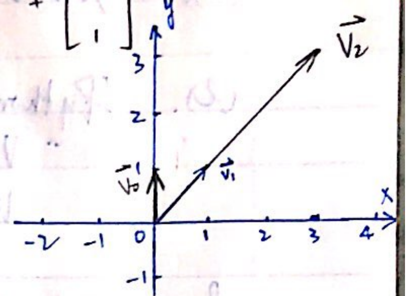
7. Image Stitching.

(a). Using the given info, we have $\vec{V_2} = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}$



so $\vec{V_2} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$\vec{V_2}$ is transformed from $\vec{V_0}$ by:

→. first rotating 45° clockwise and then scaled by 3.

(b). From Equation (5), we can get: $p_x \cdot R_{xx} + p_y \cdot R_{xy} + T_x = q_x$ (i)

$p_x \cdot R_{yx} + p_y \cdot R_{yy} + T_y = q_y$ (ii)

The known values in Eq (i) are p_x, p_y, q_x. The unknowns are R_{xx}, R_{xy}, T_x.
In Eq (ii), the known values are p_x, p_y, q_y. The unknowns are R_{yx}, R_{yy}, T_y.
So, there are $\boxed{6}$ unknowns,
and I need $\boxed{6}$ independent equations to solve them all, which
means I need $\boxed{3}$ pairs of common points $\vec{p}$ and $\vec{q}$.

(c) The vector of the unknown values is: $\begin{bmatrix} R_{xx} \\ R_{xy} \\ R_{yx} \\ R_{yy} \\ T_x \\ T_y \end{bmatrix}$

Let the three pairs of common points be $(p_1, q_1), (p_2, q_2), (p_3, q_3)$.
So, we have linear equations:

$$p_{1x} \cdot R_{xx} + p_{1y} \cdot R_{xy} + T_x = q_{1x}$$
$$p_{1x} \cdot R_{yx} + p_{1y} \cdot R_{yy} + T_y = q_{1y}.$$
$$p_{2x} \cdot R_{xx} + p_{2y} \cdot R_{xy} + T_x = q_{2x}$$
$$p_{2x} \cdot R_{yx} + p_{2y} \cdot R_{yy} + T_y = q_{2y}$$
$$p_{3x} \cdot R_{xx} + p_{3y} \cdot R_{xy} + T_x = q_{3x}$$
$$p_{3x} \cdot R_{yx} + p_{3y} \cdot R_{yy} + T_y = q_{3y}.$$

which can be transformed into:

$$\begin{bmatrix} p_{1x} & p_{1y} & 0 & 0 & 1 & 0 \\ 0 & 0 & p_{1x} & p_{1y} & 0 & 1 \\ p_{2x} & p_{2y} & 0 & 0 & 1 & 0 \\ 0 & 0 & p_{2x} & p_{2y} & 0 & 1 \\ p_{3x} & p_{3y} & 0 & 0 & 1 & 0 \\ 0 & 0 & p_{3x} & p_{3y} & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_{xx} \\ R_{xy} \\ R_{yx} \\ R_{yy} \\ T_x \\ T_y \end{bmatrix} = \begin{bmatrix} q_{1x} \\ q_{1y} \\ q_{2x} \\ q_{2y} \\ q_{3x} \\ q_{3y} \end{bmatrix}$$

(d). Solved Successfully.

(e). IPython throws out an error:
" ValueError: Rows are not linearly independent. Cannot solve system of linear equations uniquely. :) "

(f). Let $\vec{p_1}, \vec{p_2}, \vec{p_3}$ be collinear, so using the Fact, we have that:
$(\vec{p_2} - \vec{p_1}) = k(\vec{p_3} - \vec{p_1})$ for some $k \in \mathbb{R}$.

Thus, separating into values (sub-vectors) on x- and y- axis, we have:

$(p_{2x} - p_{1x}) = k(p_{3x} - p_{1x})$ and $(p_{2y} - p_{1y}) = k(p_{3y} - p_{1y})$ $\Rightarrow$ $p_{2x} = k \cdot p_{3x} - (k-1) p_{1x}$, $p_{2y} = k \cdot p_{3y} - (k-1) p_{1y}$.

Since the linear system in part (c) could be transformed into an augmented matrix:

$$\left[\begin{array}{cccccc|c} p_{1x} & p_{1y} & 0 & 0 & 1 & 0 & q_{1x} \\ 0 & 0 & p_{1x} & p_{1y} & 0 & 1 & q_{1y} \\ p_{2x} & p_{2y} & 0 & 0 & 1 & 0 & q_{2x} \\ 0 & 0 & p_{2x} & p_{2y} & 0 & 1 & q_{2y} \\ p_{3x} & p_{3y} & 0 & 0 & 1 & 0 & q_{3x} \\ 0 & 0 & p_{3x} & p_{3y} & 0 & 1 & q_{3y} \end{array}\right]$$

Row 3: Add $(k-1)\cdot$ Row 1 and Subtract $k \cdot$ Row 5. $\Rightarrow$

and since $p_{2x} + (k-1)p_{1x} - k\,p_{3x} = 0$

and $p_{2y} + (k-1)p_{2y} - k\,p_{3y} = 0$,

and $1 + (k-1) - k = 0$

and $0 + (k-1)\cdot 0 - k \cdot 0 = 0$, so $\Rightarrow$:

$$\left[\begin{array}{cccccc|c} p_{1x} & p_{1y} & 0 & 0 & 1 & 0 & q_{1x} \\ 0 & 0 & p_{1x} & p_{1y} & 0 & 1 & q_{1y} \\ 0 & 0 & 0 & 0 & 0 & 0 & q_{2x} + (k-1)q_{1x} - k \cdot q_{3x} \\ 0 & 0 & p_{2x} & p_{2y} & 0 & 1 & q_{2y} \\ p_{3x} & p_{3y} & 0 & 0 & 1 & 0 & q_{3x} \\ 0 & 0 & p_{3x} & p_{3y} & 0 & 1 & q_{3y} \end{array}\right]$$

Now, we have linearly dependent vectors.
Since we now have a row of 0s, we reached a stopping condition, and since its right-hand side $q_{2x} + (k-1)q_{1x} - k \cdot q_{3x} = 0$ by definition of collinear,
Thus, this system of equations have an infinite number of solutions.

Q.E.D.