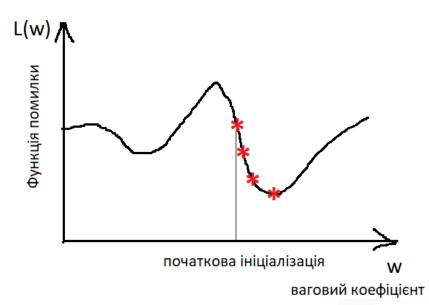
# Backpropagation

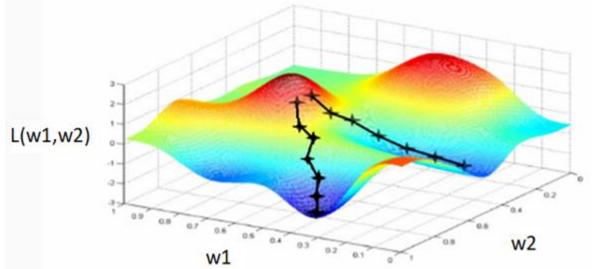
# Loss Function

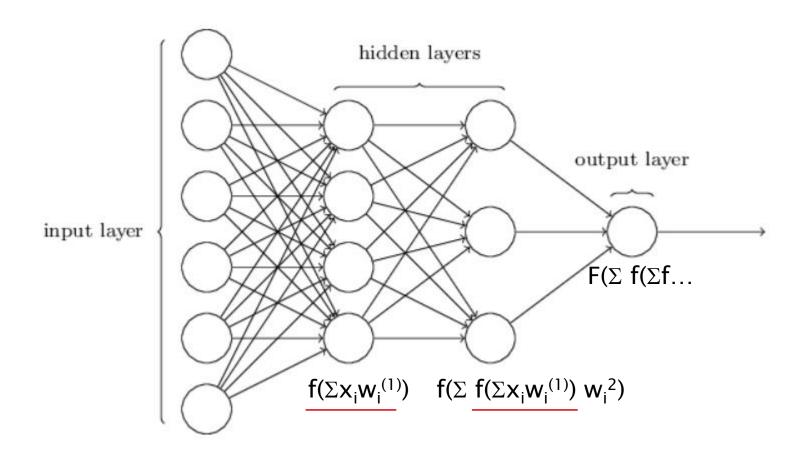
$$L = \frac{1}{N} \sum_{i} L_i(f(x_i, W), y_i)$$

### Градієнтний спуск (Gradient descent)



$$\overrightarrow{w} = \overrightarrow{w} - \eta \overrightarrow{\nabla_{w}} L$$
 
$$\overrightarrow{b} = \overrightarrow{b} - \eta \overrightarrow{\nabla}_{b} L$$





# 1986р: Д.Румельхарт, Дж.Хінтон, Р.Вільямс розробляють обчислювально ефективний алгоритм навчання нейромереж –

метод зворотного поширення помилки.



Джефрі Хінтон



Девід Румельхарт



Рональд Вільямс

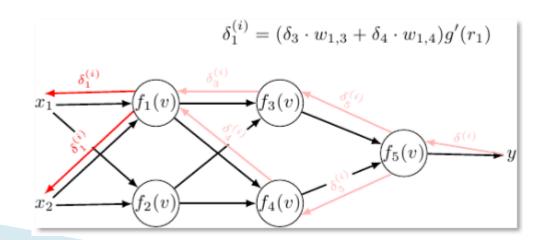
#### Learning Internal Representations by Error Propagation

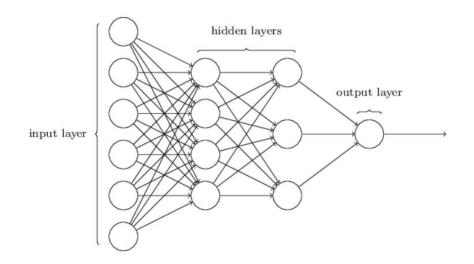
D. E. RUMELHART, G. E. HINTON, and R. J. WILLIAMS

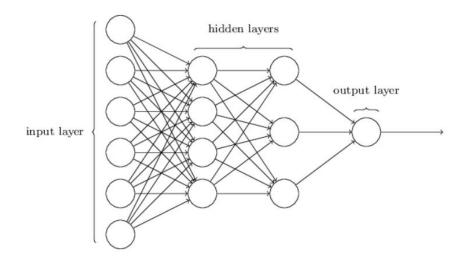
#### THE PROBLEM

We now have a suther good understanding of simple two-layer associative networks in which a set of input patterns arriving at an input layer are mapped directly to a set of output patterns at an output layer. Such networks have no hadder units. They involve only cypic and output units in these cases there is no internal representation. The coding provided by the external world must suffice. These networks have provide useful in a wide variety of applications (6f. Chapters 2, 172, and 183). Perhaps the essential character of such networks is that they may similar input patterns to similar output patterns. This is what allows these networks to make reasonable generalizations and perform reasonably on patterns in a PDP system is determined by their overlap. The overlap in such networks is determined outside the learning system itself—by whatever products the natiteness.

The constraint that similar input patterns lead to similar outputs can lead to an inability of the system to learn certain mappings from input to output. Whenever the representation provided by the outside world is such that the similarity structure of the input and output patterns are very different, a network without internal conventations. (i.e. a







forward pass прямий прохід backward pass зворотній прохід

обчислюємо значення нейронів та функцію втрат обчислюємо похідні та корегуємо вагові коефіцієнти

