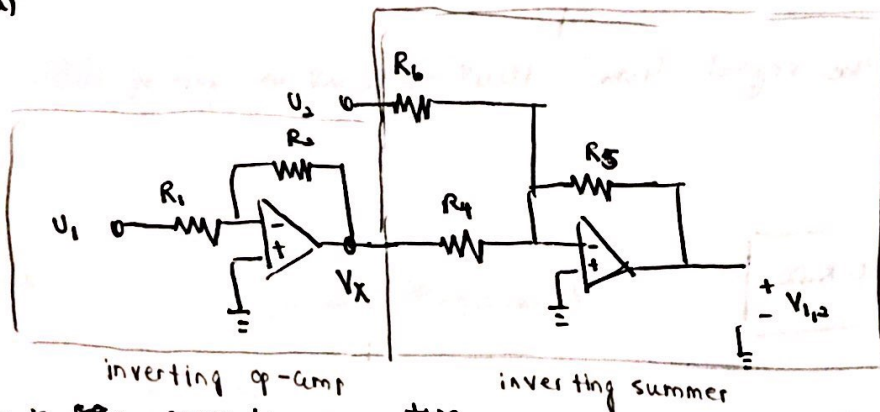


3. a)

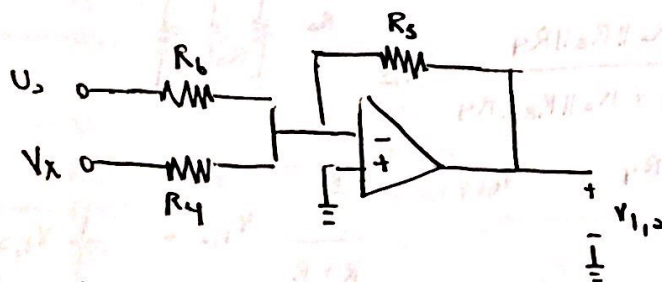


This is a cascade of ~~two~~ ^{two} op-amp circuit blocks: an inverting op amp and an inverting summer.

We can simplify by looking at the first block and solving for V_x .

$$V_x = -\frac{R_2}{R_1} U_1 \quad (\text{basic inverting op-amp})$$

We can then redraw the circuit as



Doing KCL at node v^- :

$$\frac{v^- - U_2}{R_6} + \frac{v^- - V_x}{R_4} + \frac{v^- - V_{1,2}}{R_5} = 0$$

$v^- = 0$ by Golden Rules

$$\begin{aligned} \Rightarrow V_{1,2} &= -\left(\frac{R_5}{R_4} V_x + \frac{R_5}{R_3} U_2 \right) \\ &= -\left(\frac{R_5}{R_4} \left(-\frac{R_2}{R_1} U_1 \right) + \frac{R_5}{R_3} U_2 \right) \end{aligned}$$

We are given $R_1 = R_2$, $R_3 = R_4$

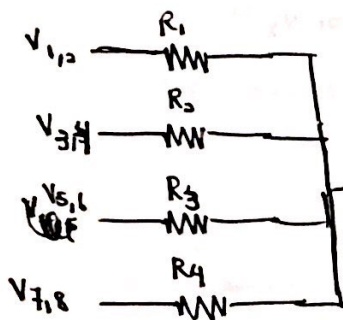
$$\Rightarrow V_{1,2} = \left(\frac{R_5}{R_4} (U_2 - U_1) \right) \quad \text{负号反了}$$

b) From part (a), we have $V_{1,2} = \frac{R_5}{R_4} (V_2 - V_1)$.

We want to amplify the signal from $10\mu V$ to $10mV \Rightarrow$ gain of 1000.
Therefore $\frac{R_5}{R_4} = 1000$.

$$R_4 = 100\Omega \Rightarrow R_5 = 100k\Omega$$

c) We want to build a circuit that takes the average of all 4 signals, and then amplifies by ~~25~~ 25. We can average by using the summing circuit from the "cookbook".

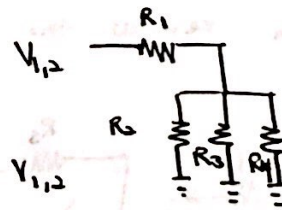


We can solve for V_{out} in two ways:

Method 1. Use superposition and voltage dividers.

