4. Filtering Out The Troll.

(a). $\vec{m_1} = \cos(45°) \cdot \vec{a} + \cos(-30°) \cdot \vec{b}$

and $\vec{m_2} = \sin(45°) \cdot \vec{a} + \sin(-30°) \cdot \vec{b}$

Thus, $\vec{m_1} = \frac{\sqrt{2}}{2} \vec{a} + \frac{\sqrt{3}}{2} \vec{b}$, $\vec{m_2} = \frac{\sqrt{2}}{2} \vec{a} - \frac{1}{2} \vec{b}$

(b) We have $\begin{cases} \frac{\sqrt{2}}{2} \vec{a} + \frac{\sqrt{3}}{2} \vec{b} = \vec{m_1} & (1) \\ \frac{\sqrt{2}}{2} \vec{a} - \frac{1}{2} \vec{b} = \vec{m_2} & (2). \end{cases}$

Eq.1 + $\sqrt{3}$ · (Eq.2) ; $\frac{\sqrt{2}}{2} \vec{a} + \frac{\sqrt{3}}{2} \vec{b} + \frac{\sqrt{6}}{2} \vec{a} - \frac{\sqrt{3}}{2} \vec{b} = \vec{m_1} + \sqrt{3} \vec{m_2}$

So, $\frac{\sqrt{6} + \sqrt{2}}{2} \vec{a} = \vec{m_1} + \sqrt{3} \vec{m_2}$

Thus, $\vec{a} = \frac{2}{\sqrt{6}+\sqrt{2}} \vec{m_1} + \frac{2\sqrt{3}}{\sqrt{6}+\sqrt{2}} \vec{m_2}$

which means that. $\vec{a} = \frac{\sqrt{6}-\sqrt{2}}{2} \vec{m_1} + \frac{3\sqrt{2}-\sqrt{6}}{2} \vec{m_2}$

Thus, $u = \frac{\sqrt{6}-\sqrt{2}}{2}$ and $v = \frac{3\sqrt{2}-\sqrt{6}}{2}$

(c). "All human beings are born free and equal in dignity and rights."

It is taken from the Universal Declaration of Human Rights.