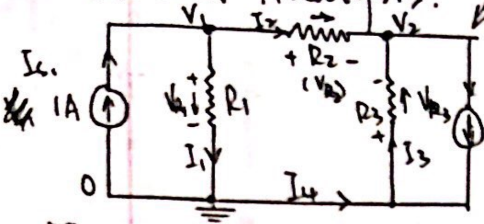
1. Circuit Analysis. Labeling. So $A\vec{x}=\vec{b}$ where



$$\vec{x} = [I_1\ I_2\ I_3\ I_4\ V_{R1}\ V_{R2}\ V_{R3}\ V_1\ V_2]^T.$$

$I_2 = 1 - I_1$, $I_3 = I_1 + 2$

Using KCL, so:

$$\begin{cases} I_{s1} = 1A = I_4 + I_2 \\ I_2 + I_3 = I_{s2} = 3A \\ I_1 = I_4 + I_{s1} = I_4 + 1A \\ (I_3 = I_4 + I_{s2} = I_4 + 3A). \end{cases} \Rightarrow \begin{cases} I_1 + I_2 = 1A & (1) \\ I_2 + I_3 = 3A & (2) \\ I_1 - I_4 = 1A & (3) \end{cases}$$

Then, using Ohm's Law, so:

and also:

$$\begin{cases} V_{R1} = R_1 \cdot I_1 = 10 I_1 \\ V_{R2} = R_2 \cdot I_2 = 20 I_2 = 20 - 20 I_1 \\ V_{R3} = R_3 \cdot I_3 = 50 I_3 = 50 I_1 + 100 \\ V_1 - 0 = V_{R1} \\ V_1 - V_2 = V_{R2} \Rightarrow V_2 = 30 I_1 - 20 \\ V_2 - 0 = -V_{R3} \Rightarrow V_2 = -50 I_1 - 100 \end{cases} \Rightarrow \begin{cases} V_{R1} - R_1 \cdot I_1 = 0 & (4) \\ V_{R2} - R_2 \cdot I_2 = 0 & (5) \\ V_{R3} - R_3 \cdot I_3 = 0 & (6) \\ V_1 - V_{R1} = 0 & (7) \\ V_1 - V_2 - V_{R2} = 0 & (8) \\ V_2 + V_{R3} = 0 & (9) \end{cases}$$

$V_2 = V_1 - 20 I_2$

Using Equations (1)-(9), we can setup $A\vec{x}=\vec{b}$ as:

$I_2 = 1 - I_1$
$I_3 = I_1 + 2$
$I_4 = I_1 - 1$
$V_1 = -10 I_1$
$V_2 = 30 I_1 - 20 = 50 I_1 - 100$
$I_1 = -1A$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -R_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -R_2 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -R_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_{R1} \\ V_{R2} \\ V_{R3} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

With $R_1 = 10\Omega$

$R_2 = 20\Omega$

$R_3 = 50\Omega$

So,

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ V_{R1} \\ V_{R2} \\ V_{R3} \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1\ A \\ 2\ A \\ 1\ A \\ -2\ A \\ -10\ V \\ 40\ V \\ 50\ V \\ -10\ V \\ -50\ V \end{bmatrix}$$

2. Cell Phone Battery

(a) 35.1 hours

Since $P = I \cdot V$, so we have

$$I = \frac{P}{V} = \frac{0.3W}{3.8V} = 7.89 * 10^{-2}A = 78.9mA$$

Then, with $C = I \cdot t$, so we have

$$t = \frac{C}{I} = \frac{2770mAh}{78.9} = 35.1hr$$

Thus, a Pixel's full battery will last 35.1 hours under regular usage conditions.

(b) $6.22 * 10^{22}$ electrons

Since 2770 mAh = 2770 mAh $\cdot \frac{3600s}{1h} = 9.972 \cdot 10^6$ mAs, and given that 1 mC = 1 mAs,

so $C_{pixel} = 2770$ mAh $= 9.972 \cdot 10^6$ mAs $= 9.972 \cdot 10^6$ mC

So, there are $\frac{C_{pixel}}{C_{electron}} = \frac{9.972 \cdot 10^3 C}{1.602 \cdot 10^{-19} C} = 6.22 * 10^{22}$ usable electrons worth of charge.

(c) $3.79 \cdot 10^4$ J

Since we could calculate that:

$$E_{discharge} = P \cdot t = 0.3\ W \cdot 35.1\ hr \cdot \frac{3600\ s}{1\ hr} = 3.79 \cdot 10^4\ Ws = 3.79 \cdot 10^4\ J$$

Thus, we have that

$$E_{charge} = E_{discharge} = 3.79 \cdot 10^4\ J$$

So, $3.79 \cdot 10^4$ J is the energy necessary for recharging a completely discharged cell phone battery.

