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3 authors:

Bo Peng
Huazhong University of Science and Technol…

Zhipeng Lü
Huazhong University of Science and Technol…

T. C. E. Cheng
The Hong Kong Polytechnic University

A Tabu Search/Path Relinking Algorithm to Solve the Job Shop Scheduling Problem

Bo Peng[a], Zhipeng Lü[a,b,*], T.C.E. Cheng[b]

[a] *SMART, School of Computer Science and Technology, Huazhong University of Science and Technology, Wuhan, 430074, P.R. China*
[b] *Department of Logistics and Maritime Studies, The Hong Kong Polytechnic University, Hung Hom, Kowloon, Hong Kong*

Abstract

We present an algorithm that incorporates a tabu search procedure into the framework of path relinking to tackle the job shop scheduling problem (JSP). This tabu search/path relinking (TS/PR) algorithm comprises several distinguishing features, such as a specific relinking procedure and a reference solution determination method. To test the performance of TS/PR, we apply it to tackle almost all of the benchmark JSP instances available in the literature. The test results show that TS/PR obtains competitive results compared with state-of-the-art algorithms for JSP in the literature, demonstrating its efficacy in terms of both solution quality and computational efficiency. In particular, TS/PR is able to improve the upper bounds for 49 out of the 205 tested instances and it solves a challenging instance that has remained unsolved for over 20 years.

Keywords: scheduling; job shop; metaheuristics; tabu search; path

*Corresponding author.
Email addresses: zhipeng.lv@hust.edu.cn (Zhipeng Lü), Edwin.Cheng@polyu.edu.hk (T.C.E. Cheng)

relinking; hybrid algorithms

1. Introduction

The job shop scheduling problem (JSP) is not only one of the most notorious and intractable NP-hard problems, but also one of the most important scheduling problems that arise in situations where a set of activities that follow irregular flow patterns have to be performed by a set of scarce resources. In job shop scheduling, we have a set $M = \{1, \ldots, m\}$ of m machines and a set $J = \{1, \ldots, n\}$ of n jobs. JSP seeks to find a feasible schedule for the operations on the machines that minimizes the makespan (the maximum job completion time), i.e., C_{max}, which means the completion time of the last completed operation in the schedule. Each job $j \in J$ consists of n_j ordered operations $O_{j,1}, \ldots, O_{j,n_j}$, each of which must be processed on one of the m machines. Let $O = \{0, 1, \ldots, o, o+1\}$ denote the set of all the operations to be scheduled, where operations 0 and $o+1$ are dummies, have no duration, and represent the initial and final operations, respectively. Each operation $k \in O$ is associated with a fixed processing duration P_k. Each machine can process at most one operation at a time and once an operation begins processing on a given machine, it must complete processing on that machine without preemption. In addition, let p_k be the predecessor operation of operation $k \in O$. Note that the first operation has no predecessor. The operations are interrelated by two kinds of constraints. First, operation $k \in O$ can only be scheduled if the machine on which it is processed is idle. Second, precedence constraints require that before each operation $k \in O$ is processed, its predecessor operation p_k must have been completed.

Furthermore, let S_o be the start time of operation o ($S_0 = 0$). JSP is to find a starting time for each operation $o \in O$. Denoting E_h as the set of operations being processed on machine $h \in M$, we can formulate JSP as follows:

$$Minimize \quad C_{max} = \max_{k \in O}\{S_k + P_k\}, \tag{1}$$

subject to

$$S_k \geq 0; \quad k = 0, \ldots, o+1, \tag{2}$$

$$S_k - S_{p_k} \geq P_{p_k}; \quad k = 1, \ldots, o+1, \tag{3}$$

$$S_i - S_j \geq P_i \quad or \quad S_j - S_i \geq P_j; \quad (i,j) \in E_h,\ h \in M. \tag{4}$$

In the above problem, the objective function (1) is to minimize the makespan. Constraints (2) require that the completion times of all the operations are non-negative. Constraints (3) stipulate the precedence relations among the operations of the same job. Constraints (4) guarantee that each machine can process no more than one single operation at a time.

Over the past few decades, JSP has attracted much attention from a significant number of researchers, who have proposed a large number of heuristic and metaheuristic algorithms to find optimal or near-optimal solutions for the problem. One of the most famous algorithms is the tabu search (TS) algorithm TSAB proposed by Nowicki and Smutnicki (1996). Nowicki and Smutnicki (2005) later extend algorithm TSAB to algorithm i-TSAB, which Beck et al. (2011) combine with a constraint programming based constructive search procedure to create algorithm CP/LS. Pardalos and Shylo (2006) propose algorithm GES, which is based on global equilibrium search techniques. Zhang et al. (2007) extend the N6 neighbourhood proposed by Balas and Vazacopoulos (1998) to the N7 neighbourhood and Zhang et al. (2008) combine TS with SA to create algorithm TS/SA, which outperforms almost all the algorithms. Nagata and Tojo (2009) present a local search framework termed guided ejection search, which always searches for an incomplete solution for JSP. Recently, Gonçalves and Resende (2013) present the biased random-key genetic algorithm BRKGA, which is able to improve the best known results for 57 instances and outperforms all the reference algorithms considered in their paper. From all these algorithms, it is apparent that the recent state-of-the-art algorithms either hybridize several strategies instead of using a single algorithm or employ a population-based algorithm instead of a single-solution based one.

Among the metaheuristic approaches used to tackle JSP, especially most of the state-of-the-art algorithms for JSP, a powerful local search procedure is always necessary. As one of the most popular local search algorithms, TS has been widely used by researchers to tackle JSP, e.g., Nowicki and Smutnicki (2005), Zhang et al. (2007), Nasiri and Kianfar (2012a), Shen and Buscher (2012), and Gonçalves and Resende (2013), among others.

On the other hand, Aiex et al. (2003) apply path relinking within a GRASP procedure as an intensification strategy to tackle JSP. Furthermore, Nowicki and Smutnicki (2005) improve their famous algorithm TSAB by introducing a new initial solution generator based on path relinking. Recently, Nasiri and Kianfar (2012b) apply i-TSAB to tackle JSP using the N1 neighbourhood as the path relinking procedure.

The above observations and considerations motivate us to develop a more robust algorithm for JSP via combining the more global relinking approach and the more intensive TS, which consists of several distinguishing features. In this vein, we design the tabu search and path relinking (TS/PR) algorithm, which is able to strike a better balance between the exploration and exploitation of the search space in a flexible manner. We summarize the main contributions of TS/PR as follows:

- Compared with the state-of-the-art algorithms for tackling JSP, TS/PR consists of several distinguishing features. In particular, it uses a specific mechanism to adaptively construct the path linking the initiating solution and the guiding solution, as well as using two kinds of improvement method to determine the reference solution.

- We test the performance of TS/PR by applying it to solve 205 benchmark JSP instances widely used in the literature. The test results show the efficacy TS/PR in terms of both solution quality and computational efficiency. In particular, TS/PR is able to improve the upper bounds for 49 out of the 205 tested instances in a reasonable time and it finds the optimal solution for the challenging instance SWV15, which has remained unsolved for over 20 years.

The remaining part of the paper is organized as follows: Section 2 describes in detail the components of TS/PR. Section 3 presents the detailed computational results and comparisons between TS/PR and some best performing algorithms in the literature for tackling six sets of a total of 205 challenging benchmark JSP instances. Finally, we conclude the paper and suggest future research topics in Section 4.

2. The TS/PR Algorithm

2.1. Main Framework

In principle, TS/PR repeatedly operates between a path relinking method that is used to generate promising solutions on the trajectory set up from an initiating solution to a guiding solution, and a TS procedure that improves the generated promising solution to a local optimum. Algorithm 1 presents the main procedure of TS/PR.

