EECS 16A Designing Information Devices and Systems I
Fall 2018 Homework 7

This homework is due October 12, 2018, at 23:59.
Self-grades are due October 16, 2018, at 23:59.

Submission Format

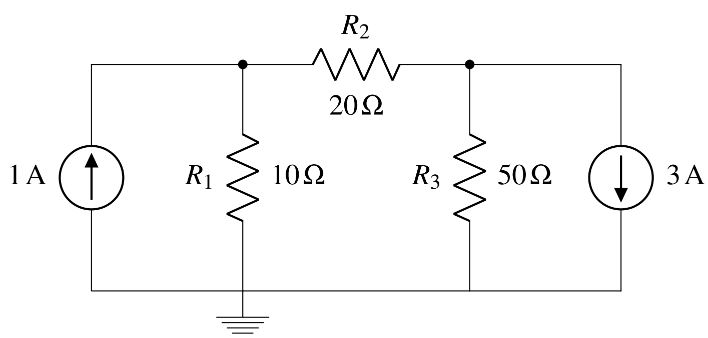
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

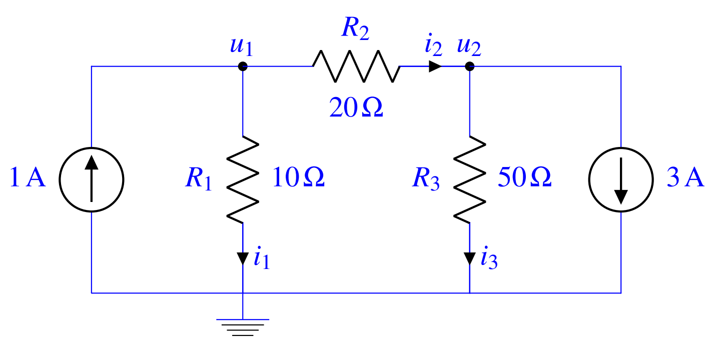
1. Circuit Analysis

Solve the circuit given below for all the currents and all the node voltages.



Solution:

The circuit above only has 3 nodes, one of which is marked as ground. We begin by labeling all of the branch currents and all of the node potentials.



We see that there are 3 unknown currents, i_1, i_2, i_3 and two unknown node potentials u_1, and u_2. Now we write the KCL equations for each of the nodes in the circuit, except for the ground node. For node 1:

$$1\,\mathrm{A} - i_1 - i_2 = 0$$

For node 2:

$$i_2 - i_3 - 3\,\mathrm{A} = 0$$

Next we write all the element equations:

$$\begin{aligned}
(u_1 - 0) = V_{R_1}, V_{R_1} = i_1 R_1 &\implies (u_1 - 0) = i_1 R_1 \implies u_1 - i_1 R_1 = 0 \\
(u_1 - u_2) = V_{R_2}, V_{R_2} = i_2 R_2 &\implies (u_1 - u_2) = i_2 R_2 \implies u_1 - u_2 - i_2 R_2 = 0 \\
(u_2 - 0) = V_{R_3}, V_{R_3} = i_3 R_3 &\implies (u_2 - 0) = i_3 R_3 \implies u_2 - i_3 R_3 = 0
\end{aligned}$$

We can represent this system of equations with the following matrix vector equation:

$$\begin{bmatrix} 1 & 0 & -R_1 & 0 & 0 \\ 1 & -1 & 0 & -R_2 & 0 \\ 0 & 1 & 0 & 0 & -R_3 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 3 \end{bmatrix}$$

Plugging in the values for all of the resistors, and solving, we find the following:

$$\begin{bmatrix} u_1 \\ u_2 \\ i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} -10 \\ -50 \\ -1 \\ 2 \\ -1 \end{bmatrix}$$

2. Cell Phone Battery

As great as smartphones are, one of their main drawbacks is that their batteries don't last a very long time. A Google Pixel, under somewhat regular usage conditions (internet, a few cat videos, etc.) uses 0.3 W of power. We will model the battery as a voltage source (which, as you know, will maintain a voltage across its terminals regardless of current through it) with one caveat: they have a limited amount of charge, or capacity. When the battery runs out of charge, it no longer provides a constant voltage, and your phone dies. Typically, engineers specify battery capacity in terms of mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel's battery has a battery capacity of 2770 mAh and operates at a voltage of 3.8 V.

(a) When a battery's capacity is depleted, it no longer operates as a voltage source. How long will a Pixel's full battery last under regular usage conditions?

Solution:

300 mW of power at 3.8 V is about 78.94 mA of current. A battery that can provide 1 mAh can provide 1 mA for an hour, so our 2770 mAh battery can source 78.94 mA for $\frac{2770\,\mathrm{mAh}}{78.94\,\mathrm{mA}} = 35.1\,\mathrm{h}$, or about a day and a half.

