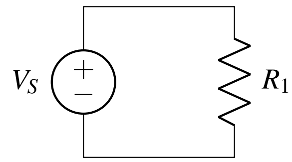
EECS 16A Designing Information Devices and Systems I
Fall 2018 Discussion 6B

1. A Simple Circuit

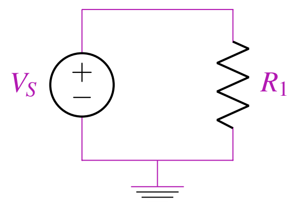
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

Answer:

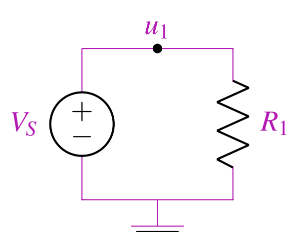
There are two nodes in this circuit and thus two choices for the ground node. The choice of ground does not matter. We will use the ground node shown below:



(b) With your choice of ground, label the node potentials for every node in the circuit.

Answer:

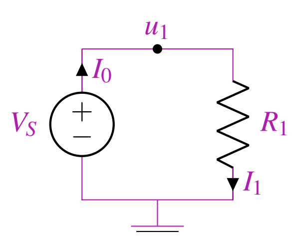
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all of the branch currents. Does the direction you pick matter?

Answer:

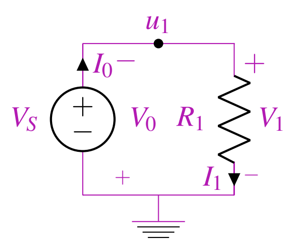
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

Answer:

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix}$$

$\mathbf{A}$ will be a 3×3 matrix since there are three unknowns in the circuit, the two currents I_0 and I_1 and the one potential u_1.

(f) Use KCL to find as many equations as you can for the matrix.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 = 0$. Thus, so far our matrix is as follows:

$$\begin{bmatrix} 1 & -1 & 0 \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \end{bmatrix}$$

(g) Use IV relations to find the remaining the equations for the matrix.

Answer:

We know that the difference in potentials across the voltage source must be the voltage on the voltage source, i.e.

$$-V_0 = V_S \tag{1}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$V_1 = I_1 R_1 \tag{2}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \end{aligned} \tag{3}$$

Substituting expressions from Equations (1) and (2) into Equation (3), we have:

$$\begin{aligned} -u_1 = -V_S &\implies u_1 = V_S \\ u_1 = I_1 R_1 &\implies -I_1 R_1 + u_1 = 0 \end{aligned} \tag{4}$$

Our matrix is then:

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & -R_1 & 1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_S \\ 0 \end{bmatrix}$$

(h) Solve the system of equations if $V_S = 5\,\text{V}$ and $R_1 = 5\,\Omega$.

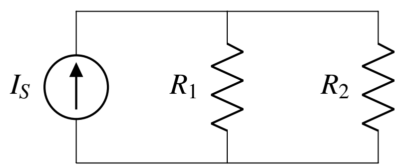
Answer:

By plugging the given values into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$$

2. A Slightly More Complicated Circuit

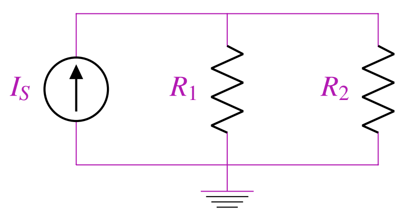
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

Answer:

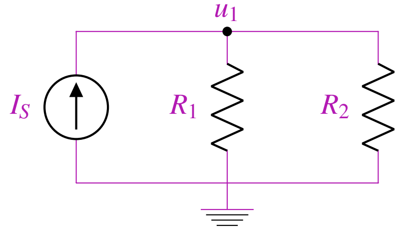
There are two nodes in this circuit and thus two choices for the ground node. The choice of ground does not matter. We will use the ground node shown below:



(b) With your choice of ground, label the node potentials for every node in the circuit.

Answer:

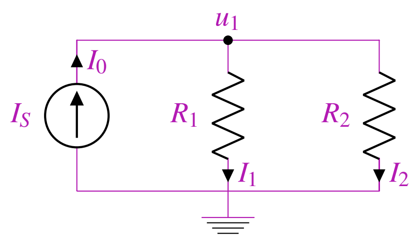
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all of the branch currents. Does the direction you pick matter?

Answer:

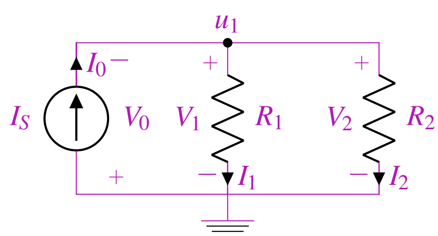
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

Answer:

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix}$$

$\mathbf{A}$ will be a 4×4 matrix since there are four unknowns in the circuit, the currents I_0, I_1, and I_2 and the one potential u_1.

