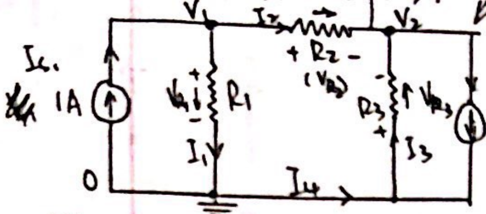
1. Circuit Analysis. Labeling. So $A\vec{x}=\vec{b}$ where



$$\vec{x}=[I_1\ I_2\ I_3\ I_4\ V_{R_1}\ V_{R_2}\ V_{R_3}\ V_1\ V_2]^T.$$

$I_2=1-I_1$
$I_3=I_1+2$

Using KCL, so:
$$\begin{cases} I_{s_1}=1A=I_4+I_2 \\ I_2+I_3=I_{s_2}=3A \\ I_1=I_4+I_{s_1}=I_4+1A \\ (I_3=I_4+I_{s_2}=I_4+3A). \end{cases} \Rightarrow \begin{cases} I_1+I_2=1A & ① \\ I_2+I_3=3A & ② \\ I_1-I_4=1A & ③ \end{cases}$$

Then, using Ohm's Law, so:
$$\begin{cases} V_{R_1}=R_1\cdot I_1 = 10I_1 \\ V_{R_2}=R_2\cdot I_2=20I_2=20-20I_1 \\ V_{R_3}=R_3\cdot I_3=50I_3=50I_1+100. \\ V_1-0=V_{R_1}. \\ V_1-V_2=V_{R_2}\Rightarrow V_2=30I_1-20 \\ V_2-0=-V_{R_3}\Rightarrow V_2=-50I_1-100 \end{cases} \Rightarrow \begin{cases} V_{R_1}-R_1\cdot I_1=0 & ④ \\ V_{R_2}-R_2\cdot I_2=0 & ⑤ \\ V_{R_3}-R_3\cdot I_3=0 & ⑥ \\ V_1-V_{R_1}=0 & ⑦ \\ V_1-V_2-V_{R_2}=0 & ⑧ \\ V_2+V_{R_3}=0 & ⑨ \end{cases}$$

and also:

$V_2=V_1-20I_2$

Using Equations ①-⑨, we can setup $A\vec{x}=\vec{b}$ as:

$I_2=1-I_1$
$I_3=I_1+2$
$I_4=I_1-1$
$V_1=-10I_1$
$V_2=30I_1-20$
$=50I_1-100$
$I_1=-1A$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -R_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -R_2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -R_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_{R_1} \\ V_{R_2} \\ V_{R_3} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

With $R_1=10\Omega$
$R_2=20\Omega$
$R_3=50\Omega$

So,
$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_{R_1} \\ V_{R_2} \\ V_{R_3} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1\ A \\ 2\ A \\ 1\ A \\ -2\ A \\ -10\ V \\ 40\ V \\ 50\ V \\ -10\ V \\ -50\ V \end{bmatrix}$$

2. Cell Phone Battery

(a) 35.1 hours

Since $P = I \cdot V$, so we have

$$I = \frac{P}{V} = \frac{0.3W}{3.8V} = 7.89 * 10^{-2}A = 78.9mA$$

Then, with $C = I \cdot t$, so we have

$$t = \frac{C}{I} = \frac{2770mAh}{78.9} = 35.1hr$$

Thus, a Pixel's full battery will last 35.1 hours under regular usage conditions.

(b) $6.22 * 10^{22}$ electrons

Since 2770 mAh = 2770 mAh $\cdot \frac{3600s}{1h} = 9.972 \cdot 10^6$ mAs, and given that 1 mC = 1 mAs,

so $C_{pixel} = 2770$ mAh $= 9.972 \cdot 10^6$ mAs $= 9.972 \cdot 10^6$ mC

So, there are $\frac{C_{pixel}}{C_{electron}} = \frac{9.972 \cdot 10^3 C}{1.602 \cdot 10^{-19} C} = 6.22 * 10^{22}$ usable electrons worth of charge.

(c) $3.79 \cdot 10^4$ J

Since we could calculate that:

$$E_{discharge} = P \cdot t = 0.3\ W \cdot 35.1\ hr \cdot \frac{3600\ s}{1\ hr} = 3.79 \cdot 10^4\ Ws = 3.79 \cdot 10^4\ J$$

Thus, we have that

$$E_{charge} = E_{discharge} = 3.79 \cdot 10^4\ J$$

So, $3.79 \cdot 10^4$ J is the energy necessary for recharging a completely discharged cell phone battery.