An alternative approach is to say 2770 mAh at 3.8 V is $2770\,\mathrm{mAh} \cdot 3.8\,\mathrm{V} = 10\,526\,\mathrm{mWh}$. 0.3 W is 300 mW, so $\frac{10\,526\,\mathrm{mWh}}{300\,\mathrm{mW}} = 35.1\,\mathrm{h}$ is how long the battery will last.

(b) How many coulombs of charge does the battery contain? Recall that $1\,\mathrm{C} = 1\,\mathrm{A} \times 1\,\mathrm{s}$, which implies that $1\,\mathrm{mC} = 1\,\mathrm{mAs}$. An electron has approximately $1.602 \times 10^{-19}\,\mathrm{C}$ of charge. How many usable electrons worth of charge are contained in the battery when it is fully charged?

Solution:

One hour has 3600 seconds, so the battery's capacity can be written as $2770\,\mathrm{mAh} \times 3600\,\frac{\mathrm{s}}{\mathrm{h}} = 9.972 \times 10^6\,\mathrm{mAs}$. To find this in coulombs, divide it by 1000 to get $9972\,\mathrm{C}$.

An electron has a charge of approximately $1.602 \times 10^{-19}\,\mathrm{C}$, so $9972\,\mathrm{C}$ is $\frac{9972\,\mathrm{C}}{1.602 \times 10^{-19}\,\mathrm{C}} \approx 6.225 \times 10^{22}$ electrons. That's a lot!

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Solution:

The battery is rated for $2770\,\mathrm{mAh}$ at $3.8\,\mathrm{V}$, which gives $2770\,\mathrm{mAh} \cdot 3.8\,\mathrm{V} = 10\,526\,\mathrm{mWh}$.

A joule is equivalent to a watt-second, and there are 3600 seconds in an hour, so our battery has $10\,526\,\mathrm{mWh} \cdot 3600\,\frac{\mathrm{s}}{\mathrm{h}} = 37\,893\,600\,\mathrm{mJ}$, or $37\,893.6\,\mathrm{J}$.

(d) Suppose PG&E charges \$0.12 per kWh. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?

Solution:

$2770\,\mathrm{mAh}$ at $3.8\,\mathrm{V}$ is $2770\,\mathrm{mAh} \cdot 3.8\,\mathrm{V} = 10\,526\,\mathrm{mWh}$, or $0.010\,526\,\mathrm{kWh}$. At \$0.12 per kWh, that is $\$0.12 \cdot 0.010526$ per day, or $\$0.12 \cdot 0.010526 \cdot 31 = \$0.0392$, or about 4 cents a month. Compare that to your cell phone data bill! Whew!

(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). The circuitry is also used to transfer power into the chemical reactions that store the energy. We will model this internal circuitry as being one resistor with resistance R_{bat}, which is typically a small, non-negative resistance. Furthermore, we'll assume that all the energy dissipated across R_{bat} goes to recharging the battery. Suppose the wall plug and wire can be modeled as a $5\,\mathrm{V}$ voltage source and $200\,\mathrm{m\Omega}$ resistor, as pictured in Figure 1. What is the power dissipated across R_{bat} for $R_{\mathrm{bat}} = 1\,\mathrm{m\Omega}$, $1\,\Omega$, and $10\,\mathrm{k\Omega}$? How long will the battery take to charge for each of those values of R_{bat}?

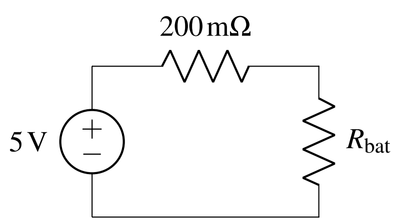


Figure 1: Model of wall plug, wire, and battery.

Solution:

The energy stored in the battery is $2770\,\mathrm{mAh}$ at $3.8\,\mathrm{V}$, which is $2.77\,\mathrm{Ah} \cdot 3.8\,\mathrm{V} = 10.526\,\mathrm{Wh}$. We can find the time to charge by dividing this energy by power in W to get time in hours.

For $R_{\mathrm{bat}} = 1\,\mathrm{m\Omega}$, the total resistance seen by the battery is $1\,\mathrm{m\Omega} + 200\,\mathrm{m\Omega} = 201\,\mathrm{m\Omega}$ (because the wire and R_{bat} are in series), so by Ohm's law, the current is $\frac{5\,\mathrm{V}}{0.201\,\Omega} = 24.88\,\mathrm{A}$. The voltage drop across R_{bat} is (again by Ohm's law) $24.88\,\mathrm{A} \cdot 0.001\,\Omega = 0.024\,88\,\mathrm{V}$. Then power is $0.024\,88\,\mathrm{V} \cdot 24.88\,\mathrm{A} = 0.619\,\mathrm{W}$, and the total time to charge the battery is $\frac{10.526\,\mathrm{Wh}}{0.619\,\mathrm{W}} = 17.00\,\mathrm{h}$.

