Circuit Analysis

- KCL: The net current flowing out of (= into) any junction of a circuit is 0.
- KVL: $\sum_{loop} V_k = 0$
- Ohm's Law: $V_{elem} = I_{elem}R$ (Volt = Ampere · Ohm Ω)
- Passive sign convention: Positive current should enter the positive terminal and exit the negative terminal of an element. Positive power means that power is dissipated, and negative power means that power is being generated.
- Resistors: in series $R_{eq} = R_1 + R_2$; in parallel $R_{eq} = R_1 || R_2 = \frac{R_1 R_2}{R_1 + R_2}$
- Voltage divider: $V_{out} = \frac{R_{out}}{R_1 + R_{out}} \cdot V_s$
- $I = \frac{dQ}{dt}$ (Coulombs/second)
- $R = \rho \cdot \frac{L}{A}$
- $P = VI = I^2 R = \frac{V^2}{R} = \frac{dE}{dt} = V\frac{dQ}{dt}$

Superposition and Equivalence

- Superposition: Zero all but 1 independent source (Voltage source: replace with a wire; Current source: replace with an open circuit)
- Def 15.1 (equivalent circuit): If we pick two terminals within a circuit, we say that another circuit is equivalent to the original circuit if it exhibits the same $I - V$ relationship at those two terminals. (In short, Two circuits are equivalent if they have the same $I - V$ relationship.)
- Thevenin equivalent circuit: 1 voltage source and 1 resistor ("in series")
  - Find V_{Th}: Connect an open circuit across the two output terminals and measure the voltage across them. This measured V_{OC} equals V_{Th}.
- Norton equivalent circuit: 1 current source and 1 resistor ("in parallel")
  - Find I_{No}: Connect a short circuit across the two output terminals and measure the current through it. This measured I_{SC} equals I_{No}.
- For R_{eq} between nodes a, b:
  1. Calculate V_{th} and I_{no} (superposition if necessary), and then $R_{eq} = \frac{V_{th}}{I_{no}}$
     N.B. Works only if $\exists$ at least one independent source in the circuit. Else, $V_{th} = I_{no} = 0$.
  2. Zero out all independent sources and apply a V_{test} or I_{test} to calculate the resulting I_{test} or V_{test} respectively. $R_{eq} = \frac{V_{test}}{I_{test}}$.
     N.B. Works for any circuit. When in doubt, use this.
- The power dissipated by the source in the original circuit is not the same as the power dissipated in the Thevenin equiv. circuit, but the power through R_{load} is the same! Thevenin equiv. can be used to calculate the power through elements that are not part of the circuit that was transformed.
- Extra: $R_{Th} = R_{No}$; Norton $\rightarrow$ Thevenin: $V_{th} = I_{No}R_{No}$; Thevenin $\rightarrow$ Norton: $I_{No} = \frac{V_{Th}}{R_{Th}}$;
- Extra: Resistors in parallel have an equivalent resistance $<$ any of the individual resistors (positive resistances).
- Extra: The power generated by the Thevenin equivalent circuit $\neq$ total power generated in the original circuit.

Capacitors

- $C = I \cdot t = \frac{P}{V}t = \frac{Energy}{V}$
- $Q = CV_c \iff C = \frac{Q}{V_c}$ unit is Farad (F), $1A \cdot 1s$
- $I = \frac{dQ}{dt} = C\frac{dV_c}{dt}$ (general), so current is flowing through the capacitor $\iff$ the voltage across the capacitor is changing with time. ($I_c(t) = C\frac{dV_c(t)}{dt}$)
- $\implies V_c(t) = \frac{I}{C}t + V_c(0)$ Only valid when the current is constant over time.
- $C_{eq} = \frac{I_{test}}{\frac{dV_{test}}{dt}}$
- Capacitors: in series $C_{eq} = C_1 || C_2 = \frac{C_1C_2}{C_1+C_2}$; in parallel $C_{eq} = C_1 + C_2$
- $C = \epsilon\frac{A}{d}$
- Energy stored in capacitor: $E = \frac{1}{2}C_{eq}V^2$
- For $V_c(0) = 0V$, then $V = \frac{I_s t}{C}$. If we measure the voltage at a known time t, we can solve for the capacitance. However, as time continues to pass, the voltage across the capacitor (and also the charge stored on the capacitor) will grow to ∞. It is very challenging to build an ideal current source that works over this large range of voltages, so our model quickly becomes unrealistic $\implies$ periodic current source.
- Consider a square current I_s
  - I_s is constant from $t = 0$ to $t = \frac{\tau}{2}$, so $V_c(t) = \frac{I_s}{C}t$ when $0 \leq t \leq \frac{\tau}{2}$
  - Similarly, $V_c(t) = -\frac{I_s}{C}(t - \frac{\tau}{2}) + \frac{I_s\tau}{2C}$ when $\frac{\tau}{2} < t < \tau$

