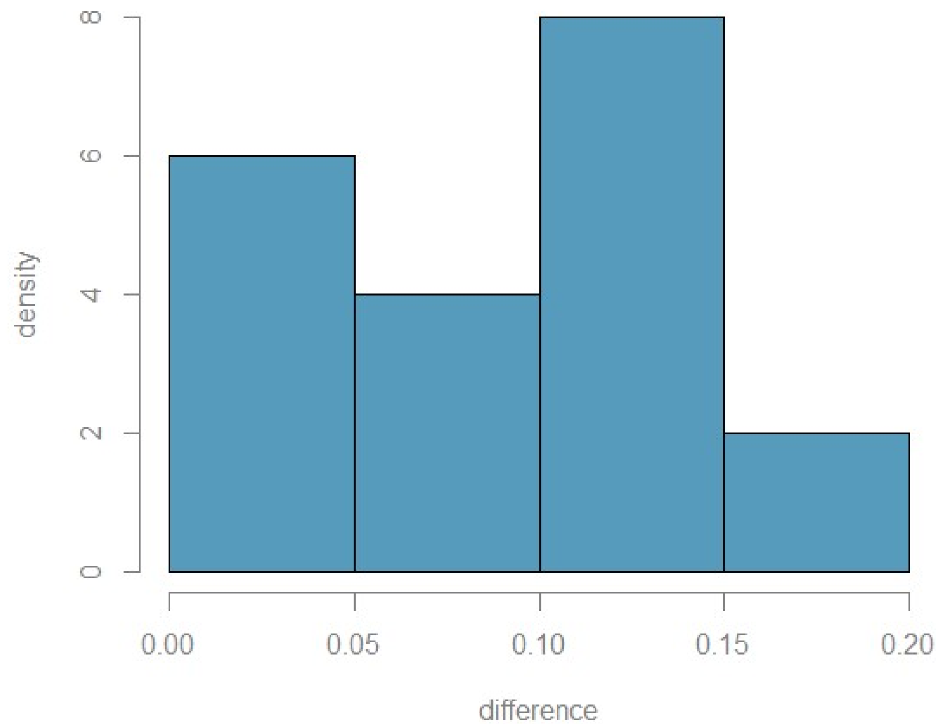
```
zinc =  read.table("https://onlinecourses.science.psu.edu/stat500/sites/onli
                   header=T, fileEncoding="UTF-16LE")
zinc$difference = zinc$bottom - zinc$surface
summary(zinc)
```

```
##      bottom          surface          difference    
##  Min.   :0.2660   Min.   :0.2380   Min.   :0.0150  
##  1st Qu.:0.4845   1st Qu.:0.4103   1st Qu.:0.0355  
##  Median :0.5780   Median :0.4690   Median :0.0840  
##  Mean   :0.5649   Mean   :0.4845   Mean   :0.0804  
##  3rd Qu.:0.6930   3rd Qu.:0.6080   3rd Qu.:0.1100  
##  Max.   :0.7230   Max.   :0.6320   Max.   :0.1770  
```

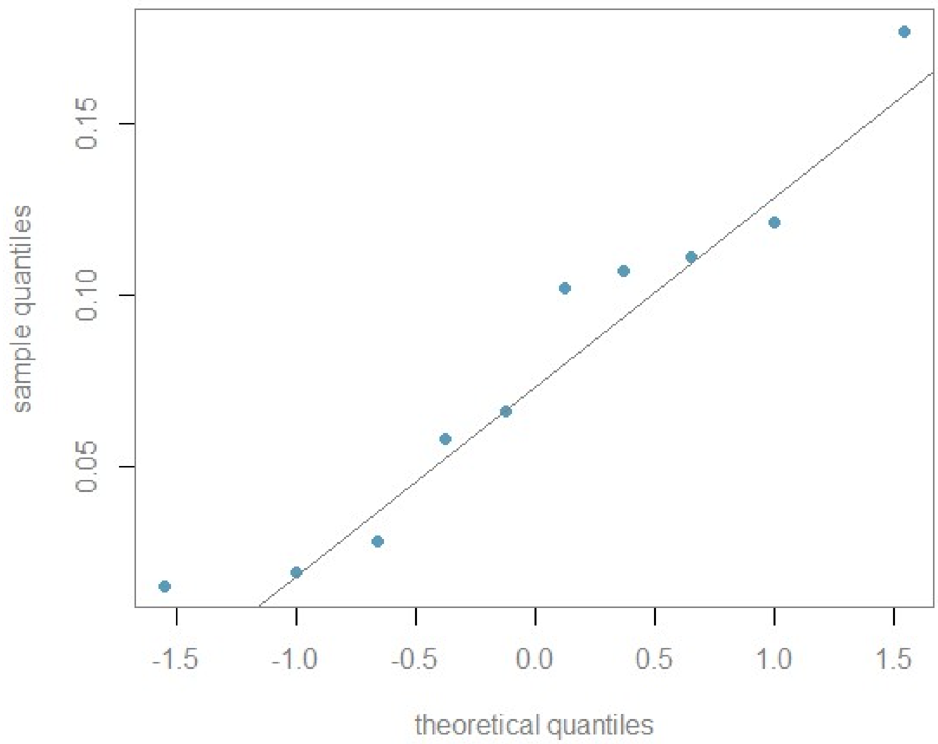
Let's look at the distribution of the sampled differences

```
myblue = rgb(86,155,189, name="myblue", max=256)
mydarkgrey = rgb(.5,.5,.5, name="mydarkgrey", max=1)
par(mar=c(5, 9, 2, 2), col.lab=mydarkgrey, col.axis=mydarkgrey, col=mydarkgr

# histogram
hist(zinc$difference, col=myblue,
     xlab="difference", lwd=3, ylab="density",
     main="", prob=T, axes=F)
axis(1,col=mydarkgrey)
axis(2,col=mydarkgrey)
```

