

2. Wheatstone Bridge.

- (a) Since this is a circuit with R_1, R_4 in series, which combination is in parallel with R_2, R_5 in series, so the voltages across both routes (R_1, R_4 and R_2, R_5) are the same.

Thus, using Voltage Divider equation, so $V_a = \frac{R_4}{R_1 + R_4} V_s + 0V$ where $V_s = 5V$.
ground.

So $V_a = \frac{5R_4}{R_1 + R_4} (V_s)$. We can apply the same logic to calculate:

$$V_b = \frac{R_5}{R_2 + R_5} V_s + 0V = \frac{5R_5}{R_2 + R_5} (V_s)$$

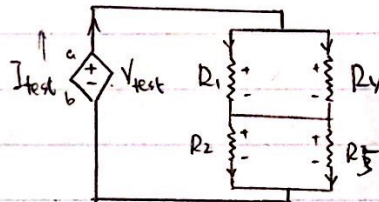
$$\text{Thus, } V_{th} = V_a - V_b = \left[\frac{5R_4}{R_1 + R_4} - \frac{5R_5}{R_2 + R_5} \right] (V_s)$$

- (b) No, it isn't.

Since there will be current flowing through R_3 , so V_{th} is actually the total voltage across nodes/terminals a and b, with V_{R_3} just being part of the voltage. In other words, since R_3 is not actually open circuit (infinite resistance), so $V_{R_3} \neq V_{th}$.

- (c) By supplying a test voltage V_{test} across terminal a and b and substituting V_s with wire, we can redraw the circuit as:

Using our equations for resistors in series and resistors in parallel,



So we have directly that, since it's $R_1 // R_4$ in series with $R_2 // R_5$,

$$\text{So } R_{th} = R_1 // R_4 + R_2 // R_5 = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_5}{R_2 + R_5}$$

- (d) Since in essence, this is a current with R_3 and R_{th} being in series, and a voltage source of supply (V_{th} volts), so we can calculate the current first:

$$I_{R_3} = \frac{V_{th}}{R_{th} + R_3} = \frac{-5R_1R_5 + 5R_2R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_2R_5R_3 + R_3R_4R_5}$$

$$\text{Thus, } V_{R_3} = R_3 \cdot I_{R_3} = \frac{-5R_1R_3R_5 + 5R_2R_3R_4}{R_1R_2R_3 + R_1R_2R_4 + R_1R_2R_5 + R_1R_3R_5 + R_1R_4R_5 + R_2R_3R_4 + R_2R_4R_5 + R_2R_5R_3 + R_3R_4R_5}$$

3.(a) Since $i_s = \frac{V_s}{R_{eq}} = \frac{V_s}{R_s + R_{motor}}$, so $P_s = V_s \cdot i_s = \boxed{\frac{V_s^2}{R_s + R_{motor}}}$

(b). Since $i_{motor} = i_s = \frac{V_s}{R_s + R_{motor}}$, so $P_{motor} = i_{motor}^2 R_{motor} = \boxed{\frac{V_s^2 \cdot R_{motor}}{(R_s + R_{motor})^2}}$

(c). Since $P_{\text{motor}} = \frac{V_s^2 \cdot R_{\text{motor}}}{(R_s + R_{\text{motor}})^2}$, so $\frac{dP_{\text{motor}}}{dR_{\text{motor}}} = V_s^2 \cdot \frac{(R_s + R_{\text{motor}})^2 \cdot 1 - R_{\text{motor}} \cdot 2(R_s + R_{\text{motor}}) \cdot 1}{(R_s + R_{\text{motor}})^4}$.

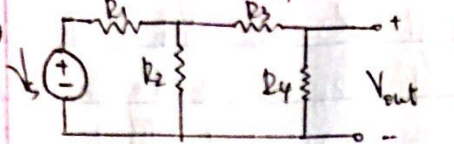
(using Quotient Rule.)

$$\text{so } \frac{dP_{\text{motor}}}{dR_{\text{motor}}} = V_s^2 \cdot \frac{R_s + R_{\text{motor}} - 2R_{\text{motor}}}{(R_s + R_{\text{motor}})^3} = V_s^2 \cdot \frac{R_s - R_{\text{motor}}}{(R_s + R_{\text{motor}})^3}$$

Set the derivative to 0 and we get that $\boxed{R_{\text{motor}} = R_s}$

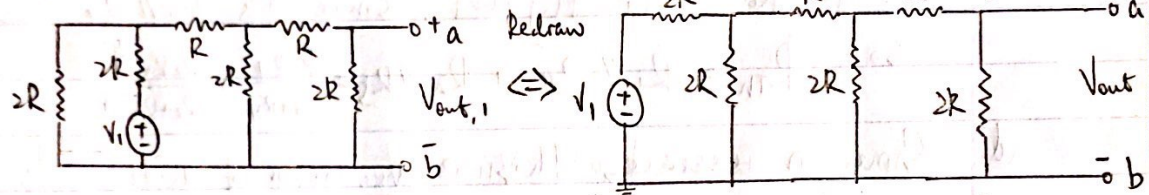
(d). Intuitively, if R_s is the only variable, since we want to maximize P_{motor} , i.e. maximize power, so the total resistance should be minimized, so optimal $\boxed{R_s = 0}$