(f) Use KCL to find as many equations as you can for the matrix.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 - I_2 = 0$. Thus, so far our matrix is as follows:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \\ ? \end{bmatrix}$$

(g) Use IV relations to find the remaining the equations for the matrix.

Answer: We know that the current through the current source must be the value of the current source, i.e.

$$I_0 = I_S \tag{5}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$\begin{aligned} V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \end{aligned} \tag{6}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \\ u_1 - 0 &= V_2 \end{aligned} \tag{7}$$

Using Equation (5) and substituting expressions from Equation (6) into Equation (7), we have:

$$\begin{aligned} I_0 &= I_S \\ u_1 = I_1 R_1 \implies -I_1 R_1 + u_1 &= 0 \\ u_1 = I_2 R_2 \implies -I_2 R_2 + u_1 &= 0 \end{aligned} \tag{8}$$

Our matrix is then:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -R_1 & 0 & 1 \\ 0 & 0 & -R_2 & 1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ I_S \\ 0 \\ 0 \end{bmatrix}$$

(h) Solve the system of equations if $I_S = 5\,\text{A}$, $R_1 = 5\,\Omega$, and $R_2 = 10\,\Omega$.

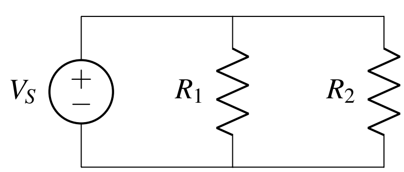
Answer:

By plugging in the values into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 5 \\ 3.33 \\ 1.67 \\ 16.67 \end{bmatrix}$$

3. (PRACTICE) Another Circuit

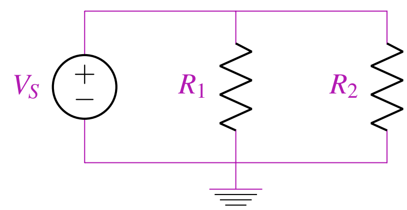
For the circuit shown below, find the voltages across all the elements and the currents through all the elements.



(a) In the above circuit, pick a ground node. Does your choice of ground matter?

Answer:

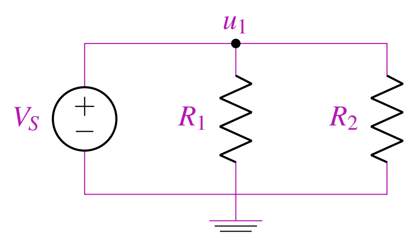
There are two nodes in this circuit and thus two choices for the ground node. The choice of ground does not matter. We will use the ground node shown below:



(b) With your choice of ground, label the node potentials for every node in the circuit.

Answer:

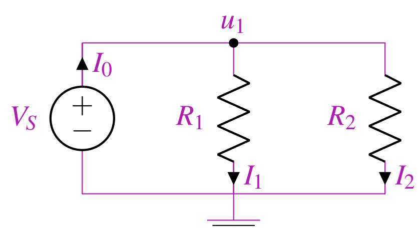
Since this circuit only has two nodes, there will only be one additional node potential.



(c) Label all the branch currents. Does the direction you pick matter?

Answer:

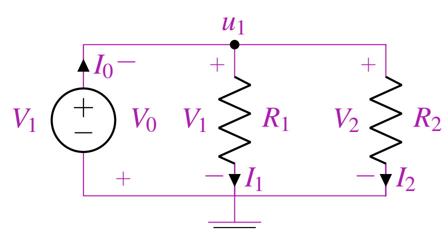
When labeling the currents through branches, the direction you pick does not matter.



(d) Draw the $+/-$ labels on every element. What convention must you follow?

Answer:

When drawing the $+/-$ labels, you must follow the passive sign convention. That is, current flows into the $+$ terminal of every element.



(e) Set up a matrix equation in the form $\mathbf{A}\vec{x} = \vec{b}$ to solve for the unknown node potentials and currents. What are the dimensions of the matrix $\mathbf{A}$?

Answer:

$$\begin{bmatrix} ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix}$$

A will be a 4×4 matrix since there are four unknowns in the circuit, the currents I_0, I_1, and I_2 and the one potential u_1.

(f) Use KCL to find as many equations as you can for the matrix.

Answer:

KCL gives us one equation for the node at the top, namely that $I_0 - I_1 - I_2 = 0$. Thus, so far our matrix is as follows:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ ? & ? & ? & ? \\ ? & ? & ? & ? \\ ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ ? \\ ? \\ ? \end{bmatrix}$$

(g) Use IV relations to find the remaining equations for the matrix.

