3. Figuring Out The Tips.

(a) No, we can't. Consider these two Cases:

(1) $T_1 = 2,\ T_2 = 2,\ T_3 = 6,\ T_4 = -2,\ T_5 = 4,\ T_6 = 0.$

which would give $P_1 = 2,\ P_2 = 4,\ P_3 = 2,\ P_4 = 1,\ P_5 = 2,\ P_6 = 1.$

(2). $T_1 = 1,\ T_2 = 3,\ T_3 = 5,\ T_4 = -1,\ T_5 = 3,\ T_6 = 1$

which would also give: $P_1 = 2,\ P_2 = 4,\ P_3 = 2,\ P_4 = 1,\ P_5 = 2,\ P_6 = 1$

So, the two different assignments of T_1 to T_6 result in the same P_1 to P_6.

(b). Yes, we can. Since we have this system of linear equations:

$$\begin{aligned} 0.5\,T_1 + 0.5\,T_2 &= P_1 \\ 0.5\,T_2 + 0.5\,T_3 &= P_2 \\ 0.5\,T_3 + 0.5\,T_4 &= P_3 \\ 0.5\,T_4 + 0.5\,T_5 &= P_4 \\ 0.5\,T_5 + 0.5\,T_1 &= P_5. \end{aligned} \Rightarrow \begin{bmatrix} 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix}$$

which leads to an augmented matrix, and by multiplying each row by 2:

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 2P_1 \\ 0 & 1 & 1 & 0 & 0 & 2P_2 \\ 0 & 0 & 1 & 1 & 0 & 2P_3 \\ 0 & 0 & 0 & 1 & 1 & 2P_4 \\ 1 & 0 & 0 & 0 & 1 & 2P_5 \end{array}\right]$$

For Row 5, subtract Row 1, Row 3 and add Row 2, Row 4. $\Rightarrow$.

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 2P_1 \\ 0 & 1 & 1 & 0 & 0 & 2P_2 \\ 0 & 0 & 1 & 1 & 0 & 2P_3 \\ 0 & 0 & 0 & 1 & 1 & 2P_4 \\ 0 & 0 & 0 & 0 & 2 & 2(P_2+P_4+P_5-P_1-P_3) \end{array}\right] \xrightarrow{\text{Divide Row 5 by 2.}} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 0 & 2P_1 \\ 0 & 1 & 1 & 0 & 0 & 2P_2 \\ 0 & 0 & 1 & 1 & 0 & 2P_3 \\ 0 & 0 & 0 & 1 & 1 & 2P_4 \\ 0 & 0 & 0 & 0 & 1 & P_2+P_4+P_5-P_1-P_3 \end{array}\right]$$

Now, Row 1: Subtract Row 2, Row 4, and Add Row 3, Row 5.

Row 2: Subtract Row 3, Row 5, Add Row 4.

Row 3: Subtract Row 4. Add Row 5.

Row 4: Subtract Row 5. $\Rightarrow$.

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & 0 & P_1+P_3+P_5-P_2-P_4 \\ 0 & 1 & 0 & 0 & 0 & P_1+P_2+P_4-P_3-P_5 \\ 0 & 0 & 1 & 0 & 0 & P_2+P_3+P_5-P_1-P_4 \\ 0 & 0 & 0 & 1 & 0 & P_1+P_3+P_4-P_2-P_5 \\ 0 & 0 & 0 & 0 & 1 & P_2+P_4+P_5-P_1-P_3 \end{array}\right]$$

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Thus, we can deduce the tips T_1 to T_5 from plates P_1 to P_5 by calculating a unique solution:

$$T_1 = P_1 - P_2 + P_3 - P_4 + P_5$$
$$T_2 = P_1 + P_2 - P_3 + P_4 - P_5$$
$$T_3 = -P_1 + P_2 + P_3 - P_4 + P_5$$
$$T_4 = P_1 - P_2 + P_3 + P_4 - P_5$$
$$T_5 = -P_1 + P_2 - P_3 + P_4 + P_5$$

(C). If and only if n is an odd integer.

For any $n \in \mathbb{Z}$ that's even, there would always be a vector that could be expressed as a linear combination of the rest of the equations, meaning that the n equations deduced are linearly dependent, so the n variables can't be solved.

On the other hand, if n is odd, then all the equations (or vectors) are linearly independent, which means that we'll have n independent equations for n variables, so we can deduce a distinct answer.

Therefore, we can determine the individual tips iff n is odd.