pgRouting Manual

Release v2.5.0-dev

pgRouting Contributors

July 22, 2017

pgRouting extends the PostGIS[1]/PostgreSQL[2] geospatial database to provide geospatial routing and other network analysis functionality.

This is the manual for pgRouting v2.5.0-dev.

[1] http://postgis.net
[2] http://postgresql.org
[3] http://creativecommons.org/licenses/by-sa/3.0/

CHAPTER 1

General

1.1 Introduction

pgRouting is an extension of PostGIS[1] and PostgreSQL[2] geospatial database and adds routing and other network analysis functionality. A predecessor of pgRouting – pgDijkstra, written by Sylvain Pasche from Camptocamp[3], was later extended by Orkney[4] and renamed to pgRouting. The project is now supported and maintained by Georepublic[5], iMaptools[6] and a broad user community.

pgRouting is an OSGeo Labs[7] project of the OSGeo Foundation[8] and included on OSGeo Live[9].

1.1.1 License

The following licenses can be found in pgRouting:

License	
GNU General Public License, version 2	Most features of pgRouting are available under GNU General Public License, version 2[10].
Boost Software License - Version 1.0	Some Boost extensions are available under Boost Software License - Version 1.0[11].
MIT-X License	Some code contributed by iMaptools.com is available under MIT-X license.
Creative Commons Attribution-Share Alike 3.0 License	The pgRouting Manual is licensed under a Creative Commons Attribution-Share Alike 3.0 License[12].

In general license information should be included in the header of each source file.

[1] http://postgis.net
[2] http://postgresql.org
[3] http://camptocamp.com
[4] http://www.orkney.co.jp
[5] http://georepublic.info
[6] http://imaptools.com/
[7] http://wiki.osgeo.org/wiki/OSGeo_Labs
[8] http://osgeo.org
[9] http://live.osgeo.org/
[10] http://www.gnu.org/licenses/gpl-2.0.html
[11] http://www.boost.org/LICENSE_1_0.txt
[12] http://creativecommons.org/licenses/by-sa/3.0/

1.1.2 Contributors

This Release Contributors

Individuals (in alphabetical order)

Virginia Vergara

And all the people that give us a little of their time making comments, finding issues, making pull requests etc.

Corporate Sponsors (in alphabetical order)

These are corporate entities that have contributed developer time, hosting, or direct monetary funding to the pgRouting project:

- Georepublic[13]
- Google Summer of Code[14]
- iMaptools[15]
- Paragon Corporation[16]

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- Camptocamp
- CSIS (University of Tokyo)
- Georepublic
- Google Summer of Code
- iMaptools
- Orkney
- Paragon Corporation

[13] https://georepublic.info/en/
[14] https://developers.google.com/open-source/gsoc/
[15] http://imaptools.com
[16] http://www.paragoncorporation.com/

1.1.3 More Information

- The latest software, documentation and news items are available at the pgRouting web site http://pgrouting.org.
- PostgreSQL database server at the PostgreSQL main site http://www.postgresql.org.
- PostGIS extension at the PostGIS project web site http://postgis.net.
- Boost C++ source libraries at http://www.boost.org.
- Computational Geometry Algorithms Library (CGAL) at http://www.cgal.org.
- The Migration guide can be found at https://github.com/pgRouting/pgrouting/wiki/Migration-Guide.

1.2 Installation

Table of Contents

Instructions for downloading and installing binaries for different Operative systems instructions and additional notes and corrections not included in this documentation can be found in Installation wiki[17]

To use pgRouting postGIS needs to be installed, please read the information about installation in this Install Guide[18]

1.2.1 Short Version

Extracting the tar ball

```
tar xvfz pgrouting-2.4.0.tar.gz
cd pgrouting-2.4.0
```

To compile assuming you have all the dependencies in your search path:

```
mkdir build
cd build
cmake  ..
make
sudo make install
```

Once pgRouting is installed, it needs to be enabled in each individual database you want to use it in.

```
createdb routing
psql routing -c 'CREATE EXTENSION postGIS'
psql routing -c 'CREATE EXTENSION pgRouting'
```

[17] https://github.com/pgRouting/pgrouting/wiki/Notes-on-Download%2C-Installation-and-building-pgRouting
[18] http://www.postgis.us/presentations/postgis_install_guide_22.html

1.2.2 Get the sources

The pgRouting latest release can be found in https://github.com/pgRouting/pgrouting/releases/latest

wget

To download this release:

```
wget -O pgrouting-v2.4.0.tar.gz https://github.com/pgRouting/pgrouting/archive/v2.4.0.tar.gz
```

Goto *Short Version* to the extract and compile instructions.

git

To download the repository

```
git clone git://github.com/pgRouting/pgrouting.git
cd pgrouting
git checkout |release|
```

Goto *Short Version* to the compile instructions (there is no tar ball).

1.2.3 Enabling and upgrading in the database

Enabling the database

pgRouting is an extension and depends on postGIS. Enabling postGIS before enabling pgRouting in the database

```
CREATE EXTENSION postgis;
CREATE EXTENSION pgrouting;
```

Upgrading the database

To upgrade pgRouting in the database to version 2.4.0 use the following command:

```
ALTER EXTENSION pgrouting UPDATE TO "2.4.0";
```

More information can be found in https://www.postgresql.org/docs/current/static/sql-createextension.html

1.2.4 Dependencies

Compilation Dependencies

To be able to compile pgRouting, make sure that the following dependencies are met:

- C and C++0x compilers * g++ version >= 4.8
- Postgresql version >= 9.1
- PostGIS version >= 2.0
- The Boost Graph Library (BGL). Version >= 1.46
- CMake >= 2.8.8
- CGAL >= 4.2

optional dependencies

For user's documentation

- Sphinx >= 1.1
- Latex

For developer's documentation

- Doxygen >= 1.7

For testing

- pgtap
- pg_prove

Example: Installing dependencies on linux

Installing the compilation dependencies

```
sudo apt-get install
    cmake \
    g++ \
    postgresql-9.3 \
    postgresql-server-dev-9.3 \
    libboost-graph-dev \
    libcgal-dev
```

Installing the optional dependencies

```
sudo apt-get install -y python-sphinx \
    texlive \
    doxygen \
    libtap-parser-sourcehandler-pgtap-perl \
    postgresql-9.3-pgtap
```

1.2.5 Configuring

pgRouting uses the *cmake* system to do the configuration.

The build directory is different from the source directory

Create the build directory

```
$ mkdir build
```

Configurable variables

To see the variables that can be configured

```
$ cd build
$ cmake -L ..
```

Configuring The Documentation

Most of the effort of the documentation has being on the HTML files. Some variables for the documentation:

Variable	Default	Comment
WITH_DOC	BOOL=OFF	Turn on/off building the documentation
BUILD_HTML	BOOL=ON	If ON, turn on/off building HTML for user's documentation
BUILD_DOXY	BOOL=ON	If ON, turn on/off building HTML for developer's documentation
BUILD_LATEX	BOOL=OFF	If ON, turn on/off building PDF
BUILD_MAN	BOOL=OFF	If ON, turn on/off building MAN pages
DOC_USE_-BOOTSTRAP	BOOL=OFF	If ON, use sphinx-bootstrap for HTML pages of the users documentation

Configuring with documentation

```
$ cmake -DWITH_DOC=ON ..
```

Note: Most of the effort of the documentation has being on the html files.

1.2.6 Building

Using `make` to build the code and the documentation

The following instructions start from *path/to/pgrouting/build*

```
$ make          # build the code but not the documentation
$ make doc      # build only the documentation
$ make all doc  # build both the code and the documentation
```

We have tested on several platforms, For installing or reinstalling all the steps are needed.

Warning: The sql signatures are configured and build in the `cmake` command.

MinGW on Windows

```
$ mkdir build
$ cd build
$ cmake -G"MSYS Makefiles" ..
$ make
$ make install
```

Linux

The following instructions start from *path/to/pgrouting*

```
mkdir build
cd build
cmake  ..
make
sudo make install
```

When the configuration changes:

```
rm -rf build
```

and start the build process as mentioned above.

1.2.7 Testing

Currently there is no `make test` and testing is done as follows

The following instructions start from *path/to/pgrouting/*

```
tools/testers/algorithm-tester.pl
createdb  -U <user> ___pgr___test___
sh ./tools/testers/pg_prove_tests.sh <user>
dropdb  -U <user> ___pgr___test___
```

1.2.8 See Also

Indices and tables

- genindex
- search

1.3 Support

pgRouting community support is available through the pgRouting website[19], documentation[20], tutorials, mailing lists and others. If you're looking for *commercial support*, find below a list of companies providing pgRouting development and consulting services.

1.3.1 Reporting Problems

Bugs are reported and managed in an issue tracker[21]. Please follow these steps:

1. Search the tickets to see if your problem has already been reported. If so, add any extra context you might have found, or at least indicate that you too are having the problem. This will help us prioritize common issues.
2. If your problem is unreported, create a new issue[22] for it.
3. In your report include explicit instructions to replicate your issue. The best tickets include the exact SQL necessary to replicate a problem.
4. If you can test older versions of PostGIS for your problem, please do. On your ticket, note the earliest version the problem appears.
5. For the versions where you can replicate the problem, note the operating system and version of pgRouting, PostGIS and PostgreSQL.
6. It is recommended to use the following wrapper on the problem to pin point the step that is causing the problem.

```
SET client_min_messages TO debug;
  <your code>
SET client_min_messages TO notice;
```

1.3.2 Mailing List and GIS StackExchange

There are two mailing lists for pgRouting hosted on OSGeo mailing list server:

- User mailing list: http://lists.osgeo.org/mailman/listinfo/pgrouting-users
- Developer mailing list: http://lists.osgeo.org/mailman/listinfo/pgrouting-dev

[19] http://pgrouting.org/support.html
[20] http://docs.pgrouting.org
[21] https://github.com/pgrouting/pgrouting/issues
[22] https://github.com/pgRouting/pgrouting/issues/new

For general questions and topics about how to use pgRouting, please write to the user mailing list.

You can also ask at GIS StackExchange[23] and tag the question with `pgrouting`. Find all questions tagged with `pgrouting` under http://gis.stackexchange.com/questions/tagged/pgrouting or subscribe to the pgRouting questions feed[24].

1.3.3 Commercial Support

For users who require professional support, development and consulting services, consider contacting any of the following organizations, which have significantly contributed to the development of pgRouting:

Company	Offices in	Website
Georepublic	Germany, Japan	https://georepublic.info
iMaptools	United States	http://imaptools.com
Paragon Corporation	United States	http://www.paragoncorporation.com
Camptocamp	Switzerland, France	http://www.camptocamp.com

- *Sample Data* that is used in the examples of this manual.

1.4 Sample Data

The documentation provides very simple example queries based on a small sample network. To be able to execute the sample queries, run the following SQL commands to create a table with a small network data set.

Create table

```
CREATE TABLE edge_table (
    id BIGSERIAL,
    dir character varying,
    source BIGINT,
    target BIGINT,
    cost FLOAT,
    reverse_cost FLOAT,
    capacity BIGINT,
    reverse_capacity BIGINT,
    category_id INTEGER,
    reverse_category_id INTEGER,
    x1 FLOAT,
    y1 FLOAT,
    x2 FLOAT,
    y2 FLOAT,
    the_geom geometry
);
```

Insert data

```
INSERT INTO edge_table (
    category_id, reverse_category_id,
    cost, reverse_cost,
    capacity, reverse_capacity,
    x1, y1,
    x2, y2) VALUES
(3, 1,     1,  1,  80, 130,   2,   0,     2, 1),
(3, 2,    -1,  1,  -1, 100,   2,   1,     3, 1),
```

[23] http://gis.stackexchange.com/
[24] http://gis.stackexchange.com/feeds/tag?tagnames=pgrouting&sort=newest

```
(2, 1,   -1,  1,  -1, 130,   3,   1,    4, 1),
(2, 4,    1,  1, 100,  50,   2,   1,    2, 2),
(1, 4,    1, -1, 130,  -1,   3,   1,    3, 2),
(4, 2,    1,  1,  50, 100,   0,   2,    1, 2),
(4, 1,    1,  1,  50, 130,   1,   2,    2, 2),
(2, 1,    1,  1, 100, 130,   2,   2,    3, 2),
(1, 3,    1,  1, 130,  80,   3,   2,    4, 2),
(1, 4,    1,  1, 130,  50,   2,   2,    2, 3),
(1, 2,    1, -1, 130,  -1,   3,   2,    3, 3),
(2, 3,    1, -1, 100,  -1,   2,   3,    3, 3),
(2, 4,    1, -1, 100,  -1,   3,   3,    4, 3),
(3, 1,    1,  1,  80, 130,   2,   3,    2, 4),
(3, 4,    1,  1,  80,  50,   4,   2,    4, 3),
(3, 3,    1,  1,  80,  80,   4,   1,    4, 2),
(1, 2,    1,  1, 130, 100,   0.5, 3.5,  1.999999999999,3.5),
(4, 1,    1,  1,  50, 130,   3.5, 2.3,  3.5,4);

UPDATE edge_table SET the_geom = st_makeline(st_point(x1,y1),st_point(x2,y2)),
dir = CASE WHEN (cost>0 AND reverse_cost>0) THEN 'B'   -- both ways
           WHEN (cost>0 AND reverse_cost<0) THEN 'FT'  -- direction of the LINESSTRING
           WHEN (cost<0 AND reverse_cost>0) THEN 'TF'  -- reverse direction of the LINESTRING
           ELSE '' END;                                -- unknown
```

Topology

- Before you test a routing function use this query to create a topology (fills the `source` and `target` columns).

```
SELECT pgr_createTopology('edge_table',0.001);
```

Points of interest

- When points outside of the graph.
- Used with the *withPoints - Family of functions* functions.

```
CREATE TABLE pointsOfInterest(
    pid BIGSERIAL,
    x FLOAT,
    y FLOAT,
    edge_id BIGINT,
    side CHAR,
    fraction FLOAT,
    the_geom geometry,
    newPoint geometry
);

INSERT INTO pointsOfInterest (x, y, edge_id, side, fraction) VALUES
(1.8, 0.4,   1, 'l', 0.4),
(4.2, 2.4,  15, 'r', 0.4),
(2.6, 3.2,  12, 'l', 0.6),
(0.3, 1.8,   6, 'r', 0.3),
(2.9, 1.8,   5, 'l', 0.8),
(2.2, 1.7,   4, 'b', 0.7);
UPDATE pointsOfInterest SET the_geom = st_makePoint(x,y);

UPDATE pointsOfInterest
    SET newPoint = ST_LineInterpolatePoint(e.the_geom, fraction)
```

```
    FROM edge_table AS e WHERE edge_id = id;
```

Restrictions

- Used with the *pgr_trsp - Turn Restriction Shortest Path (TRSP)* functions.

```
CREATE TABLE restrictions (
    rid BIGINT NOT NULL,
    to_cost FLOAT,
    target_id BIGINT,
    from_edge BIGINT,
    via_path TEXT
);

INSERT INTO restrictions (rid, to_cost, target_id, from_edge, via_path) VALUES
(1, 100,  7,  4, NULL),
(1, 100, 11,  8, NULL),
(1, 100, 10,  7, NULL),
(2,   4,  8,  3, 5),
(3, 100,  9, 16, NULL);
```

Categories

- Used with the *Flow - Family of functions* functions.

```
/*
CREATE TABLE categories (
    category_id INTEGER,
    category text,
    capacity BIGINT
);

INSERT INTO categories VALUES
(1, 'Category 1', 130),
(2, 'Category 2', 100),
(3, 'Category 3',  80),
(4, 'Category 4',  50);
*/
```

Vertex table

- Used in some deprecated signatures or deprecated functions.

```
-- TODO check if this table is still used
CREATE TABLE vertex_table (
    id SERIAL,
    x FLOAT,
    y FLOAT
);
INSERT INTO vertex_table VALUES
(1,2,0), (2,2,1), (3,3,1), (4,4,1), (5,0,2), (6,1,2), (7,2,2),
(8,3,2), (9,4,2), (10,2,3), (11,3,3), (12,4,3), (13,2,4);
```

1.4.1 Images

- Red arrows correspond when `cost` > 0 in the edge table.
- Blue arrows correspond when `reverse_cost` > 0 in the edge table.
- Points are outside the graph.
- Click on the graph to enlarge.

Note: On all graphs,

Network for queries marked as `directed` and `cost` and `reverse_cost` columns are used:

When working with city networks, this is recommended for point of view of vehicles.

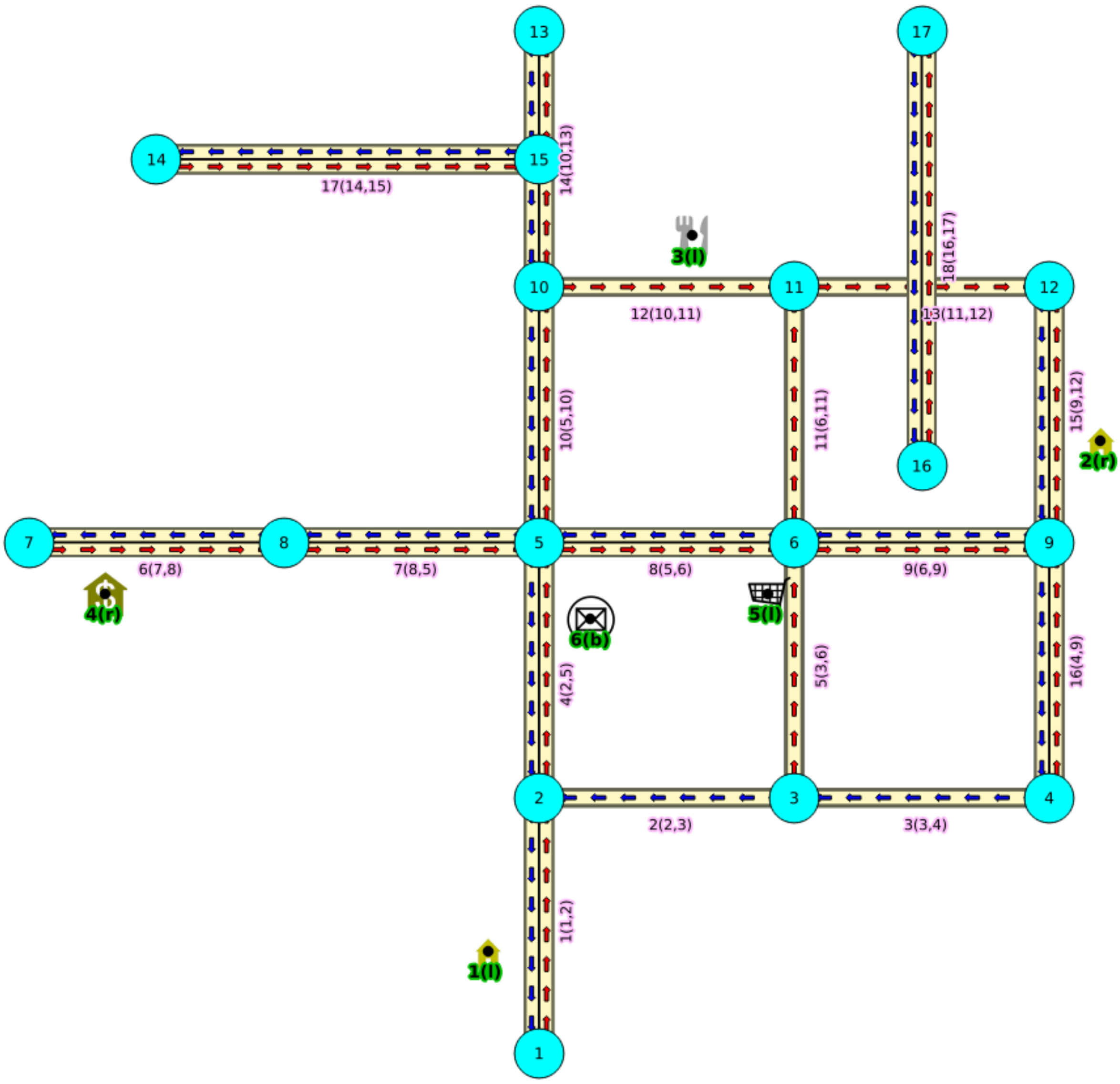


Fig. 1.1: **Graph 1: Directed, with cost and reverse cost**

Network for queries marked as `undirected` and `cost` and `reverse_cost` columns are used:

When working with city networks, this is recommended for point of view of pedestrians.

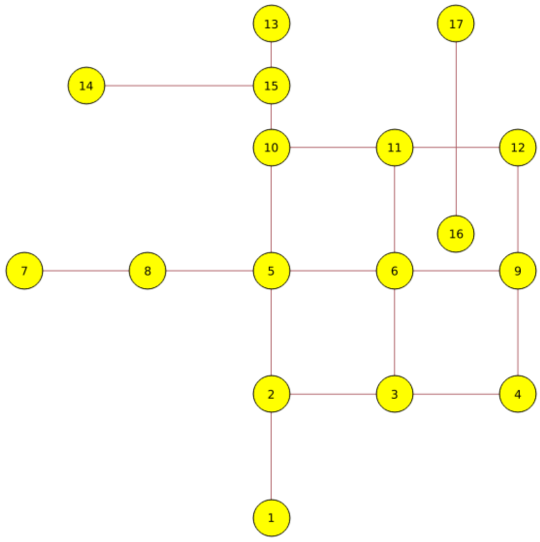


Fig. 1.2: **Graph 2: Undirected, with cost and reverse cost**

Network for queries marked as `directed` and only `cost` column is used:

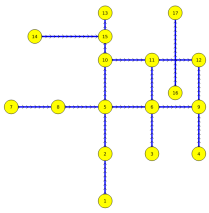


Fig. 1.3: **Graph 3: Directed, with cost**

Network for queries marked as `undirected` and only `cost` column is used:

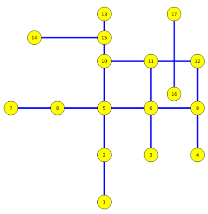


Fig. 1.4: **Graph 4: Undirected, with cost**

pickup & delivery Data

Vehicles table

```
CREATE TABLE vehicles (
      id BIGSERIAL PRIMARY KEY,
      start_node_id BIGINT,
      start_x FLOAT,
      start_y FLOAT,
      start_open FLOAT,
      start_close FLOAT,
      number integer,
      capacity FLOAT
);

INSERT INTO vehicles
(start_node_id, start_x,  start_y,  start_open,  start_close,  number,  capacity) VALUES
(            6,       3,        2,           0,           50,       2,        50);
```

Orders table

```
CREATE TABLE orders (
    id BIGSERIAL PRIMARY KEY,
    demand FLOAT,
    -- the pickups
    p_node_id BIGINT,
    p_x FLOAT,
    p_y FLOAT,
    p_open FLOAT,
    p_close FLOAT,
    p_service FLOAT,
    -- the deliveries
    d_node_id BIGINT,
    d_x FLOAT,
    d_y FLOAT,
    d_open FLOAT,
    d_close FLOAT,
    d_service FLOAT
);


INSERT INTO orders
(demand,
    p_node_id,  p_x, p_y,  p_open,  p_close,  p_service,
    d_node_id,  d_x, d_y,  d_open,  d_close,  d_service) VALUES
(10,
            3,    3,   1,       2,         10,          3,
            8,    1,   2,       6,         15,          3),
(20,
            9,    4,   2,       4,         15,          2,
            4,    4,   1,       6,         20,          3),
(30,
            5,    2,   2,       2,         10,          3,
           11,    3,   3,       3,         20,          3);
```

CHAPTER 2

Pgrouting Concepts

2.1 pgRouting Concepts

Contents

2.1.1 Getting Started

This is a simple guide to walk you through the steps of getting started with pgRouting. In this guide we will cover:

Create a routing Database

The first thing we need to do is create a database and load pgrouting in the database. Typically you will create a database for each project. Once you have a database to work in, your can load your data and build your application in that database. This makes it easy to move your project later if you want to to say a production server.

For Postgresql 9.2 and later versions

```
createdb mydatabase
psql mydatabase -c "create extension postgis"
psql mydatabase -c "create extension pgrouting"
```

Load Data

How you load your data will depend in what form it comes it. There are various OpenSource tools that can help you, like:

osm2pgrouting

- this is a tool for loading OSM data into postgresql with pgRouting requirements

shp2pgsql

- this is the postgresql shapefile loader

ogr2ogr

- this is a vector data conversion utility

osm2pgsql

- this is a tool for loading OSM data into postgresql

So these tools and probably others will allow you to read vector data so that you may then load that data into your database as a table of some kind. At this point you need to know a little about your data structure and content. One easy way to browse your new data table is with pgAdmin3 or phpPgAdmin.

Build a Routing Topology

Next we need to build a topology for our street data. What this means is that for any given edge in your street data the ends of that edge will be connected to a unique node and to other edges that are also connected to that same unique node. Once all the edges are connected to nodes we have a graph that can be used for routing with pgrouting. We provide a tool that will help with this:

Note: this step is not needed if data is loaded with *osm2pgrouting*

```
select pgr_createTopology('myroads', 0.000001);
```

- *pgr_createTopology*

Check the Routing Topology

There are lots of possible sources for errors in a graph. The data that you started with may not have been designed with routing in mind. A graph has some very specific requirements. One is that it is *NODED*, this means that except for some very specific use cases, each road segment starts and ends at a node and that in general is does not cross another road segment that it should be connected to.

There can be other errors like the direction of a one-way street being entered in the wrong direction. We do not have tools to search for all possible errors but we have some basic tools that might help.

```
select pgr_analyzegraph('myroads', 0.000001);
select pgr_analyzeoneway('myroads',  s_in_rules, s_out_rules,
                                     t_in_rules, t_out_rules
                                     direction)
select pgr_nodeNetwork('myroads', 0.001);
```

- *pgr_analyzeGraph*
- *pgr_analyzeOneway*
- *pgr_nodeNetwork*

Compute a Path

Once you have all the preparation work done above, computing a route is fairly easy. We have a lot of different algorithms that can work with your prepared road network. The general form of a route query is:

```
select pgr_dijkstra(`SELECT * FROM myroads', 1, 2)
```

As you can see this is fairly straight forward and you can look and the specific algorithms for the details of the signatures and how to use them. These results have information like edge id and/or the node id along with the cost or geometry for the step in the path from *start* to *end*. Using the ids you can join these result back to your edge table to get more information about each step in the path.

- *pgr_dijkstra*

2.1.2 Inner Queries

There are several kinds of valid inner queries and also the columns returned are depending of the function. Which kind of inner query will depend on the function(s) requirements. To simplify variety of types, `ANY-INTEGER` and `ANY-NUMERICAL` is used.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query (id is not necessary)

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default	Description
edges_sql	`TEXT`		SQL query as described above.
via_vertices	`ARRAY[ANY-INTEGER]`		Array of ordered vertices identifiers that are going to be visited.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as Undirected.
strict	`BOOLEAN`	`false`	• When `false` ignores missing paths returning all paths found • When `true` if a path is missing stops and returns *EMPTY SET*
U_turn_on_edge	`BOOLEAN`	`true`	• When `true` departing from a visited vertex will not try to avoid using the edge used to reach it. In other words, U turn using the edge with same *id* is allowed. • When `false` when a departing from a visited vertex tries to avoid using the edge used to reach it. In other words, U turn using the edge with same *id* is used when no other path is found.

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	ANY-NUMERICAL		X coordinate of *source* vertex.
y1	ANY-NUMERICAL		Y coordinate of *source* vertex.
x2	ANY-NUMERICAL		X coordinate of *target* vertex.
y2	ANY-NUMERICAL		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

2.1.3 Return columns & values

There are several kinds of columns returned are depending of the function.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

Description of the return values for a Cost function

Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, target).

2.1.4 Advanced Topics

Routing Topology

Overview

Typically when GIS files are loaded into the data database for use with pgRouting they do not have topology information associated with them. To create a useful topology the data needs to be "noded". This means that where two or more roads form an intersection there it needs to be a node at the intersection and all the road segments need to be broken at the intersection, assuming that you can navigate from any of these segments to any other segment via that intersection.

You can use the *graph analysis functions* to help you see where you might have topology problems in your data. If you need to node your data, we also have a function *pgr_nodeNetwork()* that might work for you. This function splits ALL crossing segments and nodes them. There are some cases where this might NOT be the right thing to do.

For example, when you have an overpass and underpass intersection, you do not want these noded, but pgr_-nodeNetwork does not know that is the case and will node them which is not good because then the router will be able to turn off the overpass onto the underpass like it was a flat 2D intersection. To deal with this problem some data sets use z-levels at these types of intersections and other data might not node these intersection which would be ok.

For those cases where topology needs to be added the following functions may be useful. One way to prep the data for pgRouting is to add the following columns to your table and then populate them as appropriate. This example makes a lot of assumption like that you original data tables already has certain columns in it like `one_way`, `fcc`, and possibly others and that they contain specific data values. This is only to give you an idea of what you can do with your data.

```
ALTER TABLE edge_table
    ADD COLUMN source integer,
    ADD COLUMN target integer,
    ADD COLUMN cost_len double precision,
    ADD COLUMN cost_time double precision,
    ADD COLUMN rcost_len double precision,
    ADD COLUMN rcost_time double precision,
    ADD COLUMN x1 double precision,
    ADD COLUMN y1 double precision,
    ADD COLUMN x2 double precision,
    ADD COLUMN y2 double precision,
    ADD COLUMN to_cost double precision,
    ADD COLUMN rule text,
    ADD COLUMN isolated integer;

SELECT pgr_createTopology('edge_table', 0.000001, 'the_geom', 'id');
```

The function *pgr_createTopology()* will create the `vertices_tmp` table and populate the `source` and `target` columns. The following example populated the remaining columns. In this example, the `fcc` column contains feature class code and the `CASE` statements converts it to an average speed.

```
UPDATE edge_table SET x1 = st_x(st_startpoint(the_geom)),
                      y1 = st_y(st_startpoint(the_geom)),
                      x2 = st_x(st_endpoint(the_geom)),
                      y2 = st_y(st_endpoint(the_geom)),
  cost_len  = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]'),
```

```
  rcost_len = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]'),
  len_km = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]')/1000.0,
  len_miles = st_length_spheroid(the_geom, 'SPHEROID["WGS84",6378137,298.25728]')
              / 1000.0 * 0.6213712,
  speed_mph = CASE WHEN fcc='A10' THEN 65
                   WHEN fcc='A15' THEN 65
                   WHEN fcc='A20' THEN 55
                   WHEN fcc='A25' THEN 55
                   WHEN fcc='A30' THEN 45
                   WHEN fcc='A35' THEN 45
                   WHEN fcc='A40' THEN 35
                   WHEN fcc='A45' THEN 35
                   WHEN fcc='A50' THEN 25
                   WHEN fcc='A60' THEN 25
                   WHEN fcc='A61' THEN 25
                   WHEN fcc='A62' THEN 25
                   WHEN fcc='A64' THEN 25
                   WHEN fcc='A70' THEN 15
                   WHEN fcc='A69' THEN 10
                   ELSE null END,
  speed_kmh = CASE WHEN fcc='A10' THEN 104
                   WHEN fcc='A15' THEN 104
                   WHEN fcc='A20' THEN 88
                   WHEN fcc='A25' THEN 88
                   WHEN fcc='A30' THEN 72
                   WHEN fcc='A35' THEN 72
                   WHEN fcc='A40' THEN 56
                   WHEN fcc='A45' THEN 56
                   WHEN fcc='A50' THEN 40
                   WHEN fcc='A60' THEN 50
                   WHEN fcc='A61' THEN 40
                   WHEN fcc='A62' THEN 40
                   WHEN fcc='A64' THEN 40
                   WHEN fcc='A70' THEN 25
                   WHEN fcc='A69' THEN 15
                   ELSE null END;

-- UPDATE the cost information based on oneway streets

UPDATE edge_table SET
    cost_time = CASE
        WHEN one_way='TF' THEN 10000.0
        ELSE cost_len/1000.0/speed_kmh::numeric*3600.0
        END,
    rcost_time = CASE
        WHEN one_way='FT' THEN 10000.0
        ELSE cost_len/1000.0/speed_kmh::numeric*3600.0
        END;

-- clean up the database because we have updated a lot of records

VACUUM ANALYZE VERBOSE edge_table;
```

Now your database should be ready to use any (most?) of the pgRouting algorithms.

Graph Analytics

Overview

It is common to find problems with graphs that have not been constructed fully noded or in graphs with z-levels at intersection that have been entered incorrectly. An other problem is one way streets that have been entered in the wrong direction. We can not detect errors with respect to "ground" truth, but we can look for inconsistencies and some anomalies in a graph and report them for additional inspections.

We do not current have any visualization tools for these problems, but I have used mapserver to render the graph and highlight potential problem areas. Someone familiar with graphviz might contribute tools for generating images with that.

Analyze a Graph

With *pgr_analyzeGraph* the graph can be checked for errors. For example for table "mytab" that has "mytab_-vertices_pgr" as the vertices table:

```
SELECT pgr_analyzeGraph('mytab', 0.000002);
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 158
NOTICE:                            Dead ends: 20028
NOTICE:  Potential gaps found near dead ends: 527
NOTICE:               Intersections detected: 2560
NOTICE:                      Ring geometries: 0
pgr_analyzeGraph
----------
   OK
(1 row)
```

In the vertices table "mytab_vertices_pgr":

- Deadends are identified by `cnt=1`
- Potencial gap problems are identified with `chk=1`.

```
SELECT count(*) as deadends  FROM mytab_vertices_pgr WHERE cnt = 1;
deadends
----------
    20028
 (1 row)

SELECT count(*) as gaps  FROM mytab_vertices_pgr WHERE chk = 1;
 gaps
 -----
   527
 (1 row)
```

For isolated road segments, for example, a segment where both ends are deadends. you can find these with the following query:

```
SELECT *
    FROM mytab a, mytab_vertices_pgr b, mytab_vertices_pgr c
    WHERE a.source=b.id AND b.cnt=1 AND a.target=c.id AND c.cnt=1;
```

If you want to visualize these on a graphic image, then you can use something like mapserver to render the edges and the vertices and style based on `cnt` or if they are isolated, etc. You can also do this with a tool like graphviz, or geoserver or other similar tools.

Analyze One Way Streets

pgr_analyzeOneway analyzes one way streets in a graph and identifies any flipped segments. Basically if you count the edges coming into a node and the edges exiting a node the number has to be greater than one.

This query will add two columns to the vertices_tmp table `ein int` and `eout int` and populate it with the appropriate counts. After running this on a graph you can identify nodes with potential problems with the following query.

The rules are defined as an array of text strings that if match the `col` value would be counted as true for the source or target in or out condition.

Example

Lets assume we have a table "st" of edges and a column "one_way" that might have values like:

- 'FT' - oneway from the source to the target node.
- 'TF' - oneway from the target to the source node.
- 'B' - two way street.
- '' - empty field, assume twoway.
- <NULL> - NULL field, use two_way_if_null flag.

Then we could form the following query to analyze the oneway streets for errors.

```
SELECT pgr_analyzeOneway('mytab',
            ARRAY['', 'B', 'TF'],
            ARRAY['', 'B', 'FT'],
            ARRAY['', 'B', 'FT'],
            ARRAY['', 'B', 'TF'],
            );

-- now we can see the problem nodes
SELECT * FROM mytab_vertices_pgr WHERE ein=0 OR eout=0;

-- and the problem edges connected to those nodes
SELECT gid FROM mytab a, mytab_vertices_pgr b WHERE a.source=b.id AND ein=0 OR eout=0
UNION
SELECT gid FROM mytab a, mytab_vertices_pgr b WHERE a.target=b.id AND ein=0 OR eout=0;
```

Typically these problems are generated by a break in the network, the one way direction set wrong, maybe an error related to z-levels or a network that is not properly noded.

The above tools do not detect all network issues, but they will identify some common problems. There are other problems that are hard to detect because they are more global in nature like multiple disconnected networks. Think of an island with a road network that is not connected to the mainland network because the bridge or ferry routes are missing.

2.1.5 Performance Tips

For the Routing functions

To get faster results bound your queries to the area of interest of routing to have, for example, no more than one million rows.

Use an inner query SQL that does not include some edges in the routing function

```
SELECT id, source, target from edge_table WHERE
        id < 17 and
        the_geom  && (select st_buffer(the_geom,1) as myarea FROM  edge_table where id = 5)
```

Integrating the inner query to the pgRouting function:

```
SELECT * FROM pgr_dijkstra(
        'SELECT id, source, target from edge_table WHERE
            id < 17 and
            the_geom  && (select st_buffer(the_geom,1) as myarea FROM  edge_table where id = 5)',
    1, 2)
```

For the topology functions:

When "you know" that you are going to remove a set of edges from the edges table, and without those edges you are going to use a routing function you can do the following:

Analize the new topology based on the actual topology:

```
pgr_analyzegraph('edge_table',rows_where:='id < 17');
```

Or create a new topology if the change is permanent:

```
pgr_createTopology('edge_table',rows_where:='id < 17');
pgr_analyzegraph('edge_table',rows_where:='id < 17');
```

2.1.6 How to contribute

Wiki

- Edit an existing pgRouting Wiki[1] page.
- Or create a new Wiki page
  - Create a page on the pgRouting Wiki[2]
  - Give the title an appropriate name
- Example[3]

Adding Functionaity to pgRouting

Consult the developer's documentation[4]

Indices and tables

- genindex
- search

Reference

pgr_version - to get pgRouting's version information.

[1] https://github.com/pgRouting/pgrouting/wiki
[2] https://github.com/pgRouting/pgrouting/wiki
[3] https://github.com/pgRouting/pgrouting/wiki/How-to:-Handle-parallel-edges-(KSP)
[4] http://docs.pgrouting.org/doxy/2.4/index.html

2.2 pgr_version

2.2.1 Name

`pgr_version` — Query for pgRouting version information.

2.2.2 Synopsis

Returns a table with pgRouting version information.

```
table() pgr_version();
```

2.2.3 Description

Returns a table with:

Column	Type	Description
version	`varchar`	pgRouting version
tag	`varchar`	Git tag of pgRouting build
hash	`varchar`	Git hash of pgRouting build
branch	`varchar`	Git branch of pgRouting build
boost	`varchar`	Boost version

History

- New in version 2.0.0

2.2.4 Examples

- Query for full version string

```
SELECT version FROM pgr_version();
 version
---------
 2.5.0
(1 row)
```

- Query for `version` and `boost` attribute

```
SELECT version, boost FROM pgr_version();

  version   | boost
-----------+--------
 2.2.0-dev | 1.49.0
(1 row)
```

Data Types

- *pgr_costResult[]* - A set of records to describe a path result with cost attribute.
- *pgr_costResult3[]* - A set of records to describe a path result with cost attribute.
- *pgr_geomResult* - A set of records to describe a path result with geometry attribute.

3.1 pgRouting Data Types

The following are commonly used data types for some of the pgRouting functions.

- *pgr_costResult[]* - A set of records to describe a path result with cost attribute.
- *pgr_costResult3[]* - A set of records to describe a path result with cost attribute.
- *pgr_geomResult* - A set of records to describe a path result with geometry attribute.

3.1.1 pgr_costResult[]

Name

`pgr_costResult[]` — A set of records to describe a path result with cost attribute.

Description

```
CREATE TYPE pgr_costResult AS
(
    seq integer,
    id1 integer,
    id2 integer,
    cost float8
);
```

seq sequential ID indicating the path order

id1 generic name, to be specified by the function, typically the node id

id2 generic name, to be specified by the function, typically the edge id

cost cost attribute

3.1.2 pgr_costResult3[] - Multiple Path Results with Cost

Name

`pgr_costResult3[]` — A set of records to describe a path result with cost attribute.

Description

```
CREATE TYPE pgr_costResult3 AS
(
    seq integer,
    id1 integer,
    id2 integer,
    id3 integer,
    cost float8
);
```

seq sequential ID indicating the path order

id1 generic name, to be specified by the function, typically the path id

id2 generic name, to be specified by the function, typically the node id

id3 generic name, to be specified by the function, typically the edge id

cost cost attribute

History

- New in version 2.0.0
- Replaces `path_result`

See Also

- *Introduction*

3.1.3 pgr_geomResult[]

Name

`pgr_geomResult[]` — A set of records to describe a path result with geometry attribute.

Description

```
CREATE TYPE pgr_geomResult AS
(
    seq integer,
    id1 integer,
    id2 integer,
    geom geometry
);
```

seq sequential ID indicating the path order

id1 generic name, to be specified by the function

id2 generic name, to be specified by the function

geom geometry attribute

History

- New in version 2.0.0
- Replaces `geoms`

See Also

- *Introduction*

Topology Functions

- *pgr_createTopology* - to create a topology based on the geometry.
- *pgr_createVerticesTable* - to reconstruct the vertices table based on the source and target information.
- *pgr_analyzeGraph* - to analyze the edges and vertices of the edge table.
- *pgr_analyzeOneway* - to analyze directionality of the edges.
- *pgr_nodeNetwork* -to create nodes to a not noded edge table.

4.1 Topology - Family of Functions

The pgRouting's topology of a network, represented with an edge table with source and target attributes and a vertices table associated with it. Depending on the algorithm, you can create a topology or just reconstruct the vertices table, You can analyze the topology, We also provide a function to node an unoded network.

- *pgr_createTopology* - to create a topology based on the geometry.
- *pgr_createVerticesTable* - to reconstruct the vertices table based on the source and target information.
- *pgr_analyzeGraph* - to analyze the edges and vertices of the edge table.
- *pgr_analyzeOneway* - to analyze directionality of the edges.
- *pgr_nodeNetwork* -to create nodes to a not noded edge table.

4.1.1 pgr_createTopology

Name

`pgr_createTopology` — Builds a network topology based on the geometry information.

Synopsis

The function returns:

- `OK` after the network topology has been built and the vertices table created.
- `FAIL` when the network topology was not built due to an error.

```
varchar pgr_createTopology(text edge_table, double precision tolerance,
                   text the_geom:='the_geom', text id:='id',
                   text source:='source',text target:='target',
                   text rows_where:='true', boolean clean:=false)
```

Description

Parameters

The topology creation function accepts the following parameters:

edge_table `text` Network table name. (may contain the schema name AS well)

tolerance `float8` Snapping tolerance of disconnected edges. (in projection unit)

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

id `text` Primary key column name of the network table. Default value is `id`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

rows_where `text` Condition to SELECT a subset or rows. Default value is `true` to indicate all rows that where `source` or `target` have a null value, otherwise the condition is used.

clean `boolean` Clean any previous topology. Default value is `false`.

> **Warning:** The `edge_table` will be affected
> - The `source` column values will change.
> - The `target` column values will change.
> - An index will be created, if it doesn't exists, to speed up the process to the following columns:
>   - `id`
>   - `the_geom`
>   - `source`
>   - `target`

The function returns:

- `OK` after the network topology has been built.
  - Creates a vertices table: <edge_table>_vertices_pgr.
  - Fills `id` and `the_geom` columns of the vertices table.
  - Fills the source and target columns of the edge table referencing the `id` of the vertices table.
- `FAIL` when the network topology was not built due to an error:
  - A required column of the Network table is not found or is not of the appropriate type.
  - The condition is not well formed.
  - The names of source , target or id are the same.
  - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table is a requirement of the *pgr_analyzeGraph* and the *pgr_analyzeOneway* functions.

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex. See *pgr_analyzeGraph*.

chk `integer` Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein `integer` Number of vertices in the edge_table that reference this vertex AS incoming. See *pgr_analyzeOneway*.

eout `integer` Number of vertices in the edge_table that reference this vertex AS outgoing. See *pgr_analyzeOneway*.

the_geom `geometry` Point geometry of the vertex.

History

- Renamed in version 2.0.0

Usage when the edge table's columns MATCH the default values:

The simplest way to use pgr_createTopology is:

```
SELECT  pgr_createTopology('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source', 'target', rows_where
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  18 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)
```

When the arguments are given in the order described in the parameters:

We get the same result AS the simplest way to use the function.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    'the_geom', 'id', 'source', 'target');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source', 'target', rows_where
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  18 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)
```

Warning:
An error would occur when the arguments are not given in the appropriate order:
In this example, the column `id` of the table `ege_table` is passed to the function as the geometry column, and the geometry column `the_geom` is passed to the function as the id column.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    'id', 'the_geom');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'id', 'the_geom', 'source', 'target', rows_wher
NOTICE:  Performing checks, please wait .....
NOTICE:  ----> PGR ERROR in pgr_createTopology: Wrong type of Column id:the_geom
NOTICE:  Unexpected error raise_exception
 pgr_createtopology
--------------------
 FAIL
(1 row)
```

When using the named notation

Parameters defined with a default value can be omitted, as long as the value matches the default And The order of the parameters would not matter.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    the_geom:='the_geom', id:='id', source:='source', target:='target');
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('edge_table', 0.001,
    source:='source', id:='id', target:='target', the_geom:='the_geom');
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('edge_table', 0.001, source:='source');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT  pgr_createTopology('edge_table', 0.001, rows_where:='id < 10');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting the rows where the geometry is near the geometry of row with `id = 5`.

```
SELECT  pgr_createTopology('edge_table', 0.001,
    rows_where:='the_geom && (SELECT st_buffer(the_geom, 0.05) FROM edge_table WHERE id=5)');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
CREATE TABLE otherTable AS  (SELECT 100 AS gid,  st_point(2.5, 2.5) AS other_geom);
SELECT 1
SELECT  pgr_createTopology('edge_table', 0.001,
    rows_where:='the_geom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE gid=100)');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Usage when the edge table's columns DO NOT MATCH the default values:

For the following table

```
CREATE TABLE mytable AS (SELECT id AS gid,  the_geom AS mygeom, source AS src , target AS tgt FRO
SELECT 18
```

Using positional notation:

The arguments need to be given in the order described in the parameters.

Note that this example uses clean flag. So it recreates the whole vertices table.

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt', clean := TRUE);
 pgr_createtopology
--------------------
 OK
(1 row)
```

Warning:
An error would occur when the arguments are not given in the appropiriate order:
In this example, the column `gid` of the table `mytable` is passed to the function AS the geometry column, and the geometry column `mygeom` is passed to the function AS the id column.

```
SELECT  pgr_createTopology('mytable', 0.001, 'gid', 'mygeom', 'src', 'tgt');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('mytable', 0.001, 'gid', 'mygeom', 'src', 'tgt', rows_where := 'true
NOTICE:  Performing checks, please wait .....
NOTICE:  ----> PGR ERROR in pgr_createTopology: Wrong type of Column id:mygeom
NOTICE:  Unexpected error raise_exception
 pgr_createtopology
--------------------
 FAIL
(1 row)
```

When using the named notation

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table. The order of the parameters do not matter:

```
SELECT  pgr_createTopology('mytable', 0.001, the_geom:='mygeom', id:='gid', source:='src', target
 pgr_createtopology
--------------------
 OK
(1 row)
```

```
SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_geom:='m
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting rows using rows_where parameter

Based on id:

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt', rows_where:='gid < 10
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_geom:='m
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt',
    rows_where:='mygeom && (SELECT st_buffer(mygeom, 1) FROM mytable WHERE gid=5)');
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_geom:='m
    rows_where:='mygeom && (SELECT st_buffer(mygeom, 1) FROM mytable WHERE gid=5)');
 pgr_createtopology
--------------------
 OK
(1 row)
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
SELECT  pgr_createTopology('mytable', 0.001, 'mygeom', 'gid', 'src', 'tgt',
    rows_where:='mygeom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE gid=100)');
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT  pgr_createTopology('mytable', 0.001, source:='src', id:='gid', target:='tgt', the_geom:='m
    rows_where:='mygeom && (SELECT st_buffer(other_geom, 1) FROM otherTable WHERE gid=100)');
 pgr_createtopology
```

```
--------------------
 OK
(1 row)
```

Examples with full output

This example start a clean topology, with 5 edges, and then its incremented to the rest of the edges.

```
SELECT pgr_createTopology('edge_table',  0.001, rows_where:='id < 6', clean := true);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source', 'target', rows_where
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  5 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)

SELECT pgr_createTopology('edge_table',  0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table', 0.001, 'the_geom', 'id', 'source', 'target', rows_where
NOTICE:  Performing checks, please wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  13 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------
 pgr_createtopology
--------------------
 OK
(1 row)
```

The example uses the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *pgr_createVerticesTable* to reconstruct the vertices table based on the source and target information.
- *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.

Indices and tables

- genindex
- search

4.1.2 pgr_createVerticesTable

Name

`pgr_createVerticesTable` — Reconstructs the vertices table based on the source and target information.

Synopsis

The function returns:

- `OK` after the vertices table has been reconstructed.
- `FAIL` when the vertices table was not reconstructed due to an error.

```
    pgr_createVerticesTable(edge_table, the_geom, source, target, rows_where)
RETURNS VARCHAR
```

Description

Parameters

The reconstruction of the vertices table function accepts the following parameters:

edge_table `text` Network table name. (may contain the schema name as well)

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

rows_where `text` Condition to SELECT a subset or rows. Default value is `true` to indicate all rows.

Warning: The `edge_table` will be affected

- An index will be created, if it doesn't exists, to speed up the process to the following columns:
  - `the_geom`
  - `source`
  - `target`

The function returns:

- `OK` after the vertices table has been reconstructed.
  - Creates a vertices table: <edge_table>_vertices_pgr.
  - Fills `id` and `the_geom` columns of the vertices table based on the source and target columns of the edge table.
- `FAIL` when the vertices table was not reconstructed due to an error.
  - A required column of the Network table is not found or is not of the appropriate type.
  - The condition is not well formed.
  - The names of source, target are the same.
  - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table is a requierment of the *pgr_analyzeGraph* and the *pgr_analyzeOneway* functions.

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex. See *pgr_analyzeGraph*.

chk `integer` Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein `integer` Number of vertices in the edge_table that reference this vertex as incoming. See *pgr_analyzeOneway*.

eout `integer` Number of vertices in the edge_table that reference this vertex as outgoing. See *pgr_analyzeOneway*.

the_geom `geometry` Point geometry of the vertex.

History

- Renamed in version 2.0.0

Usage when the edge table's columns MATCH the default values:

The simplest way to use pgr_createVerticesTable is:

```
SELECT  pgr_createVerticesTable('edge_table');
```

When the arguments are given in the order described in the parameters:

```
SELECT  pgr_createVerticesTable('edge_table','the_geom','source','target');
```

We get the same result as the simplest way to use the function.

> **Warning:**
> An error would occur when the arguments are not given in the appropriate order: In this example, the column source column `source` of the table `mytable` is passed to the function as the geometry column, and the geometry column `the_geom` is passed to the function as the source column.
> `SELECT pgr_createVerticesTable('edge_table','source','the_geom','target');`

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_createVerticesTable('edge_table',the_geom:='the_geom',source:='source',target:='targe
```

```
SELECT  pgr_createVerticesTable('edge_table',source:='source',target:='target',the_geom:='the_geo
```

Parameters defined with a default value can be omitted, as long as the value matches the default:

```
SELECT  pgr_createVerticesTable('edge_table',source:='source');
```

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT  pgr_createVerticesTable('edge_table',rows_where:='id < 10');
```

Selecting the rows where the geometry is near the geometry of row with `id` =5 .

```
SELECT  pgr_createVerticesTable('edge_table',rows_where:='the_geom && (select st_buffer(the_geom,
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_createVerticesTable('edge_table',rows_where:='the_geom && (select st_buffer(othergeom,
```

Usage when the edge table's columns DO NOT MATCH the default values:

For the following table

```
DROP TABLE IF EXISTS mytable;
CREATE TABLE mytable AS (SELECT id AS gid, the_geom AS mygeom,source AS src ,target AS tgt FROM ed
```

Using positional notation:

The arguments need to be given in the order described in the parameters:

```
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt');
```

Warning:
An error would occur when the arguments are not given in the appropriate order: In this example, the column `src` of the table `mytable` is passed to the function as the geometry column, and the geometry column `mygeom` is passed to the function as the source column.
`SELECT pgr_createVerticesTable('mytable','src','mygeom','tgt');`

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_createVerticesTable('mytable',the_geom:='mygeom',source:='src',target:='tgt');
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:='mygeom');
```

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table.

Selecting rows using rows_where parameter

Selecting rows based on the gid.