Answer: We know that the difference in potentials across the voltage source must be the voltage on the voltage source, i.e.

$$-V_0 = V_S. \tag{9}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$\begin{aligned} V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \end{aligned} \tag{10}$$

Writing the equations for node potentials we have:

$$\begin{aligned} 0 - u_1 &= V_0 \\ u_1 - 0 &= V_1 \\ u_1 - 0 &= V_2 \end{aligned} \tag{11}$$

Substituting expressions from Equations (9) and (10) into Equation (11), we have:

$$\begin{aligned} -u_1 = -V_S &\implies u_1 = V_S \\ u_1 = I_1 R_1 &\implies -I_1 R_1 + u_1 = 0 \\ u_1 = I_2 R_2 &\implies -I_2 R_2 + u_2 = 0 \end{aligned} \tag{12}$$

Our matrix is then:

$$\begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -R_1 & 0 & 1 \\ 0 & 0 & -R_2 & 1 \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ V_S \\ 0 \\ 0 \end{bmatrix}$$

(h) Solve the system of equations if $V_S = 5\,\mathrm{V}$, $R_1 = 5\,\Omega$, and $R_2 = 10\,\Omega$.

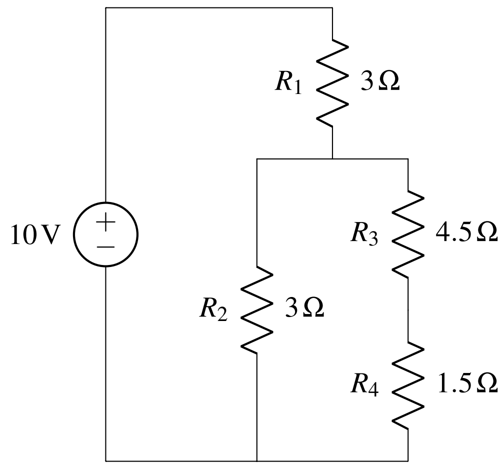
Answer:

By plugging in the values into the system of equations, we get:

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 1 \\ 0.5 \\ 5 \end{bmatrix}$$

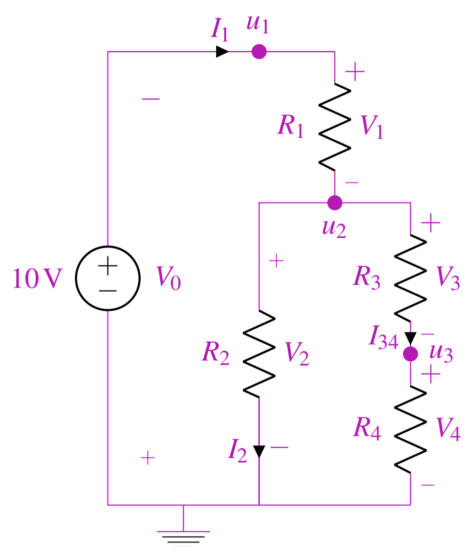
4. Mechanical Circuits

Find the voltages across and currents flowing through all of the resistors.



Answer:

First, we label the ground node. Then, we label all the node potentials, branch currents and identify $+/-$ labels for each element:



Expressing the voltage differences in terms of node potentials, we get

$$\begin{aligned} V_0 &= 0 - u_1 \\ V_1 &= u_1 - u_2 \\ V_2 &= u_2 - 0 \\ V_3 &= u_2 - u_3 \\ V_4 &= u_3 - 0 \end{aligned} \tag{13}$$

Now we set up our KCL equation:

$$I_1 - I_2 - I_{34} = 0$$

We can use KVL and IV relations to find the rest of the equations. We know that the difference in potentials across the voltage source must be the voltage on the voltage source, i.e.

$$-V_0 = 10. \tag{14}$$

We also know that the voltage across the resistor is equal to the current times the resistance, i.e.

$$\begin{aligned} V_0 &= -10 \\ V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \\ V_3 &= I_{34} R_3 \\ V_4 &= I_{34} R_4 \end{aligned} \tag{15}$$

Substituting expressions from Equations (14) and (15) into Equation (13), we have

$$\begin{aligned} u_1 &= 10 \\ u_1 - u_2 = I_1 R_1 &\implies -I_1 R_1 + u_1 - u_2 = 0 \\ u_2 = I_2 R_2 &\implies -I_2 R_2 + u_2 = 0 \\ u_2 - u_3 = I_{34} R_3 &\implies -I_{34} R_3 + u_2 - u_3 = 0 \\ u_3 = I_{34} R_4 &\implies -I_{34} R_4 + u_3 = 0 \end{aligned}$$

We can now set the system up in a matrix-vector product form and use Gaussian Elimination/IPython to solve:

$$\begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -R_1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -R_2 & 0 & 0 & 1 & 0 \\ 0 & 0 & -R_3 & 0 & 1 & -1 \\ 0 & 0 & -R_4 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_{34} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

This returns the array:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_{34} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 2 \\ \frac{4}{3} \\ \frac{2}{3} \\ 10 \\ 4 \\ 3 \end{bmatrix}.$$

Substituting the values of node potentials in Equation (13), we have

$$\begin{aligned} I_1 &= 2\,\text{A}, \\ I_2 &= 4/3\,\text{A}, \\ I_{34} &= 2/3\,\text{A}, \\ V_1 &= 6\,\text{V}, \\ V_2 &= 4\,\text{V}, \\ V_3 &= 3\,\text{V}, \\ V_4 &= 1\,\text{V}. \end{aligned}$$