(d) \$0.04

The total energy used by recharging for 31 days is:

$$E_{total} = E_{charge} \cdot 31 = 3.79 \cdot 10^4\ J \cdot 31 = 1.175 \cdot 10^6\ J = 1.175 \cdot 10^6\ Ws$$

So, we can transform its unit to get:

$$E_{total} = 1.175 \cdot 10^6\ Ws \cdot \frac{1\ kW}{1000\ W} \cdot \frac{1\ hr}{3600\ s} = 0.326\ kWh$$

Thus, I would need to pay $0.326\ kWh \cdot \frac{\$0.12}{1\ kWh} = \$0.04$ for recharging for the month of October.

(e)

First, $R = 200m\Omega = 200m\Omega \cdot \frac{1\Omega}{1000m\Omega} = 0.2\ \Omega$. We consider $R_{bat} = 1m\Omega, 1\Omega, 10k\Omega$ separately below.

Case 1 ($R_{bat} = 1\ m\Omega$): With $R_{eq} = R + R_{bat} = 200m\Omega + 1m\Omega = 201m\Omega$, so

$$I_{bat} = I = \frac{V}{R_{eq}} = \frac{5V}{201m\Omega} = 24.88A$$

So the power dissipated across R_{bat} is:

$$P_{bat} = I_{bat}V_{bat} = I_{bat}^2 R_{bat} = (24.88A)^2 \cdot 1m\Omega = 0.62\ W$$

Thus, using the results we got from part (d), so it takes the battery

$$t = \frac{E_{total}}{P} = \frac{3.79 \cdot 10^4\ Ws}{0.62W} = 6.11 \cdot 10^4 s = 6.11 \cdot 10^4 s \cdot \frac{1hr}{3600s} = 16.98\ hr$$

Case 2 ($R_{bat} = 1\ \Omega$): With $R_{eq} = R + R_{bat} = 0.2\Omega + 1m\Omega = 1.2m\Omega$, so

$$I_{bat} = I = \frac{V}{R_{eq}} = \frac{5V}{1.2\Omega} = 4.17A$$

So the power dissipated across R_{bat} is:

$$P_{bat} = I_{bat}V_{bat} = I_{bat}^2 R_{bat} = (4.17A)^2 \cdot 1\Omega = 17.39\ W$$

Thus, using the results we got from part (d), so it takes the battery

$$t = \frac{E_{total}}{P} = \frac{3.79 \cdot 10^4\ Ws}{17.39W} = 2.18 \cdot 10^3 s = 2.18 \cdot 10^3 s \cdot \frac{1hr}{3600s} = 0.605\ hr = 36.3\ min$$

Case 3 ($R_{bat} = 10\ k\Omega$): With $R_{eq} = R + R_{bat} = 0.2\Omega + 10k\Omega = 10000.2\ \Omega$, so

$$I_{bat} = I = \frac{V}{R_{eq}} = \frac{5V}{10000.2\Omega} = 5.00 \cdot 10^{-4}\ A$$

So the power dissipated across R_{bat} is:

$$P_{bat} = I_{bat}V_{bat} = I_{bat}^2 R_{bat} = (5.00 \cdot 10^{-4} A)^2 \cdot 10k\Omega = 2.5 \cdot 10^{-3}\ W$$

Thus, using the results we got from part (d), so it takes the battery

$$t = \frac{E_{total}}{P} = \frac{3.79 \cdot 10^4\ Ws}{2.5 \cdot 10^{-3} W} = 1.52 \cdot 10^7 s = 1.52 \cdot 10^7 s \cdot \frac{1hr}{3600s} = 4.22 \cdot 10^3\ hr$$

3. Fruity Fred

(a) $R_{AB} = \rho\frac{L-kF}{A_c} + \rho\frac{L-kF}{A_c} = \frac{2\rho(L-kF)}{A_c}$