Op-Amp and Golden Rules

- Apply KCL, then use Golden Rule(s) to find desired relationships.
- Op-amp (operational amplifier): $A > 1$ is the gain. Op-amp acts $\sim$ a comparator since A is very large.
- For an ideal op-amp, we have $A \to \infty$, which implies Golden Rules:
  1. $I_+ = I_- = 0$ holds regardless of whether there's negative feedback or not.
  2. $U_+ = U_-$ holds $\iff$ there's negative feedback.
- Checking if an op-amp is in negative feedback:
  1. Zero out all independent sources like we did in Thevenin-Norton Equivalences.
  2. Increase the output. If the feedback coming from the circuit as a result of increasing the output is in the opposite direction (i.e. decreasing), then the circuit is in negative feedback. If not, the circuit is in positive feedback or zero feedback.
- In this negative feedback setup, $V_{out} = V_{in}(1 + \frac{R_1}{R_2})$. Thus, As long as the two resistors are produced with the same error rate ϵ, i.e., they have resistance $(1+\epsilon)R_1$ and $(1+\epsilon)R_2$, then the ratio between their resistance will remain the same, so $\frac{V_{out}}{V_{in}}$ is the same.
- Inverting an op-amp gives $V_{out} = -\frac{R_f}{R_{in}} V_{in}$. Here, gain $A = -\frac{R_f}{R_{in}}$ can have be any negative value.
- This models an artificial neuron, withs $V_{ana} = -\frac{R_3}{R_1}V_1 - \frac{R_3}{R_2}V_2$, and so $V_{out} = -\frac{R_4}{R_4}V_{ana} = \frac{R_3}{R_1}V_1 + \frac{R_3}{R_2}V_2$
- Unity gain buffer to cancel out the effects of loading, allowing $V_{speaker} = V_{DAC}\frac{A}{1+A} \sim V_{DAC}$ as $A \to \infty$.
- Buffers are a powerful tool because they allow us to split circuits into blocks that we can analyze separately and then combine later. When circuit blocks behave the same way regardless of what they're are connected to, we don't need to worry about what's inside, making it much easier to design complex circuits.
- Extra: An op-amp cannot operate without externally supplied power. Any power an op-amp delivers to a load comes from the supply voltages since it can't generate its own energy.
- Extra: Since an op-amp has infinite input resistance, there's no current flowing into the input terminals, so it doesn't change the voltage of any circuit it's connected to.

Extra Sanity Checks

- Always return to the 7-step nodal analysis if stuck or unsure.
- Care: Units!!!
- The choice of ground does not matter. When labeling the currents through branches, the direction you pick does not matter.
- A Norton equivalent of a voltage source is not necessary, since a voltage source is a basic element. The Thevenin equivalent is just a voltage source with voltage V_s, that is, $R_{th} = 0$.
- A Thevenin equivalent of a current source is not necessary because a current source is a basic element and cannot be represented as a voltage source. The Norton equivalent is just a current source with current I_s, that is, $R_{no} = \infty$.
- Make sure $V_{DD} > V_{SS}$ (equal is not really useful...)
- We can connect an O-A to any other circuit, and the O-A will not disturb that circuit because it does not load the circuit (it is an open circuit). The output of the O-A can be connected to any other circuit (except a voltage source) to get the desired/expected voltage out of the O-A.
- $\implies$ current source should not enter a terminal directly (parallel with a resistor and then enter, Norton-Thev. equiv. into a voltage source with a resistor).
- Positive gain $\iff$ non-inverting O-A; negative gain $\iff$ inverting O-A.
- Incident Matrix is the transpose of the matrix created for nodal analysis.
- Extra: Capacitors connected in series must have the same charge Q, and thus can calculate their voltage in proportion with $Q = CV$.
- Ground is a reference, the point we define as 0 in a circuit. There's only 1 GND, so we can connect all the grounds. But, we can't even connect ground to other nodes that happens to be at $0V$.