1 Double-Check Your Intuition

(a) $8.85 \cdot 10^{-12} F/m \cdot \frac{L_{\text{train}} W}{h}$ for both capacitances

Since $\epsilon_{air} = 8.85 \cdot 10^{-12}\ F/m$, so we have that $C = \epsilon \frac{A}{d} = 8.85 \cdot 10^{-12} F/m \cdot \frac{L_{\text{train}} W}{h}$

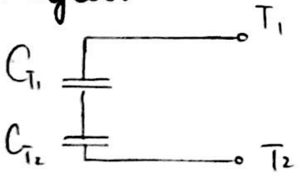
This is the capacitance between both T_1, M and T_2, M

Parts (b), (c), and more explanation/graph of (d) on the next page. Below is just the concluded equation for $V_{C_{eq}}(t)$ for more clarity.

(d) Equation:

$V_{C_{eq}}(t) =$ when $k\tau \leq t \leq (k+\frac{1}{2})\tau$, then $\frac{I_1}{C_{eq}}(t-k\tau)$; when $(k+\frac{1}{2})\tau < t < (k+1)\tau$, then $-\frac{I_1}{C_{eq}}(t-k\tau-\frac{\tau}{2})+\frac{I_1\tau}{2C_{eq}}$

1. **Maglev.**

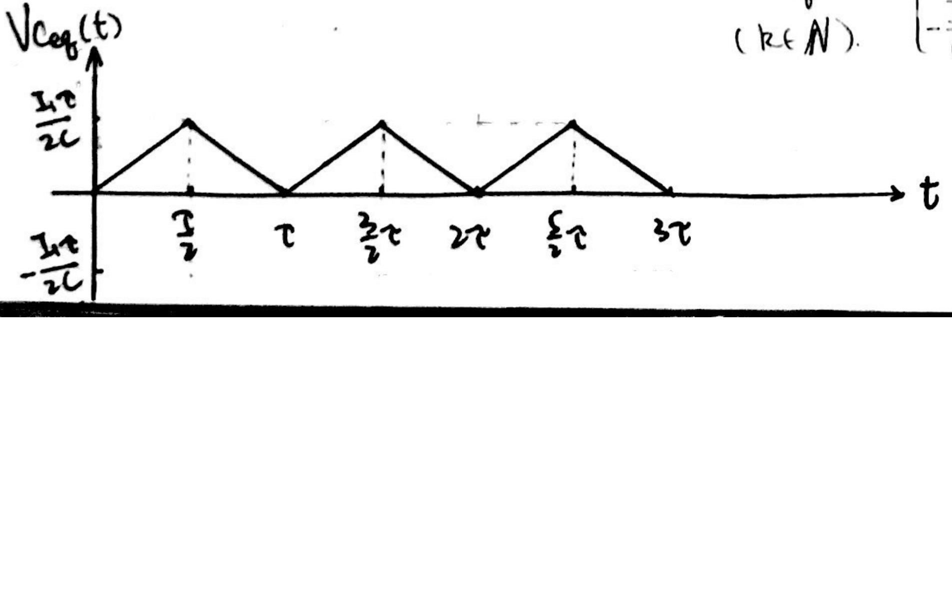
(b) 

They're connected to each other in <u>series</u>.

(c). $C_{eq} = C_{T_1} \parallel C_{T_2} = \dfrac{C_{T_1} C_{T_2}}{C_{T_1} + C_{T_2}} = \dfrac{C_{T_1}^2}{2C_{T_1}}$ with $C_{T_1} = C_{T_2} \neq 0$, so cancelling.

Thus, $C_{eq} = \dfrac{C_{T_1}}{2} = \boxed{4.425 \cdot 10^{-12}\ F/m \cdot \dfrac{L_{train} \cdot W}{h}}$

(d) Since at $t=0$, $V_{C_{eq}}(0) = 0V$, and when a constant current source is applied, we have $V_{C_{eq}}(t) = \frac{1}{C} t + V_{C_{eq}}(0)$, and since I_s is constant from $t=0$ to $t = \frac{\tau}{2}$, so $V_{C_{eq}}(t) = \frac{1}{C} t$ for $t \in [0, \frac{\tau}{2}]$. Using similar logic as Note 17, we have $V_{C_{eq}}(t) = \frac{1}{C}(t - t_0) + V_{C_{eq}}(t_0)$ and so here we supply each period $(\frac{\tau}{2})$ of constant current, so the voltage pattern mimics the one on the note 17 and thus is: $V_{C_{eq}}(t) = \begin{cases} \frac{I_1}{C_{eq}}(t - k\tau) & \text{when } k\tau \leq t \leq (\frac{1}{2} + k)\tau \\ -\frac{I_1}{C_{eq}}(t - k\tau - \frac{\tau}{2}) + \frac{I_1 \tau}{2C_{eq}} & \text{when } (k + \frac{1}{2})\tau < t < (k+1)\tau \end{cases}$ $(k \in \mathbb{N})$.