(b) $F = \frac{A_c V_{out} + 2\rho L(V_{out}-1)}{2\rho k(V_{out}-1)}$

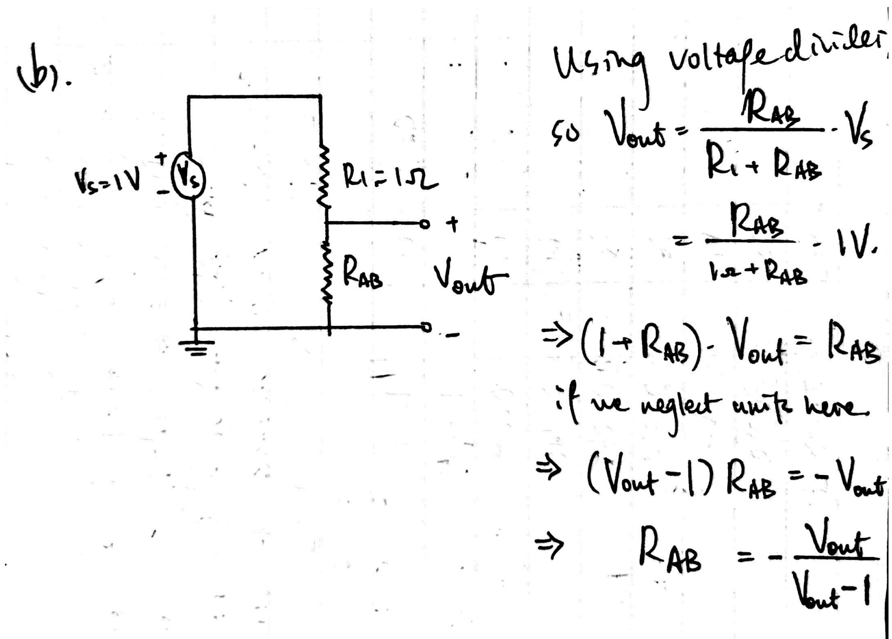


Figure 1: Circuit Designed

As deduced from the process on the picture, so $R_{AB} = -\frac{V_{out}}{V_{out}-1}$. Also, as we derived from part (a), which gives $R_{AB} = \frac{2\rho(L-kF)}{A_c}$. Thus, we have that:

$$R_{AB} = \frac{2\rho(L-kF)}{A_c} = -\frac{V_{out}}{V_{out}-1}$$

$$\implies 2\rho(L-kF) \cdot -(V_{out}-1) = A_c \cdot V_{out}$$

$$\implies (2\rho L - 2\rho kF) \cdot -(V_{out}-1) = A_c \cdot V_{out}$$

$$\implies -2\rho L(V_{out}-1) + 2\rho k(V_{out}-1)F = A_c \cdot V_{out}$$

$$\implies 2\rho k(V_{out}-1)F = A_c \cdot V_{out} + 2\rho L(V_{out}-1)$$

$$\implies F = \frac{A_c V_{out} + 2\rho L(V_{out}-1)}{2\rho k(V_{out}-1)}$$

4. Temperature Sensor

(a) $V_{out} = \frac{V_s R_2}{R_1+R_2}$

The current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1 + R_2$, so $I = \frac{V_s}{R_1+R_2}$

So, V_{out}, which measures the voltage drop over R_2, is equal to (or we could've used the Voltage Divider formula directly to obtain):

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_2} \cdot R_2 = \frac{V_s R_2}{R_1 + R_2}$$

Thus, $V_{out} = V_2 = \frac{V_s R_2}{R_1+R_2}$

(b) $T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{(V_{out}-V_s) R_o \alpha}$

Similarly, the current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1 + R_2 = R_1 + R_o(1 + \alpha T)$, so $I = \frac{V_s}{R_1+R_o(1+\alpha T)}$

So, V_{out}, which measures the voltage drop over R_2, is equal to:

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1 + R_o(1 + \alpha T)} \cdot R_o(1 + \alpha T) = \frac{V_s R_o(1 + \alpha T)}{R_1 + R_o(1 + \alpha T)}$$

Thus, $V_{out} = V_2 = \frac{V_s R_o(1+\alpha T)}{R_1+R_o(1+\alpha T)}$, which gives us that: $V_{out} \cdot (R_1 + R_o(1 + \alpha T)) = V_s R_o(1 + \alpha T)$

So, $V_{out} R_1 + V_{out} R_o + V_{out} R_o \alpha T = V_s R_o + V_s R_o \alpha T$, which gives:

$$V_{out} R_o \alpha T - V_s R_o \alpha T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies (V_{out} - V_s) R_o \alpha \cdot T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{(V_{out} - V_s) R_o \alpha}$$

(c) $T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{-V_s R_o \alpha + V_{out} R_1 \beta + V_{out} R_o \alpha}$

Again, similarly, the current through the circuit is $I = \frac{V_s}{R_{eq}}$, where $R_{eq} = R_1' + R_2 = R_1(1 + \beta T) + R_o(1 + \alpha T)$, so $I = \frac{V_s}{R_1(1+\beta T)+R_o(1+\alpha T)}$