Algorithm 1 Outline of algorithm TS/PR for JSP

Input: J, M, and P_k
Output: C_{max} and the best solution S^* found so far
$P = \{S^1, \ldots, S^p\} \leftarrow$ Population_Initialization() /* Section 2.2 */
for $i = \{1, \ldots, p\}$ **do**
  $S^i \leftarrow$ Tabu_Search(S^i) /* Section 2.3 */
end for
$S^* = arg\ min\{f(S^i) \mid i = 1, \ldots, p\}$
PairSet $\leftarrow$ $\{(S^i, S^j) \mid S^i \in P,\ S^j \in P$ and $S^i \neq S^j\}$
repeat
  Randomly select one solution pair $\{S^i, S^j\}$ from *PairSet*
  $S^{p+1} \leftarrow$ Path_Relinking(S^i, S^j) , $S^{p+2} \leftarrow$ Path_Relinking(S^j, S^i) /* Section2.4 */
  $S^{p+1} \leftarrow$ Tabu_Search(S^{p+1}), $S^{p+2} \leftarrow$ Tabu_Search(S^{p+2}) /* Section 2.3 */
  if S^{p+1}(or S^{p+2}) is better than S^* **then**
    $S^* = S^{p+1}$(or S^{p+2})
  end if
  Tentatively add S^{p+1} and S^{p+2} to population P: $P' = P \cup \{S^{p+1}, S^{p+2}\}$
  PairSet $\leftarrow$ *PairSet* $\cup \{(S^{p+1}, S^k) \mid S^k \in P$ and $S^k \neq S^{p+1}\}$
  PairSet $\leftarrow$ *PairSet* $\cup \{(S^{p+2}, S^k) \mid S^k \in P$ and $S^k \neq S^{p+2}\}$
  Identify the two worst solutions S^u and S^v in the temporary population P'
  Generate new population by removing the two worst solutions S^u and S^v:
  $P = \{S^1, \ldots, S^p, S^{p+1}, S^{p+2}\} \backslash \{S^u, S^v\}$
  Update *PairSet*:
    PairSet $\leftarrow$ *PairSet* $\backslash \{(S^u, S^k) \mid S^k \in P$ and $S^k \neq S^u\}$
    PairSet $\leftarrow$ *PairSet* $\backslash \{(S^v, S^k) \mid S^k \in P$ and $S^k \neq S^v\}$
until a stop criterion is met

2.2. Initial Population

In TS/PR, the initial population is constructed as follows: Starting from scratch, we randomly generate a feasible solution and then optimize the solution to become a local optimum using our improvement method (see Section 2.3). The resulting improved solution is added to the population if it does not duplicate any solution currently in the population. This procedure is repeated until the size of the population reaches the cardinality p.

2.3. Tabu Search Procedure

Our TS procedure uses the N7 neighbourhood proposed by Zhang et al. (2007). It stops if the optimal solution is found or the best objective value has not been improved for a given number of TS iterations, called the tabu search *cutoff*. The interested reader may refer to the hybrid evolutionary algorithm HEA presented in Cheng et al. (2013) for more details.

2.4. Path Relinking Procedure

The relinking procedure is used to generate new solutions by exploring trajectories (confined to the neighbourhood space) that connect high-quality solutions. The solution that begins the path is called the initiating solution while the solution that the path leads to is called the guiding solution. The $PathSet$ is a list of candidate solutions that stores all the solutions generated during the path relinking procedure. After the relinking procedure, a so-called reference solution is chosen from the $PathSet$ that serves to update the population. In order to better describe the relinking procedure, we give some definitions in Table 1.

Table 1: The description of the symbols used in TS/PR

Symbols	Description
$j_{k,i}$	The ith operation executed on machine k.
S	A schedule for JSP is represented by permutations of operations on the machines: $\{(j_{1,1}, j_{1,2}, \ldots, j_{1,n}), (j_{2,1}, j_{2,2}, \ldots, j_{2,n}), \ldots, (j_{m,1}, j_{m,2}, \ldots, j_{m,n})\}$.
S^I	The initiating solution for the relinking procedure.
S^G	The guiding solution in the relinking procedure.
S^C	The current solution during the relinking procedure.
$CS(S^I, S^G)$	The common sequence between S^I and S^G: $\{j^I_{k,i} \mid j^I_{k,i} = j^G_{k,i}, k \in M, i \in N\}$.
$NCS(S^I, S^G)$	The set of elements not in the common sequence between S^I and S^G: $\{j^I_{k,i} \mid j^I_{k,i} \neq j^G_{k,i}, k \in M, i \in N\}$.
$Dis(S^I, S^G)$	The distance between S^I and S^G: $\lvert NCS(S^I, S^G)\rvert$.
PairSet	A set that stores the candidate solution pairs for path building.
PathSet	A set that stores the candidate solutions on a single path that will be optimized by the improvement method.
α	The minimum distance between the initiating (or guiding) solution and the first (or last) solution in the *PathSet*.
β	The interval for choosing the candidate solutions in *PathSet*.

Contrary to previous studies, our proposed path relinking process mainly integrates two complementary key components to ensure search efficiency. The first one is the constructing approach used for establishing the path between the initiating and the guiding solutions. In the related literature, Nasiri and Kianfar (2012b)'s relinking swaps adjacent operations on a machine, while GRASP/PR by Aiex et al. (2003) swaps different operations on each machine in turn. However, in this study we swap two different operations on one machine randomly, where both the operations and the responding machines are randomly chosen. More details will be presented in Section 2. The second one is the method used to choose the reference solution. In related studies, Aiex et al. (2003) simply consider the solution with the best makespan in the path as the reference one, while Nasiri and Kianfar (2012b) follow Nowicki and Smutnicki (2005)'s i-TSAB whereby it goes from the initiating solution, then stops at a specific iteration and returns the current solution as the reference solution. In contrast, we devise a dedicated strategy based on the adaptive distance-control mechanism to obtain the most promising solution. Therefore, the path relinking approach plays the important role of diversification in coordinating with the efficient TS procedure.

2.4.1. The relinking procedure:

Algorithm 2 presents the relinking procedure in detail. Section 2.4.2 presents how we construct a path from the initiating solution S^I to the guiding solution S^G. Section 2.4.3 explains how we choose a subset of candidate solutions, denoted by the $PathSet$, possibly as the reference solution. In Section 2.4.4 the reference solution is determined by applying both *slight* and *strong* TS procedures to the candidate solutions in the $PathSet$.

2.4.2. Path construction:

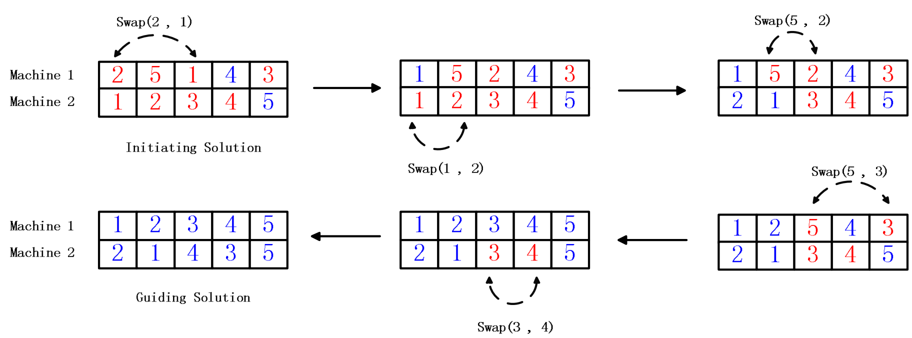


Figure 1: Path construction procedure

We employ the swap operation, which swaps two elements on the same machine, to build a path from the initiating solution S^I to the guiding solution S^G. At the beginning, the current solution S^C is assigned as S^I. In each iteration, S^C is changed by a swap operation towards the guiding solution