### Chain rule

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x}$$

upstream gradient

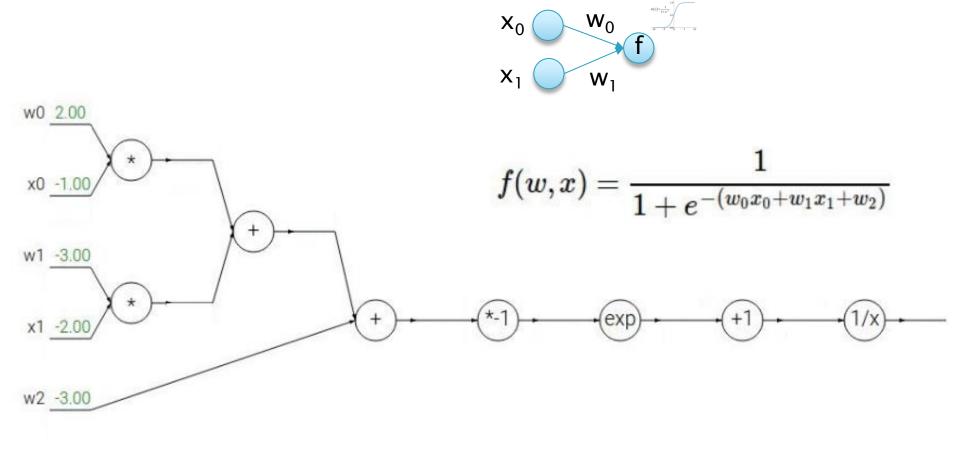
local gradient

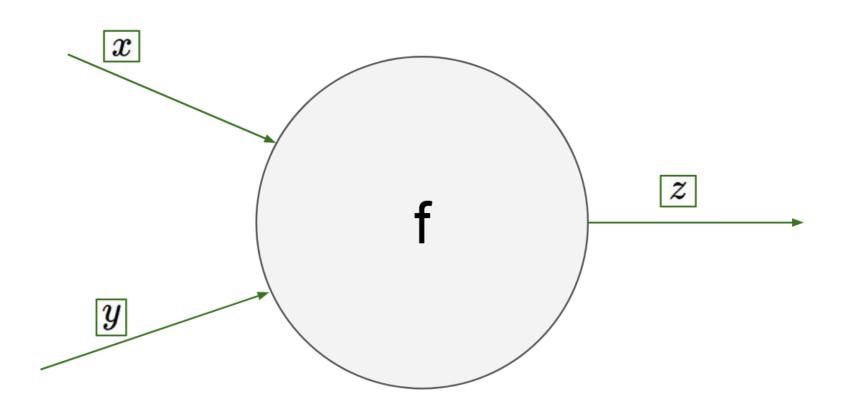
$$\frac{\partial f}{\partial a} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial a} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial a}$$

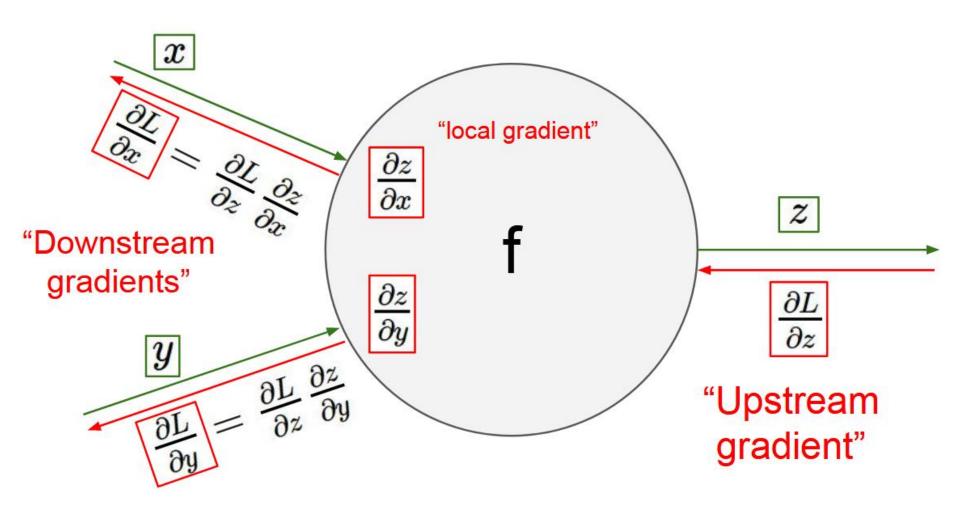
$$\frac{\partial f}{\partial a} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial a}$$

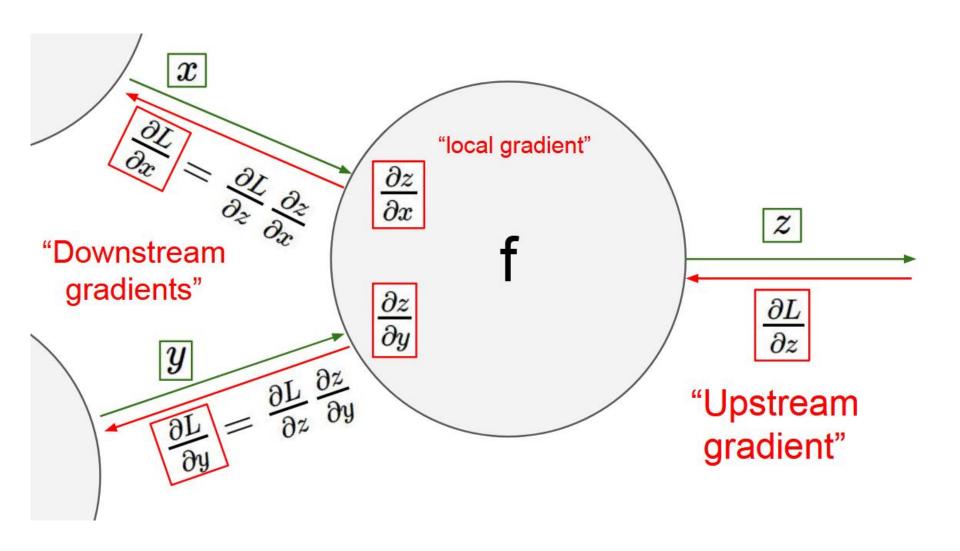
$$\frac{\partial f}{\partial b} = \frac{\partial f}{\partial g} \cdot \frac{\partial g}{\partial x} \cdot \frac{\partial x}{\partial b}$$

