2. Wheatstone Bridge.

(a). Since this is a circuit with R_1, R_4 in series, which combination is in parallel with R_2, R_5 in series, so the voltages across both routes (R_1, R_4 and R_2, R_5) are the same.

Thus, using Voltage Divider equation, so $V_a = \frac{R_4}{R_1+R_4} V_s + \underset{\text{ground}}{0V}$ where $V_s = 5V$.

So $V_a = \frac{5R_4}{R_1+R_4}$ (V). We can apply the same logic to calculate:

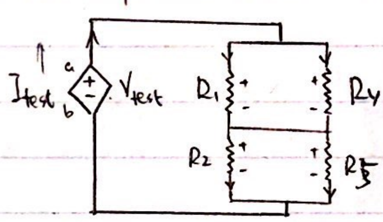
$$V_b = \frac{R_5}{R_2+R_5} \cdot V_s + 0V = \frac{5R_5}{R_2+R_5} \text{ (V)}.$$

Thus, $V_{Th} = V_a - V_b = \boxed{\frac{5R_4}{R_1+R_4} - \frac{5R_5}{R_2+R_5} \text{ (V)}}$

(b) $\boxed{\text{No}}$, it isn't.

Since there will be current flowing through R_3, so V_{Th} is actually the total voltage across nodes/terminals a and b, with V_{R_3} just being part of the voltage. In other words, since R_3 is not actually open circuit (infinite resistance), so $V_{R_3} \neq V_{Th}$.

(c). By supplying a test voltage V_{test} across terminal a and b and substituting V_s with wire, we can redraw the circuit as:



Using our equations for resistors in series and resistors in parallel,

So we have directly that, since it's $R_1 // R_4$ in series with $R_2 // R_5$,

$$\text{so } R_{Th} = R_1 // R_4 + R_2 // R_5 = \frac{R_1R_4}{R_1+R_4} + \frac{R_2R_5}{R_2+R_5}$$

(d) Since in essence, this is a current with R_3 and R_{Th} being in series, and a voltage source of supply (V_{Th} volts), so we can calculate the current first:

$$I_{R_3} = \frac{V_{Th}}{R_{Th}+R_3} = \boxed{\frac{-5R_1R_5 + 5R_2R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_3R_4R_5}}$$

$$\text{Thus, } V_{R_3} = R_3 \cdot I_{R_3} = \boxed{\frac{-5R_1R_3R_5 + 5R_2R_3R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_3R_4R_5}}$$

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3.(a). Since $i_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_s + R_{motor}}$, so $P_s = V_s \cdot i_s = \boxed{\frac{V_s^2}{R_s + R_{motor}}}$.

(b). Since $i_{motor} = i_s = \frac{V_s}{R_s + R_{motor}}$, so $P_{motor} = i_{motor}^2 R_{motor} = \boxed{\frac{V_s^2 \cdot R_{motor}}{(R_s + R_{motor})^2}}$

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(c). Since $P_{motor} = \frac{V_S^2 \cdot R_{motor}}{(R_s + R_{motor})^2}$, so $\frac{dP_{motor}}{dR_{motor}} = V_S^2 \cdot \frac{(R_s+R_{motor})^2 \cdot 1 - R_{motor} \cdot 2(R_s + R_{motor}) \cdot 1}{(R_s+R_{motor})^4}$

(using Quotient Rule.)

so $\frac{dP_{motor}}{dR_{motor}} = V_S^2 \cdot \frac{R_s + R_{motor} - 2R_{motor}}{(R_s + R_{motor})^3} = V_S^2 \cdot \frac{R_s - R_{motor}}{(R_s + R_{motor})^3}$

Set the derivative to 0 and we get that, $\boxed{R_{motor} = R_s}$

(d). Intuitively, if R_s is the only variable, since we want to maximize P_{motor}, i.e. maximize power, so the total resistance should be minimized, so optimal $\boxed{R_s = 0}$

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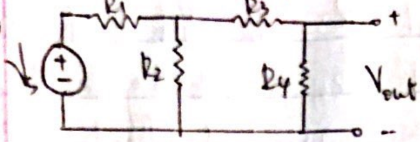
4. D to A Converter (DAC).

(a)



This is a circuit with R_1 in series with $R_2 \parallel (R_3$ series with $R_4)$

so $R_{eq_{2,3,4}} = R_2 \parallel (R_3 + R_4)$ and with $R_1 = R_2 = R_3 = R_4 = R$,

so $R_{eq_{2,3,4}} = R \parallel 2R = \frac{2}{3}R$. Now we can use Voltage Divider,

so $V_{out} = V_{eq_{2,3,4}} = \frac{R_{eq_{2,3,4}}}{R_1 + R_{eq_{2,3,4}}} \cdot V_s = \boxed{\frac{2}{5}V_s} \neq \frac{1}{4}V_s$. $\boxed{\text{No,}}$ it isn't.