Algorithm 2 Pseudo-code of the relinking procedure

1: **Input**: Initiating solution S^I and guiding solution S^G
2: **Output**: A reference solution S^R
3: Identify the elements not in common sequence between S^I and S^G, denoted as $NCS(S^I, S^G)$
4: $S^C = S^I$, $PathSet = \emptyset$
5: //Lines 6-10: Change S^C to S^G by α consecutive swap moves /*Section 2.4.2*/
6: **for** $k = \{1, \ldots, \alpha\}$ **do**
7:     Randomly select an element S_i^G in $NCS(S^C, S^G)$
8:     Swap element S_i^G and another one in S^C such that S_i^G's position in S^C is the same as that in S^G
9:     $NCS(S^C, S^G) \leftarrow NCS(S^C, S^G) \setminus S_i^G$
10: **end for**
11: $PathSet \leftarrow PathSet \cup S^C$
12: //Lines 13-20: Construct the path with an interval β until its distance to S^G is less than α /* Section 2.4.2*/
13: **while** $Dis(S^C, S^G) > \alpha$ **do**
14:     **for** $k = \{1, \ldots, \beta\}$ **do**
15:         Randomly select an element S_i^G in $NCS(S^C, S^G)$
16:         Swap element S_i^G and another one in S^C such that S_i^G's position in S^C is the same as in S^G
17:         $NCS(S^C, S^G) \leftarrow NCS(S^C, S^G) \setminus S_i^G$
18:     **end for**
19:     $PathSet \leftarrow PathSet \cup S^C$
20: **end while**
21: //Lines 22-30: Choose the reference solution from $PathSet$ /* Section 2.4.3*/
22: let q be the cardinality of $PathSet$: $q = |PathSet|$
23: **for** $S_k \in PathSet$, $k = \{1, \ldots, q\}$ **do**
24:     **if** solution S_k is an infeasible solution **then**
25:         $S_k \leftarrow$ Repair(S_k)
26:     **end if**
27:     $S_k \leftarrow$ Tabu_Search(S_k) with a small number of iteration si
28: **end for**
29: Record the best solution in $PathSet$ as the reference solution S^R: S^R= arg $min\{f(S_k), k = 1, \ldots, q\}$
30: $S^R \leftarrow$ Tabu_Search(S^R) with a large number of iteration li
31: **return** S^R

S^G. Specifically, in S^C we iteratively swap two (random) elements that are in different order in S^C and S^G. Figure 1 gives an example of executing the path construction procedure on two machines. In this example, the initiating solution S^I is transformed into the guiding solution S^G in five swap moves. It should be noted that Aiex et al. (2003)'s GRASP/PR only swaps the operations on one machine in each iteration. In other words, only if the operations on one machine are the same will the next machine be taken into consideration, whereas in our study both the operations and machines are randomly chosen. Despite this subtle difference, our approach enhances the possibility of constructing a diversified path.

2.4.3. Path solution selection:

Since two consecutive solutions on a relinking path differ only by swapping two elements on a machine, it is not productive to apply an improvement method to each solution on the path since many of these solutions would lead to the same local optimum. More importantly, the improvement method is very time consuming, so we should restrict its use to only a subset of promising solutions, denoted as the $PathSet$.

We construct the $PathSet$ as follows: First, we choose the first solution at a distance of at least α from the initiating solution in the $PathSet$. Then, for each interval of β swap moves, we add a solution to the $PathSet$ until the distance between the current solution and the guiding solution is less than α. In such a way, the candidate solution list $PathSet$ is constructed. Figure 2 gives an illustration of the path solution selection procedure.

2.4.4. Reference solution determination:

As soon as the $PathSet$ is built, we need to determine the reference solution to update the population. For this purpose, we first employ a TS

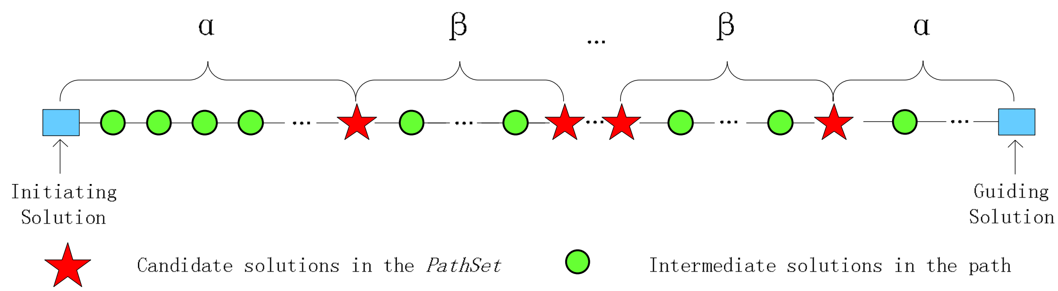


Figure 2: Illustration of the path solution selection procedure

with short iterations (we label it as a *slight* TS) to optimize each solution in the $PathSet$ to become a local optimum. Then, the best optimized solution is selected and further optimized using a TS with long iterations (we label it as a *strong* TS). This optimized solution is chosen as the reference solution. The reason is that a slight TS is not too time consuming but can optimize a solution to some extent, from which we can judge which solution is more promising than the others. As long as a reference solution is chosen, it is necessary to optimize it as far as possible, so we utilize a *strong* TS to optimize it.

It should be noted that it is possible that the solutions in the $PathSet$ are infeasible solutions because of the random swap operations during the path construction procedure, which may violate the precedence constraints. In this case, the previous literature is often inclined to abandon or delete them, e.g., GRASP/PR from Aiex et al. (2003). However, we utilize a special technique proposed by Qing-dao-er ji and Wang (2012) to repair the infeasible solutions to feasible ones (lines 24-26 in Algorithm 2).

3. Computational Results

In this section we report extensive experimental results of applying TS/PR to tackle six sets of a total of 205 benchmark JSP instances widely used in the literature. We coded TS/PR in C++ and ran it on a PC with a Quad-Core AMD Athlon 3.0GHz CPU and 2Gb RAM under the Windows 7 operating system. Table 2 gives the descriptions and settings of the parameters used in TS/PR, in which the last column denotes the settings for the set of all the instances. Given the stochastic nature of TS/PR, we solved each problem instance ten times independently. For each run, in view of the different levels of difficulty of the benchmark instances, we set different total time limits for applying TS/PR to tackle them. Table 3 gives the set time limits.

Table 2: The settings of some important parameters in TS/PR

Parameter	Value	Description
α	$\frac{dis}{5}$	Minimum distance between solutions in $PathSet$ and S^I and S^G
β	$max(\frac{dis}{10}, 2)$	Interval for choosing the path solutions
si	500	The number of iterations for the slight tabu search
li	12500	The number of iterations for the strong tabu search
p	30	Population size

Table 3: The settings of the time limit for different categories of instances

Instance Name :	SWV12(15)	DMU56-65	DMU66-70	DMU71-80	Other instances
Time limit :	2 hours	2 hours	4 hours	5 hours	1 hour

In this paper we report the detailed results on testing TS/PR (such as the best and relative values, and the time required to obtain the results) to faciliate future comparisons. All these benchmark instances can be downloaded from the OR-Library[1] and Shylo's webpage[2].

To test the performance of TS/PR, we consider the following well-known JSP instance classes:

- The first set of benchmark instances consists of 13 basic instances, including the three instances FT6, FT10, and FT20 due to Fisher and Thompson (1963), and the ten instances ORB01-10 due to Applegate and Cook (1991);

- The second set of benchmark instances consists of the 40 classic instances LA01-40 due to Lawrence (1984).