Similarly, for $1\,\Omega$, the total resistance seen by the battery is $1\,\Omega + 0.2\,\Omega = 1.2\,\Omega$, the current through the battery is $\frac{5\,\mathrm{V}}{1.2\,\Omega} = 4.167\,\mathrm{A}$, and the voltage across the battery is by Ohm's law $4.167\,\mathrm{A} \cdot 1\,\Omega = 4.167\,\mathrm{V}$.

Then the power is $4.167\,\mathrm{A} \cdot 4.167\,\mathrm{V} = 17.36\,\mathrm{W}$, and the total time to charge the battery is $\frac{10.526\,\mathrm{Wh}}{17.36\,\mathrm{W}} = 0.606\,\mathrm{h}$, about $36\,\mathrm{min}$.

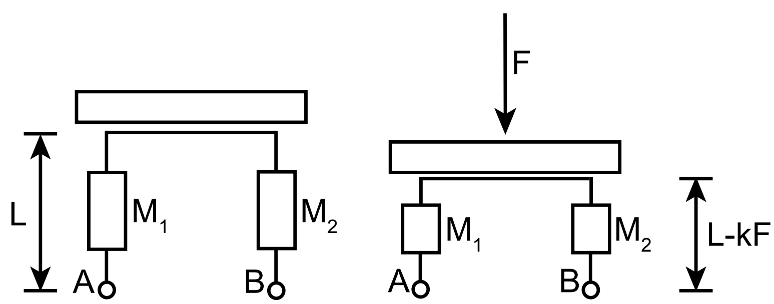
For $10\,\mathrm{k\Omega}$, the total resistance seen by the battery is $10\,000\,\Omega + 0.2\,\Omega = 10\,000.2\,\Omega$, the current through the battery is $\frac{5\,\mathrm{V}}{10\,000.2\,\Omega} \approx 0.5\,\mathrm{mA}$, and the voltage across the battery is by Ohm's law $0.5\,\mathrm{mA} \cdot 10\,\mathrm{k\Omega} \approx 5\,\mathrm{V}$ (up to 2 significant figures). Then the power is $5\,\mathrm{V} \cdot 0.5\,\mathrm{mA} = 2.5\,\mathrm{mW}$, and the total time to charge the battery is $\frac{10.526\,\mathrm{Wh}}{0.0025\,\mathrm{W}} = 4210\,\mathrm{h}$.

3. Fruity Fred

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EE16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (meters) and cross-sectional area A_c (meters^2), which are made of a material with resistivity ρ. He knows that the length of these bars decreases by k meters per Newton of force applied, while the cross-sectional area remains constant.

He builds his scale as shown below, where the top of the bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons), causing the length to decrease by kF meters. Fred's mangoes are not very heavy, so $L \gg kF$.



(a) Let R_{AB} be the resistance between nodes A and B. Write an expression for R_{AB} as a function of A_c, L, ρ, F, and k.

Solution:

The length of each spring as a function of F is $L - kF$.

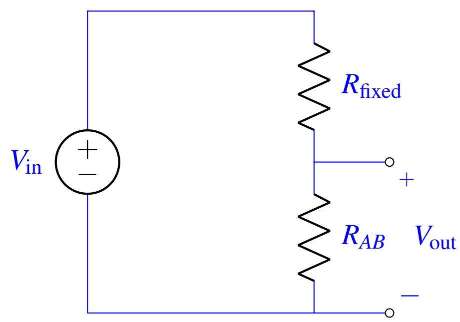
The combination of R_1 and R_2 has a resistance $R_{AB} = R_1 + R_2 = \frac{2\rho(L-kF)}{A_c}$.

(b) Fred's scale design is such that the resistance R_{AB} changes depending on how much weight is placed on it. However, he really wants to measure a voltage rather than a resistance.

Design a circuit for Fred that outputs a voltage that is some function of the weight. Your circuit should include R_{AB}, and you may use any number of voltage sources and resistors in your design. Be sure to label where the voltage should be measured in your circuit. Also provide an expression relating the output voltage of your circuit to the force applied on the scale.

Solution:

One possible solution: use a voltage divider.



$$V_{\text{out}} = \frac{R_{AB}}{R_{\text{fixed}} + R_{AB}} V_{\text{in}}$$

$$V_{\text{out}} = \frac{\frac{2\rho(L-kF)}{A_c}}{R_{\text{fixed}} + \frac{2\rho(L-kF)}{A_c}} V_{\text{in}} = \frac{2\rho(L-kF)}{R_{\text{fixed}}A_c + 2\rho(L-kF)} V_{\text{in}}$$

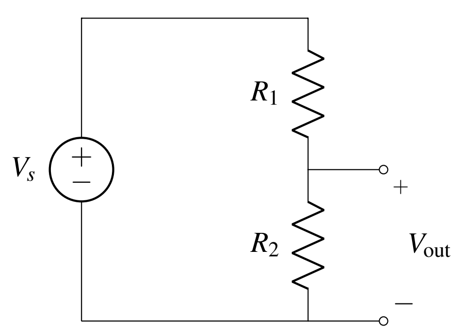
4. Temperature Sensor

Measuring quantities in the physical world is the job of sensors. This means somehow extracting that information from the world and then converting it into a form that can be observed and processed. Electrical circuits can be very useful for doing this.