### Computational graph

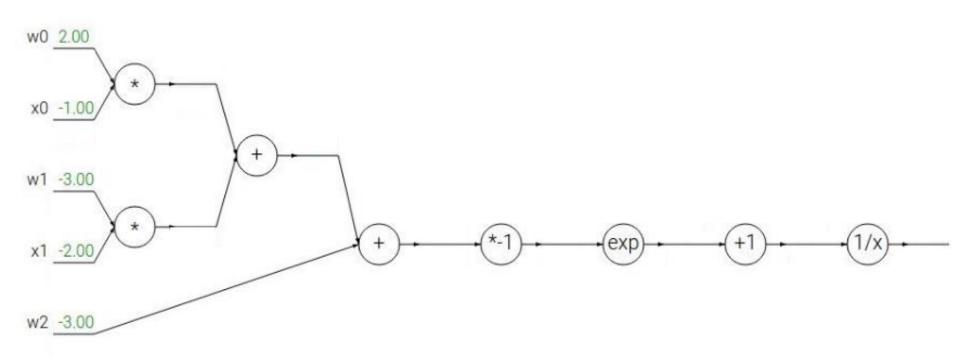




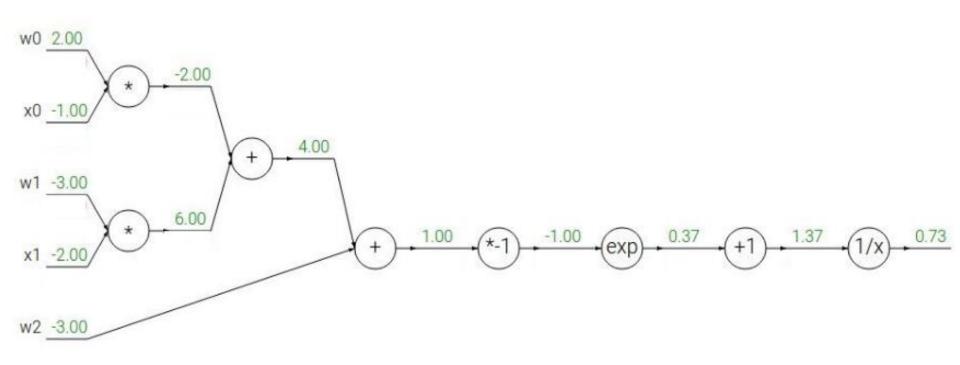




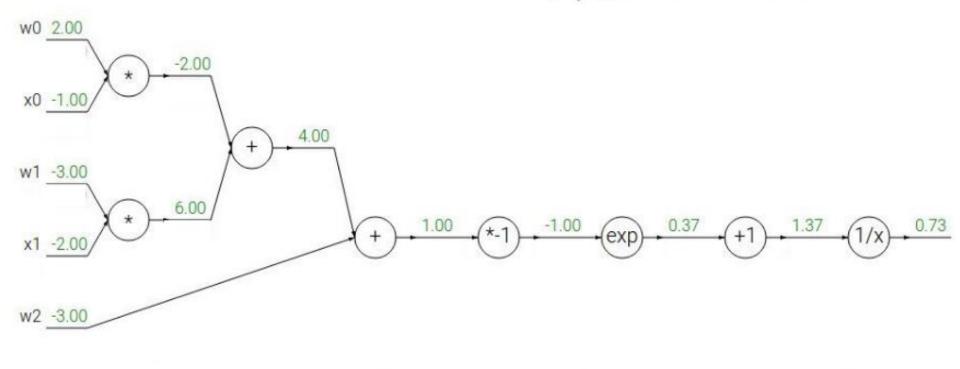
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



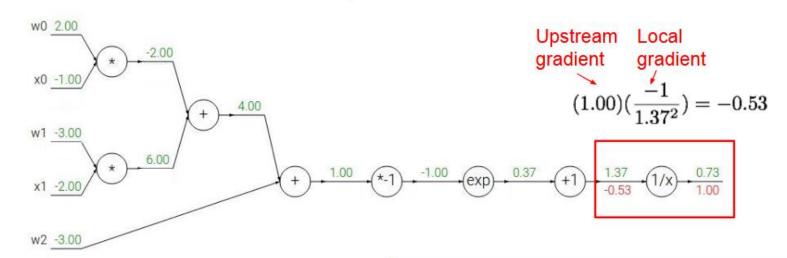
$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

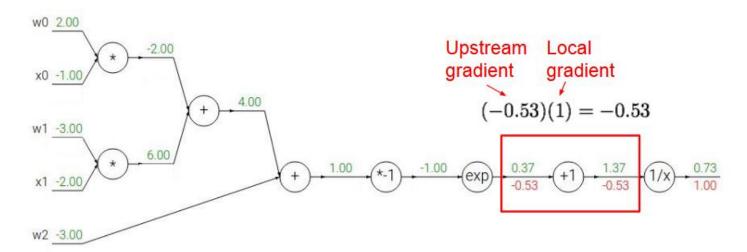


$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



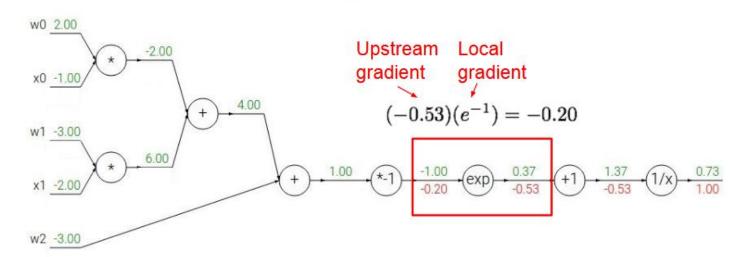
$$f(x)=rac{1}{x} \qquad \qquad \qquad rac{df}{dx}=-1/x^2 \ f_c(x)=c+x \qquad \qquad \qquad \qquad rac{df}{dx}=1$$

