2. Elementary Matrices.

(a). i.

$$E_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

ii.

$$E_{ii} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii

$$E_{iii} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{bmatrix}$$

(b) First, reduce R_4 to the form of $[0\ 0\ 0\ 1]$:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \quad \left(\text{Since } [0\,1\,0\,3] - [0\,1\,0\,2] = [0\,0\,0\,1]\right)$$

Then, to reduce R_3 of the resulting matrix to $[0\ 0\ 1\ 0]$:

$$E_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 7 & 1 & -5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \left(\begin{array}{l} \text{Since } 2\cdot[1\ -2\ 0\ -5] \\ \quad +7\cdot[0\ 1\ 0\ 3] \\ \quad +1\cdot[-2\ -3\ 1\ -6] \\ \quad -5\,[0\ 0\ 0\ 1] \end{array} = [0\ 0\ 1\ 0]\right)$$

Then, to reduce R_2 of the resulting matrix to $[0\ 1\ 0\ 0]$.

$$E_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \left(\text{Since } [0\,1\,0\,3] - 3\cdot[0\,0\,0\,1] = [0\,1\,0\,0]\right)$$

Lastly, to reduce R_1 to desired $[1\ 0\ 0\ 0]$, similarly so:

$$E_4 = \begin{bmatrix} 1 & 2 & 0 & 5 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using IPython, we then have:

i.

$$E = E_4 \cdot E_3 \cdot E_2 \cdot E_1 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 0 & 3 \\ 2 & 2 & 1 & 5 \\ 0 & 1 & 0 & -1 \end{bmatrix}$$

ii.

$$EA = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & -2 & 0 & 3 \\ 2 & 2 & 1 & 5 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \left[\begin{array}{cccc|c} 1 & -2 & 0 & -5 & 15 \\ 0 & 1 & 0 & 3 & -7 \\ -2 & -3 & 1 & -6 & 9 \\ 0 & 1 & 0 & 2 & -5 \end{array}\right] = \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right]$$

is an identity matrix with constants Verified Q.E.D.