For most materials, resistance increases with increasing temperature; that is, a resistor has higher resistance when it is hot than when it is cold. This is often an annoyance to circuit designers who want their circuits to work the same way at different temperatures, but this fact can also be useful. It allows us to convert temperature, a "physical" quantity, into resistance, an "electrical" quantity, to build an electronic thermometer.

In this problem, we are going to explore how effective a particular circuit made out of various types of resistors is at allowing us to measure temperature.

(a) Let's begin by analyzing a common topology, the voltage divider shown below. Find an equation for the voltage V_{out} in terms of R_1, R_2, and V_s.

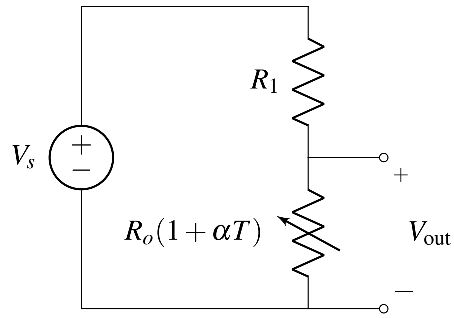


Solution:

We recognize that this circuit is a voltage divider, we can directly write:

$$V_{\text{out}} = \frac{R_2}{R_1 + R_2} V_s$$

(b) Now let's suppose that R_1 is an ideal resistor that does not depend on temperature, but R_2 is a temperature-dependent resistor whose resistance R is set by $R = R_o(1+\alpha T)$, where T is the absolute temperature. Find an equation for the temperature T in terms of the voltage V_{out}, V_s, R_1, R_o, and α.



Solution:

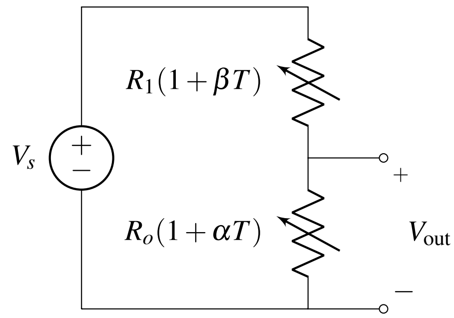
Using the relationship from the earlier part:

$$V_{\text{out}} = \frac{R_o(1+\alpha T)}{R_1 + R_o(1+\alpha T)} V_s$$

$$R_1 V_{\text{out}} + R_o V_{\text{out}} + R_o \alpha T V_{\text{out}} = R_o V_s + R_o \alpha T V_s$$

$$T = \frac{(R_1+R_o)V_{\text{out}} - R_o V_s}{R_o \alpha (V_s - V_{\text{out}})}$$

(c) It turns out that almost all resisitors have some temperature dependence. Consider the same circuit as before, but now, R_1' has a temperature dependence given by $R_1' = R_1(1+\beta T)$. Find an equation for the temperature T in terms of the voltage V_{out}, R_1, R_o, V_s, α, and β.



Solution:

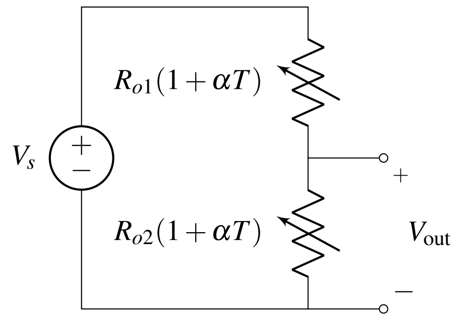
Once again using the equation for the voltage divider:

$$V_{\text{out}} = \frac{R_o(1+\alpha T)}{R_1(1+\beta T) + R_o(1+\alpha T)} V_s$$

$$R_1 V_{\text{out}} + R_1 \beta T V_{\text{out}} + R_o V_{\text{out}} + R_o \alpha T V_{\text{out}} = R_o V_s + R_o \alpha T V_s$$

$$T = \frac{(R_1+R_o)V_{\text{out}} - R_o V_s}{R_o \alpha (V_s - V_{\text{out}}) - R_1 \beta V_{\text{out}}}$$

(d) Your colleague who has not taken EE16A thinks that they can improve this circuit's ability to measure temperature by making both resistors depend on temperature in the same way. He hence came up with the circuit shown below, where both R_1 and R_2 have nominally different values, but both vary with temperature as a function of $(1+\alpha T)$. Can this circuit be used to measure temperature? Why or why not?



