

This homework is due August 31, 2018, at 23:59.

Self-grades are due September 4, 2018, at 23:59.

Submission Format

Your homework submission should consist of **two** files.

- `hw1.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

If you do not attach a PDF “printout” of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

- `hw1.ipynb`: A single IPython notebook with all of your code in it.

Submit each file to its respective assignment on Gradescope.

1. Student Information

Fill out your relevant student information at the link [here](https://goo.gl/forms/TyHbUnL7n6AJHIdu2) (goo.gl/forms/TyHbUnL7n6AJHIdu2). Please double check that you are filling out all of the fields correctly. Once you click submit, you will receive an email with a copy of your responses. **Solution:** [Fill out the relevant fields in the google form.](#)

2. Administrivia

If you want to be eligible for midterm clobbering, what do you need to do?

Solution:

You need to (1) attend at least 75% of the discussions (not including discussions in the first week of class) and (2) perform better on the corresponding part of the final than your lowest midterm.

3. Stojanovic’s Optimal Smoothies

Solution: `#SystemsOfEquations #GaussianElimination`

Stojanovic’s Optimal Smoothies has a unique way of serving its customers. To ensure the best customer experience, each customer gets a smoothie personalized to their tastes. Professor Stojanovic knows that a lot of customers don’t know what they want, so when the customer walks up to the counter, they are asked to taste four standard smoothies that cover the entire range of flavors found in the smoothies.

Each smoothie is made of $\frac{1}{2}$ cup Greek yogurt, $\frac{1}{8}$ cup vanilla soy milk, $\frac{1}{2}$ cup crushed ice, and 1 cup mystery fruit. The four standard smoothies have the following recipes for the cup of mystery fruit:

Fruit [cups]	Banana Berry	Caribbean Passion	Mango-a-go-go	Strawberries Wild
Strawberries	$\frac{1}{3}$	$\frac{1}{3}$	0	$\frac{2}{3}$
Bananas	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{3}$
Mangos	0	$\frac{1}{3}$	$\frac{3}{5}$	0
Blueberries	$\frac{1}{3}$	0	0	0

Each customer is assumed to have a score (from 0 to 10) for each fruit, and the total score for the smoothie is computed by multiplying the score for a fruit with its proportion in the smoothie. The other ingredients do not influence the customer's score. For example, if a customer's score for strawberries is 6 and bananas is 3, then the total score for the Strawberries Wild smoothie would be $6 \cdot \frac{2}{3} + 3 \cdot \frac{1}{3} = 5$.

After a customer gives a score (from 0 to 10) for each smoothie, Professor Stojanovic then calculates (on the spot!) how much the customer likes each fruit. Then Professor Stojanovic blends up a special smoothie that will maximize the customer's score.

Professor Ranade was thirsty after giving the first lecture, so Professor Ranade decided to take a drink break at Stojanovic's Optimal Smoothies. Professor Ranade walked in and gave the following ratings:

Smoothie	Score
Banana Berry	7
Caribbean Passion	7
Mango-a-go-go	$7\frac{2}{5}$
Strawberries Wild	$6\frac{1}{3}$

- (a) What were Professor Ranade's ratings for each fruit? **Work this problem out by hand in terms of the steps. You may use a calculator to do algebra.**

Solution:

Using Professor Ranade's ratings, Professor Stojanovic mentally records the following system of equations:

$$\begin{aligned}
 \text{Banana Berry:} \quad 7 &= \frac{1}{3}x_S + \frac{1}{3}x_{Ba} + \frac{1}{3}x_{Bb} \\
 \text{Caribbean Passion:} \quad 7 &= \frac{1}{3}x_S + \frac{1}{3}x_{Ba} + \frac{1}{3}x_M \\
 \text{Mango-a-go-go:} \quad 7\frac{2}{5} &= \frac{2}{5}x_{Ba} + \frac{3}{5}x_M \\
 \text{Strawberries Wild:} \quad 6\frac{1}{3} &= \frac{2}{3}x_S + \frac{1}{3}x_{Ba}
 \end{aligned}$$