(d) \$0.04

The total energy used by recharging for 31 days is:

$$E_{total} = E_{charge} \cdot 31 = 3.79 \cdot 10^4\ J \cdot 31 = 1.175 \cdot 10^6\ J = 1.175 \cdot 10^6\ Ws$$

So, we can transform its unit to get:

$$E_{total} = 1.175 \cdot 10^6\ Ws \cdot \frac{1\ kW}{1000\ W} \cdot \frac{1\ hr}{3600\ s} = 0.326\ kWh$$

Thus, I would need to pay $0.326\ kWh \cdot \frac{\$0.12}{1\ kWh} = \$0.04$ for recharging for the month of October.

(e)

First, $R = 200m\Omega = 200m\Omega \cdot \frac{1\Omega}{1000m\Omega} = 0.2\ \Omega$. We consider $R_{bat} = 1m\Omega, 1\Omega, 10k\Omega$ separately below.

Case 1 ($R_{bat} = 1\ m\Omega$): With $R_{eq} = R + R_{bat} = 200m\Omega + 1m\Omega = 201m\Omega$, so

$$I_{bat} = I = \frac{V}{R_{eq}} = \frac{5V}{201m\Omega} = 24.88A$$

So the power dissipated across R_{bat} is:

$$P_{bat} = I_{bat}V_{bat} = I_{bat}^2 R_{bat} = (24.88A)^2 \cdot 1m\Omega = 0.62\ W$$

Thus, using the results we got from part (d), so it takes the battery

$$t = \frac{E_{total}}{P} = \frac{3.79 \cdot 10^4\ Ws}{0.62W} = 6.11 \cdot 10^4 s = 6.11 \cdot 10^4 s \cdot \frac{1hr}{3600s} = 16.98\ hr$$

Case 2 ($R_{bat} = 1\ \Omega$): With $R_{eq} = R + R_{bat} = 0.2\Omega + 1m\Omega = 1.2m\Omega$, so

$$I_{bat} = I = \frac{V}{R_{eq}} = \frac{5V}{1.2\Omega} = 4.17A$$

So the power dissipated across R_{bat} is:

$$P_{bat} = I_{bat}V_{bat} = I_{bat}^2 R_{bat} = (4.17A)^2 \cdot 1\Omega = 17.39\ W$$

Thus, using the results we got from part (d), so it takes the battery

$$t = \frac{E_{total}}{P} = \frac{3.79 \cdot 10^4\ Ws}{17.39W} = 2.18 \cdot 10^3 s = 2.18 \cdot 10^3 s \cdot \frac{1hr}{3600s} = 0.605\ hr = 36.3\ min$$

Case 3 ($R_{bat} = 10\ k\Omega$): With $R_{eq} = R + R_{bat} = 0.2\Omega + 10k\Omega = 10000.2\ \Omega$, so

$$I_{bat} = I = \frac{V}{R_{eq}} = \frac{5V}{10000.2\Omega} = 5.00 \cdot 10^{-4}\ A$$

So the power dissipated across R_{bat} is:

$$P_{bat} = I_{bat}V_{bat} = I_{bat}^2 R_{bat} = (5.00 \cdot 10^{-4} A)^2 \cdot 10k\Omega = 2.5 \cdot 10^{-3}\ W$$

Thus, using the results we got from part (d), so it takes the battery

$$t = \frac{E_{total}}{P} = \frac{3.79 \cdot 10^4\ Ws}{2.5 \cdot 10^{-3} W} = 1.52 \cdot 10^7 s = 1.52 \cdot 10^7 s \cdot \frac{1hr}{3600s} = 4.22 \cdot 10^3\ hr$$

3. Fruity Fred

(a) $R_{AB} = \rho\frac{L-kF}{A_c} + \rho\frac{L-kF}{A_c} = \frac{2\rho(L-kF)}{A_c}$