Null $V_{3,4}, V_{5,6}, V_{7,8}$:

$$V_{out,1,2} = \frac{R_2 \parallel R_3 \parallel R_4}{R_1 + R_2 \parallel R_3 \parallel R_4} V_{1,2}$$



$$\text{If } R_1 = R_2 = R_3 = R_4 : V_{out,1,2} = \frac{\frac{R}{3}}{R + \frac{R}{3}} V_{1,2} = \frac{1}{4} V_{1,2}$$

Repeating this process for $V_{out,3,4}, V_{out,5,6}, V_{out,7,8}$

$$\text{we get } V_{out} = \frac{1}{4} [V_{1,2} + V_{3,4} + V_{5,6} + V_{7,8}]$$

This performs the averaging operation we wanted!

Method 2:

$$I_{out} = \frac{V_{1,2}}{R_1} + \frac{V_{3,4}}{R_2} + \frac{V_{5,6}}{R_3} + \frac{V_{7,8}}{R_4}$$

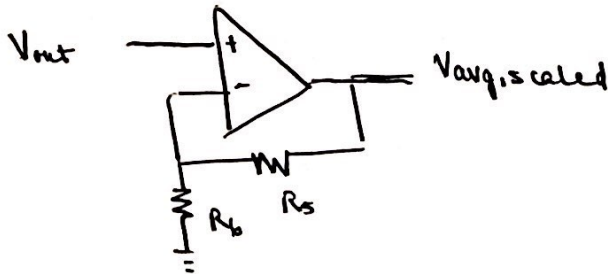
$$R_{out} = R_1 \parallel R_2 \parallel R_3 \parallel R_4$$

$$V_{out} = I_{out} R_{out} = \left[\frac{V_{1,2}}{R_1} + \frac{V_{3,4}}{R_2} + \frac{V_{5,6}}{R_3} + \frac{V_{7,8}}{R_4} \right] \left[\frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}} \right]$$

$$\text{If } R_1 = R_2 = R_3 = R_4$$

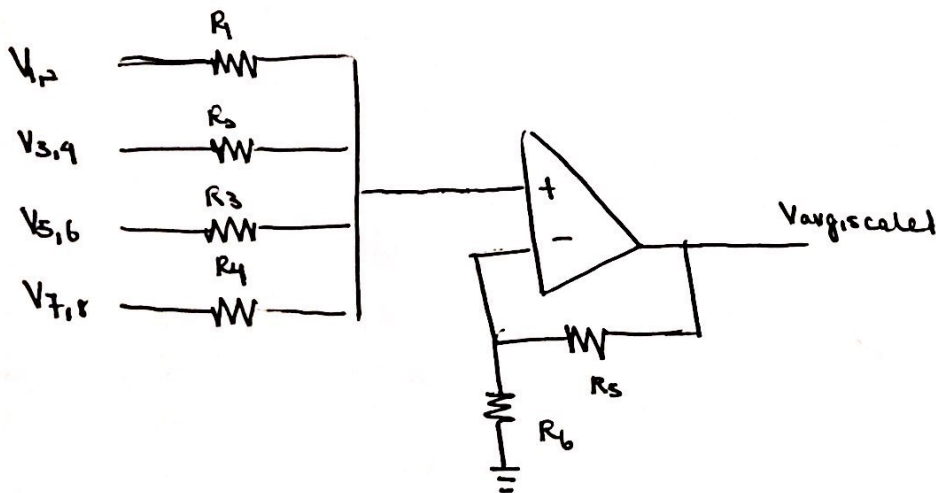
$$V_{out} = \frac{1}{R} [V_{1,2} + V_{3,4} + V_{5,6} + V_{7,8}] \left[\frac{1}{\frac{4}{R}} \right] = \frac{1}{4} [V_{1,2} + V_{3,4} + V_{5,6} + V_{7,8}]$$

Now we need a gain of 25. We can simply feed V_{out} into a non-inverting op amp configuration.



$$\frac{V_{avg,scaled}}{V_{out}} = 1 + \frac{R_5}{R_6} = 25 \Rightarrow \frac{R_5}{R_6} = 24$$

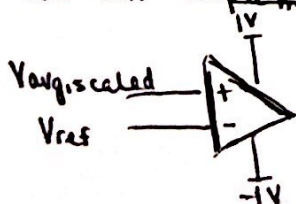
So, the overall circuit looks like this:



$$R_1 : R_2 : R_3 : R_4 : R_5 : R_6 = 1 : 1 : 1 : 1 : 24 : 1$$

($R_1 - R_4$ need to be the same, $R_5 : R_6 = 24 : 1$)

- d) The max value $V_{avg,scaled}$ will reach when AWAKE is 125 mV.
 The max value $V_{avg,scaled}$ will reach when SLEEPY is 375 mV.
 We can ~~implement~~ use a comparator.



$$125 \text{ mV} < V_{ref} < 375 \text{ mV}$$