```
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt',rows_where:='gid < 10');
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:='mygeom',rows_whe
```

Selecting the rows where the geometry is near the geometry of row with `gid` =5 .

```
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt',
                                    rows_where:='the_geom && (SELECT st_buffer(mygeom,0.5) FROM mytable WHE
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:='mygeom',
                                    rows_where:='mygeom && (SELECT st_buffer(mygeom,0.5) FROM mytable WHERE
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_createVerticesTable('mytable','mygeom','src','tgt',
                                      rows_where:='the_geom && (SELECT st_buffer(othergeom,0.5) FROM otherTa
```

```
SELECT  pgr_createVerticesTable('mytable',source:='src',target:='tgt',the_geom:='mygeom',
                                        rows_where:='the_geom && (SELECT st_buffer(othergeom,0.5) FROM otherTa
```

Examples

```
    SELECT pgr_createVerticesTable('edge_table');
    NOTICE:  PROCESSING:
NOTICE:  pgr_createVerticesTable('edge_table','the_geom','source','target','true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Populating public.edge_table_vertices_pgr, please wait...
NOTICE:    ----->   VERTICES TABLE CREATED WITH  17 VERTICES
NOTICE:                                          FOR   18  EDGES
NOTICE:    Edges with NULL geometry,source or target: 0
NOTICE:                              Edges processed: 18
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------

     pgr_createVerticesTable
    --------------------
     OK
    (1 row)
```

The example uses the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *pgr_createTopology* to create a topology based on the geometry.
- *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.
- *pgr_analyzeOneway* to analyze directionality of the edges.

4.1.3 pgr_analyzeGraph

Name

`pgr_analyzeGraph` — Analyzes the network topology.

Synopsis

The function returns:

- `OK` after the analysis has finished.
- `FAIL` when the analysis was not completed due to an error.

```
varchar pgr_analyzeGraph(text edge_table, double precision tolerance,
                   text the_geom:='the_geom', text id:='id',
                   text source:='source',text target:='target',text rows_where:='true')
```

Description

Prerequisites

The edge table to be analyzed must contain a source column and a target column filled with id's of the vertices of the segments and the corresponding vertices table <edge_table>_vertices_pgr that stores the vertices information.

- Use *pgr_createVerticesTable* to create the vertices table.
- Use *pgr_createTopology* to create the topology and the vertices table.

Parameters

The analyze graph function accepts the following parameters:

edge_table `text` Network table name. (may contain the schema name as well)

tolerance `float8` Snapping tolerance of disconnected edges. (in projection unit)

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

id `text` Primary key column name of the network table. Default value is `id`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

rows_where `text` Condition to select a subset or rows. Default value is `true` to indicate all rows.

The function returns:

- `OK` after the analysis has finished.
  - Uses the vertices table: <edge_table>_vertices_pgr.
  - Fills completely the `cnt` and `chk` columns of the vertices table.
  - Returns the analysis of the section of the network defined by `rows_where`
- `FAIL` when the analysis was not completed due to an error.
  - The vertices table is not found.
  - A required column of the Network table is not found or is not of the appropriate type.
  - The condition is not well formed.
  - The names of source , target or id are the same.
  - The SRID of the geometry could not be determined.

The Vertices Table

The vertices table can be created with *pgr_createVerticesTable* or *pgr_createTopology*

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex.

chk `integer` Indicator that the vertex might have a problem.

ein `integer` Number of vertices in the edge_table that reference this vertex as incoming. See *pgr_analyzeOneway*.

eout `integer` Number of vertices in the edge_table that reference this vertex as outgoing. See *pgr_analyzeOneway*.

the_geom `geometry` Point geometry of the vertex.

History

- New in version 2.0.0

Usage when the edge table's columns MATCH the default values:

The simplest way to use pgr_analyzeGraph is:

```
SELECT  pgr_createTopology('edge_table',0.001);
SELECT  pgr_analyzeGraph('edge_table',0.001);
```

When the arguments are given in the order described in the parameters:

```
SELECT  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target');
```

We get the same result as the simplest way to use the function.

> **Warning:**
> An error would occur when the arguments are not given in the appropriate order: In this example, the column `id` of the table `mytable` is passed to the function as the geometry column, and the geometry column `the_geom` is passed to the function as the id column.
> `SELECT pgr_analyzeGraph('edge_table',0.001,'id','the_geom','source','target');`
> ERROR: Can not determine the srid of the geometry "id" in table public.edge_table

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_analyzeGraph('edge_table',0.001,the_geom:='the_geom',id:='id',source:='source',target
```

```
SELECT  pgr_analyzeGraph('edge_table',0.001,source:='source',id:='id',target:='target',the_geom:=
```

Parameters defined with a default value can be omitted, as long as the value matches the default:

```
SELECT  pgr_analyzeGraph('edge_table',0.001,source:='source');
```

Selecting rows using rows_where parameter

Selecting rows based on the id. Displays the analysis a the section of the network.

```
SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='id < 10');
```

Selecting the rows where the geometry is near the geometry of row with `id` =5 .

```
SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='the_geom && (SELECT st_buffer(the_geom,0
```

Selecting the rows where the geometry is near the geometry of the row with `gid` =100 of the table `othertable`.

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 100 AS gid, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='the_geom && (SELECT st_buffer(other_geom,
```

Usage when the edge table's columns DO NOT MATCH the default values:

For the following table

```
DROP TABLE IF EXISTS mytable;
CREATE TABLE mytable AS (SELECT id AS gid, source AS src ,target AS tgt , the_geom AS mygeom FROM
SELECT pgr_createTopology('mytable',0.001,'mygeom','gid','src','tgt');
```

Using positional notation:

The arguments need to be given in the order described in the parameters:

```
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt');
```

Warning:
An error would occur when the arguments are not given in the appropriate order: In this example, the column `gid` of the table `mytable` is passed to the function as the geometry column, and the geometry column `mygeom` is passed to the function as the id column.
`SELECT pgr_analyzeGraph('mytable',0.001,'gid','mygeom','src','tgt');`
ERROR: Can not determine the srid of the geometry "gid" in table public.mytable

When using the named notation

The order of the parameters do not matter:

```
SELECT  pgr_analyzeGraph('mytable',0.001,the_geom:='mygeom',id:='gid',source:='src',target:='tgt'
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:='mygeom'
```

In this scenario omitting a parameter would create an error because the default values for the column names do not match the column names of the table.

Selecting rows using rows_where parameter

Selecting rows based on the id.

```
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',rows_where:='gid < 10');
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:='mygeom',
```

Selecting the rows WHERE the geometry is near the geometry of row with `id` =5 .

```
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',
                           rows_where:='mygeom && (SELECT st_buffer(mygeom,1) FROM mytable WHERE 
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:='mygeom',
                           rows_where:='mygeom && (SELECT st_buffer(mygeom,1) FROM mytable WHERE 
```

Selecting the rows WHERE the geometry is near the place='myhouse' of the table `othertable`. (note the use of quote_literal)

```
DROP TABLE IF EXISTS otherTable;
CREATE TABLE otherTable AS  (SELECT 'myhouse'::text AS place, st_point(2.5,2.5) AS other_geom) ;
SELECT  pgr_analyzeGraph('mytable',0.001,'mygeom','gid','src','tgt',
         rows_where:='mygeom && (SELECT st_buffer(other_geom,1) FROM otherTable WHERE place='||quo
```

```
SELECT  pgr_analyzeGraph('mytable',0.001,source:='src',id:='gid',target:='tgt',the_geom:='mygeom',
        rows_where:='mygeom && (SELECT st_buffer(other_geom,1) FROM otherTable WHERE place='||quot
```

Examples

```
    SELECT  pgr_createTopology('edge_table',0.001);
    SELECT pgr_analyzeGraph('edge_table', 0.001);
    NOTICE:  PROCESSING:
    NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
```

```
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 2
NOTICE:                            Dead ends: 7
NOTICE:  Potential gaps found near dead ends: 1
NOTICE:               Intersections detected: 1
NOTICE:                      Ring geometries: 0

 pgr_analyzeGraph
--------------------
 OK
(1 row)

SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='id < 10');
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','id < 10')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 4
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 0
NOTICE:                      Ring geometries: 0

 pgr_analyzeGraph
--------------------
 OK
(1 row)

SELECT  pgr_analyzeGraph('edge_table',0.001,rows_where:='id >= 10');
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','id >= 10')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 2
NOTICE:                            Dead ends: 8
NOTICE:  Potential gaps found near dead ends: 1
NOTICE:               Intersections detected: 1
NOTICE:                      Ring geometries: 0

 pgr_analyzeGraph
--------------------
 OK
(1 row)

-- Simulate removal of edges
SELECT pgr_createTopology('edge_table', 0.001,rows_where:='id <17');
SELECT pgr_analyzeGraph('edge_table', 0.001);
```

```
    NOTICE:  PROCESSING:
    NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
    NOTICE:  Performing checks, pelase wait...
    NOTICE:  Analyzing for dead ends. Please wait...
    NOTICE:  Analyzing for gaps. Please wait...
    NOTICE:  Analyzing for isolated edges. Please wait...
    NOTICE:  Analyzing for ring geometries. Please wait...
    NOTICE:  Analyzing for intersections. Please wait...
    NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
    NOTICE:                    Isolated segments: 0
                                       Dead ends: 3
    NOTICE:  Potential gaps found near dead ends: 0
    NOTICE:               Intersections detected: 0
    NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)
SELECT pgr_createTopology('edge_table', 0.001,rows_where:='id <17');
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table',0.001,'the_geom','id','source','target','id <17')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  16 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)

SELECT pgr_analyzeGraph('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 3
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 0
NOTICE:                      Ring geometries: 0

     pgr_analyzeGraph
    --------------------
     OK
    (1 row)
```

The examples use the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *pgr_analyzeOneway* to analyze directionality of the edges.

- *pgr_createVerticesTable* to reconstruct the vertices table based on the source and target information.
- *pgr_nodeNetwork* to create nodes to a not noded edge table.

4.1.4 pgr_analyzeOneway

Name

`pgr_analyzeOneway` — Analyzes oneway Sstreets and identifies flipped segments.

Synopsis

This function analyzes oneway streets in a graph and identifies any flipped segments.

```
text pgr_analyzeOneway(geom_table text,
                       text[] s_in_rules, text[] s_out_rules,
                       text[] t_in_rules, text[] t_out_rules,
                       text oneway='oneway', text source='source', text target='target',
                       boolean two_way_if_null=true);
```

Description

The analyses of one way segments is pretty simple but can be a powerful tools to identifying some the potential problems created by setting the direction of a segment the wrong way. A node is a *source* if it has edges the exit from that node and no edges enter that node. Conversely, a node is a *sink* if all edges enter the node but none exit that node. For a *source* type node it is logically impossible to exist because no vehicle can exit the node if no vehicle and enter the node. Likewise, if you had a *sink* node you would have an infinite number of vehicle piling up on this node because you can enter it but not leave it.

So why do we care if the are not feasible? Well if the direction of an edge was reversed by mistake we could generate exactly these conditions. Think about a divided highway and on the north bound lane one segment got entered wrong or maybe a sequence of multiple segments got entered wrong or maybe this happened on a round-about. The result would be potentially a *source* and/or a *sink* node.

So by counting the number of edges entering and exiting each node we can identify both *source* and *sink* nodes so that you can look at those areas of your network to make repairs and/or report the problem back to your data vendor.

Prerequisites

The edge table to be analyzed must contain a source column and a target column filled with id's of the vertices of the segments and the corresponding vertices table <edge_table>_vertices_pgr that stores the vertices information.

- Use *pgr_createVerticesTable* to create the vertices table.
- Use *pgr_createTopology* to create the topology and the vertices table.

Parameters

edge_table `text` Network table name. (may contain the schema name as well)

s_in_rules `text[]` source node **in** rules

s_out_rules `text[]` source node **out** rules

t_in_rules `text[]` target node **in** rules

t_out_rules `text[]` target node **out** rules

oneway `text` oneway column name name of the network table. Default value is `oneway`.

source `text` Source column name of the network table. Default value is `source`.

target `text` Target column name of the network table. Default value is `target`.

two_way_if_null `boolean` flag to treat oneway NULL values as bi-directional. Default value is `true`.

Note: It is strongly recommended to use the named notation. See *pgr_createVerticesTable* or *pgr_createTopology* for examples.

The function returns:

- `OK` after the analysis has finished.
  - Uses the vertices table: <edge_table>_vertices_pgr.
  - Fills completely the `ein` and `eout` columns of the vertices table.
- `FAIL` when the analysis was not completed due to an error.
  - The vertices table is not found.
  - A required column of the Network table is not found or is not of the appropriate type.
  - The names of source , target or oneway are the same.

The rules are defined as an array of text strings that if match the `oneway` value would be counted as `true` for the source or target **in** or **out** condition.

The Vertices Table

The vertices table can be created with *pgr_createVerticesTable* or *pgr_createTopology*

The structure of the vertices table is:

id `bigint` Identifier of the vertex.

cnt `integer` Number of vertices in the edge_table that reference this vertex. See *pgr_analyzeGraph*.

chk `integer` Indicator that the vertex might have a problem. See *pgr_analyzeGraph*.

ein `integer` Number of vertices in the edge_table that reference this vertex as incoming.

eout `integer` Number of vertices in the edge_table that reference this vertex as outgoing.

the_geom `geometry` Point geometry of the vertex.

History

- New in version 2.0.0

Examples

```
SELECT pgr_analyzeOneway('edge_table',
ARRAY['', 'B', 'TF'],
ARRAY['', 'B', 'FT'],
ARRAY['', 'B', 'FT'],
ARRAY['', 'B', 'TF'],
oneway:='dir');
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table','{"",B,TF}','{"",B,FT}','{"",B,FT}','{"",B,TF}','dir','sou
NOTICE:  Analyzing graph for one way street errors.
```

```
NOTICE:  Analysis 25% complete ...
NOTICE:  Analysis 50% complete ...
NOTICE:  Analysis 75% complete ...
NOTICE:  Analysis 100% complete ...
NOTICE:  Found 0 potential problems in directionality

pgr_analyzeoneway
-------------------
OK
(1 row)
```

The queries use the *Sample Data* network.

See Also

- *Routing Topology* for an overview of a topology for routing algorithms.
- *Graph Analytics* for an overview of the analysis of a graph.
- *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.
- *pgr_createVerticesTable* to reconstruct the vertices table based on the source and target information.

4.1.5 pgr_nodeNetwork

Name

`pgr_nodeNetwork` - Nodes an network edge table.

Author Nicolas Ribot

Copyright Nicolas Ribot, The source code is released under the MIT-X license.

Synopsis

The function reads edges from a not "noded" network table and writes the "noded" edges into a new table.

```
pgr_nodenetwork(edge_table, tolerance, id, text the_geom, table_ending, rows_where, outall)
RETURNS TEXT
```

Description

A common problem associated with bringing GIS data into pgRouting is the fact that the data is often not "noded" correctly. This will create invalid topologies, which will result in routes that are incorrect.

What we mean by "noded" is that at every intersection in the road network all the edges will be broken into separate road segments. There are cases like an over-pass and under-pass intersection where you can not traverse from the over-pass to the under-pass, but this function does not have the ability to detect and accommodate those situations.

This function reads the `edge_table` table, that has a primary key column `id` and geometry column named `the_geom` and intersect all the segments in it against all the other segments and then creates a table `edge_table_noded`. It uses the `tolerance` for deciding that multiple nodes within the tolerance are considered the same node.

Parameters

edge_table `text` Network table name. (may contain the schema name as well)

tolerance `float8` tolerance for coincident points (in projection unit)dd

id `text` Primary key column name of the network table. Default value is `id`.

the_geom `text` Geometry column name of the network table. Default value is `the_geom`.

table_ending `text` Suffix for the new table's. Default value is `noded`.

The output table will have for `edge_table_noded`

id `bigint` Unique identifier for the table

old_id `bigint` Identifier of the edge in original table

sub_id `integer` Segment number of the original edge

source `integer` Empty source column to be used with *pgr_createTopology* function

target `integer` Empty target column to be used with *pgr_createTopology* function

the geom `geometry` Geometry column of the noded network

History

- New in version 2.0.0

Example

Let's create the topology for the data in *Sample Data*

```
SELECT pgr_createTopology('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  18 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------
pgr_createtopology
--------------------
 OK
(1 row)
```

Now we can analyze the network.

```
SELECT pgr_analyzegraph('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 2
NOTICE:                            Dead ends: 7
NOTICE:  Potential gaps found near dead ends: 1
NOTICE:               Intersections detected: 1
NOTICE:                      Ring geometries: 0
pgr_analyzegraph
------------------
 OK
(1 row)
```

The analysis tell us that the network has a gap and an intersection. We try to fix the problem using:

```
SELECT pgr_nodeNetwork('edge_table', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_nodeNetwork('edge_table',0.001,'the_geom','id','noded')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Processing, pelase wait .....
NOTICE:    Split Edges: 3
NOTICE:   Untouched Edges: 15
NOTICE:       Total original Edges: 18
NOTICE:   Edges generated: 6
NOTICE:   Untouched Edges: 15
NOTICE:         Total New segments: 21
NOTICE:   New Table: public.edge_table_noded
NOTICE:  ----------------------------------
pgr_nodenetwork
-----------------
 OK
(1 row)
```

Inspecting the generated table, we can see that edges 13,14 and 18 has been segmented

```
SELECT old_id,sub_id FROM edge_table_noded ORDER BY old_id,sub_id;
 old_id | sub_id
--------+--------
 1      |      1
 2      |      1
 3      |      1
 4      |      1
 5      |      1
 6      |      1
 7      |      1
 8      |      1
 9      |      1
 10     |      1
 11     |      1
 12     |      1
 13     |      1
 13     |      2
 14     |      1
 14     |      2
 15     |      1
 16     |      1
 17     |      1
 18     |      1
 18     |      2
(21 rows)
```

We can create the topology of the new network

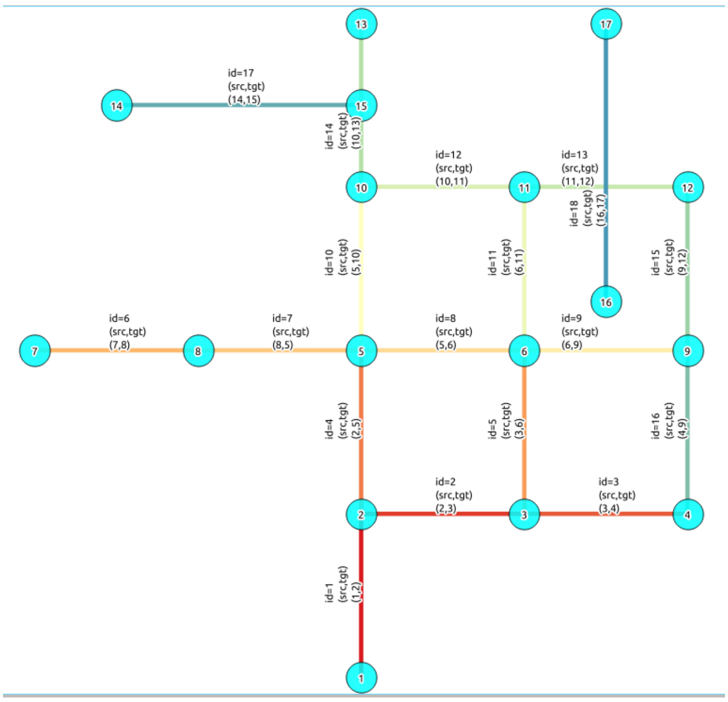
```
SELECT pgr_createTopology('edge_table_noded', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table_noded',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  21 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table_noded is: public.edge_table_noded_vertices_pgr
NOTICE:  ----------------------------------------------
pgr_createtopology
--------------------
 OK
(1 row)
```

Now let's analyze the new topology

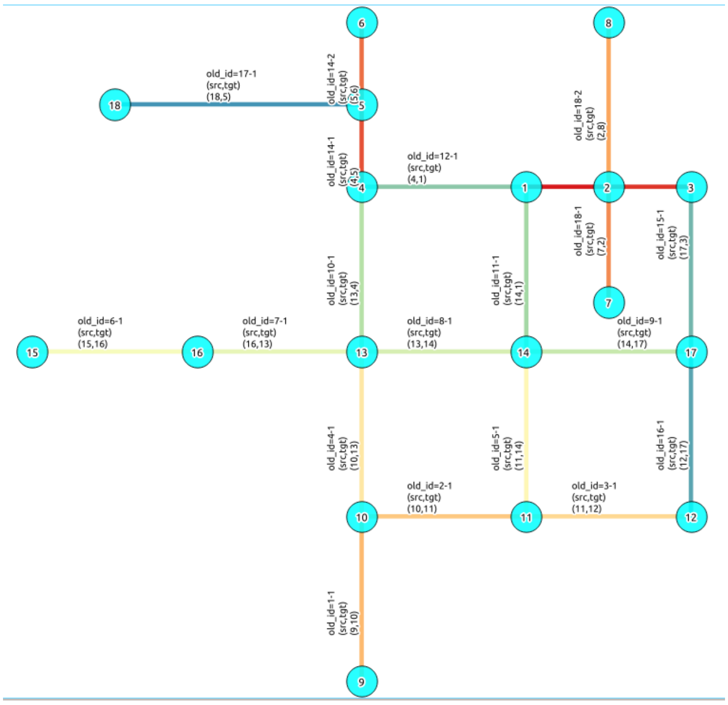
```
SELECT pgr_analyzegraph('edge_table_noded', 0.001);
NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table_noded',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 6
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 0
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
 OK
(1 row)
```

Images

Before Image



After Image



Comparing the results

Comparing with the Analysis in the original edge_table, we see that.

	Before	After
Table name	edge_table	edge_table_noded
Fields	All original fields	Has only basic fields to do a topology analysis
Dead ends	• Edges with 1 dead end: 1,6,24 • Edges with 2 dead ends 17,18 Edge 17's right node is a dead end because there is no other edge sharing that same node. (cnt=1)	Edges with 1 dead end: 1-1 ,6-1,14-2, 18-1 17-1 18-2
Isolated segments	two isolated segments: 17 and 18 both they have 2 dead ends	**No Isolated segments** • Edge 17 now shares a node with edges 14-1 and 14-2 • Edges 18-1 and 18-2 share a node with edges 13-1 and 13-2
Gaps	There is a gap between edge 17 and 14 because edge 14 is near to the right node of edge 17	Edge 14 was segmented Now edges: 14-1 14-2 17 share the same node The tolerance value was taken in account
Intersections	Edges 13 and 18 were intersecting	Edges were segmented, So, now in the interection's point there is a node and the following edges share it: 13-1 13-2 18-1 18-2

Now, we are going to include the segments 13-1, 13-2 14-1, 14-2 ,18-1 and 18-2 into our edge-table, copying the data for dir,cost,and reverse cost with tho following steps:

- Add a column old_id into edge_table, this column is going to keep track the id of the original edge
- Insert only the segmented edges, that is, the ones whose max(sub_id) >1

```
alter table edge_table drop column if exists old_id;
alter table edge_table add column old_id integer;
insert into edge_table (old_id,dir,cost,reverse_cost,the_geom)
        (with
        segmented as (select old_id,count(*) as i from edge_table_noded group by old_id)
        select  segments.old_id,dir,cost,reverse_cost,segments.the_geom
                from edge_table as edges join edge_table_noded as segments on (edges.id = segment
                where edges.id in (select old_id from segmented where i>1) );
```

We recreate the topology:

```
SELECT pgr_createTopology('edge_table', 0.001);

NOTICE:  PROCESSING:
NOTICE:  pgr_createTopology('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait .....
NOTICE:  Creating Topology, Please wait...
NOTICE:  -------------> TOPOLOGY CREATED FOR  24 edges
NOTICE:  Rows with NULL geometry or NULL id: 0
NOTICE:  Vertices table for table public.edge_table is: public.edge_table_vertices_pgr
NOTICE:  ----------------------------------------------
pgr_createtopology
--------------------
OK
(1 row)
```

To get the same analysis results as the topology of edge_table_noded, we do the following query:

```
SELECT pgr_analyzegraph('edge_table', 0.001,rows_where:='id not in (select old_id from edge_table

NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target',
                           'id not in (select old_id from edge_table where old_id is not null)')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:             ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                   Isolated segments: 0
NOTICE:                           Dead ends: 6
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:              Intersections detected: 0
NOTICE:                     Ring geometries: 0
pgr_createtopology
--------------------
OK
(1 row)
```

To get the same analysis results as the original edge_table, we do the following query:

```
SELECT pgr_analyzegraph('edge_table', 0.001,rows_where:='old_id is null')

NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','old_id is null')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
```

```
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 2
NOTICE:                            Dead ends: 7
NOTICE:  Potential gaps found near dead ends: 1
NOTICE:               Intersections detected: 1
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
OK
(1 row)
```

Or we can analyze everything because, maybe edge 18 is an overpass, edge 14 is an under pass and there is also a street level juction, and the same happens with edges 17 and 13.

```
SELECT pgr_analyzegraph('edge_table', 0.001);

NOTICE:  PROCESSING:
NOTICE:  pgr_analyzeGraph('edge_table',0.001,'the_geom','id','source','target','true')
NOTICE:  Performing checks, pelase wait...
NOTICE:  Analyzing for dead ends. Please wait...
NOTICE:  Analyzing for gaps. Please wait...
NOTICE:  Analyzing for isolated edges. Please wait...
NOTICE:  Analyzing for ring geometries. Please wait...
NOTICE:  Analyzing for intersections. Please wait...
NOTICE:              ANALYSIS RESULTS FOR SELECTED EDGES:
NOTICE:                    Isolated segments: 0
NOTICE:                            Dead ends: 3
NOTICE:  Potential gaps found near dead ends: 0
NOTICE:               Intersections detected: 5
NOTICE:                      Ring geometries: 0
pgr_createtopology
--------------------
OK
(1 row)
```

See Also

Routing Topology for an overview of a topology for routing algorithms. *pgr_analyzeOneway* to analyze directionality of the edges. *pgr_createTopology* to create a topology based on the geometry. *pgr_analyzeGraph* to analyze the edges and vertices of the edge table.

CHAPTER 5

Routing functions

5.1 Routing Functions

All Pairs - Family of Functions

- *pgr_floydWarshall* - Floyd-Warshall's Algorithm
- *pgr_johnson*- Johnson's Algorithm

pgr_aStar - Shortest Path A*

pgr_bdAstar - Bi-directional A* Shortest Path

pgr_bdDijkstra - Bi-directional Dijkstra Shortest Path

Dijkstra - Family of functions

- *pgr_dijkstra* - Dijkstra's algorithm for the shortest paths.
- *pgr_dijkstraCost* - Get the aggregate cost of the shortest paths.
- *pgr_dijkstraCostMatrix - proposed* - Use pgr_dijkstra to create a costs matrix.
- *pgr_drivingDistance* - Use pgr_dijkstra to calculate catchament information.
- *pgr_KSP* - Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- *pgr_dijkstraVia - Proposed* - Get a route of a seuence of vertices.

pgr_KSP - K-Shortest Path

pgr_trsp - Turn Restriction Shortest Path (TRSP)

Traveling Sales Person - Family of functions

- *pgr_TSP* - When input is given as matrix cell information.
- *pgr_eucledianTSP* - When input are coordinates.

Driving Distance - Category

- *pgr_drivingDistance* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsDD - Proposed* - Driving Distance based on pgr_withPoints
- Post pocessing
  - *pgr_alphaShape* - Alpha shape computation
  - *pgr_pointsAsPolygon* - Polygon around set of points

5.1.1 All Pairs - Family of Functions

The following functions work an all vertices pair combinations

pgr_floydWarshall

Synopsis

`pgr_floydWarshall` - Returns the sum of the costs of the shortest path for each pair of nodes in the graph using Floyd-Warshall algorithm.

[1]

Fig. 5.1: Boost Graph Inside

Availability: 2.0.0

- Renamed on 2.2.0, previous name pgr_apspWarshall

The Floyd-Warshall algorithm, also known as Floyd's algorithm, is a good choice to calculate the sum of the costs of the shortest path for each pair of nodes in the graph, for *dense graphs*. We make use of the Boost's implementation which runs in $\Theta(V^3)$ time,

Characteristics

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for each pair of nodes in the graph.
- Process is done only on edges with positive costs.
- Boost returns a $V \times V$ matrix, where the infinity values. Represent the distance between vertices for which there is no path.
  - We return only the non infinity values in form of a set of *(start_vid, end_vid, agg_cost)*.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_-vid, end_vid)*.
- For the undirected graph, the results are symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- When *start_vid* = *end_vid*, the *agg_cost* = 0.
- **Recommended, use a bounding box of no more than 3500 edges.**

Signature Summary

```
pgr_floydWarshall(edges_sql)
pgr_floydWarshall(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_floydWarshall(edges_sql)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 1 On a directed graph.

```
SELECT * FROM pgr_floydWarshall(
    'SELECT id, source, target, cost FROM edge_table where id < 5'
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       5 |        1
(3 rows)
```

Complete Signature

```
pgr_floydWarshall(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 2 On an undirected graph.

```
SELECT * FROM pgr_floydWarshall(
    'SELECT id, source, target, cost FROM edge_table where id < 5',
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       1 |        1
         2 |       5 |        1
         5 |       1 |        2
         5 |       2 |        1
(6 rows)
```

Description of the Signatures

Description of the edges_sql query (id is not necessary)

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures Receives `(edges_sql, directed)`

Parameter	Type	Description
edges_sql	TEXT	SQL query as described above.
directed	BOOLEAN	(optional) Default is true (is directed). When set to false the graph is considered as Undirected

Description of the return values Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex.
end_vid	BIGINT	Identifier of the ending vertex.
agg_cost	FLOAT	Total cost from `start_vid` to `end_vid`.

History

- Re-design of pgr_apspWarshall in Version 2.2.0

See Also

- *pgr_johnson*
- Boost floyd-Warshall[2] algorithm
- Queries uses the *Sample Data* network.

[2] http://www.boost.org/libs/graph/doc/floyd_warshall_shortest.html

Indices and tables

- genindex
- search

pgr_johnson

Synopsis

`pgr_johnson` - Returns the sum of the costs of the shortest path for each pair of nodes in the graph using Floyd-Warshall algorithm.

[3]

Fig. 5.2: Boost Graph Inside

Availability: 2.0.0

- Renamed on 2.2.0, previous name pgr_apspJohnson

The Johnson algorithm, is a good choice to calculate the sum of the costs of the shortest path for each pair of nodes in the graph, for *sparse graphs*. It usees the Boost's implementation which runs in $O(V E \log V)$ time,

Characteristics

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for each pair of nodes in the graph.
- Process is done only on edges with positive costs.
- Boost returns a $V \times V$ matrix, where the infinity values. Represent the distance between vertices for which there is no path.
  - We return only the non infinity values in form of a set of *(start_vid, end_vid, agg_cost)*.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- For the undirected graph, the results are symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- When *start_vid* = *end_vid*, the *agg_cost* = 0.

Signature Summary

```
pgr_johnson(edges_sql)
pgr_johnson(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_johnson(edges_sql)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 1 On a directed graph.

```
SELECT * FROM pgr_johnson(
    'SELECT source, target, cost FROM edge_table WHERE id < 5
         ORDER BY id'
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       5 |        1
(3 rows)
```

Complete Signature

```
pgr_johnson(edges_sql, directed)
RETURNS SET OF (start_vid, end_vid,  agg_cost) or EMPTY SET
```

Example 2 On an undirected graph.

```
SELECT * FROM pgr_johnson(
    'SELECT source, target, cost FROM edge_table WHERE id < 5
         ORDER BY id',
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       5 |        2
         2 |       1 |        1
         2 |       5 |        1
         5 |       1 |        2
         5 |       2 |        1
(6 rows)
```

Description of the Signatures

Description of the edges_sql query (id is not necessary)

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures Receives `(edges_sql, directed)`

Parameter	Type	Description
edges_sql	`TEXT`	SQL query as described above.
directed	`BOOLEAN`	(optional) Default is true (is directed). When set to false the graph is considered as Undirected

Description of the return values Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex.
end_vid	`BIGINT`	Identifier of the ending vertex.
agg_cost	`FLOAT`	Total cost from `start_vid` to `end_vid`.

History

- Re-design of pgr_apspJohnson in Version 2.2.0

See Also

- *pgr_floydWarshall*
- Boost Johnson[4] algorithm implementation.
- Queries uses the *Sample Data* network.

[4]http://www.boost.org/libs/graph/doc/johnson_all_pairs_shortest.html

Indices and tables

- genindex
- search

Performance

The following tests:

- non server computer
- with AMD 64 CPU
- 4G memory
- trusty
- posgreSQL version 9.3

Data

The following data was used

```
BBOX="-122.8,45.4,-122.5,45.6"
wget --progress=dot:mega -O "sampledata.osm" "http://www.overpass-api.de/api/xapi?*[bbox=][@meta]
```

Data processing was done with osm2pgrouting-alpha

```
createdb portland
psql -c "create extension postgis" portland
psql -c "create extension pgrouting" portland
osm2pgrouting -f sampledata.osm -d portland -s 0
```

Results

Test One

This test is not with a bounding box The density of the passed graph is extremely low. For each <SIZE> 30 tests were executed to get the average The tested query is:

```
SELECT count(*) FROM pgr_floydWarshall(
   'SELECT gid as id, source, target, cost, reverse_cost FROM ways where id <=  <SIZE>');

SELECT count(*) FROM pgr_johnson(
   'SELECT gid as id, source, target, cost, reverse_cost FROM ways where id <=  <SIZE>');
```

The results of this tests are presented as:

SIZE is the number of edges given as input.

EDGES is the total number of records in the query.

DENSITY is the density of the data $\dfrac{E}{V \times (V-1)}$.

OUT ROWS is the number of records returned by the queries.

Floyd-Warshall is the average execution time in seconds of pgr_floydWarshall.

Johnson is the average execution time in seconds of pgr_johnson.

SIZE	EDGES	DENSITY	OUT ROWS	Floyd-Warshall	Johnson
500	500	0.18E-7	1346	0.14	0.13
1000	1000	0.36E-7	2655	0.23	0.18
1500	1500	0.55E-7	4110	0.37	0.34
2000	2000	0.73E-7	5676	0.56	0.37
2500	2500	0.89E-7	7177	0.84	0.51
3000	3000	1.07E-7	8778	1.28	0.68
3500	3500	1.24E-7	10526	2.08	0.95
4000	4000	1.41E-7	12484	3.16	1.24
4500	4500	1.58E-7	14354	4.49	1.47
5000	5000	1.76E-7	16503	6.05	1.78
5500	5500	1.93E-7	18623	7.53	2.03
6000	6000	2.11E-7	20710	8.47	2.37
6500	6500	2.28E-7	22752	9.99	2.68
7000	7000	2.46E-7	24687	11.82	3.12
7500	7500	2.64E-7	26861	13.94	3.60
8000	8000	2.83E-7	29050	15.61	4.09
8500	8500	3.01E-7	31693	17.43	4.63
9000	9000	3.17E-7	33879	19.19	5.34
9500	9500	3.35E-7	36287	20.77	6.24
10000	10000	3.52E-7	38491	23.26	6.51

Test Two

This test is with a bounding box The density of the passed graph higher than of the Test One. For each <SIZE> 30 tests were executed to get the average The tested edge query is:

```
WITH
    buffer AS (SELECT ST_Buffer(ST_Centroid(ST_Extent(the_geom)), SIZE) AS geom FROM ways),
    bbox AS (SELECT ST_Envelope(ST_Extent(geom)) as box from buffer)
SELECT gid as id, source, target, cost, reverse_cost FROM ways where the_geom && (SELECT box from
```

The tested queries

```
SELECT count(*) FROM pgr_floydWarshall(<edge query>)
SELECT count(*) FROM pgr_johnson(<edge query>)
```

The results of this tests are presented as:

SIZE is the size of the bounding box.

EDGES is the total number of records in the query.

DENSITY is the density of the data $\frac{E}{V \times (V-1)}$.

OUT ROWS is the number of records returned by the queries.

Floyd-Warshall is the average execution time in seconds of pgr_floydWarshall.

Johnson is the average execution time in seconds of pgr_johnson.

SIZE	EDGES	DENSITY	OUT ROWS	Floyd-Warshall	Johnson
0.001	44	0.0608	1197	0.10	0.10
0.002	99	0.0251	4330	0.10	0.10
0.003	223	0.0122	18849	0.12	0.12
0.004	358	0.0085	71834	0.16	0.16
0.005	470	0.0070	116290	0.22	0.19
0.006	639	0.0055	207030	0.37	0.27
0.007	843	0.0043	346930	0.64	0.38
0.008	996	0.0037	469936	0.90	0.49
0.009	1146	0.0032	613135	1.26	0.62
0.010	1360	0.0027	849304	1.87	0.82
0.011	1573	0.0024	1147101	2.65	1.04
0.012	1789	0.0021	1483629	3.72	1.35
0.013	1975	0.0019	1846897	4.86	1.68
0.014	2281	0.0017	2438298	7.08	2.28
0.015	2588	0.0015	3156007	10.28	2.80
0.016	2958	0.0013	4090618	14.67	3.76
0.017	3247	0.0012	4868919	18.12	4.48

See Also

- *pgr_johnson*
- *pgr_floydWarshall*
- Boost floyd-Warshall[5] algorithm

Indices and tables

- genindex
- search

5.1.2 pgr_bdAstar

Name

`pgr_bdAstar` — Returns the shortest path using A* algorithm.

[6]

Fig. 5.3: Boost Graph Inside

Availability:

- pgr_bdAstar(one to one) 2.0.0, Signature change on 2.5.0
- pgr_bdAstar(other signatures) 2.5.0

[5] http://www.boost.org/libs/graph/doc/floyd_warshall_shortest.html

Signature Summary

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq , node, edge, cost, agg_cost)
  OR EMPTY SET
```

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

```
pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)
OR EMPTY SET
```

Using these signatures, will load once the graph and perform several one to one *pgr_bdAstar*

- The result is the union of the results of the one to one *pgr_bdAStar*.
- The extra `start_vid` and/or `end_vid` in the result is used to distinguish to which path it belongs.

Avaliability

- pgr_bdAstar(one to one) 2.0, signature change on 2.5
- pgr_bdAstar(other signatures) 2.5

Signatures

Minimal Signature

```
pgr_bdAstar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the `start_vid` to the `end_vid`

- on a **directed** graph
- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example Using the defaults

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_bdAstar One to One

```
pgr_bdAstar(edges_sql, start_vid, end_vid, directed [, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

This usage finds the shortest path from the `start_vid` to the `end_vid` allowing the user to choose

- **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Note: In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example Directed using Heuristic 2

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3,
    true, heuristic := 2
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_bdAstar One to many

```
pgr_bdAstar(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This usage finds the shortest path from the `start_vid` to each `end_vid` in `end_vids` allowing the user to choose

- if the graph is **directed** or **undirected**

- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed using Heuristic 3 and a factor of 3.5

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, ARRAY[3, 11],
    heuristic := 3, factor := 3.5
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |      11 |    2 |    4 |    1 |        0
   8 |        2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |      11 |    6 |   11 |    1 |        2
  10 |        4 |      11 |   11 |   -1 |    0 |        3
(10 rows)
```

pgr_bdAstar Many to One

```
pgr_bdAstar(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This usage finds the shortest path from each `start_vid` in `start_vids` to the `end_vid` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Undirected graph with Heuristic 4

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], 3,
    false, heuristic := 4
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    2 |    1 |        0
   2 |        2 |         2 |    3 |   -1 |    0 |        1
   3 |        1 |         7 |    7 |    6 |    1 |        0
   4 |        2 |         7 |    8 |    7 |    1 |        1
   5 |        3 |         7 |    5 |    4 |    1 |        2
   6 |        4 |         7 |    2 |    2 |    1 |        3
   7 |        5 |         7 |    3 |   -1 |    0 |        4
(7 rows)
```

pgr_bdAstar Many to Many

```
pgr_bdAstar(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This usage finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids` allowing the use

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstar(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |      11 |    2 |    4 |    1 |        0
   8 |        2 |         2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |         2 |      11 |    6 |   11 |    1 |        2
  10 |        4 |         2 |      11 |   11 |   -1 |    0 |        3
  11 |        1 |         7 |       3 |    7 |    6 |    1 |        0
  12 |        2 |         7 |       3 |    8 |    7 |    1 |        1
  13 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  14 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  15 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  16 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  17 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
  18 |        1 |         7 |      11 |    7 |    6 |    1 |        0
  19 |        2 |         7 |      11 |    8 |    7 |    1 |        1
  20 |        3 |         7 |      11 |    5 |   10 |    1 |        2
  21 |        4 |         7 |      11 |   10 |   12 |    1 |        3
  22 |        5 |         7 |      11 |   11 |   -1 |    0 |        4
(22 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
start_vids	`ARRAY[ANY-INTEGER]`	Starting vertices identifierers.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
end_vids	`ARRAY[ANY-INTEGER]`	Ending vertices identifiers.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. see *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- *Bidirectional A* - Family of functions*
- *Sample Data* network.
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

5.1.3 pgr_bdDijkstra

`pgr_bdDijkstra` — Returns the shortest path(s) using Bidirectional Dijkstra algorithm.

[7]

Fig. 5.4: Boost Graph Inside

Availability:

- pgr_bdDijkstra(one to one) 2.0.0, Signature changed 2.4.0
- pgr_bdDijkstra(other signatures) 2.5.0

Signature Summary

```
pgr_bdDijkstra(edges_sql, start_vid,  end_vid)
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)
OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`:

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_bdDijkstra One to One

```
pgr_bdDijkstra(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    false
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)
```

pgr_bdDijkstra One to many

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_dijkstra* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_dijkstra*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3, 11]);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |      11 |    2 |    4 |    1 |        0
   8 |        2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |      11 |    6 |   11 |    1 |        2
  10 |        4 |      11 |   11 |   -1 |    0 |        3
(10 rows)
```

pgr_bdDijkstra Many to One

```
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_dijkstra* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], 3);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |    8 |    1 |        1
   3 |        3 |         2 |    6 |    9 |    1 |        2
```

```
   4  |          4  |           2  |      9  |    16  |     1  |          3
   5  |          5  |           2  |      4  |     3  |     1  |          4
   6  |          6  |           2  |      3  |    -1  |     0  |          5
   7  |          1  |           7  |      7  |     6  |     1  |          0
   8  |          2  |           7  |      8  |     7  |     1  |          1
   9  |          3  |           7  |      5  |     8  |     1  |          2
  10  |          4  |           7  |      6  |     9  |     1  |          3
  11  |          5  |           7  |      9  |    16  |     1  |          4
  12  |          6  |           7  |      4  |     3  |     1  |          5
  13  |          7  |           7  |      3  |    -1  |     0  |          6
(13 rows)
```

pgr_bdDijkstra Many to Many

```
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11]);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |      11 |    2 |    4 |    1 |        0
   8 |        2 |         2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |         2 |      11 |    6 |   11 |    1 |        2
  10 |        4 |         2 |      11 |   11 |   -1 |    0 |        3
  11 |        1 |         7 |       3 |    7 |    6 |    1 |        0
  12 |        2 |         7 |       3 |    8 |    7 |    1 |        1
  13 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  14 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  15 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  16 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  17 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
  18 |        1 |         7 |      11 |    7 |    6 |    1 |        0
  19 |        2 |         7 |      11 |    8 |    7 |    1 |        1
  20 |        3 |         7 |      11 |    5 |   10 |    1 |        2
  21 |        4 |         7 |      11 |   10 |   12 |    1 |        3
  22 |        5 |         7 |      11 |   11 |   -1 |    0 |        4
(22 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default	Description
sql	`TEXT`		SQL query as described above.
start_vid	`BIGINT`		Identifier of the starting vertex of the path.
start_vids	`ARRAY[BIGINT]`		Array of identifiers of starting vertices.
end_vid	`BIGINT`		Identifier of the ending vertex of the path.
end_vids	`ARRAY[BIGINT]`		Array of identifiers of ending vertices.
directed	`BOOLEAN`	`true`	• When `true` Graph is considered *Directed* • When `false` the graph is considered as *Undirected*.

Description of the return values for a path

Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- The queries use the *Sample Data* network.
- *Bidirectional Dijkstra - Family of functions*
- http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP%20shortest%20path%20algorithms.pdf
- https://en.wikipedia.org/wiki/Bidirectional_search

Indices and tables

- genindex
- search

5.1.4 Dijkstra - Family of functions

- *pgr_dijkstra* - Dijkstra's algorithm for the shortest paths.
- *pgr_dijkstraCost* - Get the aggregate cost of the shortest paths.
- *pgr_dijkstraCostMatrix - proposed* - Use pgr_dijkstra to create a costs matrix.
- *pgr_drivingDistance* - Use pgr_dijkstra to calculate catchament information.
- *pgr_KSP* - Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- *pgr_dijkstraVia - Proposed* - Get a route of a seuence of vertices.

pgr_dijkstra

`pgr_dijkstra` — Returns the shortest path(s) using Dijkstra algorithm. In particular, the Dijkstra algorithm implemented by Boost.Graph.

[8]

Fig. 5.5: Boost Graph Inside

Availability

- pgr_dijkstra(one to one) 2.0.0, signature change 2.1.0
- pgr_dijkstra(other signatures) 2.1.0

Synopsis

Dijkstra's algorithm, conceived by Dutch computer scientist Edsger Dijkstra in 1956. It is a graph search algorithm that solves the shortest path problem for a graph with non-negative edge path costs, producing a shortest path from a starting vertex (`start_vid`) to an ending vertex (`end_vid`). This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* the non included values *(v, v)* is *0*
  - When the starting vertex and ending vertex are the different and there is no path:
    - The *agg_cost* the non included values *(u, v)* is ∞
- For optimization purposes, any duplicated value in the *start_vids* or *end_vids* are ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending
- Running time: $O(|start\_vids| * (V \log V + E))$

Signature Summary

```
pgr_dijkstra(edges_sql, start_vid,  end_vid)
pgr_dijkstra(edges_sql, start_vid,  end_vid,  directed:=true)
pgr_dijkstra(edges_sql, start_vid,  end_vids, directed:=true)
pgr_dijkstra(edges_sql, start_vids, end_vid,  directed:=true)
pgr_dijkstra(edges_sql, start_vids, end_vids, directed:=true)

RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)
    OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)
```

pgr_dijkstra One to One

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)
```

pgr_dijkstra One to many

```
pgr_dijkstra(TEXT edges_sql, BIGINT start_vid, ARRAY[ANY_INTEGER] end_vids,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_dijkstra* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_dijkstra*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    5 |    1 |        2
   4 |        4 |       3 |    3 |   -1 |    0 |        3
   5 |        1 |       5 |    2 |    4 |    1 |        0
   6 |        2 |       5 |    5 |   -1 |    0 |        1
(6 rows)
```

pgr_dijkstra Many to One

```
pgr_dijkstra(TEXT edges_sql, ARRAY[ANY_INTEGER] start_vids, BIGINT end_vid,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_dijkstra* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   13 |    1 |        0
   4 |        2 |        11 |   12 |   15 |    1 |        1
   5 |        3 |        11 |    9 |    9 |    1 |        2
   6 |        4 |        11 |    6 |    8 |    1 |        3
   7 |        5 |        11 |    5 |   -1 |    0 |        4
(7 rows)
```

pgr_dijkstra Many to Many

```
pgr_dijkstra(TEXT edges_sql, ARRAY[ANY_INTEGER] start_vids, ARRAY[ANY_INTEGER] end_vids,
    BOOLEAN directed:=true);
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], ARRAY[3,5],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    2 |    1 |        0
   2 |        2 |         2 |       3 |    3 |   -1 |    0 |        1
   3 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   4 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
   5 |        1 |        11 |       3 |   11 |   11 |    1 |        0
   6 |        2 |        11 |       3 |    6 |    5 |    1 |        1
   7 |        3 |        11 |       3 |    3 |   -1 |    0 |        2
   8 |        1 |        11 |       5 |   11 |   11 |    1 |        0
   9 |        2 |        11 |       5 |    6 |    8 |    1 |        1
  10 |        3 |        11 |       5 |    5 |   -1 |    0 |        2
(10 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default
sql	TEXT	
start_vid	BIGINT	
start_vids	ARRAY[BIGINT]	
end_vid	BIGINT	
end_vids	ARRAY[BIGINT]	
directed	BOOLEAN	true

Description of the return values for a path Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	INT	Sequential value starting from **1**.
path_id	INT	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same start_vid to end_vid combination.
path_seq	INT	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	BIGINT	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	BIGINT	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	BIGINT	Identifier of the node in the path from start_vid to end_vid.
edge	BIGINT	Identifier of the edge used to go from node to the next node in the path sequence. -1 for the last node of the path.
cost	FLOAT	Cost to traverse from node using edge to the next node in the path sequence.
agg_cost	FLOAT	Aggregate cost from start_v to node.

Additional Examples

The examples of this section are based on the *Sample Data* network.

The examples include combinations from starting vertices 2 and 11 to ending vertices 3 and 5 in a directed and undirected graph with and with out reverse_cost.

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns The examples in this section use the following *Graph 1: Directed, with cost and reverse cost*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3,5]
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
```

```
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |       5 |    2 |    4 |    1 |        0
   8 |        2 |       5 |    5 |   -1 |    0 |        1
(8 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   13 |    1 |        0
   2 |        2 |   12 |   15 |    1 |        1
   3 |        3 |    9 |   16 |    1 |        2
   4 |        4 |    4 |    3 |    1 |        3
   5 |        5 |    3 |   -1 |    0 |        4
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   13 |    1 |        0
   2 |        2 |   12 |   15 |    1 |        1
   3 |        3 |    9 |    9 |    1 |        2
   4 |        4 |    6 |    8 |    1 |        3
   5 |        5 |    5 |   -1 |    0 |        4
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   13 |    1 |        0
   4 |        2 |        11 |   12 |   15 |    1 |        1
   5 |        3 |        11 |    9 |    9 |    1 |        2
   6 |        4 |        11 |    6 |    8 |    1 |        3
   7 |        5 |        11 |    5 |   -1 |    0 |        4
(7 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   8 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
```

```
   9 |        1 |        11 |        3 |   11 |   13 |    1 |         0
  10 |        2 |        11 |        3 |   12 |   15 |    1 |         1
  11 |        3 |        11 |        3 |    9 |   16 |    1 |         2
  12 |        4 |        11 |        3 |    4 |    3 |    1 |         3
  13 |        5 |        11 |        3 |    3 |   -1 |    0 |         4
  14 |        1 |        11 |        5 |   11 |   13 |    1 |         0
  15 |        2 |        11 |        5 |   12 |   15 |    1 |         1
  16 |        3 |        11 |        5 |    9 |    9 |    1 |         2
  17 |        4 |        11 |        5 |    6 |    8 |    1 |         3
  18 |        5 |        11 |        5 |    5 |   -1 |    0 |         4
(18 rows)
```

Examples for queries marked as `undirected` with `cost` and `reverse_cost` columns The examples in this section use the following *Graph 2: Undirected, with cost and reverse cost*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    5 |    1 |        1
   3 |        3 |    3 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    11, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    8 |    1 |        1
   3 |        3 |    5 |   -1 |    0 |        2
(3 rows)
```

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2,11], 5,
    FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   12 |    1 |        0
   4 |        2 |        11 |   10 |   10 |    1 |        1
   5 |        3 |        11 |    5 |   -1 |    0 |        2
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3,5],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    2 |    1 |        0
   2 |        2 |       3 |    3 |   -1 |    0 |        1
   3 |        1 |       5 |    2 |    4 |    1 |        0
   4 |        2 |       5 |    5 |   -1 |    0 |        1
(4 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    2 |    1 |        0
   2 |        2 |         2 |       3 |    3 |   -1 |    0 |        1
   3 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   4 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
   5 |        1 |        11 |       3 |   11 |   11 |    1 |        0
   6 |        2 |        11 |       3 |    6 |    5 |    1 |        1
   7 |        3 |        11 |       3 |    3 |   -1 |    0 |        2
   8 |        1 |        11 |       5 |   11 |   11 |    1 |        0
   9 |        2 |        11 |       5 |    6 |    8 |    1 |        1
  10 |        3 |        11 |       5 |    5 |   -1 |    0 |        2
(10 rows)
```

Examples for queries marked as `directed` with `cost` column The examples in this section use the following *Graph 3: Directed, with cost*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
(0 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 5
);
```

```
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
(0 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 5
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
(0 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2,11], 5
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5]
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       5 |    2 |    4 |    1 |        0
   2 |        2 |       5 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
(2 rows)
```

Examples for queries marked as `undirected` with `cost` column The examples in this section use the following *Graph 4: Undirected, with cost*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
```

```
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    5 |    1 |        2
   4 |        4 |    3 |   -1 |    0 |        3
(4 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    5 |    1 |        1
   3 |        3 |    3 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    11, 5,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   11 |   11 |    1 |        0
   2 |        2 |    6 |    8 |    1 |        1
   3 |        3 |    5 |   -1 |    0 |        2
(3 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2,11], 5,
    FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   -1 |    0 |        1
   3 |        1 |        11 |   11 |   12 |    1 |        0
   4 |        2 |        11 |   10 |   10 |    1 |        1
   5 |        3 |        11 |    5 |   -1 |    0 |        2
(5 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    2, ARRAY[3,5],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
```

```
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    5 |    1 |        2
   4 |        4 |       3 |    3 |   -1 |    0 |        3
   5 |        1 |       5 |    2 |    4 |    1 |        0
   6 |        2 |       5 |    5 |   -1 |    0 |        1
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost FROM edge_table',
    ARRAY[2, 11], ARRAY[3,5],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    5 |    1 |        2
   4 |        4 |         2 |       3 |    3 |   -1 |    0 |        3
   5 |        1 |         2 |       5 |    2 |    4 |    1 |        0
   6 |        2 |         2 |       5 |    5 |   -1 |    0 |        1
   7 |        1 |        11 |       3 |   11 |   11 |    1 |        0
   8 |        2 |        11 |       3 |    6 |    5 |    1 |        1
   9 |        3 |        11 |       3 |    3 |   -1 |    0 |        2
  10 |        1 |        11 |       5 |   11 |   11 |    1 |        0
  11 |        2 |        11 |       5 |    6 |    8 |    1 |        1
  12 |        3 |        11 |       5 |    5 |   -1 |    0 |        2
(12 rows)
```

Equvalences between signatures

Examples For queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following:

- *Graph 1: Directed, with cost and reverse cost*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    TRUE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2,3
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |    8 |    1 |        1
   3 |        3 |    6 |    9 |    1 |        2
   4 |        4 |    9 |   16 |    1 |        3
```

```
   5 |        5 |    4 |    3 |    1 |        4
   6 |        6 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3],
    TRUE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3]
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], ARRAY[3],
    TRUE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
(6 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], ARRAY[3]
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
(6 rows)
```

Examples For queries marked as `undirected` with `cost` and `reverse_cost` columns

The examples in this section use the following:

- *Graph 2: Undirected, with cost and reverse cost*

```
SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    FALSE
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3],
    FALSE
);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    2 |    1 |        0
   2 |        2 |       3 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], 3,
    FALSE
);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    2 |    1 |        0
   2 |        2 |         2 |    3 |   -1 |    0 |        1
(2 rows)

SELECT * FROM pgr_dijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2], ARRAY[3],
    FALSE
);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    2 |    1 |        0
   2 |        2 |         2 |       3 |    3 |   -1 |    0 |        1
(2 rows)
```

See Also

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- The queries use the *Sample Data* network.

Indices and tables

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pgr_dijkstraCost

Synopsis

`pgr_dijkstraCost`

Using Dijkstra algorithm implemented by Boost.Graph, and extract only the aggregate cost of the shortest path(s) found, for the combination of vertices given.

[9]

Fig. 5.6: Boost Graph Inside

Availability

- pgr_dijkstraCost(all signatures) 2.2.0

The `pgr_dijkstraCost` algorithm, is a good choice to calculate the sum of the costs of the shortest path for a subset of pairs of nodes of the graph. We make use of the Boost's implementation of dijkstra which runs in $O(V \log V + E)$ time.

Characteristics

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* int the non included values *(v, v)* is *0*
  - When the starting vertex and ending vertex are the different and there is no path.
    - The *agg_cost* in the non included values *(u, v)* is ∞
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_-vid, end_vid)*.
- For undirected graphs, the results are symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- Any duplicated value in the *start_vids* or *end_vids* is ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending
- Running time: $O(|start\_vids| * (V \log V + E))$

Signature Summary

```
pgr_dijkstraCost(edges_sql, start_vid, end_vid);
pgr_dijkstraCost(edges_sql, start_vid, end_vid, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vid, directed);
pgr_dijkstraCost(edges_sql, start_vid, end_vids, directed);
pgr_dijkstraCost(edges_sql, start_vids, end_vids, directed);

    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Signatures

Minimal signature The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`:

```
pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid)
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    2, 3);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
(1 row)
```

pgr_dijkstraCost One to One

This signature performs a Dijkstra from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

```
pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, BIGINT end_vid,
                 BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    2, 3, false);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        1
(1 row)
```

pgr_dijkstraCost One to Many

```
pgr_dijkstraCost(TEXT edges_sql, BIGINT start_vid, array[ANY_INTEGER] end_vids,
        BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

This signature performs a Dijkstra from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.

- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    2, ARRAY[3, 11]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
(2 rows)
```

pgr_dijkstraCost Many to One

```
pgr_dijkstraCost(TEXT edges_sql, array[ANY_INTEGER] start_vids, BIGINT end_vid,
                     BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

This signature performs a Dijkstra from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    ARRAY[2, 7], 3);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         7 |       3 |        6
(2 rows)
```

pgr_dijkstraCost Many to Many

```
pgr_dijkstraCost(TEXT edges_sql, array[ANY_INTEGER] start_vids, array[ANY_INTEGER] end_vids,
        BOOLEAN directed:=true);
    RETURNS SET OF (start_vid, end_vid, agg_cost) or EMPTY SET
```

This signature performs a Dijkstra from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_dijkstraCost(
    'select id, source, target, cost, reverse_cost from edge_table',
    ARRAY[2, 7], ARRAY[3, 11]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
         7 |       3 |        6
         7 |      11 |        4
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default
sql	TEXT	
start_vid	BIGINT	
start_vids	ARRAY[BIGINT]	
end_vid	BIGINT	
end_vids	ARRAY[BIGINT]	
directed	BOOLEAN	true

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Additional Examples

Example 1 Demonstration of repeated values are ignored, and result is sorted.

```
SELECT * FROM pgr_dijkstraCost(
        'select id, source, target, cost, reverse_cost from edge_table',
            ARRAY[5, 3, 4, 3, 3, 4], ARRAY[3, 5, 3, 4]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         3 |       4 |        3
         3 |       5 |        2
         4 |       3 |        1
         4 |       5 |        3
         5 |       3 |        4
         5 |       4 |        3
(6 rows)
```

Example 2 Making *start_vids* the same as *end_vids*

```
SELECT * FROM pgr_dijkstraCost(
        'select id, source, target, cost, reverse_cost from edge_table',
            ARRAY[5, 3, 4], ARRAY[5, 3, 4]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         3 |       4 |        3
         3 |       5 |        2
         4 |       3 |        1
         4 |       5 |        3
         5 |       3 |        4
         5 |       4 |        3
(6 rows)
```

See Also

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- *Sample Data* network.

