2. Technical Issues.

(a). Suppose originally, A is (x_a, y_a). Using my Basis, so $\vec{a_1} = x_a \vec{V_1} + x_b \vec{V_2}$
Since my view, $\vec{a_1} = \begin{bmatrix} -2 \\ 3 \end{bmatrix} = x_a \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y_a \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ so $x_a = -2$, $y_a = 3$

and we have that $A(-2, 3)$ originally. Similarly, we obtain $B(0,2)$. $C(2,-3)$. $D(0,-2)$ before applying my Basis.

Now, consider my partner's view.

$$\vec{a_2} = -2 \cdot \vec{u_1} + 3 \cdot \vec{u_2} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{Eq (1)}$$
$$\vec{b_2} = 0 \cdot \vec{u_1} + 2 \cdot \vec{u_2} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \quad (2)$$
$$\vec{c_2} = 2 \cdot \vec{u_1} + (-3) \cdot \vec{u_2} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} \quad (3)$$
$$\vec{d_2} = 0 \cdot \vec{u_1} + (-2) \cdot \vec{u_2} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad (4)$$

So we can obtain that $\vec{u_2} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ from Eq (2). and then from Eq (1), we have $\vec{u_1} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$

Thus, $\boxed{\vec{u_1} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}, \quad \vec{u_2} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}}$

(b). Since we have that $A_{v \to u} \cdot \vec{V} = \vec{u}$, so $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{3}{2} & -1 \end{bmatrix}$

so we obtain equations:

$$\begin{aligned} 1 \cdot a_{11} + 0 \cdot a_{12} &= -\frac{1}{2} \\ 0 \cdot a_{11} + 1 \cdot a_{12} &= 0 \\ 1 \cdot a_{21} + 0 \cdot a_{22} &= -\frac{3}{2} \\ 0 \cdot a_{21} + 1 \cdot a_{22} &= -1. \end{aligned} \quad \Rightarrow \quad \begin{aligned} a_{11} &= -\frac{1}{2} \\ a_{12} &= 0 \\ a_{21} &= -\frac{3}{2} \\ a_{22} &= -1. \end{aligned}$$

Thus, $A_{v \to u} = \boxed{\begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{3}{2} & -1 \end{bmatrix}}$

(c). Using the given information, so

$$-2 \cdot \vec{w_1} + 3 \cdot \vec{w_2} = \vec{A_{new}} = \begin{bmatrix} -2 \\ -3 \end{bmatrix} \quad (5)$$
$$0 \cdot \vec{w_1} + 2 \cdot \vec{w_2} = \vec{B_{new}} = \begin{bmatrix} 0 \\ 2 \end{bmatrix} \quad (6)$$
$$2 \cdot \vec{w_1} + (-3) \cdot \vec{w_2} = \vec{C_{new}} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad (7)$$
$$0 \cdot \vec{w_1} + (-2) \cdot \vec{w_2} = \vec{D_{new}} = \begin{bmatrix} 0 \\ -2 \end{bmatrix} \quad (8).$$

From Eq (6). we can solve that $\vec{w_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, substitute in Eq (5), so $\vec{w_1} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$
Thus, the new basis vectors are $\boxed{\vec{w_1} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \quad \vec{w_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$

(d). Since we have that $A_{w\to u} \cdot \vec{W} = \vec{U}$, so $\begin{bmatrix} a_{w1} & a_{w2} \\ a_{w3} & a_{w4} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{3}{2} & -1 \end{bmatrix}$

so we obtain equations:

$$\begin{aligned} 1 \cdot a_{w1} - 3 a_{w2} &= -\tfrac{1}{2} \\ 0 \cdot a_{w1} + 1 \cdot a_{w2} &= 0. \\ 1 \cdot a_{w3} - 3 \cdot a_{w4} &= -\tfrac{3}{2} \\ 0 \cdot a_{w3} + 1 \cdot a_{w4} &= -1 \end{aligned} \quad \Rightarrow. \quad \begin{aligned} a_{w1} &= -\tfrac{1}{2} \\ a_{w2} &= 0 \\ a_{w3} &= -\tfrac{9}{2} \\ a_{w4} &= -1 \end{aligned}$$

Thus, $A_{w\to u} = \boxed{\begin{bmatrix} -\frac{1}{2} & 0 \\ -\frac{9}{2} & -1 \end{bmatrix}}$