- The third set of benchmark instances consists of the three difficult instances ABZ07-09 due to Adams et al. (1988), and the four instances YN01-04 due to Yamada and Nakano (1992). Although this instance set is not large, the optimal values of these instances are still unknown.

- The fourth set of benchmark instances consists of the 15 instances SWV01-15 due to Storer et al. (1992).

- The fifth set of benchmark instances consists of 50 of the most difficult instances TA01-50 due to Taillard (1994).

- The sixth set of benchmark instances consist of 80 of the most difficult instances DMU01-80 due to Demirkol et al. (1997).

[1] `http://people.brunel.ac.uk/~mastjjb/jeb/orlib/jobshopinfo.html`

[2] `http://plaza.ufl.edu/shylo/jobshopinfo.html`

To measure the performance of TS/PR, we calculate the relative error (RE) using the relative deviation formula: RE = 100 $\times$ (UB$_{solve}$ - LB$_{best}$)/ LB$_{best}$, for each instance, where LB$_{best}$ is the best known lower bound and UB$_{solve}$ is the best makespan found by all of the tested algorithms. Subsequently, we calculate the mean relative error (MRE) for a given algorithm as the mean RE over all the tested instances.

In our experiments, the best known LB and UB were obtained from the following papers and website pages. Note that these algorithms can generate the upper bounds for almost all of the instances and can be considered as the current state-of-the-art algorithms for JSP. In the context of performance evaluation, we compare TS/PR mainly with these state-of-the-art algorithms in detail, which include: i-TSAB by Nowicki and Smutnicki (2005), GES by Pardalos and Shylo (2006), TS by Zhang et al. (2007), TS/SA by Zhang et al. (2008), AlgFix by Pardalos et al. (2010), CP/LS by Beck et al. (2011), GES/TS by Nasiri and Kianfar (2012a), HGA by Qing-dao-er ji and Wang (2012), BRKGA by Gonçalves and Resende (2013), Taillard's URL[3], and Shylo's webpage[4].

3.1. Computational results on the first two sets of instances

We conducted the first experiment to evaluate the performance of TS/PR in tackling the sets of 53 benchmark JSP instances: FT06, 10, 20, ORB01-10, and LA01-40. The number of operations for these instances ranges from 36 to 300. Tables 4 and 5 provide a summary of the performance comparisons

[3] http://mistic.heig-vd.ch/taillard/problemes.dir/ordonnancement.dir/jobshop.dir/best_lb_up.txt

[4] http://plaza.ufl.edu/shylo/jobshopinfo.html

Table 4: Computational results and comparisons for the first set of instances (FT series and ORB01-10)

Problem	Size	OPT	TS/PR			BRKGA		TS/SA	GES	TS
			Best	M_{av}	T_{av}	Best	M_{av}			
FT06	6×6	55	55	55	0.03	55	55	55	-	55
FT10	10×10	930	930	930	4.75	930	930	930	-	930
FT20	20×5	1165	1165	1165	0.18	1165	1165	1165	-	1165
ORB01	20×20	1059	1059	1059	0.51	1059	1059	1059	1059	-
ORB02	20×20	888	888	888	1.69	888	888	888	888	-
ORB03	20×20	1005	1005	1005	1.46	1005	1005	1005	1005	-
ORB04	20×20	1005	1005	1005	3.71	1005	1005	1005	1005	-
ORB05	20×20	887	887	887	7.28	887	887	887	887	-
ORB06	20×20	1010	1010	1010	1.81	1010	1010	1010	1010	-
ORB07	20×20	397	397	397	0.13	397	397	397	397	-
ORB08	20×20	899	899	899	3.99	899	899	899	899	-
ORB09	20×20	934	934	934	0.47	934	934	934	934	-
ORB10	20×20	944	944	944	0.09	944	944	944	944	-
MRE		0	0	0	2.01	0	0	0	0	0

between TS/PR and BRKGA, TS/SA, GES, and TS for instances of the FT, ORB01-10, and LA classes. In both tables, the column OPT lists the optimal solution for each instance. The following three columns Best, M_{av}, and T_{av} show the best makespan, average makespan, and average computing time in seconds to obtain the best value, respectively, by TS/PR over ten runs. The next two column presents the best makespan and average makespan in BRKGA, and the last three columns give the best results of the reference algorithms TS/SA, GES, and TS. The last row presents the MRE value averaged over one set of instances. In addition, the row TS/PR reports the MRE value for part of the instances since some reference algorithms only give results for part of the instances. For each class of instances, the best MRE (Best), the average MRE (M_{av}), and the average running time (T_{av}) are listed for each algorithm. Due to their relatively small sizes, most of these instances, except for LA29, are very easy to solve by TS/PR and the reference algorithms.

From Table 4, we see that TS/PR can easily reach the optima within 2.01 seconds on average for the 13 FT and ORB instances. From Table 5, we see that TS/PR can easily reach the optima for all the 40 LA instances, except for LA29, within 13.9 seconds on average. TS/PR is only slightly worse than algorithm GES but is better than the other three algorithms TS/SA, TS, and HGA in terms of solution quality.

3.2. Computational results on the third set of instances

In order to further evaluate the performance of TS/PR, we tested it on the third set of benchmark JSP instances ABZ07-09 and YN01-04

Table 6 summarizes the results of this experiment. In this table, the column UB(LB) lists the best known upper bound (lower bound) and the next three columns Best, M_{av}, and T_{av} show the best makespan, average makespan, and average computing time in seconds to obtain the best value, respectively, by TS/PR over ten independent runs. The next two columns present the best makespan and average makespan of BRKGA. The last three columns show the best results of GES/TS, TS/SA, and TS, respectively. From Table 6, we see that TS/PR outperforms GES/TS, TS/SA, and TS, while it is only slightly worse than BRKGA in terms of solution quality.

3.3. Computational results on the fourth set of instances

The set of benchmark JSP instances SWV01-15 was first reported by Storer et al. (1992). As this set contains some of the most difficult JSP

Table 5: Detailed computational results and comparisons for the second set of instances LA01-40