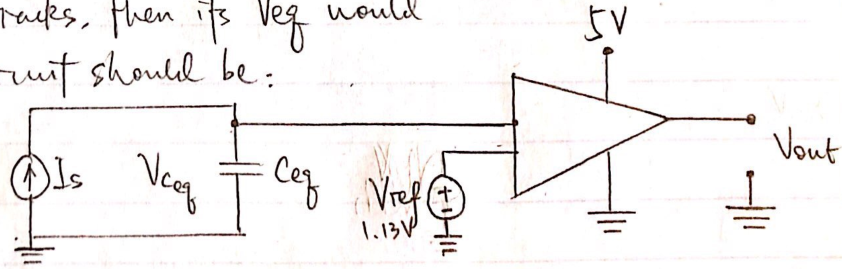
(e). When $d < 1\text{cm}$, then $C_{eq} = 4.425 \cdot 10^{-12}\,\text{F/m} \cdot \frac{L_{train} \cdot w}{d} > 4.425 \cdot 10^{-12}\,\text{F/m} \cdot \frac{100\text{m} \cdot \cancel{1\text{cm}}}{\cancel{1\text{cm}}}$

with $L_{train} = 100\text{m}$, $w = 1\text{cm}$, so, $C_{eq} > 4.425 \cdot 10^{-10}\,\text{F}$.

Thus, $V_{C_{eq}} \leq \frac{I_1 T}{2 C_{eq}} < \frac{1\text{mA} \cdot 1\mu\text{s}}{2 \cdot 4.425 \cdot 10^{-10}\,\text{F}} = 1.13\,\text{V}$

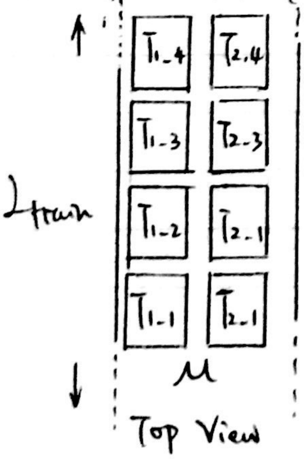
which gives that the maximum V_{eq} for the train is $1.13\,\text{V}$.
and when it's too high above the tracks, then its V_{eq} would increase. In other words, the circuit should be:

$V_{ref} = 1.13\text{V}$.



(f). The design should be that the strips of metal, T_1 and T_2, should be segmented into four pieces, and have sensors attached to each piece so that each op-amp comparator works independently to check the location they're measuring, respectively.

i.e. The strip T_1 should be broken into one smaller strip per desired location, and similarly (symmetrically) for T_2, which should look like:



Then, for any two strips T_{1-i}, T_{2-i}, $i \in [1,4]$, have an individual circuit hooked up to the two strips.

In this way, we could measure the height of the train at 4 different locations if the magnets (smaller strips) are placed at these 4 locations.

2. DRAM.

Given that $V_c(t) = \frac{I \cdot t}{C} + V_c(0)$, $V_c(0) = 1.2V$, $C = 18fF$

and $I = I_{leak}$ as we could view the right half of the diagram as 

which gives this relationship using KCL.

Thus, for $V_{bit} = V_c(t) > 0.8V$ and $t > 1ms$,

so $$I = \frac{(V_c(t) - V_c(0)) \cdot C}{t} < \frac{(1.2V - 0.8V) \cdot 18 \cdot 10^{-15} F}{1 \cdot 10^{-3} s} = 7.2 pA$$

Thus, the maximum value of I_{leak} is $\boxed{7.2pA}$ ($7.2 \times 10^{-12} A$).

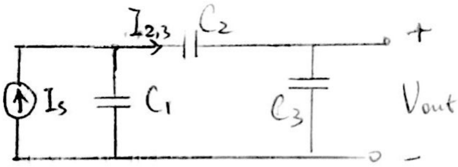
3\. (a). Since it's a current source and the capacitors are in series, using KCL,

so $I_{C_2} = I_s$, and so $\Rightarrow V_{out}(t) = \dfrac{I_{C_2} \cdot t}{C_2} + V_{C_2}(0) = \boxed{\dfrac{I_s \cdot t}{C_2}}$

and with $V_{C_2}(0) = 0V$,

(b) Since $C_{eq_{2,3}} = C_2 // C_3 = \frac{C_2 C_3}{C_2 + C_3}$, so $C_{eq} = C_1 + C_{eq_{2,3}}$,

so first, $\frac{dV_s}{dt} = \frac{I_s}{C_{eq}}$ and so $I_s = C_{eq} \cdot \frac{dV_s}{dt}$