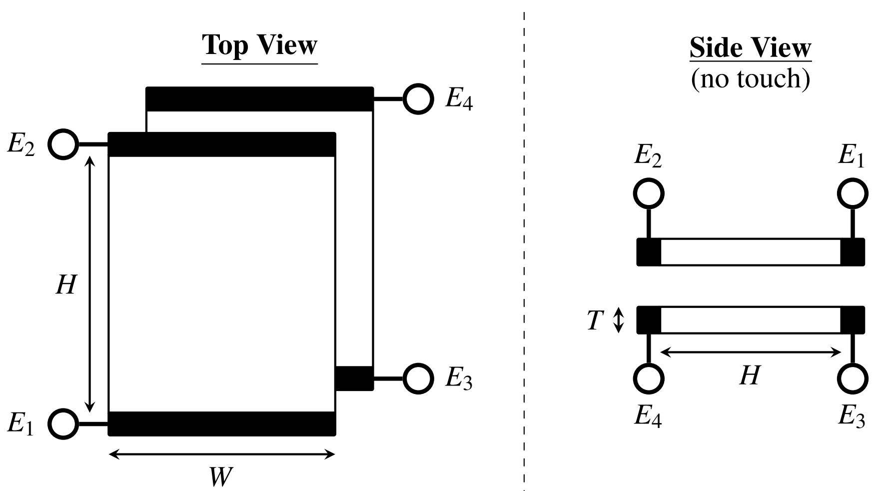
Solution: Using the equation for a voltage divider:

$$V_{\text{out}} = \frac{R_{o2}(1+\alpha T)}{R_{o1}(1+\alpha T)+R_{o2}(1+\alpha T)}V_s = \frac{R_{o2}}{R_{o1}+R_{o2}}V_s$$

Notice this circuit cannot be used to measure temperature because the output voltage is independent of temperature.

5. Multitouch Resistive Touchscreen

In this problem, we will look at a simplified version of the multitouch resistive touchscreen. In particular, rather than measuring the position of two potential touch points in both dimensions (i.e. a pair of coordinates (x_1, y_1) and (x_2, y_2) corresponding to two touch positions), let's think about a version where we are interested in measuring only the vertical position of the two touch points (i.e. y_1 and y_2). Therefore, unlike the touchscreens we looked at in class, both of the resistive plates (i.e. both the top and the bottom plate) would have conductive strips placed along their top and bottom edges, as shown below.



(a) Assuming that both of the plates are made out of a material with $\rho = 1\,\Omega\,\text{m}$ and that the dimensions of the plates are $W = 3\,\text{cm}$, $H = 12\,\text{cm}$, and $T = 1\,\text{mm}$, with no touches at all, what is the resistance between terminals E_1 and E_2 (which would be the same as the resistance between terminals E_3 and E_4)?

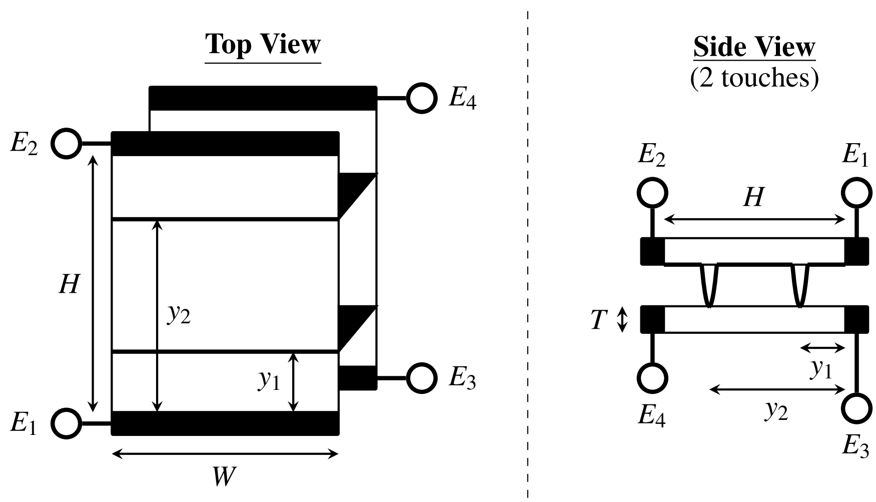
Solution:

$$R = \rho \cdot \frac{L}{A} \implies R_{E1-E2} = \rho\left(\frac{H}{W \cdot T}\right)$$

