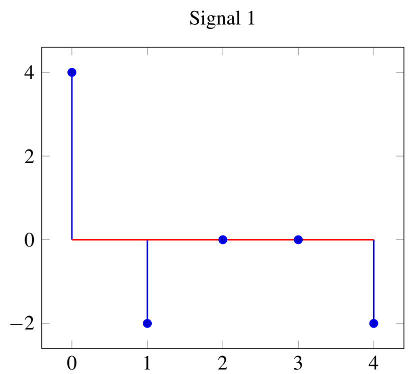
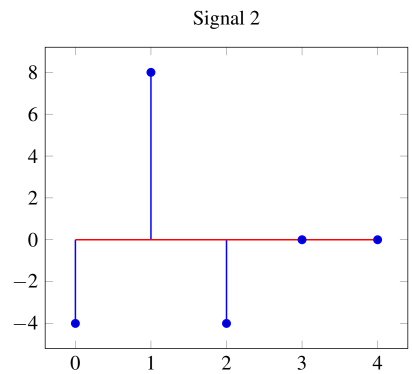
EECS 16A Designing Information Devices and Systems I
Fall 2018 Discussion 11B

1. Correlation You are given the following two signals:





(a) Assume the two signals are periodic with period 5. Find their linear cross correlation, that is find $\text{corr}(\vec{s}_1, \vec{s}_2)$.

Answer: For $\vec{x}$, $\vec{y}$ that are periodic with period N: $\text{corr}_N(\vec{x}, \vec{y})[k] = \sum_{i=0}^{N-1} x[i]y[i-k]$

Since the signals are periodic they continue on to $+\infty$ and $-\infty$. We'll start by just performing the linear correlation over one period, and ignore the signals outside of this period. Shifting the signal back will bring the next period of the signal into our range of interest. Thus we calculate the linear cross-correlation assuming the signals are periodic as below:

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n]$	-4		8		-4		0		0	
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	-16	$+$	-16	$+$	0	$+$	0	$+$	0	$=-32$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-1]$	0		-4		8		-4		0	
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	0	$+$	8	$+$	0	$+$	0	$+$	0	$=8$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-2]$	0		0		-4		8		-4	
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	0	$+$	0	$+$	0	$+$	0	$+$	8	$=8$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-3]$	-4		0		0		-4		8	
$\langle \vec{s}_1, \vec{s}_2[n-3] \rangle$	-16	$+$	0	$+$	0	$+$	0	$+$	-16	$=-32$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-4]$	8		-4		0		0		-4	
$\langle \vec{s}_1, \vec{s}_2[n-4] \rangle$	32	$+$	8	$+$	0	$+$	0	$+$	8	$=48$

Let’s continue to calculate the values of the inner product with more shifts.

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-5]$	-4		8		-4		0		0	
$\langle \vec{s}_1, \vec{s}_2[n-5] \rangle$	-16	$+$	016	$+$	0	$+$	0	$+$	0	$=-32$

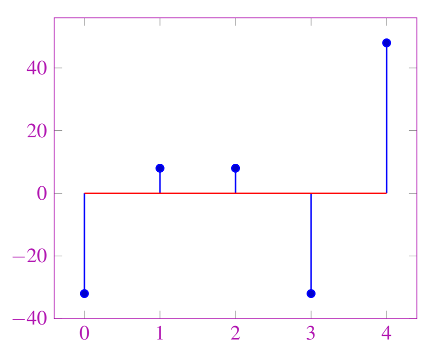
$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-6]$	0		-4		8		-4		0	
$\langle \vec{s}_1, \vec{s}_2[n-6] \rangle$	0	$+$	8	$+$	0	$+$	0	$+$	0	$=8$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-7]$	0		0		-4		8		-4	
$\langle \vec{s}_1, \vec{s}_2[n-7] \rangle$	0	$+$	0	$+$	0	$+$	0	$+$	8	$=8$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-8]$	-4		0		0		-4		8	
$\langle \vec{s}_1, \vec{s}_2[n-8] \rangle$	-16	$+$	0	$+$	0	$+$	0	$+$	-16	$=-32$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-9]$	8		-4		0		0		-4	
$\langle \vec{s}_1, \vec{s}_2[n-9] \rangle$	32	$+$	8	$+$	0	$+$	0	$+$	8	$=48$

Non-periodic Cross-correlation of Signals 1 and 2



Notice that the pattern repeats, this leads to the definition of circular correlation, which we will explore in the later part.