Problem	Size	OPT	TS/PR			BRKGA		GES	TS/SA	TS	HGA
			Best	M_{av}	T_{av}	Best	M_{av}				
LA01	10×5	666	666	666	0.05	666	666	666	-	-	666
LA02	10×5	655	655	655	0.05	655	655	655	-	-	655
LA03	10×5	597	597	597	0.06	597	597	597	-	-	597
LA04	10×5	590	590	590	0.05	590	590	590	-	-	590
LA05	10×5	593	593	593	0.06	593	593	593	-	-	593
LA06	15×5	926	926	926	0.09	926	926	926	-	-	926
LA07	15×5	890	890	890	0.06	890	890	890	-	-	890
LA08	15×5	863	863	863	0.08	863	863	863	-	-	863
LA09	15×5	951	951	951	0.09	951	951	951	-	-	951
LA10	15×5	958	958	958	0.09	958	958	958	-	-	958
LA11	20×5	1222	1222	1222	0.11	1222	1222	1222	-	-	1222
LA12	20×5	1039	1039	1039	0.12	1039	1039	1039	-	-	1039
LA13	20×5	1150	1150	1150	0.12	1150	1150	1150	-	-	1150
LA14	20×5	1292	1292	1292	0.12	1292	1292	1292	-	-	1292
LA15	20×5	1207	1207	1207	0.11	1207	1207	1207	-	-	1207
LA16	10×10	945	945	945	0.15	945	945	945	-	-	945
LA17	10×10	784	784	784	0.08	784	784	784	-	-	784
LA18	10×10	848	848	848	0.09	848	848	848	-	-	848
LA19	10×10	842	842	842	0.16	842	842	842	842	842	844
LA20	10×10	902	902	902	0.11	902	902	902	-	-	907
LA21	15×10	1046	1046	1046	7.33	1046	1046	1046	1046	1046	1046
LA22	15×10	927	927	927	3.94	927	927	927	-	-	935
LA23	15×10	1032	1032	1032	0.13	1032	1032	1032	-	-	1032
LA24	15×10	935	935	935	3.09	935	935	935	935	935	953
LA25	15×10	977	977	977	1.38	977	977	977	977	977	981
LA26	20×10	1218	1218	1218	0.28	1218	1218	1218	-	-	1218
LA27	20×10	1235	1235	1235	2.19	1235	1235	1235	1235	1235	1236
LA28	20×10	1216	1216	1216	0.35	1216	1216	1216	-	-	1216
LA29	20×10	1152	1153	1153	73.78	1153	1154.7	1152	1153	1156	1160
LA30	20×10	1355	1355	1355	0.31	1355	1355	1355	-	-	1355
LA31	30×10	1784	1784	1784	0.27	1784	1784	1784	-	-	1784
LA32	30×10	1850	1850	1850	0.29	1850	1850	1850	-	-	1850
LA33	30×10	1719	1719	1719	0.27	1719	1719	1719	-	-	1719
LA34	30×10	1721	1721	1721	0.28	1721	1721	1721	-	-	1721
LA35	30×10	1888	1888	1888	0.27	1888	1888	1888	-	-	1888
LA36	15×15	1268	1268	1268	4.53	1268	1268	1268	1268	1268	1287
LA37	15×15	1397	1397	1397	26.24	1397	1397	1397	1397	1397	1407
LA38	15×15	1196	1196	1196	32.61	1196	1196	1196	1196	1196	1196
LA39	15×15	1233	1233	1233	11.63	1233	1233	1233	1233	1233	1233
LA40	15×15	1222	1222	1222	384.8	1222	1223.2	1222	1224	1224	1229
MRE		0	0.002		13.90	0.002		0.000	0.023	0.046	0.189
TS/PR						0.002		0.002	0.008	0.008	0.002

Table 6: Computational results and comparisons for the third set of instances ABZ07-09 and YN01-04

Problem	Size	UB(LB)	TS/PR			BRKGA		GES/TS	TS/SA	TS
			Best	M_{av}	T_{av}	Best	M_{av}			
ABZ07	20×15	656(656)	657	657.1	438.01	656	658	658	658	657
ABZ08	20×15	665(645)	667	667.8	138.97	667	667.7	669	669	669
ABZ09	20×15	678(661)	678	678	90.22	678	678.9	679	678	680
YN01	20×20	884(826)	884	885.5	169.29	884	886	884	884	-
YN02	20×20	904(861)	904	907.7	202.22	904	906.5	905	907	-
YN03	20×20	892(827)	892	893.8	344.15	892	893.1	892	892	-
YN04	20×20	968(918)	968	969.1	320.51	968	973	969	969	-
MRE			4.494		243.34	4.472		4.614	4.625	2.249
TS/PR						4.494		4.494	4.494	2.045

Table 7: Computational results and comparisons for the fourth set of instances SWV01-15

Problem	Size	UB(LB)	TS/PR			BRKGA		GES/TS	TS/SA	TS
			Best	M_{av}	T_{av}	Best	M_{av}			
SWV01	20×10	1407(1407)	1407	1411.4	575.76	1407	1408.9	1412	1412	-
SWV02	20×10	1475(1475)	1475	1475.1	294.13	1475	1478.2	1475	1475	-
SWV03	20×10	1398(1369)	1398	1398.9	613.00	1398	1400	1398	1398	-
SWV04	20×10	1470(1450)	1470	1473.5	257.63	1470	1472.8	1471	1470	-
SWV05	20×10	1424(1424)	1425	1426	612.78	1425	1431.4	1426	1425	-
SWV06	20×15	1672(1591)	**1671**	1675.9	385.73	1675	1682.1	1677	1679	-
SWV07	20×15	1594(1446)	1595	1605	626.46	1594	1601.2	1595	1603	-
SWV08	20×15	1752(1640)	1752	1760.4	503.00	1755	1764.3	1766	1756	-
SWV09	20×15	1656(1604)	**1655**	1661.8	521.91	1656	1667.9	1660	1661	-
SWV10	20×15	1743(1631)	1743	1756.6	441.40	1743	1754.6	1760	1754	-
SWV11	50×10	2983(2983)	2983	2984.5	940.68	2983	2985.9	-	-	2983
SWV12	50×10	2979(2972)	**2977**	2985.3	6097.35	2979	2989.7	-	-	2979
SWV13	50×10	3104(3104)	3104	3104	1111.22	3104	3111.6	-	-	3104
SWV14	50×10	2968(2968)	2968	2968	422.81	2968	2968	-	-	2968
SWV15	50×10	2886(2885)	**2885***	2889.4	6000.57	2901	2902.9	-	-	2886
MRE			2.396		1293.63	2.466		3.886	3.848	0.054
TS/PR						2.396		3.578	3.578	0.034

Newly found upper bounds by TS/PR are indicated in bold.

$*$: the best solution found by the TS/PR is equal to the lower bound.

instances, many powerful algorithms have been proposed for solving them. However, 60% of these instances have not been solved until now.

From Table 7, it is easy to see that TS/PR outperforms all of the five reference algorithms in terms of the MRE value. Specifically, TS/PR matches nine best known solutions and improves the upper bounds for the four instances SWV06, SWV09, SWV12, and SWV15. Even if TS/PR cannot reach the best upper bound for the two instances SWV05 and SWV07, the gaps between our solutions and the best upper bounds are only one unit. In comparison, BRKGA, which is one of the best performing algorithms in the literature, reaches the upper bounds for 11 out of 15 instances. It is worth noting that the best makespan obtained by TS/PR is better than that of BRKGA in four cases, while TS/PR is worse than BRKGA for one instance.

In particular, TS/PR is able to find better upper bounds for the instances SWV06-1671, SWV09-1655, SWV12-2977, and SWV15-2885, while the previous best upper bounds were for the instances SWV06-1672, SWV09-1656, SWV12-2979, and SWV15-2886. More strikingly, TS/PR is able to reach the best lower bound for the instance SWV15, meaning that TS/PR solves this instance, which has remained unsolved for over 20 years.

3.4. Computational results on the fifth set of instances

The fifth set of 50 benchmark JSP instances TA is one of the most widely used set of instances and is also part of the most difficult JSP instances over the last 20 years.

Table 8 presents the results of applying TS/PR to tackle the set of TA instances and comparisons with the reference algorithms. As can be seen, TS/PR is able to find the current best known solutions for 34 of the

50 instances and in addition improve upon the solutions for five instances. Specifically, TS/PR finds better upper bounds for the instances TA43-1846, TA44-1982, TA47-1889, TA49-1963, and TA50-1923, while the previous best upper bounds were for the instances TA43-1848, TA44-1983, TA47-1894, TA49-1964, and TA50-1924. Although TS/PR performs slightly worse than BRKGA in terms of the MRE value, it outperforms all the other reference algorithms in the literature.

3.5. Computational results on the sixth set of instances

Our last experiment was based on the DMU set of instances, which are considered to be one of the hardest JSP instances. In particular, the instances DMU41-80 are considered to be extremely challenging (Demirkol et al., 1997). However, our computational experiments show that TS/PR yields high-quality solutions for these instances and can even improve many of the best upper bounds. The detailed results for these instances are presented in Tables 9 and 10.

