7.

Question: Solve the inverse for $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. If there is, solve; if there isn't, explain.

Solution: Let P be A's inverse, so we have that $AP = I$.

- So we have. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} P_1 & P_2 & P_3 \\ P_4 & P_5 & P_6 \\ P_7 & P_8 & P_9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

which can be represented as this augmented matrix:

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 9 & 0 & 0 & 1 \end{array}\right]$$

$R_3$: Add $R_1$ and Subtract $2 \times R_2$.

and we have:

$$\Rightarrow \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 \end{array}\right]$$

Now, we have a row of 0s, but the right side is not 0.

Thus, we reached an ending condition, and we've shown/proved that A does not have an inverse Q.E.D.