3. Mechanical Inverses.

(a) Yes.

By definition A's inverse, P_1, have that $AP_1 = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \cdot \begin{bmatrix}P_{11} & P_{12}\\ P_{13} & P_{14}\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

So, $\Rightarrow \left[\begin{array}{cc|cc}0 & 1 & 1 & 0\\ 1 & 0 & 0 & 1\end{array}\right]$ By swapping R_1 and R_2, we have that $\Rightarrow \left[\begin{array}{cc|cc}1 & 0 & 0 & 1\\ 0 & 1 & 1 & 0\end{array}\right]$
with Gauss-Jordan Elimination,

So, $P_1 = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$ and if $A\vec{b} = \vec{c}$, then A changes a vector $\vec{b}$ by reflecting $\vec{b}$ across the line $x=y$.

(d). Yes.

Let P_4 be A's inverse, so we have that $AP_4 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} P_{41} & P_{42} \\ P_{43} & P_{44} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Since the determinant of A is $\cos\theta \cdot \cos\theta - (-\sin\theta \cdot \sin\theta) = \cos^2\theta + \sin^2\theta = 1$,

So, its inverse, $P_4 = \begin{bmatrix} \frac{\cos\theta}{1} & -\frac{-\sin\theta}{1} \\ -\frac{\sin\theta}{1} & \frac{\cos\theta}{1} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Scanned with CamScanner

Apply the original matrix. if $A\vec{b} = \vec{c}$, then A would rotate $\vec{b}$ by (-90°).
(rotate 90° counterclockwise).

(e). Yes.

Let P_5 be A's inverse, so we have that $AP_5 = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{51} & P_{52} \\ P_{53} & P_{54} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

So, $\Rightarrow \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 \end{array}\right]$ R_1: Subtract $(R_2/2)$. and R_2: Divide by 2. then, swap R_1 and R_2 $\Rightarrow \left[\begin{array}{cc|cc} 1 & 0 & 0 & 0.5 \\ 0 & 1 & 1 & -0.5 \end{array}\right]$

So, $P_5 = \begin{bmatrix} 0 & 0.5 \\ 1 & -0.5 \end{bmatrix}$

(f) No.

Let P_6 be A's inverse, so by definition. $AP_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 1 & 4 & 4 \end{bmatrix} \cdot \begin{bmatrix} P_{61} & P_{62} & P_{63} \\ P_{64} & P_{65} & P_{66} \\ P_{67} & P_{68} & P_{69} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So, $\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 1 & 4 & 4 & 0 & 0 & 1 \end{array}\right]$ R_3: Subtract R_1, Subtract $2 \times R_2$.

$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & -2 & 1 \end{array}\right]$ Since we end up with a row of 0s, and the right side is not 0, so there is no solution to the system of equations.

Thus, no inverse of A exists.