Indices and tables

- genindex
- search

pgr_dijkstraCostMatrix - proposed

Name

`pgr_dijkstraCostMatrix` - Calculates the a cost matrix using pgr_dijktras.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[10]

Fig. 5.7: Boost Graph Inside

Availability: 2.3.0

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_dijkstraCostMatrix(edges_sql, start_vids)
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.

```
pgr_dijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_dijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        6
         1 |       4 |        5
         2 |       1 |        1
         2 |       3 |        5
         2 |       4 |        4
         3 |       1 |        2
```

```
         3 |       2 |        1
         3 |       4 |        3
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Complete Signature

```
pgr_dijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_dijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        2
         1 |       4 |        3
         2 |       1 |        1
         2 |       3 |        1
         2 |       4 |        2
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        1
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of the vertices.
directed	`BOOLEAN`	(optional). When `false` the graph is consider `true` which considers the graph as Directed.

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_dijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
```

```
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
(5 rows)
```

See Also

- *Dijkstra - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_drivingDistance

Name

`pgr_drivingDistance` - Returns the driving distance from a start node.

[11]

Fig. 5.8: Boost Graph Inside

Availability

- pgr_drivingDistance(single vertex) 2.0.0, signature change 2.1.0
- pgr_drivingDistance(multiple vertices) 2.1.0

Synopsis

Using Dijkstra algorithm, extracts all the nodes that have costs less than or equal to the value `distance`. The edges extracted will conform the corresponding spanning tree.

Signature Summary

```
pgr_drivingDistance(edges_sql, start_vid, distance)
pgr_drivingDistance(edges_sql, start_vid, distance, directed)
pgr_drivingDistance(edges_sql, start_vids, distance, directed, equicost)

RETURNS SET OF (seq, [start_vid,] node, edge, cost, agg_cost)
```

Signatures

Minimal Use

```
pgr_drivingDistance(edges_sql, start_vid, distance)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Driving Distance From A Single Starting Vertex

```
pgr_drivingDistance(edges_sql, start_vid, distance, directed)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Driving Distance From Multiple Starting Vertices

```
pgr_drivingDistance(edges_sql, start_vids, distance, directed, equicost)
RETURNS SET OF (seq, start_vid, node, edge, cost, agg_cost)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	`TEXT`	SQL query as described above.
start_vid	`BIGINT`	Identifier of the starting vertex.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of starting vertices.
distance	`FLOAT`	Upper limit for the inclusion of the node in the result
directed	`BOOLEAN`	(optional). When `false` the graph is considered as U which considers the graph as Directed.
equicost	`BOOLEAN`	(optional). When `true` the node will only appear in Default is `false` which resembles several calls usin signatures. Tie brakes are arbitrarely.

Description of the return values Returns set of `(seq [, start_v], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INTEGER`	Sequential value starting from **1**.
start_vid	`INTEGER`	Identifier of the starting vertex.
node	`BIGINT`	Identifier of the node in the path within the limits from `start_vid`.
edge	`BIGINT`	Identifier of the edge used to arrive to `node`. `0` when the `node` is the `start_vid`.
cost	`FLOAT`	Cost to traverse `edge`.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `node`.

Additional Examples

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns The examples in this section use the following *Graph 1: Directed, with cost and reverse cost*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        2, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    1 |    1 |    1 |        1
   3 |    5 |    4 |    1 |        1
   4 |    6 |    8 |    1 |        2
   5 |    8 |    7 |    1 |        2
   6 |   10 |   10 |    1 |        2
   7 |    7 |    6 |    1 |        3
   8 |    9 |    9 |    1 |        3
   9 |   11 |   12 |    1 |        3
  10 |   13 |   14 |    1 |        3
(10 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        13, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
   2 |   10 |   14 |    1 |        1
   3 |    5 |   10 |    1 |        2
```

```
   4 |    11 |    12 |     1 |         2
   5 |     2 |     4 |     1 |         3
   6 |     6 |     8 |     1 |         3
   7 |     8 |     7 |     1 |         3
   8 |    12 |    13 |     1 |         3
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    5 |    4 |    1 |        1
   4 |      2 |    6 |    8 |    1 |        2
   5 |      2 |    8 |    7 |    1 |        2
   6 |      2 |   10 |   10 |    1 |        2
   7 |      2 |    7 |    6 |    1 |        3
   8 |      2 |    9 |    9 |    1 |        3
   9 |      2 |   11 |   12 |    1 |        3
  10 |      2 |   13 |   14 |    1 |        3
  11 |     13 |   13 |   -1 |    0 |        0
  12 |     13 |   10 |   14 |    1 |        1
  13 |     13 |    5 |   10 |    1 |        2
  14 |     13 |   11 |   12 |    1 |        2
  15 |     13 |    2 |    4 |    1 |        3
  16 |     13 |    6 |    8 |    1 |        3
  17 |     13 |    8 |    7 |    1 |        3
  18 |     13 |   12 |   13 |    1 |        3
(18 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    5 |    4 |    1 |        1
   4 |      2 |    6 |    8 |    1 |        2
   5 |      2 |    8 |    7 |    1 |        2
   6 |      2 |    7 |    6 |    1 |        3
   7 |      2 |    9 |    9 |    1 |        3
   8 |     13 |   13 |   -1 |    0 |        0
   9 |     13 |   10 |   14 |    1 |        1
  10 |     13 |   11 |   12 |    1 |        2
  11 |     13 |   12 |   13 |    1 |        3
(11 rows)
```

Examples for queries marked as `undirected` with `cost` and `reverse_cost` columns The examples in this section use the following *Graph 2: Undirected, with cost and reverse cost*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        2, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
```

```
   1 |    2 |   -1 |    0 |        0
   2 |    1 |    1 |    1 |        1
   3 |    3 |    2 |    1 |        1
   4 |    5 |    4 |    1 |        1
   5 |    4 |    3 |    1 |        2
   6 |    6 |    8 |    1 |        2
   7 |    8 |    7 |    1 |        2
   8 |   10 |   10 |    1 |        2
   9 |    7 |    6 |    1 |        3
  10 |    9 |   16 |    1 |        3
  11 |   11 |   12 |    1 |        3
  12 |   13 |   14 |    1 |        3
(12 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        13, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
   2 |   10 |   14 |    1 |        1
   3 |    5 |   10 |    1 |        2
   4 |   11 |   12 |    1 |        2
   5 |    2 |    4 |    1 |        3
   6 |    6 |    8 |    1 |        3
   7 |    8 |    7 |    1 |        3
   8 |   12 |   13 |    1 |        3
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, false
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    3 |    2 |    1 |        1
   4 |      2 |    5 |    4 |    1 |        1
   5 |      2 |    4 |    3 |    1 |        2
   6 |      2 |    6 |    8 |    1 |        2
   7 |      2 |    8 |    7 |    1 |        2
   8 |      2 |   10 |   10 |    1 |        2
   9 |      2 |    7 |    6 |    1 |        3
  10 |      2 |    9 |   16 |    1 |        3
  11 |      2 |   11 |   12 |    1 |        3
  12 |      2 |   13 |   14 |    1 |        3
  13 |     13 |   13 |   -1 |    0 |        0
  14 |     13 |   10 |   14 |    1 |        1
  15 |     13 |    5 |   10 |    1 |        2
  16 |     13 |   11 |   12 |    1 |        2
  17 |     13 |    2 |    4 |    1 |        3
  18 |     13 |    6 |    8 |    1 |        3
  19 |     13 |    8 |    7 |    1 |        3
  20 |     13 |   12 |   13 |    1 |        3
(20 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        array[2,13], 3, false, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
```

```
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    3 |    2 |    1 |        1
   4 |      2 |    5 |    4 |    1 |        1
   5 |      2 |    4 |    3 |    1 |        2
   6 |      2 |    6 |    8 |    1 |        2
   7 |      2 |    8 |    7 |    1 |        2
   8 |      2 |    7 |    6 |    1 |        3
   9 |      2 |    9 |   16 |    1 |        3
  10 |     13 |   13 |   -1 |    0 |        0
  11 |     13 |   10 |   14 |    1 |        1
  12 |     13 |   11 |   12 |    1 |        2
  13 |     13 |   12 |   13 |    1 |        3
(13 rows)
```

Examples for queries marked as `directed` with `cost` column The examples in this section use the following *Graph 3: Directed, with cost*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        2, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    5 |    4 |    1 |        1
   3 |    6 |    8 |    1 |        2
   4 |   10 |   10 |    1 |        2
   5 |    9 |    9 |    1 |        3
   6 |   11 |   11 |    1 |        3
   7 |   13 |   14 |    1 |        3
(7 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        13, 3
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
(1 row)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    5 |    4 |    1 |        1
   3 |      2 |    6 |    8 |    1 |        2
   4 |      2 |   10 |   10 |    1 |        2
   5 |      2 |    9 |    9 |    1 |        3
   6 |      2 |   11 |   11 |    1 |        3
   7 |      2 |   13 |   14 |    1 |        3
   8 |     13 |   13 |   -1 |    0 |        0
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
```

```
        array[2,13], 3, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    5 |    4 |    1 |        1
   3 |      2 |    6 |    8 |    1 |        2
   4 |      2 |   10 |   10 |    1 |        2
   5 |      2 |    9 |    9 |    1 |        3
   6 |      2 |   11 |   11 |    1 |        3
   7 |     13 |   13 |   -1 |    0 |        0
(7 rows)
```

Examples for queries marked as `undirected` with `cost` column The examples in this section use the following *Graph 4: Undirected, with cost*

```
SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        2, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |    2 |   -1 |    0 |        0
   2 |    1 |    1 |    1 |        1
   3 |    5 |    4 |    1 |        1
   4 |    6 |    8 |    1 |        2
   5 |    8 |    7 |    1 |        2
   6 |   10 |   10 |    1 |        2
   7 |    3 |    5 |    1 |        3
   8 |    7 |    6 |    1 |        3
   9 |    9 |    9 |    1 |        3
  10 |   11 |   12 |    1 |        3
  11 |   13 |   14 |    1 |        3
(11 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        13, 3, false
      );
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   13 |   -1 |    0 |        0
   2 |   10 |   14 |    1 |        1
   3 |    5 |   10 |    1 |        2
   4 |   11 |   12 |    1 |        2
   5 |    2 |    4 |    1 |        3
   6 |    6 |    8 |    1 |        3
   7 |    8 |    7 |    1 |        3
   8 |   12 |   13 |    1 |        3
(8 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3, false
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |      2 |    2 |   -1 |    0 |        0
   2 |      2 |    1 |    1 |    1 |        1
   3 |      2 |    5 |    4 |    1 |        1
   4 |      2 |    6 |    8 |    1 |        2
```

```
   5 |       2 |    8 |    7 |    1 |        2
   6 |       2 |   10 |   10 |    1 |        2
   7 |       2 |    3 |    5 |    1 |        3
   8 |       2 |    7 |    6 |    1 |        3
   9 |       2 |    9 |    9 |    1 |        3
  10 |       2 |   11 |   12 |    1 |        3
  11 |       2 |   13 |   14 |    1 |        3
  12 |      13 |   13 |   -1 |    0 |        0
  13 |      13 |   10 |   14 |    1 |        1
  14 |      13 |    5 |   10 |    1 |        2
  15 |      13 |   11 |   12 |    1 |        2
  16 |      13 |    2 |    4 |    1 |        3
  17 |      13 |    6 |    8 |    1 |        3
  18 |      13 |    8 |    7 |    1 |        3
  19 |      13 |   12 |   13 |    1 |        3
(19 rows)

SELECT * FROM pgr_drivingDistance(
        'SELECT id, source, target, cost FROM edge_table',
        array[2,13], 3, false, equicost:=true
      );
 seq | from_v | node | edge | cost | agg_cost
-----+--------+------+------+------+----------
   1 |       2 |    2 |   -1 |    0 |        0
   2 |       2 |    1 |    1 |    1 |        1
   3 |       2 |    5 |    4 |    1 |        1
   4 |       2 |    6 |    8 |    1 |        2
   5 |       2 |    8 |    7 |    1 |        2
   6 |       2 |    3 |    5 |    1 |        3
   7 |       2 |    7 |    6 |    1 |        3
   8 |       2 |    9 |    9 |    1 |        3
   9 |      13 |   13 |   -1 |    0 |        0
  10 |      13 |   10 |   14 |    1 |        1
  11 |      13 |   11 |   12 |    1 |        2
  12 |      13 |   12 |   13 |    1 |        3
(12 rows)
```

See Also

- *pgr_alphaShape* - Alpha shape computation
- *pgr_pointsAsPolygon* - Polygon around set of points
- *Sample Data* network.

Indices and tables

- genindex
- search

pgr_KSP

Name

`pgr_KSP` — Returns the “K” shortest paths.

[12]

Fig. 5.9: Boost Graph Inside

Availability: 2.0.0

- Signature change 2.1.0

Synopsis

The K shortest path routing algorithm based on Yen's algorithm. "K" is the number of shortest paths desired.

Signature Summary

```
pgr_KSP(edges_sql, start_vid, end_vid, K);
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_ksp(edges_sql, start_vid, end_vid, K);
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Complete Signature

```
pgr_KSP(edges_sql, start_vid, end_vid, k, directed, heap_paths)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	TEXT	SQL query as described above.
start_vid	BIGINT	Identifier of the starting vertex.
end_vid	BIGINT	Identifier of the ending vertex.
k	INTEGER	The desiered number of paths.
directed	BOOLEAN	(optional). When `false` the graph is considered as Un which considers the graph as Directed.
heap_paths	BOOLEAN	(optional). When `true` returns all the paths stored in th `false` which only returns `k` pahts.

Roughly, if the shortest path has `N` edges, the heap will contain about than `N * k` paths for small value of `k` and `k > 1`.

Description of the return values Returns set of `(seq, path_seq, path_id, node, edge, cost, agg_cost)`

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
path_seq	INTEGER	Relative position in the path of node and edge. Has value **1** for the beginning of a path.
path_id	BIGINT	Path identifier. The ordering of the paths For two paths i, j if i < j then agg_cost(i) <= agg_cost(j).
node	BIGINT	Identifier of the node in the path.
edge	BIGINT	Identifier of the edge used to go from node to the next node in the path sequence. -1 for the last node of the route.
cost	FLOAT	Cost to traverse from node using edge to the next node in the path sequence.
agg_cost	FLOAT	Aggregate cost from start_vid to node.

Warning: During the transition to 3.0, because pgr_ksp version 2.0 doesn't have defined a directed flag nor a heap_path flag, when pgr_ksp is used with only one flag version 2.0 signature will be used.

Additional Examples

Examples to handle the one flag to choose signatures The examples in this section use the following *Graph 1: Directed, with cost and reverse cost*

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2,
      directed:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)
```

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns The examples in this section use the following *Graph 1: Directed, with cost and reverse cost*

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, true, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
```

```
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)
```

Examples for queries marked as `undirected` with `cost` and `reverse_cost` columns The examples in this section use the following *Graph 2: Undirected, with cost and reverse cost*

```
SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, directed:=false
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    2 |    1 |        0
   2 |       1 |        2 |    3 |    3 |    1 |        1
   3 |       1 |        3 |    4 |   16 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT * FROM pgr_KSP(
     'SELECT id, source, target, cost, reverse_cost FROM edge_table',
      2, 12, 2, false, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    2 |    1 |        0
   2 |       1 |        2 |    3 |    3 |    1 |        1
   3 |       1 |        3 |    4 |   16 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
  16 |       4 |        1 |    2 |    4 |    1 |        0
  17 |       4 |        2 |    5 |   10 |    1 |        1
  18 |       4 |        3 |   10 |   12 |    1 |        2
  19 |       4 |        4 |   11 |   11 |    1 |        3
  20 |       4 |        5 |    6 |    9 |    1 |        4
  21 |       4 |        6 |    9 |   15 |    1 |        5
  22 |       4 |        7 |   12 |   -1 |    0 |        6
(22 rows)
```

Examples for queries marked as `directed` with `cost` column The examples in this section use the following *Graph 3: Directed, with cost*

```
SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 3, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
(0 rows)

SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT   * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)

SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, true, true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
```

```
   9  |        2 |        4 |   11 |   13 |    1 |        3
  10  |        2 |        5 |   12 |   -1 |    0 |        4
  11  |        3 |        1 |    2 |    4 |    1 |        0
  12  |        3 |        2 |    5 |   10 |    1 |        1
  13  |        3 |        3 |   10 |   12 |    1 |        2
  14  |        3 |        4 |   11 |   13 |    1 |        3
  15  |        3 |        5 |   12 |   -1 |    0 |        4
(15 rows)
```

Examples for queries marked as `undirected` with `cost` column The examples in this section use the following *Graph 4: Undirected, with cost*

```
SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
(10 rows)

SELECT  * FROM pgr_KSP(
     'SELECT id, source, target, cost FROM edge_table',
      2, 12, 2, directed:=false, heap_paths:=true
   );
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    2 |    4 |    1 |        0
   2 |       1 |        2 |    5 |    8 |    1 |        1
   3 |       1 |        3 |    6 |    9 |    1 |        2
   4 |       1 |        4 |    9 |   15 |    1 |        3
   5 |       1 |        5 |   12 |   -1 |    0 |        4
   6 |       2 |        1 |    2 |    4 |    1 |        0
   7 |       2 |        2 |    5 |    8 |    1 |        1
   8 |       2 |        3 |    6 |   11 |    1 |        2
   9 |       2 |        4 |   11 |   13 |    1 |        3
  10 |       2 |        5 |   12 |   -1 |    0 |        4
  11 |       3 |        1 |    2 |    4 |    1 |        0
  12 |       3 |        2 |    5 |   10 |    1 |        1
  13 |       3 |        3 |   10 |   12 |    1 |        2
  14 |       3 |        4 |   11 |   13 |    1 |        3
  15 |       3 |        5 |   12 |   -1 |    0 |        4
(15 rows)
```

See Also

- http://en.wikipedia.org/wiki/K_shortest_path_routing
- *Sample Data* network.

Indices and tables

- genindex
- search

pgr_dijkstraVia - Proposed

Name

`pgr_dijkstraVia` — Using dijkstra algorithm, it finds the route that goes through a list of vertices.

[13]

Fig. 5.10: Boost Graph Inside

Availability: 2.2.0

Synopsis

Given a list of vertices and a graph, this function is equivalent to finding the shortest path between $vertex_i$ and $vertex_{i+1}$ for all $i < size\_of(vertex_via)$.

The paths represents the sections of the route.

Note: This is a proposed function

Signatrue Summary

```
pgr_dijkstraVia(edges_sql, via_vertices)
pgr_dijkstraVia(edges_sql, via_vertices, directed, strict, U_turn_on_edge)

RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid,
    node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET
```

Signatures

Minimal Signature

```
pgr_dijkstraVia(edges_sql, via_vertices)
RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid,
    node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET
```

Example Find the route that visits the vertices 1 3 9 in that order

```
SELECT * FROM pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 3, 9]
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost | route_agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------+----------------
   1 |       1 |        1 |         1 |       3 |    1 |    1 |    1 |        0 |              0
```

```
   2 |        1 |          2 |           1 |         3 |      2 |     4 |      1 |         1 |             1
   3 |        1 |          3 |           1 |         3 |      5 |     8 |      1 |         2 |             2
   4 |        1 |          4 |           1 |         3 |      6 |     9 |      1 |         3 |             3
   5 |        1 |          5 |           1 |         3 |      9 |    16 |      1 |         4 |             4
   6 |        1 |          6 |           1 |         3 |      4 |     3 |      1 |         5 |             5
   7 |        1 |          7 |           1 |         3 |      3 |    -1 |      0 |         6 |             6
   8 |        2 |          1 |           3 |         9 |      3 |     5 |      1 |         0 |             6
   9 |        2 |          2 |           3 |         9 |      6 |     9 |      1 |         1 |             7
  10 |        2 |          3 |           3 |         9 |      9 |    -2 |      0 |         2 |             8
(10 rows)
```

Complete Signature

```
pgr_dijkstraVia(edges_sql, via_vertices, directed, strict, U_turn_on_edge)
RETURNS SET OF (seq, path_pid, path_seq, start_vid, end_vid,
    node, edge, cost, agg_cost, route_agg_cost) or EMPTY SET
```

Example Find the route that visits the vertices 1 3 9 in that order on an undirected graph, avoiding U-turns when possible

```
SELECT * FROM pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 3, 9], false, strict:=true, U_turn_on_edge:=false
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost | route_agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------+----------------
   1 |        1 |          1 |           1 |         3 |      1 |     1 |      1 |         0 |             0
   2 |        1 |          2 |           1 |         3 |      2 |     2 |      1 |         1 |             1
   3 |        1 |          3 |           1 |         3 |      3 |    -1 |      0 |         2 |             2
   4 |        2 |          1 |           3 |         9 |      3 |     5 |      1 |         0 |             2
   5 |        2 |          2 |           3 |         9 |      6 |     9 |      1 |         1 |             3
   6 |        2 |          3 |           3 |         9 |      9 |    -2 |      0 |         2 |             4
(6 rows)
```

Description of the Signature

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Default
edges_sql	`TEXT`	
via_vertices	`ARRAY[ANY-INTEGER]`	
directed	`BOOLEAN`	`true`
strict	`BOOLEAN`	`false`
U_turn_on_edge	`BOOLEAN`	`true`

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	SQL query as described above.
via_vertices	`ARRAY[ANY-INTEGER]`	Array of vertices identifiers
directed	`BOOLEAN`	(optional) Default is true (is directed). When set to as Undirected
strict	`BOOLEAN`	(optional) ignores if a subsection of the route is m found Default is true (is directed). When set to fal Undirected
U_turn_on_edge	`BOOLEAN`	(optional) Default is true (is directed). When set to as Undirected

Description of the return values Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
seq	`BIGINT`	Sequential value starting from 1.
path_pid	`BIGINT`	Identifier of the path.
path_seq	`BIGINT`	Sequential value starting from 1 for the path.
start_vid	`BIGINT`	Identifier of the starting vertex of the path.
end_vid	`BIGINT`	Identifier of the ending vertex of the path.
node	`BIGINT`	Identifier of the node in the path from start_vid to end_vid.
edge	`BIGINT`	Identifier of the edge used to go from node to the next node in the path sequence. -1 for the last node of the path. -2 for the last node of the route.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the route sequence.
agg_cost	`FLOAT`	Total cost from `start_vid` to `end_vid` of the path.
route_agg_cost	`FLOAT`	Total cost from `start_vid` of `path_pid = 1` to `end_vid` of the current `path_pid`.

Examples

Example 1 Find the route that visits the vertices 1 5 3 9 4 in that order

```
SELECT * FROM pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost | route_agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------+-----------------
   1 |       1 |        1 |         1 |       5 |    1 |    1 |    1 |        0 |              0
   2 |       1 |        2 |         1 |       5 |    2 |    4 |    1 |        1 |              1
   3 |       1 |        3 |         1 |       5 |    5 |   -1 |    0 |        2 |              2
   4 |       2 |        1 |         5 |       3 |    5 |    8 |    1 |        0 |              2
   5 |       2 |        2 |         5 |       3 |    6 |    9 |    1 |        1 |              3
   6 |       2 |        3 |         5 |       3 |    9 |   16 |    1 |        2 |              4
   7 |       2 |        4 |         5 |       3 |    4 |    3 |    1 |        3 |              5
   8 |       2 |        5 |         5 |       3 |    3 |   -1 |    0 |        4 |              6
   9 |       3 |        1 |         3 |       9 |    3 |    5 |    1 |        0 |              6
  10 |       3 |        2 |         3 |       9 |    6 |    9 |    1 |        1 |              7
  11 |       3 |        3 |         3 |       9 |    9 |   -1 |    0 |        2 |              8
  12 |       4 |        1 |         9 |       4 |    9 |   16 |    1 |        0 |              8
  13 |       4 |        2 |         9 |       4 |    4 |   -2 |    0 |        1 |              9
(13 rows)
```

Example 2 What's the aggregate cost of the third path?

```
SELECT agg_cost FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE path_id = 3 AND edge <0;
 agg_cost
----------
        2
(1 row)
```

Example 3 What's the route's aggregate cost of the route at the end of the third path?

```
SELECT route_agg_cost FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE path_id = 3 AND edge < 0;
 route_agg_cost
----------------
              8
(1 row)
```

Example 4 How are the nodes visited in the route?

```
SELECT row_number() over () as node_seq, node
FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE edge <> -1 ORDER BY seq;
 node_seq | node
----------+------
        1 |    1
        2 |    2
        3 |    5
        4 |    6
        5 |    9
        6 |    4
        7 |    3
        8 |    6
        9 |    9
       10 |    4
(10 rows)
```

Example 5 What are the aggregate costs of the route when the visited vertices are reached?

```
SELECT path_id, route_agg_cost FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4]
)
WHERE edge < 0;
 path_id | route_agg_cost
---------+----------------
       1 |              2
       2 |              6
       3 |              8
       4 |              9
(4 rows)
```

Example 6 show the route's seq and aggregate cost and a status of "passes in front" or "visits" node

9

```
SELECT seq, route_agg_cost, node, agg_cost ,
CASE WHEN edge = -1 THEN 'visits'
ELSE 'passes in front'
END as status
FROM  pgr_dijkstraVia(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table order by id',
    ARRAY[1, 5, 3, 9, 4])
WHERE node = 9 and (agg_cost  <> 0 or seq = 1);
 seq | route_agg_cost | node | agg_cost |      status
-----+----------------+------+----------+-----------------
   6 |              4 |    9 |        2 | passes in front
  11 |              8 |    9 |        2 | visits
(2 rows)

ROLLBACK;
ROLLBACK
```

See Also

- http://en.wikipedia.org/wiki/Dijkstra%27s_algorithm
- *Sample Data* network.

Indices and tables

- genindex
- search

The problem definition (Advanced documentation)

Given the following query:

pgr_dijkstra($sql, start_{vid}, end_{vid}, directed$)

where $sql = \{(id_i, source_i, target_i, cost_i, reverse\_cost_i)\}$

and

- $source = \bigcup source_i$,
- $target = \bigcup target_i$,

The graphs are defined as follows:

Directed graph

The weighted directed graph, $G_d(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target \cup start_{vid} \cup end_{vid}$
- the set of edges E

– $E = \begin{cases} \text{if } reverse\_cost = & \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \\ & \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup & \{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} \\ \text{if } reverse\_cost \neq & \end{cases}$

Undirected graph

The weighted undirected graph, $G_u(V, E)$, is definied by:

- the set of vertices V
  - $V = source \cup target \cup start_v vid \cup end_{vid}$
- the set of edges E
  - $E = \begin{cases} & \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup & \{(target_i, source_i, cost_i) \text{ when } cost >= 0\} \\ \text{if } reverse\_cost = & \\ & \{(source_i, target_i, cost_i) \text{ when } cost >= 0\} \\ \cup & \{(target_i, source_i, cost_i) \text{ when } cost >= 0\} \\ \cup & \{(target_i, source_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} \\ \cup & \{(source_i, target_i, reverse\_cost_i) \text{ when } reverse\_cost_i >= 0)\} \\ \text{if } reverse\_cost \neq & \end{cases}$

The problem

Given:

- $start_{vid} \in V$ a starting vertex
- $end_{vid} \in V$ an ending vertex
- $G(V, E) = \begin{cases} G_d(V, E) & \text{if } directed = true \\ G_u(V, E) & \text{if } directed = false \end{cases}$

Then:

$$\text{pgr\_dijkstra}(sql, start_{vid}, end_{vid}, directed) = \begin{cases} \text{shortest path } \boldsymbol{\pi} \text{ between } start_{vid} \text{and } end_{vid} & \text{if } \exists \boldsymbol{\pi} \\ & \text{otherwise} \end{cases}$$

$\boldsymbol{\pi} = \{(path\_seq_i, node_i, edge_i, cost_i, agg\_cost_i)\}$

where:

- $path\_seq_i = i$
- $path\_seq_{|\pi|} = |\pi|$
- $node_i \in V$
- $node_1 = start_{vid}$
- $node_{|\pi|} = end_{vid}$
- $\forall i \neq |\pi|, \quad (node_i, node_{i+1}, cost_i) \in E$
- $edge_i = \begin{cases} id_{(node_i, node_{i+1}, cost_i)} & \text{when } i \neq |\pi| \\ -1 & \text{when } i = |\pi| \end{cases}$

- $cost_i = cost_{(node_i, node_{i+1})}$
- $agg\_cost_i = \begin{cases} 0 & \text{when } i = 1 \\ \displaystyle\sum_{k=1}^{i} cost_{(node_{k-1}, node_k)} & \text{when } i \neq 1 \end{cases}$

In other words: The algorithm returns a the shortest path between $start_{vid}$ and end_{vid} , if it exists, in terms of a sequence of

- $path\_seq$ indicates the relative position in the path of the $node$ or $edge$.
- $cost$ is the cost of the edge to be used to go to the next node.
- agg_cost is the cost from the $start_{vid}$ up to the node.

If there is no path, the resulting set is empty.

5.1.5 pgr_trsp - Turn Restriction Shortest Path (TRSP)

Name

`pgr_trsp` — Returns the shortest path with support for turn restrictions.

Synopsis

The turn restricted shorthest path (TRSP) is a shortest path algorithm that can optionally take into account complicated turn restrictions like those found in real world navigable road networks. Performamnce wise it is nearly as fast as the A* search but has many additional features like it works with edges rather than the nodes of the network. Returns a set of *pgr_costResult* (seq, id1, id2, cost) rows, that make up a path.

```
pgr_costResult[] pgr_trsp(sql text, source integer, target integer,
            directed boolean, has_rcost boolean [,restrict_sql text]);
```

```
pgr_costResult[] pgr_trsp(sql text, source_edge integer, source_pos float8,
                target_edge integer, target_pos float8,
            directed boolean, has_rcost boolean [,restrict_sql text]);
```

```
pgr_costResult3[] pgr_trspViaVertices(sql text, vids integer[],
                directed boolean, has_rcost boolean
                [, turn_restrict_sql text]);
```

```
pgr_costResult3[] pgr_trspViaEdges(sql text, eids integer[], pcts float8[],
               directed boolean, has_rcost boolean
               [, turn_restrict_sql text]);
```

Description

The Turn Restricted Shortest Path algorithm (TRSP) is similar to the shooting star in that you can specify turn restrictions.

The TRSP setup is mostly the same as *Dijkstra shortest path* with the addition of an optional turn restriction table. This provides an easy way of adding turn restrictions to a road network by placing them in a separate table.

sql a SQL query, which should return a set of rows with the following columns:

```
SELECT id, source, target, cost, [,reverse_cost] FROM edge_table
```

id `int4` identifier of the edge

source `int4` identifier of the source vertex

target `int4` identifier of the target vertex

cost `float8` value, of the edge traversal cost. A negative cost will prevent the edge from being inserted in the graph.

reverse_cost (optional) the cost for the reverse traversal of the edge. This is only used when the `directed` and `has_rcost` parameters are `true` (see the above remark about negative costs).

source `int4` **NODE id** of the start point

target `int4` **NODE id** of the end point

directed `true` if the graph is directed

has_rcost if `true`, the `reverse_cost` column of the SQL generated set of rows will be used for the cost of the traversal of the edge in the opposite direction.

restrict_sql (optional) a SQL query, which should return a set of rows with the following columns:

```
SELECT to_cost, target_id, via_path FROM restrictions
```

to_cost `float8` turn restriction cost

target_id `int4` target id

via_path `text` comma separated list of edges in the reverse order of `rule`

Another variant of TRSP allows to specify **EDGE id** of source and target together with a fraction to interpolate the position:

source_edge `int4` **EDGE id** of the start edge

source_pos `float8` fraction of 1 defines the position on the start edge

target_edge `int4` **EDGE id** of the end edge

target_pos `float8` fraction of 1 defines the position on the end edge

Returns set of *pgr_costResult[]*:

seq row sequence

id1 node ID

id2 edge ID (`-1` for the last row)

cost cost to traverse from `id1` using `id2`

History

- New in version 2.0.0

Support for Vias

Warning: The Support for Vias functions are prototypes. Not all corner cases are being considered.

We also have support for vias where you can say generate a from A to B to C, etc. We support both methods above only you pass an array of vertices or and array of edges and percentage position along the edge in two arrays.

sql a SQL query, which should return a set of rows with the following columns:

```
SELECT id, source, target, cost, [,reverse_cost] FROM edge_table
```

id `int4` identifier of the edge

source `int4` identifier of the source vertex

target `int4` identifier of the target vertex

cost `float8` value, of the edge traversal cost. A negative cost will prevent the edge from being inserted in the graph.

reverse_cost (optional) the cost for the reverse traversal of the edge. This is only used when the `directed` and `has_rcost` parameters are `true` (see the above remark about negative costs).

vids `int4[]` An ordered array of **NODE id** the path will go through from start to end.

directed `true` if the graph is directed

has_rcost if `true`, the `reverse_cost` column of the SQL generated set of rows will be used for the cost of the traversal of the edge in the opposite direction.

restrict_sql (optional) a SQL query, which should return a set of rows with the following columns:

```
SELECT to_cost, target_id, via_path FROM restrictions
```

to_cost `float8` turn restriction cost

target_id `int4` target id

via_path `text` commar separated list of edges in the reverse order of `rule`

Another variant of TRSP allows to specify **EDGE id** together with a fraction to interpolate the position:

eids `int4` An ordered array of **EDGE id** that the path has to traverse

pcts `float8` An array of fractional positions along the respective edges in `eids`, where 0.0 is the start of the edge and 1.0 is the end of the eadge.

Returns set of *pgr_costResult[]*:

seq row sequence

id1 route ID

id2 node ID

id3 edge ID (`-1` for the last row)

cost cost to traverse from `id2` using `id3`

History

- Via Support prototypes new in version 2.1.0

Examples

Without turn restrictions

```
SELECT * FROM pgr_trsp(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
        7, 12, false, false
    );
 seq | id1 | id2 | cost
-----+-----+-----+------
   0 |   7 |   6 |    1
   1 |   8 |   7 |    1
   2 |   5 |   8 |    1
   3 |   6 |   9 |    1
   4 |   9 |  15 |    1
   5 |  12 |  -1 |    0
(6 rows)
```

With turn restrictions

Then a query with turn restrictions is created as:

```
SELECT * FROM pgr_trsp(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
        2, 7, false, false,
        'SELECT to_cost, target_id::int4,
        from_edge || coalesce('','' || via_path, '''') AS via_path
        FROM restrictions'
    );
 seq | id1 | id2 | cost
-----+-----+-----+------
   0 |   2 |   4 |    1
   1 |   5 |  10 |    1
   2 |  10 |  12 |    1
   3 |  11 |  11 |    1
   4 |   6 |   8 |    1
   5 |   5 |   7 |    1
   6 |   8 |   6 |    1
   7 |   7 |  -1 |    0
(8 rows)

SELECT * FROM pgr_trsp(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
        7, 11, false, false,
        'SELECT to_cost, target_id::int4,
        from_edge || coalesce('','' || via_path, '''') AS via_path
        FROM restrictions'
    );
 seq | id1 | id2 | cost
-----+-----+-----+------
   0 |   7 |   6 |    1
   1 |   8 |   7 |    1
   2 |   5 |   8 |    1
   3 |   6 |   9 |    1
   4 |   9 |  15 |    1
   5 |  12 |  13 |    1
   6 |  11 |  -1 |    0
(7 rows)
```

An example query using vertex ids and via points:

```
SELECT * FROM pgr_trspViaVertices(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost FROM edge_table',
        ARRAY[2,7,11]::INTEGER[],
        false,  false,
        'SELECT to_cost, target_id::int4, from_edge ||
        coalesce('',''||via_path,'''') AS via_path FROM restrictions');
 seq | id1 | id2 | id3 | cost
-----+-----+-----+-----+------
   1 |   1 |   2 |   4 |    1
   2 |   1 |   5 |  10 |    1
   3 |   1 |  10 |  12 |    1
   4 |   1 |  11 |  11 |    1
   5 |   1 |   6 |   8 |    1
   6 |   1 |   5 |   7 |    1
   7 |   1 |   8 |   6 |    1
   8 |   2 |   7 |   6 |    1
   9 |   2 |   8 |   7 |    1
  10 |   2 |   5 |   8 |    1
  11 |   2 |   6 |   9 |    1
  12 |   2 |   9 |  15 |    1
```

```
  13 |    2 |   12 |   13 |     1
  14 |    2 |   11 |   -1 |     0
(14 rows)
```

An example query using edge ids and vias:

```
SELECT * FROM pgr_trspViaEdges(
        'SELECT id::INTEGER, source::INTEGER, target::INTEGER, cost,
        reverse_cost FROM edge_table',
        ARRAY[2,7,11]::INTEGER[],
        ARRAY[0.5, 0.5, 0.5]::FLOAT[],
        true,
        true,
        'SELECT to_cost, target_id::int4, FROM_edge ||
        coalesce('','',''||via_path,'''') AS via_path FROM restrictions');
 seq | id1 | id2 | id3 | cost
-----+-----+-----+-----+------
   1 |   1 |  -1 |   2 |  0.5
   2 |   1 |   2 |   4 |    1
   3 |   1 |   5 |   8 |    1
   4 |   1 |   6 |   9 |    1
   5 |   1 |   9 |  16 |    1
   6 |   1 |   4 |   3 |    1
   7 |   1 |   3 |   5 |    1
   8 |   1 |   6 |   8 |    1
   9 |   1 |   5 |   7 |    1
  10 |   2 |   5 |   8 |    1
  11 |   2 |   6 |   9 |    1
  12 |   2 |   9 |  16 |    1
  13 |   2 |   4 |   3 |    1
  14 |   2 |   3 |   5 |    1
  15 |   2 |   6 |  11 |  0.5
(15 rows)
```

The queries use the *Sample Data* network.

See Also

- *pgr_costResult[]*

5.1.6 Traveling Sales Person - Family of functions

- *pgr_TSP* - When input is given as matrix cell information.
- *pgr_eucledianTSP* - When input are coordinates.

pgr_TSP

Name

- `pgr_TSP` - Returns a route that visits all the nodes exactly once.

Availability: 2.0.0

- Signature changed 2.3.0

Synopsis

The travelling salesman problem (TSP) or travelling salesperson problem asks the following question:

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

This implementation uses simulated annealing to return the approximate solution when the input is given in the form of matrix cell contents. The matrix information must be symmetrical.

Signature Summary

```
pgr_TSP(matrix_cell_sql)
pgr_TSP(matrix_cell_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Signatures

Basic Use

```
pgr_TSP(matrix_cell_sql)
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example

Because the documentation examples are auto generated and tested for non changing results, and the default is to have random execution, the example is wrapping the actual call.

```
WITH
query AS (
    SELECT * FROM pgr_TSP(
        $$
        SELECT * FROM pgr_dijkstraCostMatrix(
            'SELECT id, source, target, cost, reverse_cost FROM edge_table',
            (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 14),
            directed := false
        )
        $$
    )
)
SELECT agg_cost < 20 AS under_20 FROM query WHERE seq = 14;
 under_20
----------
 t
(1 row)
```

Complete Signature

```
pgr_TSP(matrix_cell_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example:

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_dijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 14),
        directed := false
    )
    $$,
    start_id := 7,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    7 |    1 |        0
   2 |    8 |    1 |        1
   3 |    5 |    1 |        2
   4 |    2 |    1 |        3
   5 |    1 |    2 |        4
   6 |    3 |    1 |        6
   7 |    4 |    1 |        7
   8 |    9 |    1 |        8
   9 |   12 |    1 |        9
  10 |   11 |    1 |       10
  11 |   10 |    1 |       11
  12 |   13 |    3 |       12
  13 |    6 |    3 |       15
  14 |    7 |    0 |       18
(14 rows)
```

Description of the Signatures

Description of the Matrix Cell SQL query

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex.
end_vid	BIGINT	Identifier of the ending vertex.
agg_cost	FLOAT	Cost for going from start_vid to end_vid

Can be Used with:

- *pgr_dijkstraCostMatrix - proposed*
- *pgr_withPointsCostMatrix - proposed*
- *pgr_floydWarshall*
- *pgr_johnson*

To generate a symmetric matrix

- directed := false.

If using directed := true, the resulting non symmetric matrix must be converted to symmetric by fixing the non symmetric values according to your application needs.

```
Description Of the Control parameters
.....................................................................

The control parameters are optional, and have a default value.

=============================== =========== ============ =========================================
Parameter                       Type         Default      Description
```

```
=============================== =========== ============ ===========================================
**start_vid**                   ``BIGINT``    `0`         The greedy part of the implementation w
**end_vid**                     ``BIGINT``    `0`         Last visiting vertex before returning t
**max_processing_time**         ``FLOAT``    `+infinity`  Stop the annealing processing when the
**tries_per_temperature**       ``INTEGER``  `500`        Maximum number of times a neighbor(s) i
**max_changes_per_temperature** ``INTEGER``  `60`         Maximum number of times the solution is
**max_consecutive_non_changes** ``INTEGER``  `100`        Maximum number of consecutive times the
**initial_temperature**         ``FLOAT``    `100`        Starting temperature.
**final_temperature**           ``FLOAT``    `0.1`        Ending temperature.
**cooling_factor**              ``FLOAT``    `0.9`        Value between between 0 and 1 (not incl
**randomize**                   ``BOOLEAN``  `true`       Choose the random seed

                                                          - true: Use current time as seed
                                                          - false: Use `1` as seed. Using this va

=============================== =========== ============ ===========================================
```

```
Description of the return columns
...............................................................................

Returns set of ``(seq, node, cost, agg_cost)``

============= =========== =================================================
Column           Type              Description
============= =========== =================================================
**seq**       ``INTEGER`` Row sequence.
**node**      ``BIGINT``  Identifier of the node/coordinate/point.
**cost**      ``FLOAT``   Cost to traverse from the current ``node`` ito the next ``node`` in the
                            - ``0`` for the last row in the path sequence.

**agg_cost**  ``FLOAT``   Aggregate cost from the ``node`` at ``seq = 1`` to the current node.
                            - ``0`` for the first row in the path sequence.

============= =========== =================================================
```

Examples

Example Using with points of interest.

To generate a symmetric matrix:

- the **side** information of pointsOfInterset is ignored by not including it in the query
- and **directed := false**

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_withPointsCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
        'SELECT pid, edge_id, fraction from pointsOfInterest',
        array[-1, 3, 5, 6, -6], directed := false);
    $$,
    start_id := 5,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    5 |    1 |        0
   2 |    6 |    1 |        1
```

```
   3 |     3 |  1.6 |         2
   4 |    -1 |  1.3 |       3.6
   5 |    -6 |  0.3 |       4.9
   6 |     5 |    0 |       5.2
(6 rows)
```

The queries use the *Sample Data* network.

See Also

- *Traveling Sales Person - Family of functions*
- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

pgr_eucledianTSP

Name

`pgr_eucledianTSP` - Returns a route that visits all the coordinates pairs exactly once.

Availability: 2.3.0

Synopsis

The travelling salesman problem (TSP) or travelling salesperson problem asks the following question:

- Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city?

This implementation uses simulated annealing to return the approximate solution when the input is given in the form of coordinates.

Signature Summary

```
pgr_eucledianTSP(coordinates_sql)
pgr_eucledianTSP(coordinates_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Signatures

Minimal Signature

```
pgr_eucledianTSP(coordinates_sql)
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example

Because the documentation examples are auto generated and tested for non changing results, and the default is to have random execution, the example is wrapping the actual call.

```
WITH
query AS (
    SELECT * FROM pgr_eucledianTSP(
        $$
        SELECT id, st_X(the_geom) AS x, st_Y(the_geom)AS y  FROM edge_table_vertices_pgr
        $$
    )
)
SELECT agg_cost < 20 AS under_20 FROM query WHERE seq = 18;
 under_20
----------
 t
(1 row)
```

Complete Signature

```
pgr_eucledianTSP(coordinates_sql,
    start_id, end_id,
    max_processing_time,
    tries_per_temperature, max_changes_per_temperature, max_consecutive_non_changes,
    initial_temperature, final_temperature, cooling_factor,
    randomize,
RETURNS SETOF (seq, node, cost, agg_cost)
```

Example:

```
SELECT* from pgr_eucledianTSP(
    $$
    SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_pgr
    $$,
    tries_per_temperature := 3,
    cooling_factor := 0.5,
    randomize := false
);
 seq | node |       cost        |     agg_cost
-----+------+-------------------+------------------
   1 |    1 |   1.4142135623731 |                0
   2 |    3 |                 1 |   1.4142135623731
   3 |    4 |                 1 |  2.41421356237309
   4 |    9 |  0.58309518948453 |  3.41421356237309
   5 |   16 |  0.58309518948453 |  3.99730875185762
   6 |    6 |                 1 |  4.58040394134215
   7 |    5 |                 1 |  5.58040394134215
   8 |    8 |                 1 |  6.58040394134215
   9 |    7 |  1.58113883008419 |  7.58040394134215
  10 |   14 |    1.499999999999 |  9.16154277142634
  11 |   15 |               0.5 |  10.6615427714253
  12 |   13 |               1.5 |  11.1615427714253
  13 |   17 |  1.11803398874989 |  12.6615427714253
  14 |   12 |                 1 |  13.7795767601752
  15 |   11 |                 1 |  14.7795767601752
  16 |   10 |                 2 |  15.7795767601752
  17 |    2 |                 1 |  17.7795767601752
  18 |    1 |                 0 |  18.7795767601752
(18 rows)
```

Description of the Signatures

Description of the coordinates SQL query

Column	Type	Description
id	BIGINT	Identifier of the coordinate. (optional)
x	FLOAT	X value of the coordinate.
y	FLOAT	Y value of the coordinate.

