4. Temperature Sensor

(a) $V_{out} = \frac{V_s R_2}{R_1+R_2}$

The current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1 + R_2$, so $I = \frac{V_s}{R_1+R_2}$

So, V_{out}, which measures the voltage drop over R_2, is equal to (or we could've used the Voltage Divider formula directly to obtain):

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_2} \cdot R_2 = \frac{V_s R_2}{R_1 + R_2}$$

Thus, $V_{out} = V_2 = \frac{V_s R_2}{R_1+R_2}$

(b) $T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{(V_{out} - V_s) R_o \alpha}$

Similarly, the current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1 + R_2 = R_1 + R_o(1 + \alpha T)$, so $I = \frac{V_s}{R_1+R_o(1+\alpha T)}$

So, V_{out}, which measures the voltage drop over R_2, is equal to:

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_o(1 + \alpha T)} \cdot R_o(1 + \alpha T) = \frac{V_s R_o(1 + \alpha T)}{R_1 + R_o(1 + \alpha T)}$$

Thus, $V_{out} = V_2 = \frac{V_s R_o(1+\alpha T)}{R_1+R_o(1+\alpha T)}$, which gives us that: $V_{out} \cdot (R_1 + R_o(1 + \alpha T)) = V_s R_o(1 + \alpha T)$

So, $V_{out} R_1 + V_{out} R_o + V_{out} R_o \alpha T = V_s R_o + V_s R_o \alpha T$, which gives:

$$V_{out} R_o \alpha T - V_s R_o \alpha T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies (V_{out} - V_s) R_o \alpha \cdot T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{(V_{out} - V_s) R_o \alpha}$$

(c) $T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{-V_s R_o \alpha + V_{out} R_1 \beta + V_{out} R_o \alpha}$

Again, similarly, the current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1' + R_2 = R_1(1 + \beta T) + R_o(1 + \alpha T)$, so $I = \frac{V_s}{R_1(1+\beta T)+R_o(1+\alpha T)}$

So, V_{out}, which measures the voltage drop over R_2, is equal to:

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1(1 + \beta T) + R_o(1 + \alpha T)} \cdot R_o(1 + \alpha T) = \frac{V_s R_o(1 + \alpha T)}{R_1(1 + \beta T) + R_o(1 + \alpha T)}$$

Thus, $V_{out} = V_2 = \frac{V_s R_o(1+\alpha T)}{R_1(1+\beta T)+R_o(1+\alpha T)}$, which gives us that:

$$V_{out} \cdot (R_1(1 + \beta T) + R_o(1 + \alpha T)) = V_s R_o(1 + \alpha T)$$

$$\implies V_{out} R_1 + V_{out} R_1 \beta T + V_{out} R_o + V_{out} R_o \alpha T = V_s R_o + V_s R_o \alpha T$$

$$\implies (V_{out} R_1 \beta + V_{out} R_o \alpha - V_s R_o \alpha) \cdot T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{-V_s R_o \alpha + V_{out} R_1 \beta + V_{out} R_o \alpha}$$

(d) No, it can't.

Here, we use the derived formula of voltage dividers directly to obtain the voltage drop over R_2:

$$V_2 = \frac{V_s \cdot R_{o2} \cdot (1 + \alpha T)}{(R_{o1} + R_{o2}) \cdot (1 + \alpha T)} = \frac{V_s R_{o2}}{R_{o1} + R_{o2}}$$

Thus, $V_{out} = V_2 = \frac{V_s R_{o2}}{R_{o1}+R_{o2}}$, which is independent of the variable T, which implies that we cannot express the temperature T as an equation in terms of the measurable variables. Therefore, this circuit (specifically the measurements of V_{out}) cannot be used to measure temperature.