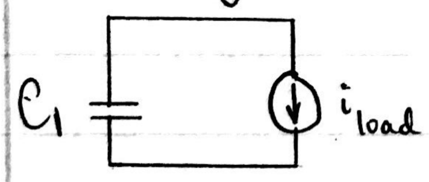


Using similar logic, so $I_{2,3} = C_{eq_{2,3}} \cdot \frac{dV_s}{dt}$, which gives the proportionality that $\frac{I_s}{C_{eq}} = \frac{I_{2,3}}{C_{eq_{2,3}}}$, so $I_{2,3} = \frac{C_2 C_3}{C_1 C_2 + C_2 C_3 + C_3 C_1} \cdot I_s$

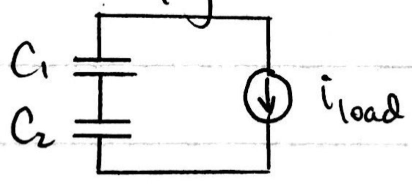
Thus, $V_{out}(t) = \frac{I_{2,3}}{C_3} \cdot t + \cancel{V_{C_3}(0)} = \boxed{\frac{C_2}{C_1 C_2 + C_2 C_3 + C_3 C_1} \cdot I_s \cdot t}$

$\downarrow$
0V

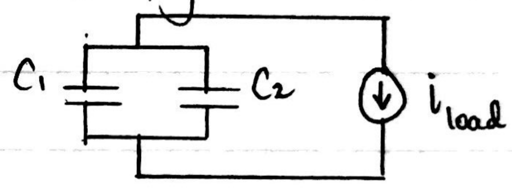
4. (a). Config 1.



Config 2.



Config 3.



(b). (i) Config 1: $V_c(t) = \frac{I}{C}(t - t_0) + V_c(t_0)$. Using given information, so:

$$V(t) = V_c(t) = \frac{-i_{load}}{C_{sc}}(t-0) + V_{init} = \boxed{-\frac{i_{load}}{C_{sc}} \cdot t + V_{init}}$$

(ii) Config 2: Similarly, with the same basic equation and the fact that parallel capacitors gives: $\begin{cases} V_c(t_0) = 2V_{init} \\ C_{eq} = C_{sc} // C_{sc} = \frac{C_{sc}}{2} \end{cases}$

So, $V(t) = V_c(t) = \frac{-i_{load}}{\frac{1}{2}C_{sc}}(t-0) + 2 \cdot V_{init} = \boxed{-\frac{2i_{load}}{C_{sc}} \cdot t + 2V_{init}}$

(iii). Config 3: Again, $V_c(t_0) = V_{sc}(t_0) = V_{init}$ and $C_{eq} = C_{sc} + C_{sc} = 2C_{sc}$ (in series)

Thus, $V(t) = V_c(t) = \frac{-i_{load}}{2C_{sc}}(t-0) + V_{init} = \boxed{-\frac{i_{load}}{2C_{sc}} \cdot t + V_{init}}$

(c) i) Config 1. For the device to function properly, so $V(t) \geq V_{min}$, so we have

$$-\frac{i_{load}}{C_{sc}} \cdot t + V_{init} \geq V_{min} \Rightarrow \frac{i_{load}}{C_{sc}} \cdot t \leq V_{init} - V_{min}$$

so, $t \leq (V_{init} - V_{min}) \cdot \frac{C_{sc}}{i_{load}}$, which is the life time is:

$$t_{life_1} = \boxed{(V_{init} - V_{min}) \cdot \frac{C_{sc}}{i_{load}}}$$

(ii) Config 2. Similarly, so: $-\frac{2 i_{load}}{C_{sc}} \cdot t + 2 V_{init} \geq V_{min}$

$$\Rightarrow t \leq (2 V_{init} - V_{min}) \cdot \frac{C_{sc}}{2 i_{load}}$$

which gives that $t_{life_2} = \boxed{(V_{init} - \frac{1}{2} V_{min}) \cdot \frac{C_{sc}}{i_{load}}}$

(iii) Config 3. Again, similarly we have $-\frac{i_{load}}{2 C_{sc}} \cdot t + V_{init} \geq V_{min}$

$$\Rightarrow t \leq (V_{init} - V_{min}) \cdot \frac{2 C_{sc}}{i_{load}}$$

Thus, $t_{life_3} = \boxed{(2 V_{init} - 2 V_{min}) \cdot \frac{C_{sc}}{i_{load}}}$

(d). To have Config 3 better than Config 2, so $t_{life_3} > t_{life_2}$,

which is equivalent to: $(2 V_{init} - 2 V_{min}) \cdot \cancel{\frac{C_{sc}}{i_{load}}} > (V_{init} - \frac{1}{2} V_{min}) \cdot \cancel{\frac{C_{sc}}{i_{load}}}$

$$\Rightarrow 2 V_{init} - 2 V_{min} > V_{init} - \frac{1}{2} V_{min}$$

$$\Rightarrow V_{init} > \frac{3}{2} V_{min}.$$

Thus, under the condition that $\boxed{V_{init} > \frac{3}{2} V_{min}}$ only is Config 3 better than 2.