Professor Stojanovic then multiplies each equation by the denominator of the fraction (in order to make them easier to read):

$$\begin{aligned}
 21 &= x_S + x_{Ba} + x_{Bb} \\
 21 &= x_S + x_{Ba} + x_M \\
 37 &= 2x_{Ba} + 3x_M \\
 19 &= 2x_S + x_{Ba}
 \end{aligned}$$

Professor Stojanovic then writes the above equations in matrix form:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 2 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_S \\ x_{Ba} \\ x_M \\ x_{Bb} \end{bmatrix} = \begin{bmatrix} 21 \\ 21 \\ 37 \\ 19 \end{bmatrix}$$

and as an augmented matrix:

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 21 \\ 1 & 1 & 1 & 0 & 21 \\ 0 & 2 & 3 & 0 & 37 \\ 2 & 1 & 0 & 0 & 19 \end{array} \right].$$

Professor Stojanovic then proceeds to row reduce the matrix into reduced row echelon form as follows. (It's fine if you solved the system of equations by hand a different way. Here, however, we will demonstrate how to do it using Gaussian elimination.)

Noting that there is a 1 in the upper left hand corner, Professor Stojanovic subtracts Row 1 from Row 2 and $2 \times$ Row 1 from Row 4.

$$\begin{array}{l} \text{Row 2: subtract Row 1} \\ \text{Row 4: subtract } 2 \times \text{Row 1} \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 21 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 2 & 3 & 0 & 37 \\ 0 & -1 & 0 & -2 & -23 \end{array} \right]$$

Since Row 2 has a 0 in the diagonal element, Professor Stojanovic multiplies Row 4 by -1 and then switches Rows 2 and 4.

$$\begin{array}{l} \text{Multiply Row 4 by } -1 \\ \text{Switch Row 2 and Row 4} \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 21 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 2 & 3 & 0 & 37 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

Professor Stojanovic then subtracts Row 2 from Row 1 and $2 \times$ Row 2 from Row 3.

$$\begin{array}{l} \text{Row 1: subtract Row 2} \\ \text{Row 3: subtract } 2 \times \text{Row 2} \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 0 & 3 & -4 & -9 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right]$$

Professor Stojanovic then switches Row 3 and Row 4 and then subtracts $3 \times$ the new Row 3 from the new Row 4.

$$\begin{array}{l} \text{Switch Row 3 and Row 4} \\ \text{Row 4: subtract } 3 \times \text{Row 3} \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -1 & -2 \\ 0 & 1 & 0 & 2 & 23 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & -9 \end{array} \right]$$

Finally, Professor Stojanovic multiplies Row 4 by -1 and then adds Row 4 to Row 1 and Row 3 and subtracts $2 \times$ Row 4 from Row 2.

$$\begin{array}{l} \text{Multiply Row 4 by } -1 \\ \text{Row 1, Row 3: add Row 4} \\ \text{Row 2: subtract } 2 \times \text{Row 4} \end{array} \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 0 & 9 \\ 0 & 0 & 0 & 1 & 9 \end{array} \right]$$

Thus, Professor Stojanovic determines that Professor Ranade's ratings for each fruit are

Fruit	Score
Strawberries	7
Bananas	5
Mangos	9
Blueberries	9

- (b) What mystery fruit combination should Professor Stojanovic put in Professor Ranade's personalized smoothie to maximize the customer's score? What score would Professor Ranade give for this smoothie? There may be more than one correct answer.

Solution:

Any linear combination of mango and blueberry is acceptable as they have equal ratings. More precisely, for any $0 \leq a \leq 1$, a cups of mango and $1 - a$ cups of blueberries. Any such combination will yield a score of 9.

How could you see this? At one level, it is somewhat obvious and it is fine if you said as much — Mangoes and Blueberries are tied for Professor Ranade's favorite fruit, so it doesn't make a difference if Professor Stojanovic substitutes one for the other in any quantity. It also doesn't make sense to substitute a less preferred fruit like Strawberries for Professor Ranade's favorite fruits.

4. Filtering Out The Troll

Solution: #SystemsOfEquations #LinearCombination

You attended a very important public speech and recorded it using a recording device that consists of two directional microphones. However, there was this particular person in the audience who was trolling around. When you went back home to listen to the recording, you realized that the two recordings were dominated by the troll and you could not hear the speech. Fortunately, since you had two microphones, you realized that there is a way to combine the two recordings such that the trolling is removed. Recollecting the scene, the locations of the speaker and the troll are shown in Figure 1.