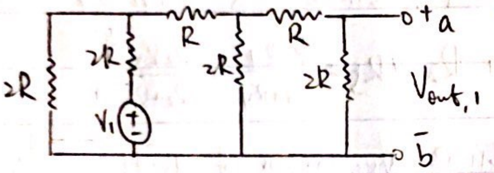
(b) i. $R_{eq} = (2R) \parallel (2R) = \boxed{R}$

ii) $R_{eq} = 2R \parallel (R + 2R \parallel 2R) = 2R \parallel 2R = \boxed{R}$

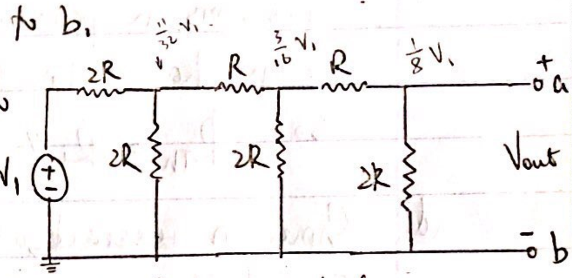
iii) $R_{eq} = 2R \parallel (R + 2R \parallel (R + 2R \parallel 2R)) = 2R \parallel (R + 2R \parallel 2R) = \boxed{R}$

These circuits all yield resistance R from point a to b.

(c) First we consider V_1 and zero V_2 and V_3:

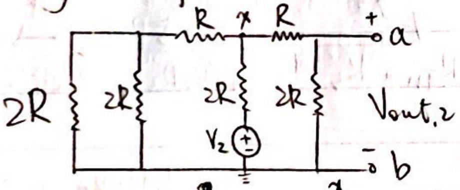


Redraw $\Leftrightarrow$



So, $V_{out,1} = \frac{1}{8}V_1$ using voltage dividers and parallel equation repeatedly.

Then we consider V_2, and zero V_1 and V_3:

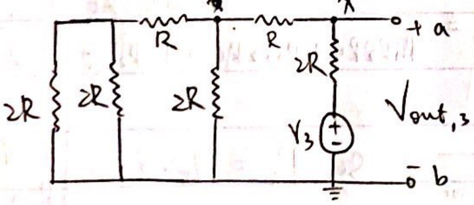


Using parallel resistors and voltage divider,

so $V_x = \frac{3}{8}V_2$

so $V_{out,2} = \frac{2R}{R+2R} \cdot V_x = \frac{1}{4}V_2$.

Then, we separate V_3, and zero V_1, V_2:



Again, using voltage divides etc, here, $V_x = \frac{2R}{2R+R+R} \cdot V_3 = \frac{1}{2}V_3$.

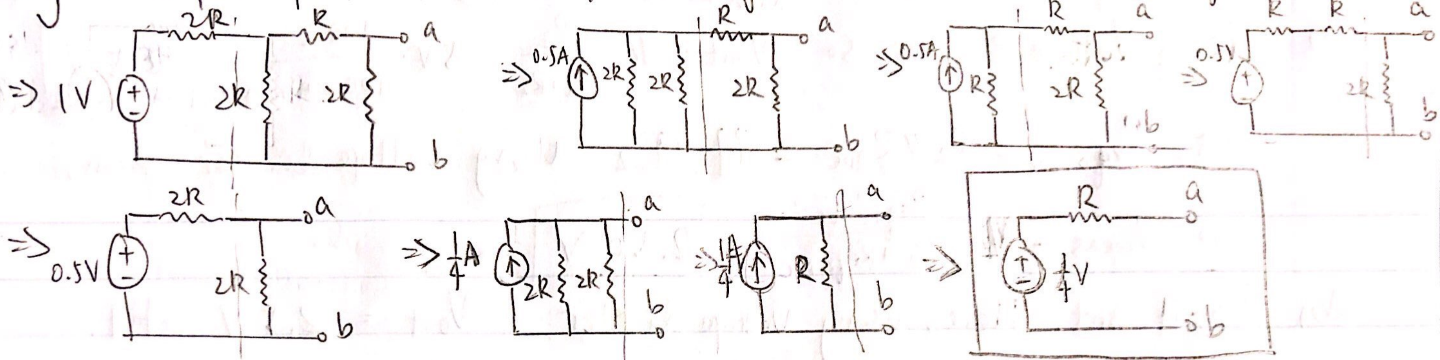
so, $V_{out,3} = V_x = \frac{1}{2}V_3$.

Thus, using superposition, so $V_{out} = V_{out,1} + V_{out,2} + V_{out,3}$

$$= \boxed{\frac{1}{8}V_1 + \frac{1}{4}V_2 + \frac{1}{2}V_3}$$

(d) Given $V_2 = 1V$, $V_1 = V_3 = 0V$, so $V_{out} = 0V + \frac{1}{4}V + 0V = \boxed{\frac{1}{4}V}$

(e). Using results from part (b), so the Thevenin equivalent of the circuit with given conditions is:



(f) Here, $V_{speaker} = V_{Th} \cdot \frac{R_{sp}}{R_{sp} + R_{Th}} = \frac{1}{4} V \cdot \frac{\frac{1}{3}\Omega}{\frac{1}{3}\Omega + 1\Omega} = \frac{1}{16} V < \frac{1}{4} V$.

