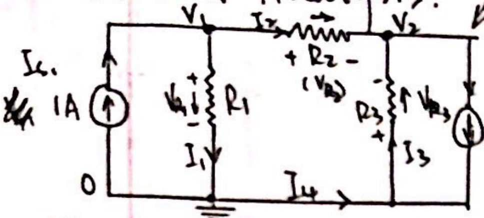
1. Circuit Analysis. Labeling. So $A\vec{x} = \vec{b}$ where



$\vec{x} = [I_1\ I_2\ I_3\ I_4\ V_{R_1}\ V_{R_2}\ V_{R_3}\ V_1\ V_2]^T$.

$I_2 = 1 - I_1$
$I_3 = I_1 + 2$

Using KCL, so:

$$\begin{cases} I_{s1} = 1A = I_4 + I_2 \\ I_2 + I_3 = I_{s2} = 3A \\ I_1 = I_4 + I_{s1} = I_4 + 1A \\ (I_3 = I_4 + I_{s2} = I_4 + 3A). \end{cases} \Rightarrow \begin{cases} I_1 + I_2 = 1A & ① \\ I_2 + I_3 = 3A & ② \\ I_1 - I_4 = 1A & ③ \end{cases}$$

Then, using Ohm's Law, so:

$$\begin{cases} V_{R_1} = R_1 \cdot I_1 = 10I_1 \\ V_{R_2} = R_2 \cdot I_2 = 20I_2 = 20 - 20I_1 \\ V_{R_3} = R_3 \cdot I_3 = 50I_3 = 50I_1 + 100 \\ V_1 - 0 = V_{R_1} \\ V_1 - V_2 = V_{R_2} \Rightarrow V_2 = 30I_1 - 20 \\ V_2 - 0 = -V_{R_3} \Rightarrow V_2 = -50I_1 - 100 \end{cases}$$

and also:

$$\Rightarrow \begin{cases} V_{R_1} - R_1 \cdot I_1 = 0 & ④ \\ V_{R_2} - R_2 \cdot I_2 = 0 & ⑤ \\ V_{R_3} - R_3 \cdot I_3 = 0 & ⑥ \\ V_1 - V_{R_1} = 0 & ⑦ \\ V_1 - V_2 - V_{R_2} = 0 & ⑧ \\ V_2 + V_{R_3} = 0 & ⑨ \end{cases}$$

$V_2 = V_1 - 20I_2$

Using Equations ①-⑨, we can setup $A\vec{x} = \vec{b}$ as:

$I_2 = 1 - I_1$
$I_3 = I_1 + 2$
$I_4 = I_1 - 1$
$V_1 = -10I_1$
$V_2 = 30I_1 - 20$
$= 50I_1 - 100$
$I_1 = -1A$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -R_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -R_2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -R_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_{R_1} \\ V_{R_2} \\ V_{R_3} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

With $R_1 = 10\Omega$

$R_2 = 20\Omega$

$R_3 = 50\Omega$

So,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_{R_1} \\ V_{R_2} \\ V_{R_3} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1 & A \\ 2 & A \\ 1 & A \\ -2 & A \\ -10 & V \\ 40 & V \\ 50 & V \\ -10 & V \\ -50 & V \end{bmatrix}$$