So, V_{out}, which measures the voltage drop over R_2, is equal to:

$$V_2 = I_2 \cdot R_2 = I \cdot R_2 = \frac{V_s}{R_1(1 + \beta T) + R_o(1 + \alpha T)} \cdot R_o(1 + \alpha T) = \frac{V_s R_o(1 + \alpha T)}{R_1(1 + \beta T) + R_o(1 + \alpha T)}$$

Thus, $V_{out} = V_2 = \frac{V_s R_o(1+\alpha T)}{R_1(1+\beta T)+R_o(1+\alpha T)}$, which gives us that:

$$V_{out} \cdot (R_1(1 + \beta T) + R_o(1 + \alpha T)) = V_s R_o(1 + \alpha T)$$

$$\implies V_{out} R_1 + V_{out} R_1 \beta T + V_{out} R_o + V_{out} R_o \alpha T = V_s R_o + V_s R_o \alpha T$$

$$\implies (V_{out} R_1 \beta + V_{out} R_o \alpha - V_s R_o \alpha) \cdot T = V_s R_o - V_{out} R_1 - V_{out} R_o$$

$$\implies T = \frac{V_s R_o - V_{out} R_1 - V_{out} R_o}{-V_s R_o \alpha + V_{out} R_1 \beta + V_{out} R_o \alpha}$$

(d) No, it can't.

Here, we use the derived formula of voltage dividers directly to obtain the voltage drop over R_2:

$$V_2 = \frac{V_s \cdot R_{o2} \cdot (1 + \alpha T)}{(R_{o1} + R_{o2}) \cdot (1 + \alpha T)} = \frac{V_s R_{o2}}{R_{o1} + R_{o2}}$$

Thus, $V_{out} = V_2 = \frac{V_s R_{o2}}{R_{o1} + R_{o2}}$, which is independent of the variable T, which implies that we cannot express the temperature T as an equation in terms of the measurable variables. Therefore, this circuit (specifically the measurements of V_{out}) cannot be used to measure temperature.

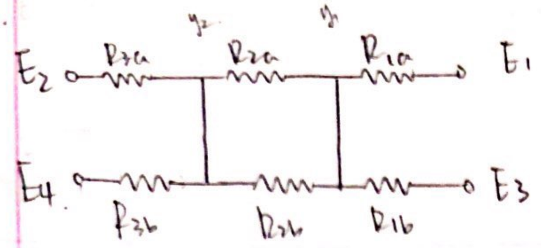
5. Multitouch Resistive Touchscreen

(a) $4k\Omega$

Since $W = 3cm = 0.03m, H = 12cm = 0.12m, T = 1mm = 1 \cdot 10^{-3}m = 0.001m$, so we can calculate the resistance between E_1 and E_2 as:

$$R = \rho \cdot \frac{L}{A} = \rho \cdot \frac{H}{W \cdot T} = 1\Omega m \cdot \frac{0.12m}{0.03m \cdot 0.001m} = 4000\Omega = 4\ k\Omega$$

(b).



Given that $R_{total} = \rho \frac{H}{W \cdot T} = 8k\Omega$, $y_1 = 3cm$ $y_2 = 7cm$ $H = 12cm$

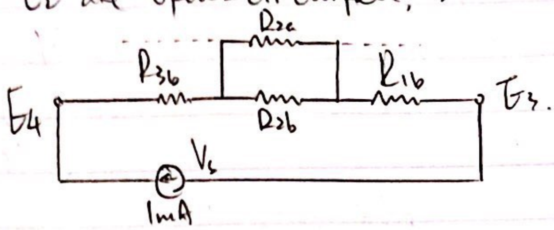
with $R_{total} = 8k\Omega$,

so $R_{1a} = R_{1b} = R_{total} \cdot \frac{y_1}{H} = 2k\Omega$

$R_{2a} = R_{2b} = R_{total} \cdot \frac{y_2 - y_1}{H} = 2.667\ k\Omega$.