(b) Sketch the linear cross-correlation of signal 1 with signal 2, that is find : $\text{corr}(\vec{s}_1, \vec{s}_2)$. Do not assume the signals are periodic.

Answer:

Represent signal 1 as the vector $\vec{s}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 4 & -2 & 0 & 0 & -2 \end{bmatrix}^T$, zero-padded so that we compute only the linear correlation. Similarly, represent signal 2 as the vector $\vec{s}_2 = \begin{bmatrix} -4 & 8 & -4 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$, where we once again zero pad the vector. Notice we zero pad the front of the vector $\vec{s}_2$ but the back of the vector $\vec{s}_1$.

The cross-correlation between two vectors is defined as follows:

$$\text{corr}(\vec{x}, \vec{y})[k] = \sum_{i=-\infty}^{\infty} \vec{x}[i]\vec{y}[i-k]$$

To compute the cross-correlation $\text{corr}(\vec{s}_1, \vec{s}_2)$, we shift the vector $\vec{s}_2$ and compute the inner product of the shifted $\vec{s}_2$ and the vector $\vec{s}_1$.

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n+4]$	-4	8	-4	0	0	0	0	0	0	
$\langle \vec{s}_1, \vec{s}_2[n+4] \rangle$	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0	= 0

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n+3]$	0	-4	8	-4	0	0	0	0	0	
$\langle \vec{s}_1, \vec{s}_2[n+3] \rangle$	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0	= 0

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n+2]$	0	0	-4	8	-4	0	0	0	0	
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	0 +	0 +	0 +	0 +	-16 +	0 +	0 +	0 +	0	= -16

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n+1]$	0	0	0	-4	8	-4	0	0	0	
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	0 +	0 +	0 +	0 +	32 +	-8 +	0 +	0 +	0	= 40

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n]$	0	0	0	0	-4	8	-4	0	0	
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	0 +	0 +	0 +	0 +	-16 +	-16 +	0 +	0 +	0	= -32

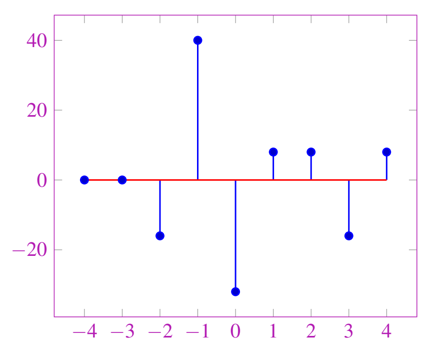
$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n-1]$	0	0	0	0	0	-4	-8	-4	0	
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	0 +	0 +	0 +	0 +	0 +	8 +	0 +	0 +	0	= 8

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n-2]$	0	0	0	0	0	0	-4	8	-4	
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0 +	8	= 8

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n-3]$	-4	0	0	0	0	0	0	-4	8	
$\langle \vec{s}_1, \vec{s}_2[n-3] \rangle$	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0 +	-16	= -16

$\vec{s}_1$	0	0	0	0	4	-2	0	0	-2	
$\vec{s}_2[n-4]$	8	-4	0	0	0	0	0	0	-4	
$\langle \vec{s}_1, \vec{s}_2[n-4] \rangle$	0 +	0 +	0 +	0 +	0 +	0 +	0 +	0 +	8	= 8

Non-periodic Cross-correlation of Signals 1 and 2



(c) Find the circular cross correlation of $\vec{s}_2$ with $\vec{s}_1$, that is find circcorr$(\vec{s}_1, \vec{s}_2)$

Answer:

Represent signal 1 as the vector $\vec{s}_1 = \begin{bmatrix}4 & -2 & 0 & 0 & -2\end{bmatrix}^T$. Similarly, represent signal 2 as the vector $\vec{s}_2 = \begin{bmatrix}-4 & 8 & -4 & 0 & 0\end{bmatrix}$

The cross-correlation between two vectors of length N is defined as follows:

$$\text{circcorr}(\vec{x}, \vec{y})[k] = \sum_{i=0}^{N-1} \vec{x}[i]\vec{y}[(i-k)_N]$$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n]$	-4		8		-4		0		0	
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	-16	$+$	-16	$+$	0	$+$	0	$+$	0	$=-32$