In general, TS/PR performs well on these DMU instances in comparison with the reference algorithms BRKGA, TS, GES, i-TSAB, and AlgFix. The results reveal that TS/PR outperforms all of these algorithms for the majority of these instances. In particular, for the first 40 DMU instances, TS/PR is able to improve the best upper bounds for five instances and solve the problem in less CPU time than BRKGA, one of the best performing algorithms for these instances.

On the other hand, TS/PR is able to obtain better solutions for the difficult instances DMU41-80. For example, TS/PR is able to improve 35 upper bounds and hit two best upper bounds for these instances.

Table 8: Computational results and comparisons for the fifth set of instances TA01-50

Problem	Size	UB(LB)	TS/PR			BRKGA		CP/LS	GES	AlgFix	i-TSAB	TS/SA
			Best	M_{av}	T_{av}	Best	M_{av}					
TA01	15×15	1231(1231)	1231	1231	2.93	1231	1231	-	1231	1231	-	1231
TA02	15×15	1244(1244)	1244	1244	38.09	1244	1244	-	1244	1244	-	1244
TA03	15×15	1218(1218)	1218	1218	43.66	1218	1218	-	1218	1218	-	1218
TA04	15×15	1175(1175)	1175	1175	38.72	1175	1175	-	1175	1175	-	1175
TA05	15×15	1224(1224)	1224	1224	11.24	1224	1224.9	-	1224	1224	-	1224
TA06	15×15	1238(1238)	1238	1238.4	178.06	1238	1238.9	-	1238	1238	-	1238
TA07	15×15	1227(1227)	1228	1228	0.60	1228	1228	-	1228	1228	-	1228
TA08	15×15	1217(1217)	1217	1217	2.43	1217	1217	-	1217	1217	-	1217
TA09	15×15	1274(1274)	1274	1274	18.66	1274	1277	-	1274	1274	-	1274
TA10	15×15	1241(1241)	1241	1241	42.25	1241	1241	-	1241	1241	-	1241
TA11	20×15	1357(1323)	1357	1359.9	186.19	1357	1360	1357	1357	1358	1361	1359
TA12	20×15	1367(1351)	1367	1369.9	206.06	1367	1372.6	1367	1367	1367	-	1371
TA13	20×15	1342(1282)	1342	1346	161.37	1344	1347.3	1342	1344	1342	-	1342
TA14	20×15	1345(1345)	1345	1345	8.28	1345	1345	1345	1345	1345	-	1345
TA15	20×15	1339(1304)	1339	1339	173.45	1339	1348.9	1339	1339	1339	-	1339
TA16	20×15	1360(1304)	1360	1360	63.41	1360	1362.1	1360	1360	1360	-	1360
TA17	20×15	1462(1462)	1463	1473	203.49	1462	1470.5	1462	1469	1473	1462	1464
TA18	20×15	1396(1369)	1396	1401	91.13	1396	1400.9	1396	1401	1396	-	1399
TA19	20×15	1332(1304)	1332	1336.6	145.42	1332	1333.2	1332	1332	1332	1335	1335
TA20	20×15	1348(1318)	1348	1351.3	216.72	1348	1350.4	1348	1348	1348	1351	1350
TA21	20×20	1642(1573)	1644	1645.2	502.99	1642	1647	1642	1647	1643	1644	1644
TA22	20×20	1600(1542)	1600	1603.8	228.90	1600	1600	1600	1602	1600	1600	1600
TA23	20×20	1557(1474)	1557	1559.6	359.79	1557	1562.6	1557	1558	1557	1557	1560
TA24	20×20	1644(1606)	1645	1647.7	779.32	1646	1650.6	1644	1653	1646	1647	1646
TA25	20×20	1595(1518)	1595	1597	416.08	1595	1602	1595	1596	1595	1595	1597
TA26	20×20	1643(1558)	1647	1651.4	267.50	1643	1652.3	1643	1647	1647	1645	1647
TA27	20×20	1680(1617)	1680	1686.7	254.74	1680	1685.6	1680	1685	1686	1680	1680
TA28	20×20	1603(1591)	1613	1616.2	326.23	1603	1611.7	1603	1614	1613	1614	1603
TA29	20×20	1625(1525)	1625	1627.4	93.53	1625	1627.4	1625	1625	1625	-	1627
TA30	20×20	1584(1485)	1584	1588.3	388.66	1584	1588.5	1584	1584	1584	1584	1584
TA31	30×15	1764(1764)	1764	1764	35.57	1764	1764.4	1764	1764	1766	-	1764
TA32	30×15	1785(1774)	1787	1803.5	703.06	1785	1794.1	1796	1793	1790	-	1795
TA33	30×15	1791(1778)	1791	1794.6	457.55	1791	1793.7	1791	1799	1791	1793	1796
TA34	30×15	1829(1828)	1829	1831.2	315.71	1829	1832.1	1829	1832	1832	1829	1831
TA35	30×15	2007(2007)	2007	2007	0.56	2007	2007	2007	2007	2007	-	2007
TA36	30×15	1819(1819)	1819	1819	122.67	1819	1822.9	1819	1819	1819	-	1819
TA37	30×15	1771(1771)	1771	1776.8	652.24	1771	1777.8	1774	1779	1784	1778	1778
TA38	30×15	1673(1673)	1673	1673	307.34	1673	1676.7	1673	1673	1673	-	1673
TA39	30×15	1795(1795)	1795	1795	115.61	1795	1801.6	1795	1795	1795	-	1795
TA40	30×15	1669(1631)	1671	1676	449.96	1669	1678.1	1673	1680	1979	1674	1676
TA41	30×20	2006(1874)	2010	2018.6	1267.78	2008	2018.7	2010	2022	2022	-	2018
TA42	30×20	1937(1867)	1949	1950.3	1556.36	1937	1949.3	1947	1956	1953	1956	1953
TA43	30×20	1848(1809)	**1846**	1865.1	1726.78	1852	1863.1	1863	1870	1869	1859	1858
TA44	30×20	1983(1927)	**1982**	1989.1	1304.66	1983	1992.4	1991	1991	1992	1984	1983
TA45	30×20	2000(1997)	2000	2000.5	1057.79	2000	2000	2000	2004	2000	2000	2000
TA46	30×20	2004(1940)	2008	2022.3	1236.03	2004	2015.5	2016	2011	2011	2021	2010
TA47	30×20	1894(1789)	**1889**	1906.2	1030.88	1894	1902.1	1906	1903	1902	1903	1903
TA48	30×20	1943(1912)	1947	1955.5	1047.42	1943	1959.2	1951	1962	1962	1953	1955
TA49	30×20	1964(1915)	**1963**	1971.5	1035.82	1964	1972.6	1966	1969	1974	-	1967
TA50	30×20	1924(1807)	**1923**	1931.4	1318.05	1925	1927	1924	1931	1927	1928	1931
MRE			2.162		423.83	2.133		2.769	2.356	2.688	3.233	2.279
TS/PR						2.162		2.701	2.162	2.162	3.046	2.162

Newly found upper bounds by TS/PR are indicated in bold.

In sum, TS/PR finds improved upper bounds for 40 out of the 80 DMU instances, i.e., 50% of this set of the hardest JSP instances. This experiment demonstrates the competitiveness of TS/PR in terms of both solution quality and computational efficiency.

3.6. Performance summary of TS/PR

Finally, we summarize in Figure 3 the overall performance of TS/PR in tackling all the 205 tested instances. Figure 3(a) presents the numbers of better, equal, and worse solutions that TS/PR is able to obtain compared with the corresponding upper bounds (UB). We see that TS/PR can improve the best upper bounds for 49 out of the 205 tested instances and tie the best upper bounds for 133 instances, while obtaining worse results for only 23 instances.