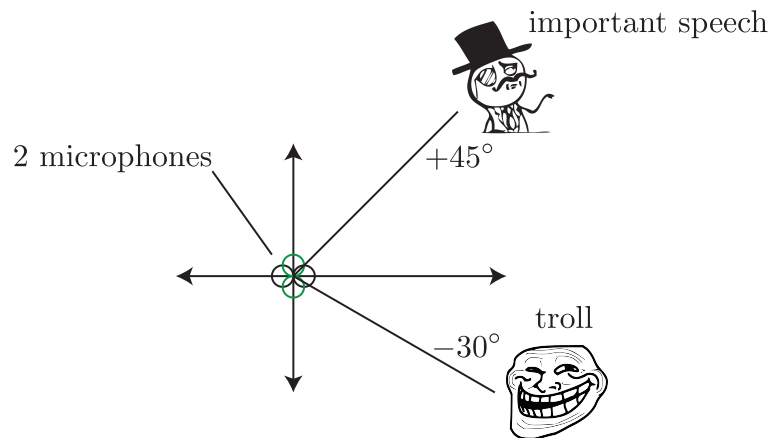


Figure 1: Locations of the speaker and the troll.

The way your recording device works is that each microphone weighs the audio signal depending on the angle of the audio source, relative to the x axis, hence the name *directional microphones*. More specifically, if the audio source is located at an angle of θ , the first microphone will record the audio signal with weight $f_1(\theta) = \cos(\theta)$, and the second microphone will record the audio signal with weight $f_2(\theta) = \sin(\theta)$. For example, an audio source that lies on the x axis will be recorded with the first microphone with weight equal

to 1 (since $\cos(0) = 1$), but will not be picked up by the second microphone (since $\sin(0) = 0$). Note that the weights can also be negative.

Graphically, the directional characteristics of the microphones are given in Figures 2 and 3 (the red and blue colors denote the positive and negative values of the weight, respectively and the distance of the red or the blue line from the midpoint is the value of the weight). Putting all of this together, assume that there are two speakers, A and B , at angles θ and ψ , respectively. Assume that speaker A produces an audio signal represented by the vector $\vec{a} \in \mathbb{R}^n$. That is, the i -th component of \vec{a} is the signal at the i -th time step. Similarly, assume speaker B produces an audio signal \vec{b} ,

Then the first microphone will record the signal

$$\vec{m}_1 = \cos(\theta) \cdot \vec{a} + \cos(\psi) \cdot \vec{b},$$

and the second microphone will record the signal

$$\vec{m}_2 = \sin(\theta) \cdot \vec{a} + \sin(\psi) \cdot \vec{b}.$$

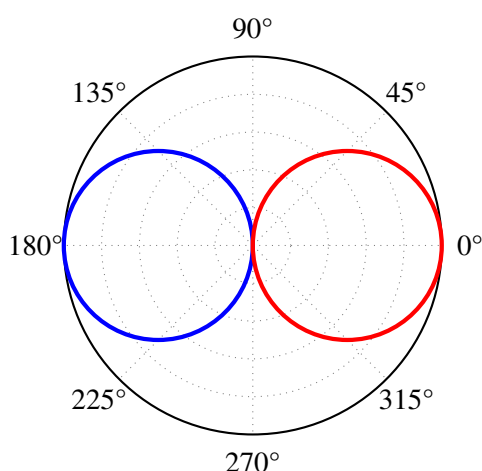


Figure 2: Directional characteristics of mic. 1. The red and blue colors denote the positive and negative values of the weight, respectively. The distance of the red or the blue line from the midpoint is the value of the weight.

