3. Traffic Flows

(a). Given that $\begin{cases} t_1 + t_3 = 0 & (1) \\ t_2 - t_1 = 0 & (2) \\ -t_3 - t_2 = 0. & (3) \end{cases}$ and that $t_1 = 10$, so substituting into Eq (2) and (3),

We have that $\boxed{t_2 = 10, \ t_3 = -10}$.

(b). Similar to part (a), we obtain this set of equations from the graph: $\boxed{\begin{cases} t_1 + t_3 - t_4 = 0 & (1) \\ -t_1 + t_2 = 0 & (2) \\ -t_2 - t_3 + t_5 = 0 & (3) \\ t_4 - t_5 = 0. & (4) \end{cases}}$

→ $\boxed{\text{Yes.}}$ it is possible with the Berkeley suggestion because, we can rewrite Eq (1), (2), (4) so that $t_2 = t_1$, $t_3 = -t_1 + t_4$, $t_5 = t_4$.

Thus, given the measurements of t_1 and t_4, we can determine all traffic flows, $[t_1, t_2, t_3, t_4, t_5]^T$.

→ $\boxed{\text{No}}$ it's not possible with the Stanford suggestion because we can write our system of linear equations as $\begin{cases} t_3 - t_4 = -t_1 \\ 0 = -t_1 + t_2 \\ -t_3 + t_5 = t_2 \\ t_4 - t_5 = 0. \end{cases} \Leftrightarrow \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} -t_1 \\ -t_1 + t_2 \\ t_2 \\ 0 \end{bmatrix}$

which can be turned into: $\left[\begin{array}{ccc|c} 1 & -1 & 0 & -t_1 \\ 0 & 0 & 0 & -t_1+t_2 \\ -1 & 0 & 1 & t_2 \\ 0 & 1 & -1 & 0 \end{array}\right]$ If only the measurements t_1, t_2 are given.

R_3: Add R_1.

Swap R_2 and R_4.

$\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & -t_1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & -t_1+t_2 \\ 0 & 0 & 0 & -t_1+t_2 \end{array}\right]$ R_2: Add R_3. $\Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 0 & -t_1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -t_1+t_2 \\ 0 & 0 & 0 & -t_1+t_2 \end{array}\right]$ Now we have two rows of 0s.

If $(-t_1+t_2) \neq 0$, then we have no solutions; if $(-t_1+t_2) = 0$, then we only have two independent equations with 3 variables, which means that we can't deduce a unique solution to determine all traffic flows $[t_1, t_2, t_3, t_4, t_5]^T$.

(c).

From the graph, we can construct and from the system of equation in part (b), so, $\boxed{B = \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}}$

(d). From the system of equations in part (b), so $\begin{cases} t_3 = -t_1 + t_4 \\ t_2 = t_1. \\ t_5 = t_4. \end{cases}$ and with $t_1 = \alpha$, $t_4 = \beta$

Thus, $\vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{bmatrix} = \begin{bmatrix} t_1 \\ t_1 \\ -t_1 + t_4 \\ t_4 \\ t_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \alpha + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} \beta$ Thus, the subspace can

be expressed as $\boxed{\left(\text{span}\left(\begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}\right), \mathbb{R}\right)}$. Justification provided in part (b).

(e). For $B\vec{t}=\vec{0}$, we can transform this system of linear equations into an augmented matrix with our calculated matrix B and the definition of matrix-vector multiplication, So:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

R_3: Add R_1 and R_2.
then R_2: Add R_1 $\Rightarrow$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

R_4: Add R_3 $\Rightarrow$.
R_3: $\times(-1)$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

So, t_3, t_5 are free variables and we have that

$$\begin{cases} t_1+t_3-t_5=0 & (1) \\ t_2+t_3-t_4=0 & (2) \\ t_4-t_5=0 \Rightarrow t_4=t_5 & (3) \end{cases}$$

Substitute Eq(3) into Eq(1)(2) and we have: $t_1=-t_3+t_5$ and $t_2=-t_3+t_4=-t_3+t_5$

Thus, $\text{Nullspace}(B)=\begin{bmatrix} -t_3+t_5 \\ -t_3+t_5 \\ t_3 \\ t_5 \\ t_5 \end{bmatrix}=\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} t_3+\begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} t_5$

which gives us that the dimension of Nullspace(B) is $\boxed{2.}$

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and a basis for it is $\boxed{\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}}$

e) $\boxed{\text{Yes.}}$ it does match my answer in part (d) since $\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$,

So the two basis representations are essentially the same

f). $\boxed{\text{No,}}$ this doesn't. Consider the network of Figure 5 used in part (b), graph G. We have shown that its incidence matrix B_G has a 2-dimensional null space. Yet, if we measure just t_1 and t_2 (as the Stanford suggestion), we have proved in part (b) that we can't recover the exact (unique) flows.

(g). In essence, the null space of M and the null space of B_G must have $\boxed{\text{one and only one intersection}}$.

In other words, if we concatenate M and B_G in axis = 0, then the null space of the resulting matrix should have a unique solution.