When the value of **id** is not given then the coordinates will receive an **id** starting from 1, in the order given.

```
Description Of the Control parameters
.....................................................................

The control parameters are optional, and have a default value.

=============================== =========== ============ =========================================
Parameter                       Type        Default      Description
=============================== =========== ============ =========================================
**start_vid**                   ``BIGINT``   `0`         The greedy part of the implementation w
**end_vid**                     ``BIGINT``   `0`         Last visiting vertex before returning t
**max_processing_time**         ``FLOAT``   `+infinity`  Stop the annealing processing when the
**tries_per_temperature**       ``INTEGER`` `500`        Maximum number of times a neighbor(s) i
**max_changes_per_temperature** ``INTEGER`` `60`         Maximum number of times the solution is
**max_consecutive_non_changes** ``INTEGER`` `100`        Maximum number of consecutive times the
**initial_temperature**         ``FLOAT``   `100`        Starting temperature.
**final_temperature**           ``FLOAT``   `0.1`        Ending temperature.
**cooling_factor**              ``FLOAT``   `0.9`        Value between between 0 and 1 (not inc
**randomize**                   ``BOOLEAN`` `true`       Choose the random seed

                                                         - true: Use current time as seed
                                                         - false: Use `1` as seed. Using this va

=============================== =========== ============ =========================================
```

```
Description of the return columns
...............................................................................

Returns set of ``(seq, node, cost, agg_cost)``

============= =========== =================================================
Column          Type           Description
============= =========== =================================================
**seq**       ``INTEGER`` Row sequence.
**node**      ``BIGINT``  Identifier of the node/coordinate/point.
**cost**      ``FLOAT``   Cost to traverse from the current ``node`` ito the next ``node`` in the
                            - ``0`` for the last row in the path sequence.

**agg_cost**  ``FLOAT``   Aggregate cost from the ``node`` at ``seq = 1`` to the current node.
                            - ``0`` for the first row in the path sequence.

============= =========== =================================================
```

Examples

Example Skipping the Simulated Annealing & showing some process information

```
SET client_min_messages TO DEBUG1;
SET
SELECT* from pgr_eucledianTSP(
    $$
    SELECT id, st_X(the_geom) AS x, st_Y(the_geom) AS y FROM edge_table_vertices_pgr
```

```
    $$,
    tries_per_temperature := 0,
    randomize := false
);
DEBUG:  pgr_eucledianTSP Processing Information
Initializing tsp class ---> tsp.greedyInitial ---> tsp.annealing ---> OK

Cycle(100)        total changes =0        0 were because  delta energy < 0
Total swaps: 3
Total slides: 0
Total reverses: 0
Times best tour changed: 4
Best cost reached = 18.7796
 seq | node |        cost        |     agg_cost
-----+------+------------------+------------------
   1 |    1 |  1.4142135623731 |                0
   2 |    3 |                1 |  1.4142135623731
   3 |    4 |                1 | 2.41421356237309
   4 |    9 | 0.58309518948453 | 3.41421356237309
   5 |   16 | 0.58309518948453 | 3.99730875185762
   6 |    6 |                1 | 4.58040394134215
   7 |    5 |                1 | 5.58040394134215
   8 |    8 |                1 | 6.58040394134215
   9 |    7 | 1.58113883008419 | 7.58040394134215
  10 |   14 |   1.499999999999 | 9.16154277142634
  11 |   15 |              0.5 | 10.6615427714253
  12 |   13 |              1.5 | 11.1615427714253
  13 |   17 | 1.11803398874989 | 12.6615427714253
  14 |   12 |                1 | 13.7795767601752
  15 |   11 |                1 | 14.7795767601752
  16 |   10 |                2 | 15.7795767601752
  17 |    2 |                1 | 17.7795767601752
  18 |    1 |                0 | 18.7795767601752
(18 rows)
```

The queries use the *Sample Data* network.

History

- New in version 2.3.0

See Also

- *Traveling Sales Person - Family of functions*
- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

General Information

Origin

The traveling sales person problem was studied in the 18th century by mathematicians Sir William Rowam Hamilton and **Thomas Penyngton Kirkman**.

A discussion about the work of Hamilton & Kirkman can be found in the book **Graph Theory (Biggs et al. 1976)**.

- ISBN-13: 978-0198539162

- ISBN-10: 0198539169

It is believed that the general form of the TSP have been first studied by Kalr Menger in Vienna and Harvard. The problem was later promoted by Hassler, Whitney & Merrill at Princeton. A detailed description about the connection between Menger & Whitney, and the development of the TSP can be found in On the history of combinatorial optimization (till 1960)[14]

Problem Definition

Given a collection of cities and travel cost between each pair, find the cheapest way for visiting all of the cities and returning to the starting point.

Characteristics

- The travel costs are symmetric:
  - traveling costs from city A to city B are just as much as traveling from B to A.
- This problem is an NP-hard optimization problem.
- To calculate the number of different tours through n cities:
  - Given a starting city,
  - There are $n - 1$ choices for the second city,
  - And $n - 2$ choices for the third city, etc.
  - Multiplying these together we get $(n - 1)! = (n - 1)(n - 2)..1$.
  - Now since our travel costs do not depend on the direction we take around the tour:
    - this number by 2
    - $(n - 1)!/2$.

TSP & Simulated Annealing

The simulated annealing algorithm was originally inspired from the process of annealing in metal work.

Annealing involves heating and cooling a material to alter its physical properties due to the changes in its internal structure. As the metal cools its new structure becomes fixed, consequently causing the metal to retain its newly obtained properties. *[C001]*

Pseudocode

Given an initial solution, the simulated annealing process, will start with a high temperature and gradually cool down until the desired temperature is reached.

For each temperature, a neighbouring new solution **snew** is calculated. The higher the temperature the higher the probability of accepting the new solution as a possible bester solution.

Once the desired temperature is reached, the best solution found is returned

```
Solution ← initial_solution;

temperature ← initial_temperature;
while (temperature > final_temperature) {

    do tries_per_temperature times {
        snew ← neighbour(solution);
```

[14] http://www.cwi.nl/ lex/files/histco.ps

```
        If P(E(solution), E(snew), T)  random(0, 1)
            solution ← snew;
    }

    temperature ← temperature * cooling factor;
}

Output: the best solution
```

pgRouting Implementation

pgRouting's implementation adds some extra parameters to allow some exit controls within the simulated annealing process.

To cool down faster to the next temperature:

- max_changes_per_temperature: limits the number of changes in the solution per temperature
- max_consecutive_non_changes: limits the number of consecutive non changes per temperature

This is done by doing some book keeping on the times **solution ← snew;** is executed.

- max_changes_per_temperature: Increases by one when **solution** changes
- max_consecutive_non_changes: Reset to 0 when **solution** changes, and increased each **try**

Additionally to stop the algorithm at a higher temperature than the desired one:

- max_processing_time: limits the time the simulated annealing is performed.
- book keeping is done to see if there was a change in **solution** on the last temperature

Note that, if no change was found in the first **max_consecutive_non_changes** tries, then the simulated annealing will stop.

```
Solution ← initial_solution;

temperature ← initial_temperature;
while (temperature > final_temperature) {

    do tries_per_temperature times {
        snew ← neighbour(solution);
        If P(E(solution), E(snew), T)  random(0, 1)
            solution ← snew;

        when max_changes_per_temperature is reached
            or max_consecutive_non_changes is reached
            BREAK;
    }

    temperature ← temperature * cooling factor;
    when no changes were done in the current temperature
        or max_processing_time has being reached
        BREAK;
}

Output: the best solution
```

Choosing parameters

There is no exact rule on how the parameters have to be chose, it will depend on the special characteristics of the problem.

- Your computational time is crucial, then put your time limit to **max_processing_time**.

- Make the **tries_per_temperture** depending on the number of cities, for example:
  - Useful to estimate the time it takes to do one cycle: use *1*
    - this will help to set a reasonable **max_processing_time**
  - $n * (n - 1)$
  - $500 * n$
- For a faster decreasing the temperature set **cooling_factor** to a smaller number, and set to a higher number for a slower decrease.
- When for the same given data the same results are needed, set **randomize** to *false*.
  - When estimating how long it takes to do one cycle: use *false*

A recommendation is to play with the values and see what fits to the particular data.

Description Of the Control parameters

The control parameters are optional, and have a default value.

Parameter	Type	Default	Description
start_vid	`BIGINT`	*0*	The greedy part of the implementation will use this identifier.
end_vid	`BIGINT`	*0*	Last visiting vertex before returning to start_vid.
max_processing_time	`FLOAT`	*+infinity*	Stop the annealing processing when the value is reached.
tries_per_temperature	`INTEGER`	*500*	Maximum number of times a neighbor(s) is searched in each temperature.
max_changes_per_temperature	`INTEGER`	*60*	Maximum number of times the solution is changed in each temperature.
max_consecutive_non_changes	`INTEGER`	*100*	Maximum number of consecutive times the solution is not changed in each temperature.
initial_temperature	`FLOAT`	*100*	Starting temperature.
final_temperature	`FLOAT`	*0.1*	Ending temperature.
cooling_factor	`FLOAT`	*0.9*	Value between between 0 and 1 (not including) used to calculate the next temperature.
randomize	`BOOLEAN`	*true*	Choose the random seed • true: Use current time as seed • false: Use *1* as seed. Using this value will get the same results with the same data in each execution.

Description of the return columns

Returns set of `(seq, node, cost, agg_cost)`

Column	Type	Description
seq	`INTEGER`	Row sequence.
node	`BIGINT`	Identifier of the node/coordinate/point.
cost	`FLOAT`	**Cost to traverse from the current `node` ito the n** • `0` for the last row in the path sequence.
agg_cost	`FLOAT`	**Aggregate cost from the `node` at `seq = 1` to th** • `0` for the first row in the path sequence.

See Also

References

- http://en.wikipedia.org/wiki/Traveling_salesman_problem
- http://en.wikipedia.org/wiki/Simulated_annealing

Indices and tables

- genindex
- search

5.1.7 Driving Distance - Category

- *pgr_drivingDistance* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsDD - Proposed* - Driving Distance based on pgr_withPoints
- Post pocessing
  - *pgr_alphaShape* - Alpha shape computation
  - *pgr_pointsAsPolygon* - Polygon around set of points

pgr_alphaShape

Name

`pgr_alphaShape` — Core function for alpha shape computation.

Synopsis

Returns a table with (x, y) rows that describe the vertices of an alpha shape.

```
table() pgr_alphaShape(text sql [, float8 alpha]);
```

Description

sql `text` a SQL query, which should return a set of rows with the following columns:

```
SELECT id, x, y FROM vertex_table
```

id `int4` identifier of the vertex

x `float8` x-coordinate

y `float8` y-coordinate

alpha (optional) `float8` alpha value. If specified alpha value equals 0 (default), then optimal alpha value is used. For more information, see CGAL - 2D Alpha Shapes[15].

Returns a vertex record for each row:

x x-coordinate

y y-coordinate

If a result includes multiple outer/inner rings, return those with separator row (x=NULL and y=NULL).

History

- Renamed in version 2.0.0
- Added alpha argument with default 0 (use optimal value) in version 2.1.0
- Supported to return multiple outer/inner ring coordinates with separator row (x=NULL and y=NULL) in version 2.1.0

Examples

PgRouting's alpha shape implementation has no way to control the order of the output points, so the actual output might different for the same input data. The first query, has the output ordered, he second query shows an example usage:

Example: the (ordered) results

```
SELECT * FROM pgr_alphaShape(
    'SELECT id::integer, ST_X(the_geom)::float AS x, ST_Y(the_geom)::float AS y
    FROM edge_table_vertices_pgr') ORDER BY x, y;
  x  |  y
-----+-----
   0 |   2
 0.5 | 3.5
   2 |   0
   2 |   4
 3.5 |   4
   4 |   1
   4 |   2
   4 |   3
(8 rows)
```

[15] http://doc.cgal.org/latest/Alpha_shapes_2/group__PkgAlphaShape2.html

Example: calculating the area

Steps:

- Calculates the alpha shape - the `ORDER BY` clause is not used.
- constructs a polygon
- and computes the area

```
SELECT round(ST_Area(ST_MakePolygon(ST_AddPoint(foo.openline, ST_StartPoint(foo.openline))))::nume
FROM (
    SELECT ST_MakeLine(points ORDER BY id) AS openline
    FROM (
        SELECT ST_MakePoint(x, y) AS points, row_number() over() AS id
        FROM pgr_alphaShape(
            'SELECT id::integer, ST_X(the_geom)::float AS x, ST_Y(the_geom)::float AS y
            FROM edge_table_vertices_pgr')
        ) AS a
   ) AS foo;
 st_area
---------
   11.75
(1 row)
```

The queries use the *Sample Data* network.

See Also

- *pgr_drivingDistance* - Driving Distance
- *pgr_pointsAsPolygon* - Polygon around set of points

pgr_pointsAsPolygon

Name

`pgr_pointsAsPolygon` — Draws an alpha shape around given set of points.

Synopsis

Returns the alpha shape as (multi)polygon geometry.

```
geometry pgr_pointsAsPolygon(text sql [, float8 alpha]);
```

Description

sql `text` a SQL query, which should return a set of rows with the following columns:

```
        SELECT id, x, y FROM vertex_result;
```

id `int4` identifier of the vertex

x `float8` x-coordinate

y `float8` y-coordinate

alpha (optional) `float8` alpha value. If specified alpha value equals 0 (default), then optimal alpha value is used. For more information, see CGAL - 2D Alpha Shapes[16].

Returns a (multi)polygon geometry (with holes).

History

- Renamed in version 2.0.0
- Added alpha argument with default 0 (use optimal value) in version 2.1.0
- Supported to return a (multi)polygon geometry (with holes) in version 2.1.0

Examples

In the following query there is no way to control which point in the polygon is the first in the list, so you may get similar but different results than the following which are also correct.

```
SELECT ST_AsText(pgr_pointsAsPolygon('SELECT id::integer, ST_X(the_geom)::float AS x, ST_Y(the_ge
        FROM edge_table_vertices_pgr'));
                      st_astext
------------------------------------------------------
 POLYGON((2 4,3.5 4,4 3,4 2,4 1,2 0,0 2,0.5 3.5,2 4))
(1 row)
```

The query use the *Sample Data* network.

See Also

- *pgr_drivingDistance* - Driving Distance
- *pgr_alphaShape* - Alpha shape computation

All Pairs - Family of Functions

- *pgr_floydWarshall* - Floyd-Warshall's Algorithm
- *pgr_johnson*- Johnson's Algorithm

pgr_aStar - Shortest Path A*

pgr_bdAstar - Bi-directional A* Shortest Path

pgr_bdDijkstra - Bi-directional Dijkstra Shortest Path

Dijkstra - Family of functions

- *pgr_dijkstra* - Dijkstra's algorithm for the shortest paths.
- *pgr_dijkstraCost* - Get the aggregate cost of the shortest paths.
- *pgr_dijkstraCostMatrix - proposed* - Use pgr_dijkstra to create a costs matrix.
- *pgr_drivingDistance* - Use pgr_dijkstra to calculate catchament information.
- *pgr_KSP* - Use Yen algorithm with pgr_dijkstra to get the K shortest paths.
- *pgr_dijkstraVia - Proposed* - Get a route of a seuence of vertices.

pgr_KSP - K-Shortest Path

pgr_trsp - Turn Restriction Shortest Path (TRSP)

Traveling Sales Person - Family of functions

[16]http://doc.cgal.org/latest/Alpha_shapes_2/group__PkgAlphaShape2.html

- *pgr_TSP* - When input is given as matrix cell information.
- *pgr_eucledianTSP* - When input are coordinates.

Driving Distance - Category

- *pgr_drivingDistance* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsDD - Proposed* - Driving Distance based on pgr_withPoints
- Post pocessing
  - *pgr_alphaShape* - Alpha shape computation
  - *pgr_pointsAsPolygon* - Polygon around set of points

CHAPTER 6

Available Functions but not official pgRouting functions

- *Stable proposed Functions*
- *Experimental and Proposed functions*

6.1 Stable proposed Functions

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

As part of the *Dijkstra - Family of functions*

- *pgr_dijkstraCostMatrix - proposed* Use pgr_dijkstra to calculate a cost matrix.
- *pgr_dijkstraVia - Proposed* - Use pgr_dijkstra to make a route via vertices.

Families

aStar - Family of functions

- *pgr_aStar* - A* algorithm for the shortest path.
- *pgr_aStarCost – proposed* - Get the aggregate cost of the shortest paths.
- *pgr_aStarCostMatrix - proposed* - Get the cost matrix of the shortest paths.

Bidirectional A - Family of functions*

- *pgr_bdAstar* - Bidirectional A* algorithm for obtaining paths.
- *pgr_bdAstarCost - Proposed* - Bidirectional A* algorithm to calculate the cost of the paths.
- *pgr_bdAstarCostMatrix - proposed* - Bidirectional A* algorithm to calculate a cost matrix of paths.

Bidirectional Dijkstra - Family of functions

- *pgr_bdDijkstra* - Bidirectional Dijkstra algorithm for the shortest paths.
- *pgr_bdDijkstraCost - Proposed* - Bidirectional Dijkstra to calculate the cost of the shortest paths

- *pgr_bdDijkstraCostMatrix - proposed* - Bidirectional Dijkstra algorithm to create a matrix of costs of the shortest paths.

Flow - Family of functions

- *pgr_maxFlow - Proposed* - Only the Max flow calculation using Push and Relabel algorithm.
- *pgr_boykovKolmogorov - Proposed* - Boykov and Kolmogorov with details of flow on edges.
- *pgr_edmondsKarp - Proposed* - Edmonds and Karp algorithm with details of flow on edges.
- *pgr_pushRelabel - Proposed* - Push and relabel algorithm with details of flow on edges.
- Applications
  - *pgr_edgeDisjointPaths - Proposed* - Calculates edge disjoint paths between two groups of vertices.
  - *pgr_maxCardinalityMatch - Proposed* - Calculates a maximum cardinality matching in a graph.

withPoints - Family of functions

- *pgr_withPoints - Proposed* - Route from/to points anywhere on the graph.
- *pgr_withPointsCost - Proposed* - Costs of the shortest paths.
- *pgr_withPointsCostMatrix - proposed* - Costs of the shortest paths.
- *pgr_withPointsKSP - Proposed* - K shortest paths.
- *pgr_withPointsDD - Proposed* - Driving distance.

categories

Cost - Category

- *pgr_aStarCost – proposed*
- *pgr_bdAstarCost - Proposed*
- *pgr_bdDijkstraCost - Proposed*
- *pgr_dijkstraCost*
- *pgr_withPointsCost - Proposed*

Cost Matrix - Category

- *pgr_aStarCostMatrix - proposed*
- *pgr_bdAstarCostMatrix - proposed*
- *pgr_bdDijkstraCostMatrix - proposed*
- *pgr_dijkstraCostMatrix - proposed*
- *pgr_withPointsCostMatrix - proposed*

KSP Category

- *pgr_KSP* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsKSP - Proposed* - Driving Distance based on pgr_dijkstra

6.1.1 aStar - Family of functions

The A* (pronounced "A Star") algorithm is based on Dijkstra's algorithm with a heuristic that allow it to solve most shortest path problems by evaluation only a sub-set of the overall graph.

- *pgr_aStar* - A* algorithm for the shortest path.
- *pgr_aStarCost – proposed* - Get the aggregate cost of the shortest paths.

- *pgr_aStarCostMatrix - proposed* - Get the cost matrix of the shortest paths.

pgr_aStar

Name

`pgr_aStar` — Returns the shortest path using A* algorithm.

[1]

Fig. 6.1: Boost Graph Inside

Availability:

- pgr_astar(one to one) 2.0.0, Signature changed 2.3.0
- pgr_astar(other signatures) 2.4.0

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Vertices of the graph are:
  - **positive** when it belongs to the edges_sql
- Values are returned when there is a path.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The agg_cost the non included values (v, v) is 0
  - When the starting vertex and ending vertex are the different and there is no path:
    - The agg_cost the non included values (u, v) is ∞
- When (x,y) coordinates for the same vertex identifier differ:
  - A random selection of the vertex's (x,y) coordinates is used.
- Running time: $O((E + V) * \log V)$

Signature Summary

```
pgr_aStar(edges_sql, start_vid, end_vid)
pgr_aStar(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
```

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

```
pgr_aStar(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- proposed
pgr_aStar(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- proposed
pgr_aStar(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- proposed
RETURNS SET OF (seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)
  OR EMPTY SET
```

Signatures

Minimal Signature

```
pgr_aStar(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example Using the defaults

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, 12);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    4 |    1 |        0
   2 |        2 |    5 |   10 |    1 |        1
   3 |        3 |   10 |   12 |    1 |        2
   4 |        4 |   11 |   13 |    1 |        3
   5 |        5 |   12 |   -1 |    0 |        4
(5 rows)
```

One to One

```
pgr_aStar(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example Undirected using Heuristic 2

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, 12,
    directed := false, heuristic := 2);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |    3 |    1 |        1
   3 |        3 |    4 |   16 |    1 |        2
   4 |        4 |    9 |   15 |    1 |        3
   5 |        5 |   12 |   -1 |    0 |        4
(5 rows)
```

One to many

```
pgr_aStar(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_astar* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_astar*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, ARRAY[3, 12], heuristic := 2);
 seq | path_seq | end_vid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |       3 |    2 |    4 |    1 |        0
   2 |        2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |       3 |    6 |    9 |    1 |        2
   4 |        4 |       3 |    9 |   16 |    1 |        3
   5 |        5 |       3 |    4 |    3 |    1 |        4
   6 |        6 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |      12 |    2 |    4 |    1 |        0
   8 |        2 |      12 |    5 |   10 |    1 |        1
   9 |        3 |      12 |   10 |   12 |    1 |        2
  10 |        4 |      12 |   11 |   13 |    1 |        3
  11 |        5 |      12 |   12 |   -1 |    0 |        4
(11 rows)
```

Many to One

```
pgr_aStar(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- Proposed
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_aStar* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_aStar*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    ARRAY[7, 2], 12, heuristic := 0);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |   10 |    1 |        1
   3 |        3 |         2 |   10 |   12 |    1 |        2
   4 |        4 |         2 |   11 |   13 |    1 |        3
   5 |        5 |         2 |   12 |   -1 |    0 |        4
```

```
   6 |        1 |         7 |    7 |    6 |    1 |        0
   7 |        2 |         7 |    8 |    7 |    1 |        1
   8 |        3 |         7 |    5 |   10 |    1 |        2
   9 |        4 |         7 |   10 |   12 |    1 |        3
  10 |        5 |         7 |   11 |   13 |    1 |        4
  11 |        6 |         7 |   12 |   -1 |    0 |        5
(11 rows)
```

Many to Many

```
pgr_aStar(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_astar(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    ARRAY[7, 2], ARRAY[3, 12], heuristic := 2);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         7 |       3 |    7 |    6 |    1 |        0
   8 |        2 |         7 |       3 |    8 |    7 |    1 |        1
   9 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  10 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  11 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  12 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  13 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
  14 |        1 |         2 |      12 |    2 |    4 |    1 |        0
  15 |        2 |         2 |      12 |    5 |   10 |    1 |        1
  16 |        3 |         2 |      12 |   10 |   12 |    1 |        2
  17 |        4 |         2 |      12 |   11 |   13 |    1 |        3
  18 |        5 |         2 |      12 |   12 |   -1 |    0 |        4
  19 |        1 |         7 |      12 |    7 |    6 |    1 |        0
  20 |        2 |         7 |      12 |    8 |    7 |    1 |        1
  21 |        3 |         7 |      12 |    5 |   10 |    1 |        2
  22 |        4 |         7 |      12 |   10 |   12 |    1 |        3
  23 |        5 |         7 |      12 |   11 |   13 |    1 |        4
  24 |        6 |         7 |      12 |   12 |   -1 |    0 |        5
(24 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
start_vid	`ANY-INTEGER`
end_vid	`ANY-INTEGER`
directed	`BOOLEAN`
heuristic	`INTEGER`
factor	`FLOAT`
epsilon	`FLOAT`

Description of the return values for a path Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- *aStar - Family of functions*
- *Sample Data*
- http://www.boost.org/libs/graph/doc/astar_search.html

- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

pgr_aStarCost – proposed

Name

`pgr_aStarCost` — Returns the aggregate cost shortest path using *aStar - Family of functions* algorithm.

[2]

Fig. 6.2: Boost Graph Inside

Availability: 2.4.0

Signature Summary

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

```
pgr_aStarCost(edges_sql, start_vid, end_vid) -- Proposed
pgr_aStarCost(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon) -- Proposed
pgr_aStarCost(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed
pgr_aStarCost(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- Proposed
pgr_aStarCost(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed

RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

Signatures

Minimal Signature

```
pgr_aStarCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

Example Using the defaults

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, 12);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |      12 |        4
(1 row)
```

One to One

```
pgr_aStarCost(edges_sql, start_vid, end_vid, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

Example Setting a Heuristic

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, 12,
    directed := false, heuristic := 2);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |      12 |        4
(1 row)
```

One to many

```
pgr_aStarCost(edges_sql, start_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

This signature finds a path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_astar* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_astar*.
- The extra `end_vid` column in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    2, ARRAY[3, 12], heuristic := 2);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      12 |        4
(2 rows)
```

Many to One

```
pgr_aStarCost(edges_sql, starts_vid, end_vid, directed, heuristic, factor, epsilon) -- Proposed
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_aStar* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_aStar*.
- The extra `start_vid` column in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    ARRAY[7, 2], 12, heuristic := 0);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |      12 |        4
         7 |      12 |        5
(2 rows)
```

Many to Many

```
pgr_aStarCost(edges_sql, starts_vid, end_vids, directed, heuristic, factor, epsilon) -- Proposed
RETURNS SET OF (start_vid, end_vid, agg_cost) OR EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_aStarCost(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    ARRAY[7, 2], ARRAY[3, 12], heuristic := 2);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      12 |        4
         7 |       3 |        6
         7 |      12 |        5
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	ANY-NUMERICAL		X coordinate of *source* vertex.
y1	ANY-NUMERICAL		Y coordinate of *source* vertex.
x2	ANY-NUMERICAL		X coordinate of *target* vertex.
y2	ANY-NUMERICAL		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type
edges_sql	TEXT
start_vid	ANY-INTEGER
end_vid	ANY-INTEGER
directed	BOOLEAN
heuristic	INTEGER
factor	FLOAT
epsilon	FLOAT

Description of the return values for a Cost function Returns set of (start_vid, end_vid, agg_cost)

Column	Type	Description
start_vid	BIGINT	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	BIGINT	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	FLOAT	Aggregate cost from start_vid to end_vid.

See Also

- *aStar - Family of functions*.
- *Sample Data* network.
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

pgr_aStarCostMatrix - proposed

Name

pgr_aStarCostMatrix - Calculates the a cost matrix using *pgr_aStar*.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[3]

Fig. 6.3: Boost Graph Inside

Availability: 2.4.0

Synopsis

Using aStar algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_aStarCostMatrix(edges_sql, vids)
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.

```
pgr_aStarCostMatrix(edges_sql, vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_aStarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       1 |        1
         3 |       1 |        2
         4 |       1 |        3
         1 |       2 |        1
         3 |       2 |        1
         4 |       2 |        2
         1 |       3 |        6
```

```
         2 |         3 |          5
         4 |         3 |          1
         1 |         4 |          5
         2 |         4 |          4
         3 |         4 |          3
(12 rows)
```

Complete Signature

```
pgr_aStarCostMatrix(edges_sql, vids, directed, heuristic, factor, epsilon)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_aStarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    directed := false, heuristic := 2
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |         1 |          1
         3 |         1 |          2
         4 |         1 |          3
         1 |         2 |          1
         3 |         2 |          1
         4 |         2 |          2
         1 |         3 |          2
         2 |         3 |          1
         4 |         3 |          1
         1 |         4 |          3
         2 |         4 |          2
         3 |         4 |          1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
vids	`ARRAY[ANY-INTEGER]`
directed	`BOOLEAN`
heuristic	`INTEGER`
factor	`FLOAT`
epsilon	`FLOAT`

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_aStarCostMatrix(
        'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        directed:= false, heuristic := 2
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
```

```
(5 rows)
```

See Also

- *aStar - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

The problem definition (Advanced documentation)

The A* (pronounced "A Star") algorithm is based on Dijkstra's algorithm with a heuristic, that is an estimation of the remaining cost from the vertex to the goal, that allows to solve most shortest path problems by evaluation only a sub-set of the overall graph. Running time: $O((E + V) * \log V)$

Heuristic

Currently the heuristic functions available are:

- 0: $h(v) = 0$ (Use this value to compare with pgr_dijkstra)
- 1: $h(v) = abs(max(\Delta x, \Delta y))$
- 2: $h(v) = abs(min(\Delta x, \Delta y))$
- 3: $h(v) = \Delta x * \Delta x + \Delta y * \Delta y$
- 4: $h(v) = sqrt(\Delta x * \Delta x + \Delta y * \Delta y)$
- 5: $h(v) = abs(\Delta x) + abs(\Delta y)$

where $\Delta x = x_1 - x_0$ and $\Delta y = y_1 - y_0$

Factor

Analysis 1

Working with cost/reverse_cost as length in degrees, x/y in lat/lon: Factor = 1 (no need to change units)

Analysis 2

Working with cost/reverse_cost as length in meters, x/y in lat/lon: Factor = would depend on the location of the points:

latitude	conversion	Factor
45	1 longitude degree is 78846.81 m	78846
0	1 longitude degree is 111319.46 m	111319

Analysis 3

Working with cost/reverse_cost as time in seconds, x/y in lat/lon: Factor: would depend on the location of the points and on the average speed say 25m/s is the speed.

latitude	conversion	Factor
45	1 longitude degree is (78846.81m)/(25m/s)	3153 s
0	1 longitude degree is (111319.46 m)/(25m/s)	4452 s

See Also

- *pgr_aStar*
- *pgr_aStarCost – proposed*
- *pgr_aStarCostMatrix - proposed*
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

6.1.2 Bidirectional A* - Family of functions

- *pgr_bdAstar* - Bidirectional A* algorithm for obtaining paths.
- *pgr_bdAstarCost - Proposed* - Bidirectional A* algorithm to calculate the cost of the paths.
- *pgr_bdAstarCostMatrix - proposed* - Bidirectional A* algorithm to calculate a cost matrix of paths.

pgr_bdAstarCost - Proposed

Name

`pgr_bdAstarCost` — Returns the shortest path using A* algorithm.

[4]

Fig. 6.4: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Signature Summary

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid)
pgr_bdAstarCost(edges_sql, start_vid, end_vid [, directed , heuristic, factor, epsilon])
pgr_bdAstarCost(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
pgr_bdAstarCost(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
pgr_bdAstarCost(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])

RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Using these signatures, will load once the graph and perform several one to one *pgr_bdAstarCost*

- The result is the union of the results of the one to one *pgr_bdAstarCost*.
- The extra `start_vid` and/or `end_vid` in the result is used to distinguish to which path it belongs.

Signatures

Minimal Signature

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the `start_vid` to the `end_vid`

- on a **directed** graph
- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example Using the defaults

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
```

```
         2 |       3 |        5
(1 row)
```

pgr_bdAstarCost One to One

```
pgr_bdAstarCost(edges_sql, start_vid, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the `start_vid` to each `end_vid` in `end_vids` allowing the user to choose

- if the graph is **directed** or **undirected**
- **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Note: In the One to One signature, because of the deprecated signature existence, it is compulsory to indicate if the graph is **directed** or **undirected**.

Example Directed using Heuristic 2

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, 3,
    true, heuristic := 2
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
(1 row)
```

pgr_bdAstarCost One to many

```
pgr_bdAstarCost(edges_sql, start_vid, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from the `start_vid` to each `end_vid` in `end_vids` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed using Heuristic 3 and a factor of 3.5

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    2, ARRAY[3, 11],
    heuristic := 3, factor := 3.5
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
```

```
(2 rows)
```

pgr_bdAstarCost Many to One

```
pgr_bdAstarCost(edges_sql, start_vids, end_vid [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from each `start_vid` in `start_vids` to the `end_vid` allowing the user to choose

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Undirected graph with Heuristic 4

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], 3,
    false, heuristic := 4
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        1
         7 |       3 |        4
(2 rows)
```

pgr_bdAstarCost Many to Many

```
pgr_bdAstarCost(edges_sql, start_vids, end_vids [, directed, heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

This usage finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids` allowing the use

- if the graph is **directed** or **undirected**
- and/or **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Directed graph with a factor of 0.5

```
SELECT * FROM pgr_bdAstarCost(
    'SELECT id, source, target, cost, reverse_cost, x1,y1,x2,y2
    FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11],
    factor := 0.5
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
         7 |       3 |        6
         7 |      11 |        4
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
start_vid	`ANY-INTEGER`
start_vids	`ARRAY[ANY-INTEGER]`
end_vid	`ANY-INTEGER`
end_vids	`ARRAY[ANY-INTEGER]`
directed	`BOOLEAN`
heuristic	`INTEGER`
factor	`FLOAT`
epsilon	`FLOAT`

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

See Also

- *Bidirectional A* - Family of functions*
- *Sample Data* network.
- Migration Guide[5]
- http://www.boost.org/libs/graph/doc/astar_search.html
- http://en.wikipedia.org/wiki/A*_search_algorithm

Indices and tables

- genindex
- search

[5] https://github.com/cvvergara/pgrouting/wiki/Migration-Guide#pgr_bdastar

pgr_bdAstarCostMatrix - proposed

Name

`pgr_bdAstarCostMatrix` - Calculates the a cost matrix using *pgr_bdAstar*.

[6]

Fig. 6.5: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Signatures

Minimal Signature

```
pgr_bdAstarCostMatrix(edges_sql, start_vids)
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each `start_vid` in `start_vids` to each `start_vid` in `start_vids`

- on a **directed** graph
- with **heuristic**'s value 5
- with **factor**'s value 1
- with **epsilon**'s value 1

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdAstarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        6
         1 |       4 |        5
         2 |       1 |        1
         2 |       3 |        5
         2 |       4 |        4
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        3
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Complete Signature

```
pgr_bdAstarCostMatrix(edges_sql, start_vids, [, directed , heuristic, factor, epsilon])
RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

This usage calculates the cost from the each `start_vid` in `start_vids` to each `start_vid` in `start_vids` allowing th

- if the graph is **directed** or **undirected**
- **heuristic**,
- and/or **factor**
- and/or **epsilon**.

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_bdAstarCostMatrix(
    'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        2
         1 |       4 |        3
         2 |       1 |        1
         2 |       3 |        1
         2 |       4 |        2
         3 |       1 |        2
         3 |       2 |        1
```

```
         3 |         4 |          1
         4 |         1 |          3
         4 |         2 |          2
         4 |         3 |          1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
cost	ANY-NUMERICAL		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	ANY-NUMERICAL	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	ANY-NUMERICAL		X coordinate of *source* vertex.
y1	ANY-NUMERICAL		Y coordinate of *source* vertex.
x2	ANY-NUMERICAL		X coordinate of *target* vertex.
y2	ANY-NUMERICAL		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
start_vid	`ANY-INTEGER`
start_vids	`ARRAY[ANY-INTEGER]`
end_vid	`ANY-INTEGER`
end_vids	`ARRAY[ANY-INTEGER]`
directed	`BOOLEAN`
heuristic	`INTEGER`
factor	`FLOAT`
epsilon	`FLOAT`

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdAstarCostMatrix(
        'SELECT id, source, target, cost, reverse_cost, x1, y1, x2, y2 FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
```

```
   4 |     4 |     3 |          3
   5 |     1 |     0 |          6
(5 rows)
```

See Also

- *Bidirectional A* - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

Synopsis

Based on A* algorithm, the bidirectional search finds a shortest path from a starting vertex (`start_vid`) to an ending vertex (`end_vid`). It runs two simultaneous searches: one forward from the `start_vid`, and one backward from the `end_vid`, stopping when the two meet in the middle. This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
- When the starting vertex and ending vertex are the same, there is no path.
  - The *agg_cost* the non included values *(v, v)* is *0*
- When the starting vertex and ending vertex are the different and there is no path:
  - The *agg_cost* the non included values *(u, v)* is ∞
- Running time (worse case scenario): $O((E + V) * \log V)$
- For large graphs where there is a path bewtween the starting vertex and ending vertex:
  - It is expected to terminate faster than pgr_astar

Description of the Signatures

Description of the edges_sql query for astar like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.
x1	`ANY-NUMERICAL`		X coordinate of *source* vertex.
y1	`ANY-NUMERICAL`		Y coordinate of *source* vertex.
x2	`ANY-NUMERICAL`		X coordinate of *target* vertex.
y2	`ANY-NUMERICAL`		Y coordinate of *target* vertex.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vid	`ANY-INTEGER`	Starting vertex identifier.
start_vids	`ARRAY[ANY-INTEGER]`	Starting vertices identifierers.
end_vid	`ANY-INTEGER`	Ending vertex identifier.
end_vids	`ARRAY[ANY-INTEGER]`	Ending vertices identifiers.
directed	`BOOLEAN`	• Optional. – When `false` the graph is considered as Undirected. – Default is `true` which considers the graph as Directed.
heuristic	`INTEGER`	(optional). Heuristic number. Current valid values 0~5. Default `5` • 0: h(v) = 0 (Use this value to compare with pgr_dijkstra) • 1: h(v) abs(max(dx, dy)) • 2: h(v) abs(min(dx, dy)) • 3: h(v) = dx * dx + dy * dy • 4: h(v) = sqrt(dx * dx + dy * dy) • 5: h(v) = abs(dx) + abs(dy)
factor	`FLOAT`	(optional). For units manipulation. $factor > 0$. Default `1`. see *Factor*
epsilon	`FLOAT`	(optional). For less restricted results. $epsilon >= 1$. Default `1`.

6.1.3 Bidirectional Dijkstra - Family of functions

- *pgr_bdDijkstra* - Bidirectional Dijkstra algorithm for the shortest paths.
- *pgr_bdDijkstraCost - Proposed* - Bidirectional Dijkstra to calculate the cost of the shortest paths
- *pgr_bdDijkstraCostMatrix - proposed* - Bidirectional Dijkstra algorithm to create a matrix of costs of the shortest paths.

pgr_bdDijkstraCost - Proposed

`pgr_bdDijkstraCost` — Returns the shortest path(s)'s cost using Bidirectional Dijkstra algorithm.

[7]

Fig. 6.6: Boost Graph Inside

Availability: 2.5.0

> **Warning:** Experimental functions
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

Signature Summary

```
pgr_dijkstraCost(edges_sql, start_vid,  end_vid)
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid, directed)
pgr_bdDijkstraCost(edges_sql, start_vid, end_vids, directed)
pgr_bdDijkstraCost(edges_sql, start_vids, end_vid, directed)
pgr_bdDijkstraCost(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (start_vid, end_vid, agg_cost)
OR EMPTY SET
```

Signatures

Minimal signature

```
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

The minimal signature is for a **directed** graph from one `start_vid` to one `end_vid`:

Example

```
SELECT * FROM pgr_bdDijkstraCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
(1 row)
```

pgr_bdDijkstraCost One to One

```
pgr_bdDijkstraCost(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.

- on an **undirected** graph when `directed` flag is set to `false`.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, 3,
    false
);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    2 |    2 |    1 |        0
   2 |        2 |    3 |   -1 |    0 |        1
(2 rows)
```

pgr_bdDijkstraCost One to many

```
pgr_bdDijkstra(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from one `start_vid` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform a one to one *pgr_dijkstra* where the starting vertex is fixed, and stop when all `end_vids` are reached.

- The result is equivalent to the union of the results of the one to one *pgr_dijkstra*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstraCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    2, ARRAY[3, 11]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         2 |       3 |        5
         2 |      11 |        3
(2 rows)
```

pgr_bdDijkstraCost Many to One

```
pgr_bdDijkstra(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to one *pgr_dijkstra* where the ending vertex is fixed.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], 3);
 seq | path_seq | start_vid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |         2 |    2 |    4 |    1 |        0
   2 |        2 |         2 |    5 |    8 |    1 |        1
   3 |        3 |         2 |    6 |    9 |    1 |        2
   4 |        4 |         2 |    9 |   16 |    1 |        3
   5 |        5 |         2 |    4 |    3 |    1 |        4
   6 |        6 |         2 |    3 |   -1 |    0 |        5
   7 |        1 |         7 |    7 |    6 |    1 |        0
   8 |        2 |         7 |    8 |    7 |    1 |        1
   9 |        3 |         7 |    5 |    8 |    1 |        2
  10 |        4 |         7 |    6 |    9 |    1 |        3
  11 |        5 |         7 |    9 |   16 |    1 |        4
  12 |        6 |         7 |    4 |    3 |    1 |        5
  13 |        7 |         7 |    3 |   -1 |    0 |        6
(13 rows)
```

pgr_bdDijkstraCost Many to Many

```
pgr_bdDijkstra(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost) or EMPTY SET
```

This signature finds the shortest path from each `start_vid` in `start_vids` to each `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

Using this signature, will load once the graph and perform several one to Many *pgr_dijkstra* for all `start_vids`.

- The result is the union of the results of the one to one *pgr_dijkstra*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

Example

```
SELECT * FROM pgr_bdDijkstra(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[2, 7], ARRAY[3, 11]);
 seq | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |         2 |       3 |    2 |    4 |    1 |        0
   2 |        2 |         2 |       3 |    5 |    8 |    1 |        1
   3 |        3 |         2 |       3 |    6 |    9 |    1 |        2
   4 |        4 |         2 |       3 |    9 |   16 |    1 |        3
   5 |        5 |         2 |       3 |    4 |    3 |    1 |        4
   6 |        6 |         2 |       3 |    3 |   -1 |    0 |        5
   7 |        1 |         2 |      11 |    2 |    4 |    1 |        0
   8 |        2 |         2 |      11 |    5 |    8 |    1 |        1
   9 |        3 |         2 |      11 |    6 |   11 |    1 |        2
  10 |        4 |         2 |      11 |   11 |   -1 |    0 |        3
  11 |        1 |         7 |       3 |    7 |    6 |    1 |        0
  12 |        2 |         7 |       3 |    8 |    7 |    1 |        1
  13 |        3 |         7 |       3 |    5 |    8 |    1 |        2
  14 |        4 |         7 |       3 |    6 |    9 |    1 |        3
  15 |        5 |         7 |       3 |    9 |   16 |    1 |        4
  16 |        6 |         7 |       3 |    4 |    3 |    1 |        5
  17 |        7 |         7 |       3 |    3 |   -1 |    0 |        6
```

```
  18 |          1 |          7 |        11 |     7 |     6 |     1 |          0
  19 |          2 |          7 |        11 |     8 |     7 |     1 |          1
  20 |          3 |          7 |        11 |     5 |    10 |     1 |          2
  21 |          4 |          7 |        11 |    10 |    12 |     1 |          3
  22 |          5 |          7 |        11 |    11 |    -1 |     0 |          4
(22 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default
sql	`TEXT`	
start_vid	`BIGINT`	
start_vids	`ARRAY[BIGINT]`	
end_vid	`BIGINT`	
end_vids	`ARRAY[BIGINT]`	
directed	`BOOLEAN`	`true`

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

See Also

- The queries use the *Sample Data* network.
- *pgr_bdDijkstra*
- http://www.cs.princeton.edu/courses/archive/spr06/cos423/Handouts/EPP%20shortest%20path%20algorithms.pdf
- https://en.wikipedia.org/wiki/Bidirectional_search

Indices and tables

- genindex
- search

pgr_bdDijkstraCostMatrix - proposed

Name

`pgr_bdDijkstraCostMatrix` - Calculates the a cost matrix using *pgr_bdDijkstra*.

[8]

Fig. 6.7: Boost Graph Inside

Availability: 2.5.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

Using Dijkstra algorithm, calculate and return a cost matrix.

Signature Summary

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids)
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for vertices 1, 2, 3, and 4.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5)
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        6
         1 |       4 |        5
         2 |       1 |        1
         2 |       3 |        5
         2 |       4 |        4
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        3
         4 |       1 |        3
```

```
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Complete Signature

```
pgr_bdDijkstraCostMatrix(edges_sql, start_vids, directed:=true)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example Cost matrix for an undirected graph for vertices 1, 2, 3, and 4.

This example returns a symmetric cost matrix.

```
SELECT * FROM pgr_bdDijkstraCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
    false
);
 start_vid | end_vid | agg_cost
-----------+---------+----------
         1 |       2 |        1
         1 |       3 |        2
         1 |       4 |        3
         2 |       1 |        1
         2 |       3 |        1
         2 |       4 |        2
         3 |       1 |        2
         3 |       2 |        1
         3 |       4 |        1
         4 |       1 |        3
         4 |       2 |        2
         4 |       3 |        1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Parameter	Type	Description
edges_sql	`TEXT`	Edges SQL query as described above.
start_vids	`ARRAY[ANY-INTEGER]`	Array of identifiers of the vertices.
directed	`BOOLEAN`	(optional). When `false` the graph is consider `true` which considers the graph as Directed.

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_bdDijkstraCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table',
        (SELECT array_agg(id) FROM edge_table_vertices_pgr WHERE id < 5),
        false
    )
```

```
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |    1 |    1 |        0
   2 |    2 |    1 |        1
   3 |    3 |    1 |        2
   4 |    4 |    3 |        3
   5 |    1 |    0 |        6
(5 rows)
```

See Also

- *Bidirectional Dijkstra - Family of functions*
- *Cost Matrix - Category*
- *Traveling Sales Person - Family of functions*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

Synopsis

Based on Dijkstra's algorithm, the bidirectional search finds a shortest path a starting vertex (`start_vid`) to an ending vertex (`end_vid`). It runs two simultaneous searches: one forward from the source, and one backward from the target, stopping when the two meet in the middle. This implementation can be used with a directed graph and an undirected graph.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- Values are returned when there is a path.
- When the starting vertex and ending vertex are the same, there is no path.
  - The *agg_cost* the non included values *(v, v)* is *0*
- When the starting vertex and ending vertex are the different and there is no path:
  - The *agg_cost* the non included values *(u, v)* is ∞
- Running time (worse case scenario): $O((V \log V + E))$
- For large graphs where there is a path bewtween the starting vertex and ending vertex:
  - It is expected to terminate faster than pgr_dijkstra

6.1.4 withPoints - Family of functions

When points are also given as input:

- *pgr_withPoints - Proposed* - Route from/to points anywhere on the graph.
- *pgr_withPointsCost - Proposed* - Costs of the shortest paths.
- *pgr_withPointsCostMatrix - proposed* - Costs of the shortest paths.
- *pgr_withPointsKSP - Proposed* - K shortest paths.
- *pgr_withPointsDD - Proposed* - Driving distance.

pgr_withPoints - Proposed

Name

`pgr_withPoints` - Returns the shortest path in a graph with additional temporary vertices.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[9]

Fig. 6.8: Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points defined by points_sql. Using Dijkstra algorithm, find the shortest path(s)

Characteristics:

The main Characteristics are:

- Process is done only on edges with positive costs.
- Vertices of the graph are:
  - **positive** when it belongs to the edges_sql
  - **negative** when it belongs to the points_sql
- Values are returned when there is a path.
  - When the starting vertex and ending vertex are the same, there is no path.

  - The agg_cost the non included values (v, v) is 0
  - When the starting vertex and ending vertex are the different and there is no path:
  - The agg_cost the non included values (u, v) is ∞
- For optimization purposes, any duplicated value in the start_vids or end_vids are ignored.
- The returned values are ordered:
  - start_vid ascending
  - end_vid ascending
- Running time: $O(|start\_vids|(V \log V + E))$

Signature Summary

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid)
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid, directed, driving_side, details)
pgr_withPoints(edges_sql, points_sql, start_vid, end_vids, directed, driving_side, details)
pgr_withPoints(edges_sql, points_sql, start_vids, end_vid, directed, driving_side, details)
pgr_withPoints(edges_sql, points_sql, start_vids, end_vids, directed, driving_side, details)
RETURNS SET OF (seq, path_seq, [start_vid,] [end_vid,] node, edge, cost, agg_cost)
```

Signatures

Minimal Use

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of points_sql query.

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example From point 1 to point 3

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -3);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   -1 |    1 |  0.6 |        0
   2 |        2 |    2 |    4 |    1 |      0.6
   3 |        3 |    5 |   10 |    1 |      1.6
   4 |        4 |   10 |   12 |  0.6 |      2.6
   5 |        5 |   -3 |   -1 |    0 |      3.2
(5 rows)
```

One to One

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vid,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, node, edge, cost, agg_cost)
```

Example From point 1 to vertex 3

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3,
    details := true);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |   -1 |    1 |  0.6 |        0
   2 |        2 |    2 |    4 |  0.7 |      0.6
   3 |        3 |   -6 |    4 |  0.3 |      1.3
   4 |        4 |    5 |    8 |    1 |      1.6
   5 |        5 |    6 |    9 |    1 |      2.6
   6 |        6 |    9 |   16 |    1 |      3.6
   7 |        7 |    4 |    3 |    1 |      4.6
   8 |        8 |    3 |   -1 |    0 |      5.6
(8 rows)
```

One to Many

```
pgr_withPoints(edges_sql, points_sql, start_vid, end_vids,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, end_vid, node, edge, cost, agg_cost)
```

Example From point 1 to point 3 and vertex 5

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, ARRAY[-3,5]);
 seq | path_seq | end_pid | node | edge | cost | agg_cost
-----+----------+---------+------+------+------+----------
   1 |        1 |      -3 |   -1 |    1 |  0.6 |        0
   2 |        2 |      -3 |    2 |    4 |    1 |      0.6
   3 |        3 |      -3 |    5 |   10 |    1 |      1.6
   4 |        4 |      -3 |   10 |   12 |  0.6 |      2.6
   5 |        5 |      -3 |   -3 |   -1 |    0 |      3.2
   6 |        1 |       5 |   -1 |    1 |  0.6 |        0
   7 |        2 |       5 |    2 |    4 |    1 |      0.6
   8 |        3 |       5 |    5 |   -1 |    0 |      1.6
(8 rows)
```

Many to One

```
pgr_withPoints(edges_sql, points_sql, start_vids, end_vid,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, start_vid, node, edge, cost, agg_cost)
```

Example From point 1 and vertex 2 to point 3

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], -3);
 seq | path_seq | start_pid | node | edge | cost | agg_cost
-----+----------+-----------+------+------+------+----------
   1 |        1 |        -1 |   -1 |    1 |  0.6 |        0
   2 |        2 |        -1 |    2 |    4 |    1 |      0.6
   3 |        3 |        -1 |    5 |   10 |    1 |      1.6
   4 |        4 |        -1 |   10 |   12 |  0.6 |      2.6
   5 |        5 |        -1 |   -3 |   -1 |    0 |      3.2
   6 |        1 |         2 |    2 |    4 |    1 |        0
```

```
   7 |        2 |         2 |    5 |   10 |    1 |        1
   8 |        3 |         2 |   10 |   12 |  0.6 |        2
   9 |        4 |         2 |   -3 |   -1 |    0 |      2.6
(9 rows)
```

Many to Many

```
pgr_withPoints(edges_sql, points_sql, start_vids, end_vids,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost)
```

Example From point 1 and vertex 2 to point 3 and vertex 7

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7]);
 seq | path_seq | start_pid | end_pid | node | edge | cost | agg_cost
-----+----------+-----------+---------+------+------+------+----------
   1 |        1 |        -1 |      -3 |   -1 |    1 |  0.6 |        0
   2 |        2 |        -1 |      -3 |    2 |    4 |    1 |      0.6
   3 |        3 |        -1 |      -3 |    5 |   10 |    1 |      1.6
   4 |        4 |        -1 |      -3 |   10 |   12 |  0.6 |      2.6
   5 |        5 |        -1 |      -3 |   -3 |   -1 |    0 |      3.2
   6 |        1 |        -1 |       7 |   -1 |    1 |  0.6 |        0
   7 |        2 |        -1 |       7 |    2 |    4 |    1 |      0.6
   8 |        3 |        -1 |       7 |    5 |    7 |    1 |      1.6
   9 |        4 |        -1 |       7 |    8 |    6 |    1 |      2.6
  10 |        5 |        -1 |       7 |    7 |   -1 |    0 |      3.6
  11 |        1 |         2 |      -3 |    2 |    4 |    1 |        0
  12 |        2 |         2 |      -3 |    5 |   10 |    1 |        1
  13 |        3 |         2 |      -3 |   10 |   12 |  0.6 |        2
  14 |        4 |         2 |      -3 |   -3 |   -1 |    0 |      2.6
  15 |        1 |         2 |       7 |    2 |    4 |    1 |        0
  16 |        2 |         2 |       7 |    5 |    7 |    1 |        1
  17 |        3 |         2 |       7 |    8 |    6 |    1 |        2
  18 |        4 |         2 |       7 |    7 |   -1 |    0 |        3
(18 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
points_sql	`TEXT`
start_vid	`ANY-INTEGER`
end_vid	`ANY-INTEGER`
start_vids	`ARRAY[ANY-INTEGER]`
end_vids	`ARRAY[ANY-INTEGER]`
directed	`BOOLEAN`
driving_side	`CHAR`
details	`BOOLEAN`

Description of the return values Returns set of `(seq, [path_seq,] [start_vid,] [end_vid,] node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INTEGER`	Row sequence.
path_seq	`INTEGER`	Path sequence that indicates the relative position on the path.
start_vid	`BIGINT`	Identifier of the starting vertex. When negative: is a point's pid.
end_vid	`BIGINT`	Identifier of the ending vertex. When negative: is a point's pid.
node	`BIGINT`	**Identifier of the node:** • A positive value indicates the node is a vertex of edges_sql. • A negative value indicates the node is a point of points_sql.
edge	`BIGINT`	**Identifier of the edge used to go from `node` to th** • `-1` for the last row in the path sequence.
cost	`FLOAT`	**Cost to traverse from `node` using `edge` to the ne** • `0` for the last row in the path sequence.
agg_cost	`FLOAT`	**Aggregate cost from `start_pid` to `node`.** • `0` for the first row in the path sequence.