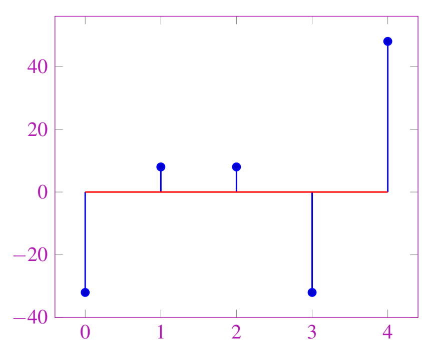
$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-1]$	0		-4		8		-4		0	
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	0	$+$	8	$+$	0	$+$	0	$+$	0	$=8$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-2]$	0		0		-4		8		-4	
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	0	$+$	0	$+$	0	$+$	0	$+$	8	$=8$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-3]$	-4		0		0		-4		8	
$\langle \vec{s}_1, \vec{s}_2[n-3] \rangle$	-16	$+$	0	$+$	0	$+$	0	$+$	-16	$=-32$

$\vec{s}_1$	4		-2		0		0		-2	
$\vec{s}_2[n-4]$	8		-4		0		0		-4	
$\langle \vec{s}_1, \vec{s}_2[n-4] \rangle$	32	$+$	8	$+$	0	$+$	0	$+$	8	$=48$

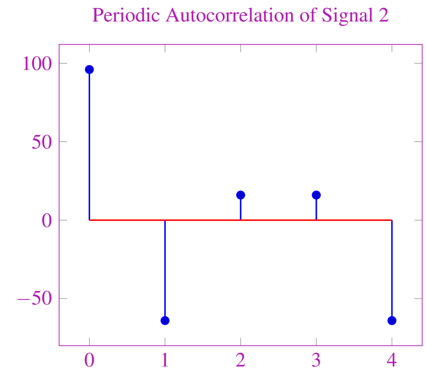
Periodic Cross-correlation of Signals 1 and 2



(d) Sketch the periodic autocorrelation (correlation with itself) of signal 2 assuming a period of 5.

Answer:

The autocorrelation is as follows. Autocorrelation is a special case of cross-correlation (it is the cross-correlation of a signal with itself). See the answer for part (c) for an example of how to compute cross-correlation.



2. Search and Rescue Dogs

Berkeley's Puppy Pound needs your help! While Mr. Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the pound have a collar that sends a bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of 3 city blocks. Can you help the pound locate their lost puppy?

Note: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map.) Mr. Muffin is constrained to running wild in the streets, meaning he won't be found in any buildings. If your TA asks 'Where is Mr. Muffin?' it is sufficient to answer with his intersection or 'between these two intersections.'

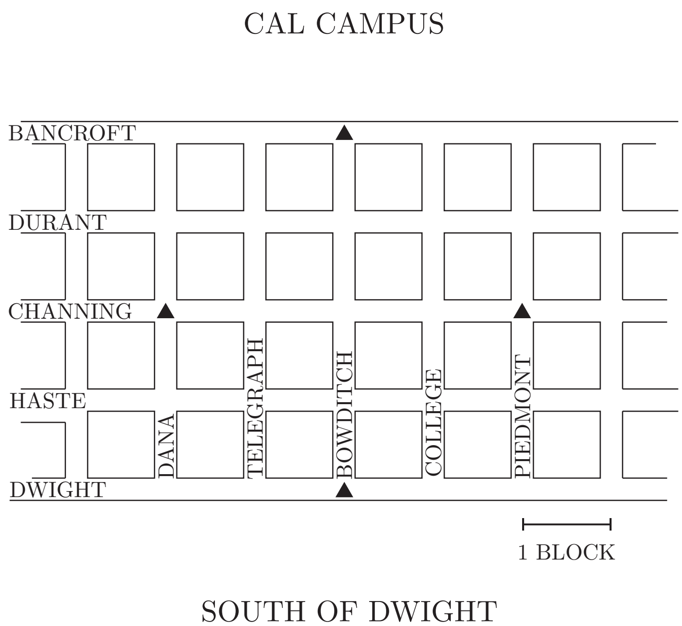
[1]

(a) You check the logs of the cell towers, and they have received the following messages:

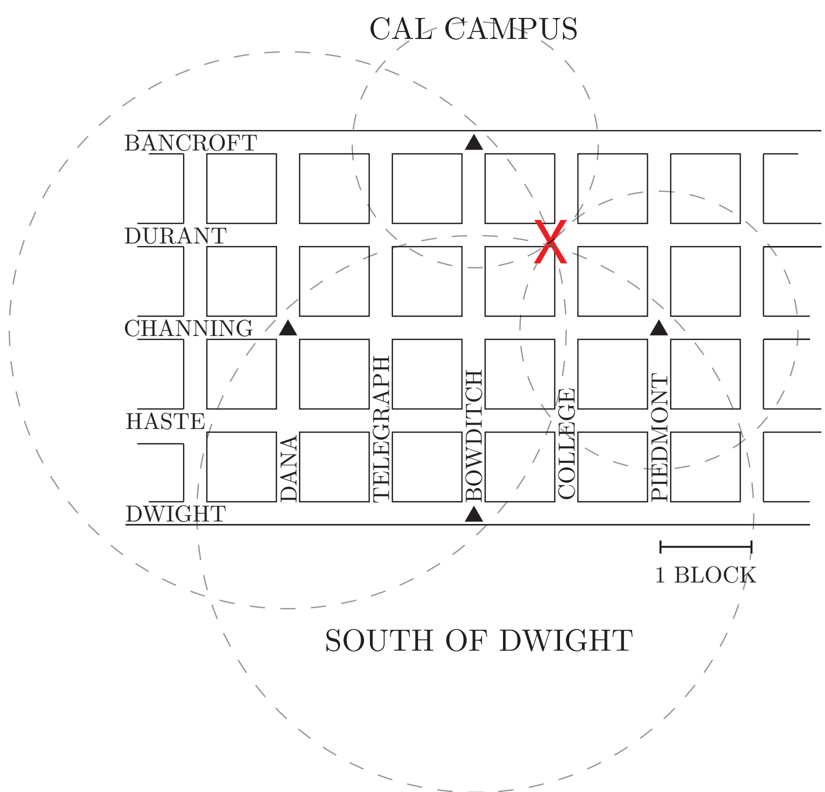
Sensor	Distance
N	1.3
W	3
E	1.5
S	3

On the map provided, identify where Mr. Muffin is!

[1] http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg



Answer:



(b) Can you set this up as a system of equations? Is it linear? If it's not linear, can you think of a way to make it linear? Now, how do you set this up in matrix form?
Hint: Set $(0,0)$ to be Channing and Bowditch.
Hint 2: Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
Hint 3: You don't need all 4 equations. You have two unknowns, x and y. You know from lecture that you need three circles to uniquely find a point. How can you use the third circle/equation to get two

equations and two unknowns?

Answer:

First, set up the system of equations:

$$(x-0)^2+(y-2)^2 = 1.3^2$$
$$(x+2)^2+(y-0)^2 = 3.0^2$$
$$(x-2)^2+(y-0)^2 = 1.5^2$$

Simplify out:

$$x^2+y^2-4y+4 = 1.3^2$$
$$x^2+4x+4+y^2 = 3.0^2$$
$$x^2-4x+4+y^2 = 1.5^2$$

Then subtract equation (1) from equations (2) and (3):

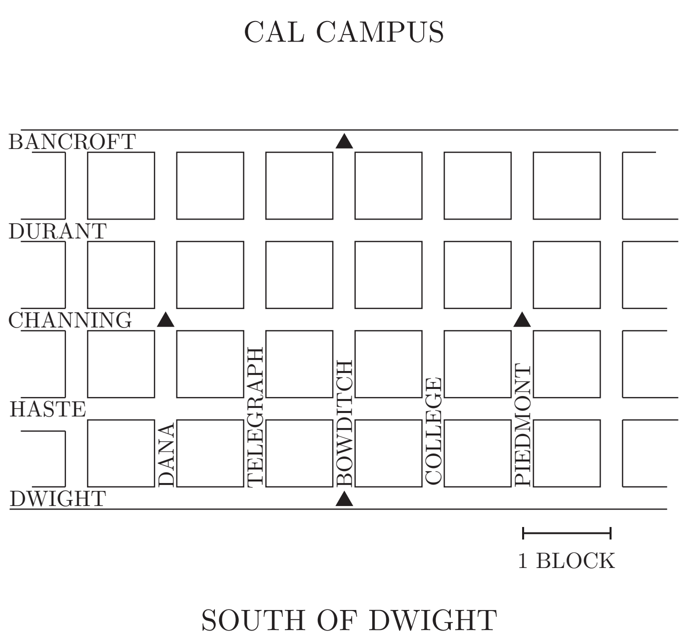
$$4x+4y = 3.0^2-1.3^2$$
$$-4x+4y = 1.5^2-1.3^2$$

This solves to $x = 0.84, y = 0.98$ which is roughly College and Durant.