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

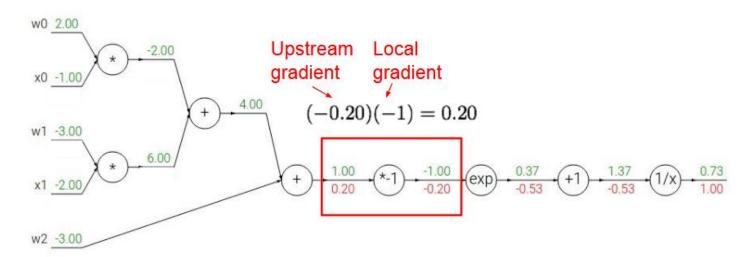


$$f(x) = e^x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = e^x \hspace{1cm} f(x) = rac{1}{x} \hspace{1cm} o \hspace{1cm} rac{df}{dx} = -1/x^2 \ f_a(x) = ax \hspace{1cm} o \hspace{1cm} rac{df}{dx} = a \hspace{1cm} f_c(x) = c + x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = 1$$

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



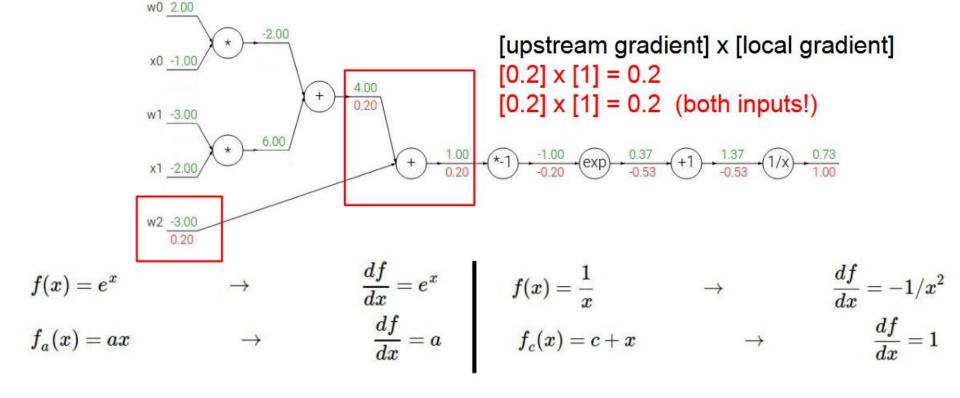
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



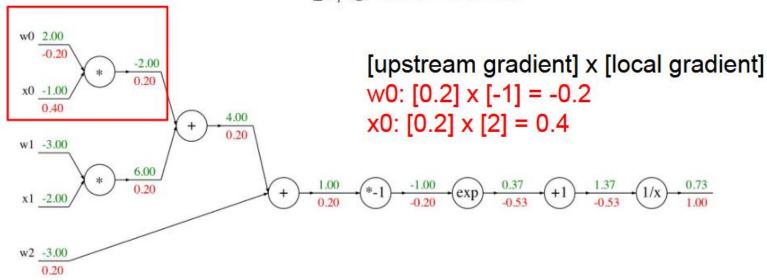
$$f(x) = e^x \hspace{1cm} o \hspace{1cm} rac{df}{dx} = e^x \ f_a(x) = ax \hspace{1cm} o \hspace{1cm} rac{df}{dx} = a$$

$$f(x)=rac{1}{x} \qquad \qquad 
ightarrow \qquad rac{df}{dx}=-1/x^2 \ f_c(x)=c+x \qquad \qquad 
ightarrow \qquad rac{df}{dx}=1$$

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



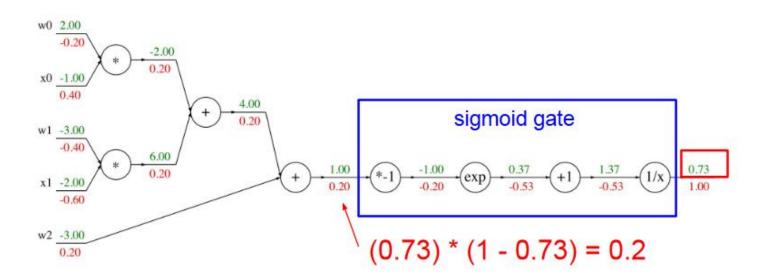
$$egin{aligned} f(x) = e^x & 
ightarrow & rac{df}{dx} = e^x & f(x) = rac{1}{x} & 
ightarrow & rac{df}{dx} = -1/x^2 \ f_a(x) = ax & 
ightarrow & rac{df}{dx} = a & f_c(x) = c + x & 
ightarrow & rac{df}{dx} = 1 \end{aligned}$$

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = rac{1}{1+e^{-x}}$$

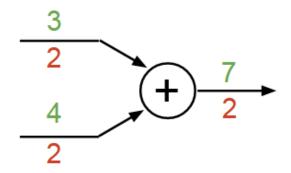
sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right) \left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$$

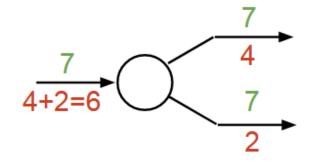


### Patterns in gradient flow

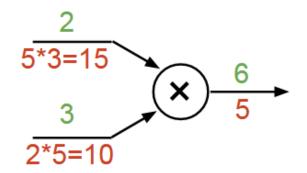
add gate: gradient distributor



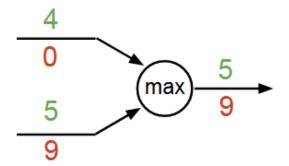
copy gate: gradient adder



mul gate: "swap multiplier"



max gate: gradient router



$$\begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{pmatrix}$$

$$\frac{\partial L}{\partial W} - ?$$
  $\frac{\partial L}{\partial X} - ?$ 

$$\frac{\partial L}{\partial X} - ?$$

$$\begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix} \qquad \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \end{pmatrix} \qquad \frac{\partial L}{\partial W} = \frac{\partial L}{\partial Y} \frac{\partial Y}{\partial W} \qquad \frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \frac{\partial Y}{\partial X}$$

$$\begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \end{pmatrix}$$

$$\frac{\partial L}{\partial W} = \frac{\partial L}{\partial Y} \frac{\partial Y}{\partial W}$$