Examples

Example Which path (if any) passes in front of point 6 or vertex 6 with **right** side driving topology.

```
SELECT ('(' || start_pid || ' => ' || end_pid ||') at ' || path_seq || 'th step:')::TEXT AS path_a
        CASE WHEN edge = -1 THEN ' visits'
            ELSE ' passes in front of'
        END as status,
        CASE WHEN node < 0 THEN 'Point'
            ELSE 'Vertex'
        END as is_a,
        abs(node) as id
    FROM pgr_withPoints(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
        'SELECT pid, edge_id, fraction, side from pointsOfInterest',
        ARRAY[1,-1], ARRAY[-2,-3,-6,3,6],
        driving_side := 'r',
        details := true)
    WHERE node IN (-6,6);
         path_at         |       status        |  is_a  | id
-------------------------+---------------------+--------+----
 (-1 => -6) at 4th step: |  visits             | Point  |  6
 (-1 => -3) at 4th step: |  passes in front of | Point  |  6
 (-1 => -2) at 4th step: |  passes in front of | Point  |  6
```

```
 (-1 => -2) at 6th step: |  passes in front of | Vertex |  6
 (-1 => 3) at 4th step:  |  passes in front of | Point  |  6
 (-1 => 3) at 6th step:  |  passes in front of | Vertex |  6
 (-1 => 6) at 4th step:  |  passes in front of | Point  |  6
 (-1 => 6) at 6th step:  |  visits             | Vertex |  6
 (1 => -6) at 3th step:  |  visits             | Point  |  6
 (1 => -3) at 3th step:  |  passes in front of | Point  |  6
 (1 => -2) at 3th step:  |  passes in front of | Point  |  6
 (1 => -2) at 5th step:  |  passes in front of | Vertex |  6
 (1 => 3) at 3th step:   |  passes in front of | Point  |  6
 (1 => 3) at 5th step:   |  passes in front of | Vertex |  6
 (1 => 6) at 3th step:   |  passes in front of | Point  |  6
 (1 => 6) at 5th step:   |  visits             | Vertex |  6
(16 rows)
```

Example Which path (if any) passes in front of point 6 or vertex 6 with **left** side driving topology.

```
SELECT ('(' || start_pid || ' => ' || end_pid ||') at ' || path_seq || 'th step:')::TEXT AS path_a
        CASE WHEN edge = -1 THEN ' visits'
            ELSE ' passes in front of'
        END as status,
        CASE WHEN node < 0 THEN 'Point'
            ELSE 'Vertex'
        END as is_a,
        abs(node) as id
    FROM pgr_withPoints(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
        'SELECT pid, edge_id, fraction, side from pointsOfInterest',
        ARRAY[1,-1], ARRAY[-2,-3,-6,3,6],
        driving_side := 'l',
        details := true)
    WHERE node IN (-6,6);
         path_at         |       status        |  is_a  | id
-------------------------+---------------------+--------+----
 (-1 => -6) at 3th step: |  visits             | Point  |  6
 (-1 => -3) at 3th step: |  passes in front of | Point  |  6
 (-1 => -2) at 3th step: |  passes in front of | Point  |  6
 (-1 => -2) at 5th step: |  passes in front of | Vertex |  6
 (-1 => 3) at 3th step:  |  passes in front of | Point  |  6
 (-1 => 3) at 5th step:  |  passes in front of | Vertex |  6
 (-1 => 6) at 3th step:  |  passes in front of | Point  |  6
 (-1 => 6) at 5th step:  |  visits             | Vertex |  6
 (1 => -6) at 4th step:  |  visits             | Point  |  6
 (1 => -3) at 4th step:  |  passes in front of | Point  |  6
 (1 => -2) at 4th step:  |  passes in front of | Point  |  6
 (1 => -2) at 6th step:  |  passes in front of | Vertex |  6
 (1 => 3) at 4th step:   |  passes in front of | Point  |  6
 (1 => 3) at 6th step:   |  passes in front of | Vertex |  6
 (1 => 6) at 4th step:   |  passes in front of | Point  |  6
 (1 => 6) at 6th step:   |  visits             | Vertex |  6
(16 rows)
```

Example Many to many example with a twist: on undirected graph and showing details.

```
SELECT * FROM pgr_withPoints(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    directed := false,
    details := true);
 seq | path_seq | start_pid | end_pid | node | edge | cost | agg_cost
```

```
-----+-----------+------------+----------+-------+-------+-------+----------
   1 |         1 |         -1 |       -3 |    -1 |     1 |   0.6 |         0
   2 |         2 |         -1 |       -3 |     2 |     4 |   0.7 |       0.6
   3 |         3 |         -1 |       -3 |    -6 |     4 |   0.3 |       1.3
   4 |         4 |         -1 |       -3 |     5 |    10 |     1 |       1.6
   5 |         5 |         -1 |       -3 |    10 |    12 |   0.6 |       2.6
   6 |         6 |         -1 |       -3 |    -3 |    -1 |     0 |       3.2
   7 |         1 |         -1 |        7 |    -1 |     1 |   0.6 |         0
   8 |         2 |         -1 |        7 |     2 |     4 |   0.7 |       0.6
   9 |         3 |         -1 |        7 |    -6 |     4 |   0.3 |       1.3
  10 |         4 |         -1 |        7 |     5 |     7 |     1 |       1.6
  11 |         5 |         -1 |        7 |     8 |     6 |   0.7 |       2.6
  12 |         6 |         -1 |        7 |    -4 |     6 |   0.3 |       3.3
  13 |         7 |         -1 |        7 |     7 |    -1 |     0 |       3.6
  14 |         1 |          2 |       -3 |     2 |     4 |   0.7 |         0
  15 |         2 |          2 |       -3 |    -6 |     4 |   0.3 |       0.7
  16 |         3 |          2 |       -3 |     5 |    10 |     1 |         1
  17 |         4 |          2 |       -3 |    10 |    12 |   0.6 |         2
  18 |         5 |          2 |       -3 |    -3 |    -1 |     0 |       2.6
  19 |         1 |          2 |        7 |     2 |     4 |   0.7 |         0
  20 |         2 |          2 |        7 |    -6 |     4 |   0.3 |       0.7
  21 |         3 |          2 |        7 |     5 |     7 |     1 |         1
  22 |         4 |          2 |        7 |     8 |     6 |   0.7 |         2
  23 |         5 |          2 |        7 |    -4 |     6 |   0.3 |       2.7
  24 |         6 |          2 |        7 |     7 |    -1 |     0 |         3
(24 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *withPoints - Family of functions*

Indices and tables

- genindex
- search

pgr_withPointsCost - Proposed

Name

`pgr_withPointsCost` - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[10]

Fig. 6.9: Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points defined by points_sql. Using Dijkstra algorithm, return only the aggregate cost of the shortest path(s) found.

Characteristics:

The main Characteristics are:

- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of vertices in the modified graph.
- Vertices of the graph are:
  - **positive** when it belongs to the edges_sql
  - **negative** when it belongs to the points_sql
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* in the non included values *(v, v)* is *0*
  - When the starting vertex and ending vertex are the different and there is no path.
    - The *agg_cost* in the non included values *(u, v)* is ∞
- If the values returned are stored in a table, the unique index would be the pair: *(start_vid, end_vid)*.
- For undirected graphs, the results are symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- For optimization purposes, any duplicated value in the *start_vids* or *end_vids* is ignored.
- The returned values are ordered:
  - *start_vid* ascending

  – *end_vid* ascending

- Running time: $O(|start\_vids| * (V \log V + E))$

Signature Summary

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid, directed, driving_side)
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vids, directed, driving_side)
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vid, directed, driving_side)
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vids, directed, driving_side)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Note: There is no **details** flag, unlike the other members of the withPoints family of functions.

Signatures

Minimal Use

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -3);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
(1 row)
```

One to One

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vid,
    directed:=true, driving_side:='b')
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3,
    directed := false);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |       3 |      1.6
(1 row)
```

One to Many

```
pgr_withPointsCost(edges_sql, points_sql, start_vid, end_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, ARRAY[-3,5]);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       5 |      1.6
(2 rows)
```

Many to One

```
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vid,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], -3);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
         2 |      -3 |      2.6
(2 rows)
```

Many to Many

```
pgr_withPointsCost(edges_sql, points_sql, start_vids, end_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7]);
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       7 |      3.6
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
points_sql	`TEXT`
start_vid	`ANY-INTEGER`
end_vid	`ANY-INTEGER`
start_vids	`ARRAY[ANY-INTEGER]`
end_vids	`ARRAY[ANY-INTEGER]`
directed	`BOOLEAN`
driving_side	`CHAR`

Description of the return values Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. When negative: is a point's pid.
end_vid	`BIGINT`	Identifier of the ending point. When negative: is a point's pid.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example With **right** side driving topology.

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'l');
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       7 |      3.6
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

Example With **left** side driving topology.

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
```

```
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'r');
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |        4
        -1 |       7 |      4.4
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

Example Does not matter driving side.

```
SELECT * FROM pgr_withPointsCost(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    ARRAY[-1,2], ARRAY[-3,7],
    driving_side := 'b');
 start_pid | end_pid | agg_cost
-----------+---------+----------
        -1 |      -3 |      3.2
        -1 |       7 |      3.6
         2 |      -3 |      2.6
         2 |       7 |        3
(4 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *withPoints - Family of functions*

Indices and tables

- genindex
- search

pgr_withPointsCostMatrix - proposed

Name

`pgr_withPointsCostMatrix` - Calculates the shortest path and returns only the aggregate cost of the shortest path(s) found, for the combination of points given.

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

[11]

Fig. 6.10: Boost Graph Inside

Availability: 2.2.0

Signature Summary

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids)
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids, directed, driving_side)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Note: There is no **details** flag, unlike the other members of the withPoints family of functions.

Signatures

Minimal Signature

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vid)
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction from pointsOfInterest',
    array[-1, 3, 6, -6]);
 start_vid | end_vid | agg_cost
-----------+---------+----------
        -6 |      -1 |      1.3
        -6 |       3 |      4.3
        -6 |       6 |      1.3
        -1 |      -6 |      1.3
        -1 |       3 |      5.6
        -1 |       6 |      2.6
         3 |      -6 |      1.7
         3 |      -1 |      1.6
```

```
         3 |        6 |         1
         6 |       -6 |       1.3
         6 |       -1 |       2.6
         6 |        3 |         3
(12 rows)
```

Complete Signature

```
pgr_withPointsCostMatrix(edges_sql, points_sql, start_vids,
    directed:=true, driving_side:='b')
RETURNS SET OF (start_vid, end_vid, agg_cost)
```

Example returning a symmetrical cost matrix

- Using the default **side** value on the **points_sql** query
- Using an undirected graph
- Using the default **driving_side** value

```
SELECT * FROM pgr_withPointsCostMatrix(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction from pointsOfInterest',
    array[-1, 3, 6, -6], directed := false);
 start_vid | end_vid | agg_cost
-----------+---------+----------
        -6 |       -1 |       1.3
        -6 |        3 |       1.7
        -6 |        6 |       1.3
        -1 |       -6 |       1.3
        -1 |        3 |       1.6
        -1 |        6 |       2.6
         3 |       -6 |       1.7
         3 |       -1 |       1.6
         3 |        6 |         1
         6 |       -6 |       1.3
         6 |       -1 |       2.6
         6 |        3 |         1
(12 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
points_sql	`TEXT`
start_vids	`ARRAY[ANY-INTEGER]`
directed	`BOOLEAN`
driving_side	`CHAR`

Description of the return values for a Cost function Returns set of `(start_vid, end_vid, agg_cost)`

Column	Type	Description
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
agg_cost	`FLOAT`	Aggregate cost from `start_vid` to `end_vid`.

Examples

Example Use with tsp

```
SELECT * FROM pgr_TSP(
    $$
    SELECT * FROM pgr_withPointsCostMatrix(
        'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
        'SELECT pid, edge_id, fraction from pointsOfInterest',
        array[-1, 3, 6, -6], directed := false);
    $$,
    randomize := false
);
 seq | node | cost | agg_cost
-----+------+------+----------
   1 |   -6 |  1.3 |        0
   2 |   -1 |  1.6 |      1.3
   3 |    3 |    1 |      2.9
   4 |    6 |  1.3 |      3.9
   5 |   -6 |    0 |      5.2
(5 rows)
```

See Also

- *withPoints - Family of functions*
- *Cost Matrix - Category*

- *Traveling Sales Person - Family of functions*
- *sampledata* network.

Indices and tables

- genindex
- search

pgr_withPointsKSP - Proposed

Name

`pgr_withPointsKSP` - Find the K shortest paths using Yen's algorithm.

> **Warning:** Proposed functions for next mayor release.
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

[12]

Fig. 6.11: Boost Graph Inside

Availability: 2.2.0

Synopsis

Modifies the graph to include the points defined in the `points_sql` and using Yen algorithm, finds the K shortest paths.

Signature Summary

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K)
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K, directed, heap_paths, driving_sid
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Signatures

Minimal Usage

The minimal usage:

- Is for a **directed** graph.

- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of the query.
- No **heap paths** are returned.

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -2, 2);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.6 |        0
   2 |       1 |        2 |    2 |    4 |    1 |      0.6
   3 |       1 |        3 |    5 |    8 |    1 |      1.6
   4 |       1 |        4 |    6 |    9 |    1 |      2.6
   5 |       1 |        5 |    9 |   15 |  0.4 |      3.6
   6 |       1 |        6 |   -2 |   -1 |    0 |        4
   7 |       2 |        1 |   -1 |    1 |  0.6 |        0
   8 |       2 |        2 |    2 |    4 |    1 |      0.6
   9 |       2 |        3 |    5 |    8 |    1 |      1.6
  10 |       2 |        4 |    6 |   11 |    1 |      2.6
  11 |       2 |        5 |   11 |   13 |    1 |      3.6
  12 |       2 |        6 |   12 |   15 |  0.6 |      4.6
  13 |       2 |        7 |   -2 |   -1 |    0 |      5.2
(13 rows)
```

Complete Signature Finds the K shortest paths depending on the optional parameters setup.

```
pgr_withPointsKSP(edges_sql, points_sql, start_pid, end_pid, K,
    directed:=true, heap_paths:=false, driving_side:='b', details:=false)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
```

Example With details.

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 6, 2, details := true);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.6 |        0
   2 |       1 |        2 |    2 |    4 |  0.7 |      0.6
   3 |       1 |        3 |   -6 |    4 |  0.3 |      1.3
   4 |       1 |        4 |    5 |    8 |    1 |      1.6
   5 |       1 |        5 |    6 |   -1 |    0 |      2.6
   6 |       2 |        1 |   -1 |    1 |  0.6 |        0
   7 |       2 |        2 |    2 |    4 |  0.7 |      0.6
   8 |       2 |        3 |   -6 |    4 |  0.3 |      1.3
   9 |       2 |        4 |    5 |   10 |    1 |      1.6
  10 |       2 |        5 |   10 |   12 |  0.6 |      2.6
  11 |       2 |        6 |   -3 |   12 |  0.4 |      3.2
  12 |       2 |        7 |   11 |   13 |    1 |      3.6
  13 |       2 |        8 |   12 |   15 |  0.6 |      4.6
  14 |       2 |        9 |   -2 |   15 |  0.4 |      5.2
  15 |       2 |       10 |    9 |    9 |    1 |      5.6
  16 |       2 |       11 |    6 |   -1 |    0 |      6.6
```

```
(16 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
points_sql	`TEXT`
start_pid	`ANY-INTEGER`
end_pid	`ANY-INTEGER`
K	`INTEGER`
directed	`BOOLEAN`
heap_paths	`BOOLEAN`
driving_side	`CHAR`
details	`BOOLEAN`

Description of the return values Returns set of `(seq, path_id, path_seq, node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INTEGER`	Row sequence.
path_seq	`INTEGER`	Relative position in the path of node and edge. Has value 1 for the beginning of a path.
path_id	`INTEGER`	Path identifier. The ordering of the paths: For two paths i, j if i < j then agg_cost(i) <= agg_cost(j).
node	`BIGINT`	Identifier of the node in the path. Negative values are the identifiers of a point.
edge	`BIGINT`	**Identifier of the edge used to go from `node` to th** • `-1` for the last row in the path sequence.
cost	`FLOAT`	**Cost to traverse from `node` using edge to the ne** • `0` for the last row in the path sequence.
agg_cost	`FLOAT`	**Aggregate cost from `start_pid` to `node`.** • `0` for the first row in the path sequence.

Examples

Example Left side driving topology with details.

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -2, 2,
    driving_side := 'l', details := true);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.6 |        0
   2 |       1 |        2 |    2 |    4 |  0.7 |      0.6
   3 |       1 |        3 |   -6 |    4 |  0.3 |      1.3
   4 |       1 |        4 |    5 |    8 |    1 |      1.6
   5 |       1 |        5 |    6 |    9 |    1 |      2.6
   6 |       1 |        6 |    9 |   15 |    1 |      3.6
   7 |       1 |        7 |   12 |   15 |  0.6 |      4.6
   8 |       1 |        8 |   -2 |   -1 |    0 |      5.2
   9 |       2 |        1 |   -1 |    1 |  0.6 |        0
  10 |       2 |        2 |    2 |    4 |  0.7 |      0.6
  11 |       2 |        3 |   -6 |    4 |  0.3 |      1.3
  12 |       2 |        4 |    5 |    8 |    1 |      1.6
  13 |       2 |        5 |    6 |   11 |    1 |      2.6
  14 |       2 |        6 |   11 |   13 |    1 |      3.6
  15 |       2 |        7 |   12 |   15 |  0.6 |      4.6
  16 |       2 |        8 |   -2 |   -1 |    0 |      5.2
(16 rows)
```

Example Right side driving topology with heap paths and details.

```
SELECT * FROM pgr_withPointsKSP(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, -2, 2,
    heap_paths := true, driving_side := 'r', details := true);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |   -1 |    1 |  0.4 |        0
   2 |       1 |        2 |    1 |    1 |    1 |      0.4
   3 |       1 |        3 |    2 |    4 |  0.7 |      1.4
   4 |       1 |        4 |   -6 |    4 |  0.3 |      2.1
   5 |       1 |        5 |    5 |    8 |    1 |      2.4
   6 |       1 |        6 |    6 |    9 |    1 |      3.4
   7 |       1 |        7 |    9 |   15 |  0.4 |      4.4
   8 |       1 |        8 |   -2 |   -1 |    0 |      4.8
   9 |       2 |        1 |   -1 |    1 |  0.4 |        0
  10 |       2 |        2 |    1 |    1 |    1 |      0.4
  11 |       2 |        3 |    2 |    4 |  0.7 |      1.4
  12 |       2 |        4 |   -6 |    4 |  0.3 |      2.1
  13 |       2 |        5 |    5 |    8 |    1 |      2.4
  14 |       2 |        6 |    6 |   11 |    1 |      3.4
  15 |       2 |        7 |   11 |   13 |    1 |      4.4
  16 |       2 |        8 |   12 |   15 |    1 |      5.4
  17 |       2 |        9 |    9 |   15 |  0.4 |      6.4
  18 |       2 |       10 |   -2 |   -1 |    0 |      6.8
  19 |       3 |        1 |   -1 |    1 |  0.4 |        0
  20 |       3 |        2 |    1 |    1 |    1 |      0.4
  21 |       3 |        3 |    2 |    4 |  0.7 |      1.4
  22 |       3 |        4 |   -6 |    4 |  0.3 |      2.1
  23 |       3 |        5 |    5 |   10 |    1 |      2.4
  24 |       3 |        6 |   10 |   12 |  0.6 |      3.4
  25 |       3 |        7 |   -3 |   12 |  0.4 |        4
  26 |       3 |        8 |   11 |   13 |    1 |      4.4
  27 |       3 |        9 |   12 |   15 |    1 |      5.4
  28 |       3 |       10 |    9 |   15 |  0.4 |      6.4
  29 |       3 |       11 |   -2 |   -1 |    0 |      6.8
(29 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *withPoints - Family of functions*

Indices and tables

- genindex
- search

pgr_withPointsDD - Proposed

Name

`pgr_withPointsDD` - Returns the driving distance from a starting point.

> **Warning:** Proposed functions for next mayor release.
> - They are not officially in the current release.
> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

[13]

Fig. 6.12: Boost Graph Inside

Availability: 2.2.0

Synopsis

Modify the graph to include points and using Dijkstra algorithm, extracts all the nodes and points that have costs less than or equal to the value `distance` from the starting point. The edges extracted will conform the corresponding spanning tree.

Signature Summary

```
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance)
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance, directed, driving_side, details)
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance, directed, driving_side, details, eq
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Signatures

Minimal Use

The minimal signature:

- Is for a **directed** graph.
- The driving side is set as **b** both. So arriving/departing to/from the point(s) can be in any direction.
- No **details** are given about distance of other points of the query.

```
pgr_withPointsDD(edges_sql, points_sql, start_vid, distance)
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Example

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    1 |    1 |  0.4 |      0.4
   3 |    2 |    1 |  0.6 |      0.6
   4 |    5 |    4 |  0.3 |      1.6
   5 |    6 |    8 |    1 |      2.6
   6 |    8 |    7 |    1 |      2.6
   7 |   10 |   10 |    1 |      2.6
   8 |    7 |    6 |  0.3 |      3.6
   9 |    9 |    9 |    1 |      3.6
  10 |   11 |   11 |    1 |      3.6
  11 |   13 |   14 |    1 |      3.6
(11 rows)
```

Driving distance from a single point Finds the driving distance depending on the optional parameters setup.

```
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance,
    directed:=true, driving_side:='b', details:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Example Right side driving topology

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'r',
    details := true);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    1 |    1 |  0.4 |      0.4
   3 |    2 |    1 |    1 |      1.4
   4 |   -6 |    4 |  0.7 |      2.1
   5 |    5 |    4 |  0.3 |      2.4
   6 |    6 |    8 |    1 |      3.4
   7 |    8 |    7 |    1 |      3.4
   8 |   10 |   10 |    1 |      3.4
(8 rows)
```

Driving distance from many starting points Finds the driving distance depending on the optional parameters setup.

```
pgr_withPointsDD(edges_sql, points_sql, start_vids, distance,
    directed:=true, driving_side:='b', details:=false, equicost:=false)
RETURNS SET OF (seq, node, edge, cost, agg_cost)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the Points SQL query

points_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
pid	`ANY-INTEGER`	(optional) Identifier of the point. • If column present, it can not be NULL. • If column not present, a sequential identifier will be given automatically.
edge_id	`ANY-INTEGER`	Identifier of the "closest" edge to the point.
fraction	`ANY-NUMERICAL`	Value in <0,1> that indicates the relative postition from the first end point of the edge.
side	`CHAR`	(optional) Value in ['b', 'r', 'l', NULL] indicating if the point is: • In the right, left of the edge or • If it doesn't matter with 'b' or NULL. • If column not present 'b' is considered.

Where:

ANY-INTEGER smallint, int, bigint

ANY-NUMERICAL smallint, int, bigint, real, float

Description of the parameters of the signatures

Parameter	Type
edges_sql	`TEXT`
points_sql	`TEXT`
start_vid	`ANY-INTEGER`
distance	`ANY-NUMERICAL`
directed	`BOOLEAN`
driving_side	`CHAR`
details	`BOOLEAN`
equicost	`BOOLEAN`

Description of the return values Returns set of `(seq, node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	row sequence.
node	`BIGINT`	Identifier of the node within the Distance from `start_pid`. If `details =: true` a negative value is the identifier of a point.
edge	`BIGINT`	**Identifier of the edge used to go from `node` to th** • `-1` when `start_vid` = `node`.
cost	`FLOAT`	**Cost to traverse `edge`.** • `0` when `start_vid` = `node`.
agg_cost	`FLOAT`	**Aggregate cost from `start_vid` to `node`.** • `0` when `start_vid` = `node`.

Examples for queries marked as `directed` with `cost` and `reverse_cost` columns

The examples in this section use the following *Graph 1: Directed, with cost and reverse cost*

Example Left side driving topology

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'l',
    details := true);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    2 |    1 |  0.6 |      0.6
   3 |   -6 |    4 |  0.7 |      1.3
   4 |    5 |    4 |  0.3 |      1.6
   5 |    1 |    1 |    1 |      1.6
   6 |    6 |    8 |    1 |      2.6
   7 |    8 |    7 |    1 |      2.6
   8 |   10 |   10 |    1 |      2.6
   9 |   -3 |   12 |  0.6 |      3.2
  10 |   -4 |    6 |  0.7 |      3.3
  11 |    7 |    6 |  0.3 |      3.6
  12 |    9 |    9 |    1 |      3.6
  13 |   11 |   11 |    1 |      3.6
  14 |   13 |   14 |    1 |      3.6
(14 rows)
```

Example Does not matter driving side.

```
SELECT * FROM pgr_withPointsDD(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table ORDER BY id',
    'SELECT pid, edge_id, fraction, side from pointsOfInterest',
    -1, 3.8,
    driving_side := 'b',
    details := true);
 seq | node | edge | cost | agg_cost
-----+------+------+------+----------
   1 |   -1 |   -1 |    0 |        0
   2 |    1 |    1 |  0.4 |      0.4
   3 |    2 |    1 |  0.6 |      0.6
   4 |   -6 |    4 |  0.7 |      1.3
   5 |    5 |    4 |  0.3 |      1.6
   6 |    6 |    8 |    1 |      2.6
   7 |    8 |    7 |    1 |      2.6
   8 |   10 |   10 |    1 |      2.6
   9 |   -3 |   12 |  0.6 |      3.2
  10 |   -4 |    6 |  0.7 |      3.3
  11 |    7 |    6 |  0.3 |      3.6
  12 |    9 |    9 |    1 |      3.6
  13 |   11 |   11 |    1 |      3.6
  14 |   13 |   14 |    1 |      3.6
(14 rows)
```

The queries use the *Sample Data* network.

History

- Proposed in version 2.2

See Also

- *pgr_drivingDistance* - Driving distance using dijkstra.
- *pgr_alphaShape* - Alpha shape computation.
- *pgr_pointsAsPolygon* - Polygon around set of points.

Indices and tables

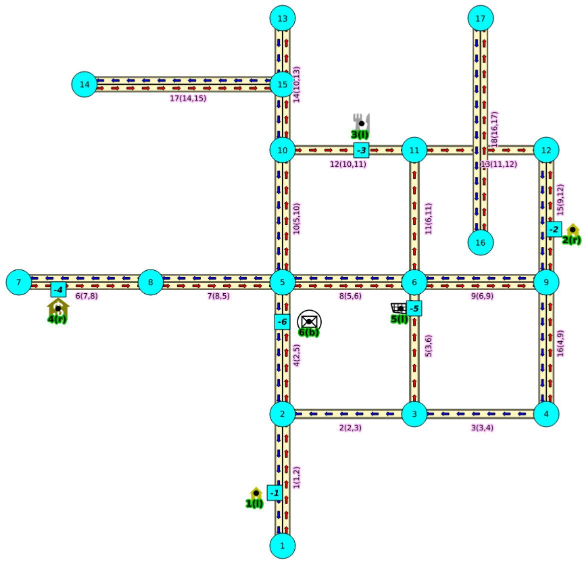
- genindex
- search

Warning: Proposed functions for next mayor release.
- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

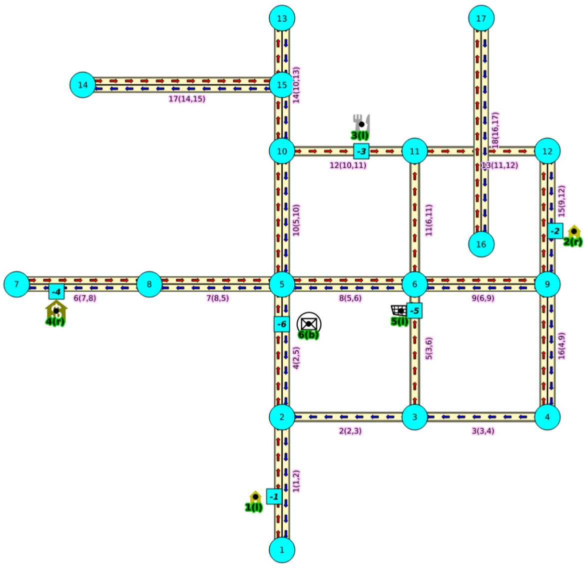
Images

The squared vertices are the temporary vertices, The temporary vertices are added acordng to the dirving side, The following images visualy show the diferences on how depending on the driving side the data is interpreted.

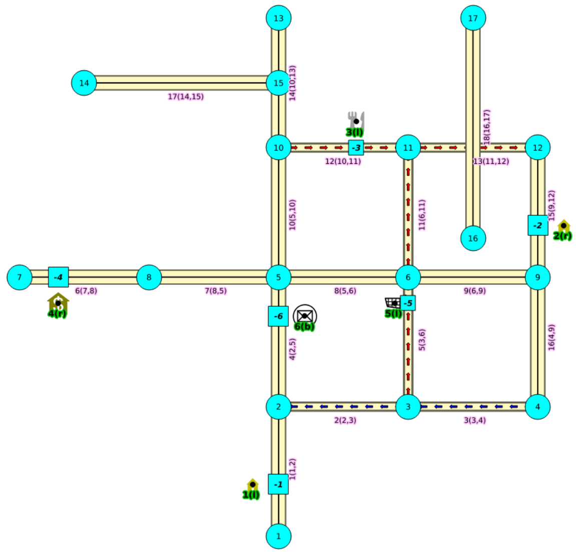
Right driving side



Left driving side



doesn't matter the driving side



Introduction

This famly of functions was thought for routing vehicles, but might as well work for some other application that we can not think of.

The with points family of function give you the ability to route between arbitrary points located outside the original graph.

When given a point identified with a *pid* that its being mapped to and edge with an identifier *edge_id*, with a *fraction* along that edge (from the source to the target of the edge) and some additional information about which *side* of the edge the point is on, then routing from arbitrary points more accurately reflect routing vehicles in road networks,

I talk about a family of functions because it includes different functionalities.

- pgr_withPoints is pgr_dijkstra based
- pgr_withPointsCost is pgr_dijkstraCost based
- pgr_withPointsKSP is pgr_ksp based
- pgr_withPointsDD is pgr_drivingDistance based

In all this functions we have to take care of as many aspects as possible:

- Must work for routing:
  - Cars (directed graph)
  - Pedestrians (undirected graph)
- Arriving at the point:
  - In either side of the street.
  - Compulsory arrival on the side of the street where the point is located.
- Countries with:
  - Right side driving
  - Left side driving
- Some points are:
  - Permanent, for example the set of points of clients stored in a table in the data base
  - Temporal, for example points given through a web application
- The numbering of the points are handled with negative sign.
  - Original point identifiers are to be positive.
  - Transformation to negative is done internally.
  - For results for involving vertices identifiers
    * positive sign is a vertex of the original grpah
    * negative sign is a point of the temporary points

The reason for doing this is to avoid confusion when there is a vertex with the same number as identifier as the points identifier.

Graph & edges

- Let $G_d(V, E)$ where V is the set of vertices and E is the set of edges be the original directed graph.
  - An edge of the original *edges_sql* is $(id, source, target, cost, reverse\_cost)$ will generate internally
    * $(id, source, target, cost)$
    * $(id, target, source, reverse\_cost)$

Point Definition

- A point is defined by the quadruplet: $(pid, eid, fraction, side)$
  - **ped** is the point identifier
  - **eid** is an edge id of the *edges_sql*
  - **fraction** represents where the edge *eid* will be cut.
  - **side** Indicates the side of the edge where the point is located.

Creating Temporary Vertices in the Graph

For edge (15, 9,12 10, 20), & lets insert point (2, 12, 0.3, r)

On a right hand side driving network

From first image above:

- We can arrive to the point only via vertex 9.
- It only afects the edge (15, 9,12, 10) so that edge is removed.
- Edge (15, 12,9, 20) is kept.
- Create new edges:
  - (15, 9,-1, 3) edge from vertex 9 to point 1 has cost 3
  - (15, -1,12, 7) edge from point 1 to vertex 12 has cost 7

On a left hand side driving network

From second image above:

- We can arrive to the point only via vertex 12.
- It only afects the edge (15, 12,9 20) so that edge is removed.
- Edge (15, 9,12, 10) is kept.
- Create new edges:
  - (15, 12,-1, 14) edge from vertex 12 to point 1 has cost 14
  - (15, -1,9, 6) edge from point 1 to vertex 9 has cost 6

  Remember that fraction is from vertex 9 to vertex 12

When driving side does not matter

From third image above:

- We can arrive to the point either via vertex 12 or via vertex 9
- Edge (15, 12,9 20) is removed.
- Edge (15, 9,12, 10) is removed.
- Create new edges:
  - (15, 12,-1, 14) edge from vertex 12 to point 1 has cost 14
  - (15, -1,9, 6) edge from point 1 to vertex 9 has cost 6
  - (15, 9,-1, 3) edge from vertex 9 to point 1 has cost 3
  - (15, -1,12, 7) edge from point 1 to vertex 12 has cost 7

6.1.5 Cost - Category

- *pgr_aStarCost – proposed*
- *pgr_bdAstarCost - Proposed*
- *pgr_bdDijkstraCost - Proposed*

- *pgr_dijkstraCost*
- *pgr_withPointsCost - Proposed*

> **Warning:** Proposed functions for next mayor release.
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> - They will likely officially be part of the next mayor release:
>   - The functions make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might not change. (But still can)
>   - Signature might not change. (But still can)
>   - Functionality might not change. (But still can)
>   - pgTap tests have being done. But might need more.
>   - Documentation might need refinement.

General Information

Characteristics

The main Characteristics are:

- Each function works as part of the family it belongs to.
- It does not return a path.
- Returns the sum of the costs of the resulting path(s) for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* int the non included values *(v, v)* is *0*.
  - When the starting vertex and ending vertex are the different and there is no path.
    - The *agg_cost* in the non included values *(u, v)* is ∞.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- Depending on the function and its parameters, the results can be symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- Any duplicated value in the *start_vids* or in *end_vids* are ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending

See Also

Indices and tables

- genindex
- search

6.1.6 Cost Matrix - Category

- *pgr_aStarCostMatrix - proposed*
- *pgr_bdAstarCostMatrix - proposed*
- *pgr_bdDijkstraCostMatrix - proposed*
- *pgr_dijkstraCostMatrix - proposed*
- *pgr_withPointsCostMatrix - proposed*

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

General Information

Synopsis

Traveling Sales Person - Family of functions needs as input a symmetric cost matrix and no edge *(u, v)* must value ∞.

This collection of functions will return a cost matrix in form of a table.

Characteristics

The main Characteristics are:

- Can be used as input to *pgr_TSP*.
  - **directly** when the resulting matrix is symmetric and there is no ∞ value.
  - It will be the users responsibility to make the matrix symmetric.
    - By using geometric or harmonic average of the non symmetric values.
    - By using max or min the non symmetric values.
    - By setting the upper triangle to be the mirror image of the lower triangle.
    - By setting the lower triangle to be the mirror image of the upper triangle.
  - It is also the users responsibility to fix an ∞ value.
- Each function works as part of the family it belongs to.
- It does not return a path.
- Returns the sum of the costs of the shortest path for pair combination of nodes in the graph.
- Process is done only on edges with positive costs.
- Values are returned when there is a path.
  - The returned values are in the form of a set of *(start_vid, end_vid, agg_cost)*.
  - When the starting vertex and ending vertex are the same, there is no path.
    - The *agg_cost* int the non included values *(v, v)* is *0*.

  - When the starting vertex and ending vertex are the different and there is no path.
    * The *agg_cost* in the non included values *(u, v)* is ∞.
- Let be the case the values returned are stored in a table, so the unique index would be the pair: *(start_vid, end_vid)*.
- Depending on the function and its parameters, the results can be symmetric.
  - The *agg_cost* of *(u, v)* is the same as for *(v, u)*.
- Any duplicated value in the *start_vids* are ignored.
- The returned values are ordered:
  - *start_vid* ascending
  - *end_vid* ascending
- Running time: approximately $O(|start\_vids| * (V \log V + E))$

See Also

- *pgr_TSP*

Indices and tables

- genindex
- search

6.1.7 KSP Category

- *pgr_KSP* - Driving Distance based on pgr_dijkstra
- *pgr_withPointsKSP - Proposed* - Driving Distance based on pgr_dijkstra

6.2 Experimental and Proposed functions

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Contraction - Family of functions - Reduce network size using contraction techniques

- *pgr_contractGraph - Proposed* - Reduce network size using contraction techniques

Graph Analysis

- *pgr_labelGraph - Proposed* - Analyze / label subgraphs within a network

Vehicle Routing Functions Category

Pickup and delivery problem

- pgr_pickDeliver - Proposed - Pickup & Delivery using a Cost Matrix
- pgr_pickDeliverEuclidean - Proposed - Pickup & Delivery with Euclidean distances

Experimental functions

- pgr_vrpOneDepot - experimental - VRP One Depot

6.2.1 Contraction - Family of functions

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

pgr_contractGraph - Proposed

pgr_contractGraph - Proposed

`pgr_contractGraph` — Performs graph contraction and returns the contracted vertices and edges.

[14]

Fig. 6.13: Boost Graph Inside

Availability: 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Synopsis

Contraction reduces the size of the graph by removing some of the vertices and edges and, for example, might add edges that represent a sequence of original edges decreasing the total time and space used in graph algorithms.

Characteristics

The main Characteristics are:

- Process is done only on edges with positive costs.
- There are two types of contraction methods used namely,
  - Dead End Contraction
  - Linear Contraction
- The values returned include the added edges and contracted vertices.
- The returned values are ordered as follows:
  - column *id* ascending when type = *v*
  - column *id* descending when type = *e*

Signature Summary:

The pgr_contractGraph function has the following signatures:

```
pgr_contractGraph(edges_sql, contraction_order)
pgr_contractGraph(edges_sql, contraction_order, max_cycles, forbidden_vertices, directed)

RETURNS SETOF (seq, type, id, contracted_vertices, source, target, cost)
```

Signatures

Minimal signature

```
pgr_contractGraph(edges_sql, contraction_order)
```

Example Making a dead end contraction and a linear contraction.

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[1, 2]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   2 | v    | 15 | {14}                |     -1 |     -1 |   -1
   3 | v    | 17 | {16}                |     -1 |     -1 |   -1
   4 | e    | -1 | {1,2}               |      3 |      5 |    2
   5 | e    | -2 | {4}                 |      9 |      3 |    2
   6 | e    | -3 | {10,13}             |      5 |     11 |    2
   7 | e    | -4 | {12}                |     11 |      9 |    2
(7 rows)
```

Complete signature

```
pgr_contractGraph(edges_sql, contraction_order, max_cycles, forbidden_vertices, directed)
```

Example Making a dead end contraction and a linear contraction and vertex 2 is forbidden from contraction

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
ARRAY[1, 2], forbidden_vertices:=ARRAY[2]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  2 | {1}                 |     -1 |     -1 |   -1
   2 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   3 | v    | 15 | {14}                |     -1 |     -1 |   -1
   4 | v    | 17 | {16}                |     -1 |     -1 |   -1
   5 | e    | -1 | {4}                 |      9 |      3 |    2
   6 | e    | -2 | {10,13}             |      5 |     11 |    2
   7 | e    | -3 | {12}                |     11 |      9 |    2
(7 rows)
```

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Description
edges_sql	`TEXT`	SQL query as described above.
contraction_order	`ARRAY[ANY-INTEGER]`	**Ordered contraction operations.** • 1 = Dead end contraction • 2 = Linear contraction
forbidden_vertices	`ARRAY[ANY-INTEGER]`	(optional). Identifiers of vertices forbidden from contraction. Default is an empty array.
max_cycles	`INTEGER`	(optional). Number of times the contraction operations on *contraction_order* will be performed. Default is 1.
directed	`BOOLEAN`	• When `true` the graph is considered as *Directed*. • When `false` the graph is considered as *Undirected*.

Description of the return values

RETURNS SETOF (seq, type, id, contracted_vertices, source, target, cost)

The function returns a single row. The columns of the row are:

Column	Type	Description
seq	`INTEGER`	Sequential value starting from **1**.
type	`TEXT`	**Type of the *id*.** • 'v' when *id* is an identifier of a vertex. • 'e' when *id* is an identifier of an edge.
id	`BIGINT`	**Identifier of:** • the vertex when *type* = *'v'*. – The vertex belongs to the edge_table passed as a parameter. • the edge when *type* = *'e'*. – The *id* is a decreasing sequence starting from **-1**. – Representing a pseudo *id* as is not incorporated into the edge_table.
contracted_vertices	`ARRAY[BIGINT]`	Array of contracted vertex identifiers.
source	`BIGINT`	Identifier of the source vertex of the current edge *id*. Valid values when *type* = *'e'*.
target	`BIGINT`	Identifier of the target vertex of the current edge *id*. Valid values when *type* = *'e'*.
cost	`FLOAT`	Weight of the edge (*source*, *target*). Valid values when *type* = *'e'*.

Examples

Example Only dead end contraction

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
ARRAY[1]);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  2 | {1}                 |     -1 |     -1 |   -1
   2 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   3 | v    | 10 | {13}                |     -1 |     -1 |   -1
   4 | v    | 15 | {14}                |     -1 |     -1 |   -1
   5 | v    | 17 | {16}                |     -1 |     -1 |   -1
(5 rows)
```

Example Only linear contraction

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
ARRAY[2]);
```

```
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | e    | -1 | {4}                 |      9 |      3 |    2
   2 | e    | -2 | {8}                 |      5 |      7 |    2
   3 | e    | -3 | {8}                 |      7 |      5 |    2
   4 | e    | -4 | {12}                |     11 |      9 |    2
(4 rows)
```

Indices and tables

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Introduction

In big graphs, like the road graphs, or electric networks, graph contraction can be used to speed up some graph algorithms. Contraction reduces the size of the graph by removing some of the vertices and edges and, for example, might add edges that represent a sequence of original edges decreasing the total time and space used in graph algorithms.

This implementation gives a flexible framework for adding contraction algorithms in the future, currently, it supports two algorithms:

1. Dead end contraction
2. Linear contraction

Allowing the user to:

- Forbid contraction on a set of nodes.
- Decide the order of the contraction algorithms and set the maximum number of times they are to be executed.

Note: UNDER DISCUSSION: Forbid contraction on a set of edges

Dead end contraction

In the algorithm, dead end contraction is represented by 1.

Dead end nodes

The definition of a dead end node is different for a directed and an undirected graph.

In case of a undirected graph, a node is considered a dead end node if

- The number of adjacent vertices is 1.

In case of an directed graph, a node is considered a dead end node if

- There are no outgoing edges and has at least one incoming edge.
- There is one incoming and one outgoing edge with the same identifier.

Examples

- The green node `B` represents a dead end node
- The node `A` is the only node connecting to `B`.
- Node `A` is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Directed graph

Operation: Dead End Contraction

The dead end contraction will stop until there are no more dead end nodes. For example from the following graph:

- Node `A` is connected to the rest of the graph by an unlimited number of edges.
- Node `B` is connected to the rest of the graph with one incoming edge.
- Node `B` is the only node connecting to `C`.
- The green node `C` represents a *Dead End* node

After contracting `C`, node `B` is now a *Dead End* node and is contracted:

Node `B` gets contracted

Nodes `B` and `C` belong to node `A`.

Not Dead End nodes

In this graph `B` is not a *dead end* node.

Linear contraction

In the algorithm, linear contraction is represented by 2.

Linear nodes

A node is considered a linear node if satisfies the following:

- The number of adjacent vertices are 2.
- Should have at least one incoming edge and one outgoing edge.

Examples

- The green node `B` represents a linear node
- The nodes `A` and `C` are the only nodes connecting to `B`.
- Node `A` is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Node `C` is part of the rest of the graph and has an unlimited number of incoming and outgoing edges.
- Directed graph

Operation: Linear Contraction

The linear contraction will stop until there are no more linear nodes. For example from the following graph:

- Node `A` is connected to the rest of the graph by an unlimited number of edges.
- Node `B` is connected to the rest of the graph with one incoming edge and one outgoing edge.
- Node `C` is connected to the rest of the graph with one incoming edge and one outgoing edge.
- Node `D` is connected to the rest of the graph by an unlimited number of edges.
- The green nodes `B` and `C` represents *Linear* nodes.

After contracting `B`, a new edge gets inserted between `A` and `C` which is represented by red color.

Node `C` is *linear node* and gets contracted.

Nodes `B` and `C` belong to edge connecting `A` and `D` which is represented by red color.

Not Linear nodes

In this graph `B` is not a *linear* node.

The cycle

Contracting a graph, can be done with more than one operation. The order of the operations affect the resulting contracted graph, after applying one operation, the set of vertices that can be contracted by another operation changes.

This implementation, cycles `max_cycles` times through `operations_order` .

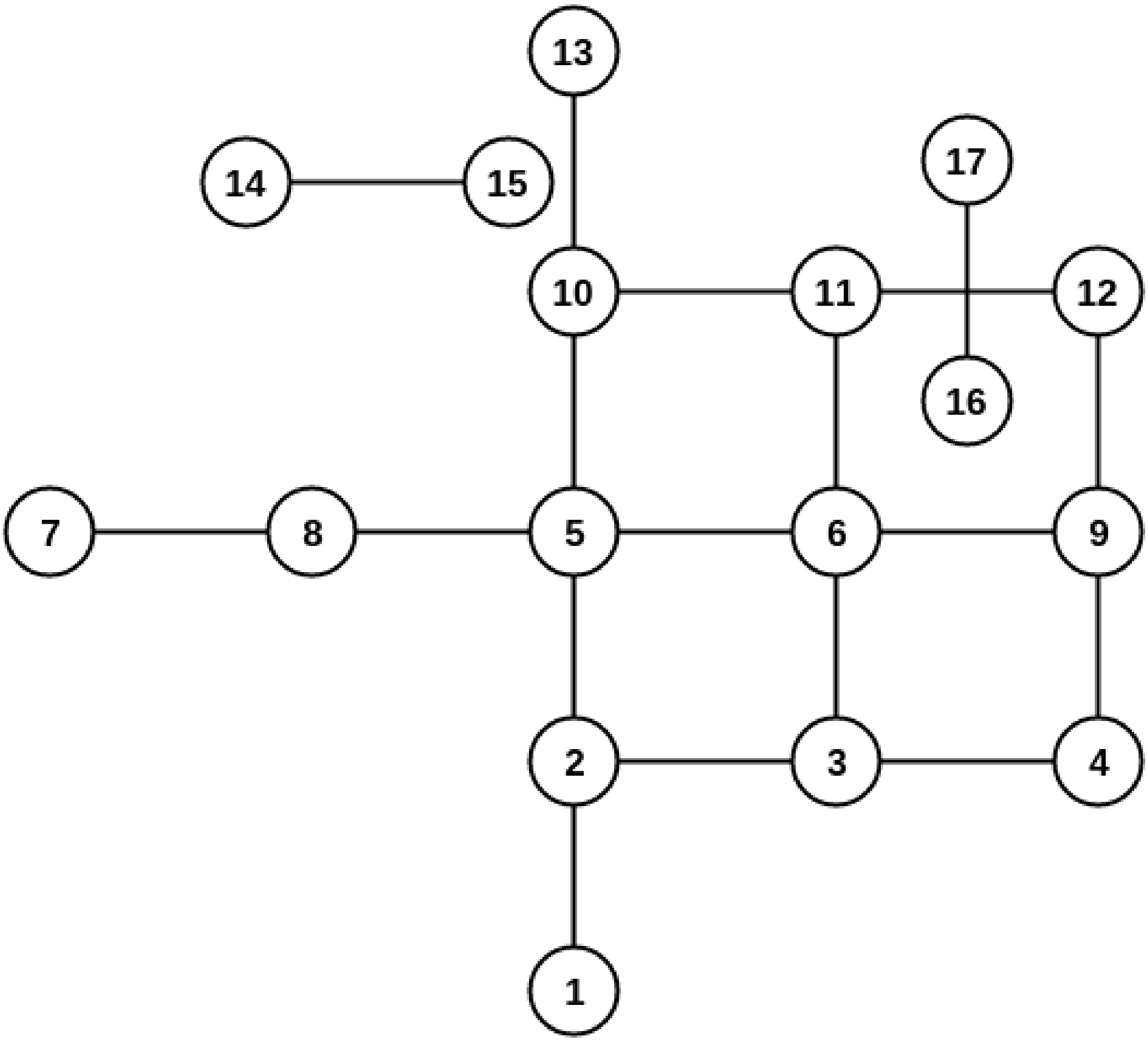
```
<input>
do max_cycles times {
    for (operation in operations_order)
     { do operation }
}
<output>
```

Contracting Sample Data

In this section, building and using a contracted graph will be shown by example.

- The *Sample Data* for an undirected graph is used
- a dead end operation first followed by a linear operation.

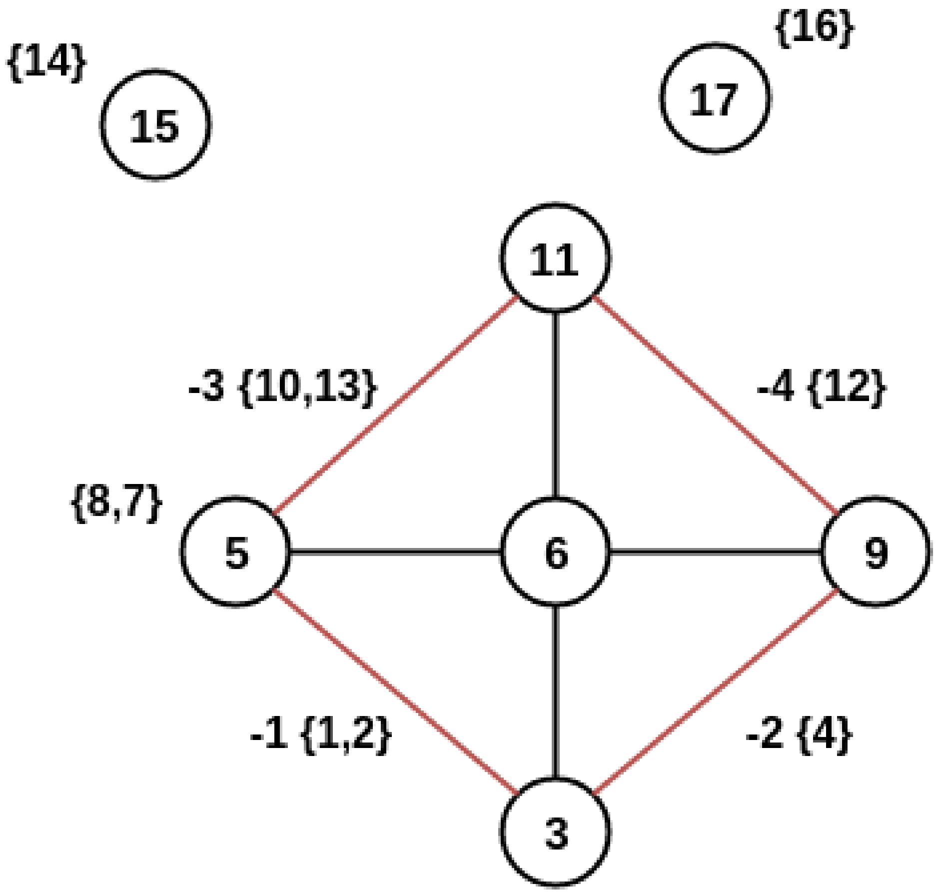
The original graph:



After doing a dead end contraction operation:

undirected-sampledata-b.png

Doing a linear contraction operation to the graph above



There are five cases, in this documentation, which arise when calculating the shortest path between a given source and target. In this examples, `pgr_dijkstra` is used.

- **Case 1**: Both source and target belong to the contracted graph.
- **Case 2**: Source belongs to a contracted graph, while target belongs to a edge subgraph.
- **Case 3**: Source belongs to a vertex subgraph, while target belongs to an edge subgraph.
- **Case 4**: Source belongs to a contracted graph, while target belongs to an vertex subgraph.
- **Case 5**: The path contains a new edge added by the contraction algorithm.

Construction of the graph in the database

Original Data

The following query shows the original data involved in the contraction operation.

Contraction Results

```
SELECT * FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    array[1,2], directed:=true);
 seq | type | id | contracted_vertices | source | target | cost
-----+------+----+---------------------+--------+--------+------
   1 | v    |  5 | {7,8}               |     -1 |     -1 |   -1
   2 | v    | 15 | {14}                |     -1 |     -1 |   -1
```

```
   3 | v    | 17 | {16}                    |      -1 |      -1 |   -1
   4 | e    | -1 | {1,2}                   |       3 |       5 |    2
   5 | e    | -2 | {4}                     |       9 |       3 |    2
   6 | e    | -3 | {10,13}                 |       5 |      11 |    2
   7 | e    | -4 | {12}                    |      11 |       9 |    2
(7 rows)
```

The above results do not represent the contracted graph. They represent the changes done to the graph after applying the contraction algorithm. We can see that vertices like 6 and 11 do not appear in the contraction results because they were not affected by the contraction algorithm.

step 1

Adding extra columns to the `edge_table` and `edge_table_vertices_pgr` tables:

Column	Description
contracted_-vertices	The vertices set belonging to the vertex/edge
is_contracted	On a *vertex* table: when `true` the vertex is contracted, so is not part of the contracted graph.
is_contracted	On an *edge* table: when `true` the edge was generated by the contraction algorithm.

Using the following queries:

```
ALTER TABLE edge_table ADD contracted_vertices BIGINT[];
ALTER TABLE
ALTER TABLE edge_table_vertices_pgr ADD contracted_vertices BIGINT[];
ALTER TABLE
ALTER TABLE edge_table ADD is_contracted BOOLEAN DEFAULT false;
ALTER TABLE
ALTER TABLE edge_table_vertices_pgr ADD is_contracted BOOLEAN DEFAULT false;
ALTER TABLE
SET client_min_messages TO NOTICE;
SET
```

step 2

For simplicity, in this documentation, store the results of the call to pgr_contractGraph in a temporary table

```
SELECT * INTO contraction_results
FROM pgr_contractGraph(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    array[1,2], directed:=true);
SELECT 7
```

step 3

Update the *vertex* and *edge* tables using the results of the call to pgr_contraction

- In *edge_table_vertices_pgr.is_contracted* indicate the vertices that are contracted.