4. D to A Converter (DAC).

- (a)  This is a circuit with R_1 in series with $R_2 \parallel (R_3 \text{ series with } R_4)$.
 So $R_{eq2,3,4} = R_2 \parallel (R_3 + R_4)$ and with $R_1 = R_2 = R_3 = R_4 = R$,
 So $R_{eq2,3,4} = R \parallel 2R = \frac{2}{3} R$. Now we can use Voltage Divider,
 So $V_{out} = V_{eq2,3,4} = \frac{R_{eq2,3,4}}{R_1 + R_{eq2,3,4}} \cdot V_s = \boxed{\frac{2}{5} V_s} \neq \frac{1}{4} V_s$. No, it isn't.

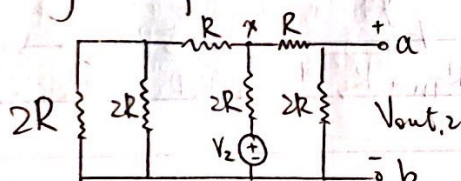
- (b) i. $R_{eq} = (2R) \parallel (2R) = \boxed{R}$
 ii. $R_{eq} = 2R \parallel (R + 2R \parallel 2R) = 2R \parallel 2R = \boxed{R}$
 iii. $R_{eq} = 2R \parallel (R + 2R \parallel (R + 2R \parallel 2R)) = 2R \parallel (R + 2R \parallel 2R) = \boxed{R}$
 These circuits all yield resistance R from point a to b.

- (c) First we consider V_1 and zero V_2 and V_3 :



So, $V_{out,1} = \frac{1}{8} V_1$ using voltage dividers and parallel equation repeatedly.

Then, we consider V_2 and zero V_1 and V_3 :

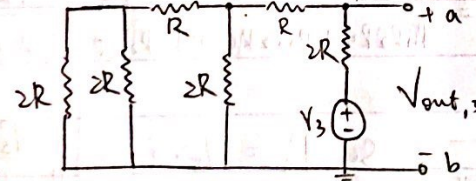


Using parallel resistors and voltage divider,

$$\text{So } V_x = \frac{3}{8} V_2$$

$$\text{So } V_{out,2} = \frac{2R}{2R+2R} \cdot V_x = \frac{1}{4} V_2$$

Then, we separate V_3 and zero V_1, V_2 :



Again, using voltage dividers etc, here, $V_x = \frac{2R}{2R+R+R} \cdot V_3 = \frac{1}{2} V_3$.

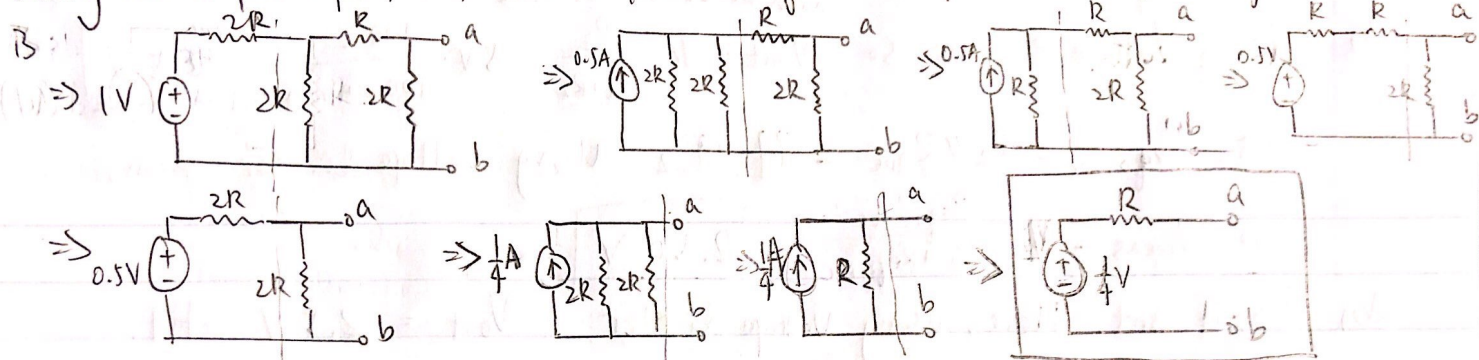
$$\text{So, } V_{out,3} = V_x = \frac{1}{2} V_3$$

Thus, using superposition, so $V_{out} = V_{out,1} + V_{out,2} + V_{out,3}$

$$= \boxed{\frac{1}{8} V_1 + \frac{1}{4} V_2 + \frac{1}{2} V_3}$$

- (d) Given $V_2 = 1V, V_1 = V_3 = 0V$, so $V_{out} = 0V + \frac{1}{4} V + 0V = \boxed{\frac{1}{4} V}$

(e). Using results from part (b), so the Thevenin equivalent of the circuit with given conditions is:



(f) Here, $V_{\text{speaker}} = V_{\text{Th}} \cdot \frac{R_{\text{sp}}}{R_{\text{sp}} + R_{\text{Th}}} = \frac{1}{4} \text{ V} \cdot \frac{\frac{1}{3} R}{\frac{1}{3} R + 1R} = \frac{1}{16} \text{ V} < \frac{1}{4} \text{ V}$.