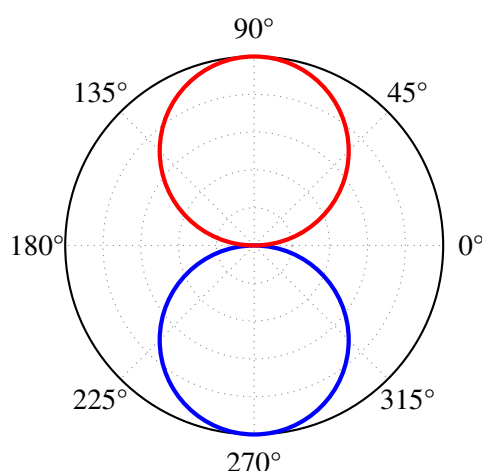


Figure 3: Directional characteristics of mic. 2. The red and blue colors denote the positive and negative values of the weight, respectively. The distance of the red or the blue line from the midpoint is the value of the weight.

- (a) Using the notation above, let the important speaker be speaker A (with signal \vec{a}) and let the person trolling be speaker B (with signal \vec{b}). Express the recordings of the two microphones \vec{m}_1 and \vec{m}_2 (i.e. the signals recorded by the first and the second microphones, respectively) as a linear combination of \vec{a} and \vec{b} .

Solution:

$$\begin{aligned}
\vec{m}_1 &= \cos\left(\frac{\pi}{4}\right) \cdot \vec{a} + \cos\left(-\frac{\pi}{6}\right) \cdot \vec{b} \\
&= \frac{1}{\sqrt{2}} \cdot \vec{a} + \frac{\sqrt{3}}{2} \cdot \vec{b} \\
\vec{m}_2 &= \sin\left(\frac{\pi}{4}\right) \cdot \vec{a} + \sin\left(-\frac{\pi}{6}\right) \cdot \vec{b} \\
&= \frac{1}{\sqrt{2}} \cdot \vec{a} - \frac{1}{2} \cdot \vec{b}
\end{aligned}$$

- (b) Recover the important speech \vec{a} , as a weighted combination of \vec{m}_1 and \vec{m}_2 . In other words, write $\vec{a} = u \cdot \vec{m}_1 + v \cdot \vec{m}_2$ (where u and v are scalars). What are the values of u and v ?

Solution:

Solving the system of linear equations yields

$$\vec{a} = \frac{\sqrt{2}}{1 + \sqrt{3}} \cdot (\vec{m}_1 + \sqrt{3}\vec{m}_2).$$

Therefore, the values are $u = \frac{\sqrt{2}}{1 + \sqrt{3}}$ and $v = \frac{\sqrt{6}}{1 + \sqrt{3}}$.

It is fine if you solved this either using IPython or by hand using any valid technique. The easiest approach is to subtract either of the two equations from the other and immediately see that $\vec{b} = \frac{2}{\sqrt{3}+1}(\vec{m}_1 - \vec{m}_2)$. Substituting \vec{b} back into the second equation and multiplying through by $\sqrt{2}$ gives that $\vec{a} = \sqrt{2}(\vec{m}_2 + \frac{1}{\sqrt{3}+1}(\vec{m}_1 - \vec{m}_2))$, which simplifies to the expression given above.

Notice that subtracting one equation from the other is natural given the symmetry of the microphone patterns and the fact that the patterns intersect at the 45 degree line where the important speech is happening. So we know that the result of subtraction will only contain the troll. Once we have the troll contribution, we can remove it.

- (c) Partial IPython code can be found in `prob1.ipynb`. Complete the code to get a clean signal of the important speech. What does the speaker say? (Optional: Where is the speech taken from?)

Note: You may have noticed that the recordings of the two microphones sounded remarkably similar. This means that you could recover the real speech from two “trolled” recordings that sounded almost identical! Leave out the fact that the recordings are actually different, and have some fun with your friends who aren’t lucky enough to be taking EE16A.

Solution:

The solution code can be found in `sol1.ipynb`. The speaker says: “All human beings are born free and equal in dignity and rights.” and the speech was taken from the Universal Declaration of Human Rights.

The idea of using multiple microphones to isolate speech is interesting and is increasingly used in practice. Furthermore, similar techniques are used in wireless communication both by cellular systems like LTE and increasingly by WiFi hotspots. (This is why they often have multiple antennas).

5. Homework Process and Study Group

Who else did you work with on this homework? List names and student ID’s. (In case of homework party, you can also just describe the group.) How did you work on this homework?

Solution:

I worked on this homework with...

I first worked by myself for 2 hours, but got stuck on problem 5, so I went to office hours on...

Then I went to homework party for a few hours, where I finished the homework.