```
UPDATE edge_table_vertices_pgr
SET is_contracted = true
WHERE id IN (SELECT  unnest(contracted_vertices) FROM  contraction_results);
UPDATE 10
```

- Add to *edge_table_vertices_pgr.contracted_vertices* the contracted vertices belonging to the vertices.

```
UPDATE edge_table_vertices_pgr
SET contracted_vertices = contraction_results.contracted_vertices
FROM contraction_results
WHERE type = 'v' AND edge_table_vertices_pgr.id = contraction_results.id;
UPDATE 3
```

- Insert the new edges generated by pgr_contractGraph.

```
INSERT INTO edge_table(source, target, cost, reverse_cost, contracted_vertices, is_contracted)
SELECT source, target, cost, -1, contracted_vertices, true
FROM contraction_results
WHERE type = 'e';
INSERT 0 4
```

step 3.1

Verify visually the updates.

- On the *edge_table_vertices_pgr*

```
SELECT id, contracted_vertices, is_contracted
FROM edge_table_vertices_pgr
ORDER BY id;
 id | contracted_vertices | is_contracted
----+---------------------+---------------
  1 |                     | t
  2 |                     | t
  3 |                     | f
  4 |                     | t
  5 | {7,8}               | f
  6 |                     | f
  7 |                     | t
  8 |                     | t
  9 |                     | f
 10 |                     | t
 11 |                     | f
 12 |                     | t
 13 |                     | t
 14 |                     | t
 15 | {14}                | f
 16 |                     | t
 17 | {16}                | f
(17 rows)
```

- On the *edge_table*

```
SELECT id, source, target, cost, reverse_cost, contracted_vertices, is_contracted
FROM edge_table
ORDER BY id;
 id | source | target | cost | reverse_cost | contracted_vertices | is_contracted
----+--------+--------+------+--------------+---------------------+---------------
  1 |      1 |      2 |    1 |            1 |                     | f
  2 |      2 |      3 |   -1 |            1 |                     | f
  3 |      3 |      4 |   -1 |            1 |                     | f
  4 |      2 |      5 |    1 |            1 |                     | f
  5 |      3 |      6 |    1 |           -1 |                     | f
  6 |      7 |      8 |    1 |            1 |                     | f
  7 |      8 |      5 |    1 |            1 |                     | f
  8 |      5 |      6 |    1 |            1 |                     | f
  9 |      6 |      9 |    1 |            1 |                     | f
 10 |      5 |     10 |    1 |            1 |                     | f
 11 |      6 |     11 |    1 |           -1 |                     | f
```

```
 12 |      10 |      11 |     1 |            -1 |                       | f
 13 |      11 |      12 |     1 |            -1 |                       | f
 14 |      10 |      13 |     1 |             1 |                       | f
 15 |       9 |      12 |     1 |             1 |                       | f
 16 |       4 |       9 |     1 |             1 |                       | f
 17 |      14 |      15 |     1 |             1 |                       | f
 18 |      16 |      17 |     1 |             1 |                       | f
 19 |       3 |       5 |     2 |            -1 | {1,2}                 | t
 20 |       9 |       3 |     2 |            -1 | {4}                   | t
 21 |       5 |      11 |     2 |            -1 | {10,13}               | t
 22 |      11 |       9 |     2 |            -1 | {12}                  | t
(22 rows)
```

- vertices that belong to the contracted graph are the non contracted vertices

```
SELECT id  FROM edge_table_vertices_pgr
WHERE is_contracted = false
ORDER BY id;
 id
----
  3
  5
  6
  9
 11
 15
 17
(7 rows)
```

case 1: Both source and target belong to the contracted graph.

Inspecting the contracted graph above, vertex 3 and vertex 11 are part of the contracted graph. In the following query:

- vertices_in_graph hold the vertices that belong to the contracted graph.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 11: 3 -> 6 -> 11, and in the contracted graph, it is also 3 -> 6 -> 11. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH
    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    3, 11, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |    5 |    1 |        0
   2 |        2 |    6 |   11 |    1 |        1
   3 |        3 |   11 |   -1 |    0 |        2
(3 rows)
```

case 2: Source belongs to the contracted graph, while target belongs to a edge subgraph.

Inspecting the contracted graph above, vertex 3 is part of the contracted graph and vertex 1 belongs to the contracted subgr

- expand1 holds the contracted vertices of the edge where vertex 1 belongs. (belongs to edge 19).
- vertices_in_graph hold the vertices that belong to the contracted graph and also the contracted vertices of edge 19.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 1: 3 -> 2 -> 1, and in the contracted graph, it is also 3 -> 2 -> 1. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH
    expand_edges AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table),
    expand1 AS (SELECT contracted_vertices FROM edge_table
        WHERE id IN (SELECT id FROM expand_edges WHERE vertex = 1)),
    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand1)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    3, 1, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |    2 |    1 |        0
   2 |        2 |    2 |    1 |    1 |        1
   3 |        3 |    1 |   -1 |    0 |        2
(3 rows)
```

case 3: Source belongs to a vertex subgraph, while target belongs to an edge subgraph.

Inspecting the contracted graph above, vertex 7 belongs to the contracted subgraph of vertex 5 and vertex 13 belongs to the contracted subgraph of edge 21. In the following query:

- expand7 holds the contracted vertices of vertex where vertex 7 belongs. (belongs to vertex 5)
- expand13 holds the contracted vertices of edge where vertex 13 belongs. (belongs to edge 21)
- vertices_in_graph hold the vertices that belong to the contracted graph, contracted vertices of vertex 5 and contracted vertices of edge 21.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 7 to 13: 7 -> 8 -> 5 -> 10 -> 13, and in the contracted graph, it is also 7 -> 8 -> 5 -> 10 -> 13. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra(
    $$
    WITH

    expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table_vertices_
```

```
    expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
        WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),

    expand_edges AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table),
    expand13 AS (SELECT contracted_vertices FROM edge_table
        WHERE id IN (SELECT id FROM expand_edges WHERE vertex = 13)),

    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand13
        UNION
        SELECT unnest(contracted_vertices) FROM expand7)

    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
    7, 13, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    7 |    6 |    1 |        0
   2 |        2 |    8 |    7 |    1 |        1
   3 |        3 |    5 |   10 |    1 |        2
   4 |        4 |   10 |   14 |    1 |        3
   5 |        5 |   13 |   -1 |    0 |        4
(5 rows)
```

case 4: Source belongs to the contracted graph, while target belongs to an vertex subgraph.

Inspecting the contracted graph above, vertex 3 is part of the contracted graph and vertex 7 belongs to the contracted subgraph of vertex 5. In the following query:

- expand7 holds the contracted vertices of vertex where vertex 7 belongs. (belongs to vertex 5)
- vertices_in_graph hold the vertices that belong to the contracted graph and the contracted vertices of vertex 5.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 7: 3 -> 2 -> 5 -> 8 -> 7, but in the contracted graph, it is 3 -> 5 -> 8 -> 7. The results, on the contracted graph do not match the results as if it was done on the original graph. This is because the path contains edge 19 which is added by the contraction algorithm.

```
SELECT * FROM  pgr_dijkstra(
    $$
    WITH
    expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table_vertices_
    expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
        WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
    vertices_in_graph AS (
        SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
        UNION
        SELECT unnest(contracted_vertices) FROM expand7)
    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    $$,
```

```
    3, 7, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |   19 |    2 |        0
   2 |        2 |    5 |    7 |    1 |        2
   3 |        3 |    8 |    6 |    1 |        3
   4 |        4 |    7 |   -1 |    0 |        4
(4 rows)
```

case 5: The path contains an edge added by the contraction algorithm.

In the previous example we can see that the path from vertex 3 to vertex 7 contains an edge which is added by the contraction algorithm.

```
WITH
first_dijkstra AS (
    SELECT * FROM  pgr_dijkstra(
        $$
        WITH
        expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table_verti
        expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
            WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
        vertices_in_graph AS (
            SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
            UNION
            SELECT unnest(contracted_vertices) FROM expand7)
        SELECT id, source, target, cost, reverse_cost
        FROM edge_table
        WHERE source IN (SELECT * FROM vertices_in_graph)
        AND target IN (SELECT * FROM vertices_in_graph)
        $$,
        3, 7, false))
SELECT edge, contracted_vertices
    FROM first_dijkstra JOIN edge_table
    ON (edge = id)
    WHERE is_contracted = true;
 edge | contracted_vertices
------+---------------------
   19 | {1,2}
(1 row)
```

Inspecting the contracted graph above, edge 19 should be expanded. In the following query:

- first_dijkstra holds the results of the dijkstra query.
- edges_to_expand holds the edges added by the contraction algorithm and included in the path.
- vertices_in_graph hold the vertices that belong to the contracted graph, vertices of the contracted solution and the contracted vertices of the edges added by the contraction algorithm and included in the contracted solution.
- when selecting the edges, only edges that have the source and the target in that set are the edges belonging to the contracted graph, that is done in the WHERE clause.

Visually, looking at the original graph, going from 3 to 7: 3 -> 2 -> 5 -> 8 -> 7, and in the contracted graph, it is also 3 -> 2 -> 5 -> 8 -> 7. The results, on the contracted graph match the results as if it was done on the original graph.

```
SELECT * FROM pgr_dijkstra($$
    WITH
    -- This returns the results from case 2
```

```
    first_dijkstra AS (
        SELECT * FROM  pgr_dijkstra(
            '
            WITH
            expand_vertices AS (SELECT id, unnest(contracted_vertices) AS vertex FROM edge_table_v
            expand7 AS (SELECT contracted_vertices FROM edge_table_vertices_pgr
                WHERE id IN (SELECT id FROM expand_vertices WHERE vertex = 7)),
            vertices_in_graph AS (
                SELECT id  FROM edge_table_vertices_pgr WHERE is_contracted = false
                UNION
                SELECT unnest(contracted_vertices) FROM expand7)
            SELECT id, source, target, cost, reverse_cost
            FROM edge_table
            WHERE source IN (SELECT * FROM vertices_in_graph)
            AND target IN (SELECT * FROM vertices_in_graph)
            ',
            3, 7, false)),

    -- edges that need expansion and the vertices to be expanded.
    edges_to_expand AS (
        SELECT edge, contracted_vertices
        FROM first_dijkstra JOIN edge_table
        ON (edge = id)
        WHERE is_contracted = true),

    vertices_in_graph AS (
        -- the nodes of the contracted solution
        SELECT node FROM first_dijkstra
        UNION
        -- the nodes of the expanding sections
        SELECT unnest(contracted_vertices) FROM edges_to_expand)

    SELECT id, source, target, cost, reverse_cost
    FROM edge_table
    WHERE source IN (SELECT * FROM vertices_in_graph)
    AND target IN (SELECT * FROM vertices_in_graph)
    -- not including the expanded edges
    AND id NOT IN (SELECT edge FROM edges_to_expand)
    $$,
    3, 7, false);
 seq | path_seq | node | edge | cost | agg_cost
-----+----------+------+------+------+----------
   1 |        1 |    3 |    2 |    1 |        0
   2 |        2 |    2 |    4 |    1 |        1
   3 |        3 |    5 |    7 |    1 |        2
   4 |        4 |    8 |    6 |    1 |        3
   5 |        5 |    7 |   -1 |    0 |        4
(5 rows)
```

See Also

- http://www.cs.cmu.edu/afs/cs/academic/class/15210-f12/www/lectures/lecture16.pdf
- http://algo2.iti.kit.edu/documents/routeplanning/geisberger_dipl.pdf
- The queries use *pgr_contractGraph - Proposed* function and the *Sample Data* network.

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6.2.2 Flow - Family of functions

- *pgr_maxFlow - Proposed* - Only the Max flow calculation using Push and Relabel algorithm.
- *pgr_boykovKolmogorov - Proposed* - Boykov and Kolmogorov with details of flow on edges.
- *pgr_edmondsKarp - Proposed* - Edmonds and Karp algorithm with details of flow on edges.
- *pgr_pushRelabel - Proposed* - Push and relabel algorithm with details of flow on edges.
- Applications
  - *pgr_edgeDisjointPaths - Proposed* - Calculates edge disjoint paths between two groups of vertices.
  - *pgr_maxCardinalityMatch - Proposed* - Calculates a maximum cardinality matching in a graph.

> **Warning:** Experimental functions
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

pgr_maxFlow - Proposed

Synopsis

`pgr_maxFlow` — Calculates the maximum flow in a directed graph from the source(s) to the targets(s) using the Push Relabel algorithm.

[15]

Fig. 6.14: Boost Graph Inside

Availability: 2.4.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- When the maximum flow is 0 then there is no flow and **0** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Uses the *pgr_pushRelabel* algorithm.
- Running time: $O(V^3)$

Signature Summary

```
pgr_maxFlow(edges_sql, source,  target)
pgr_maxFlow(edges_sql, sources,  target)
pgr_maxFlow(edges_sql, source,  targets)
pgr_maxFlow(edges_sql, sources,  targets)
RETURNS BIGINT
```

One to One Calculates the maximum flow from the *source* to the *target*.

```
pgr_maxFlow(edges_sql, source,  target)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 pgr_maxflow
-------------
```

```
         230
(1 row)
```

One to Many Calculates the maximum flow from the *source* to all of the *targets*.

```
pgr_maxFlow(edges_sql, source,  targets)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, ARRAY[11, 1, 13]
);
 pgr_maxflow
-------------
         340
(1 row)
```

Many to One Calculates the maximum flow from all the *sources* to the *target*.

```
pgr_maxFlow(edges_sql, sources,  target)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
 pgr_maxflow
-------------
         230
(1 row)
```

Many to Many Calculates the maximum flow from all of the *sources* to all of the *targets*.

```
pgr_maxFlow(edges_sql, sources,  targets)
RETURNS BIGINT
```

Example

```
SELECT * FROM pgr_maxFlow(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
```

```
    FROM edge_table'
    , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 pgr_maxflow
-------------
         360
(1 row)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
capacity	ANY-INTEGER		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	ANY-INTEGER	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	TEXT		The edges SQL query as
source	BIGINT		Identifier of the starting
sources	ARRAY[BIGINT]		Array of identifiers of th
target	BIGINT		Identifier of the ending v
targets	ARRAY[BIGINT]		Array of identifiers of th

Description of the return value

Type	Description
BIGINT	Maximum flow possible from the source(s) to the target(s)

See Also

- *Flow - Family of functions*
- http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html

- https://en.wikipedia.org/wiki/Push%E2%80%93relabel_maximum_flow_algorithm

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pgr_pushRelabel - Proposed

Synopsis

`pgr_pushRelabel` — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Push Relabel Algorithm.

[16]

Fig. 6.15: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowPushRelabel
- New in 2.3.0

> **Warning:** Experimental functions
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.

- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets
- Running time: $O(V^3)$

Signature Summary

```
pgr_pushRelabel(edges_sql, source,  target) - Proposed
pgr_pushRelabel(edges_sql, sources, target) - Proposed
pgr_pushRelabel(edges_sql, source,  targets) - Proposed
pgr_pushRelabel(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One Calculates the flow on the graph edges that maximizes the flow from the *source* to the *target*.

```
pgr_pushRelabel(edges_sql, source,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
(4 rows)
```

One to Many Calculates the flow on the graph edges that maximizes the flow from the *source* to all of the *targets*.

```
pgr_pushRelabel(edges_sql, source,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
```

```
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, ARRAY[11, 1, 13]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |  130 |                 0
   2 |    2 |         3 |       2 |   80 |                20
   3 |    3 |         4 |       3 |   80 |                50
   4 |    4 |         5 |       2 |   50 |                 0
   5 |    7 |         5 |       8 |   50 |                80
   6 |   10 |         5 |      10 |   80 |                50
   7 |    8 |         6 |       5 |  130 |                 0
   8 |    9 |         6 |       9 |   80 |                50
   9 |   11 |         6 |      11 |  130 |                 0
  10 |    6 |         7 |       8 |   50 |                 0
  11 |    6 |         8 |       7 |   50 |                50
  12 |    7 |         8 |       5 |   50 |                 0
  13 |   16 |         9 |       4 |   80 |                 0
  14 |   12 |        10 |      11 |   80 |                20
(14 rows)
```

Many to One Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to the *target*.

```
pgr_pushRelabel(edges_sql, sources,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
(4 rows)
```

Many to Many Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to all of the *targets*.

```
pgr_pushRelabel(edges_sql, sources,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_pushRelabel(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |  100 |                30
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   30 |               100
   7 |   11 |         6 |      11 |  130 |                 0
   8 |    7 |         8 |       5 |   20 |                30
   9 |   16 |         9 |       4 |   80 |                 0
  10 |   12 |        10 |      11 |  100 |                 0
  11 |   15 |        12 |       9 |   50 |                 0
(11 rows)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	`TEXT`		The edges SQL query as
source	`BIGINT`		Identifier of the starting
sources	`ARRAY[BIGINT]`		Array of identifiers of th
target	`BIGINT`		Identifier of the ending v
targets	`ARRAY[BIGINT]`		Array of identifiers of th

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, targe

See Also

- *Flow - Family of functions*, *pgr_boykovKolmogorov*, *pgr_edmondsKarp*
- http://www.boost.org/libs/graph/doc/push_relabel_max_flow.html
- https://en.wikipedia.org/wiki/Push%E2%80%93relabel_maximum_flow_algorithm

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pgr_edmondsKarp - Proposed

Synopsis

`pgr_edmondsKarp` — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Push Relabel Algorithm.

[17]

Fig. 6.16: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowEdmondsKarp
- New in 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets
- Running time: $O(V * E^2)$

Signature Summary

```
pgr_edmondsKarp(edges_sql, source,  target) - Proposed
pgr_edmondsKarp(edges_sql, sources, target) - Proposed
pgr_edmondsKarp(edges_sql, source,  targets) - Proposed
pgr_edmondsKarp(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One Calculates the flow on the graph edges that maximizes the flow from the *source* to the *target*.

```
pgr_edmondsKarp(edges_sql, source,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
(4 rows)
```

One to Many Calculates the flow on the graph edges that maximizes the flow from the *source* to all of the *targets*.

```
pgr_edmondsKarp(edges_sql, source,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
   , 6, ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |   80 |                50
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   80 |                50
   7 |   11 |         6 |      11 |  130 |                 0
   8 |   16 |         9 |       4 |   80 |                 0
   9 |   12 |        10 |      11 |   80 |                20
(9 rows)
```

Many to One Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to the *target*.

```
pgr_edmondsKarp(edges_sql, sources,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
```

```
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
   , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
(4 rows)
```

Many to Many Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to all of the *targets*.

```
pgr_edmondsKarp(edges_sql, sources,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edmondsKarp(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
   , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
   3 |    4 |         5 |       2 |   50 |                 0
   4 |   10 |         5 |      10 |  100 |                30
   5 |    8 |         6 |       5 |  130 |                 0
   6 |    9 |         6 |       9 |   80 |                50
   7 |   11 |         6 |      11 |  130 |                 0
   8 |    7 |         8 |       5 |   20 |                30
   9 |   16 |         9 |       4 |   80 |                 0
  10 |   12 |        10 |      11 |  100 |                 0
(10 rows)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the edge.
source	ANY-INTEGER		Identifier of the first end point vertex of the edge.
target	ANY-INTEGER		Identifier of the second end point vertex of the edge.
capacity	ANY-INTEGER		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	ANY-INTEGER	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	TEXT		The edges SQL query as
source	BIGINT		Identifier of the starting
sources	ARRAY[BIGINT]		Array of identifiers of th
target	BIGINT		Identifier of the ending v
targets	ARRAY[BIGINT]		Array of identifiers of th

Description of the Return Values

Column	Type	Description
seq	INT	Sequential value starting from **1**.
edge_id	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point vertex of the edge.
target	BIGINT	Identifier of the second end point vertex of the edge.
flow	BIGINT	Flow through the edge in the direction (source, target).
residual_capacity	BIGINT	Residual capacity of the edge in the direction (source, targe

See Also

- *Flow - Family of functions*, *pgr_boykovKolmogorov*, *pgr_PushRelabel*
- http://www.boost.org/libs/graph/doc/edmonds_karp_max_flow.html
- https://en.wikipedia.org/wiki/Edmonds%E2%80%93Karp_algorithm

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pgr_boykovKolmogorov - Proposed

Synopsis

`pgr_boykovKolmogorov` — Calculates the flow on the graph edges that maximizes the flow from the sources to the targets using Boykov Kolmogorov algorithm.

[18]

Fig. 6.17: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maxFlowBoykovKolmogorov
- New in 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Characteristics

- The graph is **directed**.
- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources

  - By aggregation of the incoming flow to the targets
- Running time: Polynomial

Signature Summary

```
pgr_boykovKolmogorov(edges_sql, source,  target) - Proposed
pgr_boykovKolmogorov(edges_sql, sources, target) - Proposed
pgr_boykovKolmogorov(edges_sql, source,  targets) - Proposed
pgr_boykovKolmogorov(edges_sql, sources, targets) - Proposed
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

One to One Calculates the flow on the graph edges that maximizes the flow from the *source* to the *target*.

```
pgr_boykovKolmogorov(edges_sql, source,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                30
   2 |    8 |         6 |       5 |  100 |                30
   3 |   11 |         6 |      11 |  130 |                 0
   4 |   12 |        10 |      11 |  100 |                 0
(4 rows)
```

One to Many Calculates the flow on the graph edges that maximizes the flow from the *source* to all of the *targets*.

```
pgr_boykovKolmogorov(edges_sql, source,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , 6, ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                80
   2 |    3 |         4 |       3 |   80 |                50
```

```
   3 |    4 |         5 |       2 |   50 |                  0
   4 |   10 |         5 |      10 |   80 |                 50
   5 |    8 |         6 |       5 |  130 |                  0
   6 |    9 |         6 |       9 |   80 |                 50
   7 |   11 |         6 |      11 |  130 |                  0
   8 |   16 |         9 |       4 |   80 |                  0
   9 |   12 |        10 |      11 |   80 |                 20
(9 rows)
```

Many to One Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to the *target*.

```
pgr_boykovKolmogorov(edges_sql, sources,  target)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], 11
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |   10 |         5 |      10 |  100 |                 30
   2 |    8 |         6 |       5 |  100 |                 30
   3 |   11 |         6 |      11 |  130 |                  0
   4 |   12 |        10 |      11 |  100 |                  0
(4 rows)
```

Many to Many Calculates the flow on the graph edges that maximizes the flow from all of the *sources* to all of the *targets*.

```
pgr_boykovKolmogorov(edges_sql, sources,  targets)
RETURNS SET OF (seq, edge, start_vid, end_vid, flow, residual_capacity)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_boykovKolmogorov(
    'SELECT id,
            source,
            target,
            capacity,
            reverse_capacity
    FROM edge_table'
    , ARRAY[6, 8, 12], ARRAY[1, 3, 11]
);
 seq | edge | start_vid | end_vid | flow | residual_capacity
-----+------+-----------+---------+------+-------------------
   1 |    1 |         2 |       1 |   50 |                 80
   2 |    3 |         4 |       3 |   80 |                 50
   3 |    4 |         5 |       2 |   50 |                  0
   4 |   10 |         5 |      10 |  100 |                 30
   5 |    8 |         6 |       5 |  130 |                  0
```

```
   6 |     9 |          6 |        9 |   80 |                50
   7 |    11 |          6 |       11 |  130 |                 0
   8 |     7 |          8 |        5 |   20 |                30
   9 |    16 |          9 |        4 |   80 |                 0
  10 |    12 |         10 |       11 |  100 |                 0
(10 rows)
```

Description of the Signatures

Description of the edges_sql query for Max-flow like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
capacity	`ANY-INTEGER`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_capacity	`ANY-INTEGER`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

Description of the Parameters of the Flow Signatures

Column	Type	Default	Description
edges_sql	`TEXT`		The edges SQL query as
source	`BIGINT`		Identifier of the starting
sources	`ARRAY[BIGINT]`		Array of identifiers of th
target	`BIGINT`		Identifier of the ending v
targets	`ARRAY[BIGINT]`		Array of identifiers of th

Description of the Return Values

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
edge_id	`BIGINT`	Identifier of the edge in the original query(edges_sql).
source	`BIGINT`	Identifier of the first end point vertex of the edge.
target	`BIGINT`	Identifier of the second end point vertex of the edge.
flow	`BIGINT`	Flow through the edge in the direction (source, target).
residual_capacity	`BIGINT`	Residual capacity of the edge in the direction (source, targe

See Also

- *Flow - Family of functions*, *pgr_pushRelabel*, *pgr_EdmondsKarp*
- http://www.boost.org/libs/graph/doc/boykov_kolmogorov_max_flow.html

Indices and tables

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pgr_maxCardinalityMatch - Proposed

Synopsis

`pgr_maxCardinalityMatch` — Calculates a maximum cardinality matching in a graph.

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

[19]

Fig. 6.18: Boost Graph Inside

Availability:

- Renamed 2.5.0, Previous name pgr_maximumCardinalityMatching
- New in 2.3.0

Characteristics

- A matching or independent edge set in a graph is a set of edges without common vertices.
- A maximum matching is a matching that contains the largest possible number of edges.
  - There may be many maximum matchings.

   - Calculates **one** possible maximum cardinality matching in a graph.

- The graph can be **directed** or **undirected**.
- Running time: $O(E * V * \alpha(E, V))$
   - $\alpha(E, V)$ is the inverse of the Ackermann function[20].

Signature Summary

```
pgr_MaximumCardinalityMatching(edges_sql) - Proposed
pgr_MaximumCardinalityMatching(edges_sql, directed) - Proposed

RETURNS SET OF (seq, edge_id, source, target)
    OR EMPTY SET
```

Minimal Use

```
pgr_MaximumCardinalityMatching(edges_sql)
RETURNS SET OF (seq, edge_id, source, target) OR EMPTY SET
```

The minimal use calculates one possible maximum cardinality matching on a **directed** graph.

Example

```
SELECT * FROM pgr_maxCardinalityMatch(
    'SELECT id, source, target, cost AS going, reverse_cost AS coming FROM edge_table'
);
 seq | edge | source | target
-----+------+--------+--------
   1 |    1 |      1 |      2
   2 |    3 |      4 |      3
   3 |    9 |      6 |      9
   4 |    6 |      7 |      8
   5 |   14 |     10 |     13
   6 |   13 |     11 |     12
   7 |   17 |     14 |     15
   8 |   18 |     16 |     17
(8 rows)
```

Complete signature

```
pgr_MaximumCardinalityMatching(edges_sql, directed)
RETURNS SET OF (seq, edge_id, source, target) OR EMPTY SET
```

The complete signature calculates one possible maximum cardinality matching.

Example

```
SELECT * FROM pgr_maxCardinalityMatch(
    'SELECT id, source, target, cost AS going, reverse_cost AS coming FROM edge_table',
    directed := false
);
 seq | edge | source | target
-----+------+--------+--------
   1 |    1 |      1 |      2
   2 |    3 |      3 |      4
   3 |    9 |      6 |      9
   4 |    6 |      7 |      8
   5 |   14 |     10 |     13
   6 |   13 |     11 |     12
```

[20] https://en.wikipedia.org/wiki/Ackermann_function

```
   7 |    17 |      14 |      15
   8 |    18 |      16 |      17
(8 rows)
```

Description of the Signatures

Description of the SQL query

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Description
id	ANY-INTEGER	Identifier of the edge.
source	ANY-INTEGER	Identifier of the first end point vertex of the edge.
target	ANY-INTEGER	Identifier of the second end point vertex of the edge.
going	ANY-NUMERIC	A positive value represents the existence of the edge (source, target).
coming	ANY-NUMERIC	A positive value represents the existence of the edge (target, source).

Where:

- **ANY-INTEGER** SMALLINT, INTEGER, BIGINT
- **ANY-NUMERIC** SMALLINT, INTEGER, BIGINT, REAL, DOUBLE PRECISION

Description of the parameters of the signatures

Column	Type	Description
edges_sql	TEXT	SQL query as described above.
directed	BOOLEAN	(optional) Determines the type of the graph. Defaul

Description of the Result

Column	Type	Description
seq	INT	Sequential value starting from **1**.
edge	BIGINT	Identifier of the edge in the original query(edges_sql).
source	BIGINT	Identifier of the first end point of the edge.
target	BIGINT	Identifier of the second end point of the edge.

See Also

- *Flow - Family of functions*
- http://www.boost.org/libs/graph/doc/maximum_matching.html
- https://en.wikipedia.org/wiki/Matching_%28graph_theory%29
- https://en.wikipedia.org/wiki/Ackermann_function

Indices and tables

- genindex
- search

pgr_edgeDisjointPaths - Proposed

Name

`pgr_edgeDisjointPaths` — Calculates edge disjoint paths between two groups of vertices.

[21]

Fig. 6.19: Boost Graph Inside

Availability: 2.3.0

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  – The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  – Name might change.
  – Signature might change.
  – Functionality might change.
  – pgTap tests might be missing.
  – Might need c/c++ coding.
  – May lack documentation.
  – Documentation if any might need to be rewritten.
  – Documentation examples might need to be automatically generated.
  – Might need a lot of feedback from the comunity.
  – Might depend on a proposed function of pgRouting
  – Might depend on a deprecated function of pgRouting

Synopsis

Calculates the edge disjoint paths between two groups of vertices. Utilizes underlying maximum flow algorithms to calculate the paths.

Characteristics:

The main characterics are:

- Calculates the edge disjoint paths between any two groups of vertices.
- Returns EMPTY SET when source and destination are the same, or cannot be reached.
- The graph can be directed or undirected.
- One to many, many to one, many to many versions are also supported.
- Uses *pgr_boykovKolmogorov - Proposed* to calculate the paths.

Signature Summary

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid)
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid, directed)
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vids, directed)
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vid, directed)
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vids, directed)

RETURNS SET OF (seq, path_id, path_seq, [start_vid,] [end_vid,] node, edge, cost, agg_cost)
OR EMPTY SET
```

Signatures

Minimal use

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

The minimal use is for a **directed** graph from one `start_vid` to one `end_vid`.

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, 5
);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    3 |    2 |    1 |        0
   2 |       1 |        2 |    2 |    4 |    1 |        1
   3 |       1 |        3 |    5 |   -1 |    0 |        2
   4 |       2 |        1 |    3 |    5 |    1 |        0
   5 |       2 |        2 |    6 |    8 |    1 |        1
   6 |       2 |        3 |    5 |   -1 |    0 |        2
(6 rows)
```

One to One

This signature finds the set of dijoint paths from one `start_vid` to one `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vid, directed)
RETURNS SET OF (seq, path_id, path_seq, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, 5,
    directed := false
);
 seq | path_id | path_seq | node | edge | cost | agg_cost
-----+---------+----------+------+------+------+----------
   1 |       1 |        1 |    3 |    2 |    1 |        0
   2 |       1 |        2 |    2 |    4 |    1 |        1
   3 |       1 |        3 |    5 |   -1 |    0 |        2
   4 |       2 |        1 |    3 |    3 |   -1 |        0
   5 |       2 |        2 |    4 |   16 |    1 |       -1
   6 |       2 |        3 |    9 |    9 |    1 |        0
   7 |       2 |        4 |    6 |    8 |    1 |        1
   8 |       2 |        5 |    5 |   -1 |    0 |        2
   9 |       3 |        1 |    3 |    5 |    1 |        0
  10 |       3 |        2 |    6 |   11 |    1 |        1
  11 |       3 |        3 |   11 |   12 |   -1 |        2
  12 |       3 |        4 |   10 |   10 |    1 |        1
  13 |       3 |        5 |    5 |   -1 |    0 |        2
(13 rows)
```

One to Many

This signature finds the sset of disjoint paths from the `start_vid` to each one of the `end_vid` in `end_vids`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.
- The result is equivalent to the union of the results of the one to one *pgr_edgeDisjointPaths*.
- The extra `end_vid` in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vid, end_vids, directed)
RETURNS SET OF (seq, path_id, path_seq, end_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    3, ARRAY[4, 5, 10]
);
 seq | path_id | path_seq | end_vid | node | edge | cost | agg_cost
-----+---------+----------+---------+------+------+------+----------
   1 |       1 |        1 |       4 |    3 |    5 |    1 |        0
   2 |       1 |        2 |       4 |    6 |    9 |    1 |        1
   3 |       1 |        3 |       4 |    9 |   16 |    1 |        2
   4 |       1 |        4 |       4 |    4 |   -1 |    0 |        3
   5 |       2 |        1 |       5 |    3 |    2 |    1 |        0
   6 |       2 |        2 |       5 |    2 |    4 |    1 |        1
   7 |       2 |        3 |       5 |    5 |   -1 |    0 |        2
   8 |       3 |        1 |       5 |    3 |    5 |    1 |        0
   9 |       3 |        2 |       5 |    6 |    8 |    1 |        1
  10 |       3 |        3 |       5 |    5 |   -1 |    0 |        2
  11 |       4 |        1 |      10 |    3 |    2 |    1 |        0
  12 |       4 |        2 |      10 |    2 |    4 |    1 |        1
  13 |       4 |        3 |      10 |    5 |   10 |    1 |        2
  14 |       4 |        4 |      10 |   10 |   -1 |    0 |        3
(14 rows)
```

Many to One

This signature finds the set of disjoint paths from each one of the `start_vid` in `start_vids` to the `end_vid`:

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.
- The result is equivalent to the union of the results of the one to one *pgr_edgeDisjointPaths*.
- The extra `start_vid` in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vid, directed)
RETURNS SET OF (seq, path_id, path_seq, start_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[3, 6], 5
);
 seq | path_id | path_seq | start_vid | node | edge | cost | agg_cost
-----+---------+----------+-----------+------+------+------+----------
```

```
   1 |       1 |        1 |         0 |     3 |    2 |    1 |        0
   2 |       1 |        2 |         0 |     2 |    4 |    1 |        1
   3 |       1 |        3 |         0 |     5 |   -1 |    0 |        2
   4 |       2 |        1 |         1 |     3 |    5 |    1 |        0
   5 |       2 |        2 |         1 |     6 |    8 |    1 |        1
   6 |       2 |        3 |         1 |     5 |   -1 |    0 |        2
   7 |       3 |        1 |         2 |     6 |    8 |    1 |        0
   8 |       3 |        2 |         2 |     5 |   -1 |    0 |        1
   9 |       4 |        1 |         3 |     6 |    9 |    1 |        0
  10 |       4 |        2 |         3 |     9 |   16 |    1 |        1
  11 |       4 |        3 |         3 |     4 |    3 |    1 |        2
  12 |       4 |        4 |         3 |     3 |    2 |    1 |        3
  13 |       4 |        5 |         3 |     2 |    4 |    1 |        4
  14 |       4 |        6 |         3 |     5 |   -1 |    0 |        5
(14 rows)
```

Many to Many

This signature finds the set of disjoint paths from each one of the `start_vid` in `start_vids` to each one of the `end_vid`

- on a **directed** graph when `directed` flag is missing or is set to `true`.
- on an **undirected** graph when `directed` flag is set to `false`.
- The result is equivalent to the union of the results of the one to one *pgr_edgeDisjointPaths*.
- The extra `start_vid` and `end_vid` in the result is used to distinguish to which path it belongs.

```
pgr_edgeDisjointPaths(edges_sql, start_vids, end_vids, directed)
RETURNS SET OF (seq, path_seq, start_vid, end_vid, node, edge, cost, agg_cost)
OR EMPTY SET
```

Example

```
SELECT * FROM pgr_edgeDisjointPaths(
    'SELECT id, source, target, cost, reverse_cost FROM edge_table',
    ARRAY[3, 6], ARRAY[4, 5, 10]
);
 seq | path_id | path_seq | start_vid | end_vid | node | edge | cost | agg_cost
-----+---------+----------+-----------+---------+------+------+------+----------
   1 |       1 |        1 |         0 |       4 |    3 |    5 |    1 |        0
   2 |       1 |        2 |         0 |       4 |    6 |    9 |    1 |        1
   3 |       1 |        3 |         0 |       4 |    9 |   16 |    1 |        2
   4 |       1 |        4 |         0 |       4 |    4 |   -1 |    0 |        3
   5 |       2 |        1 |         1 |       5 |    3 |    2 |    1 |        0
   6 |       2 |        2 |         1 |       5 |    2 |    4 |    1 |        1
   7 |       2 |        3 |         1 |       5 |    5 |   -1 |    0 |        2
   8 |       3 |        1 |         2 |       5 |    3 |    5 |    1 |        0
   9 |       3 |        2 |         2 |       5 |    6 |    8 |    1 |        1
  10 |       3 |        3 |         2 |       5 |    5 |   -1 |    0 |        2
  11 |       4 |        1 |         3 |      10 |    3 |    2 |    1 |        0
  12 |       4 |        2 |         3 |      10 |    2 |    4 |    1 |        1
  13 |       4 |        3 |         3 |      10 |    5 |   10 |    1 |        2
  14 |       4 |        4 |         3 |      10 |   10 |   -1 |    0 |        3
  15 |       5 |        1 |         4 |       4 |    6 |    9 |    1 |        0
  16 |       5 |        2 |         4 |       4 |    9 |   16 |    1 |        1
  17 |       5 |        3 |         4 |       4 |    4 |   -1 |    0 |        2
  18 |       6 |        1 |         5 |       5 |    6 |    8 |    1 |        0
  19 |       6 |        2 |         5 |       5 |    5 |   -1 |    0 |        1
  20 |       7 |        1 |         6 |       5 |    6 |    9 |    1 |        0
  21 |       7 |        2 |         6 |       5 |    9 |   16 |    1 |        1
  22 |       7 |        3 |         6 |       5 |    4 |    3 |    1 |        2
```

```
  23 |        7 |        4 |        6 |        5 |     3 |     2 |     1 |        3
  24 |        7 |        5 |        6 |        5 |     2 |     4 |     1 |        4
  25 |        7 |        6 |        6 |        5 |     5 |    -1 |     0 |        5
  26 |        8 |        1 |        7 |       10 |     6 |     8 |     1 |        0
  27 |        8 |        2 |        7 |       10 |     5 |    10 |     1 |        1
  28 |        8 |        3 |        7 |       10 |    10 |    -1 |     0 |        2
(28 rows)
```

Description of the Signatures

Description of the edges_sql query for dijkstra like functions

edges_sql an SQL query, which should return a set of rows with the following columns:

Column	Type	Default	Description
id	`ANY-INTEGER`		Identifier of the edge.
source	`ANY-INTEGER`		Identifier of the first end point vertex of the edge.
target	`ANY-INTEGER`		Identifier of the second end point vertex of the edge.
cost	`ANY-NUMERICAL`		Weight of the edge *(source, target)* • When negative: edge *(source, target)* does not exist, therefore it's not part of the graph.
reverse_cost	`ANY-NUMERICAL`	-1	Weight of the edge *(target, source)*, • When negative: edge *(target, source)* does not exist, therefore it's not part of the graph.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Description of the parameters of the signatures

Column	Type	Default
sql	`TEXT`	
start_vid	`BIGINT`	
start_vids	`ARRAY[BIGINT]`	
end_vid	`BIGINT`	
end_vids	`ARRAY[BIGINT]`	
directed	`BOOLEAN`	`true`

Description of the return values for a path Returns set of `(seq, path_seq [, start_vid] [, end_vid], node, edge, cost, agg_cost)`

Column	Type	Description
seq	`INT`	Sequential value starting from **1**.
path_id	`INT`	Path identifier. Has value **1** for the first of a path. Used when there are multiple paths for the same `start_vid` to `end_vid` combination.
path_seq	`INT`	Relative position in the path. Has value **1** for the beginning of a path.
start_vid	`BIGINT`	Identifier of the starting vertex. Used when multiple starting vetrices are in the query.
end_vid	`BIGINT`	Identifier of the ending vertex. Used when multiple ending vertices are in the query.
node	`BIGINT`	Identifier of the node in the path from `start_vid` to `end_vid`.
edge	`BIGINT`	Identifier of the edge used to go from `node` to the next node in the path sequence. `-1` for the last node of the path.
cost	`FLOAT`	Cost to traverse from `node` using `edge` to the next node in the path sequence.
agg_cost	`FLOAT`	Aggregate cost from `start_v` to `node`.

See Also

- *Flow - Family of functions*

Indices and tables

- genindex
- search

Flow Functions General Information

Characteristics

- The graph is **directed**.

- Process is done only on edges with positive capacities.
- When the maximum flow is 0 then there is no flow and **EMPTY SET** is returned.
  - There is no flow when a **source** is the same as a **target**.
- Any duplicated value in the source(s) or target(s) are ignored.
- Calculates the flow/residual capacity for each edge. In the output
  - Edges with zero flow are omitted.
- Creates a **super source** and edges to all the source(s), and a **super target** and the edges from all the targets(s).
- The maximum flow through the graph is guaranteed to be the value returned by *pgr_maxFlow* when executed with the same parameters and can be calculated:
  - By aggregation of the outgoing flow from the sources
  - By aggregation of the incoming flow to the targets

pgr_maxFlow is the maximum Flow and that maximum is guaranteed to be the same on the functions *pgr_pushRelabel*, *pgr_edmondsKarp*, *pgr_boykovKolmogorov*, but the actual flow through each edge may vary.

Problem definition

A flow network is a directed graph where each edge has a capacity and a flow. The flow through an edge must not exceed the capacity of the edge. Additionally, the incoming and outgoing flow of a node must be equal except the for source which only has outgoing flow, and the destination(sink) which only has incoming flow.

Maximum flow algorithms calculate the maximum flow through the graph and the flow of each edge.

The maximum flow through the graph is guaranteed to be the same with all implementations, but the actual flow through each edge may vary. Given the following query:

pgr_maxFlow $(edges\_sql, source\_vertex, sink\_vertex)$

where $edges\_sql = \{(id_i, source_i, target_i, capacity_i, reverse\_capacity_i)\}$

Graph definition

The weighted directed graph, $G(V, E)$, is defined as:

- the set of vertices V
  - $source\_vertex \cup sink\_vertex \bigcup source_i \bigcup target_i$
- the set of edges E
  - $E = \begin{cases} \text{if } reverse\_capacity = & \{(source_i, target_i, capacity_i) \text{ when } capacity > 0\} \\ & \\ & \{(source_i, target_i, capacity_i) \text{ when } capacity > 0\} \\ \cup & \{(target_i, source_i, reverse\_capacity_i) \text{ when } reverse\_capacity_i > 0)\} \\ \text{if } reverse\_capacity \neq & \end{cases}$

Maximum flow problem

Given:

- $G(V, E)$
- $source\_vertex \in V$ the source vertex

- $sink\_vertex \in V$ the sink vertex

Then:

$$pgr\_maxFlow(edges\_sql, source, sink) = \boldsymbol{\Phi}$$

$$\boldsymbol{\Phi} = (id_i, edge\_id_i, source_i, target_i, flow_i, residual\_capacity_i)$$

Where:

$\boldsymbol{\Phi}$ is a subset of the original edges with their residual capacity and flow. The maximum flow through the graph can be obtained by aggregating on the source or sink and summing the flow from/to it. In particular:

- $id_i = i$
- $edge\_id = id_i$ in edges_sql
- $residual\_capacity_i = capacity_i - flow_i$

See Also

- https://en.wikipedia.org/wiki/Maximum_flow_problem

6.2.3 pgr_labelGraph - Proposed

Name

`pgr_labelGraph` — Locates and labels sub-networks within a network which are not topologically connected.

> **Warning:** Experimental functions
> - They are not officially of the current release.
> - They likely will not be officially be part of the next release:
>   - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
>   - Name might change.
>   - Signature might change.
>   - Functionality might change.
>   - pgTap tests might be missing.
>   - Might need c/c++ coding.
>   - May lack documentation.
>   - Documentation if any might need to be rewritten.
>   - Documentation examples might need to be automatically generated.
>   - Might need a lot of feedback from the comunity.
>   - Might depend on a proposed function of pgRouting
>   - Might depend on a deprecated function of pgRouting

Synopsis

Must be run after `pgr_createTopology()`. No use of `geometry` column. Only `id`, `source` and `target` columns are required.

The function returns:

- `OK` when a column with provided name has been generated and populated successfully. All connected edges will have unique similar integer values. In case of `rows_where` condition, non participating rows will have -1 integer values.
- `FAIL` when the processing cannot be finished due to some error. Notice will be thrown accordingly.
- `rows_where condition generated 0 rows` when passed SQL condition has not been fulfilled by any row.

```
varchar pgr_labelGraph(text, text, text, text, text, text)
```

Description

A network behind any routing query may consist of sub-networks completely isolated from each other. Possible reasons could be:

- An island with no bridge connecting to the mainland.
- An edge or mesh of edges failed to connect to other networks because of human negligence during data generation.
- The data is not properly noded.
- Topology creation failed to succeed.

pgr_labelGraph() will create an integer column (with the name provided by the user) and will assign same integer values to all those edges in the network which are connected topologically. Thus better analysis regarding network structure is possible. In case of `rows_where` condition, non participating rows will have -1 integer values.

Prerequisites: Must run `pgr_createTopology()` in order to generate `source` and `target` columns. Primary key column `id` should also be there in the network table.

Function accepts the following parameters:

edge_table `text` Network table name, with optional schema name.

id `text` Primary key column name of the network table. Default is `id`.

source `text` Source column name generated after `pgr_createTopology()`. Default is `source`.

target `text` Target column name generated after `pgr_createTopology()`. Default is `target`.

subgraph `text` Column name which will hold the integer labels for each sub-graph. Default is `subgraph`.

rows_where `text` The SQL where condition. Default is `true`, means the processing will be done on the whole table.

Example Usage

The sample data, has 3 subgraphs.