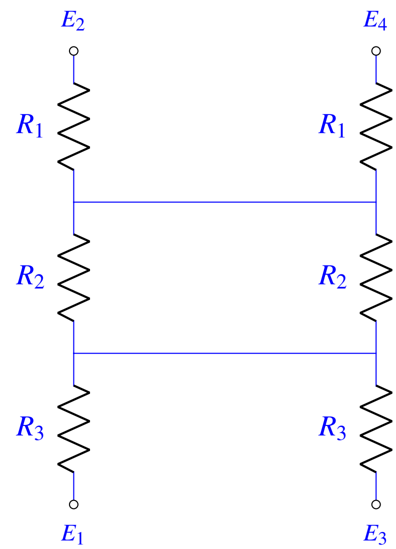
$$R_{E1-E2} = 1\,\Omega\,\text{m}\left(\frac{12\times10^{-2}\,\text{m}}{3\times10^{-2}\cdot 1\times10^{-3}\,\text{m}}\right)$$

$$R_{E1-E2} = 4\,\text{k}\Omega$$

(b) Now let's look at what happens when we have two touch points. Let's assume that at wherever height the touch occurs, a perfect contact is made between the top plate and the bottom plate along the entire width of the plates (i.e. you don't have to worry about any lateral resistors), but that otherwise none of the electrical characteristics of the plates change. Defining the bottom of the plate as being $y = 0\,\text{cm}$ (i.e. a touch at E_1 would be at $y = 0\,\text{cm}$), let's assume that the two touches happen at $y_1 = 3\,\text{cm}$ and $y_2 = 7\,\text{cm}$ and that your answer to part (a) was $8\,\text{k}\Omega$ (which may or may not be the right answer). Draw a model with 6 resistors that captures the electrical connections between E_1, E_2, E_3, and E_4 and calculate their resistances. Note that for clarity, the system has been redrawn below to depict this scenario.

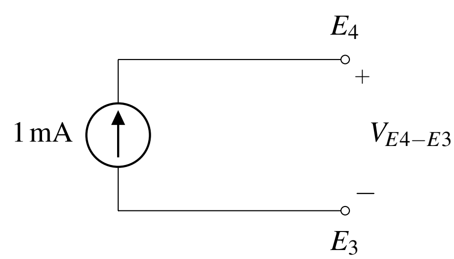


Solution:

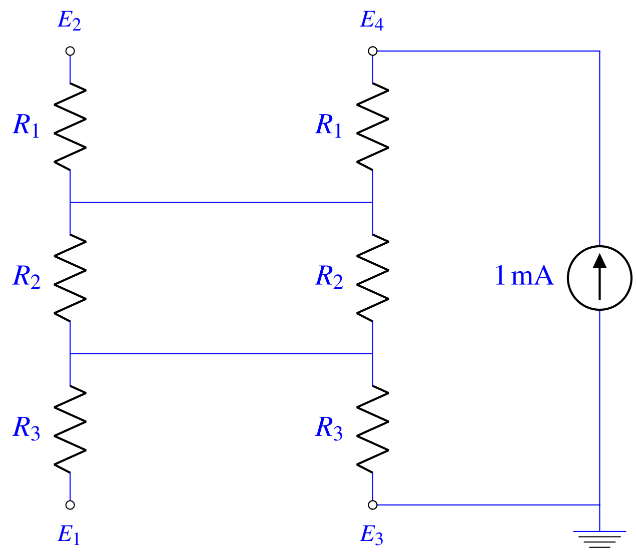


$$R_3 = \frac{3\,\text{cm}}{12\,\text{cm}} \cdot R_{E2-E1} = 2\,\text{k}\Omega$$
$$R_2 = \frac{7\,\text{cm} - 3\,\text{cm}}{12\,\text{cm}} \cdot R_{E2-E1} = 2.667\,\text{k}\Omega$$
$$R_1 = \frac{12\,\text{cm} - 7\,\text{cm}}{12\,\text{cm}} \cdot R_{E2-E1} = 3.334\,\text{k}\Omega$$

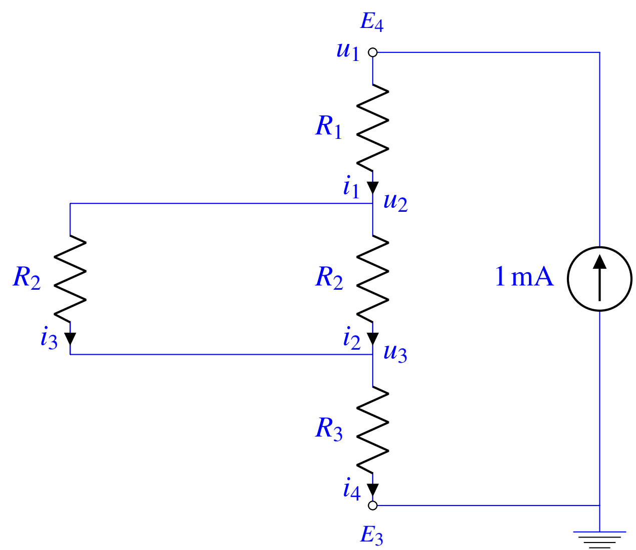
(c) Using the same assumptions as part (b), if you drove terminals E_3 and E_4 with a 1 mA current source (as shown below) but left terminals E_1 and E_2 open-circuited, what is the voltage you would measure across $E_4 - E_3$ (i.e. V_{E4-E3})?