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \frac{\partial Y}{\partial X}$$

$$\frac{\partial L}{\partial W} = \begin{pmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} & \frac{\partial L}{\partial w_{13}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} & \frac{\partial L}{\partial w_{23}} \end{pmatrix}$$

$$\frac{\partial L}{\partial X} = \begin{pmatrix} \frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} \\ \frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}} \end{pmatrix}$$

$$\frac{\partial L}{\partial W} = \begin{pmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} & \frac{\partial L}{\partial w_{13}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} & \frac{\partial L}{\partial w_{23}} \end{pmatrix} \qquad \frac{\partial L}{\partial X} = \begin{pmatrix} \frac{\partial L}{\partial x_{11}} & \frac{\partial L}{\partial x_{12}} \\ \frac{\partial L}{\partial x_{21}} & \frac{\partial L}{\partial x_{22}} \end{pmatrix} \qquad \frac{\partial L}{\partial Y} = \begin{pmatrix} \frac{\partial L}{\partial y_{11}} & \frac{\partial L}{\partial y_{12}} & \frac{\partial L}{\partial y_{13}} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{pmatrix}$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial Y} \frac{\partial Y}{\partial w_{11}} = \begin{pmatrix} \frac{\partial L}{\partial y_{11}} & \frac{\partial L}{\partial y_{12}} & \frac{\partial L}{\partial y_{13}} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{pmatrix} \begin{pmatrix} x_{11} & 0 & 0 \\ x_{21} & 0 & 0 \end{pmatrix} = \frac{\partial L}{\partial y_{11}} x_{11} + \frac{\partial L}{\partial y_{21}} x_{21}$$

$$\frac{\partial Y}{\partial w_{11}} = \begin{pmatrix} x_{11} & 0 & 0 \\ x_{21} & 0 & 0 \end{pmatrix} \quad \frac{\partial Y}{\partial w_{12}} = \begin{pmatrix} 0 & x_{11} & 0 \\ 0 & x_{21} & 0 \end{pmatrix} \quad \frac{\partial Y}{\partial w_{13}} = \begin{pmatrix} 0 & 0 & x_{11} \\ 0 & 0 & x_{21} \end{pmatrix}$$

$$\frac{\partial Y}{\partial w_{21}} = \begin{pmatrix} x_{12} & 0 & 0 \\ x_{22} & 0 & 0 \end{pmatrix} \quad \frac{\partial Y}{\partial w_{22}} = \begin{pmatrix} 0 & x_{12} & 0 \\ 0 & x_{22} & 0 \end{pmatrix} \quad \frac{\partial Y}{\partial w_{23}} = \begin{pmatrix} 0 & 0 & x_{12} \\ 0 & 0 & x_{22} \end{pmatrix}$$

$$\frac{\partial L}{\partial W} = \begin{pmatrix} \frac{\partial L}{\partial w_{11}} & \frac{\partial L}{\partial w_{12}} & \frac{\partial L}{\partial w_{13}} \\ \frac{\partial L}{\partial w_{21}} & \frac{\partial L}{\partial w_{22}} & \frac{\partial L}{\partial w_{23}} \end{pmatrix} = \begin{pmatrix} (\frac{\partial L}{\partial y_{11}} x_{11} + \frac{\partial L}{\partial y_{21}} x_{21}) & (\frac{\partial L}{\partial y_{12}} x_{11} + \frac{\partial L}{\partial y_{22}} x_{21}) & (\frac{\partial L}{\partial y_{13}} x_{11} + \frac{\partial L}{\partial y_{23}} x_{21}) \\ (\frac{\partial L}{\partial y_{11}} x_{12} + \frac{\partial L}{\partial y_{21}} x_{22}) & (\frac{\partial L}{\partial y_{12}} x_{12} + \frac{\partial L}{\partial y_{22}} x_{22}) & (\frac{\partial L}{\partial y_{13}} x_{12} + \frac{\partial L}{\partial y_{23}} x_{22}) \end{pmatrix}$$

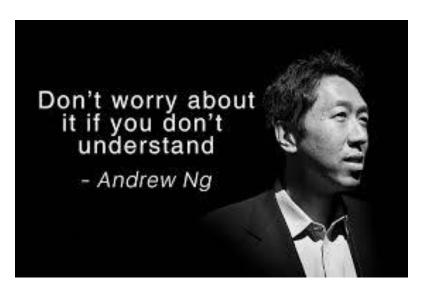
$$= \begin{pmatrix} x_{11} & x_{21} \\ x_{12} & x_{22} \end{pmatrix} \begin{pmatrix} \frac{\partial L}{\partial y_{11}} & \frac{\partial L}{\partial y_{12}} & \frac{\partial L}{\partial y_{13}} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{pmatrix} = X^{T} \frac{\partial L}{\partial Y}$$

$$= \begin{pmatrix} X_{11} & X_{21} \\ \frac{\partial L}{\partial y_{21}} & \frac{\partial L}{\partial y_{22}} & \frac{\partial L}{\partial y_{23}} \end{pmatrix} = X^{T} \frac{\partial L}{\partial Y}$$

$$\frac{\partial L}{\partial W} = X^T \frac{\partial L}{\partial Y}$$

$$\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} W^T$$







- http://cs231n.stanford.edu/handouts/linearbackprop.pdf
- http://cs231n.stanford.edu/handouts/derivat ives.pdf
- http://cs231n.stanford.edu/slides/2021/lect ure\_4.pdf
- https://cs231n.github.io/optimization-2/