(c) Suppose Mr. Muffin is moving fast, and by the time you get to destination in part (a) he's already run off! You check the logs of the cell towers again, and see the following updated messages:

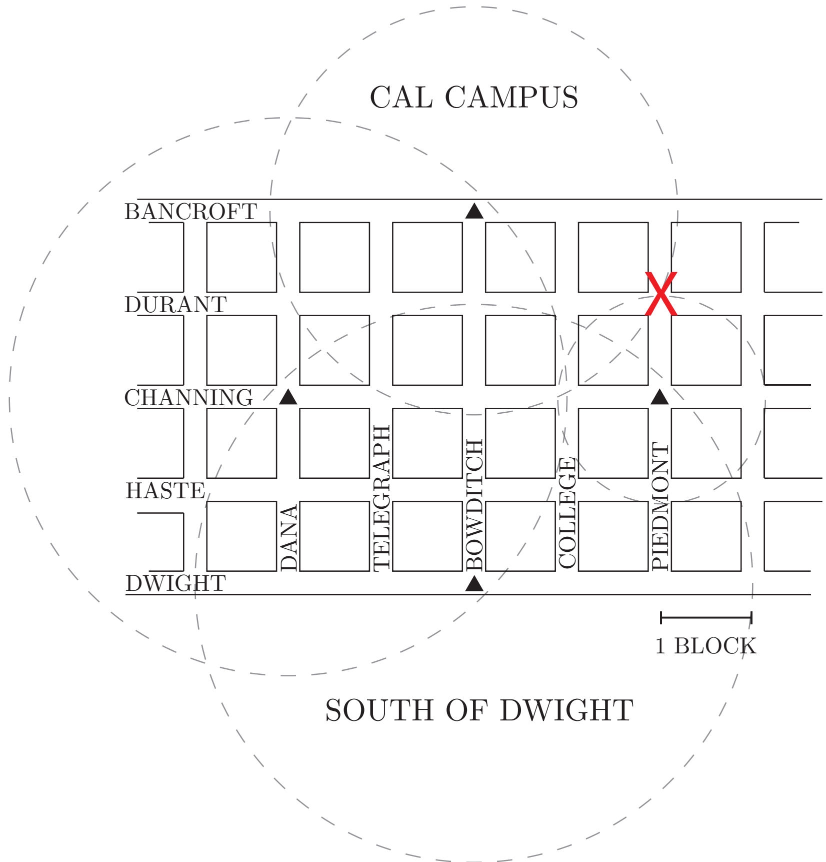
Sensor	Distance
N	2.2
W	Out of Range
E	1.1
S	Out of Range

Can you find Mr. Muffin?



Answer:

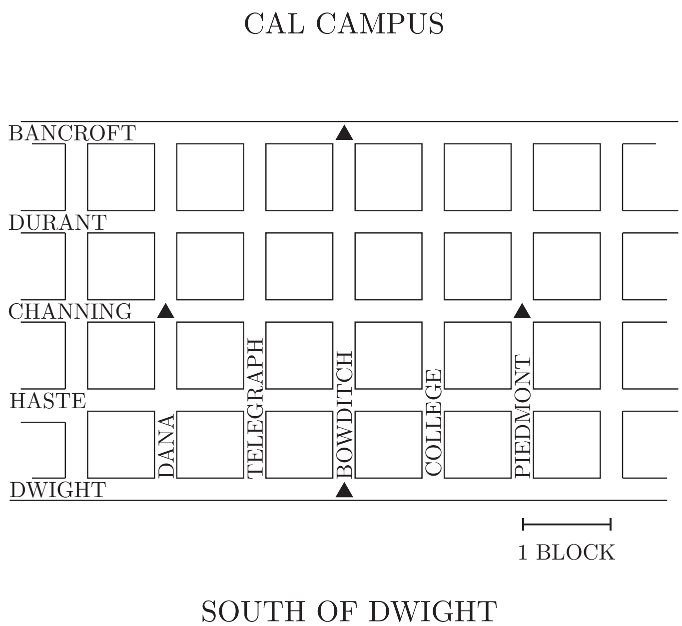
With two out of range sensors, you might think that you will not be able to find a unique solution (you need 3 circles to intersect at a point.) The trick is that out of range still provide information on where Mr. Muffin is NOT. See the diagram below.



(d) Mr. Muffin is a very mischievous puppy, and while playing and running around he damaged his collar. The transmitter on his collar will still send a signal to the receiver towers, but the distance sensor has noise. You check the logs of the cell towers, and they have received the following messages:

Sensor	Distance
N	1.7 ± 0.5
W	2.1 ± 0.2
E	Out of Range
S	Out of Range

On the map provided, identify where Mr. Muffin is! Can you find exactly where he is?



Answer:

You can't find exactly where he is, but you know he is somewhere between Dana/Telegraph and Bancroft. See the diagram below.