1 Finding Null Spaces

(a) 3

For any 3x5 matrix, the column vectors are 3x1 vectors, so they would at most span $(\mathbb{R}^3, \mathbb{R})$. Moreover, we have that $[1\ 0\ 0]^T, [0\ 1\ 0]^T, [0\ 0\ 1]^T$ is a Basis for $\mathbb{R}^3$, by definition of Basis, so this means that the maximum possible number of linearly independent column vectors is 3.

(b). $\text{Colspace}(A) = \text{span}\left(\boxed{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}}\right) = (\mathbb{R}^2, \mathbb{R})$.

$\boxed{2}$ unique vectors are required to span the column space of A.

(c). By definition, $A\vec{x}=0$. so $\begin{bmatrix}1&1&0&-2&3\\0&0&2&-2&2\\0&0&0&0&0\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$, which turn into

this augmented matrix: $\left[\begin{array}{ccccc|c}1&1&0&-2&3&0\\0&0&2&-2&2&0\\0&0&0&0&0&0\end{array}\right]$. Divide R_2 by 2: $\left[\begin{array}{ccccc|c}1&1&0&-2&3&0\\0&0&1&-1&1&0\\0&0&0&0&0&0\end{array}\right]$

So we have: $\begin{cases} x_1+x_2-2x_4+3x_5=0 \\ x_3-x_4+x_5=0\end{cases} \Rightarrow \begin{aligned} x_1 &= -x_2+2x_4-3x_5 \\ x_3 &= x_4-x_5\end{aligned}$

$$\text{so, } \vec{x} = \begin{bmatrix}-x_2+2x_4-3x_5\\x_2\\x_4-x_5\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}-1\\1\\0\\0\\0\end{bmatrix}x_2 + \begin{bmatrix}2\\0\\1\\1\\0\end{bmatrix}x_4 + \begin{bmatrix}-3\\0\\-1\\0\\1\end{bmatrix}x_5$$

Thus, vectors that span the null space of A: $\boxed{\left(\begin{bmatrix}-1\\1\\0\\0\\0\end{bmatrix}, \begin{bmatrix}2\\0\\1\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\0\\-1\\0\\1\end{bmatrix}\right)}$.

and the dimension is $\boxed{3}$.

(e). By definition, $C\vec{x}=0$, so $\begin{bmatrix}2&-4&4&8\\1&-2&3&6\\2&-4&5&10\\3&-6&7&14\end{bmatrix} \cdot \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$. equivalent to this augmented matrix:

$\left[\begin{array}{cccc|c}2&-4&4&8&0\\1&-2&3&6&0\\2&-4&5&10&0\\3&-6&7&14&0\end{array}\right]$. R_4: $3 \cdot R_3 - 2R_4$. R_3: $2 \cdot R_2 - R_3$ R_2: Subtract $\frac{1}{2} \cdot R_1$, which gives us $\Rightarrow \left[\begin{array}{cccc|c}2&-4&4&8&0\\0&0&1&2&0\\0&0&1&2&0\\0&0&1&2&0\end{array}\right]$

Then, R_1: Divide by 2, R_3: Subtract R_2, R_4: Subtract R_2. $\Rightarrow \left[\begin{array}{cccc|c}1&-2&2&4&0\\0&0&1&2&0\\0&0&0&0&0\\0&0&0&0&0\end{array}\right]$, So: $\begin{aligned} x_1-2x_2+2x_3+4x_4&=0 \\ x_3-2x_4&=0.\end{aligned}$

so, $x_3 = 2x_4$ and so $x_1 = 2x_2 - 2x_3 + 4x_4 = 2x_2 - 4x_4 + 4x_4 = 2x_2$.

$$\text{So, } \vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = \begin{bmatrix}2x_2\\x_2\\2x_4\\x_4\end{bmatrix} = \begin{bmatrix}2\\1\\0\\0\end{bmatrix}x_2 + \begin{bmatrix}0\\0\\2\\1\end{bmatrix}x_4.$$

Thus, the vectors that span N(C) is . $\boxed{\left(\begin{bmatrix}2\\1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\2\\1\end{bmatrix}\right)}$