Solution: We can represent this setup with the circuit shown below.



Notice in the above circuit that no current can flow through resistors R_1 and R_3, since E_2 and E_1 are open circuits. We can then reduce the circuit to the circuit shown below:



From the above circuit, we see we have 3 unknown nodes and 4 unknown currents. We begin by writing KCL equations for the each of the nodes. For Node u_1:

$$1\,\text{mA} - \text{i}_1 = 0$$

For Node u_2:

$$i_1 - i_2 - i_3 = 0$$

For Node u_3:

$$i_2 + i_3 - i_4 = 0$$

Notice from the equation for node u_1, we can calculate the value of i_1. In order to reduce the size of our matrix at the last step, we're going to apply this simplification moving forward. From this we see

$i_1 = i_4 = 1\,\text{mA}$, reducing our system to two unknowns and one equation

$$1\,\text{mA} = i_2 + i_3$$

Now, we continue with equations for all the elements:

$$\begin{aligned}
u_1 - u_2 = V_{R_1}, V_{R_1} = R_1 \cdot 1\,\text{mA} &\implies u_1 - u_2 = R_1 \cdot 1\,\text{mA} \\
u_2 - u_3 = V_{R_{21}}, V_{R_{21}} = i_2 R_2 &\implies u_2 - u_3 = i_2 R_2 \\
u_2 - u_3 = V_{R_{22}}, V_{R_{22}} = i_3 R_2 &\implies u_2 - u_3 = i_3 R_2 \\
u_3 - 0 = V_{R_3}, V_{R_3} = R_3 \cdot 1\,\text{mA} &\implies u_3 - 0 = R_3 \cdot 1\,\text{mA}
\end{aligned}$$

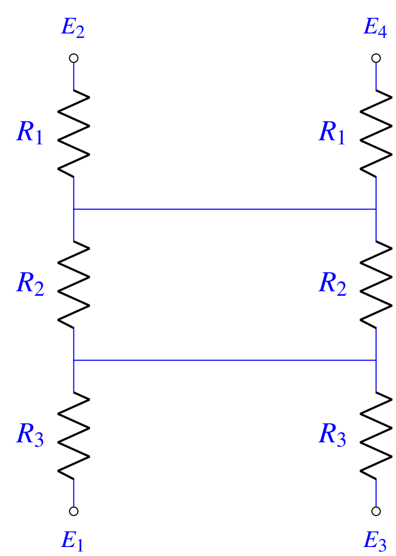
At this point we have 5 unknowns (i_2, i_2, u_1, u_2, and u_3) and 5 equations. We could setup the matrix and solve for all the unknowns. However, before we proceed, we will apply one more simplification. Notice that $i_2 = i_3 = 0.5\,\text{mA}$ from the above equations. This removes two more unknowns and we have reduced the system down to three unknowns. We can represent the system with the matrix vector product shown below:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} R_1 \cdot 1\,\text{mA} \\ R_2 \cdot 0.5\,\text{mA} \\ R_3 \cdot 1\,\text{mA} \end{bmatrix} = \begin{bmatrix} 2\,\text{V} \\ 1.333\,\text{V} \\ 3.33\,\text{V} \end{bmatrix}$$

Solving the above system, we find $V_{E4-E3} = u_1 - 0 = 6.667\,\text{V}$.

(d) Now let's try to generalize the situation by assuming that the two touches can happen at any two arbitrary points y_1 and y_2, but with y_1 defined to always be less than y_2 (i.e. y_1 is always the bottom touch point). Leaving the setup the same as in part (c) except for the arbitrary y_1 and y_2, by measuring only the voltage between E_4 and E_3, what information can you extract about the two touch positions? Please be sure to provide an equation relating V_{E4-E3} to y_1 and y_2 as a part of your answer, and note that you may want to redraw the model from part (b) to help you with this.