It's lower than predicted because the R_{Th} splits away part of the voltage.

The actual output voltage is $V_{\text{out}} = \boxed{\frac{1}{16} \text{ V}}$.

5. Measuring V and Current. with $R_1 = R_2 = 100\Omega$, $R_{ADC} = 1 \times 10^6 \Omega$
 (a) Using Voltage Divider, so $V_{out} = V_s \cdot \frac{R_2}{R_1 + R_2} = 5V \cdot \frac{100\Omega}{100\Omega + 100\Omega} = \boxed{2.5V}$

Next, $R_{eq2, ADC} = R_2 \parallel R_{ADC} = 99.99\Omega$. Using voltage divider again,

$$\text{so } V_{meas} = V_s \cdot \frac{R_{eq2, ADC}}{R_1 + R_{eq2, ADC}} = \boxed{2.50V}$$

(b) No, it isn't. Here, using voltage divider, $V_{out} = 2.5V$ still.

$$\text{But with } R_{eq2, ADC} = R_2 \parallel R_{ADC} = \frac{1}{\frac{1}{100\Omega} + \frac{1}{1M\Omega}} = 9.09 \times 10^5 \Omega.$$

$$\text{So using voltage divider, } V_{meas} = V_s \cdot \frac{R_{eq2, ADC}}{R_{eq2, ADC} + R_1} = 0.41V < V_{out}.$$

so this ADC isn't a good tool anymore.

(c). Given that $R_2 = R_1$, so using voltage divider, $V_{out} = \frac{1}{2} V_s = 2.5V$ still.

Since $R_{eq} = R_2 \parallel R_{ADC} = \frac{R_2 R_{ADC}}{R_2 + R_{ADC}} = \frac{R_1 R_{ADC}}{R_1 + R_{ADC}}$, so using voltage divider,

$$V_{meas} = V_s \cdot \frac{R_{eq}}{R_{eq} + R_1} = V_s \cdot \frac{R_{ADC}}{2R_{ADC} + R_1} \text{ and we want } 0.9V_{out} \leq V_{meas} \leq 1.1V_{out}$$

$$\text{so } 0.9 \cdot \frac{1}{2} V_s \leq \frac{R_{ADC}}{2R_{ADC} + R_1} V_s \leq 1.1 \cdot \frac{1}{2} V_s. \text{ Since } V_s > 0, \text{ and since } R_{ADC}, R_1 > 0,$$

$$\text{so } 0.9 R_{ADC} + 0.45 R_1 \leq R_{ADC} \leq 1.1 R_{ADC} + 0.55 R_1.$$

Given that $R_{ADC} = 1M\Omega = 1 \cdot 10^6 \Omega$,

$$\text{so with } 0.45 R_1 \leq 0.1 R_{ADC} \text{ and } 0.55 R_1 \geq -0.1 R_{ADC} \checkmark$$

$$\text{so } R_1 \leq 2.22 \cdot 10^5 \Omega.$$

Thus, maximum R_1 is $\boxed{2.22 \cdot 10^5 \Omega}$

$$(d) \text{ Left: } I_1 = \frac{V_s}{R_1} = \frac{5V}{1k\Omega} = \boxed{5mA}$$

$$\text{Right: } R_{eq, ADC} = R_2 \parallel R_{ADC} = \frac{1\Omega \cdot 1M\Omega}{1\Omega + 1M\Omega} = 1.00\Omega.$$

$$\text{so } I_{meas} = I = \frac{V_s}{R_1 + R_{eq, ADC}} = \frac{5V}{1k\Omega + 1.00\Omega} = \boxed{5.00mA}$$

$$(e) \text{ Here, } I_{meas} = \frac{V_s}{R_1 + R_{eq, ADC}} = \frac{5V}{R_1 + 1\Omega} \text{ while } I_1 = \frac{V_s}{R_1} = \frac{5V}{R_1}.$$

$$\text{and we want } 0.9 I_1 \leq I_{meas} \leq 1.1 I_1.$$

$$\Rightarrow \frac{4.5V}{R_1} \leq \frac{5V}{R_1 + 1\Omega} \leq \frac{5.5V}{R_1} \text{ with } R_1 > 0\Omega, \text{ so multiply by } R_1(R_1 + 1\Omega)$$

$$\Rightarrow 4.5V \cdot R_1 + 4.5V \cdot 1\Omega \leq 5V \cdot R_1 \leq 5.5V \cdot R_1 + 5.5V \cdot 1\Omega$$

$$\Rightarrow R_1 \geq 9\Omega. \text{ Thus, minimum } R_1 \text{ req. is } \boxed{9\Omega}$$