## Model

Sequential API

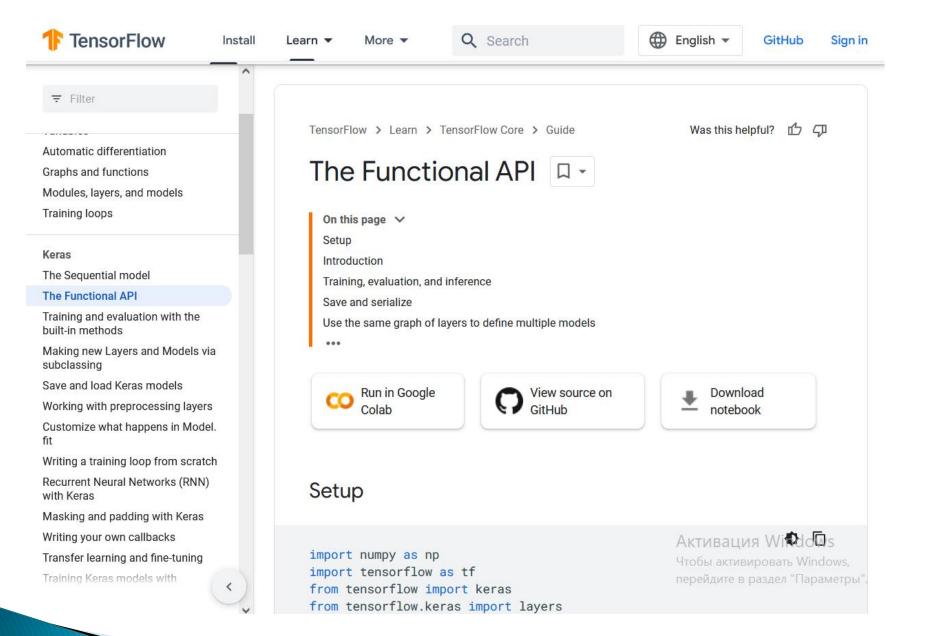
**Functional API** 

## Functional model

Input

Layers

Model



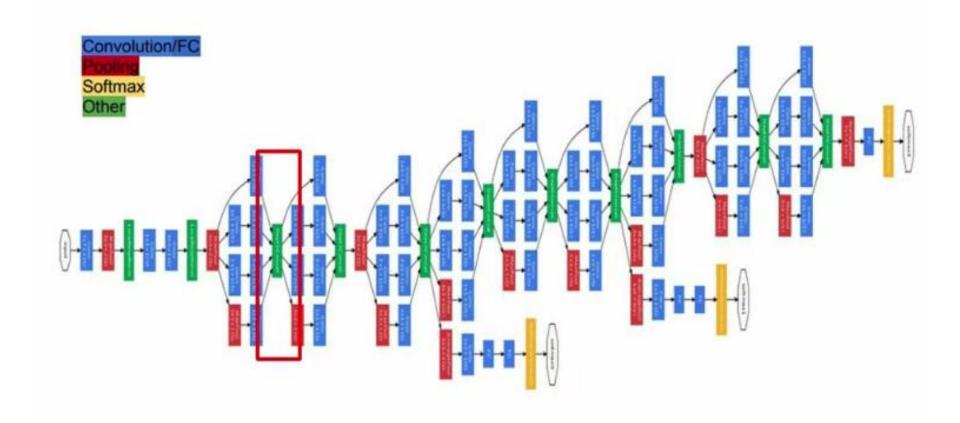
```
input = Input(shape=(28,28))
x = Flatten()(input)
x = Dense(128, activation="relu")(x)
predictions = Dense(10, activation="softmax")(x)

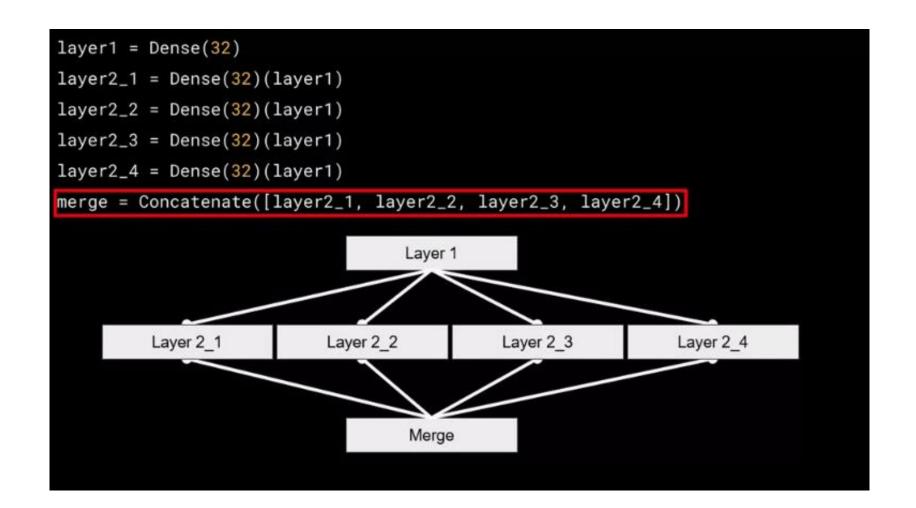
func_model = Model(inputs=input, outputs=predictions)
```

func\_model = Model(inputs=[input1, input2], outputs=[output1, output2])

```
dense = layers.Dense(64, activation="relu")
x = dense(inputs)
```