```
SET client_min_messages TO WARNING;
SET
SELECT pgr_labelGraph('edge_table', 'id', 'source', 'target', 'subgraph');
 pgr_labelgraph
----------------
 OK
(1 row)

SELECT DISTINCT subgraph FROM edge_table ORDER BY subgraph;
 subgraph
----------
        1
        2
        3
(3 rows)
```

See Also

- pgr_createTopology[22] to create the topology of a table based on its geometry and tolerance value.

6.2.4 Vehicle Routing Functions Category

Contents

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

Pickup and delivery problem

- pgr_pickDeliver - Proposed - Pickup & Delivery using a Cost Matrix
- pgr_pickDeliverEuclidean - Proposed - Pickup & Delivery with Euclidean distances

Experimental functions

- pgr_vrpOneDepot - experimental - VRP One Depot

[22] https://github.com/Zia-/pgrouting/blob/develop/src/common/sql/pgrouting_topology.sql

pgr_pickDeliver - Proposed

Warning: Documentation is being updated

`pgr_pickDeliver` - Pickup and delivery Vehicle Routing Problem

Availability

- New as proposed in 2.5.0

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

Synopsis

Problem: Distribute and optimize the pickup-delivery pairs into a fleet of vehicles.

- Optimization problem is NP-hard.
- pickup and Delivery with time windows.
- All vehicles are equal.
  - Same Starting location.
  - Same Ending location which is the same as Starting location.
  - All vehicles travel at the same speed.
- A customer is for doing a pickup or doing a deliver.
  - has an open time.
  - has a closing time.
  - has a service time.
  - has an (x, y) location.
- There is a customer where to deliver a pickup.
  - travel time between customers is distance / speed
  - pickup and delivery pair is done with the same vehicle.
  - A pickup is done before the delivery.

Characteristics

- All trucks depart at time 0.
- No multiple time windows for a location.
- Less vehicle used is considered better.

- Less total duration is better.
- Less wait time is better.
- the algorithm will raise an exception when
  - If there is a pickup-deliver pair than violates time window
  - The speed, max_cycles, ma_capacity have illegal values
- Six different initial will be optimized - the best solution found will be result

Signature

```
pgr_pickDeliver(orders_sql, vehicles_sql, matrix_sql [, factor, max_cycles, initial_sol])
RETURNS SET OF (seq, vehicle_number, vehicle_id, stop, order_id, stop_type, cargo,
                travel_time, arrival_time, wait_time, service_time, departure_time)
```

Parameters The parameters are:

```
orders_sql, vehicles_sql, matrix_sql [, factor, max_cycles, initial_sol]
```

Column	Type	Default	Description
orders_sql	`TEXT`		*Pick & Deliver Orders SQL* query contianing the orders to be processed.
vehicles_sql	`TEXT`		*Pick & Deliver Vehicles SQL* query containing the vehicles to be used.
matrix_sql	`TEXT`		*Pick & Deliver Matrix SQL* query containing the distance or travel times.
factor	`NUMERIC`	1	Travel time multiplier. See *Factor Handling*
max_cycles	`INTEGER`	10	Maximum number of cycles to perform on the optimization.
initial_sol	`INTEGER`	4	Initial solution to be used. • `1` One order per truck • `2` Push front order. • `3` Push back order. • `4` Optimize insert. • `5` Push back order that allows more orders to be inserted at the back • `6` Push front order that allows more orders to be inserted at the front

Pick & Deliver Orders SQL A *SELECT* statement that returns the following columns:

```
id, demand
p_node_id, p_open, p_close, [p_service, ]
d_node_id, d_open, d_close, [d_service, ]
```

where:

Column	Type	Default	Description
id	ANY-INTEGER (SMALLINT, INTEGER, BIGINT)		Identifier of the pick-delivery order pair.
demand	ANY-NUMERICAL (SMALLINT, INTEGER, BIGINT, REAL, FLOAT)		Number of units in the order
p_open	ANY-NUMERICAL		The time, relative to 0, when the pickup location opens.
p_close	ANY-NUMERICAL		The time, relative to 0, when the pickup location closes.
d_service	ANY-NUMERICAL	0	The duration of the loading at the pickup location.
d_open	ANY-NUMERICAL		The time, relative to 0, when the delivery location opens.
d_close	ANY-NUMERICAL		The time, relative to 0, when the delivery location closes.
d_service	ANY-NUMERICAL	0	The duration of the loading at the delivery location.

For the non euclidean implementation, the starting and ending identifiers are needed:

Column	Type	Description
p_node_id	ANY-INTEGER	The node identifier of the pickup, must match a node identifier in the matrix table.
d_node_id	ANY-INTEGER	The node identifier of the delivery, must match a node identifier in the matrix table.

Pick & Deliver Vehicles SQL A *SELECT* statement that returns the following columns:

```
id, capacity
start_node_id, start_open, start_close [, start_service, ]
[ end_node_id, end_open, end_close, end_service ]
```

where:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the pick-delivery order pair.
capacity	ANY-NUMERICAL		Number of units in the order
speed	ANY-NUMERICAL	*1*	Average speed of the vehicle.
start_open	ANY-NUMERICAL		The time, relative to 0, when the starting location opens.
start_close	ANY-NUMERICAL		The time, relative to 0, when the starting location closes.
start_service	ANY-NUMERICAL	*0*	The duration of the loading at the starting location.
end_open	ANY-NUMERICAL	*start_open*	The time, relative to 0, when the ending location opens.
end_close	ANY-NUMERICAL	*start_close*	The time, relative to 0, when the ending location closes.
end_service	ANY-NUMERICAL	*start_service*	The duration of the loading at the ending location.

For the non euclidean implementation, the starting and ending identifiers are needed:

Column	Type	Default	Description
start_node_id	ANY-INTEGER		The node identifier of the starting location, must match a node identifier in the matrix table.
end_node_id	ANY-INTEGER	*start_node_id*	The node identifier of the ending location, must match a node identifier in the matrix table.

Pick & Deliver Matrix SQL A *SELECT* statement that returns the following columns:

Warning: TODO

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Example

This example use the following data: TODO put link

```
SELECT * FROM pgr_pickDeliver(
    $$ SELECT * FROM orders ORDER BY id $$,
    $$ SELECT * FROM vehicles $$,
    $$ SELECT * from pgr_dijkstraCostMatrix(
        ' SELECT * FROM edge_table ', ARRAY[3, 4, 5, 8, 9, 11])
    $$
);
 seq | vehicle_seq | vehicle_id | stop_seq | stop_type | stop_id | order_id | cargo | travel_time
-----+-------------+------------+----------+-----------+---------+----------+-------+-------------
   1 |           1 |          1 |        1 |         1 |       6 |       -1 |     0 |           0
   2 |           1 |          1 |        2 |         2 |       5 |        3 |    30 |           1
   3 |           1 |          1 |        3 |         3 |      11 |        3 |     0 |           2
   4 |           1 |          1 |        4 |         2 |       9 |        2 |    20 |           2
   5 |           1 |          1 |        5 |         3 |       4 |        2 |     0 |           1
   6 |           1 |          1 |        6 |         6 |       6 |       -1 |     0 |           4
   7 |           2 |          1 |        1 |         1 |       6 |       -1 |     0 |           0
   8 |           2 |          1 |        2 |         2 |       3 |        1 |    10 |           5
   9 |           2 |          1 |        3 |         3 |       8 |        1 |     0 |           3
  10 |           2 |          1 |        4 |         6 |       6 |       -1 |     0 |           0
  11 |          -2 |          0 |        0 |        -1 |      -1 |       -1 |    -1 |          18
(11 rows)
```

See Also

- *Vehicle Routing Functions Category*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_pickDeliverEuclidean - Proposed

Warning: Documentation is being updated

`pgr_pickDeliverEuclidean` - Pickup and delivery Vehicle Routing Problem

Availability

- New as proposed in 2.5.0

Warning: Proposed functions for next mayor release.

- They are not officially in the current release.
- They will likely officially be part of the next mayor release:
  - The functions make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might not change. (But still can)
  - Signature might not change. (But still can)
  - Functionality might not change. (But still can)
  - pgTap tests have being done. But might need more.
  - Documentation might need refinement.

Synopsis

Problem: Distribute and optimize the pickup-delivery pairs into a fleet of vehicles.

- Optimization problem is NP-hard.
- Pickup and Delivery:
  - capacitated
  - with time windows.
- The vehicles
  - have (x, y) start and ending locations.
  - have a start and ending service times.
  - have opening and closing times for the start and ending locations.
- An order is for doing a pickup and a a deliver.
  - has (x, y) pickup and delivery locations.
  - has opening and closing times for the pickup and delivery locations.
  - has a pickup and deliver service times.
- There is a customer where to deliver a pickup.
  - travel time between customers is distance / speed
  - pickup and delivery pair is done with the same vehicle.
  - A pickup is done before the delivery.

Characteristics

- No multiple time windows for a location.
- Less vehicle used is considered better.

- Less total duration is better.
- Less wait time is better.
- Six different optional different initial solutions
  - the best solution found will be result

Signature

```
pgr_pickDeliverEuclidean(orders_sql, vehicles_sql [,factor, max_cycles, initial_sol])
RETURNS SET OF (seq, vehicle_seq, vehicle_id, stop_seq, stop_type, order_id,
    cargo, travel_time, arrival_time, wait_time, service_time, departure_time)
```

Parameters The parameters are:

```
orders_sql, vehicles_sql [,factor, max_cycles, initial_sol]
```

Where:

Column	Type	Default	Description
orders_sql	`TEXT`		*Pick & Deliver Orders SQL* query containing the orders to be processed.
vehicles_sql	`TEXT`		*Pick & Deliver Vehicles SQL* query containing the vehicles to be used.
factor	`NUMERIC`	1	(Optional) Travel time multiplier. See *Factor Handling*
max_cycles	`INTEGER`	10	(Optional) Maximum number of cycles to perform on the optimization.
initial_sol	`INTEGER`	4	(Optional) Initial solution to be used. • `1` One order per truck • `2` Push front order. • `3` Push back order. • `4` Optimize insert. • `5` Push back order that allows more orders to be inserted at the back • `6` Push front order that allows more orders to be inserted at the front

Pick & Deliver Orders SQL A *SELECT* statement that returns the following columns:

```
id, demand
p_x, p_y, p_open, p_close, [p_service, ]
d_x, d_y, d_open, d_close, [d_service, ]
```

Where:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the pick-delivery order pair.
demand	ANY-NUMERICAL		Number of units in the order
p_open	ANY-NUMERICAL		The time, relative to 0, when the pickup location opens.
p_close	ANY-NUMERICAL		The time, relative to 0, when the pickup location closes.
d_service	ANY-NUMERICAL	0	The duration of the loading at the pickup location.
d_open	ANY-NUMERICAL		The time, relative to 0, when the delivery location opens.
d_close	ANY-NUMERICAL		The time, relative to 0, when the delivery location closes.
d_service	ANY-NUMERICAL	0	The duration of the loading at the delivery location.

For the euclidean implementation, pick up and delivery (x, y) locations are needed:

Column	Type	Description
p_x	ANY-NUMERICAL	x value of the pick up location
p_y	ANY-NUMERICAL	y value of the pick up location
d_x	ANY-NUMERICAL	x value of the delivery location
d_y	ANY-NUMERICAL	y value of the delivery location

Pick & Deliver Vehicles SQL A *SELECT* statement that returns the following columns:

```
id, capacity
start_x, start_y, start_open, start_close [, start_service, ]
[ end_x, end_y, end_open, end_close, end_service ]
```

where:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the pick-delivery order pair.
capacity	ANY-NUMERICAL		Number of units in the order
speed	ANY-NUMERICAL	*1*	Average speed of the vehicle.
start_open	ANY-NUMERICAL		The time, relative to 0, when the starting location opens.
start_close	ANY-NUMERICAL		The time, relative to 0, when the starting location closes.
start_service	ANY-NUMERICAL	*0*	The duration of the loading at the starting location.
end_open	ANY-NUMERICAL	*start_open*	The time, relative to 0, when the ending location opens.
end_close	ANY-NUMERICAL	*start_close*	The time, relative to 0, when the ending location closes.
end_service	ANY-NUMERICAL	*start_service*	The duration of the loading at the ending location.

For the euclidean implementation, starting and ending (x, y) locations are needed:

Column	Type	Default	Description
start_x	ANY-NUMERICAL		x value of the coordinate of the starting location.
start_y	ANY-NUMERICAL		y value of the coordinate of the starting location.
end_x	ANY-NUMERICAL	*start_x*	x value of the coordinate of the ending location.
end_y	ANY-NUMERICAL	*start_y*	y value of the coordinate of the ending location.

Description of the result (TODO Disussion: Euclidean & Matrix)

```
RETURNS SET OF
    (seq, vehicle_seq, vehicle_id, stop_seq, stop_type,
        travel_time, arrival_time, wait_time, service_time,  departure_time)
    UNION
    (summary row)
```

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
vehicle_seq	INTEGER	Sequential value starting from **1** for current vehicles. The n_{th} vehicle in the solution.
vehicle_id	BIGINT	Current vehicle identifier.
stop_seq	INTEGER	Sequential value starting from **1** for the stops made by the current vehicle. The m_{th} stop of the current vehicle.
stop_type	INTEGER	Kind of stop location the vehicle is at: • `1`: Starting location • `2`: Pickup location • `3`: Delivery location • `6`: Ending location
order_id	BIGINT	Pickup-Delivery order pair identifier. • `-1`: When no order is involved on the current stop location.
cargo	FLOAT	Cargo units of the vehicle when leaving the stop.
travel_time	FLOAT	Travel time from previous `stop_seq` to current `stop_seq`. • `0` When `stop_type = 1`
arrival_time	FLOAT	Previous `departure_time` plus current `travel_time`.
wait_time	FLOAT	Time spent waiting for current *location* to open.
service_time	FLOAT	Service time at current *location*.
departure_time	FLOAT	$arrival\_time + wait\_time + service\_time$. • When `stop_type = 6` has the *total_time* used for the current vehicle.

Summary Row

Warning: TODO: Review the summary

Column	Type	Description
seq	INTEGER	Continues the Sequential value
vehicle_seq	INTEGER	-2 to indicate is a summary row
vehicle_id	BIGINT	*Total Capacity Violations* in the solution.
stop_seq	INTEGER	*Total Time Window Violations* in the solution.
stop_type	INTEGER	-1
order_id	BIGINT	-1
cargo	FLOAT	-1
travel_time	FLOAT	*total_travel_time* The sum of all the *travel_time*
arrival_time	FLOAT	-1
wait_time	FLOAT	*total_waiting_time* The sum of all the *wait_time*
service_time	FLOAT	*total_service_time* The sum of all the *service_time*
departure_time	FLOAT	*total_solution_time* = $total\_travel\_time + total\_wait\_time + total\_service\_time$.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Example

This example use the following data: TODO put link

```
SELECT * FROM pgr_pickDeliverEuclidean(
    'SELECT * FROM orders ORDER BY id',
    'SELECT * from vehicles'
);
 seq | vehicle_seq | vehicle_id | stop_seq | stop_type | order_id | cargo |   travel_time    |   a
-----+-------------+------------+----------+-----------+----------+-------+------------------+----
   1 |           1 |          1 |        1 |         1 |       -1 |     0 |                0 |
   2 |           1 |          1 |        2 |         2 |        3 |    30 |                1 |
   3 |           1 |          1 |        3 |         3 |        3 |     0 |  1.4142135623731 | 6.
   4 |           1 |          1 |        4 |         2 |        2 |    20 |  1.4142135623731 | 10
   5 |           1 |          1 |        5 |         3 |        2 |     0 |                1 | 13
   6 |           1 |          1 |        6 |         6 |       -1 |     0 |  1.4142135623731 | 18
   7 |           2 |          1 |        1 |         1 |       -1 |     0 |                0 |
   8 |           2 |          1 |        2 |         2 |        1 |    10 |                1 |
   9 |           2 |          1 |        3 |         3 |        1 |     0 | 2.23606797749979 | 7.
  10 |           2 |          1 |        4 |         6 |       -1 |     0 |                2 | 12
  11 |          -2 |          0 |        0 |        -1 |       -1 |    -1 | 11.4787086646191 |
(11 rows)
```

See Also

- *Vehicle Routing Functions Category*
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

pgr_vrpOneDepot - experimental

Warning: Experimental functions

- They are not officially of the current release.
- They likely will not be officially be part of the next release:
  - The functions might not make use of ANY-INTEGER and ANY-NUMERICAL
  - Name might change.
  - Signature might change.
  - Functionality might change.
  - pgTap tests might be missing.
  - Might need c/c++ coding.
  - May lack documentation.
  - Documentation if any might need to be rewritten.
  - Documentation examples might need to be automatically generated.
  - Might need a lot of feedback from the comunity.
  - Might depend on a proposed function of pgRouting
  - Might depend on a deprecated function of pgRouting

No documentation available

Example:

Under discussion

Current Result

```
BEGIN;
BEGIN
SET client_min_messages TO NOTICE;
SET
SELECT * FROM pgr_vrpOneDepot(
    'SELECT * FROM vrp_orders',
    'SELECT * FROM vrp_vehicles',
    'SELECT * FROM vrp_distance',
    1);
 oid | opos | vid | tarrival | tdepart
-----+------+-----+----------+---------
  -1 |    1 |   5 |        0 |       0
  66 |    2 |   5 |        0 |       0
  25 |    3 |   5 |        0 |       0
  21 |    4 |   5 |        0 |       0
  84 |    5 |   5 |        0 |       0
  50 |    6 |   5 |        0 |       0
  49 |    7 |   5 |        0 |       0
  24 |    8 |   5 |        0 |       0
  22 |    9 |   5 |        0 |       0
  20 |   10 |   5 |        0 |       0
  19 |   11 |   5 |        0 |       0
  66 |   12 |   5 |       11 |      21
  84 |   13 |   5 |       30 |      45
  24 |   14 |   5 |       71 |      81
  22 |   15 |   5 |       83 |      93
  20 |   16 |   5 |       98 |     108
  19 |   17 |   5 |      114 |     124
  50 |   18 |   5 |      131 |     141
  21 |   19 |   5 |      144 |     154
  25 |   20 |   5 |      158 |     168
  49 |   21 |   5 |      179 |     189
```

```
   -1 |   22 |   5 |      234 |      234
   -1 |    1 |   6 |        0 |        0
   31 |    2 |   6 |        0 |        0
   32 |    3 |   6 |        0 |        0
   81 |    4 |   6 |        0 |        0
   94 |    5 |   6 |        0 |        0
   93 |    6 |   6 |        0 |        0
   35 |    7 |   6 |        0 |        0
   33 |    8 |   6 |        0 |        0
   28 |    9 |   6 |        0 |        0
   27 |   10 |   6 |        0 |        0
   93 |   11 |   6 |       15 |       25
   32 |   12 |   6 |       61 |       71
   28 |   13 |   6 |       78 |       88
   31 |   14 |   6 |       97 |      107
   35 |   15 |   6 |      112 |      122
   27 |   16 |   6 |      134 |      144
   33 |   17 |   6 |      152 |      162
   94 |   18 |   6 |      196 |      206
   81 |   19 |   6 |      221 |      231
   -1 |   20 |   6 |      238 |      238
   -1 |    1 |   3 |        0 |        0
   16 |    2 |   3 |        0 |        0
   14 |    3 |   3 |        0 |        0
   48 |    4 |   3 |        0 |        0
   18 |    5 |   3 |        0 |        0
   17 |    6 |   3 |        0 |        0
   15 |    7 |   3 |        0 |        0
   13 |    8 |   3 |        0 |        0
   11 |    9 |   3 |        0 |        0
   10 |   10 |   3 |        0 |        0
   15 |   11 |   3 |       35 |       45
   48 |   12 |   3 |       48 |       58
   13 |   13 |   3 |       64 |       74
   16 |   14 |   3 |       82 |       92
   17 |   15 |   3 |       94 |      104
   10 |   16 |   3 |      115 |      125
   11 |   17 |   3 |      130 |      140
   14 |   18 |   3 |      147 |      157
   18 |   19 |   3 |      169 |      179
   -1 |   20 |   3 |      219 |      219
   -1 |    1 |   8 |        0 |        0
   71 |    2 |   8 |        0 |        0
   55 |    3 |   8 |        0 |        0
   44 |    4 |   8 |        0 |        0
   43 |    5 |   8 |        0 |        0
   42 |    6 |   8 |        0 |        0
   41 |    7 |   8 |        0 |        0
   40 |    8 |   8 |        0 |        0
   39 |    9 |   8 |        0 |        0
   43 |   10 |   8 |       34 |       44
   40 |   11 |   8 |       49 |       59
   39 |   12 |   8 |       61 |       85
   41 |   13 |   8 |       90 |      100
   42 |   14 |   8 |      111 |      121
   44 |   15 |   8 |      131 |      141
   55 |   16 |   8 |      166 |      176
   71 |   17 |   8 |      198 |      208
   -1 |   18 |   8 |      228 |      228
   -1 |    1 |   1 |        0 |        0
    4 |    2 |   1 |        0 |        0
  101 |    3 |   1 |        0 |        0
   46 |    4 |   1 |        0 |        0
```

```
   5 |    5 |   1 |        0 |        0
   3 |    6 |   1 |        0 |        0
  46 |    7 |   1 |       38 |       48
   3 |    8 |   1 |       55 |       65
   2 |    9 |   1 |       96 |       96
   4 |   10 |   1 |      135 |      145
   2 |   11 |   1 |      148 |      158
   5 |   12 |   1 |      165 |      175
 101 |   13 |   1 |      192 |      202
  -1 |   14 |   1 |      222 |      222
  -1 |    1 |  13 |        0 |        0
  92 |    2 |  13 |        0 |        0
  52 |    3 |  13 |        0 |        0
  57 |    4 |  13 |        0 |        0
  85 |    5 |  13 |        0 |        0
  68 |    6 |  13 |        0 |        0
  63 |    7 |  13 |        0 |        0
  63 |    8 |  13 |       29 |       62
  68 |    9 |  13 |       69 |       80
  52 |   10 |  13 |      104 |      114
  85 |   11 |  13 |      123 |      133
  57 |   12 |  13 |      142 |      152
  92 |   13 |  13 |      159 |      177
  -1 |   14 |  13 |      189 |      189
  -1 |    1 |   7 |        0 |        0
  30 |    2 |   7 |        0 |        0
  29 |    3 |   7 |        0 |        0
  38 |    4 |   7 |        0 |        0
  36 |    5 |   7 |        0 |        0
  34 |    6 |   7 |        0 |        0
  34 |    7 |   7 |       51 |       61
  29 |    8 |   7 |       70 |       80
  30 |    9 |   7 |       85 |       95
  38 |   10 |   7 |      149 |      159
  36 |   11 |   7 |      162 |      172
  -1 |   12 |   7 |      217 |      217
  -1 |    1 |   2 |        0 |        0
  89 |    2 |   2 |        0 |        0
  47 |    3 |   2 |        0 |        0
  61 |    4 |   2 |        0 |        0
   9 |    5 |   2 |        0 |        0
   8 |    6 |   2 |        0 |        0
  89 |    7 |   2 |       18 |       77
   8 |    8 |   2 |       96 |      106
   9 |    9 |   2 |      111 |      121
  47 |   10 |   2 |      124 |      134
  61 |   11 |   2 |      154 |      165
  -1 |   12 |   2 |      192 |      192
  -1 |    1 |  14 |        0 |        0
  97 |    2 |  14 |        0 |        0
  64 |    3 |  14 |        0 |        0
  51 |    4 |  14 |        0 |        0
  96 |    5 |  14 |        0 |        0
  77 |    6 |  14 |        0 |        0
  96 |    7 |  14 |       21 |       44
  64 |    8 |  14 |       63 |       73
  77 |    9 |  14 |       83 |       93
  51 |   10 |  14 |      119 |      129
  97 |   11 |  14 |      154 |      164
  -1 |   12 |  14 |      180 |      180
  -1 |    1 |  15 |        0 |        0
  67 |    2 |  15 |        0 |        0
  73 |    3 |  15 |        0 |        0
```

```
  95 |    4 |  15 |         0 |        0
  82 |    5 |  15 |         0 |        0
  72 |    6 |  15 |         0 |        0
  73 |    7 |  15 |        27 |       40
  72 |    8 |  15 |        50 |       75
  82 |    9 |  15 |        91 |      101
  95 |   10 |  15 |       114 |      124
  67 |   11 |  15 |       144 |      154
  -1 |   12 |  15 |       167 |      167
  -1 |    1 |  11 |         0 |        0
  78 |    2 |  11 |         0 |        0
  26 |    3 |  11 |         0 |        0
  87 |    4 |  11 |         0 |        0
  23 |    5 |  11 |         0 |        0
  87 |    6 |  11 |        32 |       97
  23 |    7 |  11 |       118 |      128
  78 |    8 |  11 |       149 |      160
  26 |    9 |  11 |       172 |      182
  -1 |   10 |  11 |       227 |      227
  -1 |    1 |   4 |         0 |        0
  60 |    2 |   4 |         0 |        0
  59 |    3 |   4 |         0 |        0
 100 |    4 |   4 |         0 |        0
  54 |    5 |   4 |         0 |        0
  60 |    6 |   4 |        42 |       52
 100 |    7 |   4 |        74 |       87
  54 |    8 |   4 |       103 |      113
  59 |    9 |   4 |       153 |      163
  -1 |   10 |   4 |       211 |      211
  -1 |    1 |  10 |         0 |        0
  86 |    2 |  10 |         0 |        0
  90 |    3 |  10 |         0 |        0
  65 |    4 |  10 |         0 |        0
  53 |    5 |  10 |         0 |        0
  53 |    6 |  10 |        25 |       62
  65 |    7 |  10 |        82 |       92
  86 |    8 |  10 |       111 |      121
  90 |    9 |  10 |       140 |      154
  -1 |   10 |  10 |       206 |      206
  -1 |    1 |  12 |         0 |        0
   6 |    2 |  12 |         0 |        0
  80 |    3 |  12 |         0 |        0
   7 |    4 |  12 |         0 |        0
  56 |    5 |  12 |         0 |        0
   6 |    6 |  12 |        40 |       51
  80 |    7 |  12 |        73 |       99
   7 |    8 |  12 |       113 |      123
  56 |    9 |  12 |       142 |      152
  -1 |   10 |  12 |       166 |      166
  -1 |    1 |  19 |         0 |        0
  88 |    2 |  19 |         0 |        0
  70 |    3 |  19 |         0 |        0
  58 |    4 |  19 |         0 |        0
  99 |    5 |  19 |         0 |        0
  70 |    6 |  19 |         9 |       51
  99 |    7 |  19 |        56 |       66
  88 |    8 |  19 |        97 |      107
  58 |    9 |  19 |       125 |      135
  -1 |   10 |  19 |       162 |      162
  -1 |    1 |  17 |         0 |        0
  75 |    2 |  17 |         0 |        0
  98 |    3 |  17 |         0 |        0
  76 |    4 |  17 |         0 |        0
```

```
  76 |     5 |  17 |       57 |      84
  98 |     6 |  17 |       97 |     130
  75 |     7 |  17 |      146 |     156
  -1 |     8 |  17 |      192 |     192
  -1 |     1 |  16 |        0 |       0
  69 |     2 |  16 |        0 |       0
  79 |     3 |  16 |        0 |       0
  74 |     4 |  16 |        0 |       0
  74 |     5 |  16 |       39 |      87
  79 |     6 |  16 |       94 |     104
  69 |     7 |  16 |      136 |     154
  -1 |     8 |  16 |      164 |     164
  -1 |     1 |   9 |        0 |       0
  62 |     2 |   9 |        0 |       0
  37 |     3 |   9 |        0 |       0
  45 |     4 |   9 |        0 |       0
  37 |     5 |   9 |       43 |      53
  45 |     6 |   9 |       63 |      74
  62 |     7 |   9 |       94 |     104
  -1 |     8 |   9 |      120 |     120
  -1 |     1 |  18 |        0 |       0
  91 |     2 |  18 |        0 |       0
  12 |     3 |  18 |        0 |       0
  12 |     4 |  18 |       34 |      69
  91 |     5 |  18 |       99 |     109
  -1 |     6 |  18 |      113 |     113
  -1 |     1 |  20 |        0 |       0
  83 |     2 |  20 |        0 |       0
  83 |     3 |  20 |       15 |      52
  -1 |     4 |  20 |       67 |      67
  -1 |     0 |   0 |       -1 |    3712
(241 rows)

ROLLBACK;
ROLLBACK
```

Current Result

```
BEGIN;
BEGIN
SET client_min_messages TO NOTICE;
SET
SELECT * FROM _pgr_vrpOneDepot(
    'SELECT * FROM vrp_orders',
    'SELECT * FROM vrp_vehicles',
    'SELECT src_id AS start_vid, dest_id AS end_vid, traveltime AS agg_cost FROM vrp_distance',
    1);
 seq | vehicle_seq | vehicle_id | stop_seq | stop_type | stop_id | order_id | cargo  | travel_time
-----+-------------+------------+----------+-----------+---------+----------+--------+-------------
   1 |           1 |          5 |        1 |         1 |       1 |       -1 |      0 |            0
   2 |           1 |          5 |        2 |         2 |       1 |       66 |     16 |            0
   3 |           1 |          5 |        3 |         2 |       1 |       25 |     26 |            0
   4 |           1 |          5 |        4 |         2 |       1 |       21 |     36 |            0
   5 |           1 |          5 |        5 |         2 |       1 |       84 |     50 |            0
   6 |           1 |          5 |        6 |         2 |       1 |       50 |     60 |            0
   7 |           1 |          5 |        7 |         2 |       1 |       49 |     70 |            0
   8 |           1 |          5 |        8 |         2 |       1 |       24 |    100 |            0
   9 |           1 |          5 |        9 |         2 |       1 |       22 |    110 |            0
  10 |           1 |          5 |       10 |         2 |       1 |       20 |    150 |            0
  11 |           1 |          5 |       11 |         2 |       1 |       19 |    170 |            0
  12 |           1 |          5 |       12 |         3 |      66 |       66 |    154 |     11.18034
```

```
 13 |          1 |         5 |          13 |         3 |         84 |         84 |  140 |  9.219544
 14 |          1 |         5 |          14 |         3 |         24 |         24 |  110 | 26.019224
 15 |          1 |         5 |          15 |         3 |         22 |         22 |  100 |         2
 16 |          1 |         5 |          16 |         3 |         20 |         20 |   60 |  5.385165
 17 |          1 |         5 |          17 |         3 |         19 |         19 |   40 |  5.385165
 18 |          1 |         5 |          18 |         3 |         50 |         50 |   30 |   7.28011
 19 |          1 |         5 |          19 |         3 |         21 |         21 |   20 |         3
 20 |          1 |         5 |          20 |         3 |         25 |         25 |   10 |         4
 21 |          1 |         5 |          21 |         3 |         49 |         49 |    0 |  10.77033
 22 |          1 |         5 |          22 |         6 |          1 |         -1 |    0 | 45.044423
 23 |          2 |         6 |           1 |         1 |          1 |         -1 |    0 |         0
 24 |          2 |         6 |           2 |         2 |          1 |         31 |   10 |         0
 25 |          2 |         6 |           3 |         2 |          1 |         32 |   30 |         0
 26 |          2 |         6 |           4 |         2 |          1 |         81 |   43 |         0
 27 |          2 |         6 |           5 |         2 |          1 |         94 |   46 |         0
 28 |          2 |         6 |           6 |         2 |          1 |         93 |   60 |         0
 29 |          2 |         6 |           7 |         2 |          1 |         35 |   90 |         0
 30 |          2 |         6 |           8 |         2 |          1 |         33 |  100 |         0
 31 |          2 |         6 |           9 |         2 |          1 |         28 |  120 |         0
 32 |          2 |         6 |          10 |         2 |          1 |         27 |  150 |         0
 33 |          2 |         6 |          11 |         3 |         93 |         93 |  136 | 14.764823
 34 |          2 |         6 |          12 |         3 |         32 |         32 |  116 | 35.902646
 35 |          2 |         6 |          13 |         3 |         28 |         28 |   96 |         7
 36 |          2 |         6 |          14 |         3 |         31 |         31 |   86 |  8.602325
 37 |          2 |         6 |          15 |         3 |         35 |         35 |   56 |  5.830952
 38 |          2 |         6 |          16 |         3 |         27 |         27 |   26 |  11.18034
 39 |          2 |         6 |          17 |         3 |         33 |         33 |   16 |         8
 40 |          2 |         6 |          18 |         3 |         94 |         94 |   13 | 34.058773
 41 |          2 |         6 |          19 |         3 |         81 |         81 |    0 | 14.866069
 42 |          2 |         6 |          20 |         6 |          1 |         -1 |    0 |  7.615773
 43 |          3 |         3 |           1 |         1 |          1 |         -1 |    0 |         0
 44 |          3 |         3 |           2 |         2 |          1 |         16 |   20 |         0
 45 |          3 |         3 |           3 |         2 |          1 |         14 |   30 |         0
 46 |          3 |         3 |           4 |         2 |          1 |         48 |   40 |         0
 47 |          3 |         3 |           5 |         2 |          1 |         18 |   60 |         0
 48 |          3 |         3 |           6 |         2 |          1 |         17 |   80 |         0
 49 |          3 |         3 |           7 |         2 |          1 |         15 |   90 |         0
 50 |          3 |         3 |           8 |         2 |          1 |         13 |  110 |         0
 51 |          3 |         3 |           9 |         2 |          1 |         11 |  140 |         0
 52 |          3 |         3 |          10 |         2 |          1 |         10 |  160 |         0
 53 |          3 |         3 |          11 |         3 |         15 |         15 |  150 | 35.355339
 54 |          3 |         3 |          12 |         3 |         48 |         48 |  140 |         3
 55 |          3 |         3 |          13 |         3 |         13 |         13 |  120 |         6
 56 |          3 |         3 |          14 |         3 |         16 |         16 |  100 |   7.81025
 57 |          3 |         3 |          15 |         3 |         17 |         17 |   80 |         2
 58 |          3 |         3 |          16 |         3 |         10 |         10 |   60 |  11.18034
 59 |          3 |         3 |          17 |         3 |         11 |         11 |   30 |         5
 60 |          3 |         3 |          18 |         3 |         14 |         14 |   20 |  7.071068
 61 |          3 |         3 |          19 |         3 |         18 |         18 |    0 |  11.18034
 62 |          3 |         3 |          20 |         6 |          1 |         -1 |    0 | 40.311289
 63 |          4 |         8 |           1 |         1 |          1 |         -1 |    0 |         0
 64 |          4 |         8 |           2 |         2 |          1 |         71 |   23 |         0
 65 |          4 |         8 |           3 |         2 |          1 |         55 |   39 |         0
 66 |          4 |         8 |           4 |         2 |          1 |         44 |   59 |         0
 67 |          4 |         8 |           5 |         2 |          1 |         43 |   69 |         0
 68 |          4 |         8 |           6 |         2 |          1 |         42 |   89 |         0
 69 |          4 |         8 |           7 |         2 |          1 |         41 |  119 |         0
 70 |          4 |         8 |           8 |         2 |          1 |         40 |  129 |         0
 71 |          4 |         8 |           9 |         2 |          1 |         39 |  159 |         0
 72 |          4 |         8 |          10 |         3 |         43 |         43 |  149 |  33.54102
 73 |          4 |         8 |          11 |         3 |         40 |         40 |  139 |         5
 74 |          4 |         8 |          12 |         3 |         39 |         39 |  109 |         2
 75 |          4 |         8 |          13 |         3 |         41 |         41 |   79 |  5.385165
```

```
  76 |     4 |     8 |    14 |     3 |    42 |    42 |    59 | 10.198039
  77 |     4 |     8 |    15 |     3 |    44 |    44 |    39 | 10.440307
  78 |     4 |     8 |    16 |     3 |    55 |    55 |    23 |        25
  79 |     4 |     8 |    17 |     3 |    71 |    71 |     0 | 21.931712
  80 |     4 |     8 |    18 |     6 |     1 |    -1 |     0 | 19.646883
  81 |     5 |     1 |     1 |     1 |     1 |    -1 |     0 |         0
  82 |     5 |     1 |     2 |     2 |     1 |     4 |    10 |         0
  83 |     5 |     1 |     3 |     2 |     1 |   101 |    13 |         0
  84 |     5 |     1 |     4 |     2 |     1 |    46 |    23 |         0
  85 |     5 |     1 |     5 |     2 |     1 |     5 |    63 |         0
  86 |     5 |     1 |     6 |     2 |     1 |     3 |    93 |         0
  87 |     5 |     1 |     7 |     3 |    46 |    46 |    83 | 37.735925
  88 |     5 |     1 |     8 |     3 |     3 |     3 |    53 |   7.28011
  89 |     5 |     1 |     9 |     2 |     1 |     2 |    73 | 30.805844
  90 |     5 |     1 |    10 |     3 |     4 |     4 |    63 | 39.357337
  91 |     5 |     1 |    11 |     3 |     2 |     2 |    43 |         3
  92 |     5 |     1 |    12 |     3 |     5 |     5 |     3 |  7.071068
  93 |     5 |     1 |    13 |     3 |   101 |   101 |     0 | 17.029386
  94 |     5 |     1 |    14 |     6 |     1 |    -1 |     0 | 19.235384
  95 |     6 |    13 |     1 |     1 |     1 |    -1 |     0 |         0
  96 |     6 |    13 |     2 |     2 |     1 |    92 |    13 |         0
  97 |     6 |    13 |     3 |     2 |     1 |    52 |    32 |         0
  98 |     6 |    13 |     4 |     2 |     1 |    57 |    51 |         0
  99 |     6 |    13 |     5 |     2 |     1 |    85 |    69 |         0
 100 |     6 |    13 |     6 |     2 |     1 |    68 |    78 |         0
 101 |     6 |    13 |     7 |     2 |     1 |    63 |    81 |         0
 102 |     6 |    13 |     8 |     3 |    63 |    63 |    78 | 29.154759
 103 |     6 |    13 |     9 |     3 |    68 |    68 |    69 |  7.071068
 104 |     6 |    13 |    10 |     3 |    52 |    52 |    50 | 23.769729
 105 |     6 |    13 |    11 |     3 |    85 |    85 |    32 |  9.219544
 106 |     6 |    13 |    12 |     3 |    57 |    57 |    13 |  9.219544
 107 |     6 |    13 |    13 |     3 |    92 |    92 |     0 |  7.071068
 108 |     6 |    13 |    14 |     6 |     1 |    -1 |     0 | 12.041595
 109 |     7 |     7 |     1 |     1 |     1 |    -1 |     0 |         0
 110 |     7 |     7 |     2 |     2 |     1 |    30 |    10 |         0
 111 |     7 |     7 |     3 |     2 |     1 |    29 |    20 |         0
 112 |     7 |     7 |     4 |     2 |     1 |    38 |    30 |         0
 113 |     7 |     7 |     5 |     2 |     1 |    36 |    50 |         0
 114 |     7 |     7 |     6 |     2 |     1 |    34 |    60 |         0
 115 |     7 |     7 |     7 |     3 |    34 |    34 |    50 | 51.478151
 116 |     7 |     7 |     8 |     3 |    29 |    29 |    40 |  8.602325
 117 |     7 |     7 |     9 |     3 |    30 |    30 |    30 |  5.385165
 118 |     7 |     7 |    10 |     3 |    38 |    38 |    20 | 53.235327
 119 |     7 |     7 |    11 |     3 |    36 |    36 |     0 |  3.605551
 120 |     7 |     7 |    12 |     6 |     1 |    -1 |     0 | 44.204072
 121 |     8 |     2 |     1 |     1 |     1 |    -1 |     0 |         0
 122 |     8 |     2 |     2 |     2 |     1 |    89 |    19 |         0
 123 |     8 |     2 |     3 |     2 |     1 |    47 |    29 |         0
 124 |     8 |     2 |     4 |     2 |     1 |    61 |    46 |         0
 125 |     8 |     2 |     5 |     2 |     1 |     9 |    56 |         0
 126 |     8 |     2 |     6 |     2 |     1 |     8 |    76 |         0
 127 |     8 |     2 |     7 |     3 |    89 |    89 |    57 | 17.888544
 128 |     8 |     2 |     8 |     3 |     8 |     8 |    37 | 19.235384
 129 |     8 |     2 |     9 |     3 |     9 |     9 |    27 |         5
 130 |     8 |     2 |    10 |     3 |    47 |    47 |    17 |         3
 131 |     8 |     2 |    11 |     3 |    61 |    61 |     0 | 20.223748
 132 |     8 |     2 |    12 |     6 |     1 |    -1 |     0 | 26.925824
 133 |     9 |    14 |     1 |     1 |     1 |    -1 |     0 |         0
 134 |     9 |    14 |     2 |     2 |     1 |    97 |    26 |         0
 135 |     9 |    14 |     3 |     2 |     1 |    64 |    32 |         0
 136 |     9 |    14 |     4 |     2 |     1 |    51 |    62 |         0
 137 |     9 |    14 |     5 |     2 |     1 |    96 |    68 |         0
 138 |     9 |    14 |     6 |     2 |     1 |    77 |    99 |         0
```

```
 139 |     9 |    14 |     7 |     3 |    96 |    96 |    93 | 20.615528
 140 |     9 |    14 |     8 |     3 |    64 |    64 |    87 | 19.235384
 141 |     9 |    14 |     9 |     3 |    77 |    77 |    56 |  9.433981
 142 |     9 |    14 |    10 |     3 |    51 |    51 |    26 | 25.942244
 143 |     9 |    14 |    11 |     3 |    97 |    97 |     0 | 25.495098
 144 |     9 |    14 |    12 |     6 |     1 |    -1 |     0 | 15.524175
 145 |    10 |    15 |     1 |     1 |     1 |    -1 |     0 |         0
 146 |    10 |    15 |     2 |     2 |     1 |    67 |    16 |         0
 147 |    10 |    15 |     3 |     2 |     1 |    73 |    24 |         0
 148 |    10 |    15 |     4 |     2 |     1 |    95 |    47 |         0
 149 |    10 |    15 |     5 |     2 |     1 |    82 |    57 |         0
 150 |    10 |    15 |     6 |     2 |     1 |    72 |    71 |         0
 151 |    10 |    15 |     7 |     3 |    73 |    73 |    63 |  27.45906
 152 |    10 |    15 |     8 |     3 |    72 |    72 |    49 | 10.198039
 153 |    10 |    15 |     9 |     3 |    82 |    82 |    39 | 16.278821
 154 |    10 |    15 |    10 |     3 |    95 |    95 |    16 | 12.806248
 155 |    10 |    15 |    11 |     3 |    67 |    67 |     0 | 19.416488
 156 |    10 |    15 |    12 |     6 |     1 |    -1 |     0 | 13.038405
 157 |    11 |    11 |     1 |     1 |     1 |    -1 |     0 |         0
 158 |    11 |    11 |     2 |     2 |     1 |    78 |     7 |         0
 159 |    11 |    11 |     3 |     2 |     1 |    26 |    27 |         0
 160 |    11 |    11 |     4 |     2 |     1 |    87 |    55 |         0
 161 |    11 |    11 |     5 |     2 |     1 |    23 |    95 |         0
 162 |    11 |    11 |     6 |     3 |    87 |    87 |    67 | 32.202484
 163 |    11 |    11 |     7 |     3 |    23 |    23 |    27 | 21.023796
 164 |    11 |    11 |     8 |     3 |    78 |    78 |    20 | 20.808652
 165 |    11 |    11 |     9 |     3 |    26 |    26 |     0 | 12.165525
 166 |    11 |    11 |    10 |     6 |     1 |    -1 |     0 | 45.276926
 167 |    12 |     4 |     1 |     1 |     1 |    -1 |     0 |         0
 168 |    12 |     4 |     2 |     2 |     1 |    60 |    19 |         0
 169 |    12 |     4 |     3 |     2 |     1 |    59 |    39 |         0
 170 |    12 |     4 |     4 |     2 |     1 |   100 |    54 |         0
 171 |    12 |     4 |     5 |     2 |     1 |    54 |    59 |         0
 172 |    12 |     4 |     6 |     3 |    60 |    60 |    40 | 42.426407
 173 |    12 |     4 |     7 |     3 |   100 |   100 |    25 | 21.931712
 174 |    12 |     4 |     8 |     3 |    54 |    54 |    20 | 16.155494
 175 |    12 |     4 |     9 |     3 |    59 |    59 |     0 | 40.311289
 176 |    12 |     4 |    10 |     6 |     1 |    -1 |     0 | 47.169906
 177 |    13 |    10 |     1 |     1 |     1 |    -1 |     0 |         0
 178 |    13 |    10 |     2 |     2 |     1 |    86 |     2 |         0
 179 |    13 |    10 |     3 |     2 |     1 |    90 |    27 |         0
 180 |    13 |    10 |     4 |     2 |     1 |    65 |    44 |         0
 181 |    13 |    10 |     5 |     2 |     1 |    53 |    47 |         0
 182 |    13 |    10 |     6 |     3 |    53 |    53 |    44 |        25
 183 |    13 |    10 |     7 |     3 |    65 |    65 |    27 |        20
 184 |    13 |    10 |     8 |     3 |    86 |    86 |    25 | 19.313208
 185 |    13 |    10 |     9 |     3 |    90 |    90 |     0 | 18.439089
 186 |    13 |    10 |    10 |     6 |     1 |    -1 |     0 | 52.478567
 187 |    14 |    12 |     1 |     1 |     1 |    -1 |     0 |         0
 188 |    14 |    12 |     2 |     2 |     1 |     6 |    20 |         0
 189 |    14 |    12 |     3 |     2 |     1 |    80 |    50 |         0
 190 |    14 |    12 |     4 |     2 |     1 |     7 |    70 |         0
 191 |    14 |    12 |     5 |     2 |     1 |    56 |    86 |         0
 192 |    14 |    12 |     6 |     3 |     6 |     6 |    66 | 40.311289
 193 |    14 |    12 |     7 |     3 |    80 |    80 |    36 | 22.022716
 194 |    14 |    12 |     8 |     3 |     7 |     7 |    16 | 13.892444
 195 |    14 |    12 |     9 |     3 |    56 |    56 |     0 | 19.209373
 196 |    14 |    12 |    10 |     6 |     1 |    -1 |     0 | 14.142136
 197 |    15 |    19 |     1 |     1 |     1 |    -1 |     0 |         0
 198 |    15 |    19 |     2 |     2 |     1 |    88 |    13 |         0
 199 |    15 |    19 |     3 |     2 |     1 |    70 |    40 |         0
 200 |    15 |    19 |     4 |     2 |     1 |    58 |    63 |         0
 201 |    15 |    19 |     5 |     2 |     1 |    99 |    72 |         0
```

```
 202 |          15 |          19 |          6 |          3 |       70 |       70 |    45 |    9.219544
 203 |          15 |          19 |          7 |          3 |       99 |       99 |    36 |           5
 204 |          15 |          19 |          8 |          3 |       88 |       88 |    23 |   31.304952
 205 |          15 |          19 |          9 |          3 |       58 |       58 |     0 |   18.027756
 206 |          15 |          19 |         10 |          6 |        1 |       -1 |     0 |   26.925824
 207 |          16 |          17 |          1 |          1 |        1 |       -1 |     0 |           0
 208 |          16 |          17 |          2 |          2 |        1 |       75 |     8 |           0
 209 |          16 |          17 |          3 |          2 |        1 |       98 |    43 |           0
 210 |          16 |          17 |          4 |          2 |        1 |       76 |    59 |           0
 211 |          16 |          17 |          5 |          3 |       76 |       76 |    43 |   57.008771
 212 |          16 |          17 |          6 |          3 |       98 |       98 |     8 |   13.038405
 213 |          16 |          17 |          7 |          3 |       75 |       75 |     0 |   16.124515
 214 |          16 |          17 |          8 |          6 |        1 |       -1 |     0 |   36.055513
 215 |          17 |          16 |          1 |          1 |        1 |       -1 |     0 |           0
 216 |          17 |          16 |          2 |          2 |        1 |       69 |    21 |           0
 217 |          17 |          16 |          3 |          2 |        1 |       79 |    48 |           0
 218 |          17 |          16 |          4 |          2 |        1 |       74 |    53 |           0
 219 |          17 |          16 |          5 |          3 |       74 |       74 |    48 |   39.293765
 220 |          17 |          16 |          6 |          3 |       79 |       79 |    21 |    7.211103
 221 |          17 |          16 |          7 |          3 |       69 |       69 |     0 |   32.249031
 222 |          17 |          16 |          8 |          6 |        1 |       -1 |     0 |          10
 223 |          18 |           9 |          1 |          1 |        1 |       -1 |     0 |           0
 224 |          18 |           9 |          2 |          2 |        1 |       62 |     9 |           0
 225 |          18 |           9 |          3 |          2 |        1 |       37 |    49 |           0
 226 |          18 |           9 |          4 |          2 |        1 |       45 |    59 |           0
 227 |          18 |           9 |          5 |          3 |       37 |       37 |    19 |   43.011626
 228 |          18 |           9 |          6 |          3 |       45 |       45 |     9 |   10.440307
 229 |          18 |           9 |          7 |          3 |       62 |       62 |     0 |   19.723083
 230 |          18 |           9 |          8 |          6 |        1 |       -1 |     0 |   15.811388
 231 |          19 |          18 |          1 |          1 |        1 |       -1 |     0 |           0
 232 |          19 |          18 |          2 |          2 |        1 |       91 |     6 |           0
 233 |          19 |          18 |          3 |          2 |        1 |       12 |    46 |           0
 234 |          19 |          18 |          4 |          3 |       12 |       12 |     6 |   33.526109
 235 |          19 |          18 |          5 |          3 |       91 |       91 |     0 |   29.832868
 236 |          19 |          18 |          6 |          6 |        1 |       -1 |     0 |    4.242641
 237 |          20 |          20 |          1 |          1 |        1 |       -1 |     0 |           0
 238 |          20 |          20 |          2 |          2 |        1 |       83 |     9 |           0
 239 |          20 |          20 |          3 |          3 |       83 |       83 |     0 |   14.764823
 240 |          20 |          20 |          4 |          6 |        1 |       -1 |     0 |   14.764823
 241 |          -2 |           0 |          0 |         -1 |       -1 |       -1 |    -1 | 2304.305537
(241 rows)

ROLLBACK;
ROLLBACK
```

Data

```
drop table if exists vrp_orders cascade;
create table vrp_orders (
    id integer not null primary key,
    order_unit integer,
    open_time integer,
    close_time integer,
    service_time integer,
    x float8,
    y float8

);

copy vrp_orders (id, x, y, order_unit, open_time, close_time, service_time) from stdin;
1         40.000000        50.000000        0         0        240         0
```

```
2       25.000000       85.000000       20      145     175     10
3       22.000000       75.000000       30      50      80      10
4       22.000000       85.000000       10      109     139     10
5       20.000000       80.000000       40      141     171     10
6       20.000000       85.000000       20      41      71      10
7       18.000000       75.000000       20      95      125     10
8       15.000000       75.000000       20      79      109     10
9       15.000000       80.000000       10      91      121     10
10      10.000000       35.000000       20      91      121     10
11      10.000000       40.000000       30      119     149     10
12      8.000000        40.000000       40      59      89      10
13      8.000000        45.000000       20      64      94      10
14      5.000000        35.000000       10      142     172     10
15      5.000000        45.000000       10      35      65      10
16      2.000000        40.000000       20      58      88      10
17      0.000000        40.000000       20      72      102     10
18      0.000000        45.000000       20      149     179     10
19      44.000000       5.000000        20      87      117     10
20      42.000000       10.000000       40      72      102     10
21      42.000000       15.000000       10      122     152     10
22      40.000000       5.000000        10      67      97      10
23      40.000000       15.000000       40      92      122     10
24      38.000000       5.000000        30      65      95      10
25      38.000000       15.000000       10      148     178     10
26      35.000000       5.000000        20      154     184     10
27      95.000000       30.000000       30      115     145     10
28      95.000000       35.000000       20      62      92      10
29      92.000000       30.000000       10      62      92      10
30      90.000000       35.000000       10      67      97      10
31      88.000000       30.000000       10      74      104     10
32      88.000000       35.000000       20      61      91      10
33      87.000000       30.000000       10      131     161     10
34      85.000000       25.000000       10      51      81      10
35      85.000000       35.000000       30      111     141     10
36      67.000000       85.000000       20      139     169     10
37      65.000000       85.000000       40      43      73      10
38      65.000000       82.000000       10      124     154     10
39      62.000000       80.000000       30      75      105     10
40      60.000000       80.000000       10      37      67      10
41      60.000000       85.000000       30      85      115     10
42      58.000000       75.000000       20      92      122     10
43      55.000000       80.000000       10      33      63      10
44      55.000000       85.000000       20      128     158     10
45      55.000000       82.000000       10      64      94      10
46      20.000000       82.000000       10      37      67      10
47      18.000000       80.000000       10      113     143     10
48      2.000000        45.000000       10      45      75      10
49      42.000000       5.000000        10      151     181     10
50      42.000000       12.000000       10      104     134     10
51      72.000000       35.000000       30      116     146     10
52      55.000000       20.000000       19      83      113     10
53      25.000000       30.000000       3       52      82      10
54      20.000000       50.000000       5       91      121     10
55      55.000000       60.000000       16      139     169     10
56      30.000000       60.000000       16      140     170     10
57      50.000000       35.000000       19      130     160     10
58      30.000000       25.000000       23      96      126     10
59      15.000000       10.000000       20      152     182     10
60      10.000000       20.000000       19      42      72      10
61      15.000000       60.000000       17      155     185     10
62      45.000000       65.000000       9       66      96      10
63      65.000000       35.000000       3       52      82      10
64      65.000000       20.000000       6       39      69      10
```