(b) $F = \frac{A_c V_{out} + 2\rho L(V_{out}-1)}{2\rho k(V_{out}-1)}$

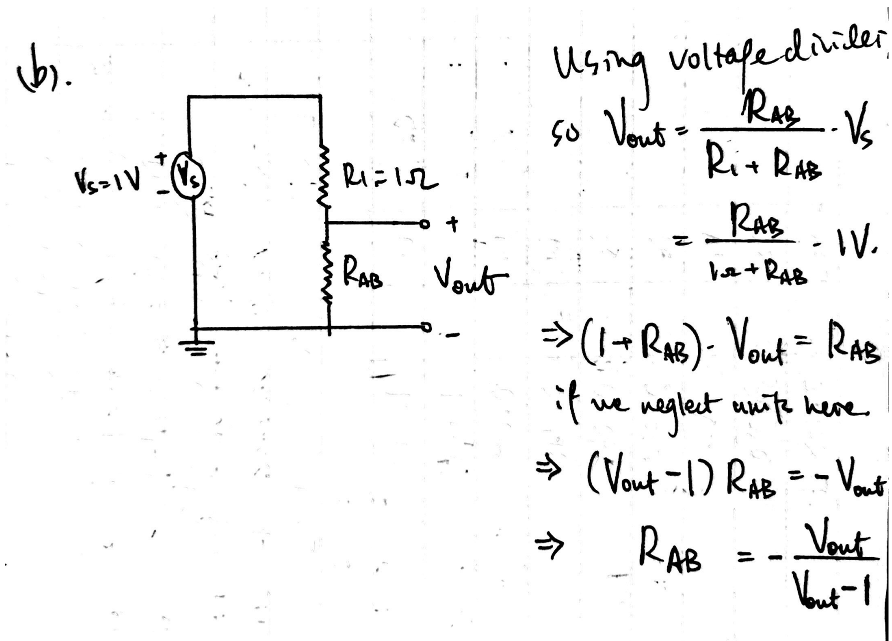


Figure 1: Circuit Designed

As deduced from the process on the picture, so $R_{AB} = -\frac{V_{out}}{V_{out}-1}$. Also, as we derived from part (a), which gives $R_{AB} = \frac{2\rho(L-kF)}{A_c}$. Thus, we have that:

$$R_{AB} = \frac{2\rho(L-kF)}{A_c} = -\frac{V_{out}}{V_{out}-1}$$

$$\implies 2\rho(L-kF)\cdot -(V_{out}-1) = A_c \cdot V_{out}$$

$$\implies (2\rho L - 2\rho kF)\cdot -(V_{out}-1) = A_c \cdot V_{out}$$

$$\implies -2\rho L(V_{out}-1) + 2\rho k(V_{out}-1)F = A_c \cdot V_{out}$$

$$\implies 2\rho k(V_{out}-1)F = A_c \cdot V_{out} + 2\rho L(V_{out}-1)$$

$$\implies F = \frac{A_c V_{out} + 2\rho L(V_{out}-1)}{2\rho k(V_{out}-1)}$$

4. Temperature Sensor

(a) $V_{out} = \frac{V_s R_2}{R_1+R_2}$

The current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1 + R_2$, so $I = \frac{V_s}{R_1+R_2}$

So, V_{out}, which measures the voltage drop over R_2, is equal to (or we could've used the Voltage Divider formula directly to obtain):

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_2} \cdot R_2 = \frac{V_s R_2}{R_1 + R_2}$$

Thus, $V_{out} = V_2 = \frac{V_s R_2}{R_1+R_2}$

(b) $T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{(V_{out} - V_s) R_o \alpha}$

Similarly, the current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1 + R_2 = R_1 + R_o(1 + \alpha T)$, so $I = \frac{V_s}{R_1+R_o(1+\alpha T)}$

So, V_{out}, which measures the voltage drop over R_2, is equal to:

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_o(1 + \alpha T)} \cdot R_o(1 + \alpha T) = \frac{V_s R_o(1 + \alpha T)}{R_1 + R_o(1 + \alpha T)}$$

Thus, $V_{out} = V_2 = \frac{V_s R_o(1+\alpha T)}{R_1+R_o(1+\alpha T)}$, which gives us that: $V_{out} \cdot (R_1 + R_o(1 + \alpha T)) = V_s R_o(1 + \alpha T)$

So, $V_{out} R_1 + V_{out} R_o + V_{out} R_o \alpha T = V_s R_o + V_s R_o \alpha T$, which gives:

$$V_{out} R_o \alpha T - V_s R_o \alpha T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies (V_{out} - V_s) R_o \alpha \cdot T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{(V_{out} - V_s) R_o \alpha}$$

