4. Image Analysis

(a)

We can find the equation of a circle that surrounds the cell by performing least squares on the points and find the best-fit quadratic equation of the form of a circle.

We first set up a system of linear equations from our data:

$$(0.3^2 + (-0.69)^2)a + 0.3d + (-0.69)e = 0.5661a + 0.3d - 0.69e \approx 1$$

$$(0.5^2 + 0.87^2)a + 0.5d + 0.87e = 1.0069a + 0.5d + 0.87e \approx 1$$

$$(0.9^2 + (-0.86)^2)a + 0.9d + (-0.86)e = 1.5496a + 0.9d - 0.86e \approx 1$$

$$(1^2 + 0.88^2)a + 1 \cdot d + 0.88e = 1.7744a + 1 \cdot d + 0.88e \approx 1$$

$$(1.2^2 + (-0.82)^2)a + 1.2d + (-0.82)e = 2.1124a + 1.2d - 0.82e \approx 1$$

$$(1.5^2 + 0.64^2)a + 1.5d + 0.64e = 2.6596a + 1.5d + 0.64e \approx 1$$

$$(1.8^2 + 0^2)a + 1.8d + 0 \cdot e = 3.24a + 1.8d + 0 \cdot e \approx 1$$

Thus, we can formulate a set of matrix equations: $\begin{bmatrix} 0.5661 & 0.3 & -0.69 \\ 1.0069 & 0.5 & 0.87 \\ 1.5496 & 0.9 & -0.86 \\ 1.7744 & 1 & 0.88 \\ 2.1124 & 1.2 & -0.82 \\ 2.6596 & 1.5 & 0.64 \\ 3.24 & 1.8 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ d \\ e \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \vec{e}$

(b)

Similar to part (a), we can find the equation of an ellipse that surrounds the cell by performing least squares on the points and find the best-fit quadratic equation of the form of an ellipse.

We first set up a system of linear equations from our data:

$$0.3^2a + 0.3 \cdot (-0.69)b + (-0.69)^2c + 0.3d + (-0.69)e \approx 1$$

$$0.5^2a + 0.5 \cdot 0.87b + 0.87^2c + 0.5d + 0.87e \approx 1$$

$$0.9^2a + 0.9 \cdot (-0.86)b + (-0.87)^2c + 0.9d + (-0.86)e \approx 1$$

$$1^2a + 1 \cdot 0.88b + 0.88^2c + 1d + 0.88e \approx 1$$

$$1.2^2a + 1.2 \cdot (-0.82)b + (-0.82)^2c + 1.2d + (-0.82)e \approx 1$$

$$1.5^2a + 1.5 \cdot 0.64b + 0.64^2c + 1.5d + 0.64e \approx 1$$

$$1.8^2a + 1.8 \cdot 0b + 0^2c + 1.8d + 0 \cdot e \approx 1$$

Thus, the set of matrix equations is: $$\begin{bmatrix} 0.09 & -0.207 & 0.4761 & 0.3 & -0.69 \\ 0.25 & 0.435 & 0.7569 & 0.5 & 0.87 \\ 0.81 & -0.774 & 0.7396 & 0.9 & -0.86 \\ 1 & 0.88 & 0.7744 & 1 & 0.88 \\ 1.44 & -0.984 & 0.6724 & 1.2 & -0.82 \\ 2.25 & 0.96 & 0.4096 & 1.5 & 0.64 \\ 3.24 & 0 & 0 & 1.8 & 0 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \vec{e}$$

(c) $\frac{\|\vec{e}\|}{N} = 0.1375$

Points and best-fit circle plotted on IPython.

(d) $\frac{\|\vec{e}\|}{N} = 0.01285$; Much smaller error; Ellipse is better.

Points and best-fit ellipse plotted on IPython.

This average error, $\frac{\|\vec{e}\|}{N} = 0.01285$ is much smaller than that of the circle's in part (c), which indicates that the method in part (d), the best-fit ellipse, is a better technique.