Solution:



For general

$$R_3 = \frac{y_1}{12\,\text{cm}} \cdot 8\,\text{k}\Omega$$
$$R_2 = \frac{y_2 - y_1}{12\,\text{cm}} \cdot 8\,\text{k}\Omega$$
$$R_1 = \frac{12\,\text{cm} - y_2}{12\,\text{cm}} \cdot 8\,\text{k}\Omega$$

Notice the circuit has not changed from the previous setup. Thus we only need to plug in these values into our eariler matrix:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} R_1 \cdot 1\,\text{mA} \\ R_2 \cdot 0.5\,\text{mA} \\ R_3 \cdot 1\,\text{mA} \end{bmatrix} = \begin{bmatrix} \frac{y_1}{12\,\text{cm}} \cdot 8\,\text{k}\Omega \cdot 1\,\text{mA} \\ \frac{y_2 - y_1}{12\,\text{cm}} \cdot 8\,\text{k}\Omega \cdot 0.5\,\text{mA} \\ \frac{12\,\text{cm} - y_2}{12\,\text{cm}} \cdot 8\,\text{k}\Omega \cdot 1\,\text{mA} \end{bmatrix} = \begin{bmatrix} \frac{y_1}{12\,\text{cm}} 8\,\text{V} \\ \frac{y_2 - y_1}{12\,\text{cm}} 4\,\text{V} \\ \frac{12\,\text{cm} - y_2}{12\,\text{cm}} 8\,\text{V} \end{bmatrix}$$

Notice that if we solve the above system for $u_1 = V_{E4-E3}$,

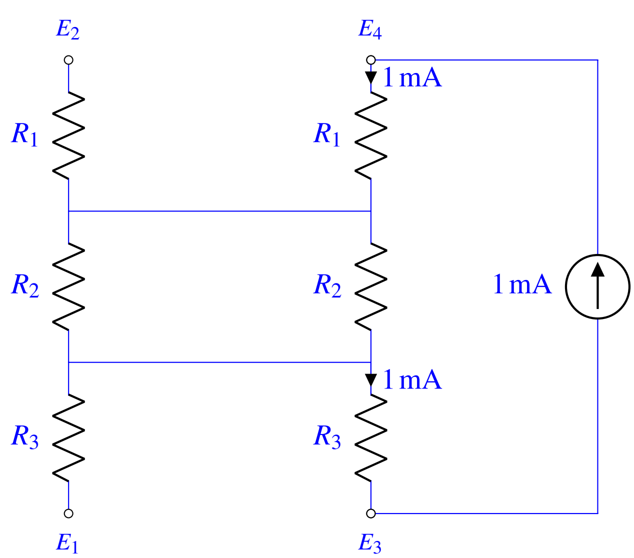
$$V_{E4-E3} = 1\,\text{mA} \cdot \frac{12\,\text{cm} - \frac{y_2 - y_1}{2}}{12\,\text{cm}} \cdot 8\,\text{k}\Omega = \frac{12\,\text{cm} - \frac{y_2 - y_1}{2}}{12\,\text{cm}} \cdot 8\,\text{V}$$

Notice we can only use this measurement to tell us the difference between the two touches, not the position of each touch.

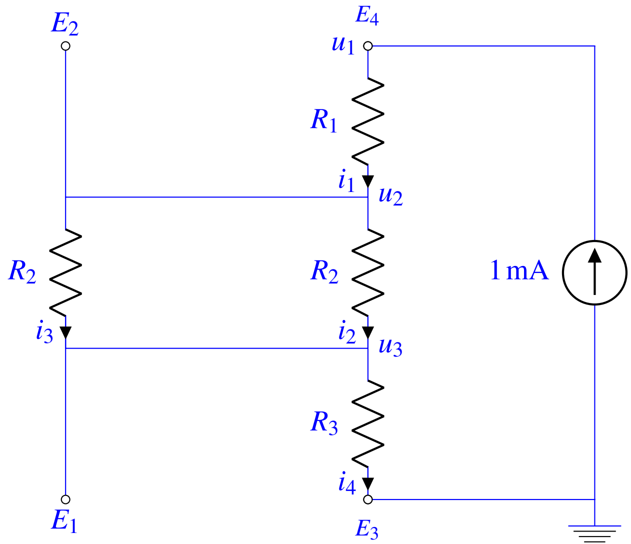
(e) One of your colleagues claims that by measuring the appropriate voltages, not only can they extract what both y_1 and y_2 are in this system, but they can even do so by formulating a system of three independent voltage equations related to y_1 and y_2. As we will see later, this will allow us to gain some robustness to noise in the voltage measurements.

In order to facilitate this, write equations relating V_{E4-E2} and V_{E1-E3} to y_1 and y_2. (The third voltage we'll use is V_{E4-E3}, which you should have already derived an equation for in the previous part of the problem.)

Solution:



Notice that R_1 in the left branch above has no current through it, therefore the potential on both sides must be equal. A similar argument can be applied to R_3 on the left branch. Thus we can redraw the circuit as shown below:



Using this drawing, and the matrix equations dervied earlier we see:

$$u_1 - u_2 = V_{E4-E2} = I \cdot R_1 = \frac{12\,\text{cm} - y_2}{12\,\text{cm}} \cdot 8\,\text{V}$$
$$u_3 = V_{E1-E3} = I \cdot R_3 = \frac{y_1}{12\,\text{cm}} \cdot 8\,\text{V}$$

6. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID's. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.