

6. Show it.

Since $n \in \mathbb{Z}^+$, and $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k\}$ is a set of linearly dependent vectors in \mathbb{R}^n , so by the definition of linear dependence, we have that:
there exists an index i and scalars α_j 's such that

$$\vec{v}_i = \sum_{j \neq i} \alpha_j \vec{v}_j \quad (1)$$

Take any $n \times n$ matrix A , multiply both sides of Eq. (1) by A , so

$$A\vec{v}_i = A\left(\sum_{j \neq i} \alpha_j \vec{v}_j\right).$$

and by distributivity of matrix-vector multiplication, so we have:

$$A\vec{v}_i = \sum_{j \neq i} (A\alpha_j \vec{v}_j) = \sum_{j \neq i} \alpha_j (A\vec{v}_j).$$

which means that there exists an index i and scalars α_j 's for the set $\{A\vec{v}_1, A\vec{v}_2, \dots, A\vec{v}_k\}$ such that $A\vec{v}_i = \sum_{j \neq i} \alpha_j (A\vec{v}_j)$

Thus, by definition of linear dependence, the set

$\{A\vec{v}_1, A\vec{v}_2, \dots, A\vec{v}_k\}$ is a set of linearly dependent vectors.

(Q.E.D.)