Figures 3(b) and 3(c) give the numbers of instances for which TS/PR outperforms and underperforms the UB, respectively. For example, the instance class for which TS/PR improves the UB by one unit is denoted by A_1 and it consists of seven instances, accounting for 14.58% of the total 205 instances. It is worthwhile to note that TS/PR improves the UB by more than ten units for two thirds of the 205 instances, whereas there are only two instances for which TS/PR underperforms the UB by more than ten units. Therefore, we conclude that TS/PR not only possesses a strong improvement capability, but it also has great improvement strength.

From Table 11, we observe that the average computing time of TS/PR is an order of magnitude less than BRKGA for the instances DMU21-40 for which the optimal solutions are known. In particular, for the instances DMU31-35, the computing time of TS/PR is 2,000 times less than that of

Table 9: Computational results and comparisons for the sixth set of instances DMU01-40

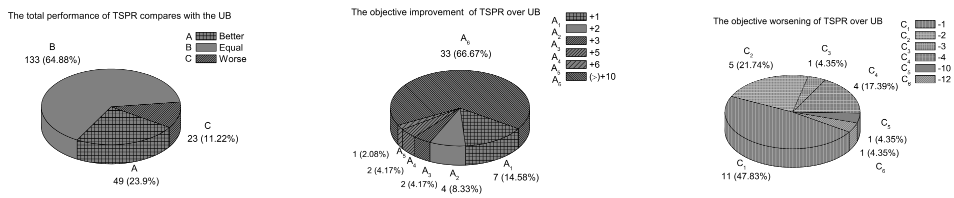
Problem	Size	UB(LB)	TS/PR			BRKGA		TS	GES	i-TSAB	AlgFix
			Best	M_{av}	T_{av}	Best	M_{av}				
DMU01	20 × 15	2563(2501)	2563	2563	332.87	2563	2563	2566	2566	2571	2563
DMU02	20 × 15	2706(2651)	2706	2713.2	179.24	2706	2714.5	2711	2706	2715	2706
DMU03	20 × 15	2731(2731)	2731	2733.1	388.59	2731	2736.5	-	2731	-	2731
DMU04	20 × 15	2669(2601)	2669	2670.2	96.54	2669	2672.4	-	2669	-	2669
DMU05	20 × 15	2749(2749)	2749	2758.6	303	2749	2755.4	-	2749	-	2749
DMU06	20 × 20	3244(2998)	3245	3249.2	823.17	3244	3246.6	3254	3250	3265	3244
DMU07	20 × 20	3046(2815)	3046	3062.3	360.58	3046	3058.6	-	3053	-	3046
DMU08	20 × 20	3188(3051)	3188	3194.3	295.81	3188	3188.3	3191	3197	3199	3188
DMU09	20 × 20	3092(2956)	3094	3097.4	148.00	3092	3094.4	-	3092	3094	3096
DMU10	20 × 20	2984(2858)	2985	2991	252.46	2984	2984.8	-	2984	2985	2984
DMU11	30 × 15	3445(3395)	**3430**	3435.2	1496.85	3445	3445.8	3455	3453	3470	3455
DMU12	30 × 15	3513(3481)	**3495**	3509.7	899.99	3513	3518.9	3516	3518	3519	3522
DMU13	30 × 15	3681(3681)	3681	3682.8	622.13	3681	3690.6	3681	3697	3698	3687
DMU14	30 × 15	3394(3394)	3394	3394	3.02	3394	3394	-	3394	3394	3394
DMU15	30 × 15	3343(3343)	3343	3343	1.77	3343	3343	-	3343	-	3343
DMU16	30 × 20	3751(3734)	3753	3765.4	1303.41	3751	3758.9	3759	3781	3787	3772
DMU17	30 × 20	3830(3709)	**3819**	3843.3	734.03	3830	3850.6	3842	3848	3854	3836
DMU18	30 × 20	3844(3844)	3844	3849.5	3787.40	3844	3845.4	3846	3849	3854	3852
DMU19	30 × 20	3770(3669)	**3768**	3787.4	718.71	3770	3791.8	3784	3807	3823	3775
DMU20	30 × 20	3712(3604)	**3710**	3726.5	701.29	3712	3715.3	3716	3739	3740	3712
DMU21	40 × 15	4380(4380)	4380	4380	0.69	4380	4380	-	4380	-	4380
DMU22	40 × 15	4725(4725)	4725	4725	1.48	4725	4725	-	4725	-	4725
DMU23	40 × 15	4668(4668)	4668	4668	1.30	4668	4668	-	4668	-	4668
DMU24	40 × 15	4648(4648)	4648	4648	0.75	4648	4648	-	4648	-	4648
DMU25	40 × 15	4164(4164)	4164	4164	0.60	4164	4164	-	4164	-	4164
DMU26	40 × 20	4647(4647)	4647	4647.3	1631.43	4647	4658.4	4647	4667	4679	4688
DMU27	40 × 20	4848(4848)	4848	4848	12.16	4848	4848	-	4848	4848	4848
DMU28	40 × 20	4692(4692)	4692	4692	17.68	4692	4692	-	4692	-	4692
DMU29	40 × 20	4691(4691)	4691	4691	63.49	4691	4691	-	4691	4691	4691
DMU30	40 × 20	4732(4732)	4732	4732	123.00	4732	4732	-	4732	4732	4749
DMU31	50 × 15	5640(5640)	5640	5640	0.84	5640	5640	-	5640	-	5640
DMU32	50 × 15	5927(5927)	5927	5927	0.62	5927	5927	-	5927	-	5927
DMU33	50 × 15	5728(5728)	5728	5728	0.43	5728	5728	-	5728	-	5728
DMU34	50 × 15	5385(5385)	5385	5385	2.22	5385	5385	-	5385	-	5385
DMU35	50 × 15	5635(5635)	5635	5635	0.71	5635	5635	-	5635	-	5635
DMU36	50 × 20	5621(5621)	5621	5621	7.83	5621	5621	-	5621	-	5621
DMU37	50 × 20	5851(5851)	5851	5851	11.38	5851	5851	-	5851	5851	5851
DMU38	50 × 20	5713(5713)	5713	5713	10.66	5713	5713	-	5713	-	5713
DMU39	50 × 20	5747(5747)	5747	5747	2.02	5747	5747	-	5747	-	5747
DMU40	50 × 20	5577(5577)	5577	5577	4.91	5577	5577	-	5577	-	5577

Newly found upper bounds by TS/PR are indicated in bold.

Table 10: Computational results and comparisons for the sixth set of instances DMU41-80