$R_{3a} = R_{3b} = R_{total} \cdot \frac{H - y_2}{H} = 3.333\ k\Omega$

(c). Using the given conditions, since E_1 and E_2 are open-circuited, so no current flow through resistors R_{3a} and R_{1a}, which means that the circuit diagram is equivalent to:



We can calculate using methods that the resistance between E_4 and E_3 is:

$$R_{eq} = R_{3b} + \frac{1}{1/R_{2a} + 1/R_{2b}} + R_{1b} = R_{total} \cdot \frac{H - y_2}{H} + \frac{1}{2} \cdot R_{total} \cdot \frac{y_2 - y_1}{H} + R_{total} \cdot \frac{y_1}{H}$$

with $y_1 = 3cm$, $y_2 = 7cm$, $H = 12cm$, so $= R_{total} \cdot \frac{H - y_2/2 + y_1/2}{H} = \frac{20}{3} k\Omega = 6.667 k\Omega$

Thus, $V_s = I_s \cdot R_{eq} = 1mA \cdot 6.667\ k\Omega = 6.667\ V$.

which means that $V_{E_4 - E_3} = V_s = \boxed{6.667 V}$.

(d) Using what we've deduced in the steps of part (c), we have that:

$$R_{eq} = R_{total} \cdot \frac{H - y_2/2 + y_1/2}{H}.$$

So, $V_{E_4 - E_3} = V_s = I_s \cdot R_{eq} = 1mA \cdot 8k\Omega \cdot \frac{H - y_2/2 + y_1/2}{H} = 8V \cdot \left(1 - \frac{y_2}{24} + \frac{y_1}{24}\right)$

So, $\boxed{V_{E_4 - E_3} = \left(8 - \frac{1}{3} y_2 + \frac{1}{3} y_1\right)\ V.}$

(e). Using similar logic from part (c) and (d), so we can ① drive E_2, E_4 with a 1mA current source and measure $V_{E_4 - E_2} = I_s \cdot R_{eq_{2,4}}$ where

$$R_{eq_{2,4}} = R_{3a} + \frac{1}{1/R_{2a} + 1/R_{2b}} + R_{3b} = R_{total} \cdot \left(\frac{H - y_2}{H} + \frac{1}{2} \cdot \frac{y_2 - y_1}{H} + \frac{H - y_2}{H}\right).$$

$$= 8k\Omega \cdot \frac{2H - \frac{3}{2} y_2 - \frac{1}{2} y_1}{H}.$$

So, $V_{E_4 - E_2} = 1mA \cdot 8k\Omega \cdot \left(2 - \frac{3}{24} y_2 - \frac{1}{24} y_1\right) = \left(16 - y_2 - \frac{1}{3} y_1\right)\ V$.

Similarly, $R_{eq1,3} = R_{1a} + \frac{1}{1/R_{2a} + 1/R_{2b}} + R_{1b} = R_{total} \cdot \left(\frac{y_1}{H} + \frac{1}{2} \cdot \frac{y_2 - y_1}{H} + \frac{y_1}{H} \right)$

$= 8k\Omega \cdot \left(\frac{y_1 + \frac{1}{2}y_2 - \frac{1}{2}y_1 + y_1}{H} \right)$ where $H = 12$ (cm)

$= 8k\Omega \cdot \left(\frac{1}{8} y_1 + \frac{1}{24} y_2 \right)$

And, providing/driving E_1, E_3 with a 1mA current source gives:

$V_{E_1-E_3} = I_s \cdot R_{eq1,3} = 1mA \cdot 8k\Omega \left(\frac{1}{8} y_1 + \frac{1}{24} y_2 \right)$

so $V_{E_1-E_3} = \left(y_1 + \frac{1}{3} y_2 \right)$ V

Thus, we have two (plus one) equations:

$$V_{E_4-E_2} = \left(16 - \frac{1}{3} y_1 - y_2 \right) \text{ V}.$$

$$V_{E_1-E_3} = \left(y_1 + \frac{1}{3} y_2 \right) \text{ V}$$

[from (?), $V_{E_4-E_3} = \left(8 + \frac{1}{3} y_1 - \frac{1}{3} y_2 \right)$ V

6. Homework Process and Study Group

I worked alone without getting any help, except asking questions and reading posts (especially answers from the GSIs) on Piazza as well as reading the Notes of the course.

5. (e) Since we're still having a current source between E_4 and E_3, so the circuit looks the same, and for $V_{E_4-E_2}$, we'll be measuring the voltage drop over R_{3b}, which is: $I \cdot R_{3b} = \frac{12um - y_2}{12um} \cdot 8V$.

Similarly, $V_{E_1-E_3}$ = Voltage drop over R_{1b}, which is: $I \cdot R_{1b} = \frac{y_1}{12um} \cdot 8V$.

Thus, $V_{E_4-E_2} = \frac{12um - y_2}{12um} \cdot 8V$

and $V_{E_1-E_3} = \frac{y_1}{12um} \cdot 8V$.