1. Energy Disaggregation

Recently, energy companies, such as PG&E, have put a lot of thought into a problem called *energy disaggregation*. The energy disaggregation problem is to take measurements of the total amount of electricity that a house uses and then try to determine which appliances are being used in the house. Energy companies want to do this because it allows them to better predict how much electricity they will need to produce on a given day, so they can offer suggestions to their customers on how they can save energy.

To get an idea for how this works, suppose you live in a very simple house with just an air conditioning unit, a refrigerator, and a television that all use power measured in watts. Suppose you want to figure out how much energy your TV and refrigerator use, but the only measuring device you have is the meter on the outside of the house that measures the total power the house is using. You can turn off the TV at any time, but you don't want to unplug the refrigerator because you don't want the food to go bad. The air conditioner stays off in the morning but then turns on in the afternoon.

(a) Design a method to determine how much power each appliance uses. How many measurements will you need to make?

Answer:

We need to get 3 sets of measurements, where each one has to give us new information. One such solution is to measure the power usage in the morning with the TV off (when only the refrigerator is running) and twice in the afternoon with the TV plugged in and unplugged.

(b) Write the system of equations you would need to solve this problem in terms of the unknowns (the power of the air conditioner x_{AC}, the power of the refrigerator x_{R}, and the power of the TV, x_{TV}) as well as the measurements you make of the total power (labeled T_1, T_2, etc.).

Answer:

The resulting three equations for the above measurements have the form:

$$\begin{aligned} x_{\text{R}} &= T_1 \\ x_{\text{AC}} + x_{\text{TV}} + x_{\text{R}} &= T_2 \\ x_{\text{AC}} + x_{\text{R}} &= T_3 \end{aligned}$$

2. Systems of Equations

Solve the following systems of equations.

(a)

$$\begin{cases} 2x + y = 6 \\ 3x - 2y = 2 \end{cases}$$

Answer:

$$x = 2, y = 2$$

(b)

$$\begin{cases} x+y+z=2 \\ x-y=1 \\ 2y+z=1 \end{cases}$$

Answer:

$$x = \frac{3}{2} - \frac{1}{2}t, y = \frac{1}{2} - \frac{1}{2}t, z = t, \forall t \in \mathbb{R}$$

(c) Systems of equations can also be interpreted graphically. We will try to build a graphical intuition for the results you found in the previous part. Follow along as your TA walks through `dis1A.ipynb`.

Answer:

i. The lines lie on top of one another (i.e. they are the same line), so there are an infinite number of solutions. This system is referred to as underdetermined, which means that there are more unknowns than equations. Though it appears we have two equations and two unknowns, dividing the top equation by 7 and the bottom one by 6 quickly reveals that they are both the same equation.

ii. The lines intersect at one point, so the solution is unique.

iii. The lines do not intersect (a little algebraic manipulation on the equations would reveal that the two lines are parallel). There is no solution.

iv. The intersection of the planes is null. There is no solution.

v. The intersection of the planes is a line, so there is an infinite number of solutions. This system is also underdetermined. Subtracting the second equation solution from the third and multiplying the result by two yields the first equation. In other words including the first equation is redundant because any point that satisfies the second and third equation will certainly satisfy the first. In effect, we have two equations and one unknown.

vi. The intersection of the planes is a single point, so there is a unique solution.