5. (a) When tank is full, the entire capacitor has permittivity of water, 81ε.

$$\text{so, } C_{eq} = C_{H_2O} = 81\varepsilon \cdot \frac{h_{tot} \cdot \cancel{w}}{\cancel{w}} = \boxed{81\varepsilon \cdot h_{tot}}$$

When it's empty, the entire capacitor is air, with permittivity ε.

$$\text{so, } C_{eq} = C_{air} = \varepsilon \cdot \frac{h_{total} \cdot \cancel{w}}{\cancel{w}} = \boxed{\varepsilon \cdot h_{tot}}$$

(b) Here, two capacitors act in parallel, and we have $C_{tank} = C_{H_2O} \parallel C_{air}$,

with $C_{H_2O} = 81\varepsilon \cdot \frac{h_{H_2O} \cdot \cancel{w}}{\cancel{w}} = 81\varepsilon \cdot h_{H_2O}$.

and $C_{air} = \varepsilon \cdot \frac{(h_{tot} - h_{H_2O}) \cdot \cancel{w}}{\cancel{w}} = \varepsilon \cdot (h_{tot} - h_{H_2O})$.

$$\text{Thus, } C_{tank} = \frac{C_{H_2O} \cdot C_{air}}{C_{H_2O} + C_{air}} = \frac{81\varepsilon^2 h_{H_2O}(h_{tot} - h_{H_2O})}{80\varepsilon h_{H_2O} + \varepsilon \cdot h_{tot}} = \boxed{\frac{81\varepsilon\, h_{H_2O}(h_{tot} - h_{H_2O})}{80\, h_{H_2O} + h_{tot}}}$$

(c) $V_c(t) = \frac{I_s}{C_{tank}} \cdot t + V_c(0)$ and since $V_c(0) = 0\,V$, so $\boxed{V_c(t) = \frac{I_s}{C_{tank}} \cdot t}$

(d). By measuring V_c for a brief amount of time, we could easily (and successfully) determine t in seconds and $V_c(t)$ in V. Since I_s is a known current source, so $C_{tank} = \frac{I_s}{V_c(t)} \cdot t$ could be calculated. ✓

Then, using the capacitance of C_{tank} calculated, with ε, h_{tot} being known, so we can determine h_{H_2O} with this equation, so h_{H_2O} can be determined as well in this way. ✓

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6. (a) Using a voltage divider, so $V_+ = \boxed{\frac{R_{fixed}}{R_{photo}+R_{fixed}} \cdot 5V}$.

(b) Using the given info, so we want $3V = \frac{R_{fixed}}{1k\Omega + R_{fixed}} \cdot 5V \Rightarrow R_{fixed} = \boxed{1.5\,k\Omega}$

(c) Since we can divide the situation into two cases, exactly one of them must be true:

(1) $V_+ > V_- = 2.5V$ or (2) $V_+ < V_- = 2.5V$, so:

Case (1). $V_+ > 2.5V$, since the op-amp is ideal, so $\boxed{V_{out} = 5V. \text{ when } V_+ > 2.5V}$

Case (2) $V_+ < 2.5V$, again similarly, so $\boxed{V_{out} = 0V \text{ when } V_+ < 2.5V}$

(d) The condition $V_+ > 2.5V$ is equivalent to $\frac{1.5k\Omega}{R_{photo}+1.5k\Omega} \cdot 5V > 2.5V \Rightarrow R_{photo} < 1.5k\Omega$

Similarly, $V_+ < 2.5V$ is equivalent to $R_{photo} > 1.5k\Omega$.

Thus, $\boxed{V_{out} = \begin{cases} 5V & \text{when } R_{photo} < 1.5k\Omega \\ 0V & \text{when } R_{photo} > 1.5k\Omega \end{cases}}$

(e) When we want $I_{LED} = 20mA$ when the photoresistor is in the "light" condition, so $V_{out} = 5V$, and we have that $R_{lim} = \frac{V_{lim}}{I_{lim}} = \frac{V_{out} - V_{LED}}{I_{LED}} = \frac{5V - 3V}{20mA} = \boxed{100\,\Omega}$

7. Homework Process and Study Group

I worked alone without getting any help, except asking questions and reading posts (especially answers from the GSIs) on Piazza as well as reading the Notes of the course.