Problem	Size	UB(LB)	TS/PR			BRKGA		TS	GES	i-TSAB	AlgFix
			Best	M_{av}	T_{av}	Best	M_{av}				
DMU41	20×15	3261(3007)	**3248**	3281.9	417.84	3261	3281.9	-	3267	3277	3278
DMU42	20×15	3395(3172)	**3390**	3409.8	448.95	3395	3403.9	3416	3401	3448	3412
DMU43	20×15	3441(3292)	3441	3450.5	399.33	3441	3452.7	3459	3443	3473	3450
DMU44	20×15	3488(3283)	3489	3509.7	371.27	3488	3510.7	3524	3489	3528	3489
DMU45	20×15	3272(3001)	3273	3287.9	709.19	3272	3287.3	3296	3273	3321	3273
DMU46	20×20	4035(3575)	4035	4051.8	984.86	4035	4043.2	4080	4099	4101	4071
DMU47	20×20	3939(3522)	3942	3963.6	829.28	3939	3968	-	3972	3973	3950
DMU48	20×20	3781(3447)	**3778**	3814.1	938.55	3781	3800.9	3795	3810	3838	3813
DMU49	20×20	3723(3403)	**3710**	3736.1	633.84	3723	3729.6	3735	3754	3780	3725
DMU50	20×20	3732(3496)	**3729**	3741.2	609.62	3732	3746.5	3761	3768	3794	3742
DMU51	30×15	4201(3917)	**4167**	4205.9	2394.25	4201	4222.9	4218	4247	4260	4202
DMU52	30×15	4341(4065)	**4311**	4353.2	2232.60	4341	4352.3	4362	4380	4383	4353
DMU53	30×15	4415(4141)	**4394**	4425.7	2161.83	4415	4420.2	4428	4450	4470	4419
DMU54	30×15	4396(4202)	**4371**	4390.5	1909.53	4396	4402.7	4405	4424	4425	4413
DMU55	30×15	4290(4140)	**4271**	4295.2	1914.37	4290	4299.4	4308	4331	4332	4321
DMU56	30×20	4961(4554)	**4941**	4990.6	3825.44	4961	4768.4	5025	5051	5079	4985
DMU57	30×20	4698(4302)	**4663**	4714	3649.41	4698	4704.9	4698	4779	4785	4709
DMU58	30×20	4751(4319)	**4708**	4779.4	3639.68	4751	4752.8	4796	4829	4834	4787
DMU59	30×20	4630(4217)	**4624**	4670.6	3614.54	4630	4633.3	4667	4694	4696	4638
DMU60	30×20	4774(4319)	**4755**	4804.3	3745.91	4774	4777	4805	4888	4904	4827
DMU61	40×15	5224(4917)	**5195**	5217.1	4739.13	5224	5233.3	5228	5293	5294	5310
DMU62	40×15	5301(5033)	**5268**	5301	4853.75	5301	5304.4	5311	5354	5354	5330
DMU63	40×15	5357(5111)	**5326**	5347.5	4122.65	5357	5386.6	5371	5439	5446	5431
DMU64	40×15	5312(5130)	**5252**	5279.8	4487.26	5312	5321.8	5330	5388	5443	5385
DMU65	40×15	5197(5105)	**5196**	5203.2	4963.8	5197	5211.5	5201	5269	5271	5322
DMU66	40×20	5796(5391)	**5717**	5788.7	9543.86	5796	5806.6	5797	5902	5911	5886
DMU67	40×20	5863(5589)	**5816**	5852.5	8431.51	5863	5881.3	5872	6012	6016	5938
DMU68	40×20	5826(5426)	**5773**	5801.8	8739.45	5826	5843.7	5834	5934	5936	5840
DMU69	40×20	5775(5423)	**5709**	5754.4	8107.63	5775	5804	5794	6002	5891	5868
DMU70	40×20	5951(5501)	**5903**	5924.2	7285.27	5951	5968.2	5954	6072	6096	6028
DMU71	50×15	6278(6080)	**6223**	6264.8	9835.11	6293	6603.8	6278	6333	6359	6437
DMU72	50×15	6503(6395)	**6483**	6510.9	10881.79	6503	6560.7	6520	6589	6586	6604
DMU73	50×15	6219(6001)	**6163**	6199.8	11475.15	6219	6250.5	6249	6291	6330	6343
DMU74	50×15	6277(6123)	**6227**	6266.4	11164.43	6277	6312.6	6316	6376	6383	6467
DMU75	50×15	6236(6029)	**6197**	6239.4	11330.86	6248	6282.4	6236	6380	6437	6397
DMU76	50×20	6876(6342)	**6813**	6854.8	9998.17	6876	6885.4	6893	6974	7082	6975
DMU77	50×20	6857(6499)	**6822**	6879.9	12062.88	6857	6892.7	6868	7006	6930	6949
DMU78	50×20	6831(6586)	**6770**	6813.2	10346.61	6831	6855.7	6846	6988	7027	6928
DMU79	50×20	7049(6650)	**6970**	7003	9818.93	7049	7060.9	7055	7158	7253	7083
DMU80	50×20	6719(6459)	**6686**	6700.1	10331.98	6736	6757.9	6719	6843	6998	6861
MRE			3.596		2791.17	3.890		5.655	4.669	6.375	4.444
TS/PR						3.577		4.829	3.577	4.589	3.577

Newly found upper bounds by TS/PR are indicated in bold.



(a) The overall performance (b) Outperforming the UB (c) Underperforming the UB

Figure 3: The overall performance of TS/PR in terms of solution quality

BRKGA. Moreover, for many of the benchmark instances for which the optimal solutions are not known, the computing times of TS/PR and BRKGA are relatively close, despite that TS/PR can usually obtain comparable or better results. In sum, TS/PR is competitive with the state-of-the-art algorithm BRKGA in terms of computational efficiency.

4. Conclusion

In this paper we present a hybrid tabu search/path relinking algorithm for tackling the notorious job shop scheduling problem, in which we incorporate a number of distinguishing features, such as a path solution construction procedure based on the distances of the solutions and a special mechanism to determine the reference solution. Based on extensive computational results of applying TS/PR to tackle six sets of a total of 205 well-known and challenging benchmark JSP instances, we demonstrate the efficacy of TS/PR in comparison with the best known results in the literature. Specifically, TS/PR is able to improve the upper bounds for 49 instances. In addition, TS/PR solves the challenging SWV15, which has remained unsolved for over

Table 11: Computational time comparisons with BRKGA (in seconds) for all the instances reported in this paper

Instance Group1	Size	TS/PR	BRKGA	Instance Group2	Size	TS/PR	BRKGA
FT06	6×6	0.03	1.0	TA21-30	20×20	361.77	143.2
FT10	10×10	4.75	10.1	TA31-40	30×15	316.03	487.6
FT20	20×5	0.18	13.4	TA41-50	30×20	1258.16	1068.3
ORB01-10	10×10	2.12	5.8	DMU01-05	20×15	260.05	68.9
LA01-05	10×5	0.05	1.4	DMU06-10	20×20	376.00	145.4
LA06-10	15×5	0.08	2.9	DMU11-15	30×15	604.75	427.3
LA11-15	20×5	0.12	5.3	DMU16-20	30×20	1448.97	1043.6
LA16-20	10×10	0.12	4.6	DMU21-25	40×15	0.96	1150.6
LA21-25	15×10	3.17	15.3	DMU26-30	40×20	369.55	3556.3
LA26-30	20×20	15.38	21.8	DMU31-35	50×15	0.96	2086.7
LA31-35	30×10	0.28	38.7	DMU36-40	50×20	7.36	9368.3
LA36-40	15×15	91.96	21.4	DMU41-45	20×15	469.32	78.9
ABZ07-09	20×15	222.40	54.6	DMU46-50	20×20	799.23	187.7
YN01-04	20×20	259.04	105.2	DMU51-55	30×15	2122.52	701.4
SWV01-05	20×10	470.66	42.5	DMU56-60	30×20	3695.00	1545.8
SWV06-10	20×15	495.70	78.7	DMU61-65	40×15	4633.32	2684.3
SWV11-15	50×10	2914.53	2304.4	DMU66-70	40×20	8421.54	5394.2
TA01-10	15×15	37.66	30.4	DMU71-75	50×15	10937.47	8070.1
TA11-20	20×15	145.55	65.8	DMU76-80	50×20	10511.71	15923.4

BRKGA was run on an AMD 2.2 GHz Opteron(2427) CPU running the Linux (Fedora release 12) operating system.

20 years. The results confirm that the relinking method is a powerful diversification tool for tackling JSP compared with other state-of-the-art algorithms for JSP. Finally, given that many of the ideas introduced in this paper are independent of JSP, it is worthwhile to test their merits in dealing with other difficult combinatorial optimization problems.

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