(c) $T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{-V_s R_o \alpha + V_{out} R_1 \beta + V_{out} R_o \alpha}$

Again, similarly, the current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1' + R_2 = R_1(1 + \beta T) + R_o(1 + \alpha T)$, so $I = \frac{V_s}{R_1(1+\beta T)+R_o(1+\alpha T)}$

So, V_{out}, which measures the voltage drop over R_2, is equal to:

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1(1 + \beta T) + R_o(1 + \alpha T)} \cdot R_o(1 + \alpha T) = \frac{V_s R_o(1 + \alpha T)}{R_1(1 + \beta T) + R_o(1 + \alpha T)}$$

Thus, $V_{out} = V_2 = \frac{V_s R_o(1+\alpha T)}{R_1(1+\beta T)+R_o(1+\alpha T)}$, which gives us that:

$$V_{out} \cdot (R_1(1 + \beta T) + R_o(1 + \alpha T)) = V_s R_o(1 + \alpha T)$$

$$\implies V_{out} R_1 + V_{out} R_1 \beta T + V_{out} R_o + V_{out} R_o \alpha T = V_s R_o + V_s R_o \alpha T$$

$$\implies (V_{out} R_1 \beta + V_{out} R_o \alpha - V_s R_o \alpha) \cdot T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{-V_s R_o \alpha + V_{out} R_1 \beta + V_{out} R_o \alpha}$$

(d) No, it can't.

Here, we use the derived formula of voltage dividers directly to obtain the voltage drop over R_2:

$$V_2 = \frac{V_s \cdot R_{o2} \cdot (1 + \alpha T)}{(R_{o1} + R_{o2}) \cdot (1 + \alpha T)} = \frac{V_s R_{o2}}{R_{o1} + R_{o2}}$$

Thus, $V_{out} = V_2 = \frac{V_s R_{o2}}{R_{o1} + R_{o2}}$, which is independent of the variable T, which implies that we cannot express the temperature T as an equation in terms of the measurable variables. Therefore, this circuit (specifically the measurements of V_{out}) cannot be used to measure temperature.

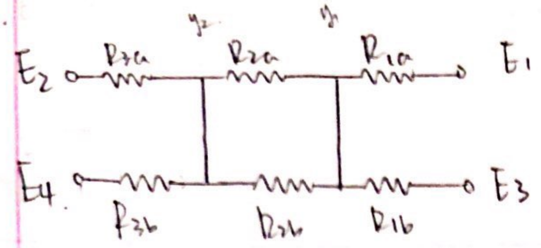
5. Multitouch Resistive Touchscreen

(a) $4k\Omega$

Since $W = 3cm = 0.03m, H = 12cm = 0.12m, T = 1mm = 1 \cdot 10^{-3}m = 0.001m$, so we can calculate the resistance between E_1 and E_2 as:

$$R = \rho \cdot \frac{L}{A} = \rho \cdot \frac{H}{W \cdot T} = 1\Omega m \cdot \frac{0.12m}{0.03m \cdot 0.001m} = 4000\Omega = 4\ k\Omega$$

(b).



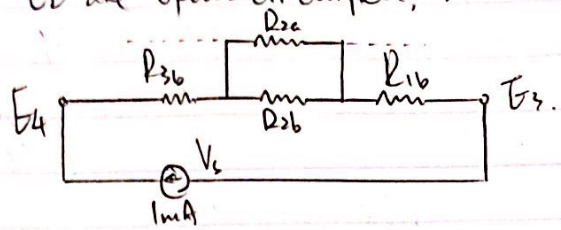
Given that $R_{total} = \rho \dfrac{H}{W \cdot T} = 8\,k\Omega$, $y_1 = 3cm$ $y_2 = 7cm$ $H = 12cm$
with $R_{total} = 8k\Omega$,

so $R_{1a} = R_{1b} = R_{total} \cdot \dfrac{y_1}{H} = 2k\Omega$.

$R_{2a} = R_{2b} = R_{total} \cdot \dfrac{y_2 - y_1}{H} = 2.667\ k\Omega$.