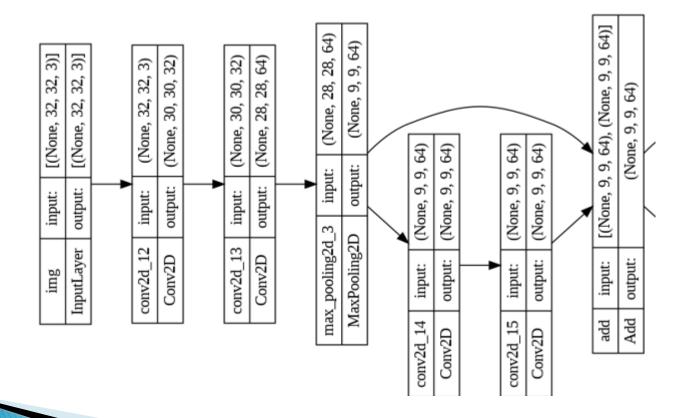
model = keras.Model(inputs=inputs, outputs=outputs, name="mnist\_model")





```
inputs = keras.Input(shape=(32, 32, 3), name="img")
x = layers.Conv2D(32, 3, activation="relu")(inputs)
x = layers.Conv2D(64, 3, activation="relu")(x)
block_1_output = layers.MaxPooling2D(3)(x)

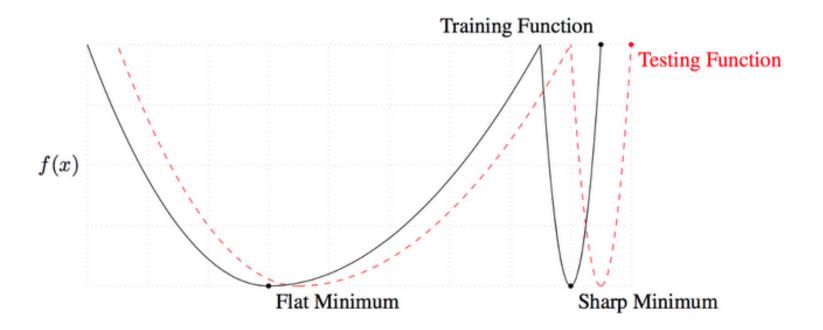
x = layers.Conv2D(64, 3, activation="relu", padding="same")(block_1_output)
x = layers.Conv2D(64, 3, activation="relu", padding="same")(x)
block_2_output = layers.add([x, block_1_output])
```



```
encoder input = keras.Input(shape=(28, 28, 1), name="img")
x = layers.Conv2D(16, 3, activation="relu")(encoder input)
x = layers.Conv2D(32, 3, activation="relu")(x)
x = layers.MaxPooling2D(3)(x)
x = layers.Conv2D(32, 3, activation="relu")(x)
x = layers.Conv2D(16, 3, activation="relu")(x)
encoder output = layers.GlobalMaxPooling2D()(x)
encoder = keras.Model(encoder input, encoder output, name="encoder")
encoder.summary()
x = layers.Reshape((4, 4, 1))(encoder output)
x = layers.Conv2DTranspose(16, 3, activation="relu")(x)
x = layers.Conv2DTranspose(32, 3, activation="relu")(x)
x = layers.UpSampling2D(3)(x)
x = layers.Conv2DTranspose(16, 3, activation="relu")(x)
decoder output = layers.Conv2DTranspose(1, 3, activation="relu")(x)
autoencoder = keras.Model(encoder input, decoder output, name="autoencoder")
autoencoder.summary()
```

```
encoder_input = keras.Input(shape=(28, 28, 1), name="original_img")
x = layers.Conv2D(16, 3, activation="relu")(encoder_input)
x = layers.Conv2D(32, 3, activation="relu")(x)
x = layers.MaxPooling2D(3)(x)
x = layers.Conv2D(32, 3, activation="relu")(x)
x = layers.Conv2D(16, 3, activation="relu")(x)
encoder_output = layers.GlobalMaxPooling2D()(x)
encoder = keras.Model(encoder_input, encoder_output, name="encoder")
encoder.summary()
decoder_input = keras.Input(shape=(16,), name="encoded_img")
x = layers.Reshape((4, 4, 1))(decoder_input)
x = layers.Conv2DTranspose(16, 3, activation="relu")(x)
x = layers.Conv2DTranspose(32, 3, activation="relu")(x)
x = layers.UpSampling2D(3)(x)
x = layers.Conv2DTranspose(16, 3, activation="relu")(x)
decoder_output = layers.Conv2DTranspose(1, 3, activation="relu")(x)
decoder = keras.Model(decoder_input, decoder_output, name="decoder")
decoder.summary()
autoencoder_input = keras.Input(shape=(28, 28, 1), name="img")
encoded_img = encoder(autoencoder_input)
decoded_img = decoder(encoded_img)
autoencoder = keras.Model(autoencoder_input, decoded_Kimgpaname=Vautoencoder")
autoencoder.summary()
                                                   Чтобы активировать Windows,
```

# Ансамблі нейронних мереж



Narrow and wide optima. Flat minimum will produce similar loss during training and testing. Narrow loss, however, will give very different results during training and testing. In other words, wide minimum is more generalizable than narrow. Source.

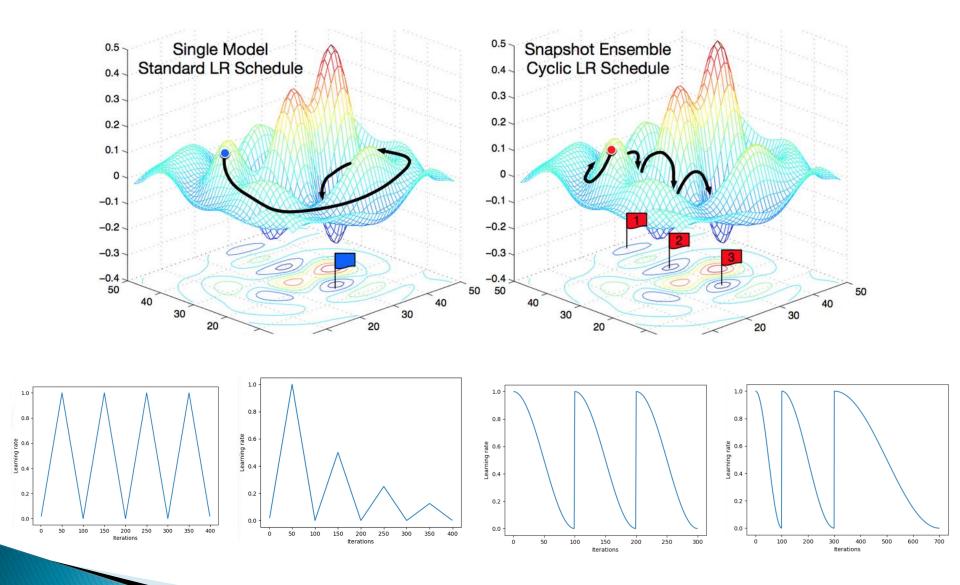
https://towardsdatascience.com/stochastic-weight-averaging-a-new-way-to-get-state-of-the-art-results-in-deep-learning-c639ccf36a

### **Model Ensembles**

- Same model, different initializations
- Top models discovered during crossvalidation
- Different checkpoints of a single model
- Running average of parameters during training

$$w_{\text{SWA}} \leftarrow \frac{w_{\text{SWA}} \cdot n_{\text{models}} + w}{n_{\text{models}} + 1},$$