```
65        45.000000        30.000000        17        53        83        10
66        35.000000        40.000000        16        11        41        10
67        41.000000        37.000000        16        133        163        10
68        64.000000        42.000000        9        70        100        10
69        40.000000        60.000000        21        144        174        10
70        31.000000        52.000000        27        41        71        10
71        35.000000        69.000000        23        180        210        10
72        65.000000        55.000000        14        65        95        10
73        63.000000        65.000000        8        30        60        10
74        2.000000        60.000000        5        77        107        10
75        20.000000        20.000000        8        141        171        10
76        5.000000        5.000000        16        74        104        10
77        60.000000        12.000000        31        75        105        10
78        23.000000        3.000000        7        150        180        10
79        8.000000        56.000000        27        90        120        10
80        6.000000        68.000000        30        89        119        10
81        47.000000        47.000000        13        192        222        10
82        49.000000        58.000000        10        86        116        10
83        27.000000        43.000000        9        42        72        10
84        37.000000        31.000000        14        35        65        10
85        57.000000        29.000000        18        96        126        10
86        63.000000        23.000000        2        87        117        10
87        21.000000        24.000000        28        87        117        10
88        12.000000        24.000000        13        90        120        10
89        24.000000        58.000000        19        67        97        10
90        67.000000        5.000000        25        144        174        10
91        37.000000        47.000000        6        86        116        10
92        49.000000        42.000000        13        167        197        10
93        53.000000        43.000000        14        14        44        10
94        61.000000        52.000000        3        178        208        10
95        57.000000        48.000000        23        95        125        10
96        56.000000        37.000000        6        34        64        10
97        55.000000        54.000000        26        132        162        10
98        4.000000        18.000000        35        120        150        10
99        26.000000        52.000000        9        46        76        10
100        26.000000        35.000000        15        77        107        10
101        31.000000        67.000000        3        180        210        10
\.

drop table if exists vrp_vehicles cascade;
create table vrp_vehicles (
    vehicle_id integer not null primary key,
    capacity integer,
    case_no integer
);

copy vrp_vehicles (vehicle_id, capacity, case_no) from stdin;
1        200        5
2        200        5
3        200        5
4        200        5
5        200        5
6        200        5
7        200        5
8        200        5
9        200        5
10        200        5
11        200        5
12        200        5
13        200        5
14        200        5
15        200        5
16        200        5
```

```
17        200        5
18        200        5
19        200        5
20        200        5
\.

drop table if exists vrp_distance cascade;
create table vrp_distance (
    src_id integer,
    dest_id integer,
    cost Float8,
    distance Float8,
    traveltime Float8
);

copy vrp_distance (src_id, dest_id, cost, distance, traveltime) from stdin;
1        2        38.078866        38.078866        38.078866
1        3        30.805844        30.805844        30.805844
1        4        39.357337        39.357337        39.357337
1        5        36.055513        36.055513        36.055513
1        6        40.311289        40.311289        40.311289
1        7        33.301652        33.301652        33.301652
1        8        35.355339        35.355339        35.355339
1        9        39.051248        39.051248        39.051248
1        10        33.541020        33.541020        33.541020
1        11        31.622777        31.622777        31.622777
1        12        33.526109        33.526109        33.526109
1        13        32.388269        32.388269        32.388269
1        14        38.078866        38.078866        38.078866
1        15        35.355339        35.355339        35.355339
1        16        39.293765        39.293765        39.293765
1        17        41.231056        41.231056        41.231056
1        18        40.311289        40.311289        40.311289
1        19        45.177428        45.177428        45.177428
1        20        40.049969        40.049969        40.049969
1        21        35.057096        35.057096        35.057096
1        22        45.000000        45.000000        45.000000
1        23        35.000000        35.000000        35.000000
1        24        45.044423        45.044423        45.044423
1        25        35.057096        35.057096        35.057096
1        26        45.276926        45.276926        45.276926
1        27        58.523500        58.523500        58.523500
1        28        57.008771        57.008771        57.008771
1        29        55.713553        55.713553        55.713553
1        30        52.201533        52.201533        52.201533
1        31        52.000000        52.000000        52.000000
1        32        50.289164        50.289164        50.289164
1        33        51.078371        51.078371        51.078371
1        34        51.478151        51.478151        51.478151
1        35        47.434165        47.434165        47.434165
1        36        44.204072        44.204072        44.204072
1        37        43.011626        43.011626        43.011626
1        38        40.607881        40.607881        40.607881
1        39        37.202150        37.202150        37.202150
1        40        36.055513        36.055513        36.055513
1        41        40.311289        40.311289        40.311289
1        42        30.805844        30.805844        30.805844
1        43        33.541020        33.541020        33.541020
1        44        38.078866        38.078866        38.078866
1        45        35.341194        35.341194        35.341194
1        46        37.735925        37.735925        37.735925
1        47        37.202150        37.202150        37.202150
1        48        38.327536        38.327536        38.327536
```

```
1        49        45.044423        45.044423        45.044423
1        50        38.052595        38.052595        38.052595
1        51        35.341194        35.341194        35.341194
1        52        33.541020        33.541020        33.541020
1        53        25.000000        25.000000        25.000000
1        54        20.000000        20.000000        20.000000
1        55        18.027756        18.027756        18.027756
1        56        14.142136        14.142136        14.142136
1        57        18.027756        18.027756        18.027756
1        58        26.925824        26.925824        26.925824
1        59        47.169906        47.169906        47.169906
1        60        42.426407        42.426407        42.426407
1        61        26.925824        26.925824        26.925824
1        62        15.811388        15.811388        15.811388
1        63        29.154759        29.154759        29.154759
1        64        39.051248        39.051248        39.051248
1        65        20.615528        20.615528        20.615528
1        66        11.180340        11.180340        11.180340
1        67        13.038405        13.038405        13.038405
1        68        25.298221        25.298221        25.298221
1        69        10.000000        10.000000        10.000000
1        70        9.219544        9.219544        9.219544
1        71        19.646883        19.646883        19.646883
1        72        25.495098        25.495098        25.495098
1        73        27.459060        27.459060        27.459060
1        74        39.293765        39.293765        39.293765
1        75        36.055513        36.055513        36.055513
1        76        57.008771        57.008771        57.008771
1        77        42.941821        42.941821        42.941821
1        78        49.979996        49.979996        49.979996
1        79        32.557641        32.557641        32.557641
1        80        38.470768        38.470768        38.470768
1        81        7.615773        7.615773        7.615773
1        82        12.041595        12.041595        12.041595
1        83        14.764823        14.764823        14.764823
1        84        19.235384        19.235384        19.235384
1        85        27.018512        27.018512        27.018512
1        86        35.468296        35.468296        35.468296
1        87        32.202484        32.202484        32.202484
1        88        38.209946        38.209946        38.209946
1        89        17.888544        17.888544        17.888544
1        90        52.478567        52.478567        52.478567
1        91        4.242641        4.242641        4.242641
1        92        12.041595        12.041595        12.041595
1        93        14.764823        14.764823        14.764823
1        94        21.095023        21.095023        21.095023
1        95        17.117243        17.117243        17.117243
1        96        20.615528        20.615528        20.615528
1        97        15.524175        15.524175        15.524175
1        98        48.166378        48.166378        48.166378
1        99        14.142136        14.142136        14.142136
1        100        20.518285        20.518285        20.518285
1        101        19.235384        19.235384        19.235384
2        1        38.078866        38.078866        38.078866
2        3        10.440307        10.440307        10.440307
2        4        3.000000        3.000000        3.000000
2        5        7.071068        7.071068        7.071068
2        6        5.000000        5.000000        5.000000
2        7        12.206556        12.206556        12.206556
2        8        14.142136        14.142136        14.142136
2        9        11.180340        11.180340        11.180340
2        10        52.201533        52.201533        52.201533
2        11        47.434165        47.434165        47.434165
```

```
2         12         48.104054         48.104054         48.104054
2         13         43.462628         43.462628         43.462628
2         14         53.851648         53.851648         53.851648
2         15         44.721360         44.721360         44.721360
2         16         50.537115         50.537115         50.537115
2         17         51.478151         51.478151         51.478151
2         18         47.169906         47.169906         47.169906
2         19         82.225300         82.225300         82.225300
2         20         76.902536         76.902536         76.902536
2         21         72.034714         72.034714         72.034714
2         22         81.394103         81.394103         81.394103
2         23         71.589105         71.589105         71.589105
2         24         81.049368         81.049368         81.049368
2         25         71.196910         71.196910         71.196910
2         26         80.622577         80.622577         80.622577
2         27         89.022469         89.022469         89.022469
2         28         86.023253         86.023253         86.023253
2         29         86.683332         86.683332         86.683332
2         30         82.006097         82.006097         82.006097
2         31         83.630138         83.630138         83.630138
2         32         80.430094         80.430094         80.430094
2         33         82.879430         82.879430         82.879430
2         34         84.852814         84.852814         84.852814
2         35         78.102497         78.102497         78.102497
2         36         42.000000         42.000000         42.000000
2         37         40.000000         40.000000         40.000000
2         38         40.112342         40.112342         40.112342
2         39         37.336309         37.336309         37.336309
2         40         35.355339         35.355339         35.355339
2         41         35.000000         35.000000         35.000000
2         42         34.481879         34.481879         34.481879
2         43         30.413813         30.413813         30.413813
2         44         30.000000         30.000000         30.000000
2         45         30.149627         30.149627         30.149627
2         46         5.830952         5.830952         5.830952
2         47         8.602325         8.602325         8.602325
2         48         46.141088         46.141088         46.141088
2         49         81.786307         81.786307         81.786307
2         50         74.953319         74.953319         74.953319
2         51         68.622154         68.622154         68.622154
2         52         71.589105         71.589105         71.589105
2         53         55.000000         55.000000         55.000000
2         54         35.355339         35.355339         35.355339
2         55         39.051248         39.051248         39.051248
2         56         25.495098         25.495098         25.495098
2         57         55.901699         55.901699         55.901699
2         58         60.207973         60.207973         60.207973
2         59         75.663730         75.663730         75.663730
2         60         66.708320         66.708320         66.708320
2         61         26.925824         26.925824         26.925824
2         62         28.284271         28.284271         28.284271
2         63         64.031242         64.031242         64.031242
2         64         76.321688         76.321688         76.321688
2         65         58.523500         58.523500         58.523500
2         66         46.097722         46.097722         46.097722
2         67         50.596443         50.596443         50.596443
2         68         58.051701         58.051701         58.051701
2         69         29.154759         29.154759         29.154759
2         70         33.541020         33.541020         33.541020
2         71         18.867962         18.867962         18.867962
2         72         50.000000         50.000000         50.000000
2         73         42.941821         42.941821         42.941821
2         74         33.970576         33.970576         33.970576
```

```
2        75       65.192024        65.192024        65.192024
2        76       82.462113        82.462113        82.462113
2        77       80.956779        80.956779        80.956779
2        78       82.024387        82.024387        82.024387
2        79       33.615473        33.615473        33.615473
2        80       25.495098        25.495098        25.495098
2        81       43.908997        43.908997        43.908997
2        82       36.124784        36.124784        36.124784
2        83       42.047592        42.047592        42.047592
2        84       55.317267        55.317267        55.317267
2        85       64.498062        64.498062        64.498062
2        86       72.718636        72.718636        72.718636
2        87       61.131007        61.131007        61.131007
2        88       62.369865        62.369865        62.369865
2        89       27.018512        27.018512        27.018512
2        90       90.354856        90.354856        90.354856
2        91       39.849718        39.849718        39.849718
2        92       49.244289        49.244289        49.244289
2        93       50.477718        50.477718        50.477718
2        94       48.836462        48.836462        48.836462
2        95       48.918299        48.918299        48.918299
2        96       57.140179        57.140179        57.140179
2        97       43.139309        43.139309        43.139309
2        98       70.213959        70.213959        70.213959
2        99       33.015148        33.015148        33.015148
2        100       50.009999        50.009999        50.009999
2        101       18.973666        18.973666        18.973666
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3        10       41.761226        41.761226        41.761226
3        11       37.000000        37.000000        37.000000
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3        13       33.105891        33.105891        33.105891
3        14       43.462628        43.462628        43.462628
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3        16       40.311289        40.311289        40.311289
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3        18       37.202150        37.202150        37.202150
3        19       73.375745        73.375745        73.375745
3        20       68.007353        68.007353        68.007353
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3        23       62.641839        62.641839        62.641839
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3        25       62.096699        62.096699        62.096699
3        26       71.196910        71.196910        71.196910
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3        28       83.240615        83.240615        83.240615
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3        30       78.892332        78.892332        78.892332
3        31       79.881162        79.881162        79.881162
3        32       77.175126        77.175126        77.175126
3        33       79.056942        79.056942        79.056942
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3        35       74.625733        74.625733        74.625733
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3       59      65.375837       65.375837       65.375837
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3       67      42.485292       42.485292       42.485292
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3       89      17.117243       17.117243       17.117243
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3       93      44.553339       44.553339       44.553339
3       94      45.276926       45.276926       45.276926
3       95      44.204072       44.204072       44.204072
3       96      50.990195       50.990195       50.990195
3       97      39.115214       39.115214       39.115214
3       98      59.774577       59.774577       59.774577
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3       100     40.199502       40.199502       40.199502
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4        15        43.462628        43.462628        43.462628
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6        84       56.612719        56.612719        56.612719
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7        6       10.198039        10.198039        10.198039
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7        9       5.830952        5.830952        5.830952
7        10       40.792156        40.792156        40.792156
7        11       35.902646        35.902646        35.902646
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18        30        90.553851        90.553851        90.553851
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18        38        74.793048        74.793048        74.793048
18        39        71.196910        71.196910        71.196910
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19        7        74.672619        74.672619        74.672619
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19        46        80.653580        80.653580        80.653580
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19        68        42.059482        42.059482        42.059482
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20        19        5.385165        5.385165        5.385165
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21        76        38.327536        38.327536        38.327536
21        77        18.248288        18.248288        18.248288
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22        3        72.277244        72.277244        72.277244
22        4        82.000000        82.000000        82.000000
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22        7        73.375745        73.375745        73.375745
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29	55	47.634021	47.634021	47.634021
29	56	68.876701	68.876701	68.876701
29	57	42.296572	42.296572	42.296572
29	58	62.201286	62.201286	62.201286
29	59	79.555012	79.555012	79.555012
29	60	82.607506	82.607506	82.607506
29	61	82.637764	82.637764	82.637764
29	62	58.600341	58.600341	58.600341
29	63	27.459060	27.459060	27.459060
29	64	28.792360	28.792360	28.792360
29	65	47.000000	47.000000	47.000000
29	66	57.870545	57.870545	57.870545
29	67	51.478151	51.478151	51.478151
29	68	30.463092	30.463092	30.463092
29	69	60.033324	60.033324	60.033324
29	70	64.845971	64.845971	64.845971
29	71	69.065187	69.065187	69.065187
29	72	36.796739	36.796739	36.796739
29	73	45.453273	45.453273	45.453273
29	74	94.868330	94.868330	94.868330
29	75	72.691127	72.691127	72.691127
29	76	90.520716	90.520716	90.520716
29	77	36.715120	36.715120	36.715120
29	78	74.094534	74.094534	74.094534
29	79	87.931792	87.931792	87.931792
29	80	94.021274	94.021274	94.021274
29	81	48.104054	48.104054	48.104054
29	82	51.312766	51.312766	51.312766
29	83	66.287254	66.287254	66.287254

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29      92      44.643029       44.643029       44.643029
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30      20      54.120237       54.120237       54.120237
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33        43       59.363288       59.363288       59.363288
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33        47       85.211502       85.211502       85.211502
33        48       86.313383       86.313383       86.313383
33        49       51.478151       51.478151       51.478151
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33        51       15.811388       15.811388       15.811388
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33        53       62.000000       62.000000       62.000000
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33        73       42.438190        42.438190        42.438190
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33        80       89.470666        89.470666        89.470666
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34        19       45.617979        45.617979        45.617979
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35          21          47.423623         47.423623         47.423623
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35          23          49.244289         49.244289         49.244289
35          24          55.758407         55.758407         55.758407
35          25          51.078371         51.078371         51.078371
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35          27          11.180340         11.180340         11.180340
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35          40          51.478151         51.478151         51.478151
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37        15        72.111026        72.111026        72.111026
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37        21        73.681748        73.681748        73.681748
37        22        83.815273        83.815273        83.815273
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37        26        85.440037        85.440037        85.440037
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37        97        32.572995        32.572995        32.572995
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38         78        89.470666        89.470666        89.470666
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38         97        29.732137        29.732137        29.732137
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40        93        37.656341        37.656341        37.656341
40        94        28.017851        28.017851        28.017851
40        95        32.140317        32.140317        32.140317
40        96        43.185646        43.185646        43.185646
40        97        26.476405        26.476405        26.476405
40        98        83.546394        83.546394        83.546394
40        99        44.045431        44.045431        44.045431
40        100        56.400355        56.400355        56.400355
40        101        31.780497        31.780497        31.780497
41        1        40.311289        40.311289        40.311289
41        2        35.000000        35.000000        35.000000
41        3        39.293765        39.293765        39.293765
41        4        38.000000        38.000000        38.000000
41        5        40.311289        40.311289        40.311289
41        6        40.000000        40.000000        40.000000
41        7        43.174066        43.174066        43.174066
41        8        46.097722        46.097722        46.097722
41        9        45.276926        45.276926        45.276926
41        10        70.710678        70.710678        70.710678
41        11        67.268120        67.268120        67.268120
41        12        68.767725        68.767725        68.767725
41        13        65.604878        65.604878        65.604878
41        14        74.330344        74.330344        74.330344
41        15        68.007353        68.007353        68.007353
41        16        73.409809        73.409809        73.409809
```

```
41        17        75.000000        75.000000        75.000000
41        18        72.111026        72.111026        72.111026
41        19        81.584312        81.584312        81.584312
41        20        77.129761        77.129761        77.129761
41        21        72.277244        72.277244        72.277244
41        22        82.462113        82.462113        82.462113
41        23        72.801099        72.801099        72.801099
41        24        82.969874        82.969874        82.969874
41        25        73.375745        73.375745        73.375745
41        26        83.815273        83.815273        83.815273
41        27        65.192024        65.192024        65.192024
41        28        61.032778        61.032778        61.032778
41        29        63.631753        63.631753        63.631753
41        30        58.309519        58.309519        58.309519
41        31        61.717096        61.717096        61.717096
41        32        57.306195        57.306195        57.306195
41        33        61.269895        61.269895        61.269895
41        34        65.000000        65.000000        65.000000
41        35        55.901699        55.901699        55.901699
41        36        7.000000        7.000000        7.000000
41        37        5.000000        5.000000        5.000000
41        38        5.830952        5.830952        5.830952
41        39        5.385165        5.385165        5.385165
41        40        5.000000        5.000000        5.000000
41        42        10.198039        10.198039        10.198039
41        43        7.071068        7.071068        7.071068
41        44        5.000000        5.000000        5.000000
41        45        5.830952        5.830952        5.830952
41        46        40.112342        40.112342        40.112342
41        47        42.296572        42.296572        42.296572
41        48        70.455660        70.455660        70.455660
41        49        82.000000        82.000000        82.000000
41        50        75.186435        75.186435        75.186435
41        51        51.419841        51.419841        51.419841
41        52        65.192024        65.192024        65.192024
41        53        65.192024        65.192024        65.192024
41        54        53.150729        53.150729        53.150729
41        55        25.495098        25.495098        25.495098
41        56        39.051248        39.051248        39.051248
41        57        50.990195        50.990195        50.990195
41        58        67.082039        67.082039        67.082039
41        59        87.464278        87.464278        87.464278
41        60        82.006097        82.006097        82.006097
41        61        51.478151        51.478151        51.478151
41        62        25.000000        25.000000        25.000000
41        63        50.249378        50.249378        50.249378
41        64        65.192024        65.192024        65.192024
41        65        57.008771        57.008771        57.008771
41        66        51.478151        51.478151        51.478151
41        67        51.623638        51.623638        51.623638
41        68        43.185646        43.185646        43.185646
41        69        32.015621        32.015621        32.015621
41        70        43.931765        43.931765        43.931765
41        71        29.681644        29.681644        29.681644
41        72        30.413813        30.413813        30.413813
41        73        20.223748        20.223748        20.223748
41        74        63.158531        63.158531        63.158531
41        75        76.321688        76.321688        76.321688
41        76        97.082439        97.082439        97.082439
41        77        73.000000        73.000000        73.000000
41        78        89.961103        89.961103        89.961103
41        79        59.539903        59.539903        59.539903
41        80        56.612719        56.612719        56.612719
```

```
41        81        40.162171        40.162171        40.162171
41        82        29.154759        29.154759        29.154759
41        83        53.413481        53.413481        53.413481
41        84        58.694122        58.694122        58.694122
41        85        56.080300        56.080300        56.080300
41        86        62.072538        62.072538        62.072538
41        87        72.401657        72.401657        72.401657
41        88        77.620873        77.620873        77.620873
41        89        45.000000        45.000000        45.000000
41        90        80.305666        80.305666        80.305666
41        91        44.418465        44.418465        44.418465
41        92        44.384682        44.384682        44.384682
41        93        42.579338        42.579338        42.579338
41        94        33.015148        33.015148        33.015148
41        95        37.121422        37.121422        37.121422
41        96        48.166378        48.166378        48.166378
41        97        31.400637        31.400637        31.400637
41        98        87.321246        87.321246        87.321246
41        99        47.381431        47.381431        47.381431
41        100        60.464866        60.464866        60.464866
41        101        34.132096        34.132096        34.132096
42        1        30.805844        30.805844        30.805844
42        2        34.481879        34.481879        34.481879
42        3        36.000000        36.000000        36.000000
42        4        37.363083        37.363083        37.363083
42        5        38.327536        38.327536        38.327536
42        6        39.293765        39.293765        39.293765
42        7        40.000000        40.000000        40.000000
42        8        43.000000        43.000000        43.000000
42        9        43.289722        43.289722        43.289722
42        10        62.481997        62.481997        62.481997
42        11        59.405387        59.405387        59.405387
42        12        61.032778        61.032778        61.032778
42        13        58.309519        58.309519        58.309519
42        14        66.400301        66.400301        66.400301
42        15        60.901560        60.901560        60.901560
42        16        66.037868        66.037868        66.037868
42        17        67.742158        67.742158        67.742158
42        18        65.299311        65.299311        65.299311
42        19        71.386273        71.386273        71.386273
42        20        66.940272        66.940272        66.940272
42        21        62.096699        62.096699        62.096699
42        22        72.277244        72.277244        72.277244
42        23        62.641839        62.641839        62.641839
42        24        72.801099        72.801099        72.801099
42        25        63.245553        63.245553        63.245553
42        26        73.681748        73.681748        73.681748
42        27        58.258047        58.258047        58.258047
42        28        54.488531        54.488531        54.488531
42        29        56.400355        56.400355        56.400355
42        30        51.224994        51.224994        51.224994
42        31        54.083269        54.083269        54.083269
42        32        50.000000        50.000000        50.000000
42        33        53.535035        53.535035        53.535035
42        34        56.824291        56.824291        56.824291
42        35        48.259714        48.259714        48.259714
42        36        13.453624        13.453624        13.453624
42        37        12.206556        12.206556        12.206556
42        38        9.899495        9.899495        9.899495
42        39        6.403124        6.403124        6.403124
42        40        5.385165        5.385165        5.385165
42        41        10.198039        10.198039        10.198039
42        43        5.830952        5.830952        5.830952
```

```
42         44        10.440307        10.440307        10.440307
42         45        7.615773         7.615773         7.615773
42         46        38.639358        38.639358        38.639358
42         47        40.311289        40.311289        40.311289
42         48        63.529521        63.529521        63.529521
42         49        71.805292        71.805292        71.805292
42         50        65.000000        65.000000        65.000000
42         51        42.379240        42.379240        42.379240
42         52        55.081757        55.081757        55.081757
42         53        55.803226        55.803226        55.803226
42         54        45.486262        45.486262        45.486262
42         55        15.297059        15.297059        15.297059
42         56        31.764760        31.764760        31.764760
42         57        40.792156        40.792156        40.792156
42         58        57.306195        57.306195        57.306195
42         59        77.935871        77.935871        77.935871
42         60        73.000000        73.000000        73.000000
42         61        45.541190        45.541190        45.541190
42         62        16.401219        16.401219        16.401219
42         63        40.607881        40.607881        40.607881
42         64        55.443665        55.443665        55.443665
42         65        46.840154        46.840154        46.840154
42         66        41.880783        41.880783        41.880783
42         67        41.629317        41.629317        41.629317
42         68        33.541020        33.541020        33.541020
42         69        23.430749        23.430749        23.430749
42         70        35.468296        35.468296        35.468296
42         71        23.769729        23.769729        23.769729
42         72        21.189620        21.189620        21.189620
42         73        11.180340        11.180340        11.180340
42         74        57.974132        57.974132        57.974132
42         75        66.850580        66.850580        66.850580
42         76        87.800911        87.800911        87.800911
42         77        63.031738        63.031738        63.031738
42         78        80.056230        80.056230        80.056230
42         79        53.488316        53.488316        53.488316
42         80        52.469038        52.469038        52.469038
42         81        30.083218        30.083218        30.083218
42         82        19.235384        19.235384        19.235384
42         83        44.553339        44.553339        44.553339
42         84        48.754487        48.754487        48.754487
42         85        46.010868        46.010868        46.010868
42         86        52.239832        52.239832        52.239832
42         87        63.007936        63.007936        63.007936
42         88        68.680419        68.680419        68.680419
42         89        38.013156        38.013156        38.013156
42         90        70.576200        70.576200        70.576200
42         91        35.000000        35.000000        35.000000
42         92        34.205263        34.205263        34.205263
42         93        32.388269        32.388269        32.388269
42         94        23.194827        23.194827        23.194827
42         95        27.018512        27.018512        27.018512
42         96        38.052595        38.052595        38.052595
42         97        21.213203        21.213203        21.213203
42         98        78.517514        78.517514        78.517514
42         99        39.408121        39.408121        39.408121
42         100       51.224994        51.224994        51.224994
42         101       28.160256        28.160256        28.160256
43         1         33.541020        33.541020        33.541020
43         2         30.413813        30.413813        30.413813
43         3         33.376639        33.376639        33.376639
43         4         33.376639        33.376639        33.376639
43         5         35.000000        35.000000        35.000000
43         6         35.355339        35.355339        35.355339
43         7         37.336309        37.336309        37.336309
43         8         40.311289        40.311289        40.311289
43         9         40.000000        40.000000        40.000000
43         10        63.639610        63.639610        63.639610
43         11        60.207973        60.207973        60.207973
43         12        61.717096        61.717096        61.717096
```

- http://en.wikipedia.org/wiki/Vehicle_routing_problem

Introduction

Vehicle Routing Problems *VRP* are **NP-hard** optimization problem, it generalises the travelling salesman problem (TSP).

- The objective of the VRP is to minimize the total route cost.
- There are several variants of the VRP problem.

pgRouting does not try to implement all variants.

Characteristics

- Capacitated Vehicle Routing Problem *CVRP* where The vehicles have limited carrying capacity of the goods.
- Vehicle Routing Problem with Time Windows *VRPTW* where the locations have time windows within which the vehicle's visits must be made.
- Vehicle Routing Problem with Pickup and Delivery *VRPPD* where a number of goods need to be moved from certain pickup locations to other delivery locations.

Limitations

- No multiple time windows for a location.
- Less vehicle used is considered better.
- Less total duration is better.
- Less wait time is better.

Pick & Delivery

Problem: *CVRPPDTW* Capacitated Pick and Delivery Vehicle Routing problem with Time Windows

- Times are relative to *0*
- The vehicles
  - have start and ending service duration times.
  - have opening and closing times for the start and ending locations.
  - have a capacity.
- The orders
  - Have pick up and delivery locations.
  - Have opening and closing times for the pickup and delivery locations.
  - Have pickup and delivery duration service times.
  - have a demand request for moving goods from the pickup location to the delivery location.
- Time based calculations:
  - Travel time between customers is $distance/speed$
  - Pickup and delivery order pair is done by the same vehicle.

– A pickup is done before the delivery.

Parameters

Pick & deliver

Both implementations use the following same parameters:

Column	Type	Default	Description
orders_sql	`TEXT`		*Pick & Deliver Orders SQL* query containing the orders to be processed.
vehicles_sql	`TEXT`		*Pick & Deliver Vehicles SQL* query containing the vehicles to be used.
factor	`NUMERIC`	1	(Optional) Travel time multiplier. See *Factor Handling*
max_cycles	`INTEGER`	10	(Optional) Maximum number of cycles to perform on the optimization.
initial_sol	`INTEGER`	4	(Optional) Initial solution to be used. • `1` One order per truck • `2` Push front order. • `3` Push back order. • `4` Optimize insert. • `5` Push back order that allows more orders to be inserted at the back • `6` Push front order that allows more orders to be inserted at the front

The non euclidean implementation, additionally has:

Column	Type	Description
matrix_sql	`TEXT`	*Pick & Deliver Matrix SQL* query containing the distance or travel times.

Inner Queries

- *Pick & Deliver Orders SQL*
- *Pick & Deliver Vehicles SQL*
- *Pick & Deliver Matrix SQL*

return columns

- *Description of return columns*
- *Description of the return columns for Euclidean version*

Pick & Deliver Orders SQL

In general, the columns for the orders SQL is the same in both implementation of pick and delivery:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the pick-delivery order pair.
demand	ANY-NUMERICAL		Number of units in the order
p_open	ANY-NUMERICAL		The time, relative to 0, when the pickup location opens.
p_close	ANY-NUMERICAL		The time, relative to 0, when the pickup location closes.
d_service	ANY-NUMERICAL	0	The duration of the loading at the pickup location.
d_open	ANY-NUMERICAL		The time, relative to 0, when the delivery location opens.
d_close	ANY-NUMERICAL		The time, relative to 0, when the delivery location closes.
d_service	ANY-NUMERICAL	0	The duration of the loading at the delivery location.

For the non euclidean implementation, the starting and ending identifiers are needed:

Column	Type	Description
p_node_-id	ANY-INTEGER	The node identifier of the pickup, must match a node identifier in the matrix table.
d_node_-id	ANY-INTEGER	The node identifier of the delivery, must match a node identifier in the matrix table.

For the euclidean implementation, pick up and delivery (x, y) locations are needed:

Column	Type	Description
p_x	ANY-NUMERICAL	x value of the pick up location
p_y	ANY-NUMERICAL	y value of the pick up location
d_x	ANY-NUMERICAL	x value of the delivery location
d_y	ANY-NUMERICAL	y value of the delivery location

Pick & Deliver Vehicles SQL

In general, the columns for the vehicles_sql is the same in both implementation of pick and delivery:

Column	Type	Default	Description
id	ANY-INTEGER		Identifier of the pick-delivery order pair.
capacity	ANY-NUMERICAL		Number of units in the order
speed	ANY-NUMERICAL	*1*	Average speed of the vehicle.
start_open	ANY-NUMERICAL		The time, relative to 0, when the starting location opens.
start_close	ANY-NUMERICAL		The time, relative to 0, when the starting location closes.
start_-service	ANY-NUMERICAL	*0*	The duration of the loading at the starting location.
end_open	ANY-NUMERICAL	*start_open*	The time, relative to 0, when the ending location opens.
end_close	ANY-NUMERICAL	*start_close*	The time, relative to 0, when the ending location closes.
end_service	ANY-NUMERICAL	*start_-service*	The duration of the loading at the ending location.

For the non euclidean implementation, the starting and ending identifiers are needed:

Column	Type	Default	Description
start_-node_id	ANY-INTEGER		The node identifier of the starting location, must match a node identifier in the matrix table.
end_-node_id	ANY-INTEGER	*start_-node_id*	The node identifier of the ending location, must match a node identifier in the matrix table.

For the euclidean implementation, starting and ending (x, y) locations are needed:

Column	Type	Default	Description
start_x	ANY-NUMERICAL		x value of the coordinate of the starting location.
start_y	ANY-NUMERICAL		y value of the coordinate of the starting location.
end_x	ANY-NUMERICAL	*start_x*	x value of the coordinate of the ending location.
end_y	ANY-NUMERICAL	*start_y*	y value of the coordinate of the ending location.

Pick & Deliver Matrix SQL

Warning: TODO

Results

Description of the result (TODO Disussion: Euclidean & Matrix)

```
RETURNS SET OF
    (seq, vehicle_seq, vehicle_id, stop_seq, stop_type,
        travel_time, arrival_time, wait_time, service_time,  departure_time)
    UNION
    (summary row)
```

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
vehicle_seq	INTEGER	Sequential value starting from **1** for current vehicles. The n_{th} vehicle in the solution.
vehicle_id	BIGINT	Current vehicle identifier.
stop_seq	INTEGER	Sequential value starting from **1** for the stops made by the current vehicle. The m_{th} stop of the current vehicle.
stop_type	INTEGER	Kind of stop location the vehicle is at: • `1`: Starting location • `2`: Pickup location • `3`: Delivery location • `6`: Ending location
order_id	BIGINT	Pickup-Delivery order pair identifier. • `-1`: When no order is involved on the current stop location.
cargo	FLOAT	Cargo units of the vehicle when leaving the stop.
travel_time	FLOAT	Travel time from previous `stop_seq` to current `stop_seq`. • `0` When `stop_type = 1`
arrival_time	FLOAT	Previous `departure_time` plus current `travel_time`.
wait_time	FLOAT	Time spent waiting for current *location* to open.
service_time	FLOAT	Service time at current *location*.
departure_time	FLOAT	$arrival\_time + wait\_time + service\_time$. • When `stop_type = 6` has the *total_time* used for the current vehicle.

Summary Row

Warning: TODO: Review the summary

Column	Type	Description
seq	INTEGER	Continues the Sequential value
vehicle_seq	INTEGER	-2 to indicate is a summary row
vehicle_id	BIGINT	*Total Capacity Violations* in the solution.
stop_seq	INTEGER	*Total Time Window Violations* in the solution.
stop_type	INTEGER	`-1`
order_id	BIGINT	`-1`
cargo	FLOAT	`-1`
travel_time	FLOAT	*total_travel_time* The sum of all the *travel_time*
arrival_time	FLOAT	`-1`
wait_time	FLOAT	*total_waiting_time* The sum of all the *wait_time*
service_time	FLOAT	*total_service_time* The sum of all the *service_time*
departure_time	FLOAT	*total_solution_time* = $total\_travel\_time + total\_wait\_time + total\_service\_time$.

Description of the result (TODO Disussion: Euclidean & Matrix)

```
RETURNS SET OF
    (seq, vehicle_seq, vehicle_id, stop_seq, stop_type,
        travel_time, arrival_time, wait_time, service_time,  departure_time)
    UNION
    (summary row)
```

Column	Type	Description
seq	INTEGER	Sequential value starting from **1**.
vehicle_seq	INTEGER	Sequential value starting from **1** for current vehicles. The n_{th} vehicle in the solution.
vehicle_id	BIGINT	Current vehicle identifier.
stop_seq	INTEGER	Sequential value starting from **1** for the stops made by the current vehicle. The m_{th} stop of the current vehicle.
stop_type	INTEGER	Kind of stop location the vehicle is at: • `1`: Starting location • `2`: Pickup location • `3`: Delivery location • `6`: Ending location
order_id	BIGINT	Pickup-Delivery order pair identifier. • `-1`: When no order is involved on the current stop location.
cargo	FLOAT	Cargo units of the vehicle when leaving the stop.
travel_time	FLOAT	Travel time from previous `stop_seq` to current `stop_seq`. • `0` When `stop_type = 1`
arrival_time	FLOAT	Previous `departure_time` plus current `travel_time`.
wait_time	FLOAT	Time spent waiting for current *location* to open.
service_time	FLOAT	Service time at current *location*.
departure_time	FLOAT	$arrival\_time + wait\_time + service\_time$. • When `stop_type = 6` has the *total_time* used for the current vehicle.

Summary Row

> **Warning:** TODO: Review the summary

Column	Type	Description
seq	INTEGER	Continues the Sequential value
vehicle_seq	INTEGER	-2 to indicate is a summary row
vehicle_id	BIGINT	*Total Capacity Violations* in the solution.
stop_seq	INTEGER	*Total Time Window Violations* in the solution.
stop_type	INTEGER	`-1`
order_id	BIGINT	`-1`
cargo	FLOAT	`-1`
travel_time	FLOAT	*total_travel_time* The sum of all the *travel_time*
arrival_time	FLOAT	`-1`
wait_time	FLOAT	*total_waiting_time* The sum of all the *wait_time*
service_time	FLOAT	*total_service_time* The sum of all the *service_time*
departure_time	FLOAT	*total_solution_time* = $total\_travel\_time + total\_wait\_time + total\_service\_time$.

Where:

ANY-INTEGER SMALLINT, INTEGER, BIGINT

ANY-NUMERICAL SMALLINT, INTEGER, BIGINT, REAL, FLOAT

Handling Parameters

To define a problem, several considerations have to be done, to get consistent results. This section gives an insight of how parameters are to be considered.

- *Capacity and Demand Units Handling*
- *Locations*
- *Time Handling*
- *Factor Handling*

Capacity and Demand Units Handling

The *capacity* of a vehicle, can be measured in:

- Volume units like m^3.
- Area units like m^2 (when no stacking is allowed).
- Weight units like kg.
- Number of boxes that fit in the vehicle.
- Number of seats in the vehicle

The *demand* request of the pickup-deliver orders must use the same units as the units used in the vehicle's *capacity*.

To handle problems like: 10 (equal dimension) boxes of apples and 5 kg of feathers that are to be transported (not packed in boxes).

If the vehicle's *capacity* is measured by *boxes*, a conversion of *kg of feathers* to *equivalent number of boxes* is needed. If the vehicle's *capacity* is measured by *kg*, a conversion of *box of apples* to *equivalent number of kg* is needed.

Showing how the 2 possible conversions can be done

Let: - f_boxes: number of boxes that would be used for *1* kg of feathers. - a_weight: weight of *1* box of apples.

Capacity Units	apples	feathers
boxes	10	$5 * f\_boxes$
kg	$10 * a\_weight$	5

Locations

- When using the Euclidean signatures:
  - The vehicles have (x, y) pairs for start and ending locations.
  - The orders Have (x, y) pairs for pickup and delivery locations.
- When using a matrix:
  - The vehicles have identifiers for the start and ending locations.
  - The orders have identifiers for the pickup and delivery locations.
  - All the identifiers are indices to the given matrix.

Time Handling

The times are relative to 0

Suppose that a vehicle's driver starts the shift at 9:00 am and ends the shift at 4:30 pm and the service time duration is 10 minutes with 30 seconds.

All time units have to be converted

Meaning of 0	time units	9:00 am	4:30 pm	10 min 30 secs
0:00 am	hours	9	16.5	$10.5/60 = 0.175$
9:00 am	hours	0	7.5	$10.5/60 = 0.175$
0:00 am	minutes	$9 * 60 = 54$	$16.5 * 60 = 990$	10.5
9:00 am	minutes	0	$7.5 * 60 = 540$	10.5

Factor Handling

Warning: TODO

See Also

- https://en.wikipedia.org/wiki/Vehicle_routing_problem
- The queries use the *Sample Data* network.

Indices and tables

- genindex
- search

CHAPTER 7

Change Log

7.1 Release Notes

To see the full list of changes check the list of Git commits[1] on Github.

Table of contents

[1] https://github.com/pgRouting/pgrouting/commits

7.1.1 pgRouting 2.5.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.5.0[2] on Github.

enhancement:

- pgr_version is now on SQL language

Breaking change on:

- pgr_edgeDisjointPaths:
  - Added path_id, cost and agg_cost columns on the result
  - Parameter names changed
  - The many version results are the union of the one to one version

New Signatures:

- pgr_bdAstar(one to one)

New Proposed functions

- pgr_bdAstar(one to many)
- pgr_bdAstar(many to one)
- pgr_bdAstar(many to many)
- pgr_bdAstarCost(one to one)
- pgr_bdAstarCost(one to many)
- pgr_bdAstarCost(many to one)
- pgr_bdAstarCost(many to many)
- pgr_bdAstarCostMatrix
- pgr_bdDijkstra(one to many)
- pgr_bdDijkstra(many to one)
- pgr_bdDijkstra(many to many)

[2] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.5.0%22+is%3Aclosed

- pgr_bdDijkstraCost(one to one)
- pgr_bdDijkstraCost(one to many)
- pgr_bdDijkstraCost(many to one)
- pgr_bdDijkstraCost(many to many)
- pgr_bdDijkstraCostMatrix
- pgr_pgr_pickDeliver
- pgr_pgr_pickDeliverEuclidean

Deprecated Signatures

- pgr_bdastar - use pgr_bdAstar instead
- pgr_gsoc_vrppdtw - use pgr_pickDeliverEuclidean instead

Renamed Functions

- pgr_maxFlowPushRelabel - use pgr_pushRelabel instead
- pgr_maxFlowEdmondsKarp -use pgr_edmondsKarp instead
- pgr_maxFlowBoykovKolmogorov - use pgr_boykovKolmogorov instead
- pgr_maximumCardinalityMatching - use pgr_maxCardinalityMatch instead

Deprecated function

- pgr_pointToEdgeNode

7.1.2 pgRouting 2.4.1 Release Notes

Bug fix

- Fixed compiling error on macOS
- Condition error on pgr_withPoints

7.1.3 pgRouting 2.4.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.4.0[3] on Github.

New Signatures

- pgr_bdDijkstra

[3]https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.4.0%22+is%3Aclosed

New Proposed Signatures

- pgr_maxFlow
- pgr_astar(one to many)
- pgr_astar(many to one)
- pgr_astar(many to many)
- pgr_astarCost(one to one)
- pgr_astarCost(one to many)
- pgr_astarCost(many to one)
- pgr_astarCost(many to many)
- pgr_astarCostMatrix

Deprecated Signatures

- pgr_bddijkstra - use pgr_bdDijkstra instead

Deprecated Functions

- pgr_pointsToVids

Bug fix

- Bug fixes on proposed functions
  - pgr_withPointsKSP: fixed ordering
- TRSP original code is used with no changes on the compilation warnings

7.1.4 pgRouting 2.3.2 Release Notes

To see the issues closed by this release see the Git closed issues for 2.3.2[4] on Github.

Bug Fixes

- Fixed pgr_gsoc_vrppdtw crash when all orders fit on one truck.
- Fixed pgr_trsp:
  - Alternate code is not executed when the point is in reality a vertex
  - Fixed ambiguity on seq

7.1.5 pgRouting 2.3.1 Release Notes

To see the issues closed by this release see the Git closed issues for 2.3.1[5] on Github.

[4] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.3.2%22+is%3Aclosed
[5] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.3.1%22+is%3Aclosed

Bug Fixes

- Leaks on proposed max_flow functions
- Regression error on pgr_trsp
- Types discrepancy on pgr_createVerticesTable

7.1.6 pgRouting 2.3.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.3.0[6] on Github.

New Signatures

- pgr_TSP
- pgr_aStar

New Functions

- pgr_eucledianTSP

New Proposed functions

- pgr_dijkstraCostMatrix
- pgr_withPointsCostMatrix
- pgr_maxFlowPushRelabel(one to one)
- pgr_maxFlowPushRelabel(one to many)
- pgr_maxFlowPushRelabel(many to one)
- pgr_maxFlowPushRelabel(many to many)
- pgr_maxFlowEdmondsKarp(one to one)
- pgr_maxFlowEdmondsKarp(one to many)
- pgr_maxFlowEdmondsKarp(many to one)
- pgr_maxFlowEdmondsKarp(many to many)
- pgr_maxFlowBoykovKolmogorov (one to one)
- pgr_maxFlowBoykovKolmogorov (one to many)
- pgr_maxFlowBoykovKolmogorov (many to one)
- pgr_maxFlowBoykovKolmogorov (many to many)
- pgr_maximumCardinalityMatching
- pgr_edgeDisjointPaths(one to one)
- pgr_edgeDisjointPaths(one to many)
- pgr_edgeDisjointPaths(many to one)
- pgr_edgeDisjointPaths(many to many)
- pgr_contractGraph

[6] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.3.0%22+is%3Aclosed

Deprecated Signatures

- pgr_tsp - use pgr_TSP or pgr_eucledianTSP instead
- pgr_astar - use pgr_aStar instead

Deprecated Functions

- pgr_flip_edges
- pgr_vidsToDmatrix
- pgr_pointsToDMatrix
- pgr_textToPoints

7.1.7 pgRouting 2.2.4 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.4[7] on Github.

Bug Fixes

- Bogus uses of extern "C"
- Build error on Fedora 24 + GCC 6.0
- Regression error pgr_nodeNetwork

7.1.8 pgRouting 2.2.3 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.3[8] on Github.

Bug Fixes

- Fixed compatibility issues with PostgreSQL 9.6.

7.1.9 pgRouting 2.2.2 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.2[9] on Github.

Bug Fixes

- Fixed regression error on pgr_drivingDistance

7.1.10 pgRouting 2.2.1 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.1[10] on Github.

[7] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.4%22+is%3Aclosed
[8] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.3%22+is%3Aclosed
[9] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.2%22+is%3Aclosed
[10] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A2.2.1+is%3Aclosed

Bug Fixes

- Server crash fix on pgr_alphaShape
- Bug fix on With Points family of functions

7.1.11 pgRouting 2.2.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.2.0[11] on Github.

Improvements

- pgr_nodeNetwork
  - Adding a row_where and outall optional parameters
- Signature fix
  - pgr_dijkstra – to match what is documented

New Functions

- pgr_floydWarshall
- pgr_Johnson
- pgr_dijkstraCost(one to one)
- pgr_dijkstraCost(one to many)
- pgr_dijkstraCost(many to one)
- pgr_dijkstraCost(many to many)

Proposed functionality

- pgr_withPoints(one to one)
- pgr_withPoints(one to many)
- pgr_withPoints(many to one)
- pgr_withPoints(many to many)
- pgr_withPointsCost(one to one)
- pgr_withPointsCost(one to many)
- pgr_withPointsCost(many to one)
- pgr_withPointsCost(many to many)
- pgr_withPointsDD(single vertex)
- pgr_withPointsDD(multiple vertices)
- pgr_withPointsKSP
- pgr_dijkstraVia

[11] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.2.0%22+is%3Aclosed

Deprecated functions:

- pgr_apspWarshall use pgr_floydWarshall instead
- pgr_apspJohnson use pgr_Johnson instead
- pgr_kDijkstraCost use pgr_dijkstraCost instead
- pgr_kDijkstraPath use pgr_dijkstra instead

Renamed and deprecated function

- pgr_makeDistanceMatrix renamed to _pgr_makeDistanceMatrix

7.1.12 pgRouting 2.1.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.1.0[12] on Github.

New Signatures

- pgr_dijkstra(one to many)
- pgr_dijkstra(many to one)
- pgr_dijkstra(many to many)
- pgr_drivingDistance(multiple vertices)

Refactored

- pgr_dijkstra(one to one)
- pgr_ksp
- pgr_drivingDistance(single vertex)

Improvements

- pgr_alphaShape function now can generate better (multi)polygon with holes and alpha parameter.

Proposed functionality

- Proposed functions from Steve Woodbridge, (Classified as Convenience by the author.)
  - pgr_pointToEdgeNode - convert a point geometry to a vertex_id based on closest edge.
  - pgr_flipEdges - flip the edges in an array of geometries so the connect end to end.
  - pgr_textToPoints - convert a string of x,y;x,y;... locations into point geometries.
  - pgr_pointsToVids - convert an array of point geometries into vertex ids.
  - pgr_pointsToDMatrix - Create a distance matrix from an array of points.
  - pgr_vidsToDMatrix - Create a distance matrix from an array of vertix_id.
  - pgr_vidsToDMatrix - Create a distance matrix from an array of vertix_id.
- Added proposed functions from GSoc Projects:

[12] https://github.com/pgRouting/pgrouting/issues?q=is%3Aissue+milestone%3A%22Release+2.1.0%22+is%3Aclosed

  - pgr_vrppdtw
  - pgr_vrponedepot

Deprecated functions

- pgr_getColumnName
- pgr_getTableName
- pgr_isColumnCndexed
- pgr_isColumnInTable
- pgr_quote_ident
- pgr_versionless
- pgr_startPoint
- pgr_endPoint
- pgr_pointToId

No longer supported

- Removed the 1.x legacy functions

Bug Fixes

- Some bug fixes in other functions

Refactoring Internal Code

- A C and C++ library for developer was created
  - encapsulates postgreSQL related functions
  - encapsulates Boost.Graph graphs
    - Directed Boost.Graph
    - Undirected Boost.graph.
  - allow any-integer in the id's
  - allow any-numerical on the cost/reverse_cost columns
- Instead of generating many libraries: - All functions are encapsulated in one library - The library has the prefix 2-1-0

7.1.13 pgRouting 2.0.1 Release Notes

Minor bug fixes.

Bug Fixes

- No track of the bug fixes were kept.

7.1.14 pgRouting 2.0.0 Release Notes

To see the issues closed by this release see the Git closed issues for 2.0.0[13] on Github.

With the release of pgRouting 2.0.0 the library has abandoned backwards compatibility to *pgRouting 1.x* releases. The main Goals for this release are:

- Major restructuring of pgRouting.
- Standardization of the function naming
- Preparation of the project for future development.

As a result of this effort:

- pgRouting has a simplified structure
- Significant new functionality has being added
- Documentation has being integrated
- Testing has being integrated
- And made it easier for multiple developers to make contributions.

Important Changes

- Graph Analytics - tools for detecting and fixing connection some problems in a graph
- A collection of useful utility functions
- Two new All Pairs Short Path algorithms (pgr_apspJohnson, pgr_apspWarshall)
- Bi-directional Dijkstra and A-star search algorithms (pgr_bdAstar, pgr_bdDijkstra)
- One to many nodes search (pgr_kDijkstra)
- K alternate paths shortest path (pgr_ksp)
- New TSP solver that simplifies the code and the build process (pgr_tsp), dropped "Gaul Library" dependency
- Turn Restricted shortest path (pgr_trsp) that replaces Shooting Star
- Dropped support for Shooting Star
- Built a test infrastructure that is run before major code changes are checked in
- Tested and fixed most all of the outstanding bugs reported against 1.x that existing in the 2.0-dev code base.
- Improved build process for Windows
- Automated testing on Linux and Windows platforms trigger by every commit
- Modular library design
- Compatibility with PostgreSQL 9.1 or newer
- Compatibility with PostGIS 2.0 or newer
- Installs as PostgreSQL EXTENSION
- Return types re factored and unified
- Support for table SCHEMA in function parameters
- Support for `st_` PostGIS function prefix
- Added `pgr_` prefix to functions and types
- Better documentation: http://docs.pgrouting.org

[13] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+2.0.0%22+is%3Aclosed

- shooting_star is discontinued

7.1.15 pgRouting 1.x Release Notes

To see the issues closed by this release see the Git closed issues for 1.x[14] on Github. The following release notes have been copied from the previous `RELEASE_NOTES` file and are kept as a reference.

Changes for release 1.05

- Bug fixes

Changes for release 1.03

- Much faster topology creation
- Bug fixes

Changes for release 1.02

- Shooting* bug fixes
- Compilation problems solved

Changes for release 1.01

- Shooting* bug fixes

Changes for release 1.0

- Core and extra functions are separated
- Cmake build process
- Bug fixes

Changes for release 1.0.0b

- Additional SQL file with more simple names for wrapper functions
- Bug fixes

Changes for release 1.0.0a

- Shooting* shortest path algorithm for real road networks
- Several SQL bugs were fixed

Changes for release 0.9.9

- PostgreSQL 8.2 support
- Shortest path functions return empty result if they could not find any path

[14] https://github.com/pgRouting/pgrouting/issues?q=milestone%3A%22Release+1.x%22+is%3Aclosed

Changes for release 0.9.8

- Renumbering scheme was added to shortest path functions
- Directed shortest path functions were added
- routing_postgis.sql was modified to use dijkstra in TSP search

Indices and tables

- genindex
- search

Bibliography

[C001] Simulated annealing algorithm for beginners[15]

[15] http://www.theprojectspot.com/tutorial-post/simulated-annealing-algorithm-for-beginners/6