It's lower than predicted because the R_{Th} splits away part of the voltage.

The actual output voltage is $V_{out} = \boxed{\frac{1}{16} V}$.

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5. Measuring V and Current. with $R_1 = R_2 = 100\Omega$, $R_{ADC} = 1\times10^6\Omega$

(a) Using Voltage Divider, so $V_{out} = V_S \cdot \frac{R_2}{R_1+R_2} = 5V \cdot \frac{100\Omega}{100\Omega + 100\Omega} = \boxed{2.5\,V}$

Next, $R_{eq_{2,ADC}} = R_2 \,//\, R_{ADC} = 99.99\Omega$. Using voltage divider again,

so $V_{meas} = V_S \cdot \frac{R_{eq_{2,ADC}}}{R_1 + R_{eq_{2,ADC}}} = \boxed{2.50\,V}$

(b) $\boxed{No}$ it isn't. Here, using Voltage divider, $V_{out} = 2.5V$ still.

But with $R_{eq_{2,ADC}} = R_2 \,//\, R_{ADC} = \frac{1}{\frac{1}{10M\Omega} + \frac{1}{1M\Omega}} = 9.09 \cdot 10^5 \Omega$.

So using voltage divider, $V_{meas} = V_S \cdot \frac{R_{eq_{2,ADC}}}{R_{eq_{2,ADC}} + R_1} = 0.41\,V < V_{out}$.

so this ADC isn't a good tool any more.

(c). Given that $R_2 = R_1$, so using voltage divider, $V_{out} = \frac{1}{2} V_S = 2.5V$ still.

Since $R_{eq} = R_2 \,//\, R_{ADC} = \frac{R_2 R_{ADC}}{R_2 + R_{ADC}} = \frac{R_1 R_{ADC}}{R_1 + R_{ADC}}$, so using voltage divider,

$V_{meas} = V_S \cdot \frac{R_{eq}}{R_{eq} + R_1} = V_S \cdot \frac{R_{ADC}}{2R_{ADC} + R_1}$ and we want $0.9\,V_{out} \le V_{meas} \le 1.1\,V_{out}$

so $0.9 \cdot \frac{1}{2} \cancel{V_S} \le \frac{R_{ADC}}{2R_{ADC} + R_1} \cancel{V_S} \le 1.1 \cdot \frac{1}{2} \cancel{V_S}$. Since $V_S > 0$.
and since $R_{ADC}, R_1 > 0$,

so $0.9\,R_{ADC} + 0.45 R_1 \le R_{ADC} \le 1.1\,R_{ADC} + 0.55 R_1$.

Given that $R_{ADC} = 1M\Omega = 1 \cdot 10^6 \Omega$,

So with $0.45 R_1 \le 0.1\,R_{ADC}$ and $0.55 R_1 \ge -0.1\,R_{ADC}$ ✓

so $R_1 \le 2.22 \cdot 10^5 \Omega$.

Thus, maximum R_1 is $\boxed{2.22 \cdot 10^5 \Omega}$

(d) Left: $I_1 = \frac{V_S}{R_1} = \frac{5V}{1k\Omega} = \boxed{5\,mA}$

Right: $R_{eq_{x,ADC}} = R_x \,//\, R_{ADC} = \frac{1\Omega \cdot 1M\Omega}{1\Omega + 1M\Omega} = 1.00\,\Omega$.

so $I_{meas} = I = \frac{V_S}{R_1 + R_{eq_{x,ADC}}} = \frac{5V}{1k\Omega + 1.00\Omega} = \boxed{5.00\,mA}$.

(e) Here, $I_{meas} = \frac{V_S}{R_1 + R_{eq_{x,ADC}}} = \frac{5V}{R_1 + 1\Omega}$ while $I_1 = \frac{V_S}{R_1} = \frac{5V}{R_1}$.

and we want $0.9\,I_1 \le I_{meas} \le 1.1\,I_1$.

$\Rightarrow \frac{4.5V}{R_1} \le \frac{5V}{R_1 + 1\Omega} \le \frac{5.5V}{R_1}$ with $R_1 > 0\Omega$, so multiply by $R_1(R_1 + 1\Omega)$

$\Rightarrow 4.5V \cdot R_1 + 4.5V \cdot \Omega \le 5V \cdot R_1 \le 5.5V \cdot R_1 + 5.5V \cdot 1\Omega$

$\Rightarrow R_1 \ge 9\Omega$. Thus, minimum R_1 req. is $\boxed{9\Omega}$

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6. Homework Process and Study Group

I worked alone without getting any help, except asking questions and reading posts (especially answers from the GSIs) on Piazza as well as reading the Notes of the course.