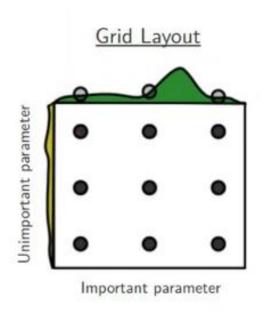
### Different checkpoints of a single model

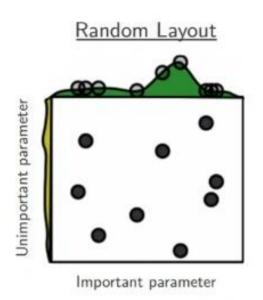


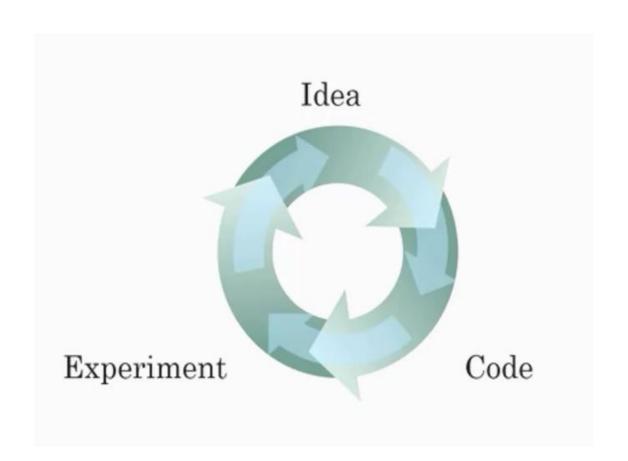
### Ensembling

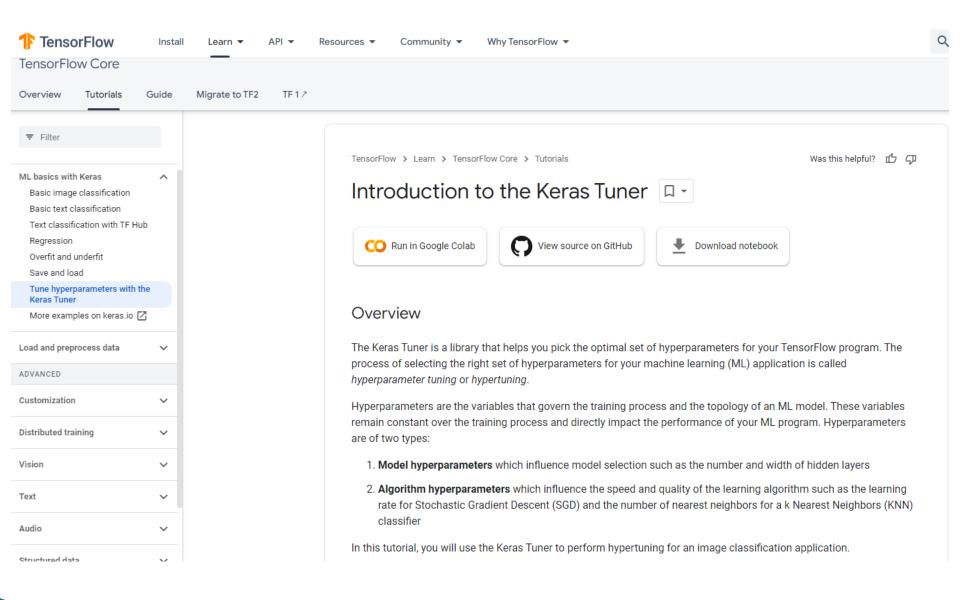
```
inputs = keras.Input(shape=(128,))
y1 = model1(inputs)
y2 = model2(inputs)
y3 = model3(inputs)
outputs = layers.average([y1, y2, y3])
ensemble_model = keras.Model(inputs=inputs, outputs=outputs)
```

## Підбір гіперпараметрів













**About Keras** 

Getting started

Developer guides

#### Keras API reference

Models API

Layers API

Callbacks API

Optimizers

Metrics

Losses

Data loading

Built-in small datasets

Keras Applications

Mixed precision

Utilities

#### KerasTuner

HyperParameters

Tuners

Search Keras documentation...

» Keras API reference / KerasTuner / Tuners / Hyperband Tuner

### **Hyperband Tuner**

Hyperband class [source]

```
keras_tuner.Hyperband(
    hypermodel=None,
   objective=None,
    max_epochs=100,
    factor=3,
    hyperband_iterations=1,
    seed=None,
    hyperparameters=None,
    tune_new_entries=True,
    allow_new_entries=True,
    max retries per trial=0,
    max_consecutive_failed_trials=3,
    **kwargs
```

Variation of HyperBand algorithm.

#### Reference

Li, Lisha, and Kevin Jamieson. "Hyperband: A Novel Bandit-Based Approach to Hyperparameter Optimization." Journal of Machine Learning Research 18 (2018): 1-52.

#### Arguments

• hypermodel: Instance of HyperModel class (or callable that takes hyperparameters and returns a Model instance). It is optional when Tuner.run\_trial() is overriden and does not use self.hvnermodel.

### get\_best\_hyperparameters method

[source]

Tuner.get\_best\_hyperparameters(num\_trials=1)

Returns the best hyperparameters, as determined by the objective.

This method can be used to reinstantiate the (untrained) best model found during the search process.

#### Example

```
best_hp = tuner.get_best_hyperparameters()[0]
model = tuner.hypermodel.build(best_hp)
```

#### **Arguments**

• num\_trials: Optional number of HyperParameters objects to return.

#### Returns

List of HyperParameter objects sorted from the best to the worst.

#### get\_best\_models method

[source]

```
Tuner.get_best_models(num_models=1)
```

Returns the best model(s), as determined by the tuner's objective.

#### The base Tuner class

set\_state method

Tuner class
get\_best\_hyperparameters method
get\_best\_models method
get\_state method
load\_model method
on\_epoch\_begin method
on\_batch\_begin method
on\_batch\_end method
on\_epoch\_end method
run\_trial method
results\_summary method
save\_model method
search\_method
search\_space\_summary method

## Save and load models