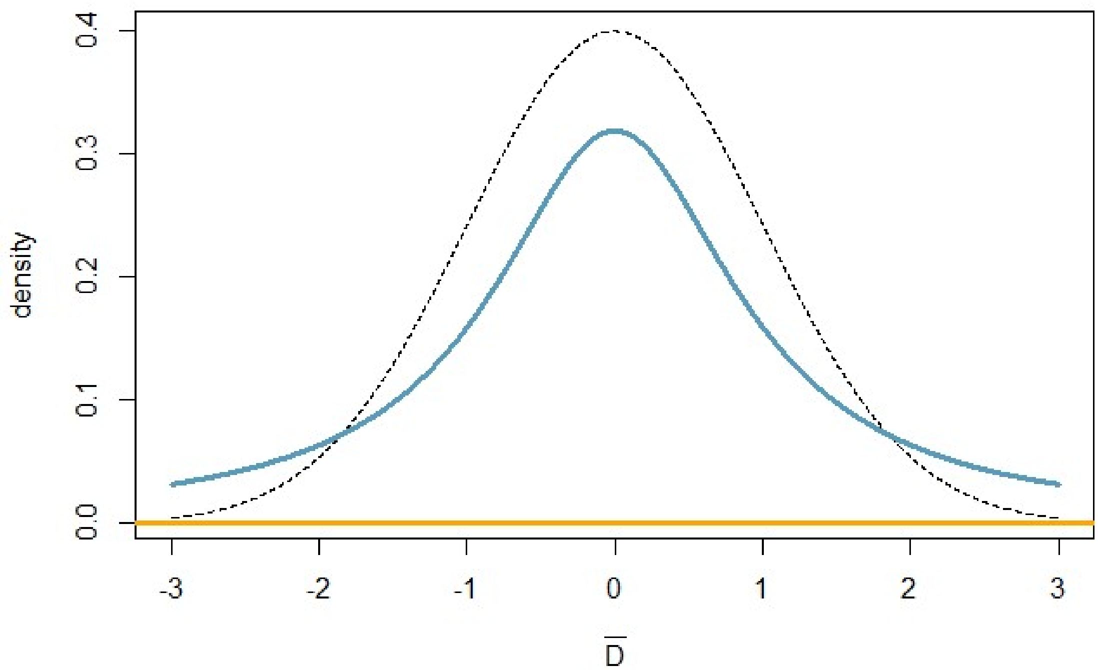



```
# Normal quantile plot
qqnorm(zinc$difference, col=myblue, pch=16, main="",
       xlab="theoretical quantiles",
       ylab="sample quantiles")
qqline(zinc$difference)
```



Prior Distributions

```
x = seq(-3, 3, length=10000)
plot(x, dnorm(x),
     xlab=expression(bar(D)), ylab="density",
     col=1, type="l", lty=2, lwd=1)
lines(x, dt(x, df=1), lty=1, lwd=3, col=myblue)
abline(h=0, lty=1, lwd=3, col="orange")
```



The black is a standard normal distribution, while the blue is a Student-t distribution with 1 degree of freedom otherwise known as the Cauchy distribution. The orange line corresponds to a limiting normal distribution as the variance or standard deviation goes to infinity.

Bayes factors and posterior probabilities

Let's define a function to help simplify the calculations of the posterior probabilities and the Bayes factor using the normal prior

$\mu \mid \sigma^2 \sim N(\mu_0, \sigma^2/n0)$

and

$p(\sigma^2) \propto 1/\sigma^2$

```
bayes.t.test = function(x, n0=1, mu0 = 0,  prior.H1=.5) {
  out = t.test(x - mu0)
  t = as.numeric(abs(out$statistic))
  n = length(x)
  df = n-1
  # BF is BF of H1 to H2
```

```
  BF=exp(.5*(log(n + n0) - log(n0) +
               (df + 1)*(log(t^2*n0/(n + n0) + df) -
                         log(t^2 + df))))
  PO= BF*prior.H1/(1 - prior.H1)
  post.prob = 1/(1 + 1/PO)
  return(list(BF.H1.H2=BF, post.prob.H1 = post.prob,
              post.prob.H2= 1 - post.prob,
              t=t, p.value=out$p.value, df=n-1))
}
```

```
out = bayes.t.test(zinc$difference)
out
```

```
## $BF.H1.H2
## [1] 0.01539321
##
## $post.prob.H1
## [1] 0.01515985
##
## $post.prob.H2
## [1] 0.9848402
##
## $t
## [1] 4.863813
##
## $p.value
## [1] 0.0008911155
##
## $df
## [1] 9
```

H1 is that the mean difference is 0 while H2 is that the mean difference is not zero. To obtain the Bayes factor for H2 to H1, we simply take 1/BF.H1.H2

```
1/out$BF.H1.H2
```

```
## [1] 64.96373
```

Note: this function could be used for any one sample hypothesis test of $\mu = \mu_0$ versus $\mu \neq \mu_0$.