$R_{3a} = R_{3b} = R_{total} \cdot \dfrac{H - y_2}{H} = 3.333\ k\Omega$

(c). Using the given conditions, since E_1 and E_2 are open-circuited, so no current flow through resistors R_{3a} and R_{1a}, which means that the circuit diagram is equivalent to:



We can calculate using methods that the resistance between E_4 and E_3 is:

$$R_{eq} = R_{3b} + \frac{1}{1/R_{2a} + 1/R_{2b}} + R_{1b} = R_{total} \cdot \frac{H - y_2}{H} + \frac{1}{2} \cdot R_{total} \cdot \frac{y_2 - y_1}{H} + R_{total} \cdot \frac{y_1}{H}$$

with $y_1 = 3cm$, $y_2 = 7cm$, $H = 12cm$, so $= R_{total} \cdot \dfrac{H - y_2/2 + y_1/2}{H} = \dfrac{20}{3}\,k\Omega = 6.667\,k\Omega$

Thus, $V_s = I_s \cdot R_{eq} = 1mA \cdot 6.667\,k\Omega = 6.667\,V$.

which means that $V_{E_4 - E_3} = V_s = \boxed{6.667\,V}$.

(d) Using what we've deduced in the steps of part (c), we have that.

$$R_{eq} = R_{total} \cdot \frac{H - y_2/2 + y_1/2}{H}.$$

So, $V_{E_4 - E_3} = V_s = I_s \cdot R_{eq} = 1mA \cdot 8k\Omega \cdot \dfrac{H - y_2/2 + y_1/2}{H} = 8V \cdot \left(1 - \dfrac{y_2}{24} + \dfrac{y_1}{24}\right)$

So, $\boxed{V_{E_4 - E_3} = \left(8 - \frac{1}{3} y_2 + \frac{1}{3} y_1\right)\ V.}$

(e). Using similar logic from part (c) and (d), so we can ① drive E_2, E_4 with a 1mA current source and measure $V_{E_4 - E_2} = I_s \cdot R_{eq_{2,4}}$ where

$$R_{eq_{2,4}} = R_{3a} + \frac{1}{1/R_{2a} + 1/R_{2b}} + R_{3b} = R_{total} \cdot \left(\frac{H - y_2}{H} + \frac{1}{2} \cdot \frac{y_2 - y_1}{H} + \frac{H - y_2}{H}\right).$$

$$= 8k\Omega \cdot \frac{2H - \frac{3}{2} y_2 - \frac{1}{2} y_1}{H}.$$

So, $V_{E_4 - E_2} = 1mA \cdot 8k\Omega \cdot \left(2 - \frac{3}{24} y_2 - \frac{1}{24} y_1\right) = \left(16 - y_2 - \frac{1}{3} y_1\right)\ V.$

Similarly, $R_{eq_{1,3}} = R_{1a} + \dfrac{1}{1/R_{2a} + 1/R_{2b}} + R_{1b} = R_{total} \cdot \left(\dfrac{y_1}{H} + \dfrac{1}{2} \cdot \dfrac{y_2 - y_1}{H} + \dfrac{y_1}{H} \right)$

$$= 8k\Omega \cdot \left(\frac{y_1 + \frac{1}{2}y_2 - \frac{1}{2}y_1 + y_1}{H} \right) \quad \text{where } H = 12 \text{ (cm)}$$

$$= 8k\Omega \cdot \left(\frac{1}{8} y_1 + \frac{1}{24} y_2 \right)$$

And, providing/driving E_1, E_3 with a 1mA current source gives:

$$V_{E_1-E_3} = I_s \cdot R_{eq_{1,3}} = 1mA \cdot 8k\Omega \left(\frac{1}{8} y_1 + \frac{1}{24} y_2 \right)$$

so $$V_{E_1-E_3} = \left(y_1 + \frac{1}{3} y_2 \right) V$$

Thus, we have two (plus one) equations:

$$V_{E_4-E_2} = \left(16 - \frac{1}{3} y_1 - y_2 \right) V.$$

$$V_{E_1-E_3} = \left(y_1 + \frac{1}{3} y_2 \right) V$$

[flowed). $$V_{E_4-E_3} = \left(8 + \frac{1}{3} y_1 - \frac{1}{3} y_2 \right) V$$

6. Homework Process and Study Group

I worked alone without getting any help, except asking questions and reading posts (especially answers from the GSIs) on Piazza as well as reading the Notes of the course.